Entanglement Entropy for integer QHE in two and higher dimensions

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Topological properties of gauge theories and their applications to high energy and condensed matter physics

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  - topological field theories (Chern-Simons effective actions)
  - bulk-edge dynamics
  - non-commutative geometries, fuzzy spaces

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- Generalization to arbitrary even (spatial) dimensions

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  QHE on $\mathbb{CP}^k$ (KARABALI AND NAIR, 2002...)
  - higher dimensionality
  - possibility of having both abelian and nonabelian magnetic fields

EFFECTIVE ACTIONS FOR $\nu = 1$ HIGHER DIMENSIONAL QHE

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- These reduce to known results in 2 dimensions.
- Common origin $\Longrightarrow$ Universal matrix action

Matrix Formulation of LLL Dynamics

- QHE on a compact space $M \Longrightarrow$ LLL defines an N-dim Hilbert space

  In the presence of confining potential $\Longrightarrow$ incompressible QH droplet

- Density matrix for ground state droplet : $\hat{\rho}_0$

$$\hat{\rho}_0 = \begin{array}{cc} \begin{bmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ & & & \ddots & & & & & \\ & & & & 1 & & & & \\ & & & & & 0 & & & \\ & & & & & & 0 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{bmatrix} & \begin{array}{l} \updownarrow K \\ \\ \updownarrow N-K \end{array} \end{array}$$

  K filled states

- Under time evolution: $\hat{\rho}_0 \to \hat{\rho} = \hat{U}\,\hat{\rho}_0\,\hat{U}^\dagger$

  $\hat{U} = N \times N$ unitary matrix ; "collective" variable describing excitations within the LLL

The action for $\hat{U}$ is

$$S_0 = \int dt \, \mathrm{Tr} \left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \; - \; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} \right]$$

which leads to the evolution equation for density matrix

$$i \frac{d\hat{\rho}}{dt} = [\hat{V}, \hat{\rho}]$$

S_0 has no explicit dependence on properties of space on which QHE is defined, abelian or nonabelian nature of fermions, etc.

Noncommutative field theory

S_0 = action of a noncommutative field theory

$$S_0 = \int dt \,\mathrm{Tr}\left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \;-\; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U}\right]$$
$$= N \int d\mu \, dt \; \left[i(\rho_0 * U^\dagger * \partial_t U) \;-\; (\rho_0 * U^\dagger * V * U)\right]$$

$$\underbrace{\hat{\rho}_0, \hat{U}, \hat{V}}_{(N \times N)\text{ matrices}} \qquad \Longrightarrow \qquad \underbrace{\rho_0(\vec{x}), U(\vec{x}, t), V(\vec{x})}_{\text{symbols}}$$

- $O(\vec{x}, t) = \frac{1}{N} \sum_{m,l} \Psi_m(\vec{x}) \hat{O}_{ml}(t) \Psi_l^*(\vec{x})$
- Matrix multiplication $\Longrightarrow$ $*$ product of symbols
- Tr $\Longrightarrow$ $N \int d\mu$

S_0 = exact bosonic action describing the dynamics of LLL fermions

Sakita: 2 dim. context

- Large N limit $\Longrightarrow$ WZW-like chiral edge action

Effective field theories for $\nu = 1$ higher dim. QHE

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- In the presence of gauge fluctuations one starts with a gauged matrix action. The corresponding NC field theory, after taking the large N limit splits into :

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- Using the Dolbeault index density we found a general formula for effective bulk topological actions in higher dimensions that account for the response to both metric and gauge background fluctuations. (KARABALI AND NAIR, 2016...)
- Calculation of entanglement entropy for higher dimensional QHE and how this compares with the 2D results.(KARABALI, 2020)

- We divide the system into two regions, D and its complementary D^C, and define the reduced density matrix

$$\rho_D = \mathrm{Tr}_{D^C} \left|GS\right\rangle \left\langle GS\right|$$

where $|GS\rangle = \prod_m c_m^\dagger |0\rangle$.

- The entanglement entropy is defined as

$$S = -\mathrm{Tr}\rho_D \log \rho_D$$

- We choose D to be the spherically symmetric region of $\mathbb{CP}^k$ satisfying $z \cdot \bar{z} \leq R^2$. For $\mathbb{CP}^1 \sim S^2$, this region is a polar cap around the north pole with $R = \tan\theta/2$ via stereographic projection.

- The LLL fermion operator can be expanded as

$$\psi = \sum_m c_m \, \Psi_m(z)$$

- Define "local" operators by

$$a_m = \frac{1}{\sqrt{\lambda_m}} \int_D d\mu \; \Psi_m^* \psi, \qquad b_m = \frac{1}{\sqrt{1-\lambda_m}} \int_{D^C} d\mu \; \Psi_m^* \psi$$

$$\lambda_m = \int_D \Psi_m^* \Psi_m$$

- $\{a_m, a_m^\dagger\}$, $\{b_m, b_m^\dagger\}$ form two independent fermionic algebras and

$$c_m = \sqrt{\lambda_m}\; a_m + \sqrt{1-\lambda_m}\; b_m \qquad c_m^\dagger = \sqrt{\lambda_m}\; a_m^\dagger + \sqrt{1-\lambda_m}\; b_m^\dagger$$

- The reduced matrix ρ_D is written as a $2^N \times 2^N$ matrix of a block diagonal form

$$\rho_D = \otimes_m \mathrm{diag}(\lambda_m,\ 1-\lambda_m)$$

- The entanglement entropy is then given by

$$S = -\mathrm{Tr}\rho_D \log \rho_D = -\sum_{m=1}^{N} [\lambda_m \log \lambda_m + (1-\lambda_m) \log(1-\lambda_m)]$$

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- λ's are eigenvalues of the two-point correlator (Peschel, 2003)

$$C(r,r') = \sum_{m=1}^{N} \Psi_m^*(z)\ \Psi_m(z')\ , \qquad z, z' \in D$$

$$\int_D C(r,r')\Psi_l^*(z')d\mu(z') = \lambda_l\ \Psi_l^*(z)$$

where

$$\lambda_l = \int_D |\Psi_l|^2 d\mu$$

2D RESULTS

- For 2d gapped systems

$$S = c\,L + \gamma + \mathcal{O}(1/L)$$

L: length of boundary

c: non-universal constant

γ: universal, topological entanglement entropy ; $\gamma = 0$ for IQHE

- For integer QHE on $S^2 = \mathbb{CP}^1$ RODRIGUEZ AND SIERRA, 2009

For $\nu = 1$: $c = 0.204$

$\mathbb{CP}^k$: 2k dim space, locally parametrized by z_i, $i = 1, \cdots, k$

$$\mathbb{CP}^k = \frac{SU(k+1)}{U(k)}$$

- $U(k) \sim U(1) \times SU(k) \Longrightarrow$ We can have both $U(1)$ and $SU(k)$ background magnetic fields
- Landau wavefunctions are functions on $SU(k+1)$ with particular transformation properties under $U(k)$.
- There are distinct Landau levels, separated by energy gap.
- Each Landau level forms an irreducible $SU(k+1)$ representation, whose degeneracy is easy to calculate.

QHE ON $\mathbb{CP}^k$ (continued)

Wavefunctions are written in terms of Wigner $\mathcal{D}$ functions

$$\Psi \sim \mathcal{D}^{(J)}_{L,R}(g) = \langle \, L \, | \hat{g} | \, R \, \rangle$$

quantum numbers of states in J rep.

$\hat{g} \in SU(k+1)$

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Left/right transformations: $\hat{L}_A\, \hat{g} = T_A\, \hat{g}, \qquad \hat{R}_A\, \hat{g} = \hat{g}\, T_A$

- $\hat{L}_A \rightarrow$ magnetic translations $\quad (A \in SU(k+1))$
- $\hat{R}_a,\ \hat{R}_{k^2+2k} \rightarrow$ gauge transformations $\quad (U(k))$
- $\hat{R}_{+i}, \hat{R}_{-i} \rightarrow$ covariant derivatives $\ (i = 1, \cdots, k)\ \ [\hat{R}_{+i}, \hat{R}_{-j}] \in U(k)$

- How Ψ transforms under gauge transformations depends on choice of background fields
- Choose "uniform" $U(1)$ or $U(k)$ background magnetic fields.

$$U(1): \qquad \bar{F} = d\bar{a} = n\,\Omega, \quad \Omega = \text{Kahler } 2-\text{form}$$

$$SU(k): \qquad \bar{F}^a \sim \bar{R}^a \sim f^{a\alpha\beta} e^{\alpha} e^{\beta}$$

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- Wavefunction for each Landau level is an $SU(k+1)$ representation J

$$\Psi^J_{m;\alpha} \sim \langle m \mid \hat{g} \mid \underbrace{\;R\;} \;\rangle$$

fixed $U(1)_R$ charge $\sim$ n and some finite $SU(k)_R$ repr. $\tilde{J}$

$m = 1, \cdots \dim J \Longrightarrow$ counts degeneracy of Landau levels

$\alpha =$ internal gauge index $= 1, \cdots, N' = \dim\tilde{J}$

Hamiltonian

$$H = \frac{1}{2Mr^2} \sum_{i=1}^{k} \hat{R}_{+i} \hat{R}_{-i} + \text{constant}$$

Lowest Landau level: $\hat{R}_{-i}\Psi = 0$ Holomorphicity condition

($|\, R \,\rangle$ is lowest weight state)

A. QHE on $\mathbb{CP}^k$ with $U(1)$ magnetic field

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The LLL wavefunctions are essentially the coherent states of $\mathbb{CP}^k$.

$$\begin{aligned}\Psi_{i_1 i_2 \cdots i_k} &= \sqrt{N}\left[\frac{n!}{i_1! i_2! \ldots i_k! (n-s)!}\right]^{\frac{1}{2}} \frac{z_1^{i_1} z_2^{i_2} \cdots z_k^{i_k}}{(1+\bar{z}\cdot z)^{\frac{n}{2}}} \ , \\ s &= i_1 + i_2 + \cdots + i_k \ , \quad 0 \le i_i \le n \ , \quad 0 \le s \le n\end{aligned}$$

They form an $SU(k+1)$ representation of dimension

$$N = \dim J = \frac{(n+k)!}{n!\, k!}$$

The volume element for $\mathbb{CP}^k$ is

$$d\mu = \frac{k!}{\pi^k} \frac{d^2 z_1 \cdots d^2 z_k}{(1+\bar{z}\cdot z)^{k+1}} \quad , \quad \int d\mu = 1$$

- The eigenvalues λ are given by

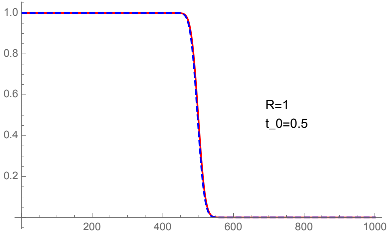
$$\lambda_{i_1 i_2 \cdots i_k} \equiv \lambda_s = \frac{(n+k)!}{(n-s)!(s+k-1)!} \int_0^{t_0} dt\, t^{s+k-1}\,(1-t)^{n-s}$$

where $t_0 = R^2/(1+R^2)$.

- The entanglement entropy is

$$\begin{aligned} S &= \sum_{s=0}^{n} \overbrace{\frac{(s+k-1)!}{s!(k-1)!}}^{\text{degeneracy}} H_s \\ H_s &= [-\lambda_s \log \lambda_s - (1-\lambda_s)\log(1-\lambda_s)] \end{aligned}$$

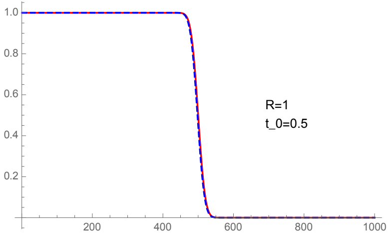
- For large n, this is amenable to a semiclassical analytical calculation for all $k \ll n$.



Graph of λ_s vs s

Transition ($\lambda = \frac{1}{2}$) at $s^* \sim n\, t_0$

$k = 1$, $k = 5$



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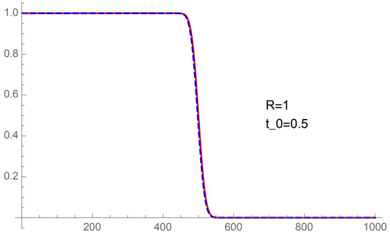

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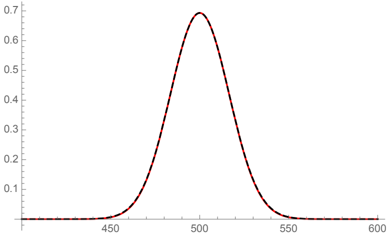

Graph of H_s vs s

—- exact

- - - - Gaussian approximation

UNIVERSAL FORM FOR ENTANGLEMENT ENTROPY FOR $\nu = 1$

From semiclassical analysis

$$S \sim n^{k-\frac{1}{2}} \frac{\pi\,(\log 2)^{3/2}}{2\,k!} \underbrace{2k\frac{R^{2k-1}}{(1+R^2)^k}}_{\text{geometric area}} \sim c_k \text{ Area}$$

In agreement with $k = 1$ result by RODRIGUEZ AND SIERRA

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- Formula for entropy becomes universal if expressed in terms of a ”phase space” area instead of a geometric area.
- $V_{\text{phase space}} = \frac{n^k}{k!} \int \Omega^k = \frac{n^k}{k!} \int d\mu$

$$A_{\text{phase space}} = \frac{n^{k-\frac{1}{2}}}{k!} A_{\text{geom}} = \frac{n^{k-\frac{1}{2}}}{k!}\, 2k \frac{R^{2k-1}}{(1+R^2)^k}$$

$$S \sim \frac{\pi}{2} (\log 2)^{3/2} A_{\text{phase space}}$$

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The LLL single particle states form an $SU(k+1)$ irreducible representation of the type (p, l) corresponding to the tensor

$$\mathcal{T}^{\gamma_1 \cdots \gamma_l}_{b_1 \ldots b_p} \equiv \mathcal{T}^l_p$$

$p : U(1)$ indices , $l : SU(k)$ indices and $p = n - j$ and $l = j\,k$, $j = 1, \cdots$

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Consider simplest case : $\mathbb{CP}^2$ and $l = 2 \Rightarrow SU(2)$ triplet ($dim\tilde{J} = 3$)

The degeneracy of the LLL is

$$N = 3\,\frac{n(n+3)}{2}$$

There are three distinct types of wavefunctions for the $SU(2)$ triplet and three corresponding λ.

$$
\begin{aligned}
\lambda^{(1)}_{s,k=2} &= \lambda^{(\mathrm{Ab})}_{s+1,k=3} \\
\lambda^{(2)}_{s,k=2} &= \frac{n+3}{n+1}\lambda^{(\mathrm{Ab})}_{s+1,k=2} - \frac{2}{n+1}\lambda^{(\mathrm{Ab})}_{s+1,k=3} \\
\lambda^{(3)}_{s,k=2} &= \frac{n+3}{n+1}\lambda^{(\mathrm{Ab})}_{s+1,k=1} - \frac{2(n+3)}{(n+1)(n+2)}\lambda^{(\mathrm{Ab})}_{s+1,k=2} + \frac{2}{(n+1)(n+2)}\lambda^{(\mathrm{Ab})}_{s+1,k=3}
\end{aligned}
$$

$\mathbb{CP}^2$ and nonabelian magnetic field

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$$
\begin{aligned}
S &= \sum_{s=0}^{p}\left[(s+1)H^{(1)}_{s,k=2} + (s+2)H^{(2)}_{s,k=2} + (s+3)H^{(3)}_{s,k=2}\right] \\
&\xrightarrow{\textit{large } n} \sum_{s=0}^{p}\left[(s+1)H^{(\mathrm{Ab})}_{s+1,k=3} + (s+2)H^{(\mathrm{Ab})}_{s+1,k=2} + (s+3)H^{(\mathrm{Ab})}_{s+1,k=1}\right] \\
&\to 3\, n^{3/2}\,\pi\,(\log 2)^{3/2}\,\frac{R^3}{(1+R^2)^2} = 3\, S^{(\mathrm{Ab})}
\end{aligned}
$$

- In general

$$N \xrightarrow{large\ n} \dim \tilde{J}\, \frac{n^k}{k!}$$

- The corresponding phase-space volume in this case is $V_{\text{phase space}} = \dim \tilde{J}\, \frac{n^k}{k!} \int d\mu$

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for any Abelian or non-Abelian background at large n.

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- Is this true for higher Landau levels?

QHE on $S^2 = \mathbb{CP}^1$; 1st excited level

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$$\lambda_s^{(q=1)} = \frac{(n+3)!(n+2)}{s!(n+2-s)!} \int_0^{t_0} dt\, t^{s-1}(1-t)^{n-s+1} \, [t - \frac{s}{n+2}]^2$$

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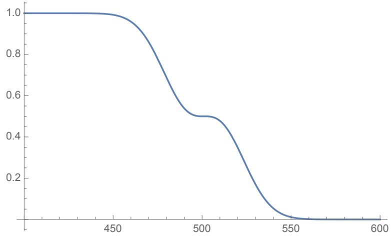

- Step-like pattern around the transition point.

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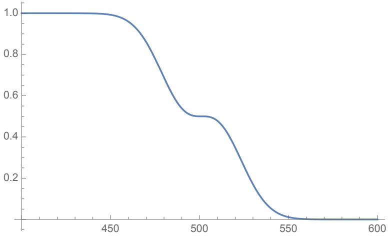
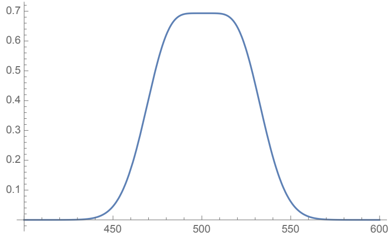


- Step-like pattern around the transition point.

  1st excited level wavefunctions have a node.

1st excited Landau level

- The step-like plateau of λ causes the broadening of the entropy H_s around $\lambda = 1/2$. H_s cannot be approximated with a simple Gaussian.



- Previous semiclassical analysis does not work.

$$S^{(q=1)} = 1.65\, S^{(q=0)}$$

What happens when both $q = 0$ and $q = 1$ Landau levels are full, namely $\nu = 2$?

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The two-point correlator now is given by

$$C(r, r') = \sum_{s=0}^{n} \Psi_s^{*0}(r)\Psi_s^0(r') + \sum_{s=0}^{n+2} \Psi_s^{*1}(r)\Psi_s^1(r')$$

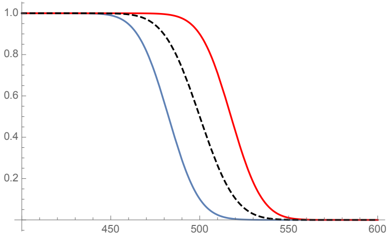
There are $2n + 4$ eigenvalues: λ_0^1 , $\tilde{\lambda}_s^{\pm}$, λ_{n+2}^1, $s = 0, \cdots, n$ and

$$\tilde{\lambda}_s^{\pm} = \frac{\lambda_s^0 + \lambda_{s+1}^1 \pm \sqrt{(\lambda_s^0 - \lambda_{s+1}^1)^2 + 4(\delta\lambda)_{s,s+1}^2}}{2}$$

where

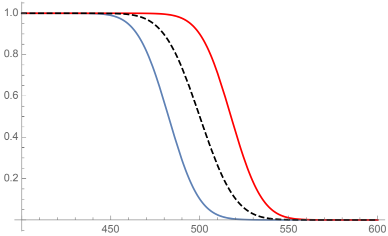
$$\delta\lambda_{s,s+1} = \int_D \Psi_s^{*(q=0)}(r)\ \Psi_{s+1}^{(q=1)}(r)\ d\mu$$

$\nu = 2$ CASE



$\tilde{\lambda}_s^+, \tilde{\lambda}_s^-$

- - - for $\nu = 1$



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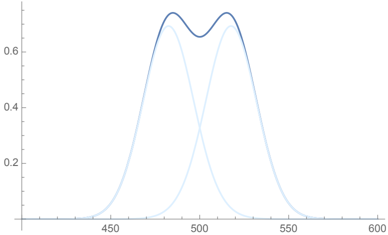
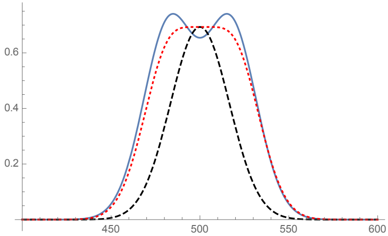


$\tilde{H}_s{}^+ + \tilde{H}_s{}^-$



- - - $H_s^{\nu=1}$

... $H_s^{q=1}$

—- $H_s^{\nu=2}$

$$S = \sum sH_s$$

$$S^{(\nu=2)} > S^{(q=1)} > S^{(\nu=1)}$$

$$S^{(q=1)} = 1.65\, S^{(\nu=1)}$$

$$S^{(\nu=2)} = 1.76\, S^{(\nu=1)}$$

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ESTIENNE AND STEPHAN; ROZON, BOLTEAU AND WITZAK-KREMPA, 2019

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  What are the higher dimensional (Abelian and non-Abelian) analogs?
- How does entanglement entropy change in presence of gauge and gravitational fluctuations? NAIR, 2020