EFFECTIVE ACTION FOR ANISOTROPIC QUANTUM HALL STATES

Carlos Hoyos
Universidad de Oviedo

GGI Workshop “Topological properties of gauge theories”
September 3, 2021

C.H., R. Lier, F. Peña-Benitez, P. Surowka 2006.14595

QUANTUM HALL TRANSPORT

QUANTUM HALL STATE: *gapped, topologically non-trivial*

Hall transport (P,T breaking) fixed by topological properties of the state

Transport coefficients related to topological terms in effective action

Hall Conductivity (filling fraction): gauge Chern-Simons $\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$

Hall Viscosity (shift)

Non-relativistic (Galilean): Wen-Zee $\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu\omega_\lambda$

Relativistic: Euler current

Golkar, Roberts, Son 1403.4279

Hall thermal conductivity (chiral central charge): gravitational Chern-Simons $\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu\omega_\lambda$

EFFECTIVE FIELD THEORIES

$$\text{LATTICE} \xrightarrow{\text{Expand close to Fermi energy}} \text{EFFECTIVE FERMION MODEL} \xrightarrow{\text{Integrate out fermions}} \text{EFFECTIVE THEORY}$$

EFFECTIVE FIELD THEORIES

LATTICE

$\downarrow$ Expand close to Fermi energy

EFFECTIVE FERMION MODEL

Usually rotational invariance is assumed

$\downarrow$ Integrate out fermions

EFFECTIVE THEORY

Breaking of rotational invariance:

- Additional transport coefficients
- Non-universal contributions

EFFECTIVE FIELD THEORIES

LATTICE

↓ Expand close to Fermi energy

EFFECTIVE FERMION MODEL

↓ Integrate out fermions

EFFECTIVE THEORY

Usually rotational invariance
is assumed

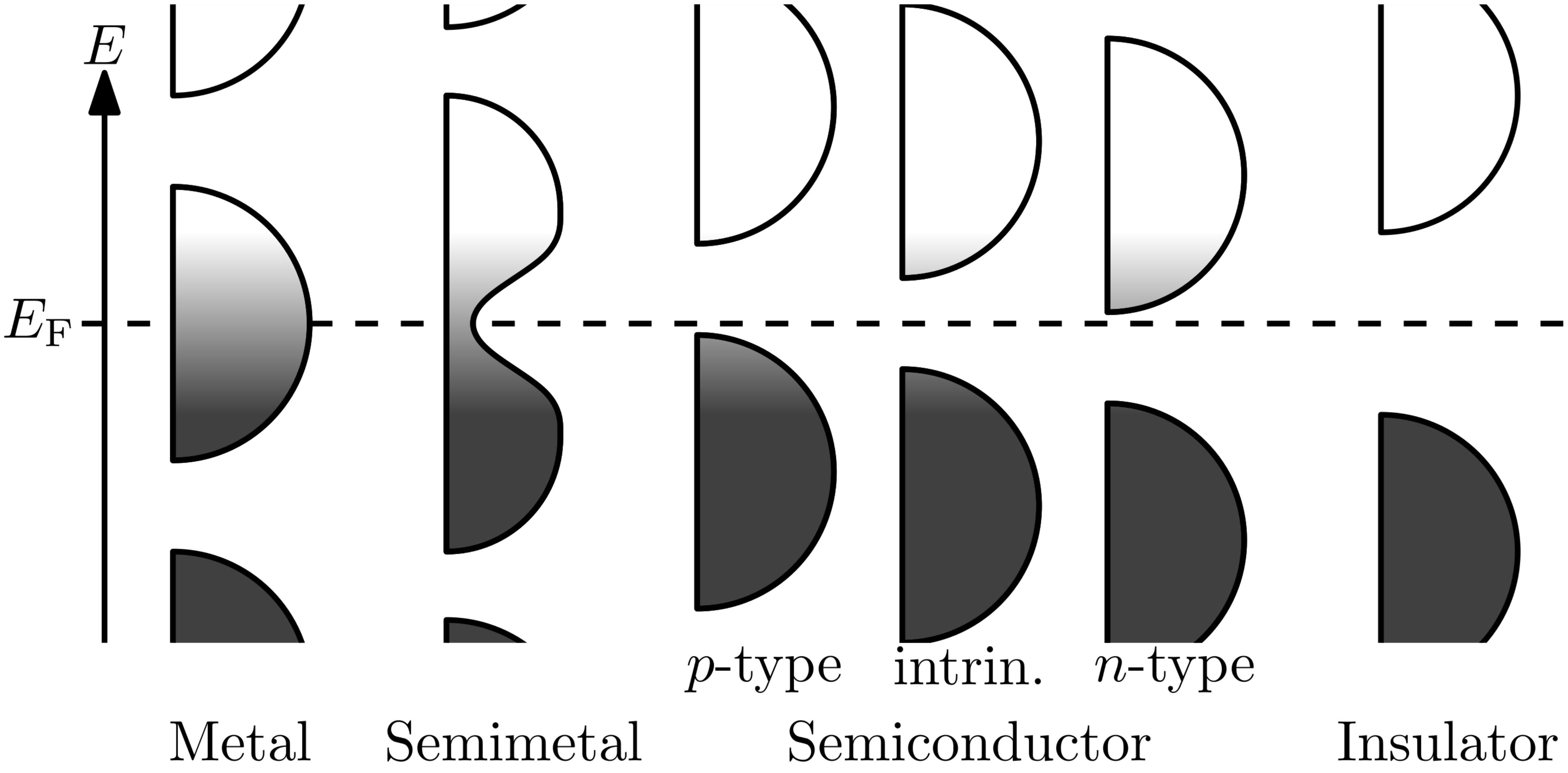
Breaking of rotational invariance:

-Additional transport coefficients

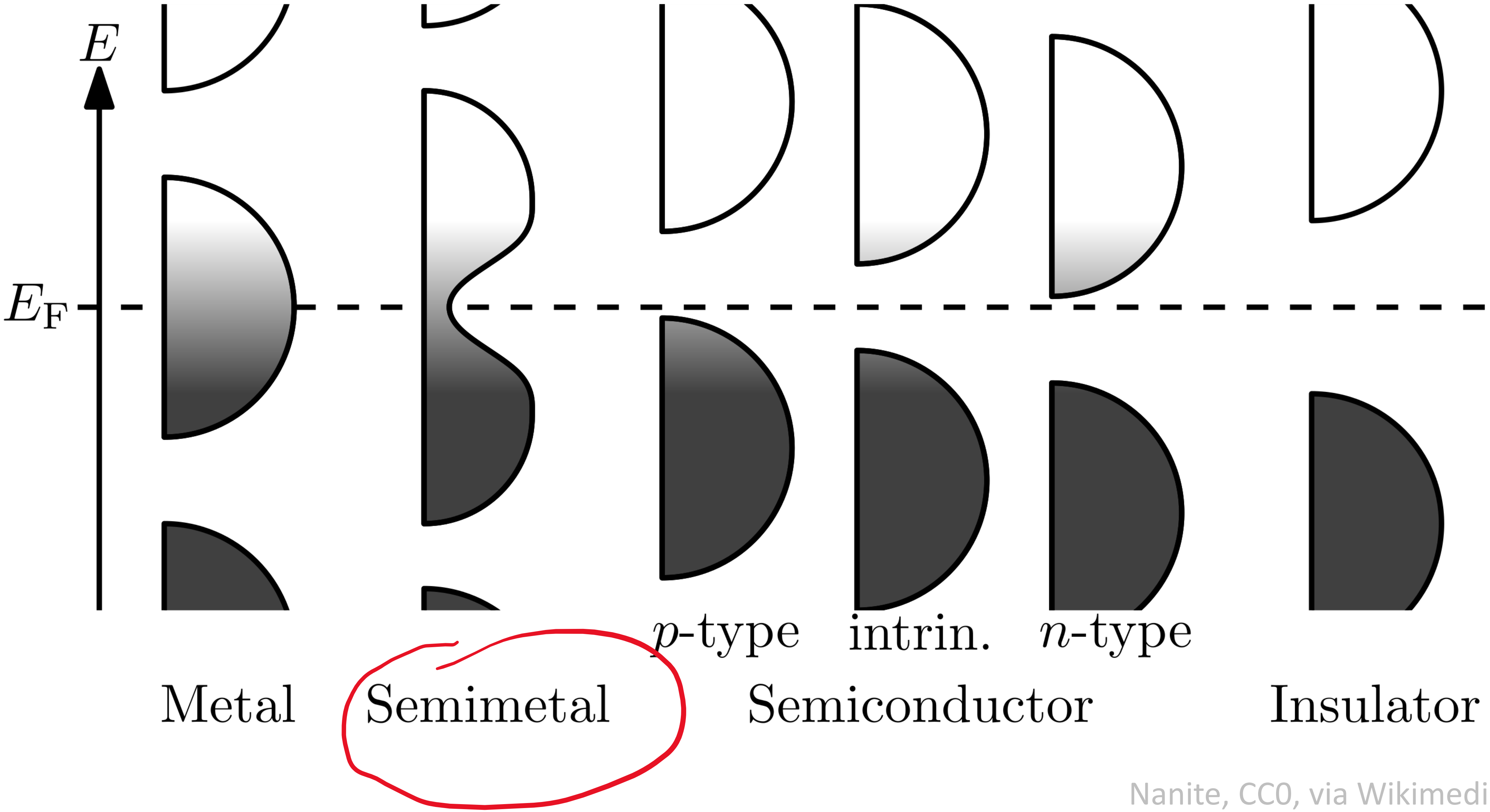
-Non-universal contributions

LABORATORY: semi-Dirac semimetal

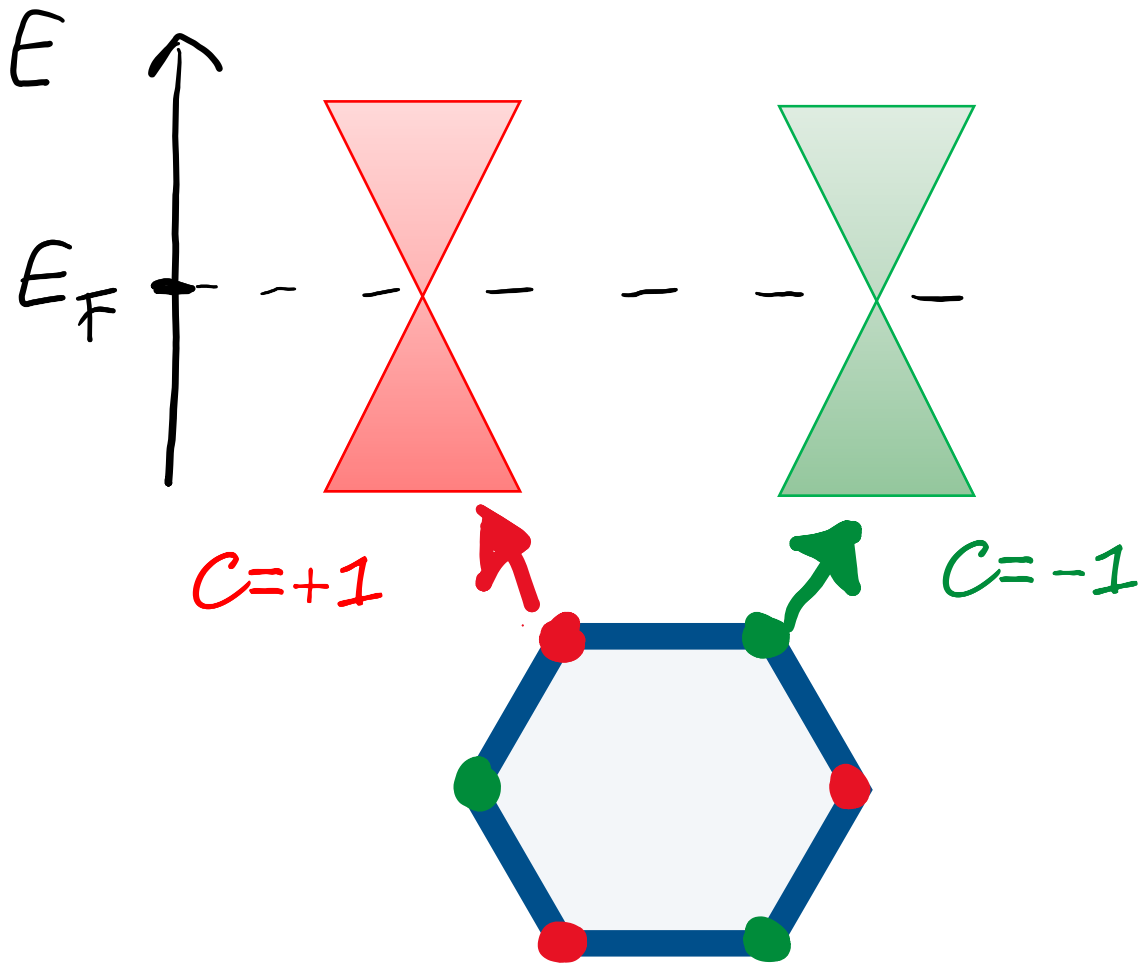
BROAD CLASSIFICATION OF SOLIDS (ACCORDING TO BAND STRUCTURE)



BROAD CLASSIFICATION OF SOLIDS (ACCORDING TO BAND STRUCTURE)



CANONICAL EXAMPLE OF TOPOLOGICAL SEMIMETAL: GRAPHENE



Berry phase (=chirality)

$$C = \frac{1}{\pi} \oint \vec{A}(\vec{k}) \cdot d\vec{k}$$

ZOO OF TOPOLOGICAL SEMIMETALS

– NODAL POINTS

- DIMENSION
  - 2D
  - 3D
- P and T SYMMETRY (degeneracy)
  - Two-fold ("Weyl"): P or T broken
  - Four-fold ("Dirac"): P and T unbroken
- LOCATION IN BRILLOUIN ZONE
  - Pinned (at symmetric point)
  - Unpinned (otherwise)

Reviews:
Armitage, Mele, Vishwanath 1705.01111
Gao, Venderbos, Kim, Rappe 1810.08186
Feng, Zhu, Wu, Yang 2103.13772

– NODAL LINES

NOTE: No odd number of Weyl fermions except at boundaries

ZOO OF TOPOLOGICAL SEMIMETALS

- NODAL POINTS
  - DIMENSION
    - 2D
    - 3D
  - P and T SYMMETRY (degeneracy)
    - Two-fold ("Weyl"): P or T broken
    - Four-fold ("Dirac"): P and T unbroken
  - LOCATION IN BRILLOUIN ZONE
    - Pinned (at symmetric point)
    - Unpinned (otherwise)
- NODAL LINES

ZOO OF TOPOLOGICAL SEMIMETALS

- NODAL POINTS
  - DIMENSION
    - 2D
    - 3D
  - P and T SYMMETRY (degeneracy)
    - Two-fold ("Weyl"): P or T broken
    - Four-fold ("Dirac"): P and T unbroken
  - LOCATION IN BRILLOUIN ZONE
    - Pinned (at symmetric point)
    - Unpinned (otherwise)

- NODAL LINES

SYMMETRIC LINE

EFFECTIVE FIELD THEORIES

LATTICE

↓ Expand close to Fermi energy

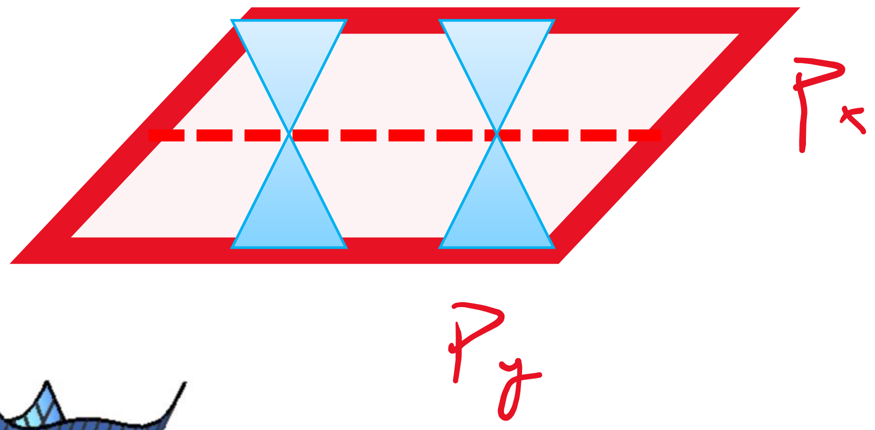
EFFECTIVE FERMION MODEL

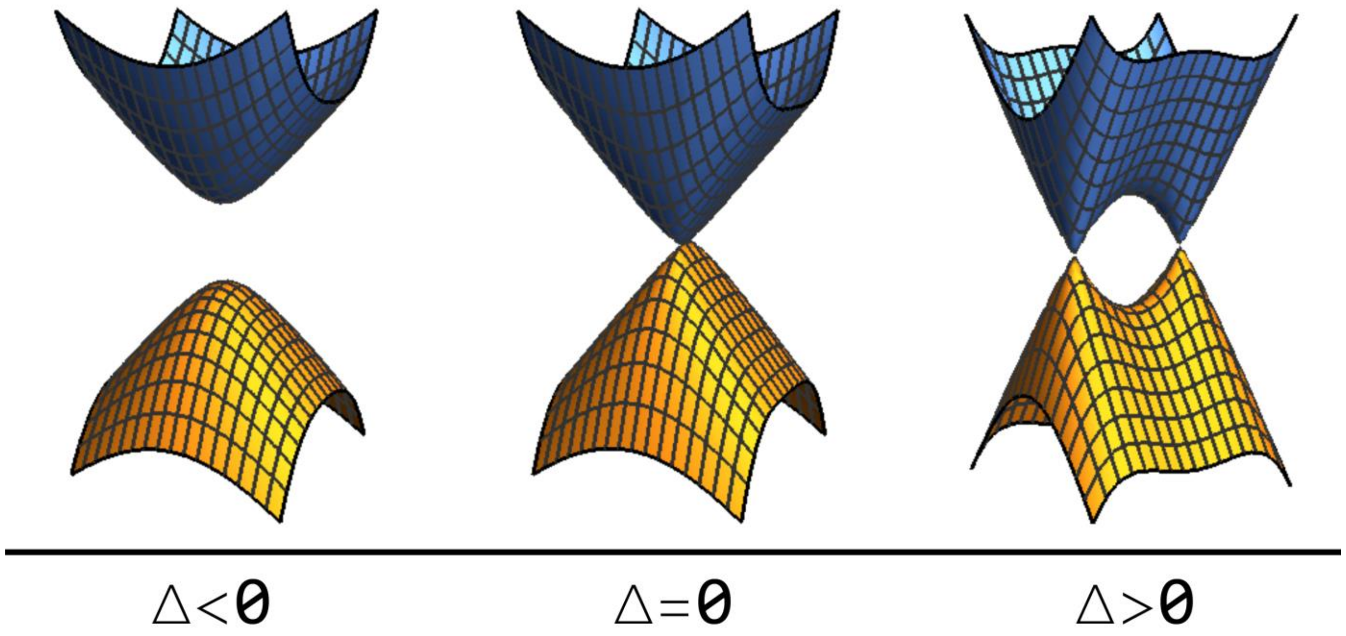
↓ Integrate out fermions

EFFECTIVE THEORY

EFFECTIVE MODEL

$$H = p_x \sigma^1 + \left(\frac{p_y^2}{2m} - \Delta \right) \sigma^3$$





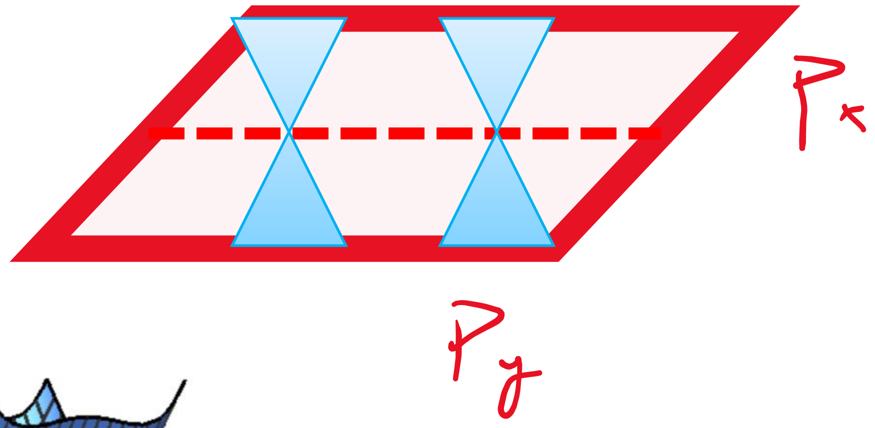
INSULATOR

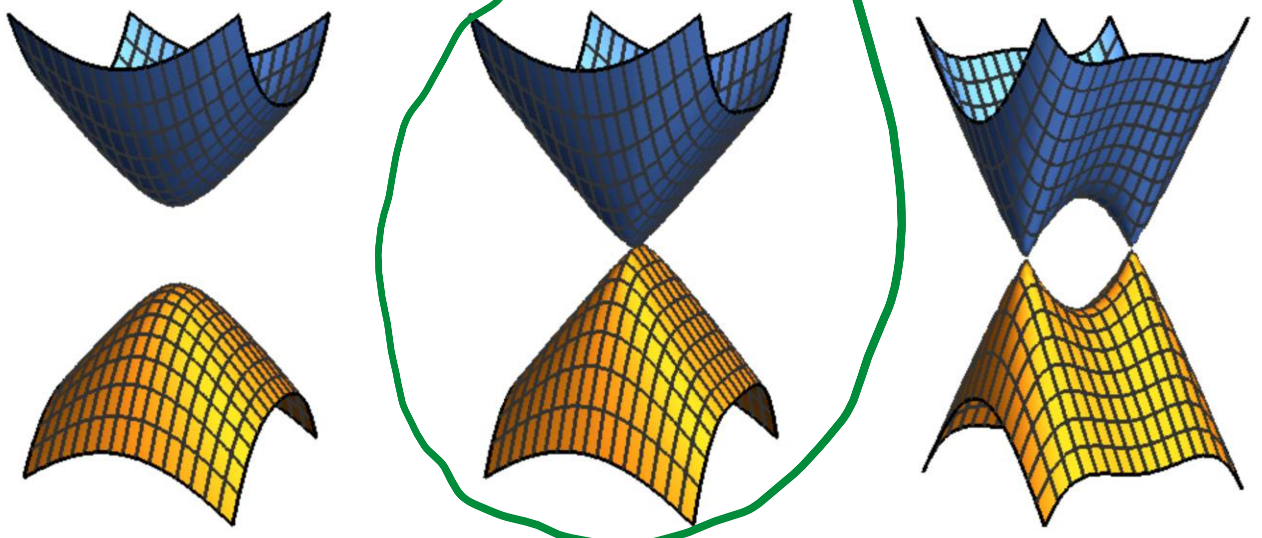
CRITICAL POINT

DIRAC SEMIMETAL

EFFECTIVE MODEL

$$H = p_x\sigma^1 + \left(\frac{p_y^2}{2m} - \Delta\right)\sigma^3$$





$\Delta<0$ $\Delta=0$ $\Delta>0$

INSULATOR

CRITICAL POINT

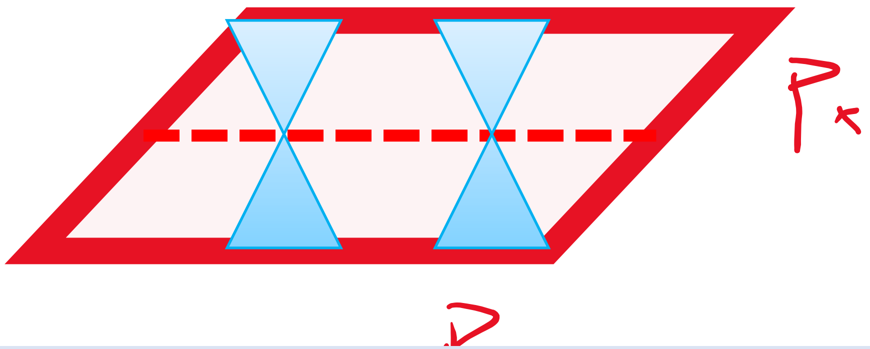
DIRAC SEMIMETAL

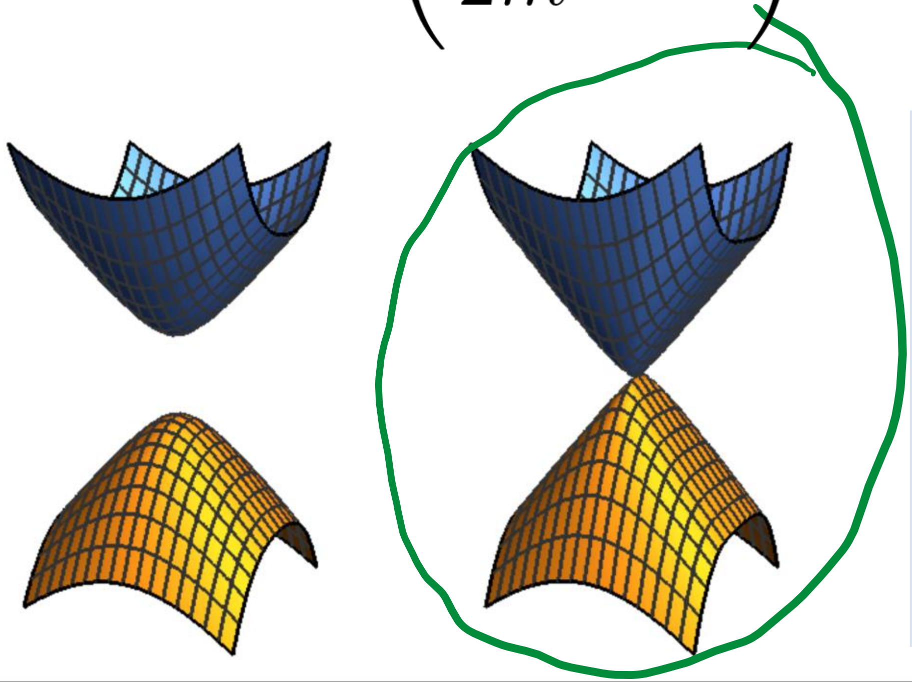
SEMI-DIRAC SEMIMETAL

$$\omega^2 = p_x^2 + \frac{p_y^4}{4m^2}$$

EFFECTIVE MODEL

$$H = p_x \sigma^1 + \left(\frac{p_y^2}{2m} - \Delta \right) \sigma^3$$





Δ<0 Δ=0 Δ>0

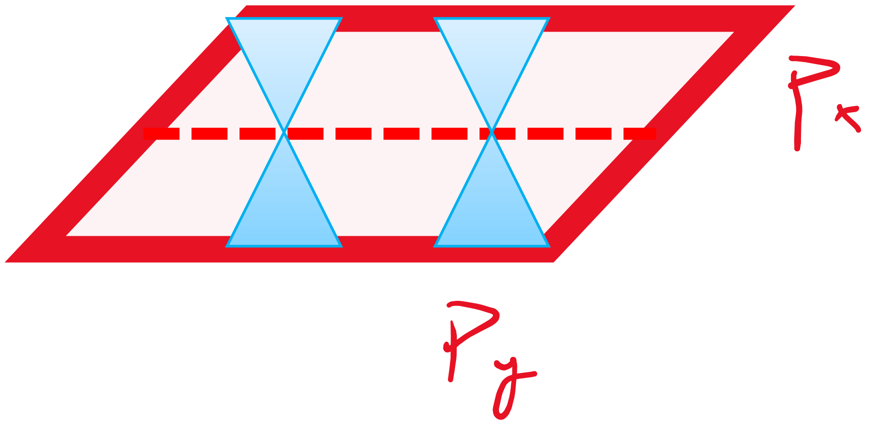
INSULATOR | CRITICAL POINT | DIRAC SEMIMETAL

TiO2/VO2 Heterostructures (multilayers);
Pardo, Pickett 0903.4820
Organic salts under pressure;
Katayama,Kobayashi, Suzumura cond-mat/0601068
Photonic metamaterials;
Wu 1312.0201
non-Hermitian systems;
Banerjee and A. Narayan 2001.11188

$\omega^2 = p_x^2 + \frac{\cdots}{4m^2}$

EFFECTIVE MODEL

$$H = p_x\sigma^1 + \left(\frac{p_y^2}{2m} - \Delta\right)\sigma^3$$



TIME-REVERSAL:

$$\mathcal{T}H(\vec{p})\mathcal{T}^{-1} = \sigma^2 H^*(-\vec{p})\sigma^2 = p_x\sigma^1 - \left(\frac{p_y^2}{2m} - \Delta\right)\sigma^3 \neq H(\vec{p})$$ Broken T

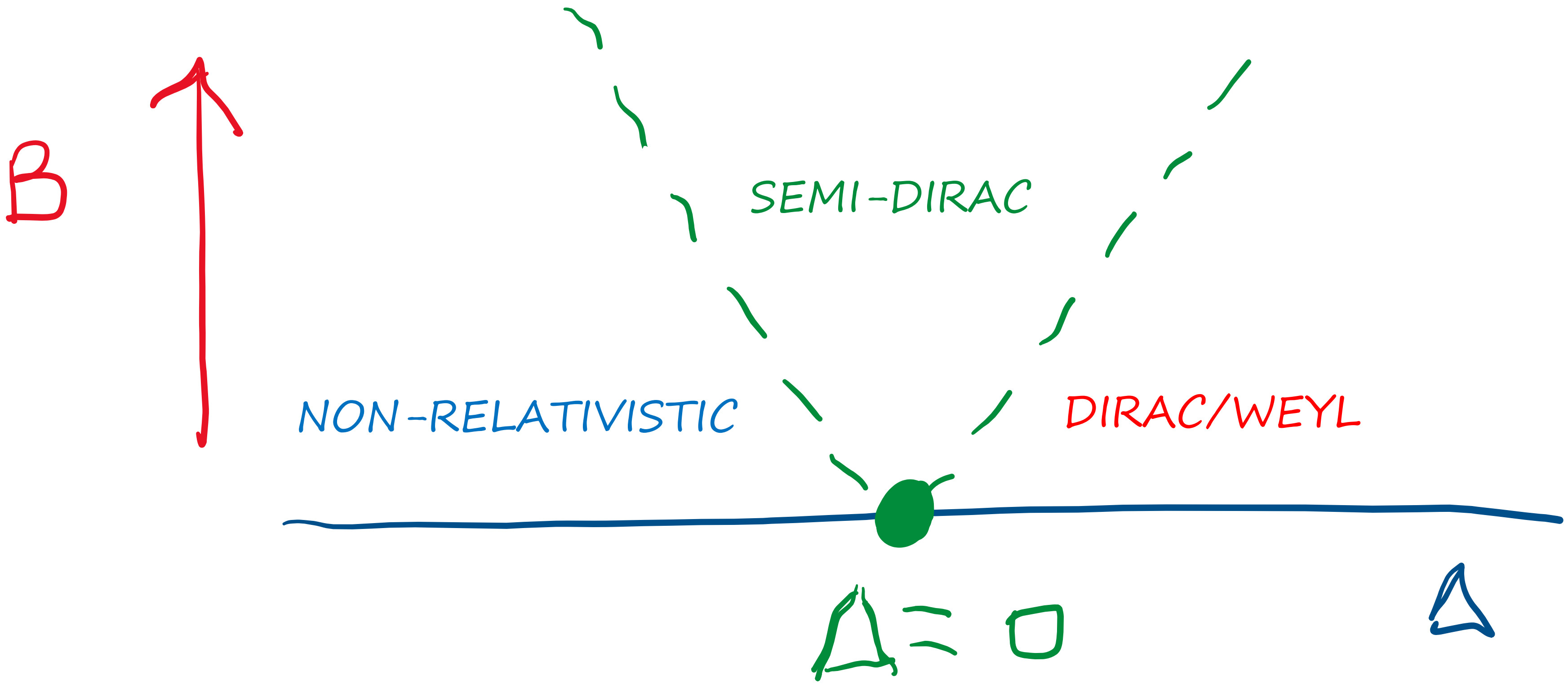
PARITY:

$$\mathcal{P}_y H(\vec{p})\mathcal{P}_y^{-1} = \sigma^1 H(p_x, -p_y)\sigma^1 = p_x\sigma^1 - \left(\frac{p_y^2}{2m} - \Delta\right)\sigma^3 \neq H(\vec{p})$$ Broken P

PT IS UNBROKEN

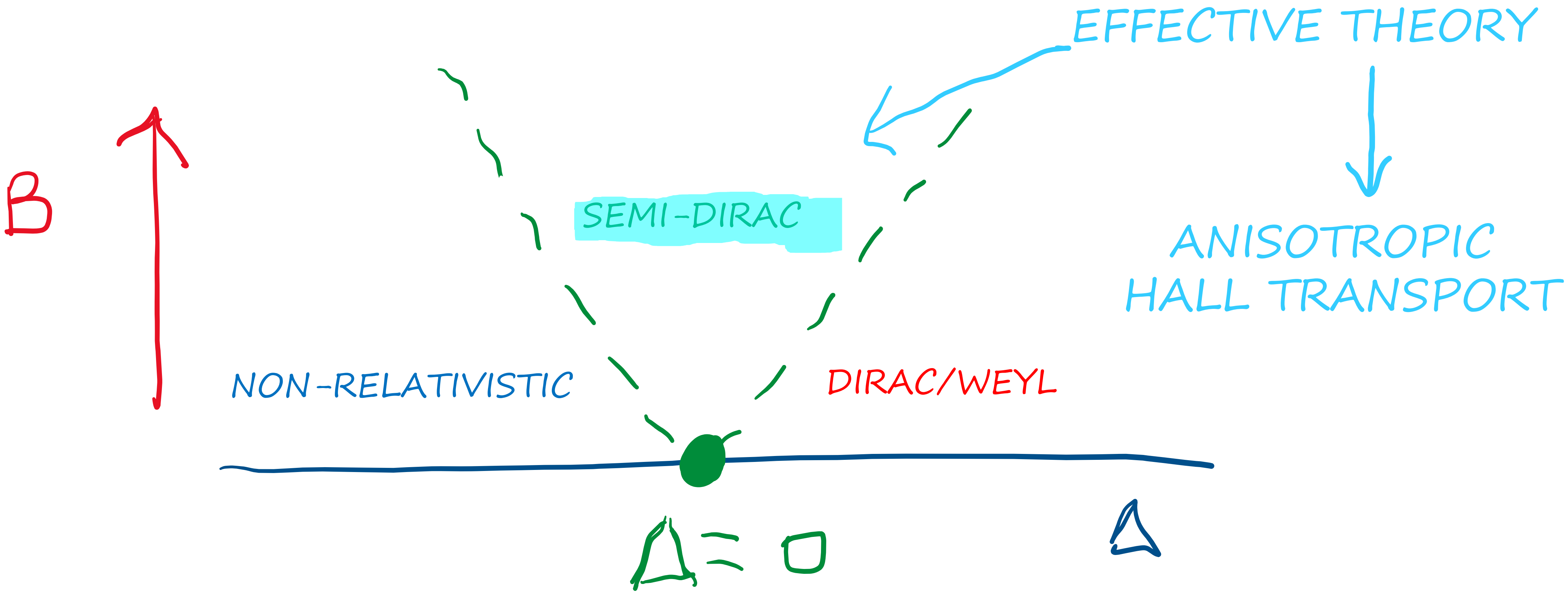
INTRODUCING A MAGNETIC FIELD:

- ASSUME FILLED LANDAU LEVELS : GAPPED STATE
- PT SYMMETRY UNBROKEN



INTRODUCING A MAGNETIC FIELD:

- ASSUME FILLED LANDAU LEVELS : GAPPED STATE
- PT SYMMETRY <u>UNBROKEN</u>



EFFECTIVE FIELD THEORIES

LATTICE

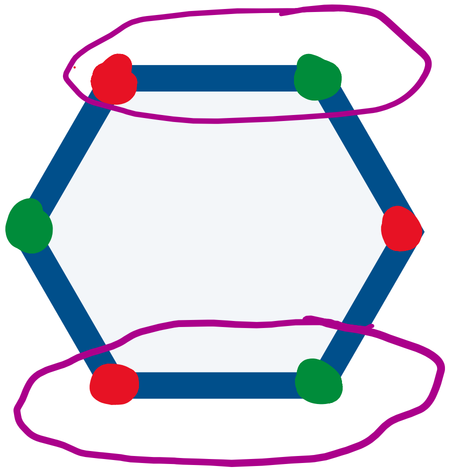
$\downarrow$ Expand close to Fermi energy

EFFECTIVE FERMION MODEL

$\downarrow$ Integrate out fermions

EFFECTIVE THEORY

LANDAU LEVELS: Hexagonal Tight-binding model



t

HOPING

$t = 1$ GRAPHENE

$t = 2$ CRITICAL POINT (SEMI-DIRAC)

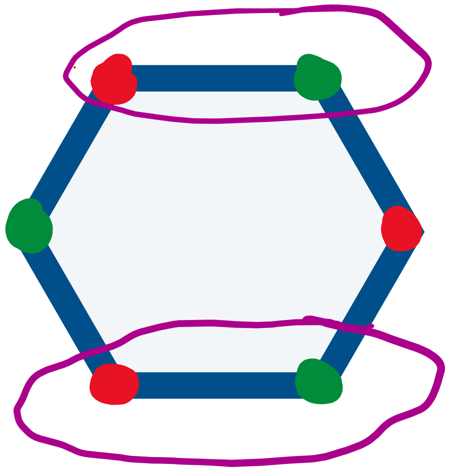
$t > 2$ INSULATOR

Sato, Tobe, Kohmoto
0808.3440

$E \sim B^{1/2}$ $E \sim B^{2/3}$ $E \sim B$

Esaki, Sato, Kohmoto, Halperin
0906.5027

LANDAU LEVELS: Hexagonal Tight-binding model



t

HOPING

Sato, Tobe, Kohmoto
0808.3440

$t = 1$ GRAPHENE

$t = 2$ CRITICAL POINT (SEMI-DIRAC)

$t > 2$ INSULATOR

$E \sim B^{1/2}$ $\quad$ $E \sim B^{2/3}$ $\quad$ $E \sim B$

SCALING SYMMETRY

Esaki, Sato, Kohmoto, Halperin
0906.5027

SCALING SYMMETRY IN LOWEST LANDAU LEVEL

MAGNETIC FIELD: $B \sim p_x \, p_y$

"RELATIVISTIC": $E \sim p_x \sim p_y$

$\rightarrow \; B \sim p_x^2 \quad \rightarrow \quad E \sim B^{1/2}$

"GALILEAN": $E \sim p_x^2 \sim p_y^2$

$\rightarrow \; B \sim p_x^2 \quad \rightarrow \quad E \sim B$

SEMI-DIRAC: $E \sim p_x \sim p_y^2$ $\quad (B \gg \Delta)$

$\rightarrow \; B \sim p_x^{3/2} \quad \rightarrow \quad E \sim B^{2/3}$

EFFECTIVE FIELD THEORIES

LATTICE

$\downarrow$ Expand close to Fermi energy

EFFECTIVE FERMION MODEL

$\downarrow$ Integrate out fermions

EFFECTIVE THEORY

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

- A COVARIANT FORMULATION LIKE NEWTON-CARTAN MAY BE USEFUL

Son 1306.0638

HOWEVER,

- NO GALILEAN BOOST INVARIANCE
- SPATIAL ANISOTROPY

IN PRINCIPLE NOT AN ISSUE BY ADDING MORE INGREDIENTS

HOWEVER,

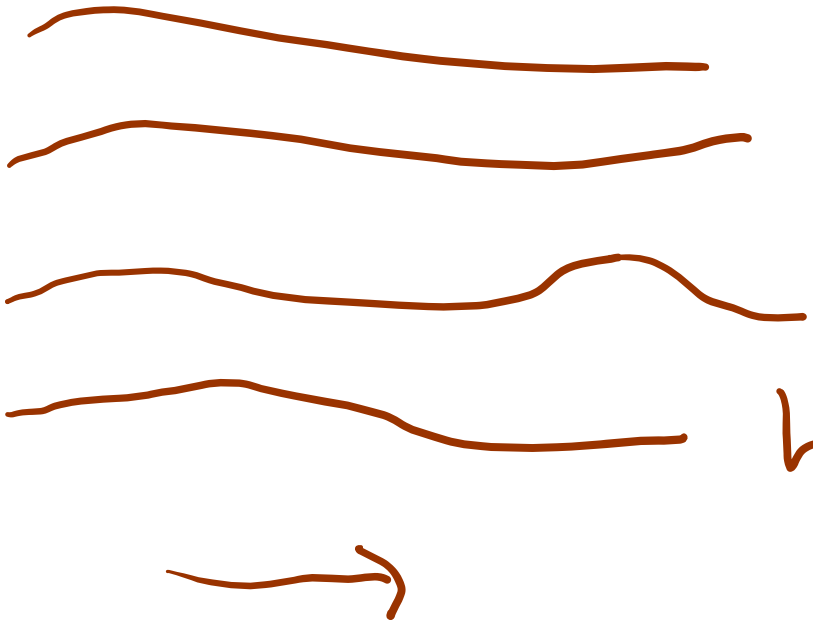
- NO INTRINSIC ADVANTAGE OF USING NEWTON-CARTAN

WE WILL USE A “SIMPLER” COVARIANT FORMULATION

See also: Copetti, Landsteiner 1901.11403

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

NEWTON-CARTAN REMINDER "TIME – SPACE ANISOTROPY"



SPACE

TIME

$$n_\mu, \quad v^\mu, \quad v^\mu n_\mu = 1$$

$$h_{\mu\nu}, \quad h^{\mu\nu}; \quad v^\mu h_{\mu\nu} = n_\mu h^{\mu\nu} = 0$$

$$h^{\mu\alpha} h_{\alpha\nu} = \delta^\mu_\nu - v^\mu n_\nu$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

NEWTON-CARTAN REMINDER

TORSIONFUL CONNECTION: $\Gamma^{\lambda}_{\mu\nu} \sim v^{\lambda}\partial_{[\mu}n_{\nu]}$

MILNE BOOSTS: $\delta v^{\mu} = h^{\mu\alpha}\psi_{\alpha}$

GALILEAN INVARIANCE

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

NEWTON-CARTAN REMINDER

TORSIONFUL CONNECTION: $\Gamma^{\lambda}_{\mu\nu} \sim v^{\lambda} \partial_{[\mu} n_{\nu]}$

MILNE BOOSTS: $\delta v^{\mu} = h^{\mu\alpha} \psi_{\alpha}$

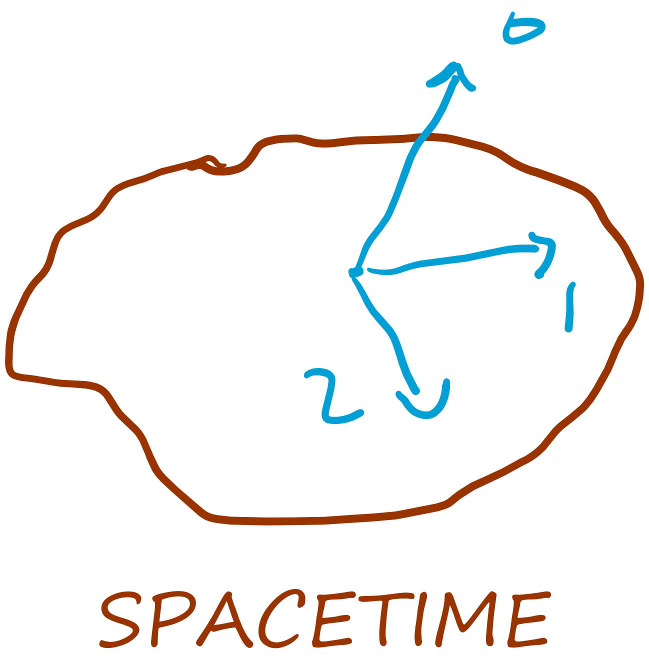
GALILEAN INVARIANCE

REWRITE NEWTON-CARTAN
USING FRAME FIELDS

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

NEWTON-CARTAN REMINDER

VIELBEINS: PICK A VECTOR BASIS AT EACH POINT



$$e_a{}^{\mu}$$

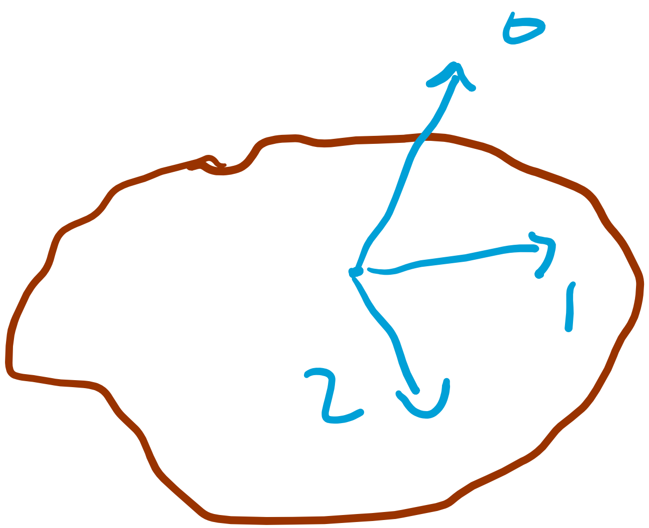
a = LABEL

μ = COMPONENTS

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

NEWTON-CARTAN REMINDER

VIELBEINS: PICK A VECTOR BASIS AT EACH POINT



SPACETIME

$e_a{}^{\mu}$

a = LABEL

μ = COMPONENTS

$E^a{}_{\mu}$ DUAL BASIS

$$E^a{}_{\mu}\, e_b{}^{\mu} = \delta^a{}_b$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

NEWTON-CARTAN REMINDER

ADD FIXED TIME-LIKE VECTOR

$$t^a = (1, 0, \dots, 0)$$

$$(t^a t_a = -1)$$

$$t_a = \eta_{ab} t^b$$

$$n_\mu \equiv t_a e^a{}_\mu$$

$$v^\mu \equiv -t^a e_a{}^\mu$$

METRIC

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$

$$h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \;, \quad h^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

WHAT WE WILL DO

$$t^a \longrightarrow \ell^a$$

TIME-LIKE SPACE-LIKE

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

$\mu = 0, 1, 2$ COORDINATE INDEX

$a = 0, 1, 2$ FRAME INDEX

DIRAC MATRICES $\quad \gamma^a = (\sigma^3, -i\sigma^2, i\sigma^1)$

DIRAC SPINOR $\quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

$$S = \int d^3x \left[\bar{\Psi}(i\gamma^0\partial_0 + i\gamma^1\partial_1)\psi + \bar{\Psi}\left(\Delta + \frac{1}{2m}\partial_2^2\right)\psi \right]$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

* SYMMETRIES:

- LORENTZ: (x^0, x^1) : $SO(1,1)$
- U(1): $\psi \to e^{i\alpha}\psi$:
- SCALE: $(x^0, x^1, x^2) \to (\lambda^2 x^0, \lambda^2 x^1, \lambda x^2)$

$(\Delta = 0)$ $\quad \psi \to \lambda^{-3/2}\psi$

$$S = \int d^3x \left[\bar{\psi}\left(i\gamma^0\partial_0 + i\gamma^1\partial_1\right)\psi + \bar{\psi}\left(\Delta + \frac{1}{2m}\partial_2^2\right)\psi \right]$$

MASS: $\cancel{P}$, $\cancel{T}$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

ANISOTROPY → BACKGROUND VECTORS

$$\ell^a = (0, 0, 1) \quad ; \quad \ell_a = (0, 0, 1)$$

$$P^a{}_b = \delta^a{}_b - \ell^a \ell_b$$

$$S = \int d^3x \left[\bar{\Psi} \left(i\gamma^0 \partial_0 + i\gamma^1 \partial_1 \right) \psi + \bar{\Psi} \left(\Delta + \frac{1}{2m} \partial_2^2 \right) \psi \right]$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

ANISOTROPY → BACKGROUND VECTORS

$$\ell^a = (0,0,1) \quad ; \quad \ell_a = (0,0,1)$$

$$P^a{}_b = \delta^a{}_b - \ell^a \ell_b$$

$$S = \int d^3x \left[\bar{\Psi}\,(\underbrace{i\gamma^0\partial_0 + i\gamma^1\partial_1}_{iP^a{}_b\gamma^b\partial_a})\psi + \bar{\Psi}\Big(\Delta + \frac{1}{2m}\underbrace{\partial_z^2}_{(\ell^a\partial_a)^2}\Big)\psi \right]$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

BACKGROUND GEOMETRY: $e_a{}^{\mu}$, $E^a{}_{\mu}$

$$E^a{}_{\mu}\, e_b{}^{\mu} = \delta^a{}_b$$

$$\ell_\mu = \ell_a E^a{}_\mu \,, \quad \ell^\mu = \ell^a e_a{}^\mu \,, \quad \partial_a = e_a{}^\mu \partial_\mu$$

BACKGROUND GAUGE FIELD: A_μ

$$\partial_\mu \Psi \longrightarrow D_\mu \Psi = \, ?$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

RELATIVISTIC 2+1 FERMION

$$D_\mu \Psi = \partial_\mu \Psi - i A_\mu \Psi + \frac{1}{2} \omega^{ab}{}_\mu \, \mathcal{J}_{ab} \Psi$$

$$\mathcal{J}^{ab} = \frac{1}{4} [\gamma^a, \gamma^b]$$

$$\omega^{ab}{}_\mu = - E^a{}_\alpha \underbrace{\nabla_\mu e^{b\alpha}}$$

$$\rightarrow \partial_\mu e^{b\alpha} - \Gamma^\alpha_{\mu\beta} e^{b\beta}$$

LEVI-CIVITA
SPIN CONNECTION

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

$$SO(2,1) \longrightarrow SO(1,1)$$

$$J_{ab} \rightarrow K = \frac{1}{2} \epsilon^{abc} \ell_c J_{ab}$$

$$j_a = J_{ab} \ell^b$$

$$\omega^{ab}{}_\mu \rightarrow \omega_\mu = \frac{1}{2} \epsilon_{abc} \ell^c \omega^{ab}{}_\mu$$

$$\theta^a{}_\mu = \omega^{ab}{}_\mu \ell_b$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

$$SO(2,1) \longrightarrow SO(1,1)$$

GAUGE FIELD FOR SO(1,1)

$$\frac{1}{2}\omega^{ab}{}_{\mu} J_{ab} \to \underbrace{\omega_\mu K}\, + \theta^a{}_\mu j_a$$

GENERAL FORM:

$$\alpha = \tilde{\alpha} - 1$$

$$D_\mu \psi = \left(\partial_\mu - iA_\mu + \omega_\mu K + \tilde{\alpha}\, \theta^a{}_\mu j_a\right)\psi$$

$$= \left(\partial_\mu - iA_\mu + \frac{1}{2}\omega_\mu{}^{ab} J_{ab} + \alpha\, \theta^a{}_\mu \bar{j}_a\right)\psi$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

$$D_\mu \psi = \left(\partial_\mu - i A_\mu + \frac{1}{2} \omega_\mu{}^{ab} J_{ab} + \alpha\, \theta^a{}_\mu \bar{j}_a \right) \psi$$

UNDETERMINED BY SYMMETRIES

- TO DETERMINE α ONE NEEDS A MICROSCOPIC MODEL

- HOWEVER, DEPENDENCE ON α HIDDEN IN COEFFICIENTS OF EFT: EXPLICIT VALUE NOT NEEDED

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

1. FERMIONIC ACTION

FULL COVARIANT ACTION:

$$S = \int d^3x \, |E| \left(\bar{\Psi} P^a{}_b \, \gamma^b e_a{}^\mu D_\mu \Psi + \bar{\Psi} M \Psi \right]$$

$$M\Psi = \left(\Delta + \frac{1}{2m} \ell^a \ell^b e_a{}^\mu e_b{}^\nu D_\mu D_\nu \right) \Psi$$

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

2. GENERATING FUNCTIONAL

$$Z[A, E, e, \ell] = \int D\Psi D\bar{\Psi}\, e^{iS_\Psi[A, E, e, \ell]}$$

BACKGROUND SOURCES $A_\mu,\ e_a{}^\mu,\ E^a{}_\mu,\ \ell^\mu$

$$Z = e^{iW}\ ; \quad \delta W = -\int d^3x\, |E| \left(\tau_\mu{}^a \delta e_a{}^\mu + j^\mu \delta A_\mu \right)$$

STRESS TENSOR

CURRENT

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

2. GENERATING FUNCTIONAL

$$Z[A, E, e, \ell] = \int D\Psi D\bar{\Psi}\, e^{iS_\Psi[A, E, e, \ell]}$$

BACKGROUND SOURCES A_μ, $e_a{}^\mu$, $E^a{}_\mu$, ℓ^μ

$$Z = e^{iW} \; ; \quad \delta W = -\int d^3x\, |E| \left(\tau_\mu{}^a \delta e_a{}^\mu + j^\mu \delta A_\mu \right)$$

KUBO FORMULAS → TRANSPORT COEFFICIENTS

CONSTRUCTING THE EFFECTIVE ACTION OF QH STATE

2. GENERATING FUNCTIONAL

$$Z[A, E, e, \ell] = \int D\psi\, D\bar{\psi}\; e^{\,i S_\psi [A, E, e, \ell]}$$

* GAP: W IS A COVARIANT LOCAL FUNCTIONAL OF THE SOURCES

$B \neq 0$

* LOCAL SYMMETRIES: SO(1,1)+U(1)

* PT SYMMETRY

CONSTRUCTING THE GENERATING FUNCTIONAL

1. DERIVATIVE EXPANSION

$$\epsilon^2 \sim \frac{\omega}{B^{2/3}} \sim \frac{p_x}{B^{2/3}} \sim \frac{p_y^2}{B^{2/3}} << 1$$

$$\ell^a,\ e_a{}^\mu,\ E^a{}_r \sim O(\epsilon^0)$$

$$\ell^a \partial_a \sim O(\epsilon),\quad P_a{}^b \partial_b \sim O(\epsilon^2)$$

$$\ell^a A_a,\ P_0{}^a A_a \sim O(\epsilon^0);\ P_i{}^a A_a \sim O(\epsilon^{-1})$$

CONSTRUCTING THE GENERATING FUNCTIONAL

1. DERIVATIVE EXPANSION

$\epsilon^2 \sim$

$e^a ,$

$e^a \partial_a$

$e^a A$ (ϵ^{-1})

In principle the effective action is insensitive to the origin of spatial anisotropy
(same form for other systems)

However, which terms are dominant depends on the scaling chosen in the derivative expansion
(may depend on the particular system)

CONSTRUCTING THE GENERATING FUNCTIONAL

1. DERIVATIVE EXPANSION

- COVARIANT TERMS CONTAIN DIFFERENT ORDERS IN DERIVATIVES

$$B^a = P^a{}_b \, \epsilon^{bcd} \partial_c A_d$$

$$\sim \underbrace{\ell^c \partial_c (P^{ab} A_b)}_{O(\epsilon^0) + O(\epsilon)} - \underbrace{P^{ab} \partial_b (\ell^c A_c)}_{O(\epsilon^2)}$$

CONSTRUCTING THE GENERATING FUNCTIONAL

1. DERIVATIVE EXPANSION

- COVARIANT TERMS CONTAIN DIFFERENT ORDERS IN DERIVATIVES

$$B^a = P^a{}_b\, \epsilon^{bcd}\, \partial_c A_d$$

$$\sim \ell^c \partial_c \left(P^{ab} A_b \right) - P^{ab} \partial_b \left(\ell^c A_c \right)$$

ASSIGN LOWEST ORDER $\Rightarrow B^a \sim O(\epsilon^0)$

CONSTRUCTING THE GENERATING FUNCTIONAL

2. COVARIANT TERMS

CURVATURE INVARIANTS:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\hat{R}_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$$

$$T_{\mu\nu} = \partial_\mu e_\nu - \partial_\nu e_\mu$$

METRIC: $g_{\mu\nu} = \eta_{ab} E^a{}_\mu E^b{}_\nu \; ; \; g^{\mu\nu} = \eta^{ab} e_a{}^\mu e_b{}^\nu$

$\longrightarrow$ RIEMANN TENSOR: $R_{\alpha\beta\mu\nu}$

CONSTRUCTING THE GENERATING FUNCTIONAL

2. COVARIANT TERMS

* SCALARS $\quad X_L = \varepsilon^{\mu\nu\lambda} \ell_\mu X_{\nu\lambda}$
* VECTORS $\quad X_\mu = \ell^\alpha X_{\alpha\mu} \; ; \; \tilde{X}^\mu = P^\mu{}_\alpha \varepsilon^{\alpha\beta\gamma} X_{\beta\gamma}$

$$X_{\mu\nu} = \left(F_{\mu\nu} \quad \hat{R}_{\mu\nu} \quad T_{\mu\nu} \right)$$

CONSTRUCTING THE GENERATING FUNCTIONAL

2. COVARIANT TERMS

- $O(\epsilon^0)$

VECTORS: $u^\mu = (\ell^\mu, \vartheta^\mu, j^\mu)$

- $\vartheta^\mu = \dfrac{\tilde{F}^\mu}{\sqrt{|\tilde{F}^2|}}$
- $j^\mu = \dfrac{F^\mu}{\sqrt{|F^2|}}$

EULER CURRENTS

$$J^\mu_E(u) = \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}\epsilon^{\alpha\beta\gamma}u_\alpha\left(\nabla_\nu u_\beta \nabla_\lambda u_\gamma \pm \frac{1}{2}R_{\nu\lambda\beta\gamma}\right)$$

$u^\mu u_\mu = \pm 1$

CONSTRUCTING THE GENERATING FUNCTIONAL

2. COVARIANT TERMS

$$O(\epsilon^0): \mathcal{L}_0 = -\mathcal{E}(B_A) + \frac{\nu}{4\pi}\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$B_0 = (F_\perp^2)^{1/2}, \quad B_y = (F_\mu F^\mu)^{1/2}$$

$$\sigma_{xy} = \frac{\nu}{2\pi} \quad \text{HALL CONDUCTIVITY}$$

CONSTRUCTING THE GENERATING FUNCTIONAL

2. COVARIANT TERMS

CONTRIBUTIONS TO HALL VISCOSITY

$\varepsilon^{\mu\nu\lambda} \ell_\mu \partial_\nu \ell_\lambda$

$O(\epsilon) \quad \mathcal{L}_1 \supset c_5 \, \varepsilon^{\mu\nu\lambda} F_\mu \partial_\nu F_\lambda$

$O(\epsilon^2) \quad \mathcal{L}_2 \supset k A_\mu J^\mu_\epsilon(\vartheta) + f_{\parallel} T_\perp^\Delta$

ANISOTROPIC CONTRIBUTIONS

ANISOTROPIC HALL VISCOSITY

$$\eta^{\mu}{}_{a}{}^{\nu}{}_{b} = \lim_{\omega \to 0} \frac{i}{\omega} \langle \tau^{\mu}{}_{a} \, \tau^{\nu}{}_{b} \rangle$$

$$\eta_H^{ijkl} = - \eta_H^{klij}$$

$$\eta^{ijkl} = \eta_{iso} P_{iso}^{ijkl} + \eta_{nem} P_{nem}^{ijkl} + \eta_{vor} P_{vor}^{ijkl}$$

THREE INDEPENDENT COEFFICIENTS

ANISOTROPIC HALL VISCOSITY

$$P_{\mathrm{iso}}^{ijkl} = -\frac{1}{2}\left(\epsilon^{ik}\delta^{jl} + (i \leftrightarrow j) + (k \leftrightarrow l) + (ik \leftrightarrow jl)\right)$$

$$P_{\mathrm{NEM}}^{ijkl} = \ell^{(i}\,\epsilon^{j)m}\,\ell_m\,\delta^{lk} - (ij \leftrightarrow kl)$$

$$P_{\mathrm{VOR}}^{ijkl} = \ell^{[i}\,\epsilon^{j]m}\,\ell_m\,\delta^{lk} - (ij \leftrightarrow kl)$$

BROKEN PT ALLOWS ADDITIONAL COEFFICIENT

$$P^{ijkl} = \ell^i \ell^j \delta^{kl} - (ij \leftrightarrow kl)$$

ANISOTROPIC HALL VISCOSITY

$$\eta_{iso} = \frac{\kappa}{4\pi} B - \frac{1}{2} c_5 B^2 - \frac{1}{2} f_{11}$$

$$\eta_{NEM} = \frac{1}{2} c_5 B^2 + \frac{1}{2} f_{11}$$

$$\eta_{vor} = \frac{1}{2} c_5 B^2 - \frac{1}{2} f_{11}$$

$\Delta = 0 \Rightarrow c_5 \sim B^{-1}, \; f_{11} \sim B$ SEMI-DIRAC POINT

ANISOTROPIC HALL VISCOSITY

$$\eta_{ISO} = \frac{\kappa}{4\pi} B - \frac{1}{2} c_5 B^2 - \frac{1}{2} f_{11}$$

$$\eta_{NEM} = \frac{1}{2} c_5 B^2 + \frac{1}{2} f_{11}$$

BERRY CURVATURES

Peña-Benitez, Saha, Surowka 1805.09827

$$\eta_{VOR} = \frac{1}{2} c_5 B^2 - \frac{1}{2} f_{11} \leftarrow$$

HAS NOT BEEN COMPUTED

$$\Delta = 0 \Rightarrow c_5 \sim B^{-1}, \; f_{11} \sim B$$

SEMI-DIRAC POINT

ANISOTROPIC HALL VISCOSITY

NON-TOPOLOGICAL

$$\eta_{iso} = \frac{\kappa}{4\pi} B - \frac{1}{2} C_5 B^2 - \frac{1}{2} f_{11}$$

"TOPOLOGICAL"
(SHIFT)

$$\eta_{nem} = \frac{1}{2} C_5 B^2 + \frac{1}{2} f_{11}$$

$$\eta_{vor} = \frac{1}{2} C_5 B^2 - \frac{1}{2} f_{11}$$

$\Delta = 0 \Rightarrow C_5 \sim B^{-1},\ f_{11} \sim B$ SEMI-DIRAC POINT

CONCLUSIONS and OUTLOOK

- Hall conductivity insensitive to breaking of rotational invariance
- Hall viscosity topological contribution independent of the breaking

$$\eta_{ISO} + \eta_{NEM} = \frac{\kappa B}{4\pi}$$

- Definite scaling with magnetic field close to the semi-Dirac point
- AC transport coefficients
- Spatial momentum dependence (inhomogeneous sources)
- Generalization to other anisotropic systems