Plaquette-Dimer Liquid Beyond Renormalization

When IR theory is not an ‘IR’ theory!

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With Roderich Moessner (MPIPKS)

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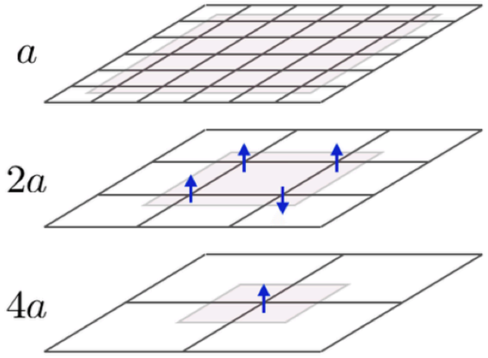
Critical phenomenon and critical liquids

- **Renormalization Group** (Wilson Fisher)

  Keep IR + coarse grain UV

- **Critical exponents** — *Universality*



Critical liquids *beyond the historic paradigm*

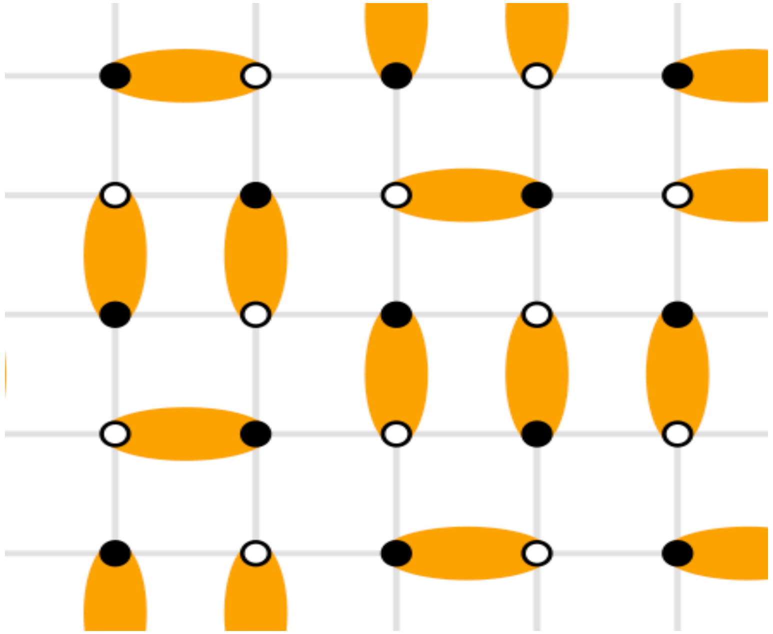
✓*Renormalization*

Long wave-length physics controls!

✓ *Beyond RG?*

Short wave-length physics controls!

Close-packed tiling problems. (close-packed dimer/trimer/plaquette…)



☆ Hilbert space subject to **<u>Hard-core constraint</u>**

e.g. Close-packed dimer configurations

☆ Extensive # of patterns: entropy driven

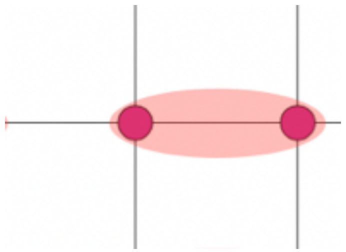
<u>What happened if we have large # of ground states with extensive entropy at T=0?</u>

Liquid phase? Order by disorder?

Closed packed Dimer = Electrostatic problem

(Moessner, Kivelson-Fradkin)

Dimer ←→ U(1) Electric field



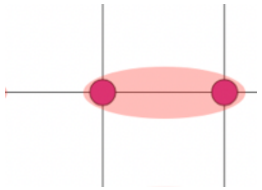
dimer coverage = ***E*** field

$$E_i(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}} D_i(\boldsymbol{r})$$

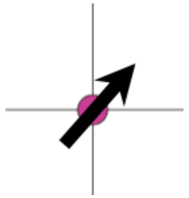
Gauss Law → Close-packed constraint

$$\partial_i E_i(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}}$$

Empty site with no dimer connectivity



q=0

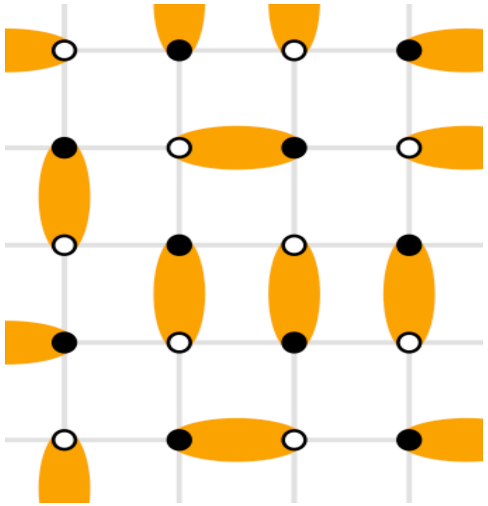


q=1

$$\partial_i E_i(\boldsymbol{r}) = (-1)^{i_{\boldsymbol{r}}}(1 - \boxed{q(\boldsymbol{r})})$$

Monomer excitation, charge

Close-packed dimer models

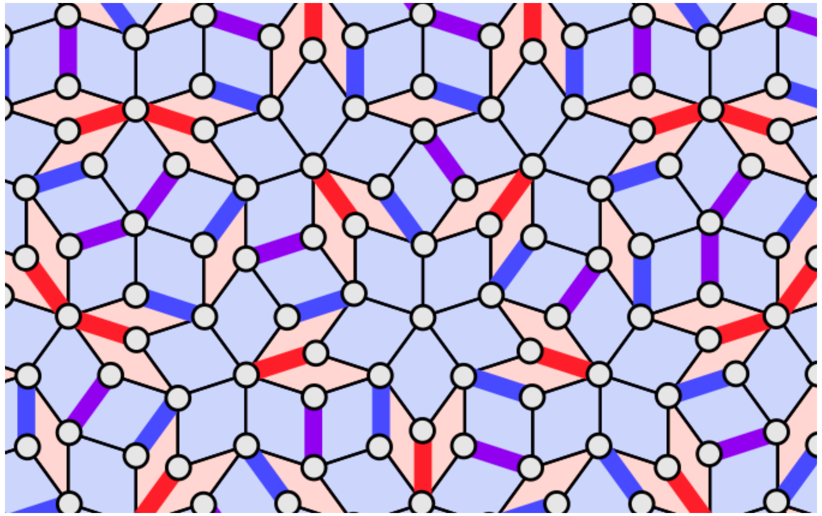


☆ Classic problem in graph combinatorics *(Fisher-Kastelyn '61)*

☆ Electrostatic problem, mapping to 'height model' *(Henley, ….)*

☆ Quantum version of dimer models *(Kivelson-Fradkin, ….)*
('Polyakov' confinement in 2D, emergent photon in 3D)

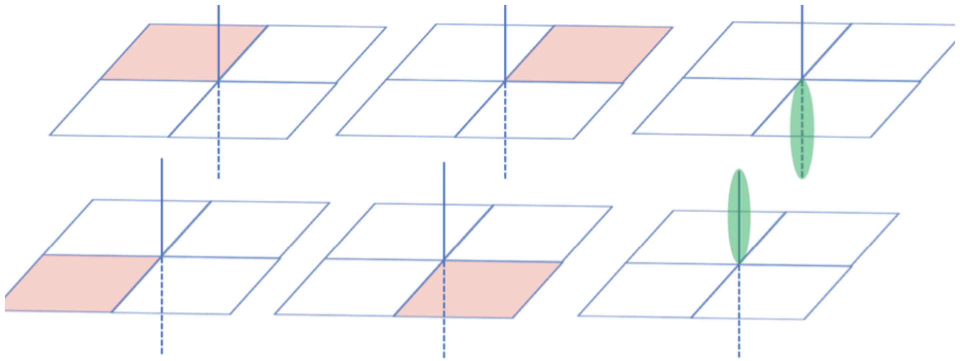
$$\begin{aligned}\hat{H} = \; & -t\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right) \\ & + V\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right)\end{aligned}$$



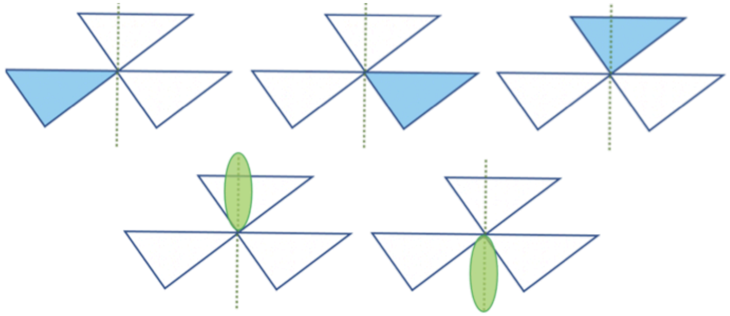
☆ non-crystals: hyperbolic lattice, quasi-crystals.
(Parameswaran,…)

Extension of close-packed dimers ?

Close-packed tiling problems in 3D: with both mixture of plaquette, trimer, dimer patterns
(Xu, Moessner-Sondhi)



Plaquette patterns



Trimer patterns

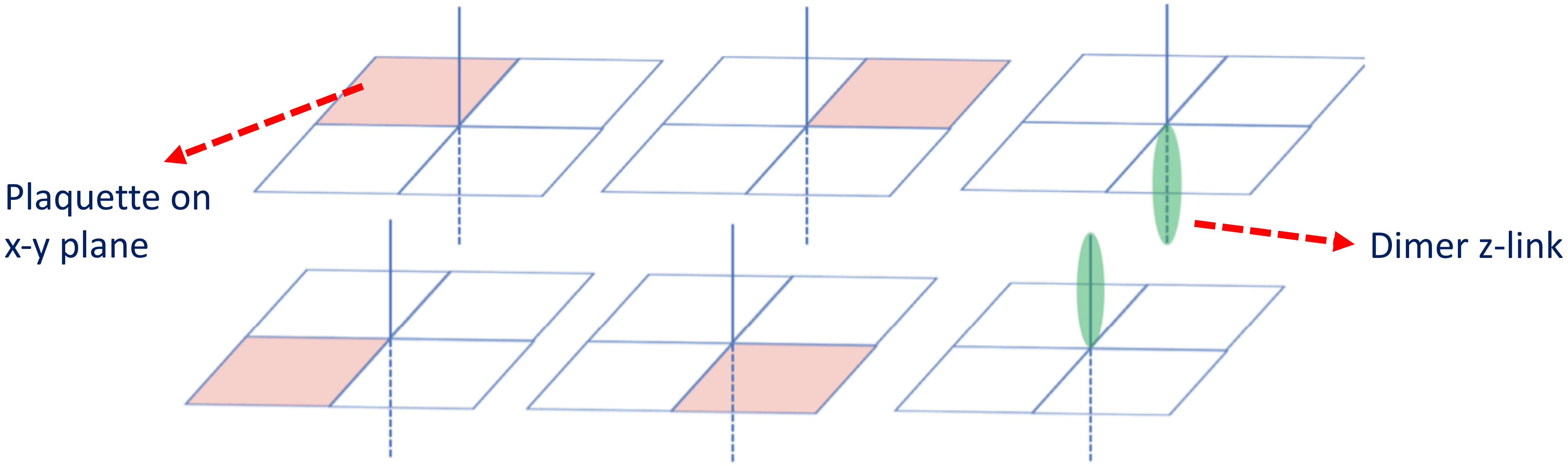
What's new here?

- How short wave-length modes control the IR theory? RG fails!

→ new paradigm for critical liquids

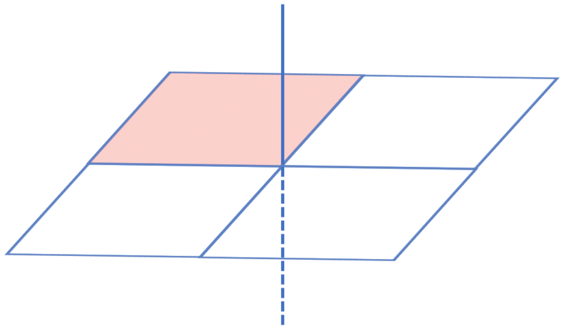
- implications for dynamics/MBL/fractons?

Close-packed plaquette-dimer in 3D



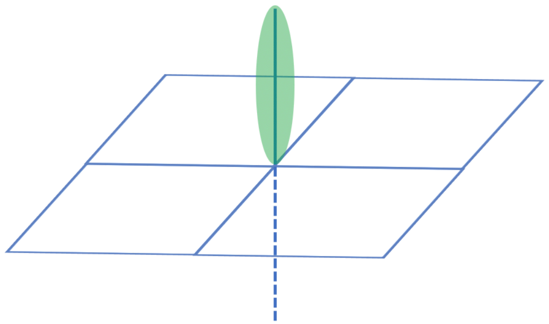
☆ Constrained patterns---each site is either adjacent the plaquettes on x-y plane or to the dimers on the z-link

☆ highly deg GS: extensive entropy at T=0



$$E_{xy} = \eta P_{xy},$$

Plaquette # = ***E*** field



$$E_z = \eta D_z$$

Dimer # = ***E*** field

Constrained patterns = local conserved quantity
= **Gauge symmetry**

Close-packed constraint → *special Gauss law*

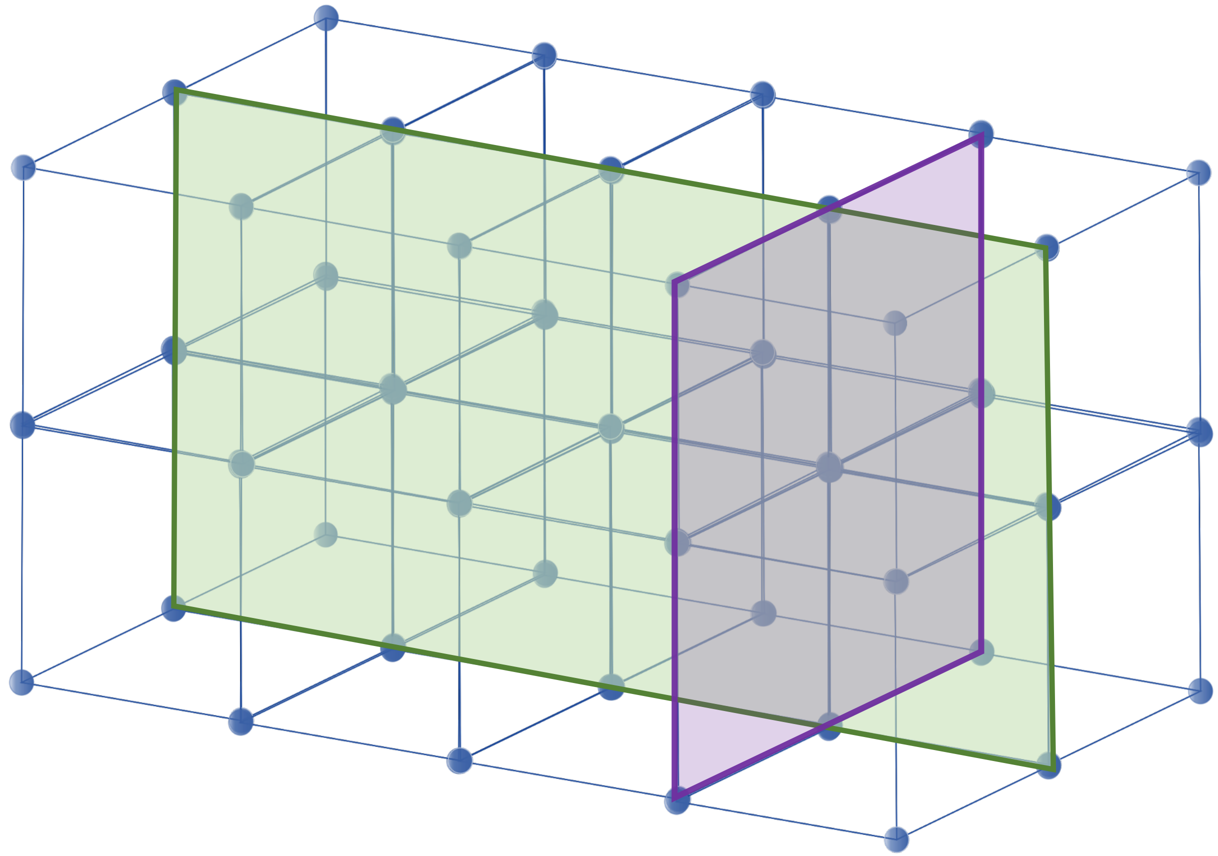
$$\Delta_x \Delta_y E_{xy} + \Delta_z E_z = \eta \qquad \eta = (-1)^{x+y+z}$$

Higher-rank Gauss law
→ an x-y plaquette or a z-dimer coverage per site

$$\left.\begin{aligned} \int dxdz\ (\Delta_x \Delta_y E_{xy} + \Delta_z E_z) &= \int dxdz\ Q = 0, \\ \int dydz\ (\Delta_x \Delta_y E_{xy} + \Delta_z E_z) &= \int dydz\ Q = 0, \end{aligned}\right\}$$

Charge is conserved on sub-planes

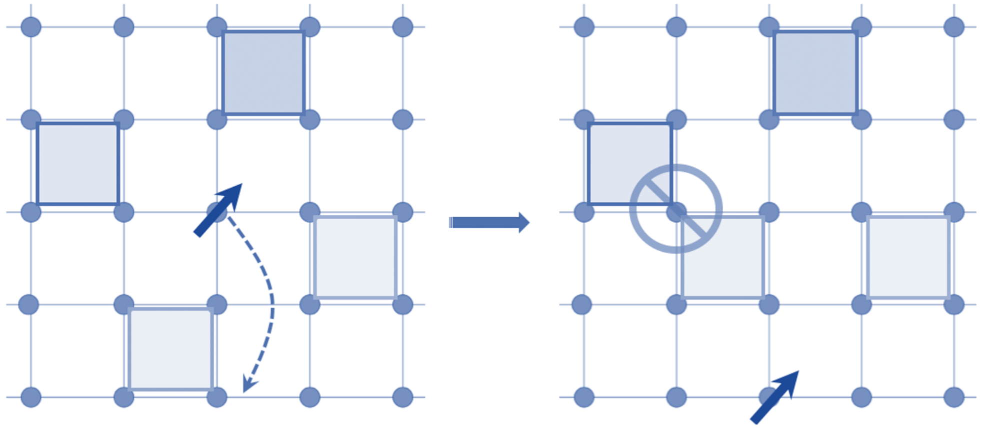
Higher-rank Gauss law: charge conserved in sub-manifolds!

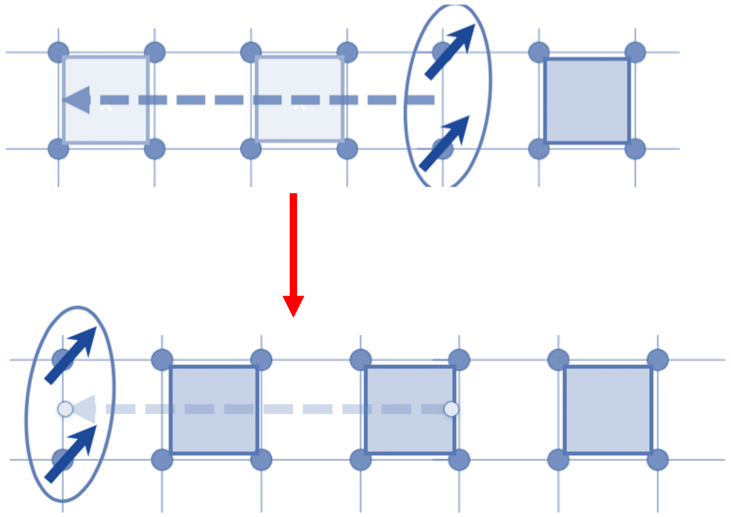


$$\int dxdz\ (\Delta_x \Delta_y E_{xy} + \Delta_z E_z) = \int dxdz\ Q = 0,$$
$$\int dydz\ (\Delta_x \Delta_y E_{xy} + \Delta_z E_z) = \int dydz\ Q = 0,$$

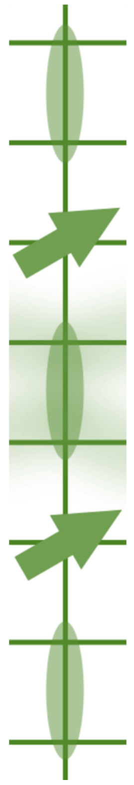
Charge is conserved on all x-z & y-z planes

Charge conserved in sub-manifolds → monomers restrict motion→ **fracton**

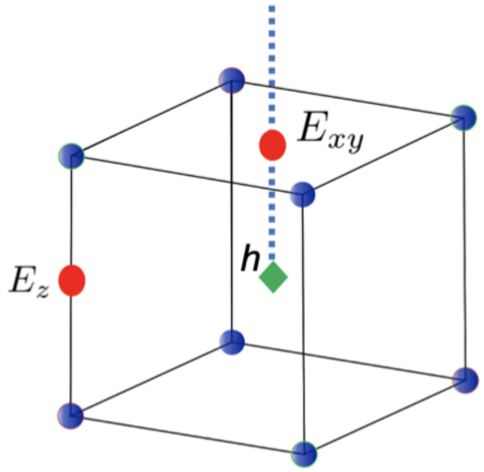




A pair of monomers can **hop along the transverse slab!**



Single monomer can **only hop along z-direction**



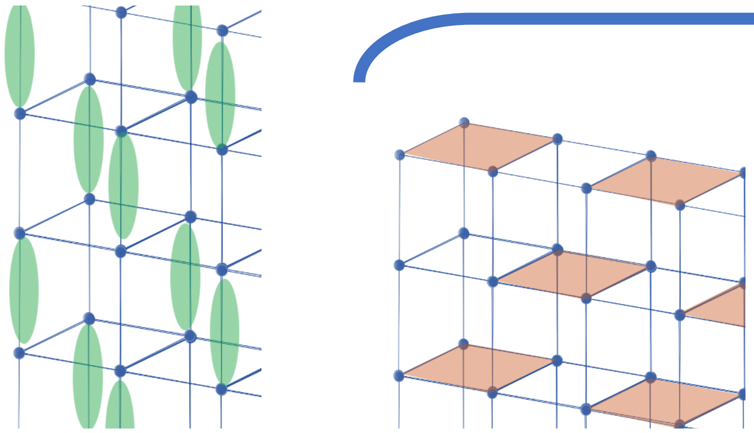
$$\Delta_x \Delta_y E_{xy} + \Delta_z E_z = \eta$$

How to interpret Gauss law?

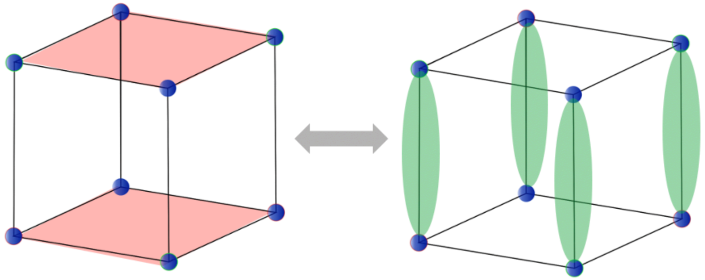
'Release constraint' by introducing "height field"

$$E_{xy} = -\Delta_z h + \bar{E}_{xy}, E_z = \Delta_x \Delta_y h + \bar{E}_z,$$

h : integer-valued field



Non-flappable patterns,
<E> nonzero



Locally -flappable patterns,
<E> =0

Theory of close packed plaquette-dimers

$$\mathcal{Z} = \int \mathcal{D}E_z \, \mathcal{D}E_{xy} \; e^{-\beta(E_z^2 + E_{xy}^2)}$$

$$= \int \mathcal{D}h \; e^{-\beta[(\partial_z h)^2 + (\partial_x \partial_y h)^2] + \alpha \underline{\cos(2\pi h)} + \dots}$$

Integer constraint!

Subsystem symmetry : h → h+ f(x)+g(y)

Shift 'h' on a plane does not change the action,
`Rough patterns ' allowed in IR

☆ **Fluctuation of E (thermal entropy of plaquette/dimer)**

→ *Spatial fluctuation of 'h' with higher-order derivative.*

☆ **β : stiffness, only parameter**

✓ *Large β: favor flappable patterns with <E> =0*
✓ *Small β: all close-packed patterns are of equal weight*

✓ *Controlled by microscopic interaction between dimer/plaquettes*

Phase diagram parameterized by β ?

$$\mathcal{Z} = \int \mathcal{D}h \; e^{-\beta[(\partial_z h)^2 + (\partial_x \partial_y h)^2] + \alpha\cos(2\pi h) + \ldots}$$

$$\langle e^{-h(0)h(z)} \rangle = e^{-\frac{1}{4\pi\beta}[\ln(z)]^2}$$

$$\langle e^{-h(0,x,y)h(0,0,0)} \rangle \to 0$$

Short-ranged correlated, $\cos(2\pi h)$ irrelevant

$$\langle e^{-(2\pi\nabla_x h(0)\; 2\pi\nabla_x h(0,y,z))} \rangle = \frac{1}{(z^2+y^2)^{\frac{\pi}{\beta}}},$$

$$\langle e^{-(2\pi\nabla_y h(0)\; 2\pi\nabla_y h(x,0,z))} \rangle = \frac{1}{(z^2+x^2)^{\frac{\pi}{\beta}}}$$

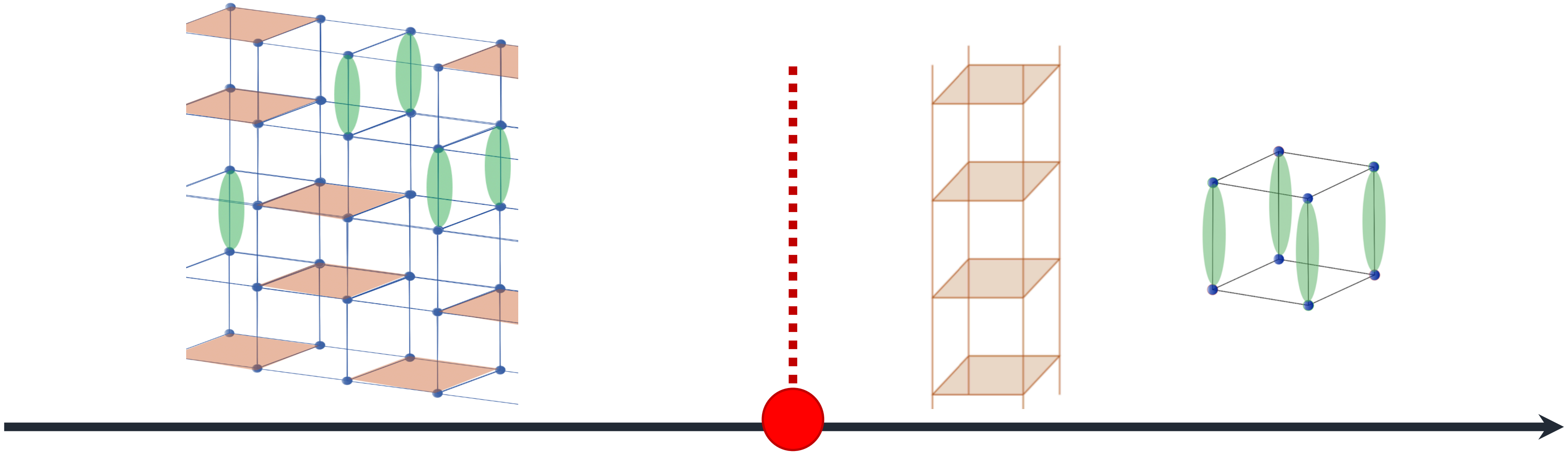
☆ Small β: liquid phase, power-law

☆ Large β: Relevant $\cos(2\pi\nabla_x h), \cos(2\pi\nabla_y h)$

Spontaneously ordered?

Phase diagram parameterized by β ?

$$\mathcal{Z} = \int \mathcal{D}h \; e^{-\beta[(\partial_z h)^2 + (\partial_x \partial_y h)^2] + \alpha \cos(2\pi h) + \dots}$$



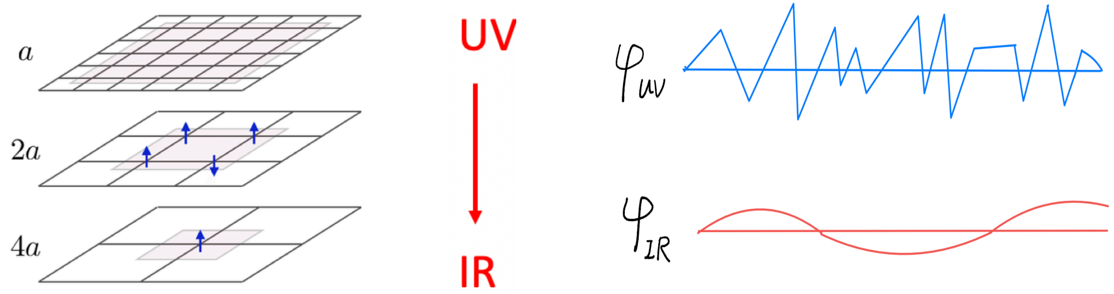
Small β: liquid phase,
power-law correlation

β_c

Large β: Spontaneously ordered
Favor flippable patterns

$\cos(2\pi \nabla_x h), \cos(2\pi \nabla_y h)$ Become relevant

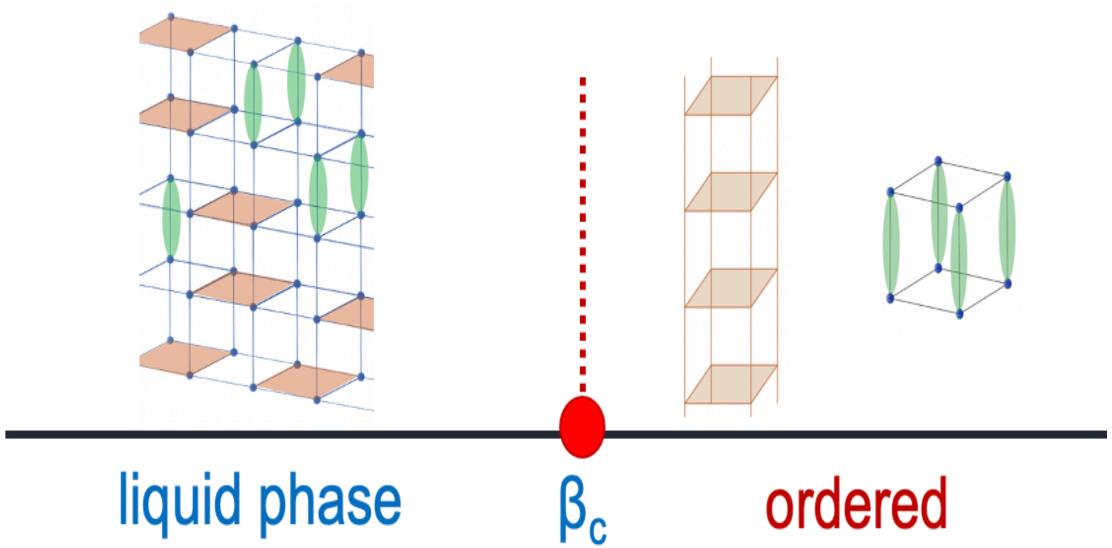
RG aspect of critical liquids



Coarse grained = keep long wave-length

→ low energy = long wave-length

How RG fails here?



$$\left.\begin{aligned}\langle e^{-h(0,x,y)h(0,0,0)}\rangle &\to 0\\ \langle e^{-(2\pi\nabla_x h(0)\ 2\pi\nabla_x h(0,y,z))}\rangle &= \frac{1}{(z^2+y^2)^{\frac{\pi}{\beta}}},\end{aligned}\right\}\ ?$$

☆ Higher order operators more relevant?
☆ Short wave-length modes plays an important role?

How RG fails here?

$$\mathcal{Z} := \int \mathcal{D}h \; e^{-\beta[(\partial_z h)^2 + (\partial_x \partial_y h)^2] + \alpha \cos(2\pi h) + \dots}$$

Subsystem symmetry : h → h+ f(x)+g(y)

$$\langle h(q)h(q)\rangle = \frac{1}{\beta(q_z^2 + q_y^2 q_x^2)}$$

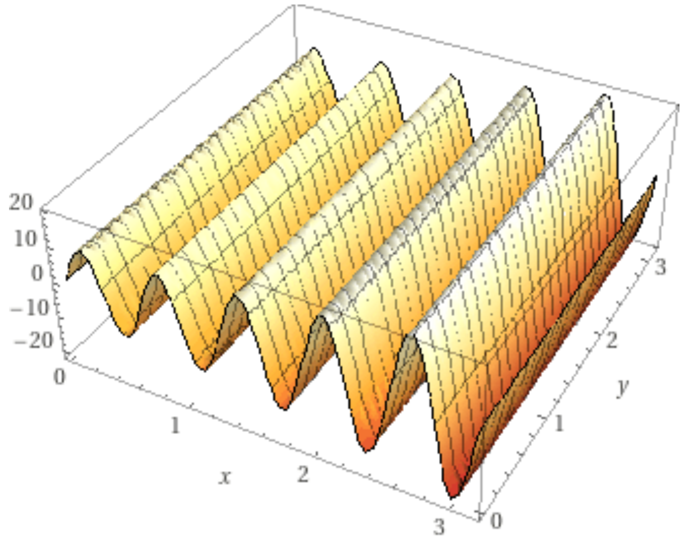
Singularity at k_x,k_y axis
→ Sub-extensive # of `low energy modes' at large momentum

local fluctuation survive at 'IR'
→ short wave-length cannot be ignored!

Rough field fluctuation at low energy

→ Short wave-length physics enter IR

UV-IR mixing! (Seiberg-*2020*)



Self Duality

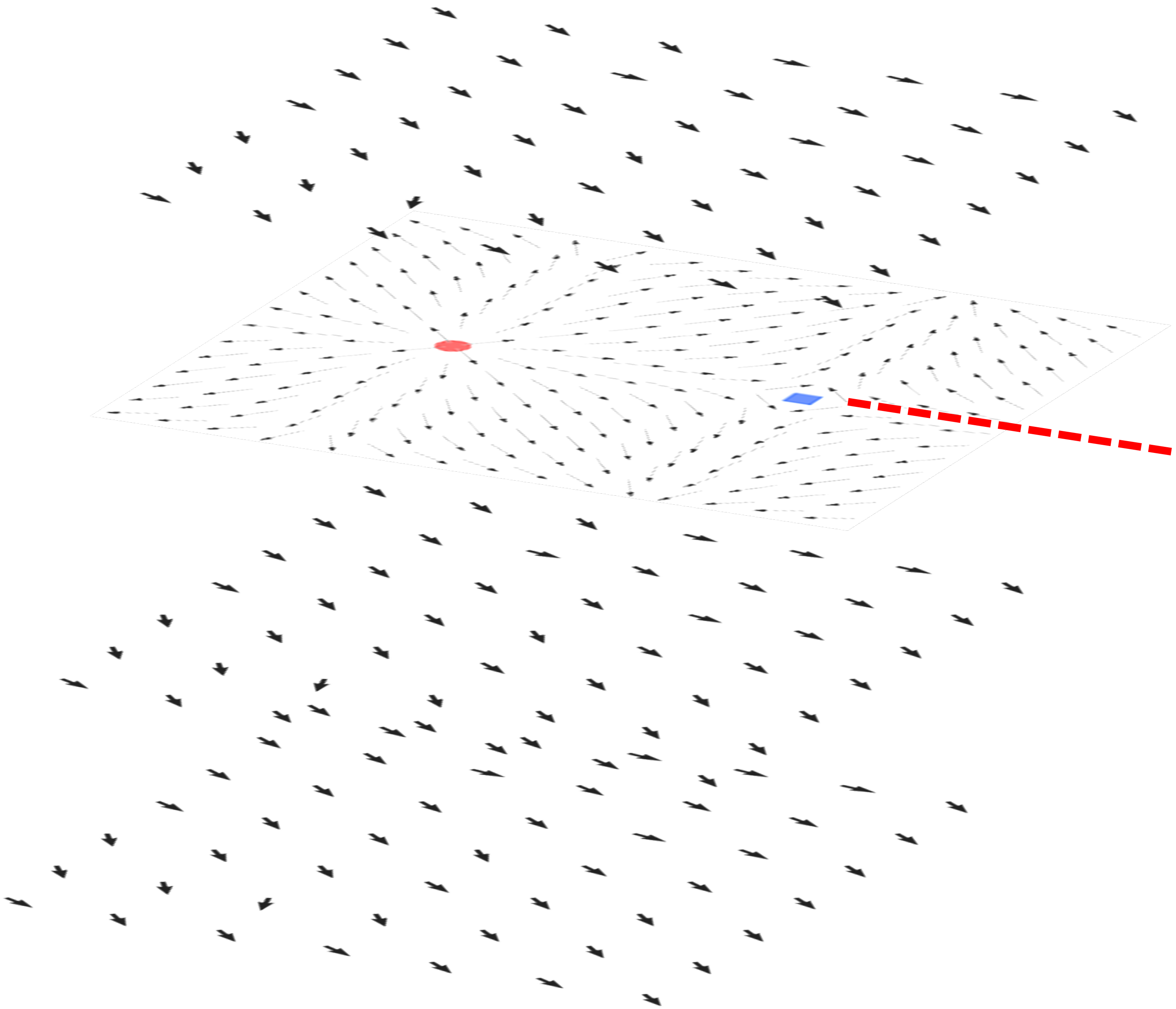
Compact

$$\mathcal{Z} = \int \mathcal{D}\theta \; e^{-\beta(\cos(\partial_z\theta)+\cos(\partial_x\partial_y\theta))+\dots}$$

Discrete

$$\mathcal{Z} = \int \mathcal{D}h \sum_n e^{-i2\pi nh+\frac{1}{4\beta}((\partial_x\partial_y h)^2+(\partial_z h)^2)\dots}$$

	Compact rotor field	**Dual height representation**
$\beta < \beta_c$	Disordered phase (short-ranged correlation)	Ordered phase (long-ranged plaquette/dimer order)
$\beta = \beta_c$	Dipole defect proliferation	$\cos(\nabla_x h)$, $\cos(\nabla_y h)$ becomes relevant
$\beta > \beta_c$	Liquid phase with quasi-long range order	Plaquette-dimer liquid with power-law correlations



$$\mathcal{Z} = \int \mathcal{D}\theta \; e^{-\beta(\cos(\partial_z\theta)+\cos(\partial_x\partial_y\theta))+\ldots}$$

Allow isolated vortex on (xz/yz) plane

Energy growth ~ ln(L)
Entropy gain ~ ln(L)

Critical β_c,
Defect proliferates→ liquid to disorder

Akin to BKT but contains UV-IR mixing

Consequence of UV-IR mixing?

UV-IR mixing → short wave-length physics survives at low energy
→ IR theory affected by UV

✓ *Critical exponent independent of spacetime, can even have emergent fractal dimension*

✓ *EFT* Depends on UV cut-off

✓ *Higher order operators could be more relevant!*

New field theory: IR blend with UV!

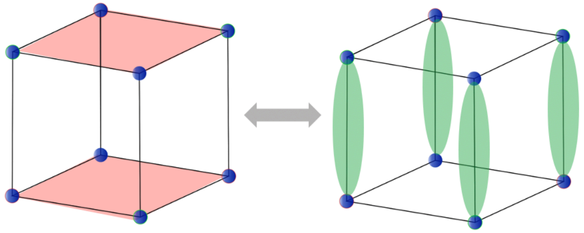
(You-2019, Seiberg-2020, Karch-2020)

More phase transition to be explored !

Outlook and Extension

✓ **If we dope the close-packed patterns: charge (monomers) have restricted motions!**

✓ **Quantum version?**



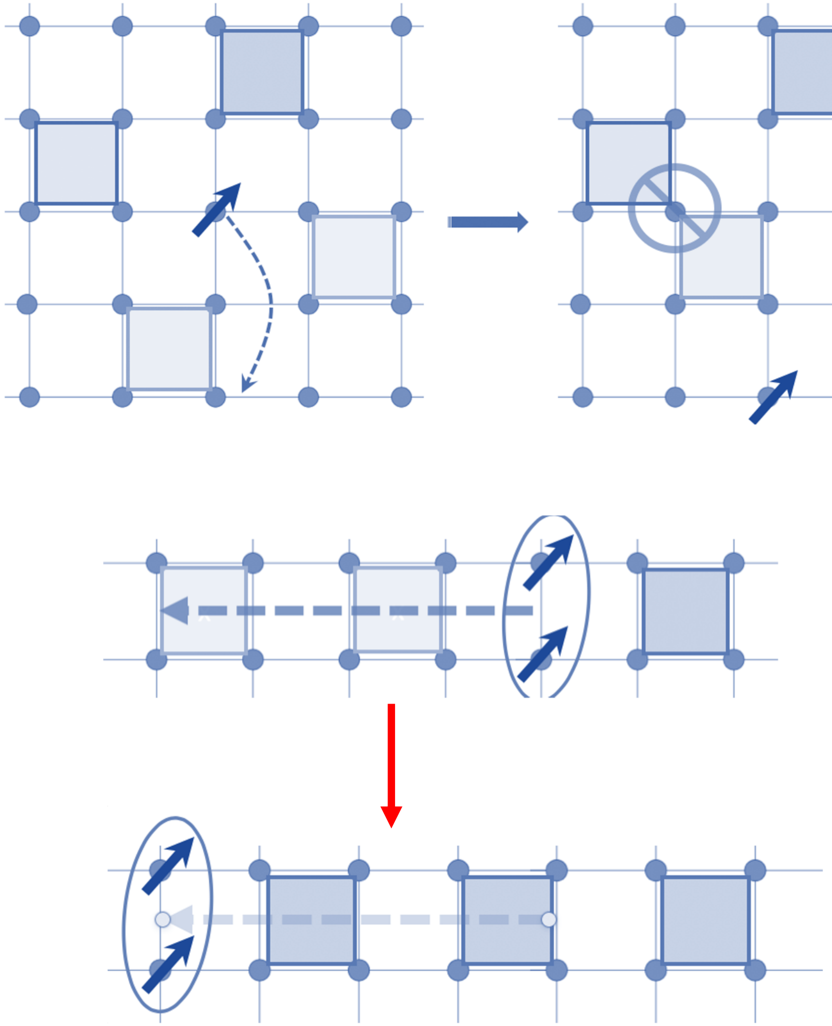
U(1) higher-rank gauge theory → Confinement!

Discretize to Z_2 → Fracton topological order *(Chen-2018)*

✓ **How to probe them?**

1) *Pinch points of dimer-plaquette correlations*
2) *Mutual information*

✓**Extensions-- include other geometric patterns, e.g. trimers**
Emergent fractal symmetry!



A pair of monomers can **hop along the transverse slab!**

Single monomer can **only hop along z-direction**

Connection and Extension

✓ **Fracton aspect: charge (monomers) have restricted motions!**

✓ **Quantum version?**

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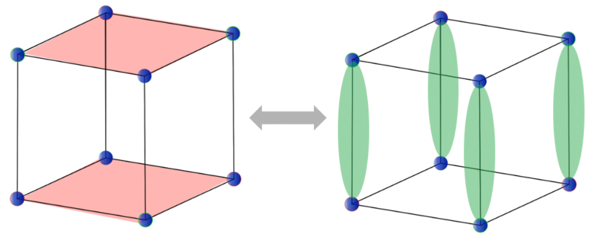

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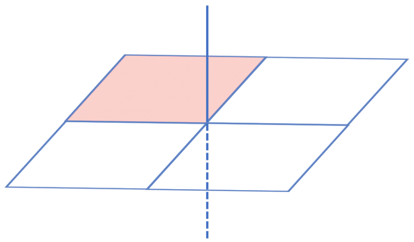
Emergent fractal symmetry!

Quantum Version

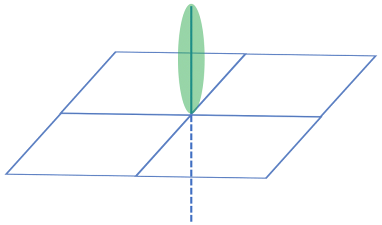
$$[\boxed{A_i(\boldsymbol{x})}, E_j(\boldsymbol{y})] = \frac{i}{2\pi}\delta_{ij}\delta_{\boldsymbol{x}\boldsymbol{y}}$$

Create/annihilate dimer/plaquette



$$E_{xy} = \eta P_{xy},$$

Plaquette # = ***E*** field

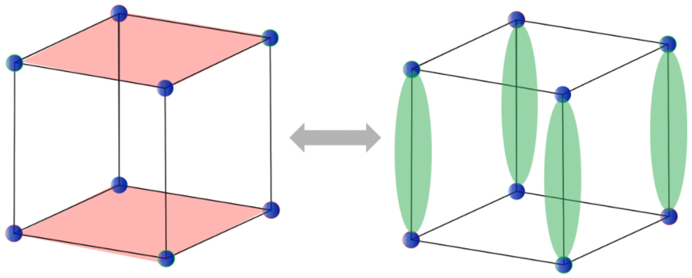


$$E_z = \eta D_z$$

Dimer # = ***E*** field

Magnetic flux $\quad B = \nabla_x \nabla_y A_z + \nabla_z A_{xy}$

Quantum flipping



$$\int dxdzB(r) = 0, \quad \int dzdyB(r) = 0,$$

Flux is also conserved on all x-z & y-z planes

Quantum Plaquette-dimer model = *Compact U(1) higher-rank gauge theory*

✓ *Confinement? Quantum liquid phase?*

☆ *Instanton* operator $e^{i2\pi h}$

Could be irrelevant due to restricted motion of ``B' flux!

☆ **Dipole of instanton** $e^{i2\pi\nabla_x h}$

Create instanton pair between links, always relevant
Proliferate → Confined phase, No quantum plaquette dimer liquid

☆ *Discretize to Z_2 → Fracton topological order (Chen-2018)*

Connection and Extension

✓ **Fracton aspect: charge (monomers) have restricted motions!**

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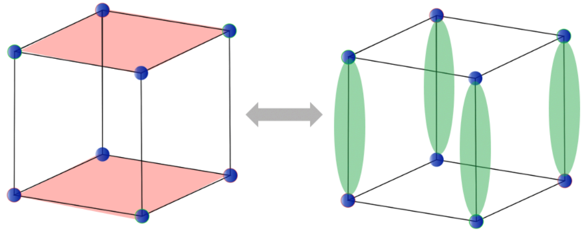

✓ **How to probe them?**

1) *Pinch points of dimer-plaquette correlations*
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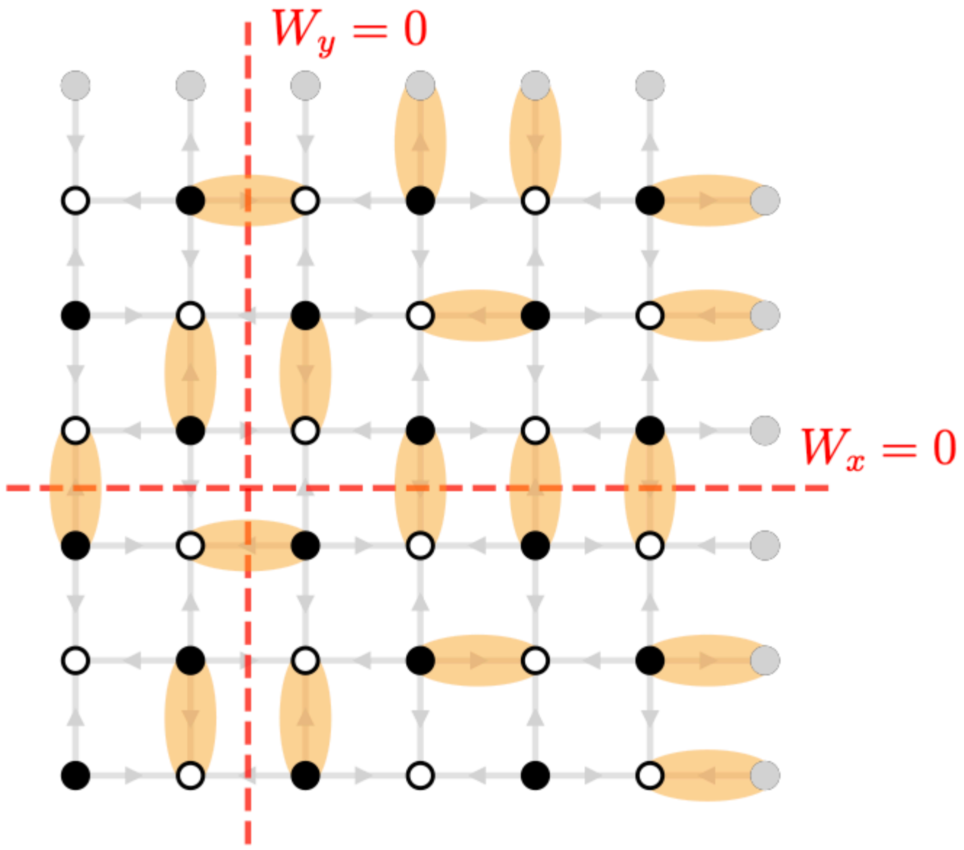
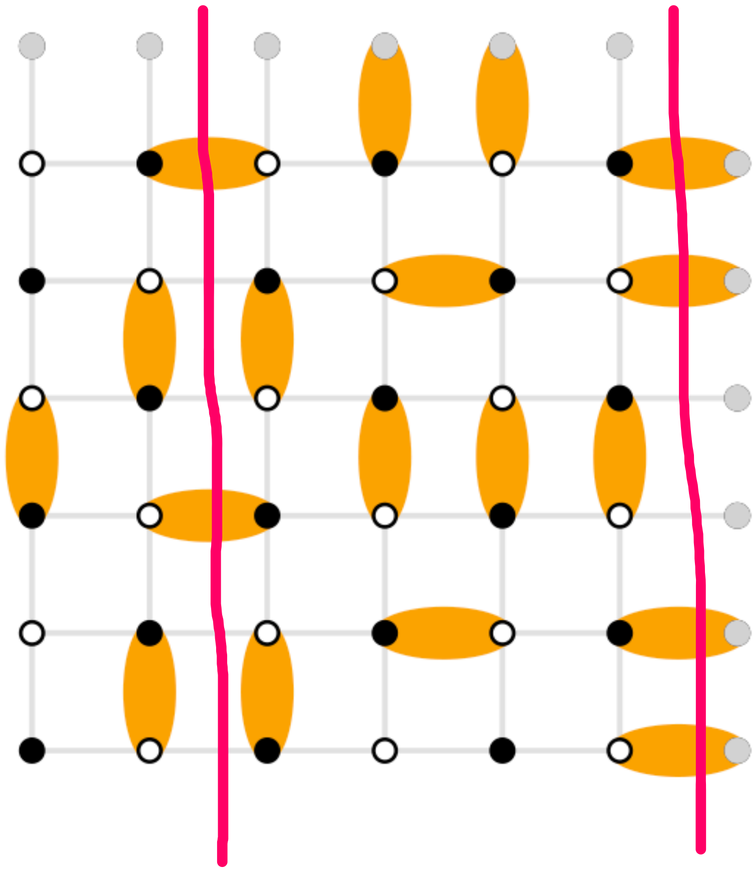
✓**Extensions-- include other geometric patterns, e.g. trimers**

Emergent fractal symmetry!

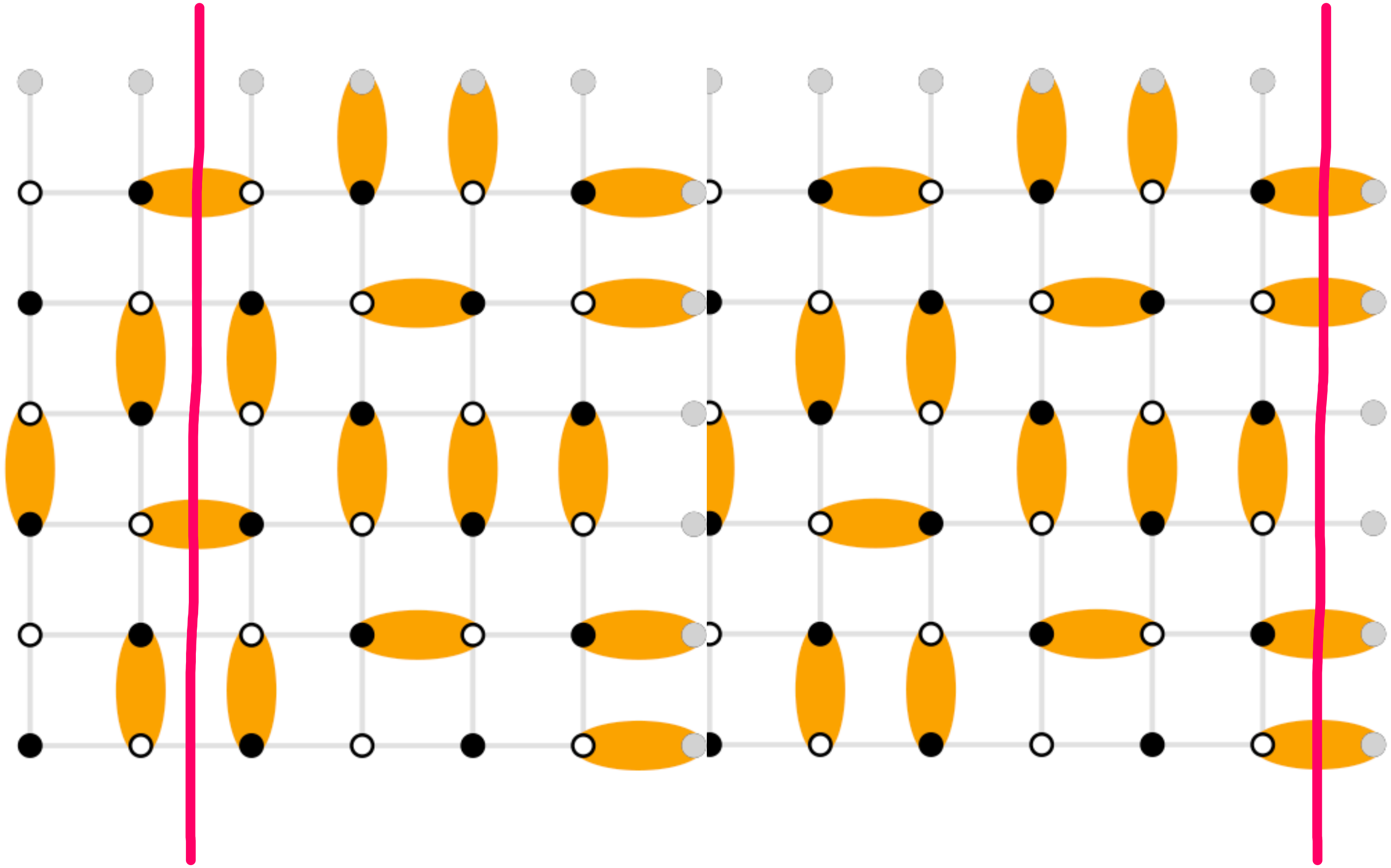
Prelude: winding number in dimers





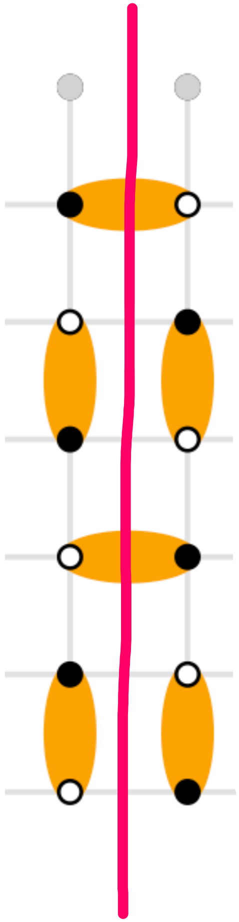
W_y is uniform

Mutual information between two stripes far apart



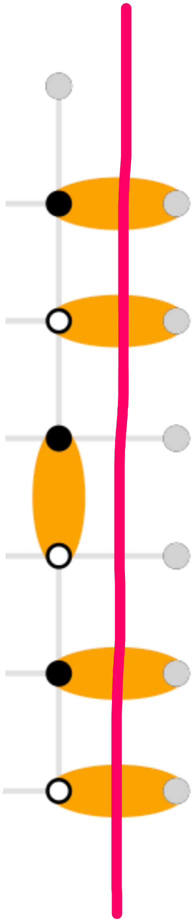
Partition function of all closed-packed configurations

Mutual information between two stripes far apart



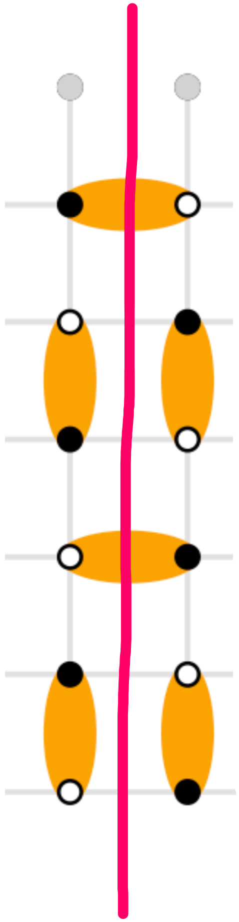
A

Trace out the rest and only keep two stripes



B

Mutual information between two stripes far apart

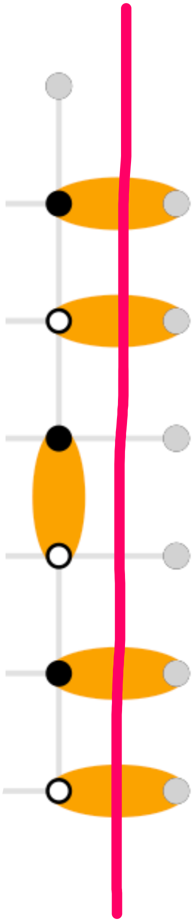


A

✓ No LRO
Local patterns are not correlated

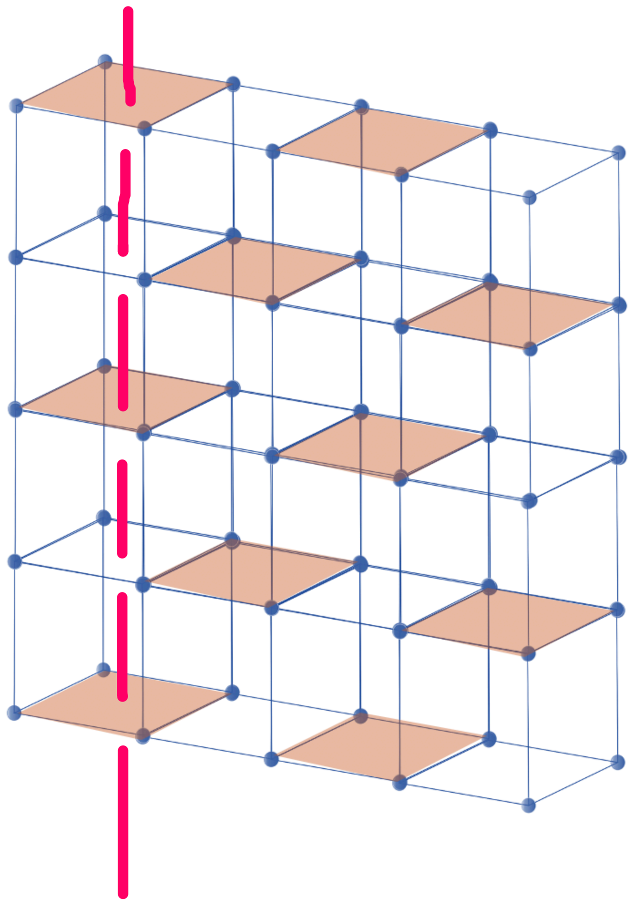
✓ W_y is uniform
Classical correlation between two striped

→ Mutual information between A & B



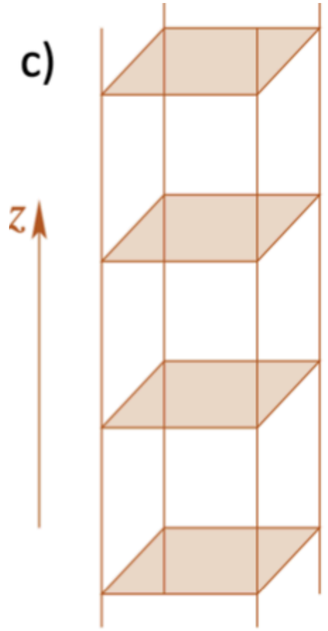
B

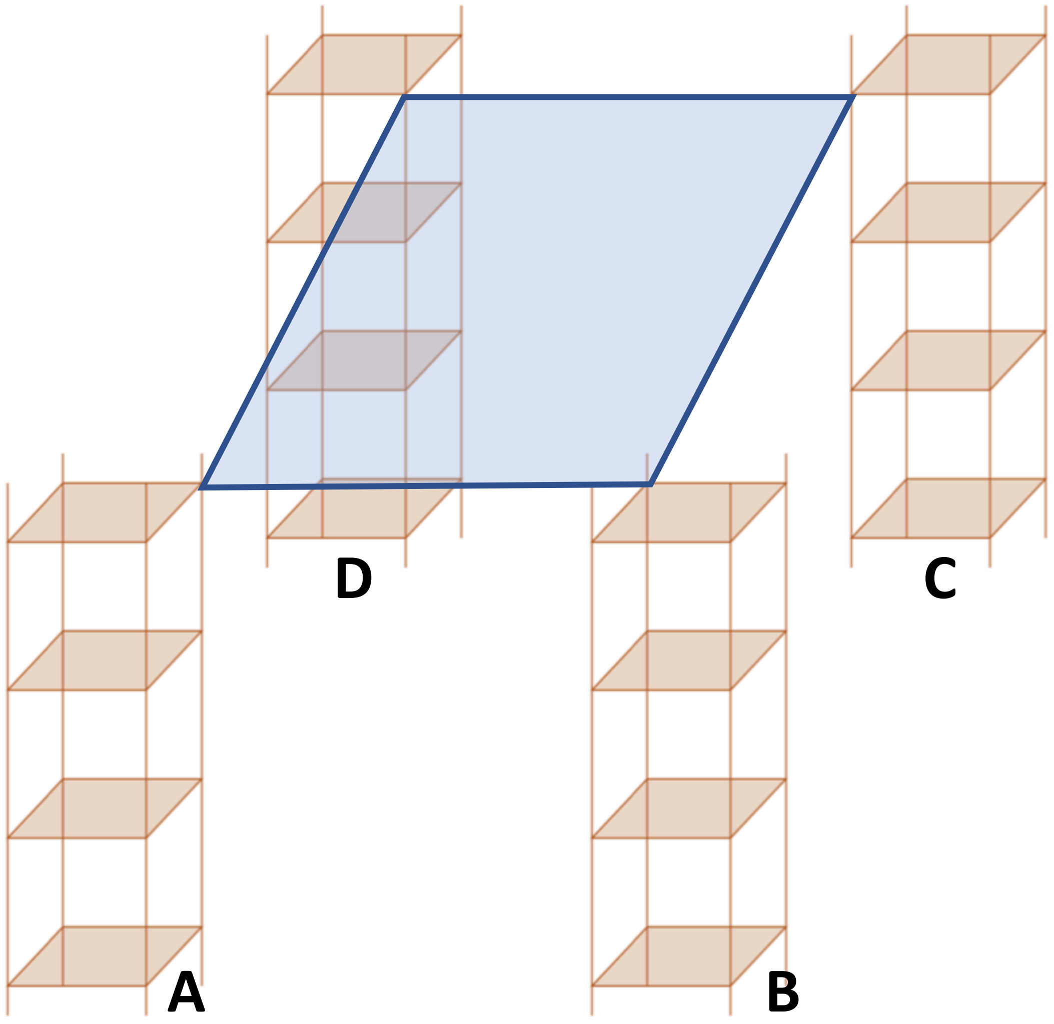
Winding number in Plaquette-Dimers



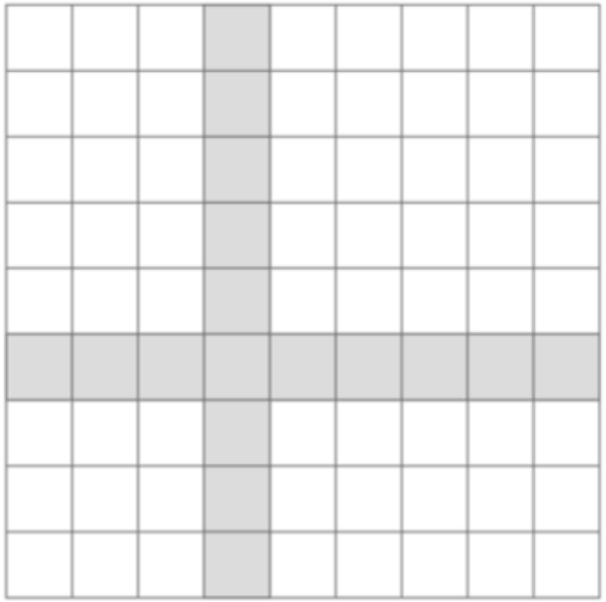
✓ Winding number m_{xy} (x,y) counting plaquettes along z-row

✓ m_{xy} (x,y) ***<u>is not uniform</u>*** in space, can change on x-y plane!





$$\nabla_x \nabla_y m_{xy}(x,y) = 0$$



m_{xy} (x,y): fixed a row and column on x-y plane, else are fixed

✓ Fixed winding number of A,B,C,
Then D is fixed
→ Four regions carry mutual information

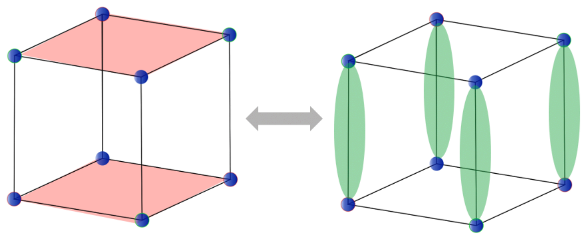
Connection and Extension

✓ **Fracton aspect: charge (monomers) have restricted motions!**

✓ **Quantum version?**

U(1) higher-rank gauge theory → Confinement!

Discretize to Z_2 → Fracton topological order *(Chen-2018)*

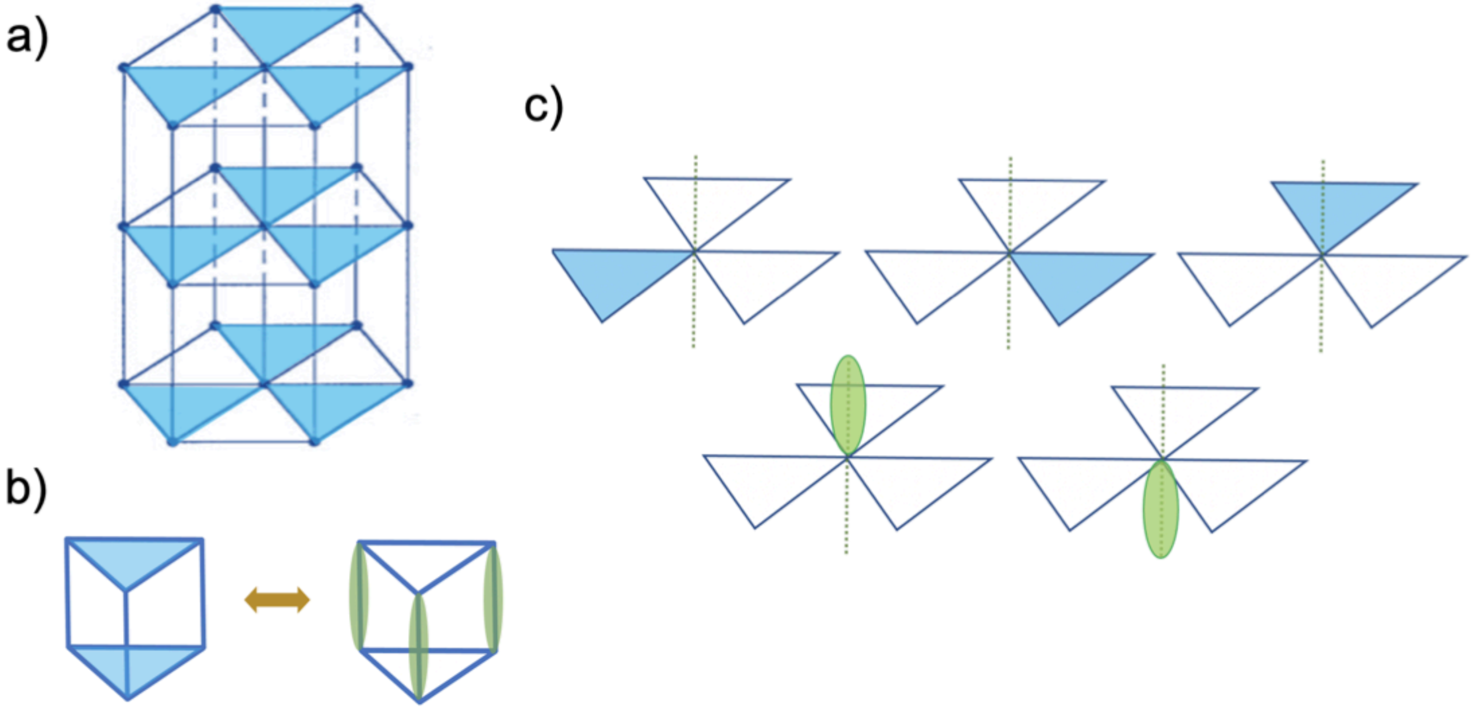


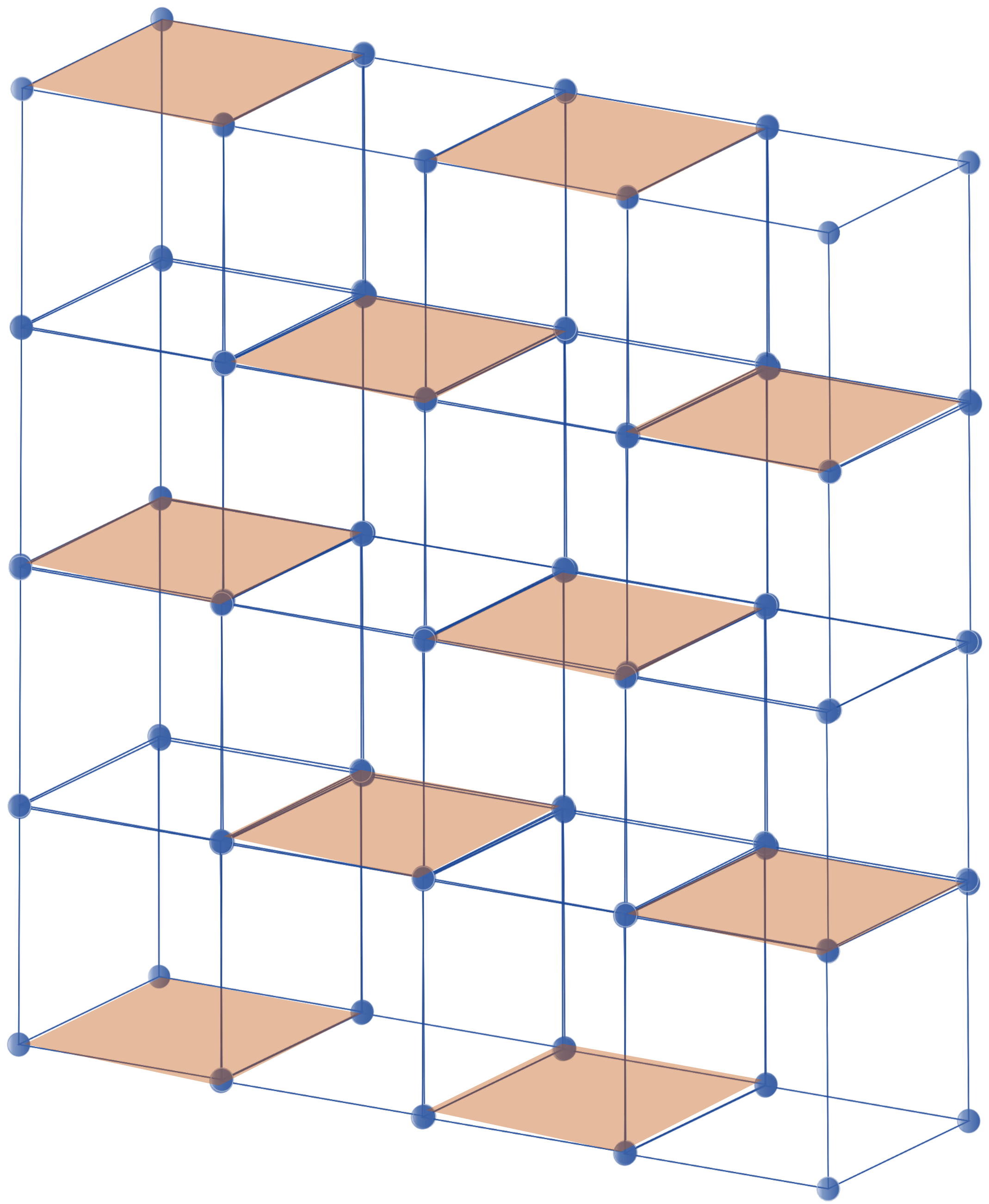
✓ **How to probe them?**

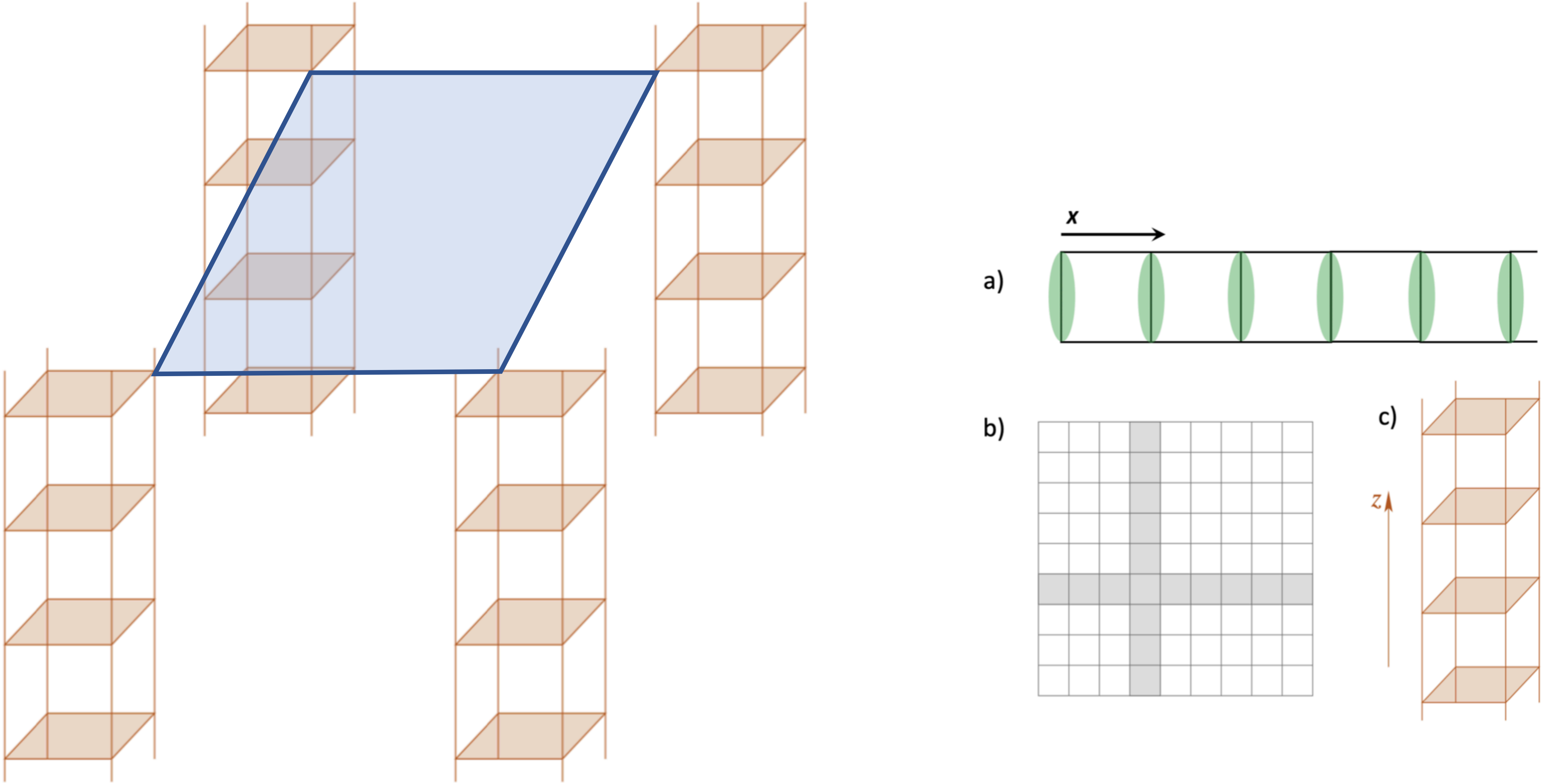
1) *Pinch points of dimer-plaquette correlations*
2) *Mutual information*

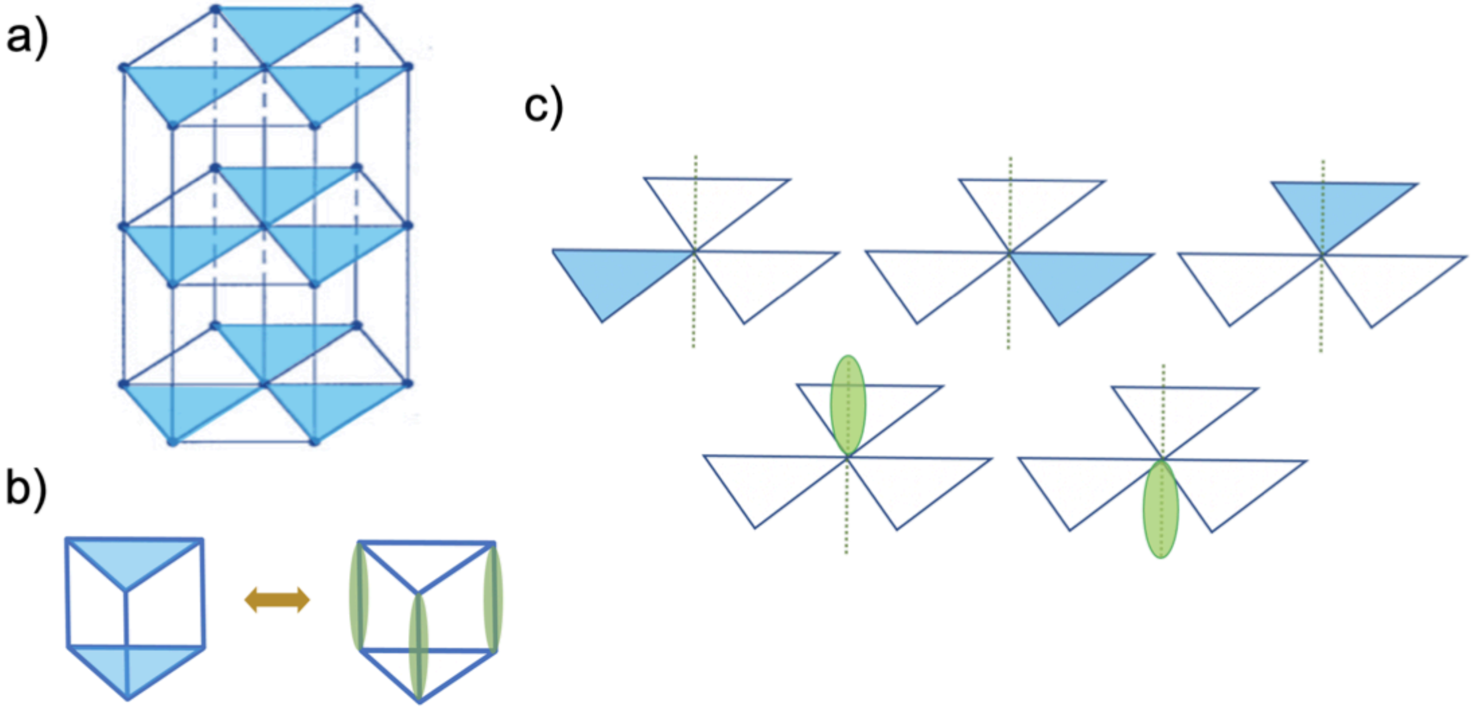
✓**Extensions-- include other geometric patterns, e.g. trimers**

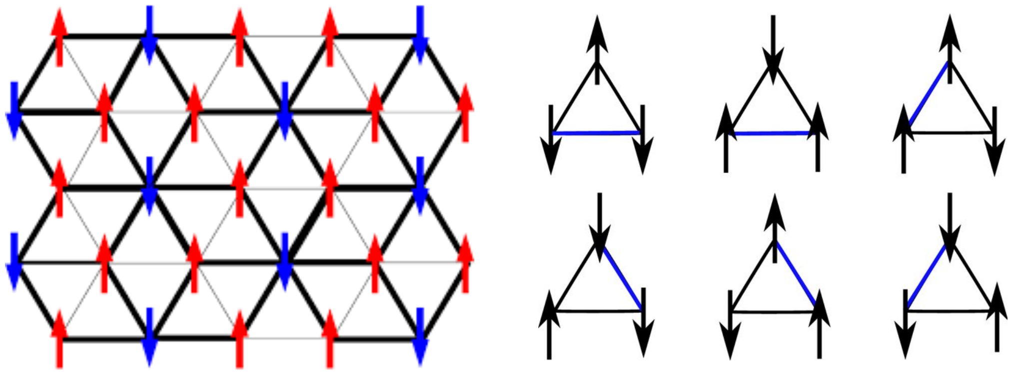
Emergent fractal symmetry!

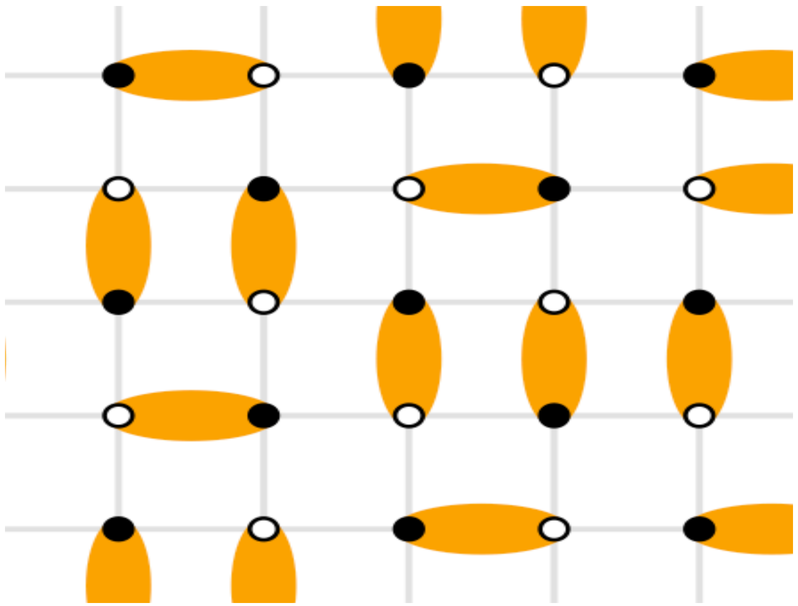

a)
b)
c)




x
a)
b)
c)
z


a)
b)
c)





Ising model on triangular lattice

☆ GS: Each triangle has one `unhappy bond'

☆ Extensive # of patterns in the GS manifold

Close-packed dimers

☆ GS: one and only one dimer connected to each site

☆ Extensive # of patterns in the GS manifold