Discrete Abelian lattice gauge theories in a ladder geometry

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2 September 2021

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Jens Nyhegn, Chia-Min Chung and M.B., Phys. Rev. Research 3 (2021)

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Jens Nyhegn, Chia-Min Chung and M.B., Phys. Rev. Research 3 (2021)

Quantum simulations

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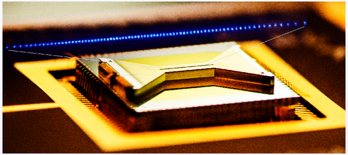
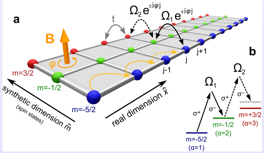


IonQ, Maryland

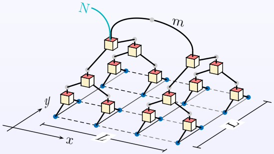
Ultracold atoms



Fallani group, Firenze

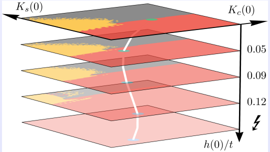
Quantum information tools

Tensor networks



Montangero group, Padova

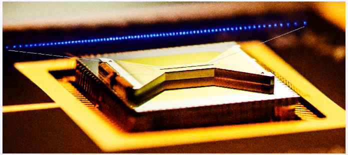
CMT tools



Quantum technologies to tackle many-body physics

Quantum simulations

Ion traps



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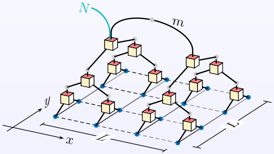
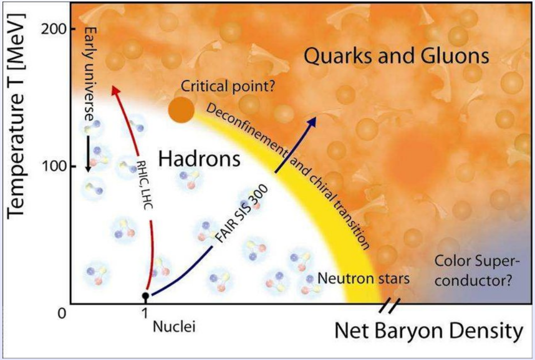


Montangero group, Padova

Most of these techniques require to work with discrete degrees of freedom!

Gauge theories

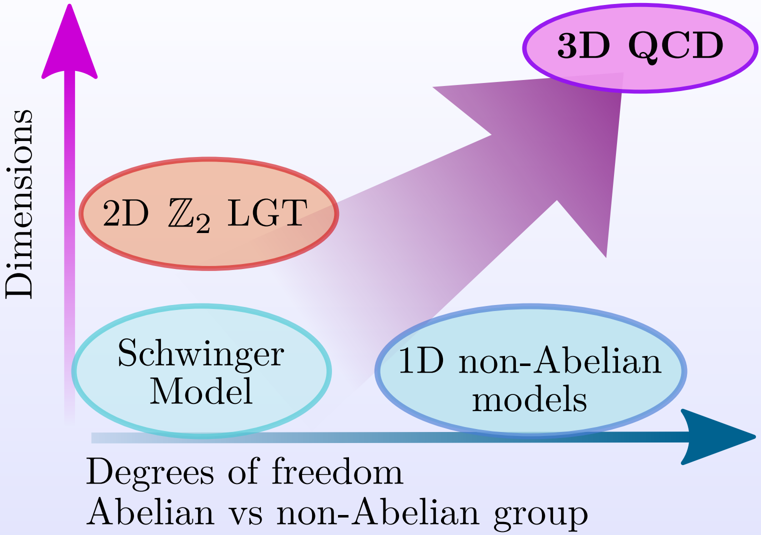
High-energy physics



Emergent gauge theories

- Spin liquids
- Resonating valence bond states for exotic superconductors
- Quantum antiferromagnets
- Topological self-correcting quantum memories

A path of increasing complexity



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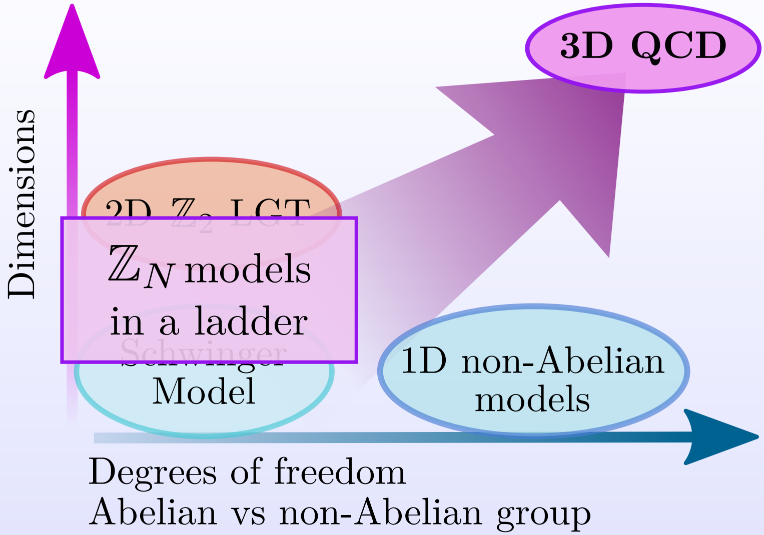
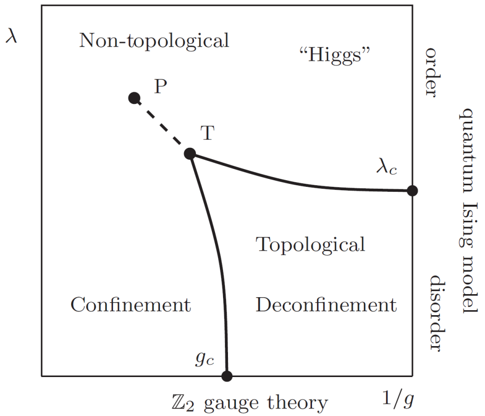
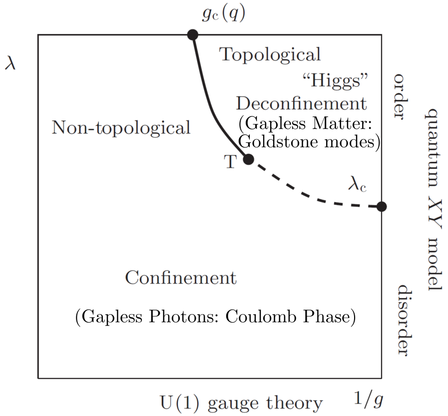


$\mathbb{Z}_N$ vs $U(1)$ Lattice Gauge Theory in 2+1D

Fradkin and Shenker (1979)





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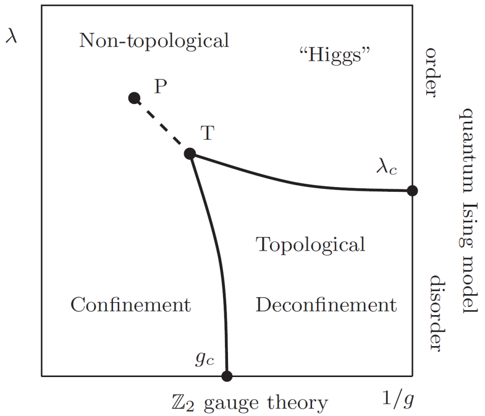
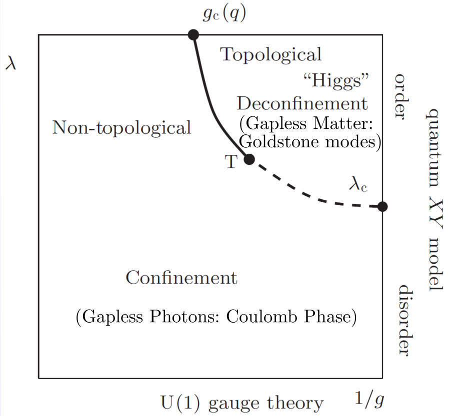




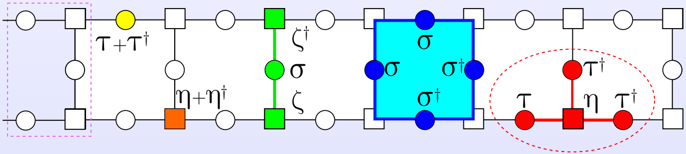
How do the gapless phase emerge from the gapped $\mathbb{Z}_N$ model to the $U(1)$ theory?

Gauge fields:

$\sigma = e^{iA}$: Connection U

$\tau = e^{i\frac{2\pi}{N}E}$: Electric field

$\sigma\tau = e^{i\frac{2\pi}{N}}\tau\sigma$



$\mathbb{Z}_N$ LGT: the model

Gauge fields:

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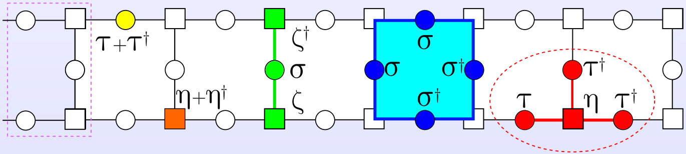
Higgs bosonic matter:

$\zeta, \zeta^\dagger$: Annihilation and creation

$\eta = e^{i\frac{2\pi}{N}q}$: Electric charge

$\zeta\eta = e^{i\frac{2\pi}{N}}\eta\zeta$



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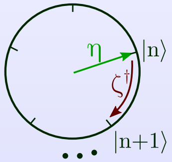
Electric basis (clock operators)

$$\tau,\,\eta = \begin{pmatrix} e^{i\frac{2\pi}{N}} & 0 & \dots & 0 \\ 0 & e^{i\frac{4\pi}{N}} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix}, \qquad \sigma,\,\zeta = \begin{pmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$



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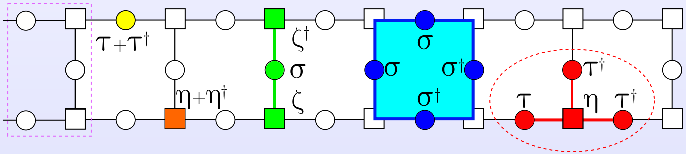
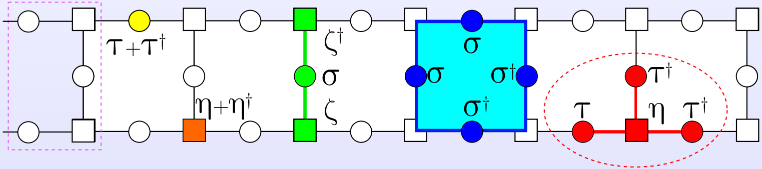
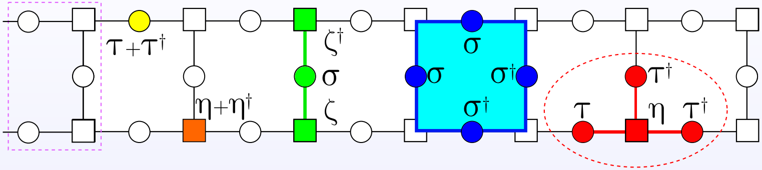


$$H = -\frac{1}{g}\sum_{r=1}^{L-1}\left(\sigma_{r,0}\sigma_{r+1,\uparrow}\sigma^\dagger_{r+1,0}\sigma^\dagger_{r+1,\downarrow} + \text{H.c.}\right) - g\sum_{s=\uparrow,\downarrow,0}\sum_{r=1}^{L}\left(\tau_{r,s}+\tau^\dagger_{r,s}\right)$$

$$-\frac{1}{\lambda}\sum_{s=\uparrow,\downarrow}\sum_{r=1}^{L}\left(\eta_{r,s}+\eta^\dagger_{r,s}\right) - \lambda\left[\sum_{s=\uparrow,\downarrow}\sum_{r=1}^{L-1}\zeta^\dagger_{r,s}\sigma^\dagger_{r+1,s}\zeta_{r+1,s} + \sum_{r=1}^{L}\zeta^\dagger_{r,\uparrow}\sigma_{r,0}\zeta_{r,\downarrow} + \text{H.c.}\right]$$



Description of the interactions



Interactions:

$$\sigma_{r,0}\sigma_{r+1,\uparrow}\sigma^\dagger_{r+1,0}\sigma^\dagger_{r+1,\downarrow} \rightarrow e^{i\oint A dl}$$

$$\zeta^\dagger_{r+1,s}\sigma_{r+1,s}\zeta_{r,s} \rightarrow \psi^\dagger(r+1)e^{i\int_r^{r+1} A dl}\psi(r)$$

$$-(\tau+\tau^\dagger) \rightarrow E^2$$

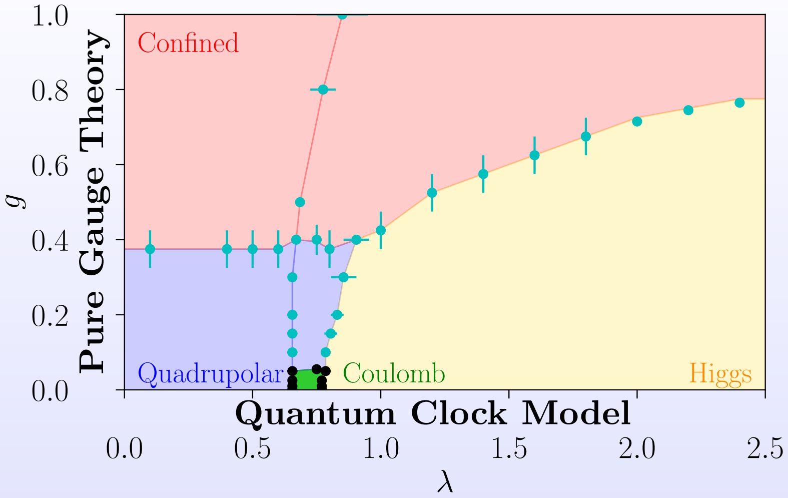
$$-(\eta+\eta^\dagger) \rightarrow \text{mass term}$$

Gauge constraint:

$$\tau_{r,\downarrow}\tau^\dagger_{r,0}\tau^\dagger_{r+1,\downarrow}\eta_{r,\downarrow}|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$\exp\left[i\frac{2\pi}{N}(\text{div}E - q)\right]|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

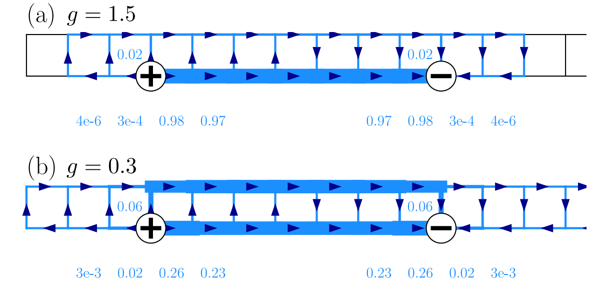
Overview of the phase diagram



The pure gauge theory limit ($\lambda \to 0$)

Single confined/disordered phase (dual to clock model in transverse and longitudinal field)

$$H_{\text{axial gauge}} = -\frac{1}{g}\sum_{r=1}^{L-1}\left(\sigma_r\sigma_{r+1}^\dagger + \text{H.c.}\right) - g\sum_{r=1}^{L}\left(\tau_r + \tau_r^\dagger\right) - 2g\sum_{r=1}^{L}\left[\prod_{j=r}^{L}\tau_j + \prod_{j=r}^{L}\tau_j^\dagger\right]$$



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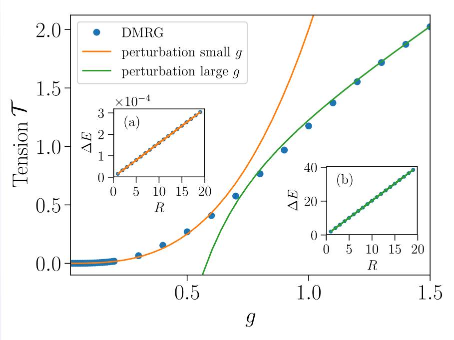

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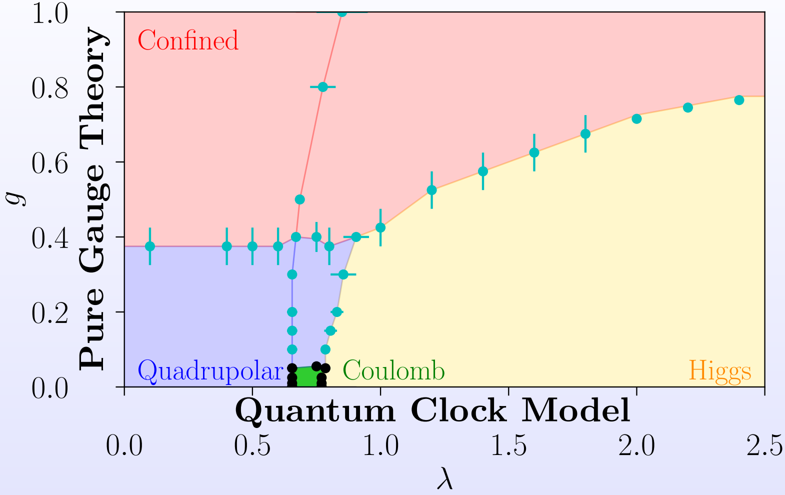
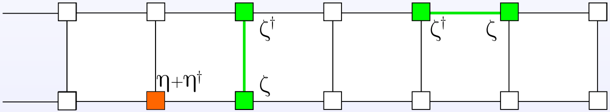
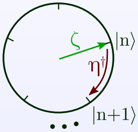


The quantum clock limit ($g \to 0$) in the ladder

Axial gauge

$$H_{\text{axial}} = -\lambda \left[\sum_{s,r} \zeta^\dagger_{r,s} \zeta_{r+1,s} + \sum_r \zeta^\dagger_{r,\uparrow} \zeta_{r,\downarrow} + \text{H.c.} \right] - \frac{1}{\lambda} \sum_{r,s} \left(\eta_{r,s} + \eta^\dagger_{r,s} \right)$$

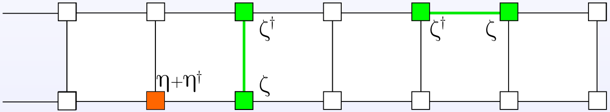


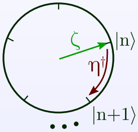


Clock model in the chain:

- $N = 2$: Ising with ordered and disordered phase
- $N = 3$: Potts with ordered and disordered phase
- $N = 4$: Ashkin-Teller with ordered and disordered phase

$$H_{\text{axial}} = -\lambda \left[\sum_{s,r} \zeta^\dagger_{r,s}\zeta_{r+1,s} + \sum_r \zeta^\dagger_{r,\uparrow}\zeta_{r,\downarrow} + \text{H.c.} \right] - \frac{1}{\lambda} \sum_{r,s} \left(\eta_{r,s} + \eta^\dagger_{r,s} \right)$$





Clock model in the chain:

- $N = 2$: Ising with ordered and disordered phase
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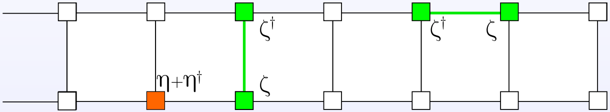
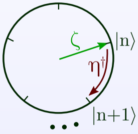
- $N \geq 5$: A critical $c = 1$ (Luttinger) phase emerges
  BKT phase transitions between
  ordered-Luttinger-disordered phases

The quantum clock limit ($g \to 0$) in the ladder

Axial gauge

$$H_{\text{axial}} = -\lambda \left[\sum_{s,r} \zeta^\dagger_{r,s}\zeta_{r+1,s} + \sum_r \zeta^\dagger_{r,\uparrow}\zeta_{r,\downarrow} + \text{H.c.} \right] - \frac{1}{\lambda}\sum_{r,s}\left(\eta_{r,s} + \eta^\dagger_{r,s}\right)$$





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Clock model in the ladder:

DMRG and Bosonization
(second order RG)

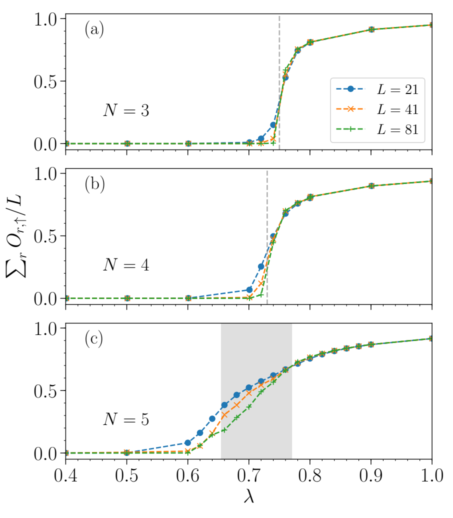
$$H_{\text{axial}} = -\lambda \left[\sum_{s,r} \zeta^\dagger_{r,s}\zeta_{r+1,s} + \sum_{r} \zeta^\dagger_{r,\uparrow}\zeta_{r,\downarrow} + \text{H.c.} \right] - \frac{1}{\lambda}\sum_{r,s}\left(\eta_{r,s} + \eta^\dagger_{r,s}\right)$$

Order parameter:

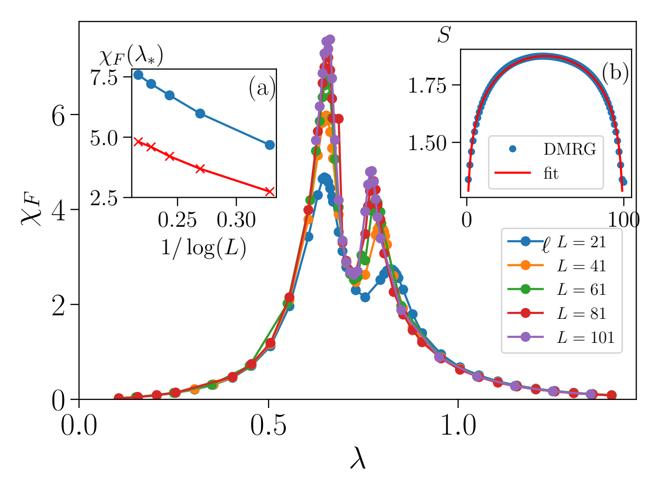
$$\text{“}\langle\zeta\rangle\text{”} \to \; O_{r,s} = \left\langle \prod_{j=1}^{r} \sigma^\dagger_{j,s}\zeta_{r,s} \right\rangle$$

Phases:

- Small λ: disordered / confined
- Large λ: ordered / Higgs
- Intermediate ($N \geq 5$): gapless / Coulomb



Signatures of the gapless phase and BKT transitions at $g = 0$ for $N = 5$



Fidelity susceptibility:

$$\chi_F = \lim_{\delta\Lambda \to 0} \frac{-2 \log |\langle \psi_0(\Lambda) | \psi_0(\Lambda + \delta\Lambda) \rangle|}{3L(\delta\Lambda)^2}$$

Calabrese-Cardy formula:

$$S_\ell = \frac{c}{6} \log \left(\frac{2L}{\pi} \sin \frac{\pi \ell}{L} \right) + \frac{c_\alpha}{2}$$

$c = 1$ in the gapless phase.

Bosonization

- Bosonic fields:

$$[\theta_s(x), \varphi_{s'}(x')] = -i\frac{2\pi}{N}\Theta\left(x - x'\right)\delta_{ss'} \,,$$

- Mapping:

$$\zeta_{j,s} \to \mathrm{e}^{-i\theta_s(ja)} \,, \quad \eta_{j,s} \to \mathrm{e}^{-i\varphi_s(ja)+i\varphi_s(ja+a)}$$

$$\sigma_{j,0} \to \mathrm{e}^{-i\theta_0(ja)} \,, \quad \tau_{j,0} \to \mathrm{e}^{-i\varphi_0(ja)+i\varphi_0(ja+a)}$$

$\varphi_{\uparrow,\downarrow}$: electric fields.

- Bare Luttinger parameters and velocity:

$$K_\uparrow = K_\downarrow = \frac{1}{\lambda} \,, \quad K_0 = g \,, \quad v = \frac{4\pi a}{N}$$

Hamiltonian

$$H = \frac{N}{4\pi}\int \mathrm{d}x \sum_{s=0,\uparrow,\downarrow} v\left[K_s\left(\partial_x\varphi_s\right)^2 + \frac{1}{K_s}\left(\partial_x\theta_s\right)^2\right]$$
$$- \frac{2\lambda}{a}\int \mathrm{d}x\, \cos\left(\theta_\uparrow - \theta_\downarrow - \theta_0\right)$$
$$- \frac{2g}{a}\int \mathrm{d}x\, \left[\cos\left(\varphi_\uparrow + \varphi_0\right) + \cos\left(\varphi_\downarrow - \varphi_0\right)\right]$$

$U(1)$ symmetries

Example: $\theta_\uparrow \to \theta_\uparrow + \alpha \,, \quad \theta_\downarrow \to \theta_\downarrow + \alpha$

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$$- \frac{2\lambda}{a}\int \mathrm{d}x\, \cos\left(\theta_\uparrow - \theta_\downarrow - \theta_0\right)$$
$$- \frac{2g}{a}\int \mathrm{d}x\, \left[\cos\left(\varphi_\uparrow + \varphi_0\right) + \cos\left(\varphi_\downarrow - \varphi_0\right)\right]$$
$$- \sum_{s=0,\uparrow,\downarrow}\int \mathrm{d}x\, \left[P_s \cos N\theta_s + Q_s \cos N\varphi_s\right]$$

$\mathbb{Z}_N$ symmetries

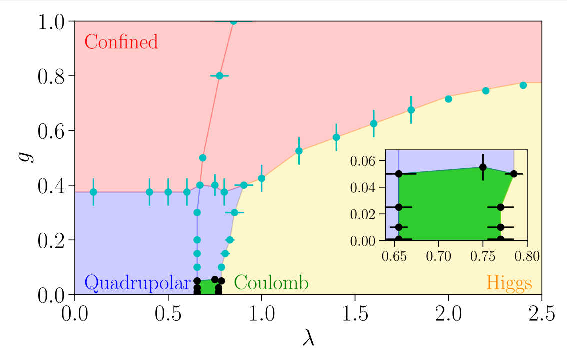
Example: $\theta_\uparrow \to \theta_\uparrow + \frac{2\pi}{N}\,, \quad \theta_\downarrow \to \theta_\downarrow + \frac{2\pi}{N}$

θ vacua: $\varphi_s \to \varphi_s + \frac{2\pi}{N}$

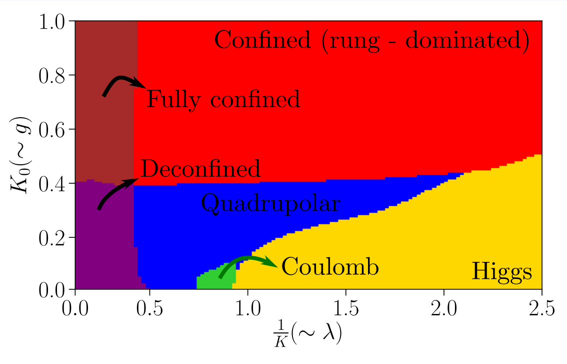
RG-analysis

Second order analysis of the $\mathbb{Z}_5$ model

DMRG



Bosonization and RG



Summary of the main properties of the phases at $N = 5$ for $g \to 0$

Spin ($\sigma = \uparrow - \downarrow$) sector. Charge ($\rho = \uparrow + \downarrow$) sector

Observables:

$$M_s(x,y) \equiv \zeta_{x,s}\left(\prod_{x\le j<y} \sigma_{j,s}\right)\zeta^\dagger_{y,s} \to \mathrm{e}^{i[\theta_s(y)-\theta_s(x)]}$$

$$M_\sigma(x,y) \equiv M_\uparrow(x,y)M^\dagger_\downarrow(x,y) \to \mathrm{e}^{i\sqrt{2}[\theta_\sigma(y)-\theta_\sigma(x)]}$$

$$R(x,y) = \zeta^\dagger_{x,\uparrow}\sigma_{x,0}\zeta_{x,\downarrow}\zeta_{y,\uparrow}\sigma^\dagger_{y,0}\zeta^\dagger_{y,\downarrow}$$
$$\to \mathrm{e}^{i\left(\sqrt{2}\theta_\sigma-\theta_0\right)(x)-i\left(\sqrt{2}\theta_\sigma-\theta_0\right)(y)}$$

Quadrupolar ($\lambda \lesssim 0.65$):

- Disordered phase $\mathcal{O} = 0$
- Mesons M_s decay exponentially
- M_σ is constant
- Ordered rungs (R constant)

Coulomb:

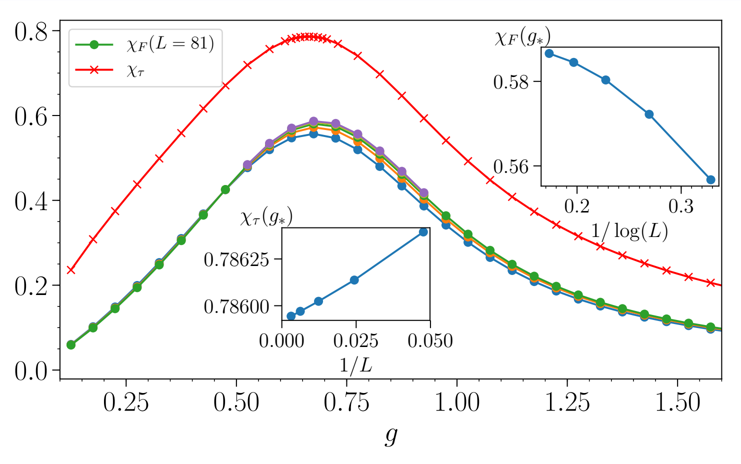
- Gapless charge sector
- $\mathcal{O}$ decays algebraically
- Mesons decay algebraically
- M_σ is constant
- Ordered rungs (R constant)

Higgs ($\lambda \gtrsim 0.77$):

- ζ are ordered, $\mathcal{O} \neq 0$
- All mesons are constant
- Ordered rungs (R constant)

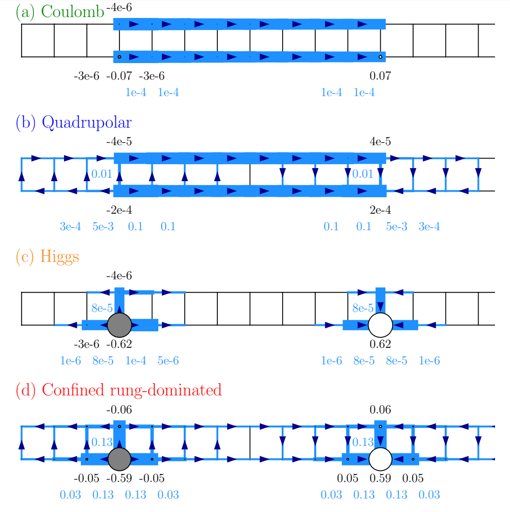
Higgs-confined crossover

Kertész line

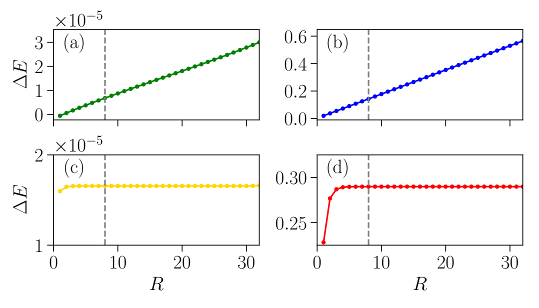


Maxima of the fidelity and electric susceptibility: Kertész line ($\lambda = 1.8$)

Screening properties

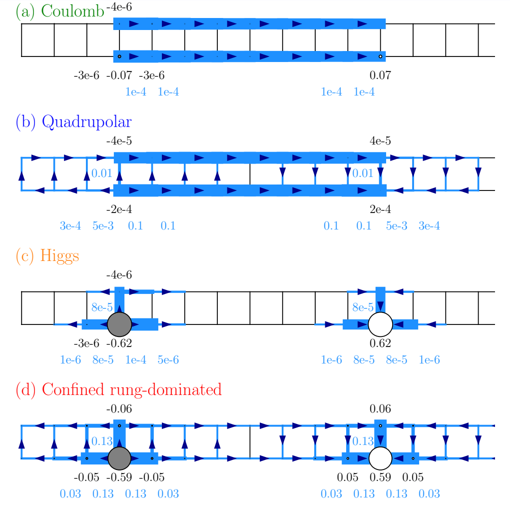


Energy of a pair of static charges:

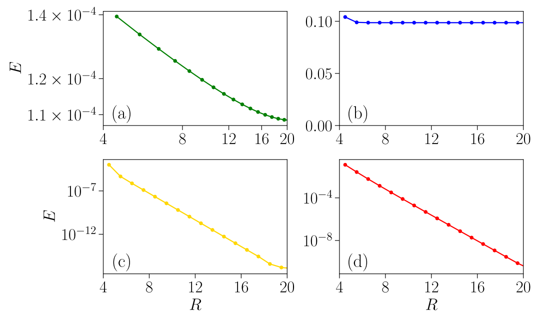


Confined and Higgs phases: strong screening

Screening properties



Electric field decay from the static charge

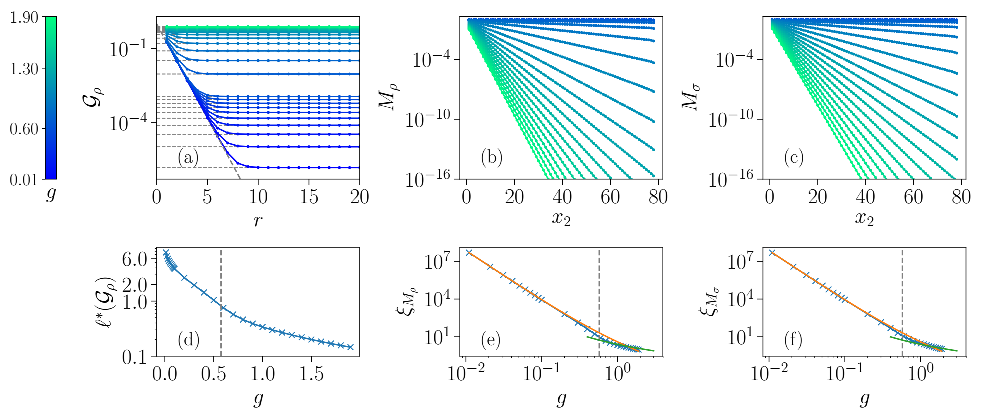


- Coulomb: power law
- Quadrupolar: not screened
- Confined and Higgs: exponential

Additional observables: electric field $\mathcal{G}_\rho$ and mesons

Higgs - confined crossover

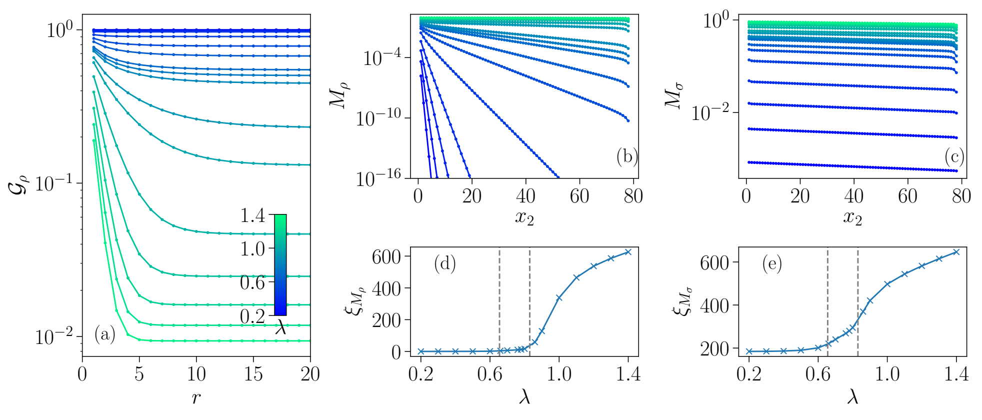
Behavior at $\lambda = 1.4$: crossover from Higgs to confined phase



Additional observables: electric field $\mathcal{G}_\rho$ and mesons

Quadrupolar - Higgs crossover

Behavior at $g = 0.2$: crossover from quadrupolar to Higgs phase



Conclusions and outlook

Jens Nyhegn, Chia-Min Chung and M.B., Phys. Rev. Research 3 (2021)

- Ladder models provide the simplest geometry with plaquettes for the analysis of LGTs
- We developed a bosonization technique to model systems with $\mathbb{Z}_N$ symmetries and predict the behavior of the main observables
- The study of $\mathbb{Z}_N$ models displays the emergence of a gapless Coulomb phase for $N \geq 5$ and extends the quantum clock models

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- We developed a bosonization technique to model systems with $\mathbb{Z}_N$ symmetries and predict the behavior of the main observables
- The study of $\mathbb{Z}_N$ models displays the emergence of a gapless Coulomb phase for $N \geq 5$ and extends the quantum clock models
- The extension to 2+1D is non-trivial: no gapless phases. The background interactions are always “dangerously irrelevant” (Oshikawa, Sandvik)
- Tensor network techniques may be extended to the 2D case (displaying topological order)

Coulomb phase at $N = 5$

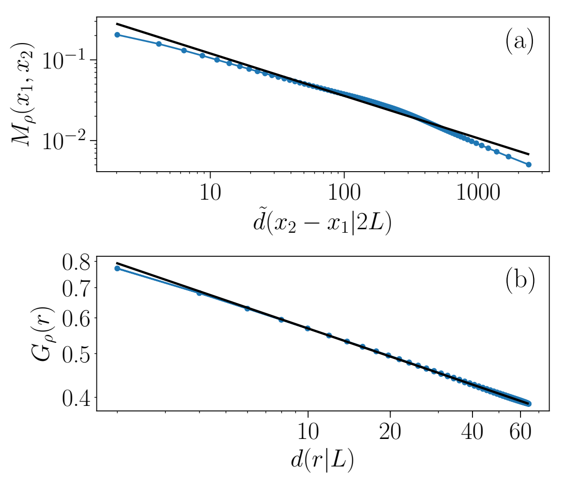
Charge sector; $L = 101,\ g = 0.001\ , \lambda = 0.75$

Observables:

$$\mathcal{G}_\rho(r) = \tau_{r,\uparrow}\tau_{r,\downarrow} \to \mathrm{e}^{i\sqrt{2}\varphi_\rho(r)}$$

$$M_\rho(x,y) \equiv M_\uparrow(x,y)M_\downarrow(x,y) \to \mathrm{e}^{i\sqrt{2}[\theta_\rho(y)-\theta_\rho(x)]}$$

The choice of the chord distance depends on the boundaries.



Higher values of N

Second order RG predictions