Experimental Platforms for phases with Fractal and other Subsystem Symmetries

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Collaborators:

Nayan E. Myerson-Jain, Stephen Yan, David Weld

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Content:

1, quick review of fractal and subsystem symmetry, and SSB of fractal symmetry (defined as “fractal order”);

2, quick review of Rydberg atoms;

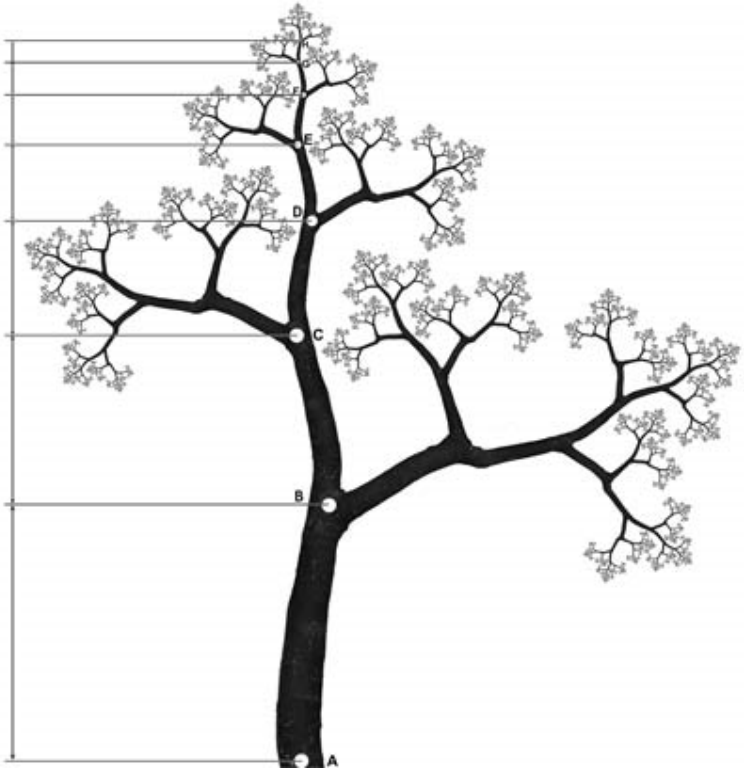
3, realizing fractal symmetry and fractal order with Rydberg atoms;

4, other fractal subsystem symmetries and realizations

Reference: arXiv:2108.07765

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Fractals in nature:



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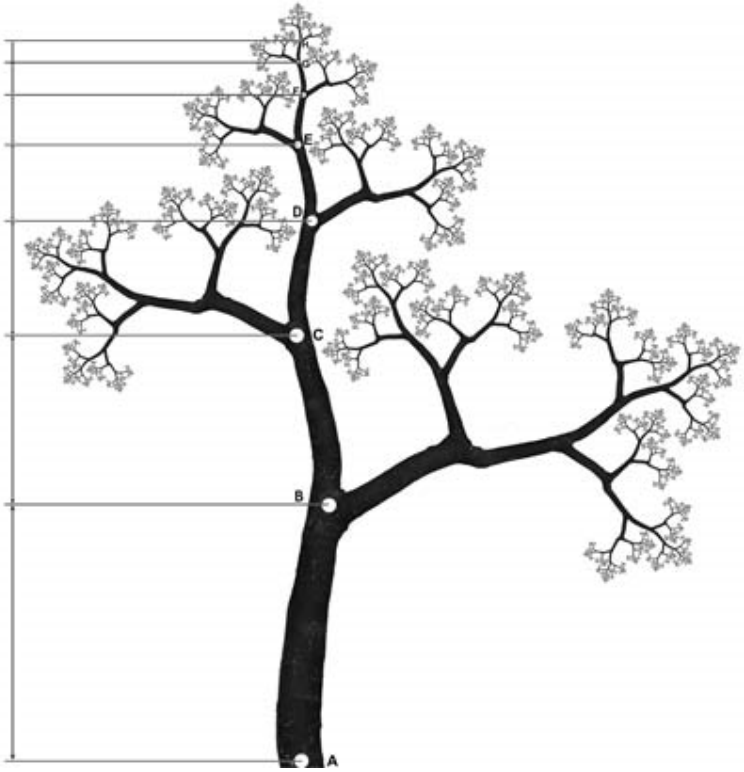
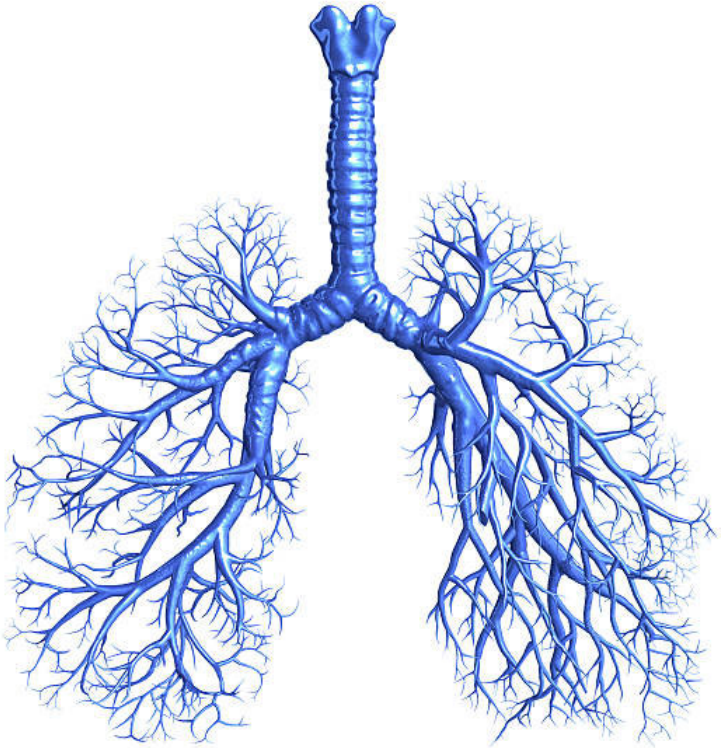
Fractals in nature:





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Fractals in nature:

Nat Rev Clin Oncol. Author manuscript; available in PMC 2016 Aug 18.
Published in final edited form as:
Nat Rev Clin Oncol. 2015 Nov; 12(11): 664–675.
Published online 2015 Jul 14. doi: 10.1038/nrclinonc.2015.108

PMCID: PMC4989864
NIHMSID: NIHMS808299
PMID: 26169924

Lung cancer—a fractal viewpoint

Frances E. Lennon, Gianguido C. Cianci, Nicole A. Cipriani, Thomas A. Hensing, Hannah J. Zhang, Chin-Tu Chen, Septimiu D. Murgu, Everett E. Vokes, Michael W. Vannier, and Ravi Salgia

"……We describe the fractal nature of the lung and explain why this organ is particularly suited to fractal analysis. Studies that have used fractal analyses to quantify changes in nuclear and chromatin FD in primary and metastatic tumour cells, and clinical imaging studies that correlated changes in the FD of tumours on CT and/or PET images with tumour growth and treatment responses are reviewed. Moreover, the potential use of these techniques in the diagnosis and therapeutic management of lung cancer are discussed."

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Subsystem symmetry:

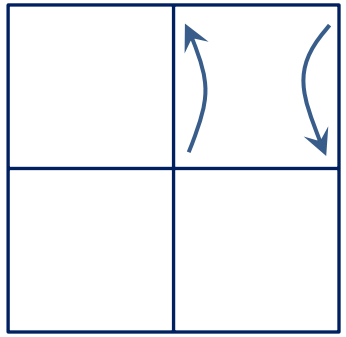
Early examples: 1, 2+1d Bose metal, Paramekanti, Balents, Fisher, 2002 (PBF)

$$\mathcal{H}_{\square} = \frac{U}{2}\sum_{\mathbf{r}}(n_{\mathbf{r}} - \bar{n})^2 - K\sum_{\mathbf{r}}\cos(\Delta_{xy}\phi_{\mathbf{r}}),$$

$$\Delta_{xy}\phi_{\mathbf{r}} \equiv \phi_{\mathbf{r}} - \phi_{\mathbf{r}+\hat{\mathbf{x}}} - \phi_{\mathbf{r}+\hat{\mathbf{y}}} + \phi_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}.$$

$$\phi_{\mathbf{r}} \to \phi_{\mathbf{r}} + f(x) + g(y)$$

1d symmetry, boson number n along each line (both directions) are conserved, as $e^{i\phi}$ is the creation operator of a boson. The system behaves a lot like 1d boson, i.e. no superfluid phase, but a power-law algebraic phase of dipoles (more discussion in part 2).



Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Subsystem symmetry:

Early examples: 2, 2+1d Ising plaquette model,

$$H = -K \sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z - h \sum_i \sigma_i^x .$$

Also has 1d subsystem symmetry which flips the spins on each line, along both directions. Prod σ^x along each line is a conserved quantity. Also behaves like 1d quantum Ising model:
1, self-duality at zero temperature;
2, quantum phase transition at $h = K$ (between an ordered phase at $K > h$ with spontaneous subsystem symmetry breaking, and a disordered phase at $h > K$);
3, no classical phase transition at finite temperature.

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Subsystem symmetry + "Topological" order: Fracton topological order. Type-I: conserved charges on an ordinary subset of the lattice such as a line, or plane (Chamon's model, X-cube model, etc.); type-II, conserved charges on a fractal subset of the lattice (Haah's code) Review of fracton: arXiv:1803.11196, Nandkishore, Hermele; arXiv:2001.01722, Pretko, You, Chen

This talk will focus on subsystem symmetries, especially type-II. The "early examples" mentioned in previous slides both had "type-I" subsystem symmetry. A simple model with the type-II subsystem symmetry is the Newman-Moore (1999)-Yoshida (2013) model, or the Sierspinki-triangle model.

$$H_{\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z,$$

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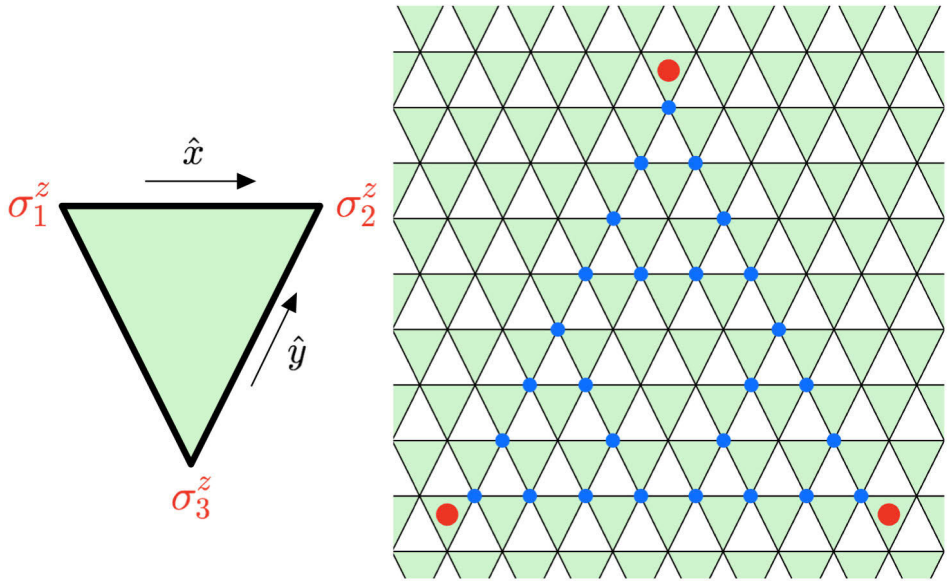
$$H_{\mathrm{ST}} = \sum_{\nabla} -K\sigma_1^z\sigma_2^z\sigma_3^z,$$



1, obvious ground state: all spins = +1. Large number of degenerate excited states: flipping spins on a Sierpinski-triangle; all down-facing triangles have either “3-up” spins, or “2-down, 1-up” spins.

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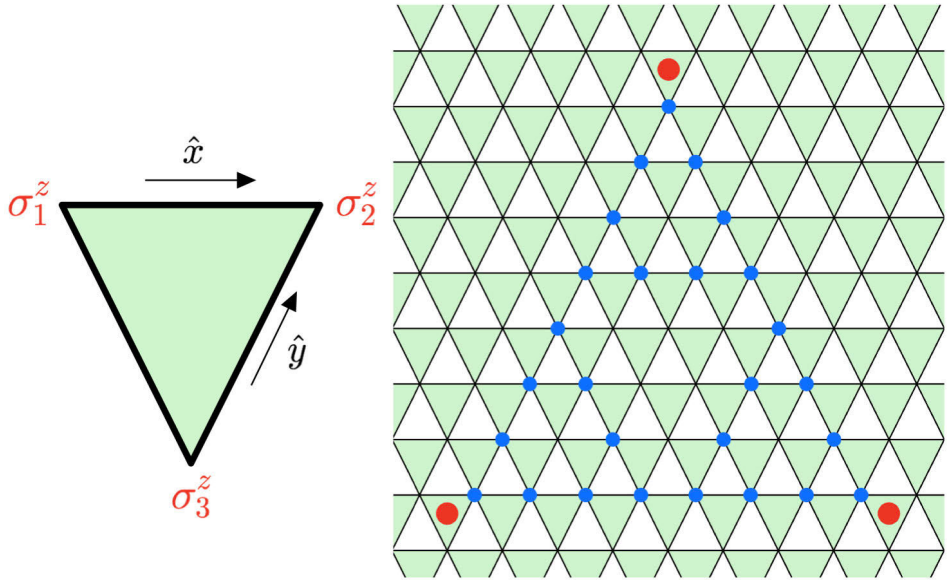
$$H_{\mathrm{ST}} = \sum_{\nabla} -K\sigma_1^z\sigma_2^z\sigma_3^z,$$



2, no phase transition at finite temperature; the three-spin correlation function is short ranged by still have fractal structure (Yoshida 2013).

$$C(r) = (1-2p)^{r^{\log 3/\log 2}}$$

$$r = 2^m$$

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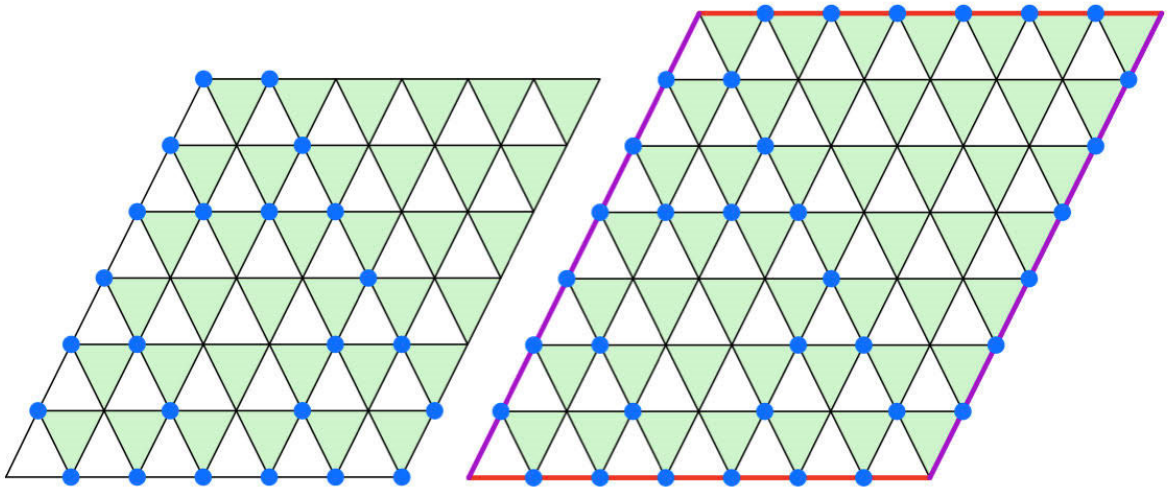
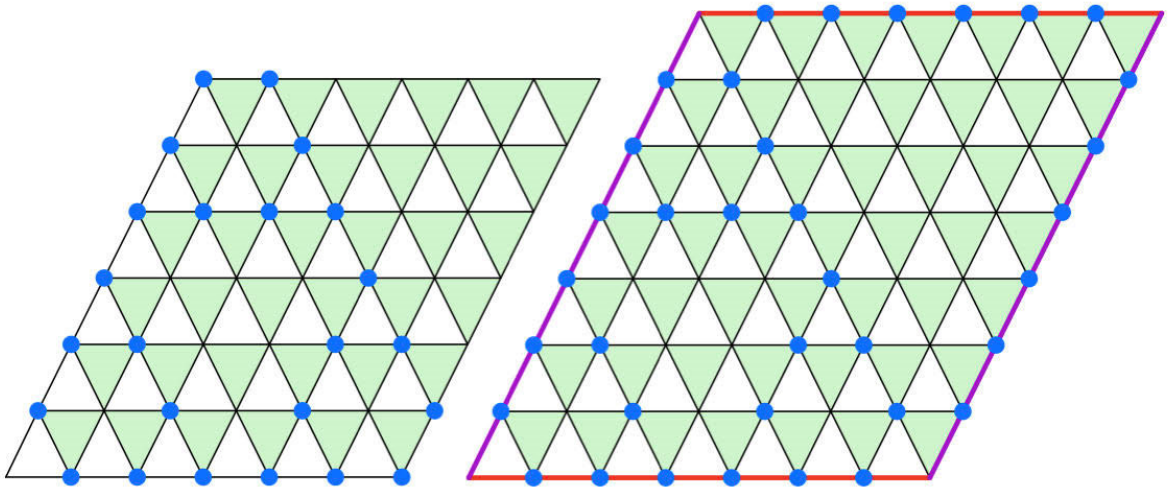
$$H_{\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z,$$



3, fractal symmetry: for a lattice (periodic bc) with size L^2, with $L = 2^k$ -1, the model has an explicit symmetry: the Hamiltonian is invariant under flipping spins on a Sierpinski triangle.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{q\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$



4, quantum version of fractal symmetry: for a lattice (periodic bc) with size L^2, with $L = 2^k$ -1, Prod σ^x on the Sierpinski triangle is a conserved quantity.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{q\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$

5, self-duality (arXiv:2105.05851, Zhou, Zhang, Pollmann, You); like the 1+1d quantum Ising model, and the 2+1d quantum plaquette model with subsystem symmetry: there is (likely) a quantum phase transition at $h = K$.

6, the quantum phase transition at $h=K$ separates two phases:
$K > h$ is a spontaneous symmetry breaking phase of the fractal symmetry, or "fractal ordered" phase, with large ground state degeneracy with system size $L = 2^k$ -1, and nonzero $<\sigma^z>$ in the ground state;
$K < h$ is a disordered phase with a nondegenerate ground state and zero $<\sigma^z>$.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{q\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$

7, nature of the quantum phase transition at $h = K$?

Theoretically, no well established field theory for spontaneous breaking of fractal symmetry yet, no RG analysis…

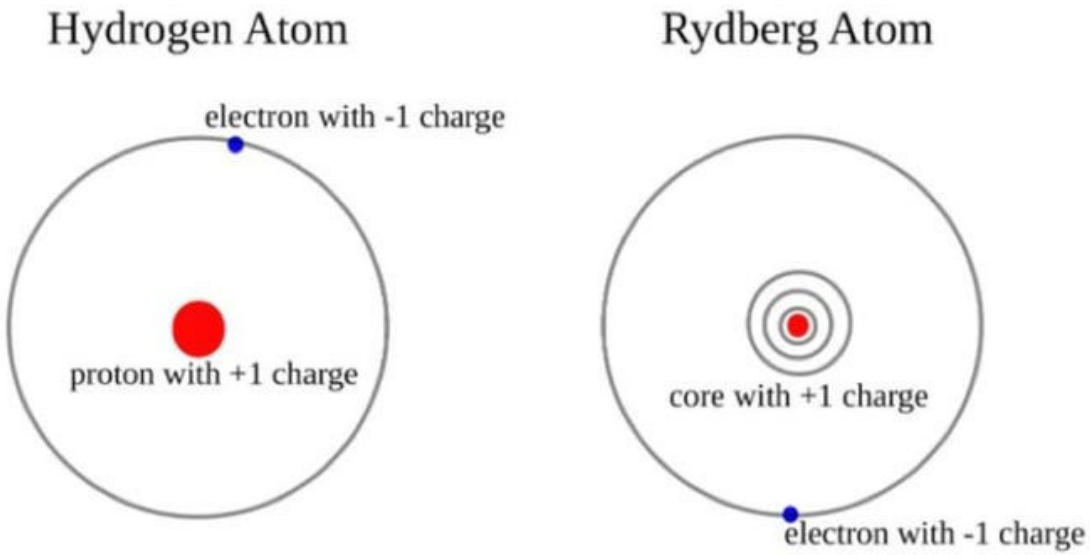
Numerically, earlier result suggests a first order transition (Vasiloiu, et.al. arXiv:1911.11739); more recent work suggests a continuous phase transition (Zhou, et.al. arXiv:2105.05851).

$$G_x(r) = \langle \sigma^x(0)\sigma^x(r)\rangle \sim \frac{1}{r^{2\Delta}}, \quad \Delta = \ln 3/\ln 2$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Rydberg atoms:

A highly excited state of an atom, one electron is excited to a state with large principal quantum number.



$$E_n = -\frac{E_0}{n^2}$$

$$E_n = -\frac{E_0}{(n-\delta_{n,l})^2}$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Rydberg atoms:

A highly excited state of an atom, one electron is excited to a state with large principal quantum number.

Two Rydberg atoms interact strongly through dipole moment fluctuation, the Van der Waals potential, which scales very strongly with the principal quantum number:

$$V(r) \sim \frac{C}{r^6}, \quad C \sim n^{11}$$

Dipole-dipole interaction decays as $1/r^3$. The Rydberg state itself may not have dipole moment; the $1/r^6$ interaction is a second order perturbation.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Rydberg atoms:

Consider an atom-photon coupled system (cavity QED), and atom is coupled with photons whose energy is close to the resonant frequency between ground state and Rydberg state. Define a number operators:

$$|\hat{n}=0\rangle = |g, N_\gamma + 1\rangle, \qquad |\hat{n}=1\rangle = |r, N_\gamma\rangle.$$

For a multi-atom system that is arranged in an array, or a lattice:

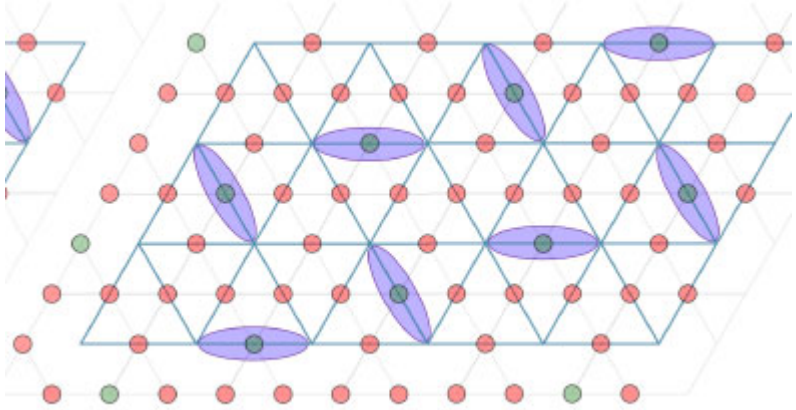
$$H = \sum_i \Omega_i \sigma_i^x + H_0, \quad H_0 = -\sum_i \delta_i \hat{n}_i + \sum_{ij} V_{ij} \hat{n}_i \hat{n}_j,$$

Review article on Rydberg: Browaeys, Lahaye, Nature Physics, 16, 132–142 (2020)

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Realizing strongly correlated phases with Rydberg atoms

Example: Z2 spin liquid/topological order, Samajdar, et.al. 2020, Verresen et.al. 2020

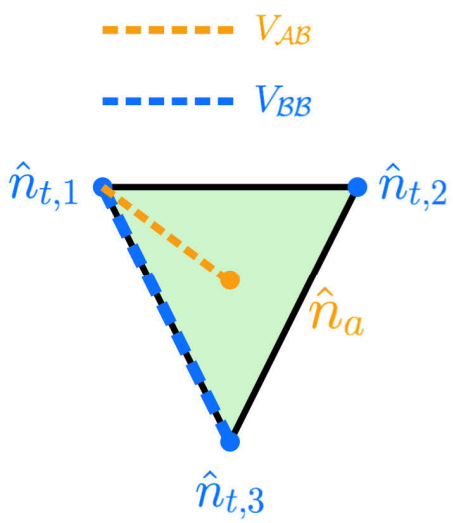


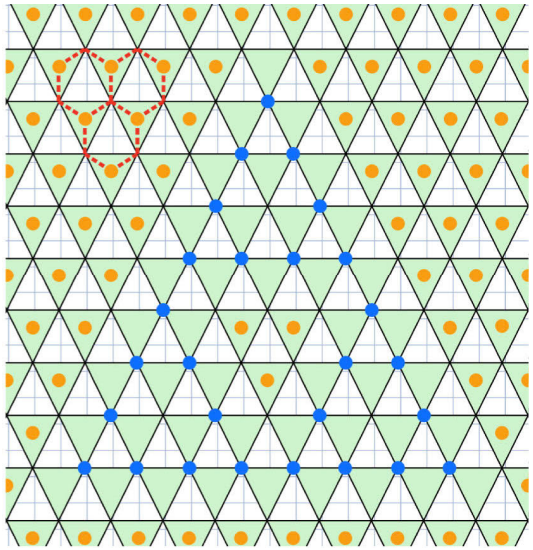
The atoms are arranged in a Kagome lattice; by tuning parameters in the Hamiltonian, neighboring Rydberg atoms are strongly suppressed; the allowed configurations of Rydberg atoms are equivalent to a quantum dimer model on a triangular lattice, which can host a Z2 topological ordered state.

Can we realize the quantum Sierpinski triangle model?

$$H_{q\mathrm{ST}} = \sum_{\triangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$

The difficulty is the 3-body interaction; as condensed matter systems are generally dominated by 2-body interactions. Let's consider a honeycomb lattice, which is a decorated triangular lattice:

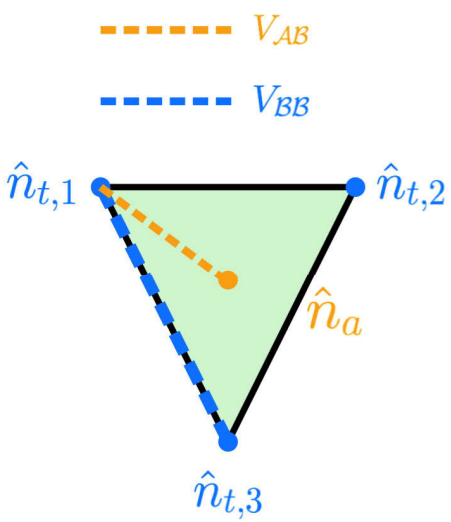


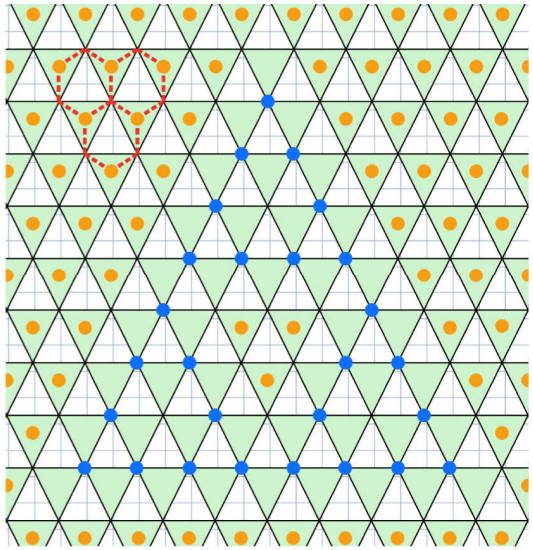


Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

We trap "target" atoms on sublattice B of the honeycomb lattice (vertices of the triangular lattice); and "auxiliary" atoms on sublattice A (centers of down-triangles). Target atoms and auxiliary atoms have principal quantum numbers n_B and n_A respectively.





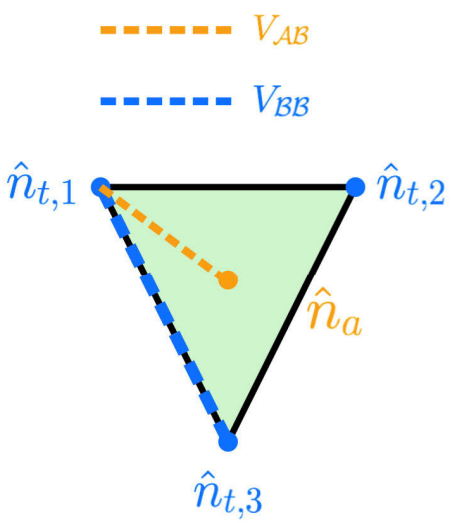
Experimental Platforms for phases with Fractal and other Subsystem Symmetries

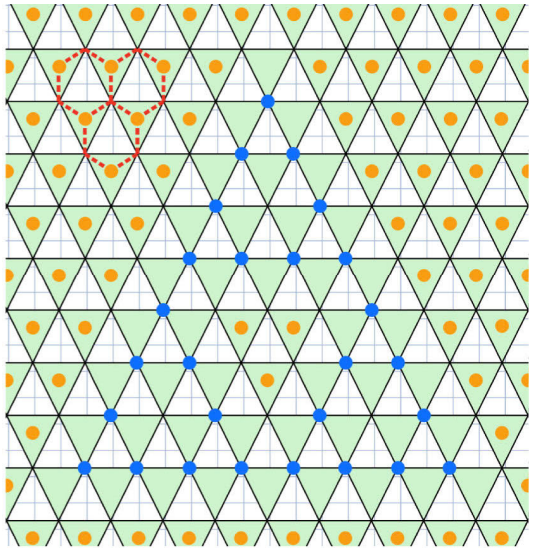
$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

Ground states of H_0: ⟶ $\sigma = 1 - 2\hat{n}_t$ ⟶ Ground states of H_{ST}:

(1) $\hat{n}_a = 1,\ \hat{n}_{t,i} = 0;$ (1) $\sigma_1^z = \sigma_2^z = \sigma_3^z = +1;$

or (2) $\hat{n}_a = 0,$ two of $\hat{n}_{t,i} = 1.$ or (2) Two of $\sigma_i^z = -1.$

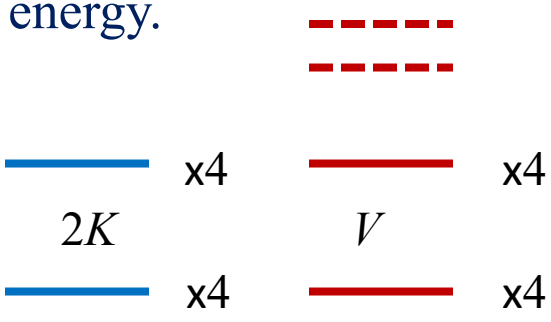




Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

All the states of the spin model is mapped to the low energy Hilbert space of the atomic system with the same degeneracy;
All the extra states of the atomic system have higher energy.



$$\sigma^z = (+1,+1,+1), \quad \text{Energy} = -K$$

$$\rightarrow \quad (\hat{n}_{t,i};\hat{n}_a) = (0,0,0;1), \quad \text{Energy} = 0;$$

$$\sigma^z = (-1,-1,+1), \quad \text{Energy} = -K,$$

$$\rightarrow \quad (\hat{n}_{t,i};\hat{n}_a) = (1,1,0;0), \quad \text{Energy} = 0;$$

$$\sigma^z = (-1,+1,+1), \quad \text{Energy} = +K,$$

$$\rightarrow \quad (\hat{n}_{t,i};\hat{n}_a) = (1,0,0;0), \quad \text{Energy} = V;$$

$$\sigma^z = (-1,-1,-1), \quad \text{Energy} = +K$$

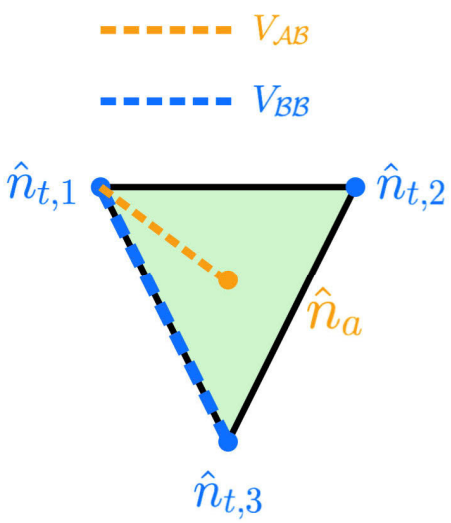
$$\rightarrow \quad (\hat{n}_{t,i};\hat{n}_a) = (1,1,1;0), \quad \text{Energy} = V.$$

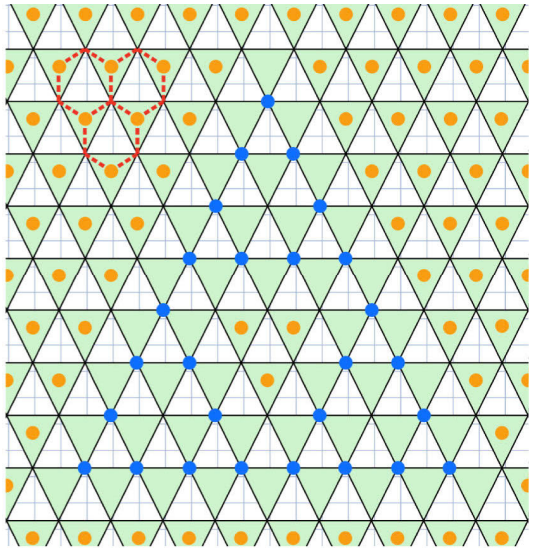
Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

$$\sim V\left(\sum_{i=1}^{3} 4\hat{n}_a\hat{n}_{t,i} + \sum_{i<j} 2\hat{n}_{t,i}\hat{n}_{t,j} - 4\hat{n}_a - \sum_{i=1}^{3} 3\hat{n}_{t,i}\right)$$

$$+ \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}\cdots . \qquad (5)$$





Experimental Platforms for phases with Fractal and other Subsystem Symmetries

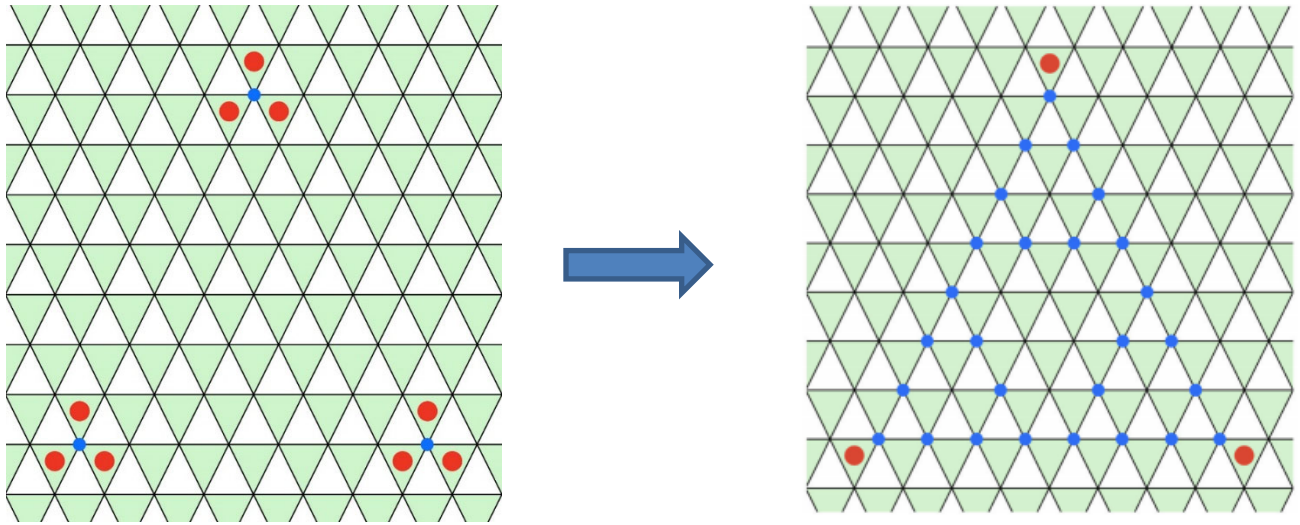
$$
\begin{aligned}
H_{0,a} &= V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i} \\
&\sim V\left(\sum_{i=1}^{3} 4\hat{n}_a\hat{n}_{t,i} + \sum_{i<j} 2\hat{n}_{t,i}\hat{n}_{t,j} - 4\hat{n}_a - \sum_{i=1}^{3} 3\hat{n}_{t,i}\right) \\
&+ \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}\cdots . \qquad (5)
\end{aligned}
$$

The physics described can be realized by choosing (for example) $n_B = 113$ and $n_A = 76$ for potassium atoms. Because the VdW interaction between Rydberg atoms decays fast with distance, perturbations (further neighbor interactions, etc.) are small. For example:

$$
\frac{V'_{\mathcal{B}\mathcal{B}}}{V_{\mathcal{B}\mathcal{B}}} = \frac{1}{(\sqrt{3})^6} \sim 0.037, \quad \frac{V'_{\mathcal{A}\mathcal{B}}}{V_{\mathcal{B}\mathcal{B}}} = \frac{1}{2^6}\frac{V_{\mathcal{A}\mathcal{B}}}{V_{\mathcal{B}\mathcal{B}}} \sim 0.041.
$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Experiment visualization of fractal shape: slow manipulation on the corners of the Sierpinski triangle can spontaneously generate a Sierpinski-triangle shape of excitations.



Fast manipulation of parameters can probe quantum dynamics of the system. Fractal subsystem symmetry can lead to exotic dynamics.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H = \sum_i \Omega_i \sigma_i^x + H_0, \quad H_0 = -\sum_i \delta_i \hat{n}_i + \sum_{ij} V_{ij} \hat{n}_i \hat{n}_j,$$

Turn on the Rabi oscillation term can potentially drive a quantum phase transition like the one in the quantum Sierpinski triangle model.

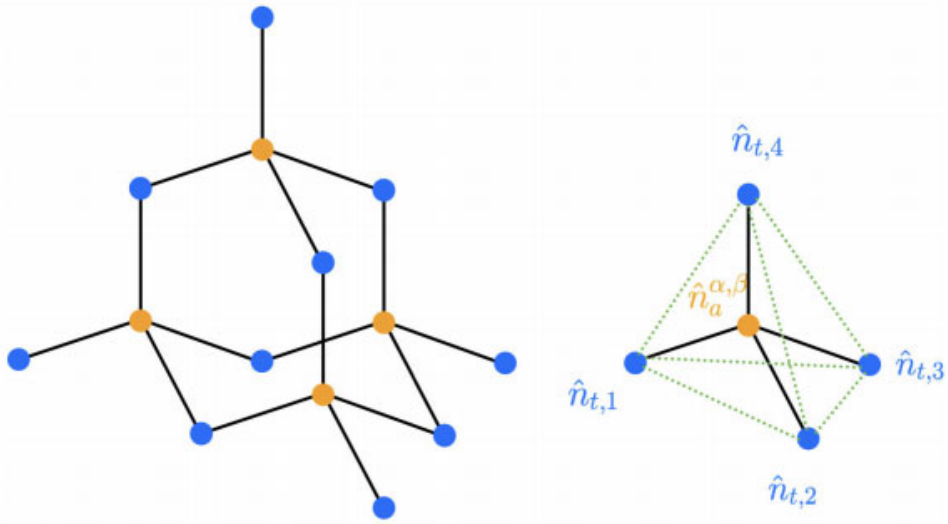
Experimentally, one can take "snapshots" of the configurations of the atoms; by averaging over many snapshots $< \sigma\, \sigma\, \sigma \ldots >$ we obtain the correlation function.

Comment: due to the lack to theoretical formalism for this quantum phase transition, we do not know whether the perturbations and the extra higher energy states mentioned before is going to be relevant or irrelevant at the quantum phase transition. Having an experimental realization may help clarify this question, and help building future theoretical paradigm for QPT involving fractal symmetries.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Models with other fractal structure

$$H_{\text{tetra}} = \sum_{\text{tetra}} -K\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z.$$



The tetrahedron model also has a fractal symmetry, and its excitations have a fractal geometry like the Sierpinski triangle model. The four body interaction can still be simulated through two body VdW interaction between Rydberg atoms after decorating the center of each tetrahedron with an auxiliary atom.

Summary for part 1:

1, quantum Sierpinski triangle model, especially its quantum phase transition involves a fractal subsystem symmetry and its spontaneous breaking, which calls for new theoretical paradigm;

2, realization of the quantum Sierpinski triangle model which involves 3-body interaction, through Rydberg atoms with only 2-body interaction.

3, the platform of Rydberg atom can directly probe the fractal structure, the correlation functions, and quantum dynamics.

Acknowledgements: Xu group: Simons Foundation; NSF-DMR. Weld group: MURI; UC Multicampus Research Programs and Initiatives; UCSB NSF Quantum Foundry (Q-AMASE-i initiative).

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Collaborators:

Matthew P. Fisher

Content:

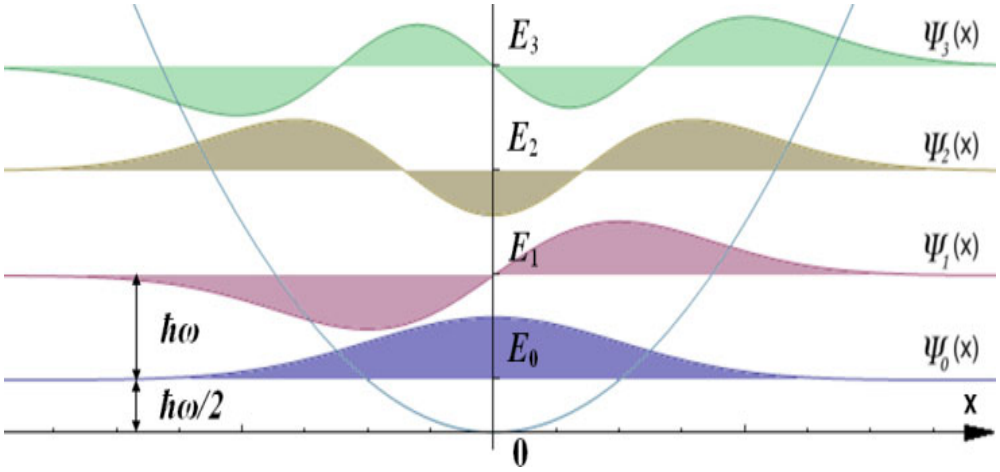
1, quick review of "*p*-band" cold atom;

2, strongly interacting p-orbital atoms in 2d, and mappings to Bose metal and quantum plaquette model at low energy;

3, strongly interacting p-orbital atoms in 3d, low energy multi flavor ring exchange model, stability of gapless phase in 2d and 3d.

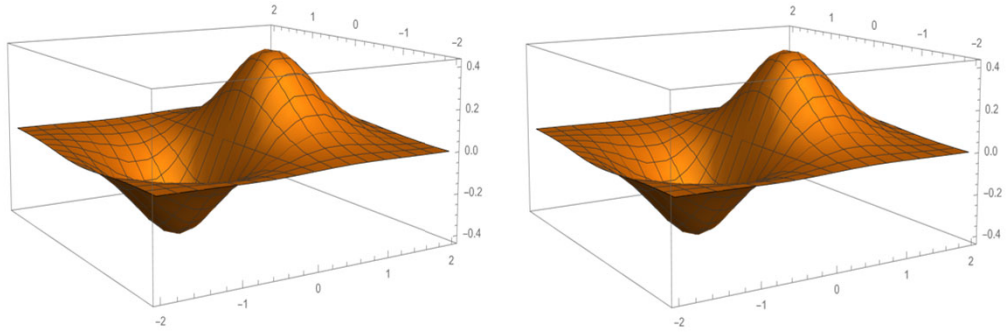
Reference: cond-mat/0611620

Quick review of "*p*-band" atom



Neutral atoms can be trapped in an optical lattice. Each site of the lattice is approximately a quadratic potential. The ground state is always an *s*-orbital; the 1st excited state is a *p*-orbital. Notice that here we are discussing the wave function of atoms, not electrons.

Quick review of "*p*-band" atom



p_x orbital wave function extends further along the x direction, hence has stronger kinetic energy / hopping along the x direction; same for the p_y orbital. Evaluation shows that the hopping anisotropy increases with the depth of the lattice site potential:

$$t(p_x, \hat{x})/t(p_x, \hat{y}) = t(p_y, \hat{y})/t(p_y, \hat{x})$$ Increases with lattice potential

Quick review of "*p*-band" atom

The atoms on the *p*-orbital, though on an exited states, may still have a long enough lifetime to reach equilibrium on *p*-orbitals before decaying into *s*-orbital and other orbitals.
Isacsson, Girvin, PRA. 72, 053604 (2005), Wu, et.al. PRL. 97, 190406 (2006), and many others theory works.
Experimental evidence of superfluid phase on *p*-orbitals: Wirth, et.al. Nature Physics, 7, 147–153 (2011)

We stat with the limit that, p_x orbital hops only along the *x*-direction, p_y orbital hops only along the *y*-direction, etc. Other hopping will be treated as perturbations later:

$$H = -t' \sum_i \sum_a (d_i^{(a)\dagger} d_{i+\hat{a}}^{(a)} + h.c.) + U(\sum_a n_i^{(a)} - \bar{n})^2 + \ldots$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

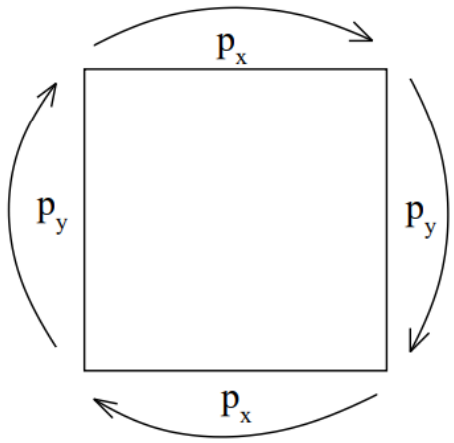
$$H = -t' \sum_i \sum_a (d_i^{(a)\dagger} d_{i+\hat{a}}^{(a)} + h.c.) + U(\sum_a n_i^{(a)} - \bar{n})^2 + \ldots$$

We consider a square optical lattice with atoms on the p_x and p_y orbitals on each site. We assume that the interaction U is the dominant energy scale, namely the total particle number, $n^{(x)} + n^{(y)}$ does not fluctuate. Then at low energy the only allowed dynamical process is

$$d^{(a)} \sim \exp(\mathrm{i}\theta_a) \quad H_{\mathrm{ring}} = -t \cos(\nabla_x \nabla_y (\theta_x - \theta_y))$$

$$\theta \sim \theta_x - \theta_y \quad n \sim n^{(x)} - n^{(y)}$$

$$H_{\mathrm{eff}} = \sum_{\mathbf{x}} -t \cos(\partial_x \partial_y \theta) + \frac{u}{2} n^2$$



which reduces to the Bose metal model (PBF, 2002) with 1d subsystem symmetry.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{\text{eff}} = \sum_{\mathbf{x}} -t\cos(\partial_x\partial_y\theta) + \frac{u}{2}n^2$$

The Bose metal phase with 1d subsystem symmetry can be constructed by the *p*-orbital cold atom.

1, ignoring other perturbations, the Bose metal model has a gapless phase with the Lagrangian

$$\mathcal{L} \sim \frac{1}{2u}(\partial_\tau\theta)^2 + \frac{t}{2}(\partial_x\partial_y\theta)^2 \qquad\qquad \omega \sim \sqrt{ut}|k_x k_y|$$

The spectrum has subsystem symmetry protected line nodes; the system behaves similarly to a 1d boson: bosons do not condense, but the spectrum remains gapless.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{\text{eff}} = \sum_{\mathbf{x}} -t\cos(\partial_x\partial_y\theta) + \frac{u}{2}n^2$$

2, property of the gapless phase: the boson correlation is not long ranged, nor does it decay exponentially (PBF 2002).

$$\mathcal{L} \sim \frac{1}{2u}(\partial_\tau\theta)^2 + \frac{t}{2}(\partial_x\partial_y\theta)^2 \qquad\qquad \omega \sim \sqrt{ut}|k_x k_y|$$

$$\langle e^{\mathrm{i}\theta_{0,0}} e^{-\mathrm{i}\theta_{x,0}} e^{\mathrm{i}\theta_{0,y}} e^{-\mathrm{i}\theta_{x,y}}\rangle \sim \exp\left(-c\sqrt{u/t}\ln|x|\ln|y|\right)$$

3, just like 1+1d boson, this algebraic phase is self-dual with dual Lagrangian (PBF).

$$\mathcal{L}_d \sim \frac{1}{2t}(\partial_\tau\varphi)^2 + \frac{u}{2}(\partial_x\partial_y\varphi)^2$$

$$\langle e^{\mathrm{i}\varphi_{0,0}} e^{-\mathrm{i}\varphi_{x,0}} e^{\mathrm{i}\varphi_{0,y}} e^{-\mathrm{i}\varphi_{x,y}}\rangle \sim \exp\left(-c\sqrt{t/u}\ln|x|\ln|y|\right)$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{\text{eff}} = \sum_{\mathbf{x}} -t\cos(\partial_x \partial_y \theta) + \frac{u}{2} n^2$$

4, in real system, one single p_x atom cannot mix with a p_y atom, due to their opposite parity; but two p_x atoms can interact, and transit into two p_y atoms. This process is described by the following term:

$$H' = \sum_{\mathbf{x}} v' \cos(2\theta_x - 2\theta_y) = \sum_{\mathbf{x}} v' \cos(2\theta)$$

When *u'* is strong enough, it pins θ to a Z2 value, and the system is further reduced to the quantum plaquette model:

$$H = -K \sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z - h \sum_i \sigma_i^x .$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{\text{eff}} = \sum_{\mathbf{x}} -t\cos(\partial_x \partial_y \theta) + \frac{u}{2} n^2$$

5, stability of the Bose metal algebraic phase against subsystem symmetry breaking perturbations?

Example of perturbation: transition from two p_x atoms to p_y atoms.

$$H' = \sum_{\mathbf{x}} v' \cos(2\theta_x - 2\theta_y) = \sum_{\mathbf{x}} v' \cos(2\theta)$$

Compare with 1d compact boson (quantum rotor model)

$$H = \sum_{x} -t\cos(\nabla_x \theta) + \frac{u}{2} n^2 \qquad\qquad \mathcal{L} = \frac{1}{2K}(\partial_\mu \theta)^2$$

Perturbation that break U(1) to Z_N: $\qquad v\cos(N\theta)$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

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Dual theory: $\quad \mathcal{L}_d = \frac{K}{2}(\partial_\mu\varphi)^2$

Perturbation in the dual theory: $\quad v'\cos(N'\varphi)$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

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Dual theory: $\mathcal{L}_d = \frac{K}{2}(\partial_\mu\varphi)^2$

Perturbation in the dual theory: $v'\cos(N'\varphi)$

Scaling dimension of *v* and *v'* in the algebraic phase is universal:

$$\Delta[\cos(N\theta)] \sim KN^2$$

$$\Delta[\cos(N'\varphi)] \sim N'^2/K$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

$$H_{\text{eff}} = \sum_{\mathbf{x}} -t\cos(\partial_x\partial_y\theta) + \frac{u}{2}n^2$$

5, stability of the Bose metal algebraic phase against subsystem symmetry breaking perturbations?

Scaling dimension of v and v' in the algebraic phase is universal:

$$\Delta[\cos(N\theta)] \sim KN^2$$

$$\Delta[\cos(N'\varphi)] \sim N'^2/K$$

The analysis for the 2+1d Bose metal algebraic phase is far less universal. It is fairly certain that terms like $\cos(2\theta)$ is irrelevant, but what could be relevant is dipole like terms (PBF, 2002)

$$v\cos(N\nabla_\mu\theta) \qquad\qquad v'\cos(N'\nabla_\mu\varphi)$$

For example, let's take $\mu = y$, then v and v' term have power-law correlation un the (x, τ) plane, but the power of the correlation depends on the details on the lattice scale.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

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For example, the "scaling dimension" depends on the UV deformation on H_{eff}, and depends on the lattice definition of

$$N\nabla_y \theta = N(\theta_{\mathbf{r}+\hat{y}} - \theta_{\mathbf{r}}), \quad \text{or} \quad N\nabla_y \theta = \theta_{\mathbf{r}+N\hat{y}} - \theta_{\mathbf{r}}$$

This "peculiarity" is called UV/IR mixing in recent years, see for example arXiv:2108.00020, Gorantla et.al.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

3d optical lattice, with degenerate p_x, p_y, p_z atomic states on each site, with the constraint $n^{(x)} + n^{(y)} + n^{(z)} =$ constant induced by interaction, the effective low energy theory is a 3d multi flavor ring exchange model:

$$H_r = -t \sum_i \sum_{a \neq b} \cos(\partial_a \partial_b (\theta_{ai} - \theta_{bi})) + \sum_a u(n_{ai} - \bar{n}_a)^2$$

There is an analogue of the gapless phase as the 2d Bose model phase, but it has two gapless modes, with 1d line nodes, and also 2d plane nodes, due to the subsystem symmetry:

$$\omega_1^2 \sim 2(k_x^2 k_y^2 + k_x^2 k_z^2 + k_y^2 k_z^2),$$

$$\omega_2^2 \sim \frac{3}{2} k_x^2 k_y^2 k_z^2 \frac{k^2}{k_x^2 k_y^2 + k_x^2 k_z^2 + k_y^2 k_z^2}.$$

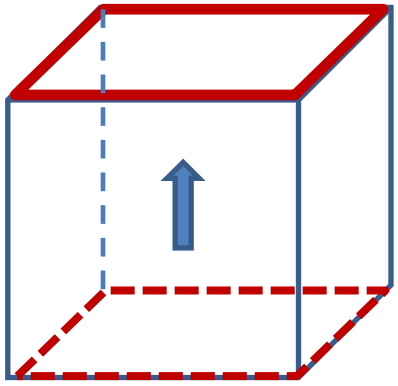
Experimental Platforms for phases with Fractal and other Subsystem Symmetries

The dual theory of the 3d multi-flavor ring exchange model, is a U(1) gauge theory (or loop model) with subsystem symmetry:

$$\mathcal{L}_d = \sum_a \frac{1}{2K}(\partial_\tau \varphi_a)^2 + \sum_{a\neq b, b\neq c, c\neq a} \frac{1}{2K}(\partial_a(\partial_b \varphi_c - \partial_c \varphi_b))^2.$$

$$\begin{aligned}
\varphi_x &\to \varphi_x + f_1(x,y) + f_2(x,z) + \partial_x \alpha(x,y,z),\\
\varphi_y &\to \varphi_y + g_1(x,y) + g_2(y,z) + \partial_y \alpha(x,y,z),\\
\varphi_z &\to \varphi_z + h_1(x,z) + h_2(y,z) + \partial_z \alpha(x,y,z).
\end{aligned}$$

The same dispersion as the original model, i.e. modes have 1d line nodes, and also 2d plane nodes.



This dual Lagrangian describes the gapless phase of a "loop model", where a unit loop hops along the direction orthogonal to it.

$$\frac{1}{2}(\partial_z(\partial_x \varphi_y - \partial_y \varphi_x))^2 \sim -\cos(\nabla_z(\nabla_x \varphi_y - \nabla_y \varphi_x))$$

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

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$$\mathcal{L}_d = \sum_a \frac{1}{2K}(\partial_\tau \varphi_a)^2 + \sum_{a\neq b, b\neq c, c\neq a} \frac{1}{2K}(\partial_a(\partial_b\varphi_c - \partial_c\varphi_b))^2.$$

$$H_r = -t\sum_i \sum_{a\neq b} \cos(\partial_a\partial_b(\theta_{ai} - \theta_{bi})) + \sum_a u(n_{ai} - \bar{n}_a)^2$$

The analysis of the subsystem symmetry breaking perturbations also depend on UV details.

Example: $\cos(\nabla_x\varphi_y - \nabla_y\varphi_x)$

The term has power-law correlation in the (z, τ) plane, but its decay power depends on the UV details.

Experimental Platforms for phases with Fractal and other Subsystem Symmetries

Summary for part 2:

1, atoms excited to the p-orbital of an optical trap may have long life time, and can form many-body states on the p-orbital;

2, interacting p-orbital atoms can form exotic states of matter with subsystem symmetries in both 2d and 3d optical lattice;

3, analysis of the stability of the algebraic "Bose metal" like phases in both 2d and 3d depends on lattice scale physics.

Reference: cond-mat/0611620