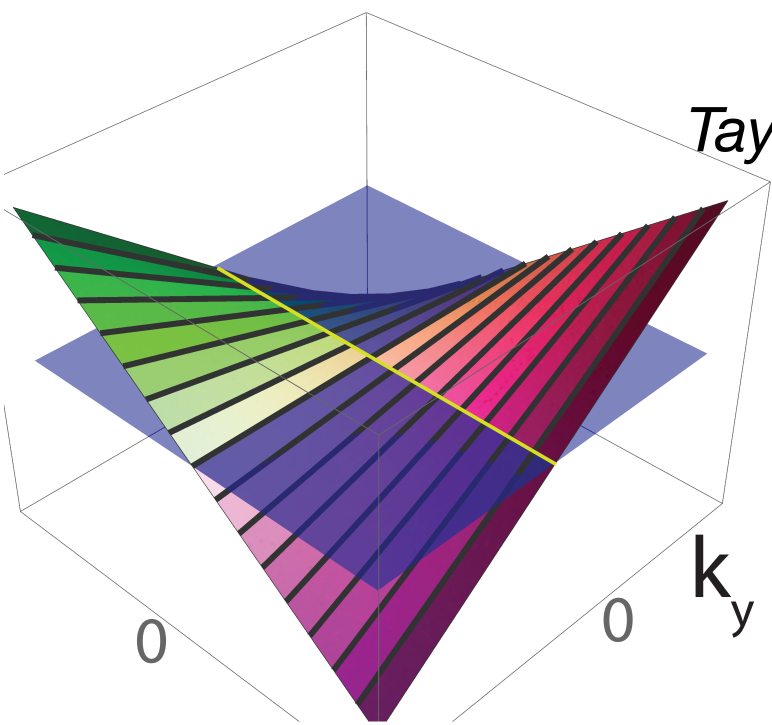
Higher-rank chirality in semi-metallic systems

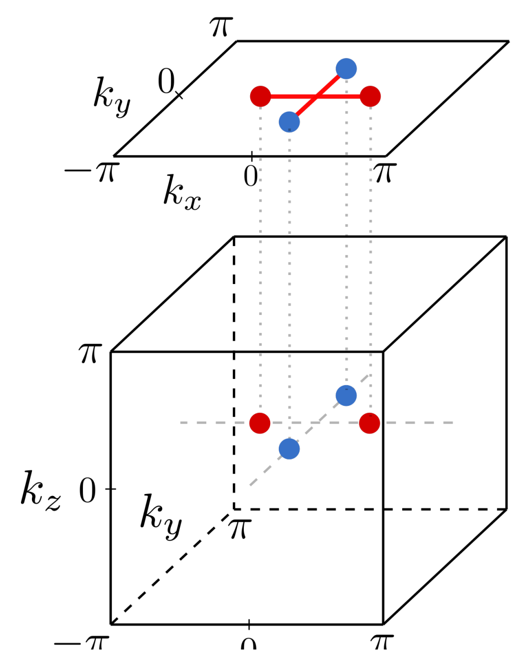
Fiona J Burnell

With

Taylor Hughes, Oleg Dubinkin



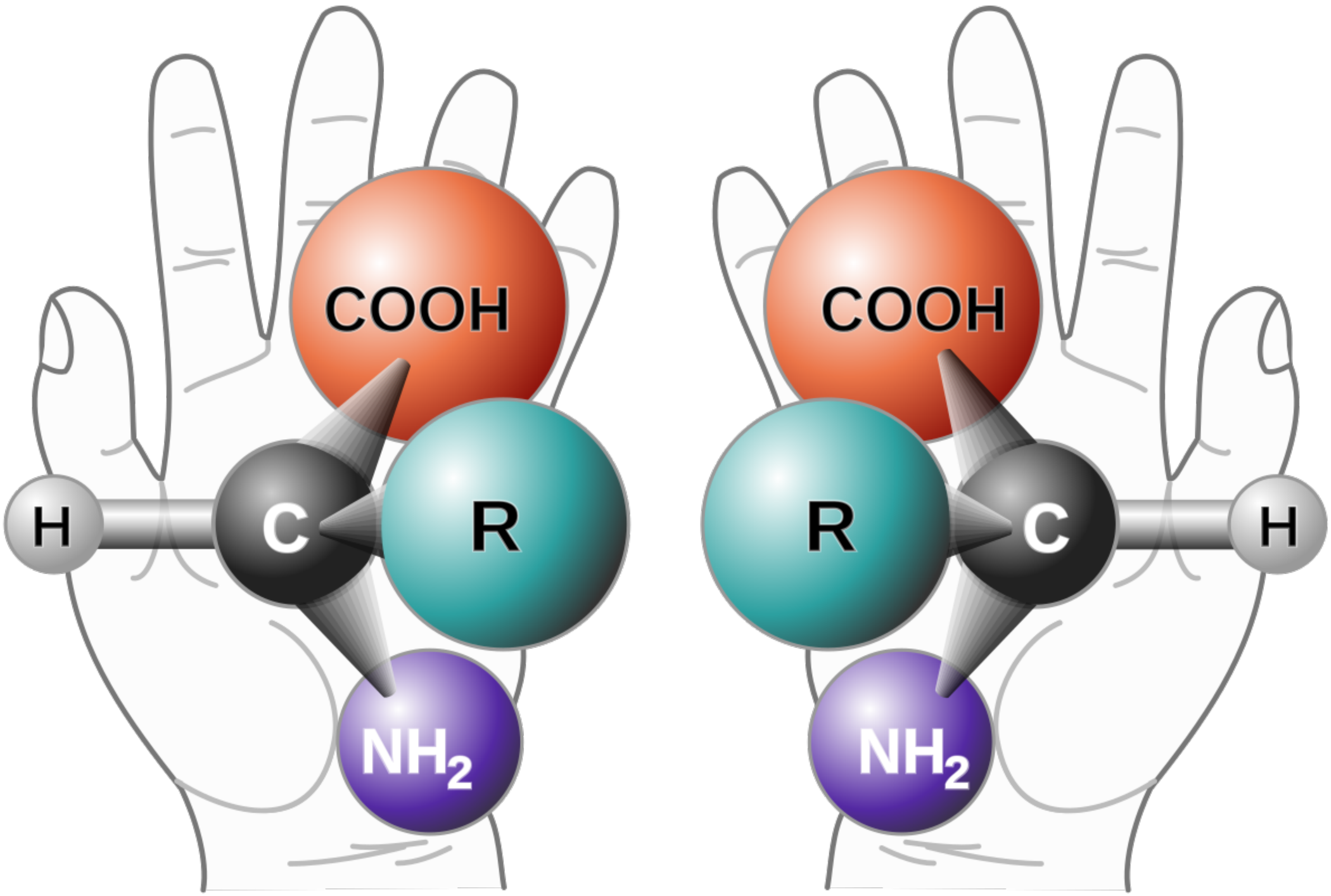
arXiv:2102.08959
(and forthcoming)



- Part 1: Chirality, the axial anomaly, and a higher-rank analog

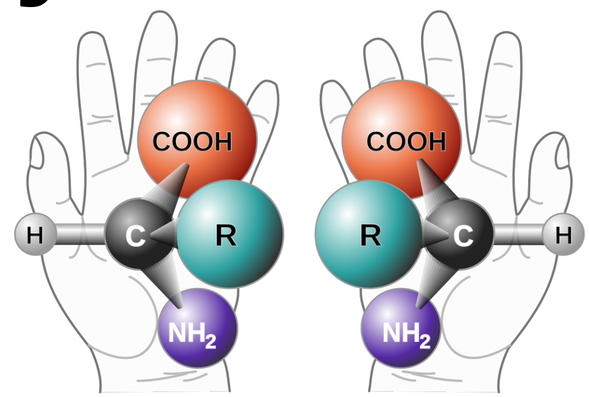
- Part 2: Quadrupolar responses of semi-metals

Part 1: Chirality, the axial anomaly, and a higher-rank analog

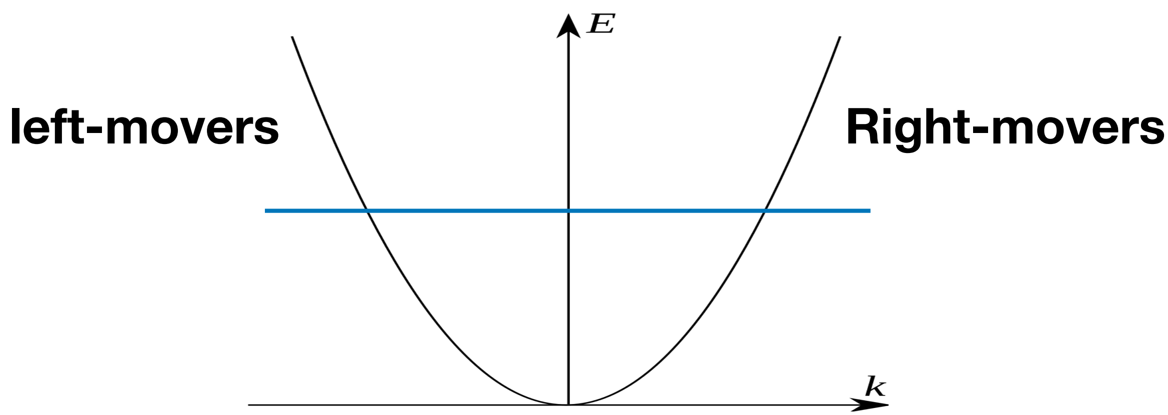


Recall: the axial anomaly and band theory

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \phi)^2\right]$$



Visualisation, as bosonized representation of
1D fermionic band structure:



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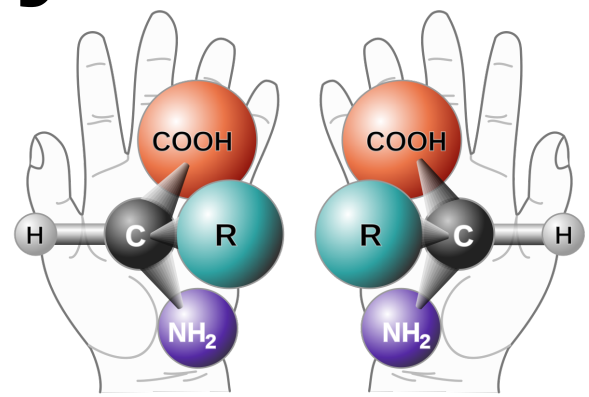


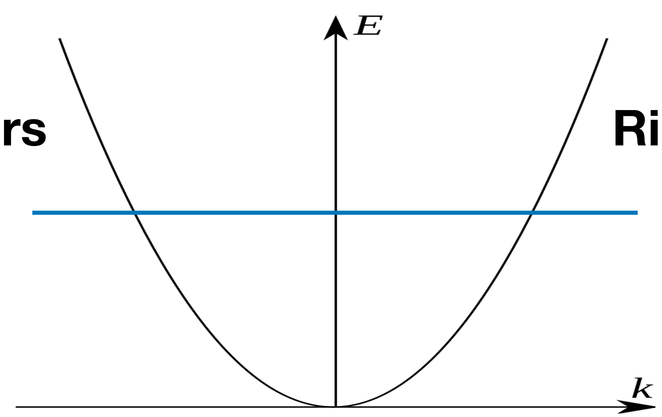
Clasically, two conserved densities:

Charge: $\rho = n_R + n_L$

Momentum: $\rho_k = k_F(n_R - n_L)$

left-movers

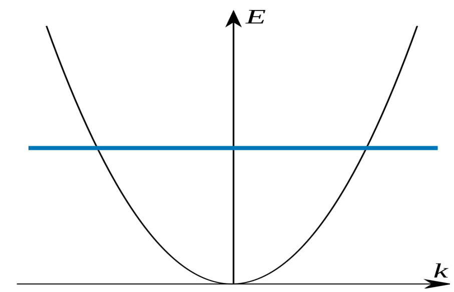
n_L



Right-movers

n_R

Two classically conserved currents:



$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \phi)^2\right]$$

$$\text{EM current: } J_0 = \frac{1}{\sqrt{\pi}}\partial_x \phi \ , \quad J_x = \frac{1}{\sqrt{\pi}}\partial_t \phi$$

- Conserved!

$$\text{Axial current: } \tilde{J}_0 = \frac{1}{\sqrt{\pi}}\partial_t \phi \ , \quad \tilde{J}_x = \frac{1}{\sqrt{\pi}}\partial_x \phi$$

- Conserved due to equations of motion.

$$\partial_t^2 \phi - \partial_x^2 \phi = 0$$

… but not in the presence of external fields!

(Argument due to Fradkin)

- Apply a (classical) electric field

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \phi)^2\right] + J_\mu A^\mu$$

- Modified equations of motion:

$$\partial_t^2 \phi - \partial_x^2 \phi - \frac{1}{\sqrt{\pi}}(\partial_t A_x - \partial_x A_0)$$

$$\partial_\mu \tilde{J}^\mu = \frac{1}{\pi} E_x$$

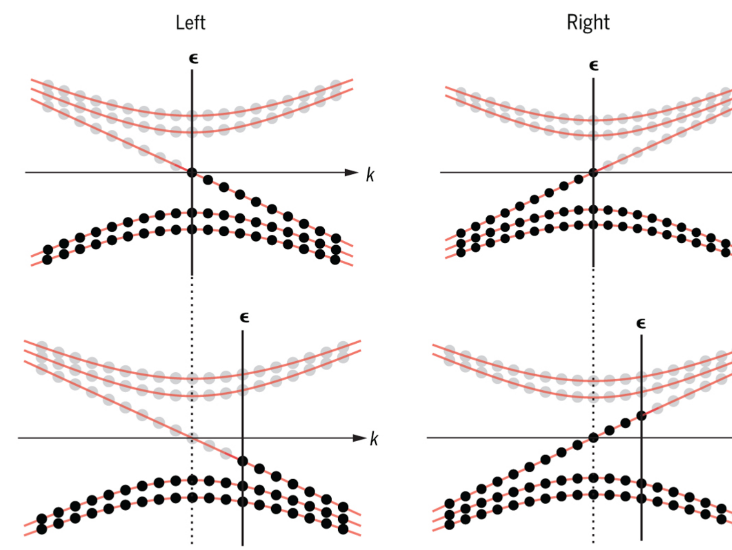


Image credit: Burkov '15

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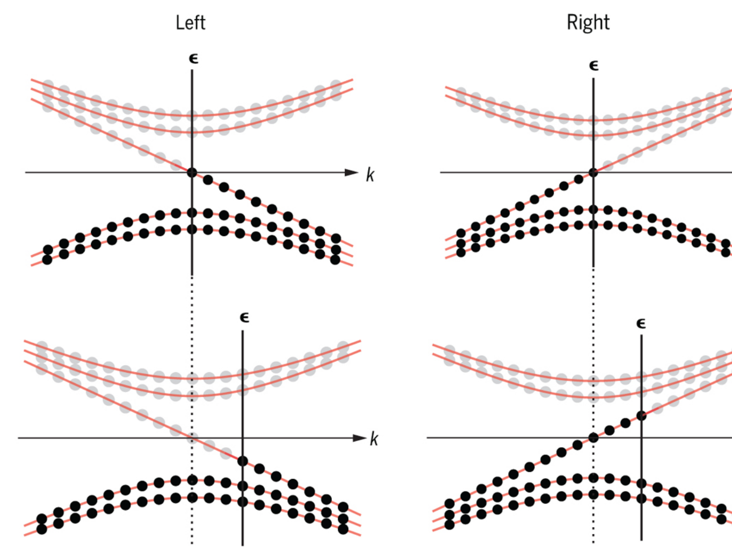


Image credit: Burkov '15

- Interpretation: *momentum is not conserved in an applied electric field!*

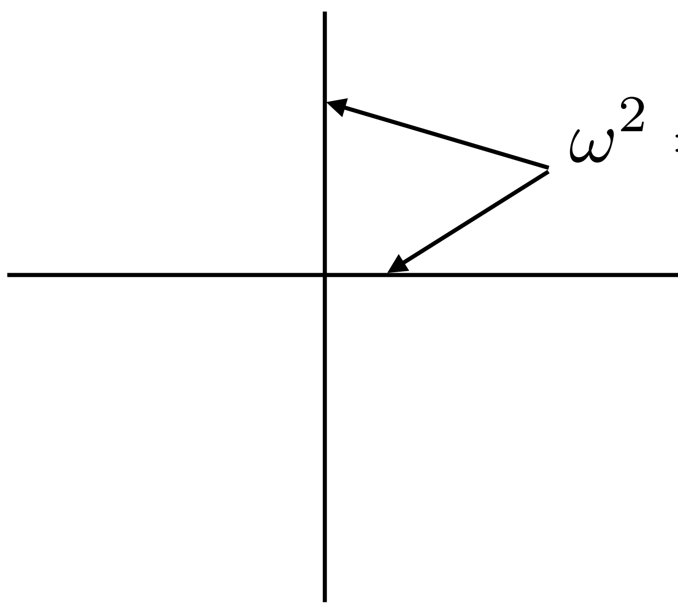
A similar anomaly, inspired by fractonic models

You, Burnell, Hughes ('19);
Gorantla, Lam, Seiberg, Shao ('21)

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \partial_y \phi)^2\right]$$

Free boson!

Unusual dispersion with lines of zeroes:



$$\omega^2 = (k_x k_y)^2 = 0$$

Subsystem symmetry:
invariant under

$$\phi \to \phi + f(x) + g(y)$$

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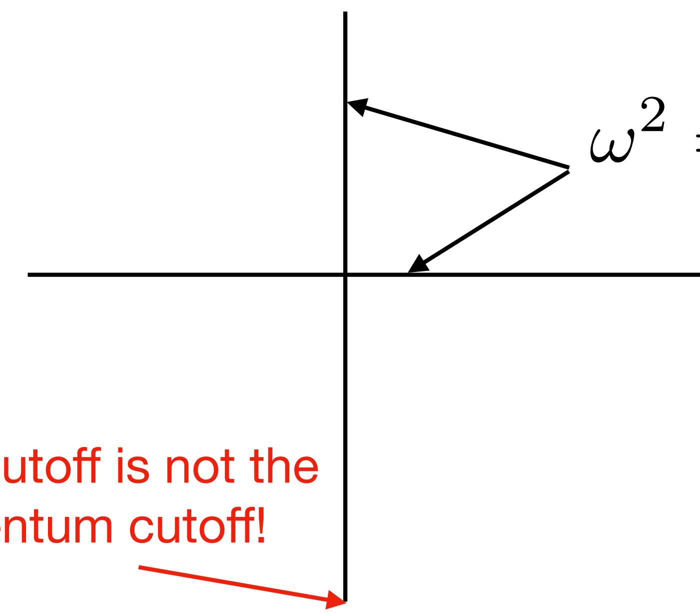

$$\omega^2 = (k_x k_y)^2 = 0$$

Caution: energy cutoff is not the same as momentum cutoff!

Subsystem symmetry: invariant under

$$\phi \to \phi + f(x) + g(y)$$

Two classically conserved currents:

You, Burnell, Hughes ('19);
Gorantla, Lam, Seiberg, Shao ('21)

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \partial_y \phi)^2\right]$$

$$J_0 = \frac{1}{\sqrt{\pi}}\partial_x \partial_y \phi \ , \quad J_{xy} = \frac{1}{\sqrt{\pi}}\partial_t \phi$$

- Conserved! $\partial_t J_0 - \partial_x \partial_y J_{xy} = 0$

$$\tilde{J}_0 = \frac{1}{\sqrt{\pi}}\partial_t \phi \ , \quad \tilde{J}_{xy} = -\frac{1}{\sqrt{\pi}}\partial_x \partial_y \phi$$

- Conserved due to equations of motion.

… but not in the presence of external applied fields!

You, Burnell, Hughes ('19);
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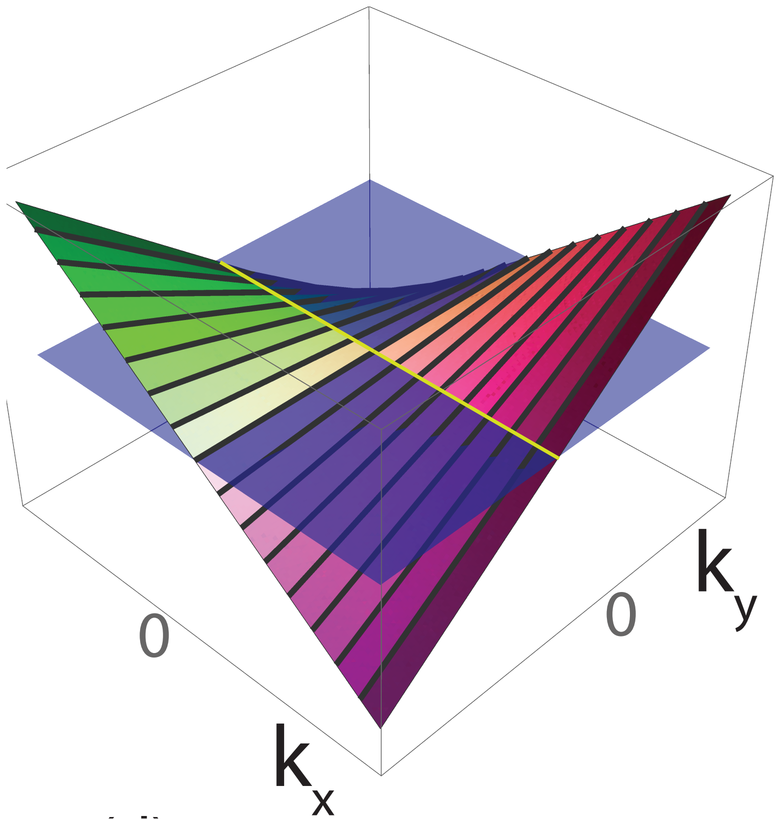
$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \partial_y \phi)^2\right] + A_0 J_0 + A_{xy} J_{xy}$$

- Modified equations of motion:

$$\partial_t^2 \phi - (\partial_x \partial_y)^2 \phi - (\partial_t A_{xy} - \partial_x \partial_y A_0) = 0$$

$$\partial_t \tilde{J}_0 - \partial_x \partial_y \tilde{J}_{xy} = \frac{1}{\pi} E_{xy}$$

Can this anomaly tell us anything about band theory?



- A Hamiltonian with the same "Fermi surface"

$$H = \psi^\dagger \partial_x \partial_y \psi$$

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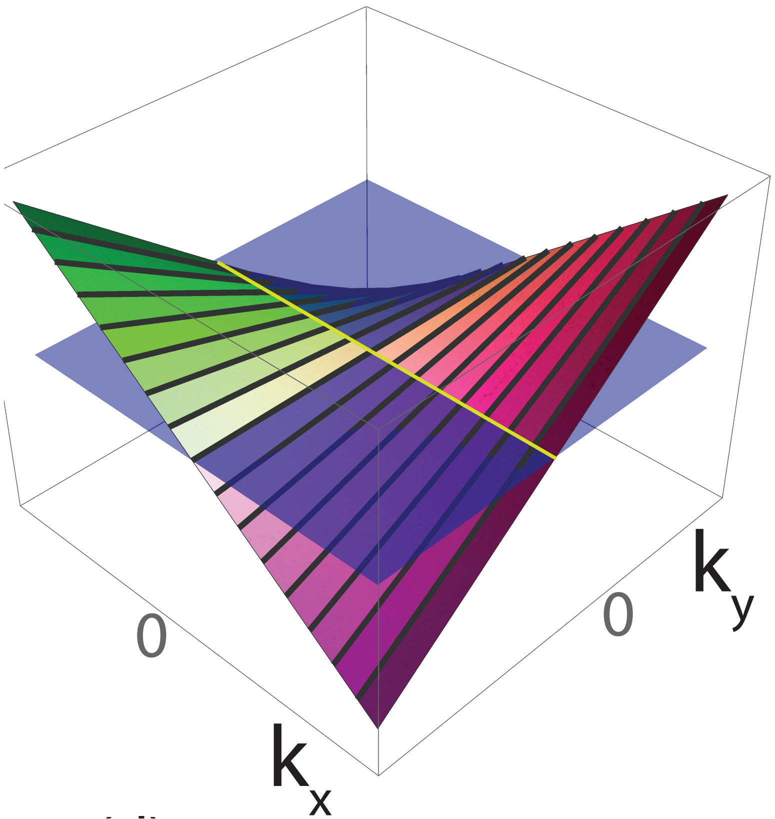

- Conserved quantities:
  - Charge
  - Momentum in x
  - Momentum in y

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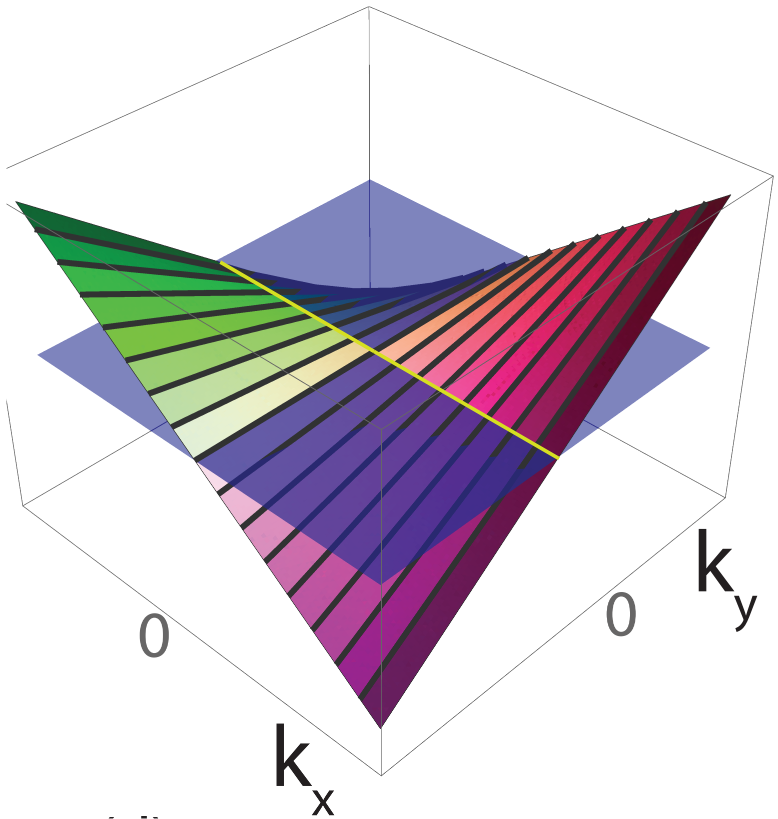


- Conserved quantities:
  - Charge
  - Momentum in x
  - Momentum in y

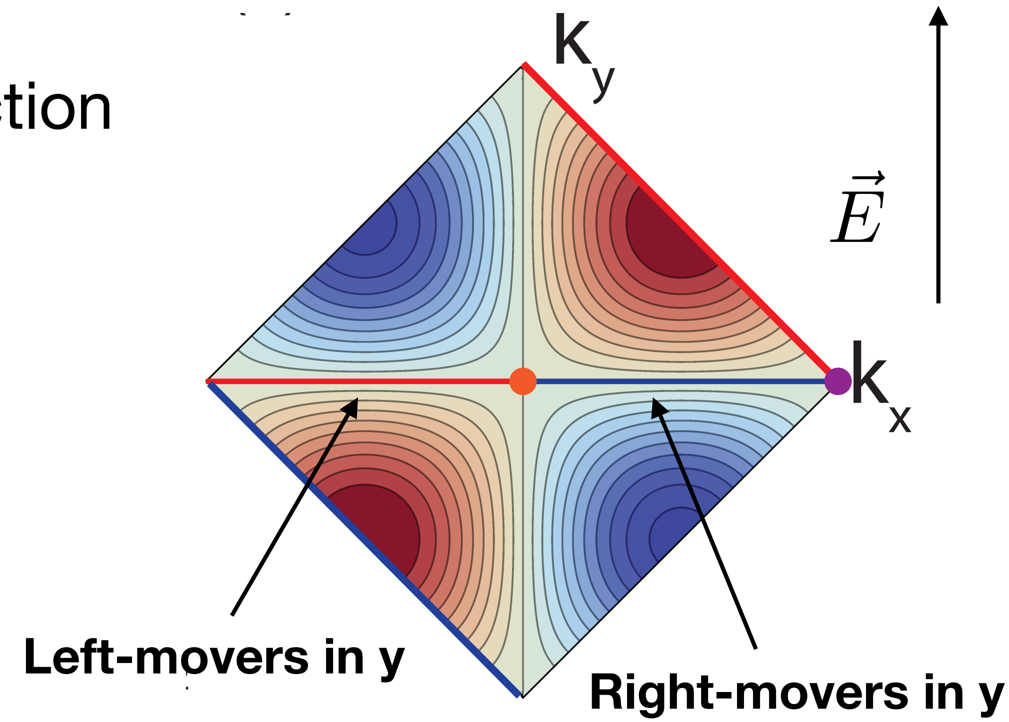
(If we impose a reflection symmetry about y=x, we could think of these as a single additional conserved current)

- A Hamiltonian with the same "Fermi surface"

$$H = \psi^\dagger \partial_x \partial_y \psi$$

Conserved currents vs. external applied fields

- Apply an electric field in y-direction



Momentum anomaly: electric fields in y violate conservation of k_x

- Apply an electric field in y-direction
- Creates particles at k_x and destroys them at $-k_x$

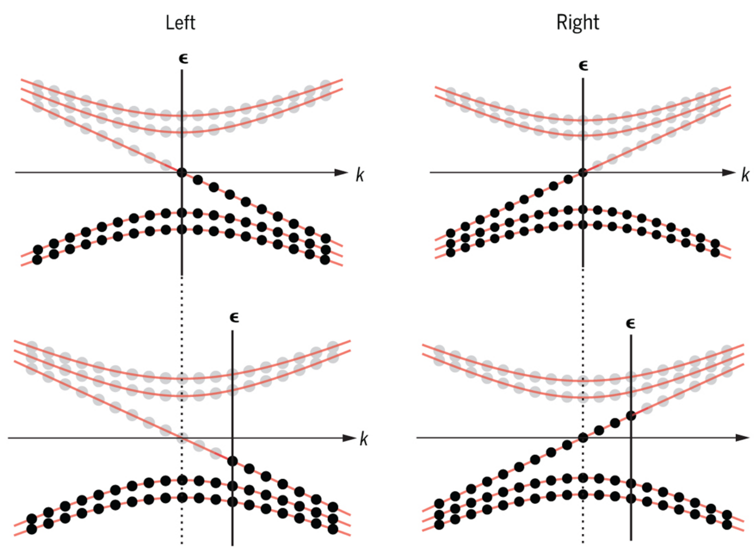
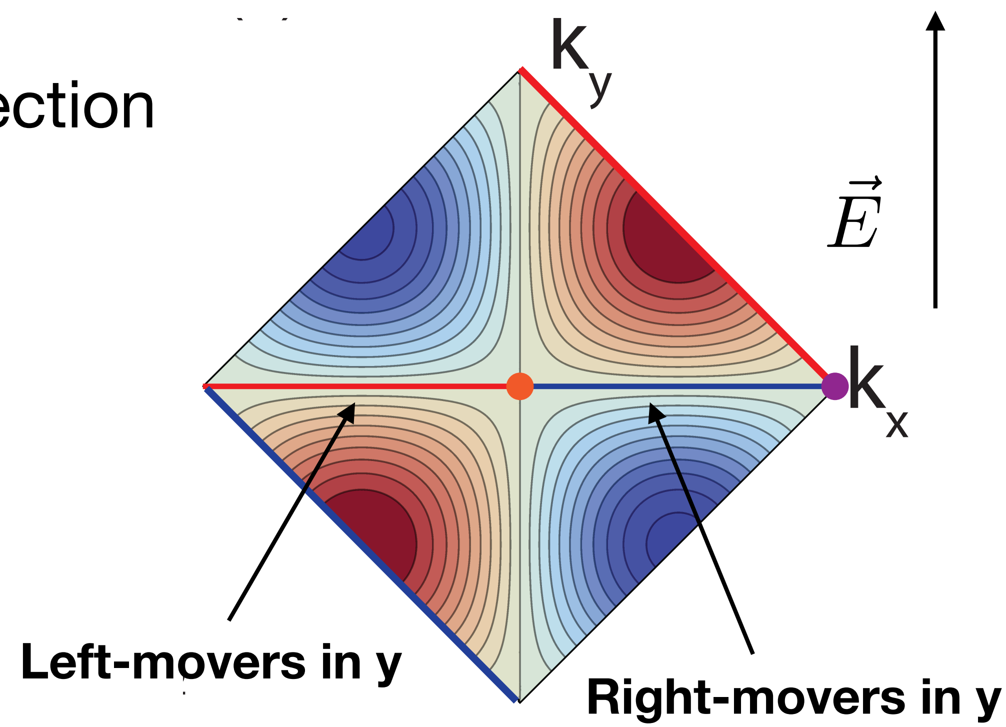


Image credit: Burkov '15



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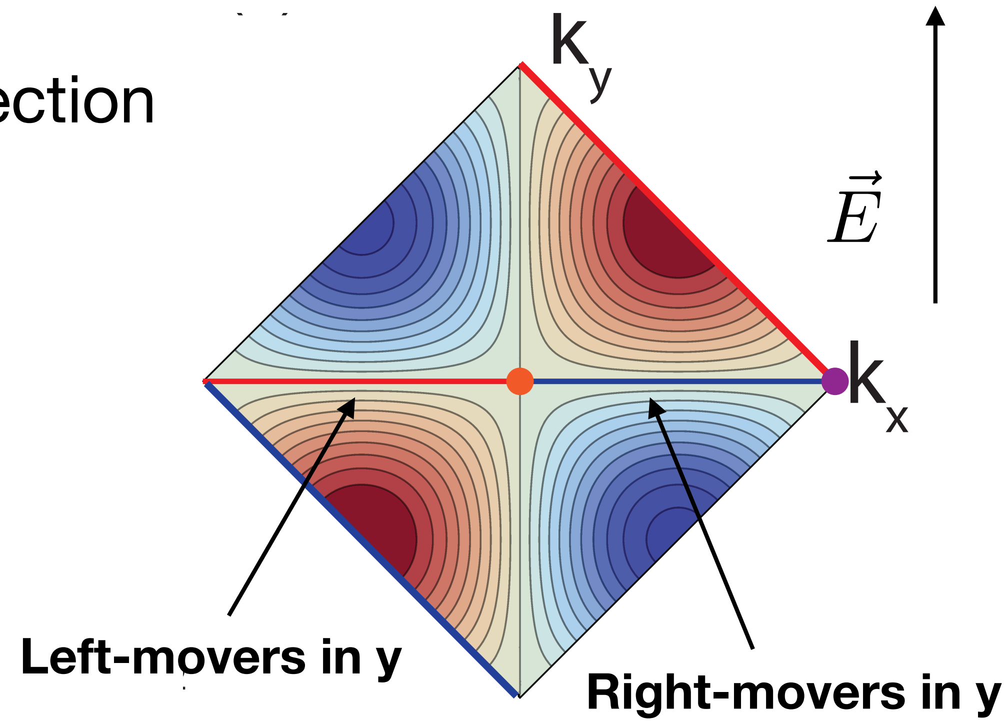
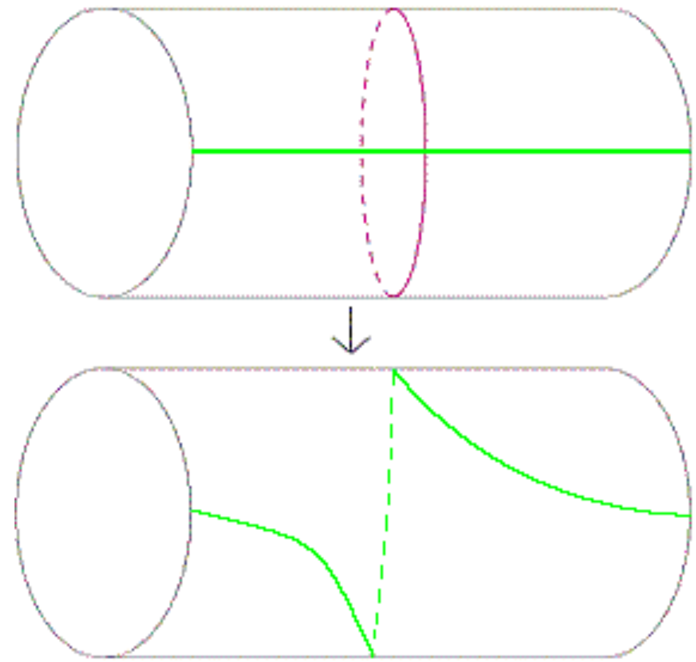


- Total charge is conserved! But …
- Total momentum in x is not conserved!

Charge anomaly: certain (unphysical) lattice defects violate charge conservation

- Twisted boundary conditions on the torus:



$$e^{i(k_x L + k_y \delta)} = 1 \ , \quad e^{ik_y L} = 1$$

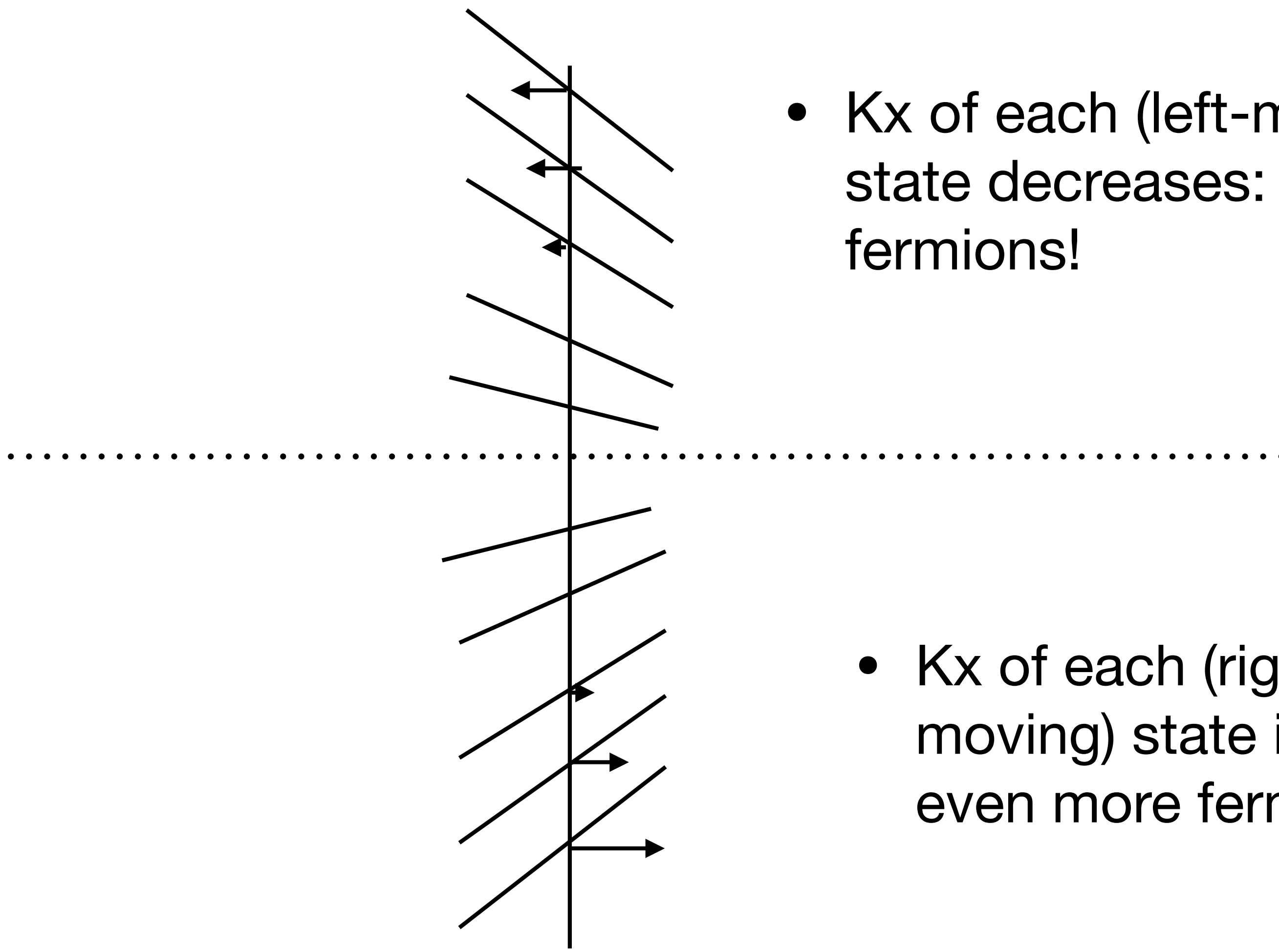
$$k_y = \frac{2\pi n_y}{L}$$

$$k_x = \frac{2\pi n_x}{L} - \frac{2\pi n_y}{L}\frac{\delta}{L}$$

- Twisting by L takes

$$k_x \rightarrow k_x - k_y$$

Twist and violation of charge conservation!



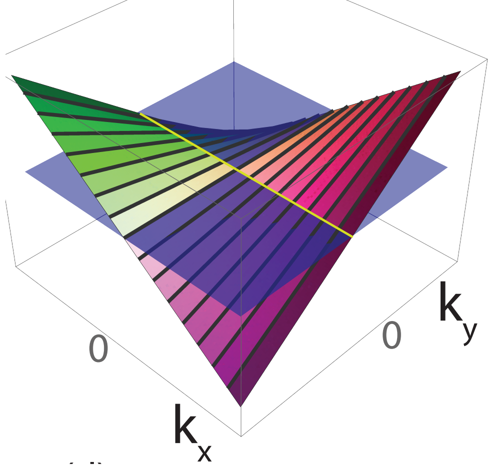
- Kx of each (left-moving) state decreases: more fermions!

- Kx of each (right-moving) state increases: even more fermions!

Higher-rank chiral anomaly: a comparison

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \partial_y \phi)^2\right] + A_0 J_0 + A_{xy} J_{xy}$$

- 2 U(1) currents.
- Applied electric field violates momentum current conservation
- Applied flux of “momentum gauge field” violates electric current conservation



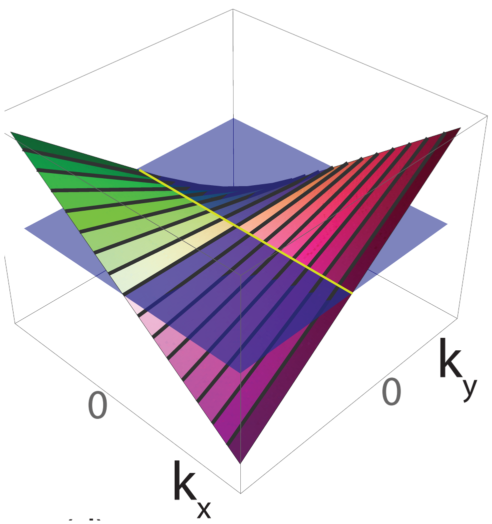
(If we impose a reflection symmetry about y=x, we could think of these as a single additional conserved current)

- 3 U(1) currents (charge, momenta in x and y).
- Applied electric field in x violates y- momentum current conservation
- Applied flux of “momentum gauge field” (twist in x or y) violates electric current conservation

Cancelling the anomaly via the bulk

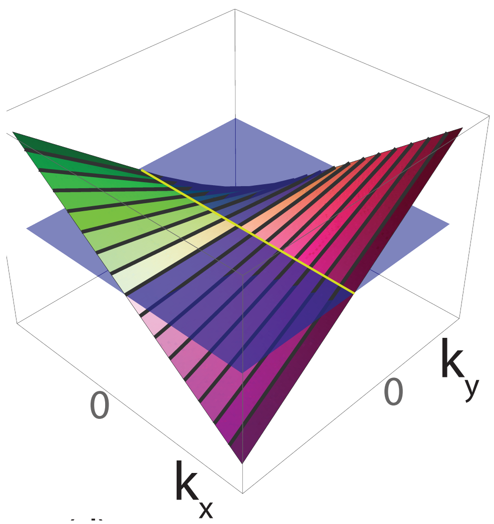
- This cannot possibly be a 2D lattice system; the Fermi surface is not closed!

- What can it be?



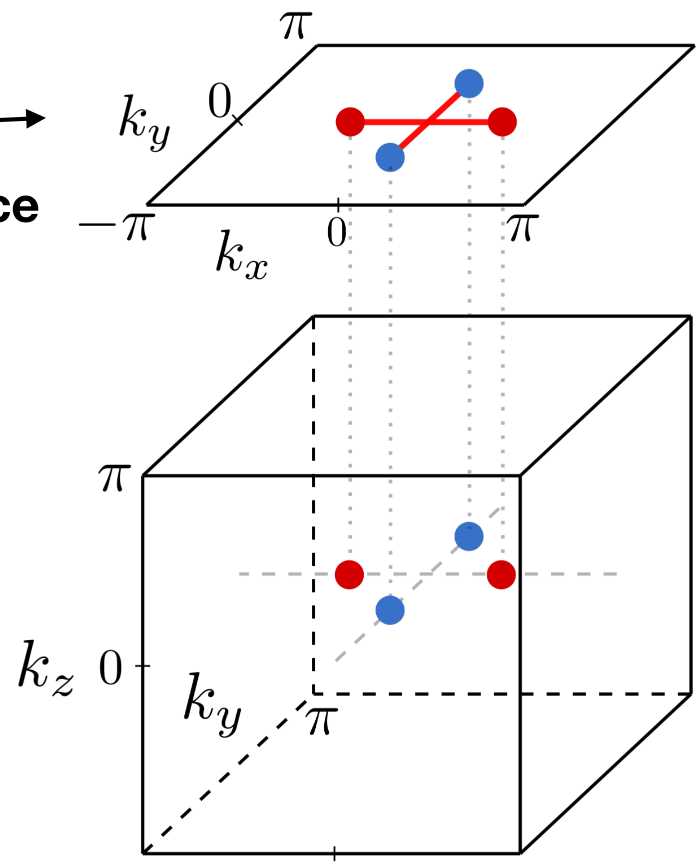
Cancelling the anomaly via the bulk

- This cannot possibly be a 2D lattice system; the Fermi surface is not closed!
- What can it be? The surface of a (quadrupolar) Weyl semi-metal!

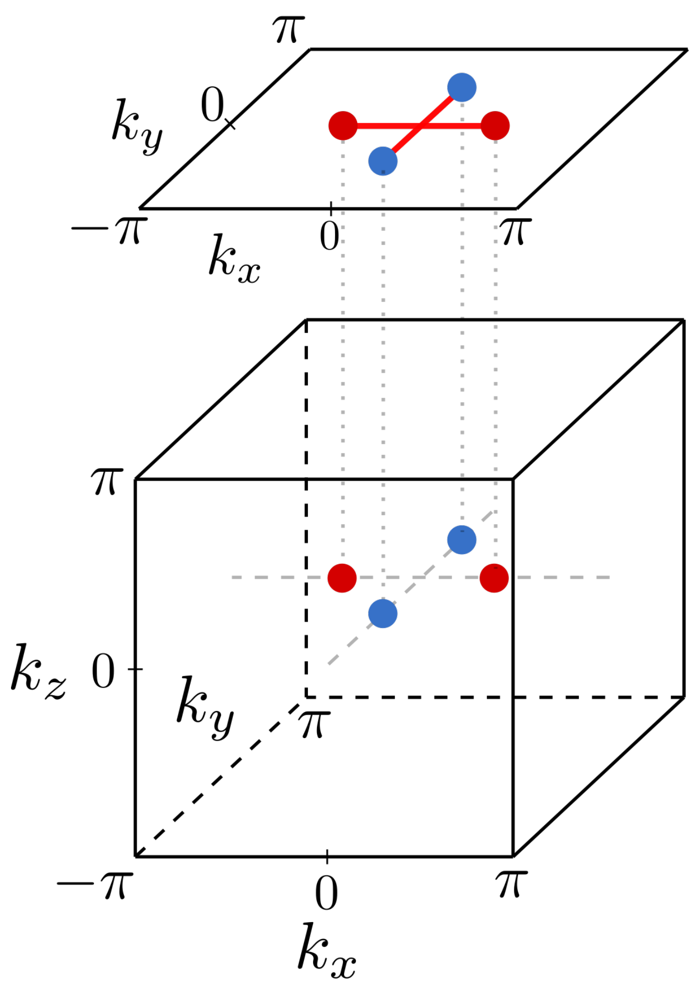


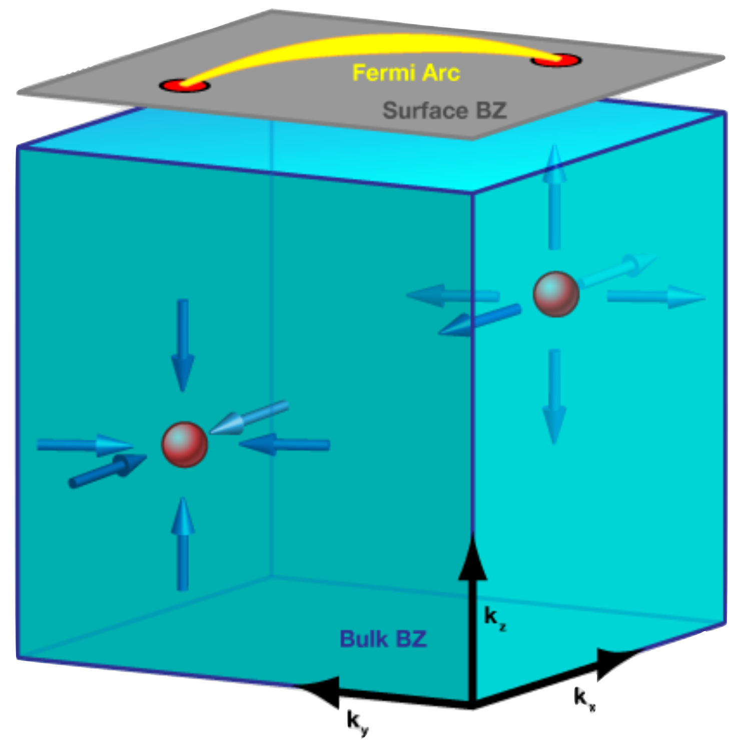
Anomalous surface

Bulk: Weyl quadrupole.



Part 2: Quadrupolar responses of semi-metals



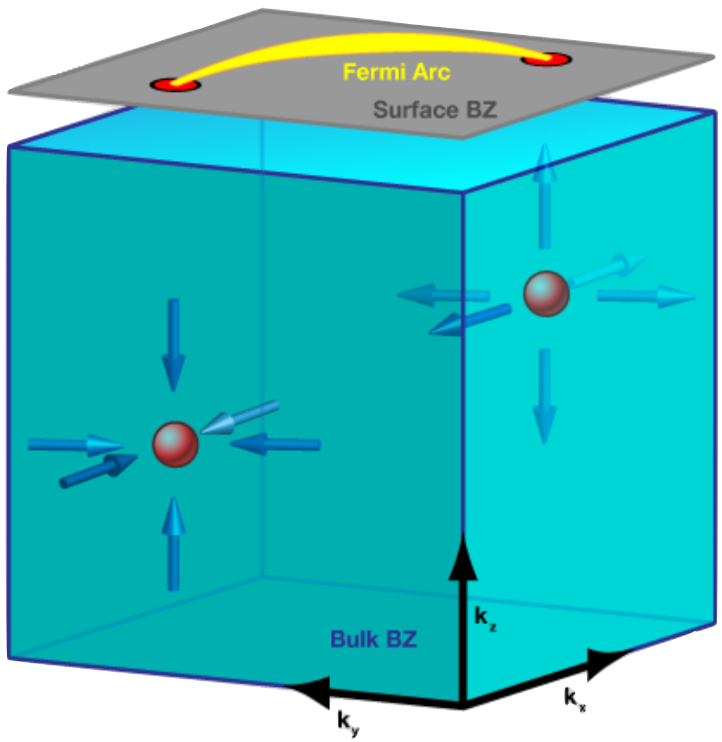


Review: dipolar response of Weyl semi-metals

- Fermi surface dipole moment:

$$P^\lambda = \sum_\alpha \chi^{(\alpha)} k_\alpha^\lambda$$

$\chi = -1$



$\chi = +1$

Review: dipolar response of Weyl semi-metals

Zyuzin & Burkov '12

- Fermi surface dipole moment determines EM response:

$$P^\lambda = \sum_\alpha \chi^{(\alpha)} k_\alpha^\lambda$$

Charge density:

$$\rho = \frac{e^2}{\pi h} \vec{P} \cdot \vec{B}$$

Current density:

$$\vec{j} = \frac{e^2}{\pi h} \left(\vec{P} \times \vec{E} - b_0 \vec{B} \right)$$

Review: dipolar response of Weyl semi-metals

Zyuzin & Burkov ’12

Topological response theory:

$$\mathcal{L} = -\frac{e^2}{2\pi h}\epsilon^{\mu\nu\rho\lambda}P_\mu(A_\nu\partial_\rho A_\lambda)$$

$\mathcal{L} - J_\mu A^\mu$ has equations of motion:

$$\rho = \frac{e^2}{\pi h}\vec{P}\cdot\vec{B}$$

$$\vec{j} = \frac{e^2}{\pi h}\left(\vec{P}\times\vec{E} - b_0\vec{B}\right)$$

Summary: dipolar response of Weyl semi-metals

Zyuzin & Burkov ’12

- Fermi surface dipole moment

$$P^{\lambda} = \sum_{\alpha} \chi^{(\alpha)} k_{\alpha}^{\lambda}$$

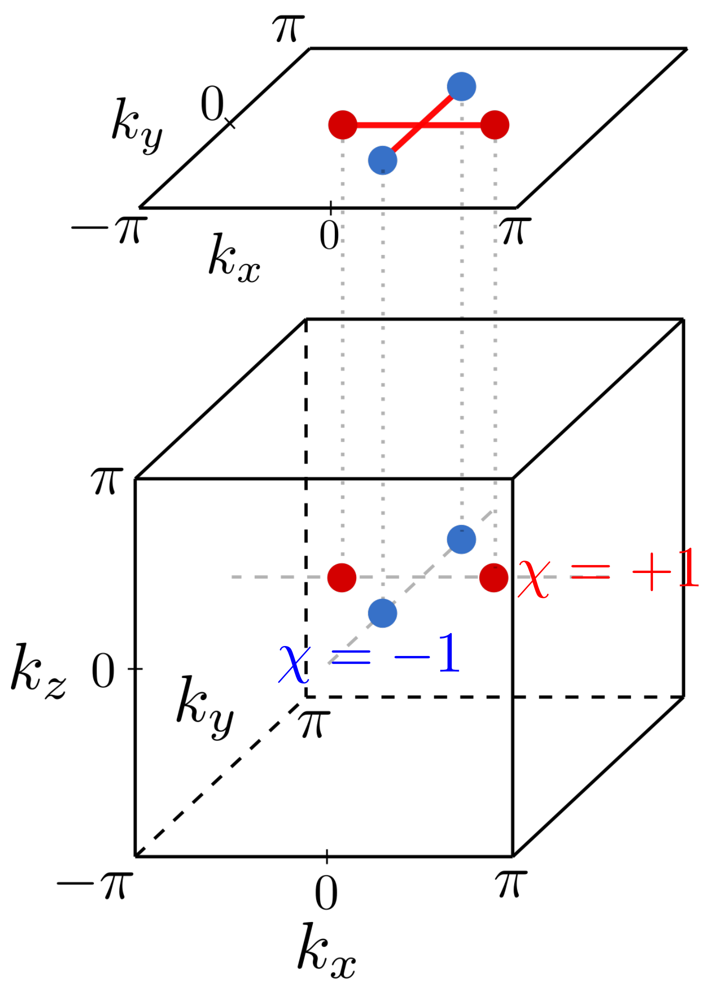
- determines EM response via:

$$\mathcal{L} = -\frac{e^2}{2\pi h} \epsilon^{\mu\nu\rho\lambda} P_{\mu} (A_{\nu} \partial_{\rho} A_{\lambda})$$

  - Note: not gauge invariant in the presence of a boundary.
  - Must add surface Fermi arc to ensure gauge invariance!

Quardupolar response

Dubinkin, FJB, Hughes ’21 (and forthcoming)
Gaoia, Wang, Burkov ‘21



- Time-reversal invariant Weyl semi-metal: dipolar response vanishes!

$$P^\lambda = \sum_\alpha \chi^{(\alpha)} k_\alpha^\lambda$$

$$= 0$$

- No net charge response to external fields. Contribution of time-reversed pairs of Weyl nodes cancels!

Quadrupolar response
(via Kubo formula)

Dubinkin, FJB, Hughes ’21

- Electromagnetic currents:

Current-current response vanishes (in clean system):

$$\langle J^\mu J^\nu \rangle = 0$$

Mixed current - momentum current response is non-vanishing:

$$\lim_{\omega\to 0} \langle \tau_a^\nu j^\mu \rangle = t_a^{\mu\nu}$$

Momentum current:

$$\tau_a^\nu(\mathbf{k}) = \frac{h}{e} k_a j^\nu(\mathbf{k})$$

Quadrupolar response
(via Kubo formula)

Dubinkin, FJB, Hughes '21

- Electromagnetic currents:

$$j^{\mu} = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda} Q_{\nu a}\partial_{\rho} e^{a}_{\lambda}$$

Quadrupole moment of Weyl nodes

Chirality of Weyl node

$$Q_{i\alpha} = \sum_{\alpha=\text{Weyl Nodes}} \chi(\alpha) K_i(\alpha) K_a(\alpha)$$

Quadrupolar response
(via Kubo formula)

Dubinkin, FJB, Hughes '21

- Electromagnetic currents:

$$j^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}Q_{\nu a}\partial_\rho e^a_\lambda$$

Gauge field for momentum current (gauged translational symmetry)

Quadrupolar response
(via Kubo formula)

Dubinkin, FJB, Hughes ’21

$$j^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda} Q_{\nu a}\partial_\rho e^a_\lambda$$

- No net electromagnetic response to E-M fields.
- But there are EM responses to lattice defects.
- E.g. charge bound to screw dislocations

Quadrupolar response
(via Kubo formula)

Dubinkin, FJB, Hughes '21

- Momentum currents:

$$\tau^{\mu}_{a} = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}Q_{\nu a}\partial_\rho A_\lambda$$

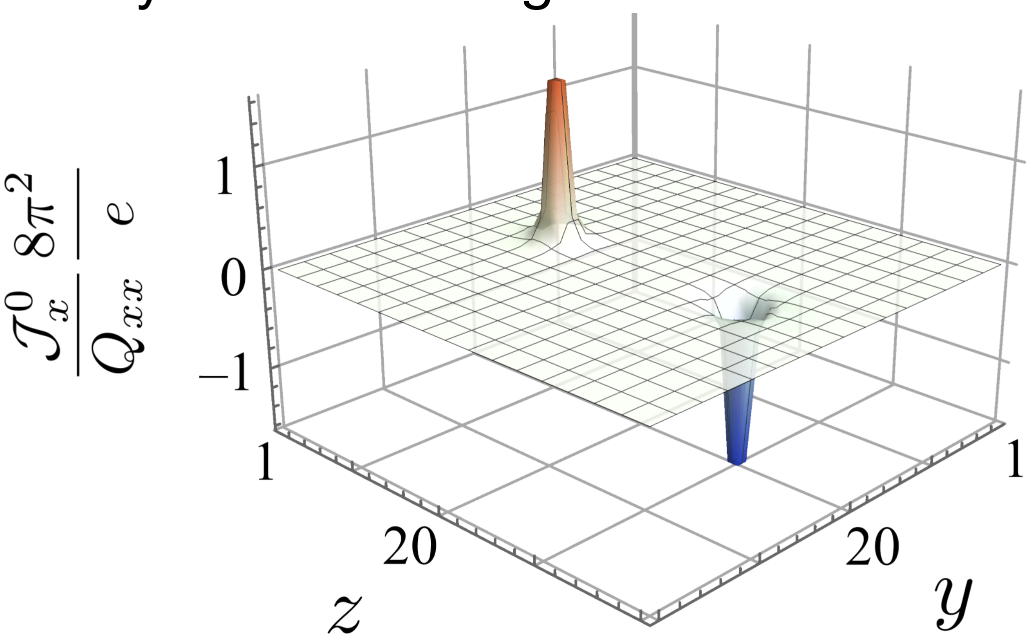
EM gauge field

Quadrupolar response
(via Kubo formula)

Dubinkin, FJB, Hughes '21

$$\tau_a^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}Q_{\nu a}\partial_\rho A_\lambda$$

- No net electromagnetic response to E-M fields. Instead, we see a momentum current response!
- E.g. momentum density bound to magnetic flux tubes!



Topological Response theory

$$\mathcal{L} = \frac{e}{8\pi^2}\epsilon^{a\mu\nu\rho}Q_{ab}A_\mu\partial_\nu e^b_\rho$$

$\mathcal{L} - J^\mu A_\mu - \tau^\mu_a e^a_\mu$ has equations of motion:

$$j^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}Q_{\nu a}\partial_\rho e^a_\lambda$$

$$\tau^\mu_a = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}Q_{\nu a}\partial_\rho A_\lambda$$

Topological Response theory: connection to higher-rank case

- Include a mirror symmetry that enforces

$$Q_{yy} = -Q_{xx} \equiv Q \ , \quad Q_{xy} = 0$$

- Obtain:

$$\mathcal{L} = -\frac{Q}{8\pi^2}\left[A_z\left(\varepsilon^y_x + \varepsilon^x_y\right) - A_y\varepsilon^x_z - A_x\varepsilon^y_z + A_0(\mathcal{B}^y_y - \mathcal{B}^x_x)\right]$$

$$\begin{aligned}\varepsilon^a_i =& \partial_i \mathbf{e}^a_0 - \partial_0 \mathbf{e}^a_i \\ \mathcal{B}^a_i =& \epsilon^{ijk}\partial_j \mathbf{e}^a_k\end{aligned}$$

Topological Response theory: connection to higher-rank case

$$\mathcal{L} = -\frac{Q}{8\pi^2}\left[A_z\left(\varepsilon_x^y + \varepsilon_y^x\right) - A_y\varepsilon_z^x - A_x\varepsilon_z^y + A_0(\mathcal{B}_y^y - \mathcal{B}_x^x)\right]$$

- Partial re-writing in terms of symmetric tensor gauge fields:

$$\mathcal{E}_{xy} \to \varepsilon_y^x + \varepsilon_x^y$$
$$e_{xy} \to e_y^x + e_x^y$$

Symmetric tensor gauge theory coupled to rank 1 gauge field

$$\mathcal{L} = -\frac{Q}{8\pi^2}\left[A_z\varepsilon_{xy} - A_0\partial_z e_{xy}\right.$$
$$\left. - A_y\varepsilon_z^x - A_x\varepsilon_z^y + A_0(\partial_y e_z^x + \partial_x e_z^y)\right]$$

The remaining terms

Topological Response theory: connection to higher-rank case

Symmetric tensor gauge theory coupled to rank 1 gauge field

$$\mathcal{L} = -\frac{Q}{8\pi^2}\left[A_z\varepsilon_{xy} - A_0\partial_z e_{xy}\right.$$
$$\left. -A_y\varepsilon^x_z - A_x\varepsilon^y_z + A_0(\partial_y e^x_z + \partial_x e^y_z)\right]$$

- Gauge-invariant up to a boundary term
- For a boundary orthogonal to z, only the first line contributes to this anomaly

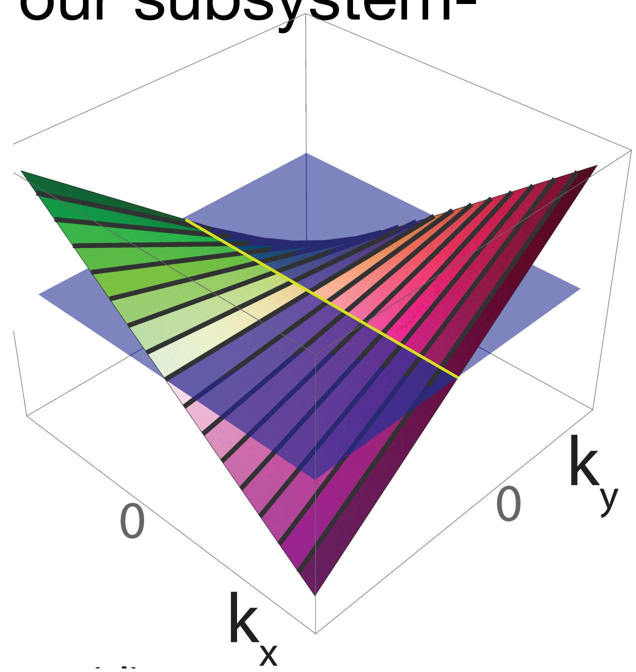
Topological Response theory: connection to higher-rank case

Symmetric tensor gauge theory coupled to rank 1 gauge field

$$\mathcal{L} = -\frac{Q}{8\pi^2}\left[A_z \varepsilon_{xy} - A_0 \partial_z e_{xy}\right]$$

- Anomaly inflow to the boundary is exactly what is needed to cancel the mixed ’t Hooft anomaly of our subsystem-symmetric scalar field theory

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 - (\partial_x \partial_y \phi)^2\right]$$



FJB, Devakul, Lam, Shao (to appear)

Quadrupolar response in 3D Weyl semi-metals: some morals

- Leading order linear EM response of semi-metals= Fermi surface dipole moment

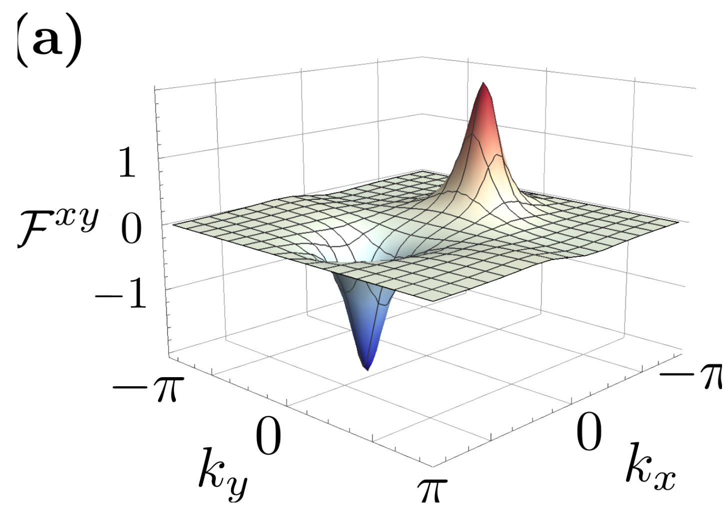
- When this vanishes, we instead obtain mixed momentum-charge responses, proportional to the Fermi surface quadrupole moment

- These responses can have boundary anomalies, indicating surface states that resemble those encountered in the study of higher-rank gauge theories

Quadrupolar responses in other systems

- 2D Dirac semi-metals: Recall Berry curvature dipole

$$\mathcal{L} = \frac{e}{8\pi}\epsilon^{\mu\nu\rho}P_\mu F_{\nu\rho}$$



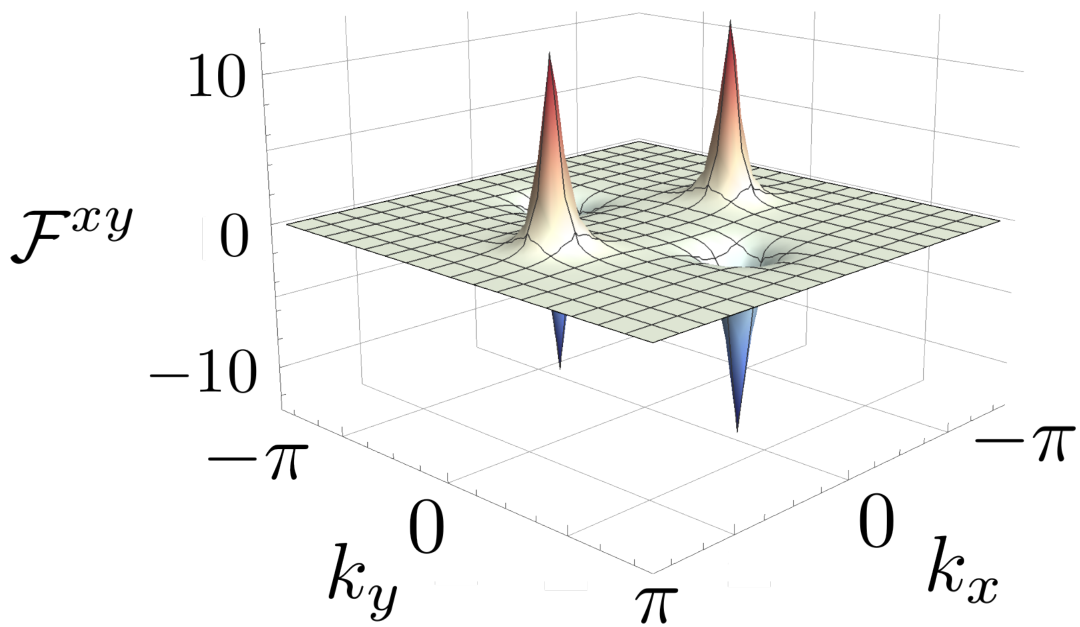
- Equations of motion predict boundary flat bands, along edges orthogonal to the dipole moment

$$\rho = -\frac{e}{4\pi}\epsilon^{ij}\partial_i P_j$$

Ramamurthy, Hughes ‘15

Quadrupolar responses in other systems

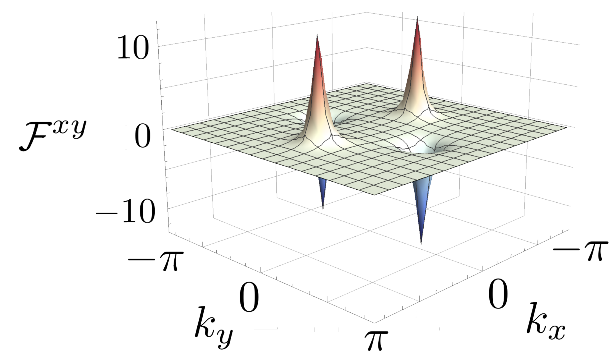
- 2D gapped systems: Berry curvature quadrupole



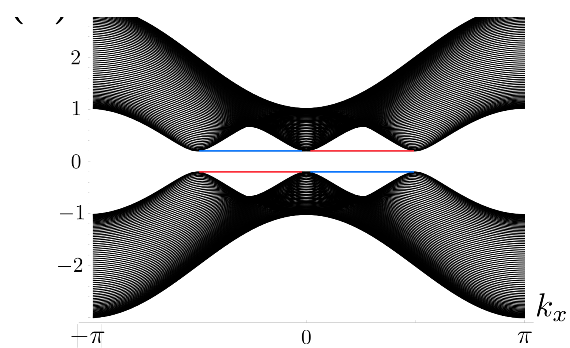
Quadrupolar responses in other systems

- 2D gapped systems: Berry curvature quadrupole

$$\mathcal{L} = \frac{\hbar}{16\pi}\epsilon^{\mu\nu\rho}Q_{\mu\sigma}\partial_\nu e^\sigma_\rho$$



- Boundary flat bands, with no net charge response, but a net momentum response



Dubinkin, FJB, Hughes

Summary

- Part 1: Chirality, the axial anomaly, and a higher-rank analog
  - A new kind of "chiral anomaly" associated with lack of momentum conservation in 2D
  - Realization in band theory, after imposing reflection symmetry
- Part 2: Quadrupolar responses of semi-metals
  - With extra symmetry, leading-order (dipolar) terms in EM response may vanish.
  - Next leading terms are quadrupolar, and describe a mixed response between EM and lattice effects
  - With appropriate symmetry, 3D response theory has the same anomalies as a symmetric tensor gauge theory, which cancels the anomaly of a subsystem-symmetric surface state.
  - Quadrupolar responses not only interesting for Weyl semi-metals