

University of Zurich<sup>UZH</sup>

# Emergent blackhole dynamics in critical Floquet systems Titus Neupert

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# How do quantum systems heat up?

#### Outline

# 1. Intro to Floquet theory, heating, and conformal field theory

2. Solving a Floquet CFT problem

3. Propagation of excitations

4. Numerics for a lattice system

Bonus: Hyperbolic space in an electric circuit

#### Floquet systems

$$\begin{array}{c} \text{state at} \\ \text{time t} \end{array} = \exp\left[-i \int_{t_0}^t \mathrm{d}t' H(t')\right] \left| \begin{array}{c} \text{initial} \\ \text{state} \end{array} \right\rangle$$

systems with time-periodic driving no energy conservation

$$H(t) = H(t + nT)$$





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**Floquet unitary:** evolution by one period





$$\exp\left[-i\int_{t_0}^{t_0+T} \mathrm{d}t' H(t')\right] \equiv \exp\left[-iT'H_{\mathrm{F}}\right]$$

**Floquet Hamiltonian:** exists, but not easy to compute and not short-ranged in generic interacting systems

## Floquet systems: heating

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Isolated, driven and interacting systems thermalize (mostly)

D'Alessio & Rigol 2014 Lazarides, Das & Moessner 2014

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Wintersperger et al., Phys. Rev. X 10, 011030 (2020)

### Floquet systems: irradiated graphene

Topological gap opening in noninteracting graphene by circularly polarized light



# Floquet systems: irradiated graphene

Topological gap opening in noninteracting graphene by circularly polarized light

realization on topological insulator surface





Wang et al. Science, 2013



Kitagawa et al. 2011



**generic case:** thermalization ETH volume law entanglement



#### scar states:

isolated sub-volume law states in the middle of the spectrum **generic case:** thermalization ETH

volume law entanglement





#### many-body localization:

all states area law entangled



entanglement entropy

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heating and non-heating phases



heating and non-heating phases



interacting particles



heating and non-heating phases



interacting particles



analytically exact calculation of floquet Hamiltonian and time evolution



heating and non-heating phases



interacting particles



analytically exact calculation of floquet Hamiltonian and time evolution



spatially structured heating phase



heating and non-heating phases



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analytically exact calculation of floquet Hamiltonian and time evolution



spatially structured heating phase



interpretation in terms of quasiparticle propagation

#### How does a driven system heat up?



Quantum critical systems in 1D - scale invariant low energy theory - conformal field theory

#### Conformal field theory



Invariance under conformal transformation (angle preserving maps)

scale invariance  $oldsymbol{x} 
ightarrow \lambda oldsymbol{x}$ 

#### Conformal field theory



Invariance under conformal transformation (angle preserving maps)

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Infinite Virasoro algebra in 1+1 D  $[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$ Central charge  $c = \frac{1}{2}$  c = 1free fermions free bosons

#### Conformal field theory



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#### Closed sub-algebra

$$\begin{split} & [L_0, L_{-1}] = L_{-1} \\ & [L_0, L_1] = -L_1 \\ & [L_1, L_{-1}] = 2L_0 \\ & L \to \overline{L} \end{split}$$

#### Generators of global conformal group

 $SL(2,\mathbb{C})$  rotations, dilations, translations, special conformal trans

CFT Hamiltonian 
$$\mathcal{H}_0 = \frac{2\pi}{L}(L_0 + \bar{L}_0)$$

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#### Floquet drive

**CFT Hamiltonian** 

$$\mathcal{H}_0 = \frac{2\pi}{L} (L_0 + \bar{L}_0)$$



Does the system heat up?



$$\mathcal{H}_0 = \sum_{j=1}^L h_{j,j+1} + \sum_{j=1}^L h_j$$
$$\mathcal{H}_{\text{SSD}} = \sum_{j=1}^L f_{j+\frac{1}{2}} h_{j,j+1} + \sum_{j=1}^L f_j h_j$$
$$f_x = \sin^2 \left[\frac{\pi}{L} \left(x - \frac{1}{2}\right)\right]$$



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Trick to eliminate boundary effects Asano & Hotta 2018



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Ground state:

Trick to eliminate boundary effects



Asano & Hotta 2018



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Asano & Hotta 2018

Hamiltonian:  $\mathcal{H}_{SSD} = L_0 - \frac{1}{2}(L_1 + L_{-1}) + \overline{L}_0 - \frac{1}{2}(\overline{L}_1 + \overline{L}_{-1})$ 

$$\mathcal{H}_0 = \frac{2\pi}{L} (L_0 + \bar{L}_0)$$
  
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#### Möbius transformation



Ishibashi, Okunishi, Tada 2016

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#### Möbius transformation



in particular

Ishibashi, Okunishi, Tada 2016

$$\hat{z} = f(z) = \frac{-\cosh(\theta)z + \sinh(\theta)}{\sinh(\theta)z - \cosh(\theta)}$$

$$\mathcal{H}_0 = \frac{2\pi}{L} (L_0 + \bar{L}_0)$$
  
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in *z* coordinates

 $\mathcal{H}_{0} = \mathcal{H}_{M\ddot{o}b(0)} \qquad \qquad \mathcal{H}_{SSD} = \mathcal{H}_{M\ddot{o}b(\theta \to \infty)}$ 

$$\mathcal{H}_{\mathrm{M\ddot{o}b}(\theta)} \propto \frac{2\pi}{L\cosh(2\theta)} (L_0 + \overline{L}_0)$$

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#### Time evolution by $\mathcal{H}_{\mathrm{M\ddot{o}b}( heta)}$

in  $\hat{z}$  coordinates simple dilation by factor  $\lambda = \exp\left\{\frac{2\pi\tau}{L\cosh 2\theta}\right\}$ 

 $\mathcal{H}_{\mathrm{M\ddot{o}b}(\theta)} \propto \frac{2\pi}{L\cosh(2\theta)} (L_0 + \overline{L}_0)$ 

in z coordinates change of coordinates  $z_{\theta}^{\text{new}}(z) = f^{-1} \left(\lambda f(z)\right)$
# Dynamics – composition of Möbius transformations







# Floquet Hamiltonian: exact

$$\mathcal{H}_{\text{eff}} = \alpha \left[ L_0 - \frac{\beta}{2} (L_1 + L_{-1}) + \bar{L}_0 - \frac{\beta}{2} (\bar{L}_1 + \bar{L}_{-1}) \right]$$

$$\beta^{-1} = \cos(\frac{\pi T_0}{L}) + \frac{L}{\pi T_1} \sin(\frac{\pi T_0}{L})$$

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 $\beta^{-1} = \cos(\frac{\pi T_0}{L}) + \frac{L}{\pi T_1} \sin(\frac{\pi T_0}{L})$ Heating phase  $|\beta| > 1$ Nonheating phase  $|\beta| < 1$ 

unbounded from below

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unbounded from below

 $SL(2,\mathbb{R})$ -related manifolds  $\alpha\beta$ 

Casimir  $SL(2, \mathbb{R})$  $c^{(2)} = \alpha^2(1 - \beta^2)$ 

# Dynamics – composition of Möbius transformations



Entanglement entropy and Loschmidt echo



	Quench	Non-heating	Heating	Phase transition
$S_A(t)$	$\log(t)$	oscillating	t	$\log(t)$
$\mathcal{L}(t)$	$1/(1+t^2)$	oscillating	$e^{-t}$	$1/(1+t^2)$

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# **3. Propagation of excitations**

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Bonus: Hyperbolic space in an electric circuit

# Quantum circuit perspective:

information spreading in interacting quantum states

translationally invariant system



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# information spreading in interacting quantum states



#### Correlations $\langle G | \phi(x,t) \phi(x_0,0) | G \rangle$

#### Dubail et al

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No driving









 $\langle \phi(x,t)\phi(0,0)\rangle \propto \frac{1}{(x-v(x)t)^2}\frac{1}{(x+v(x)t)^2}$ 





Excitations propagate in curved space

$$ds^2 = dx^2 - \left(2\sin^2\left(\frac{\pi x}{L}\right)\right)^2 dt^2$$

Dubail et al





**Emergent** period

$$T_E = 2\pi \frac{T_0 + T_1}{|\log(\eta)|}$$



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Geodesics 
$$ds^2 = dx^2 - f(x)dt^2$$



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Geodesics  $ds^2 = dx^2 - f(x)dt^2$ 

 $T_E \rightarrow \infty$  at phase boundary





Rindler (black hole) metric

$$ds^{2} = -\Theta_{H}^{2} (x - x_{c})^{2} dt^{2} + dx^{2}$$
$$ds^{2} = -\Theta_{H}^{2} (x - (L - x_{c}))^{2} dt^{2} + dx^{2}$$



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$$x_c = \frac{L}{2\pi} \arccos\left(\cos\frac{\pi T_0}{L} + \frac{L}{\pi T_1}\sin\frac{\pi T_0}{L}\right)$$



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#### Excitations stuck between the two horizons

Hawking temperature

$$\Theta_{\rm H} = \frac{|\log\left(\eta\right)|}{2\pi(T_0 + T_1)}$$

How does the system heat up?

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#### How does the system heat up?



Effective local temperature  $\beta_{eff}$ 

$$\beta_{\text{eff}}(x) = \Theta_H \frac{\sin\left(\frac{\pi}{L}(x - x_c)\right)\sin\left(\frac{\pi}{L}(x + x_c)\right)}{\sin\left(\frac{2\pi x_c}{L}\right)}$$

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# Lattice systems - XXZ spin chain

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$$H = \frac{u}{2\pi} \int_0^L \left[ K\Pi^2 + \frac{1}{K} (\partial_x \phi)^2 \right]$$

CFT c=1, compact boson field

MPS simulations

Gapless Phase





T0 = 2, T1 = 8

(Non-Heating)

T0 = -2, T1 = 8(Heating)

# Energy absorption

 $\epsilon(x,t) = \langle \psi(t) | T_{00}(x) | \psi(t) \rangle$ 



#### Accumulation of energy black hole singularities

Energy density oscillates with period  $T_E$ 

# Comparison lattice model and CFT


## Summary











SSD Hamiltonians with periodic driving shows transition between heating and non-heating phase

heating in 'hot spots', comparably slow

excitations propagate analogous to curved space-time with black holes

picture valid beyond CFT, beyond periodic drive



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# **Electric-circuit realization of a hyperbolic drum**

arxiv:2109.01148

with:

Ronny Thomale group, Würzburg Tomas Bzdusek group, PSI Switzerland Igor Boettcher, Alberta

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# How to probe hyperbolic space?

### How to probe hyperbolic space?

#### Energetic ordering of Laplacian eigenstates

Euclidean drum



#### Hyperbolic drum



Geodesics



## **Experimental realization: Electric circuits**

2+1D hyperbolic space cannot be embedded in 3+1D Euklidean space

Electrical circuits have loose locality constraint: circuit element instead of physical distance





[Ling Lu, Nature Physics (2018)]

## **Experimental realization: Electric circuits**

2+1D hyperbolic space cannot be embedded in 3+1D Euklidean space

Electrical circuits have loose locality constraint: circuit element instead of physical distance

 $I_a$ 

Kirchhoff's law

$$(\omega) = \sum_{b=1,2,\cdots} J_{ab}(\omega) V_b(\omega)$$

circuit Laplacian



[Ling Lu, Nature Physics (2018)]

#### Hyperbolic circuit



#### Measured eigenmodes





## **Signal propagation**





Electrical circuits are ideal platform for studying classical physics in hyperbolic space, test hyperbolic band theory, ...

#### Quasiperiodic drive



Fibonacci sequence

 $U_{n+2} = U_n U_{n+1}$ 

Fractal phase diagram with lines of exactly zero heating



#### Quasiperiodic drive

heating regimes: particles locate at only few fixed positions, independent of initial conditions



0.6

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#### Outlook

Internal symmetries Micromotion Higher dimensions Holographic perspective Open systems

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#### Outlook

Internal symmetries Micromotion Higher dimensions Holographic perspective Open systems

#### It may be worth to study HOW systems heat up!

Phys. Rev. Research 2, 023085 (2020) Phys. Rev. Research 2, 033461 (2020)