Effective field theory of the Fractional Quantum Hall Effect

Refs: Yi-Hsien Du, Umang Mehta, Dung X Nguyen, DTS
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Y.-H. Du, U. Mehta, DTS to appear.

Plan

- Fractional Quantum Hall effect
- Lowest Landau Level limit.
- Volume preserving diff
- Effective field theory
- Noncommutative gauge symmetry (if time permits)

FQHE :

$$H = \frac{1}{2m} \sum_{a=1}^{N} \left(\vec{p}_a - \vec{A}(\vec{x}_a) \right)^2 + \sum_{\langle a,b \rangle} V(\vec{x}_a - \vec{x}_b)$$

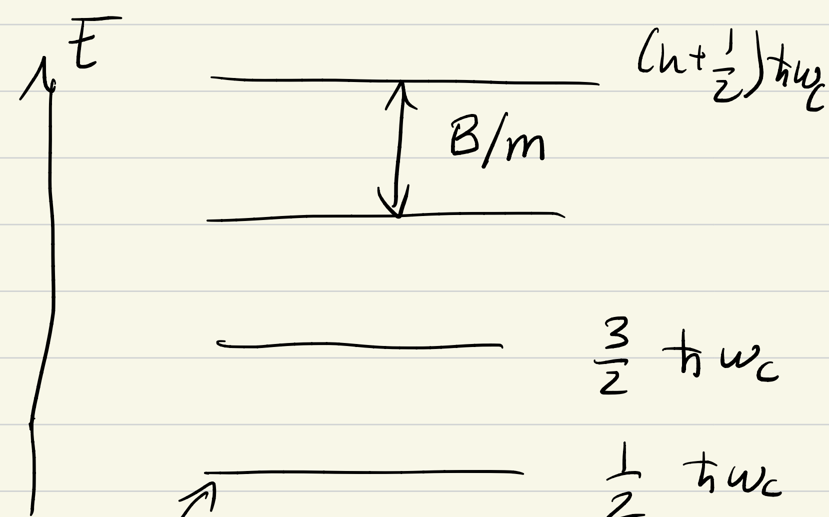
$\hat{A}$ = external $\qquad \vec{\nabla} \times \vec{A} = \vec{B}$.

ignore $V = 0 \Rightarrow$ Landau levels

$$\omega_c = \frac{eB}{mc} \qquad c = 1$$

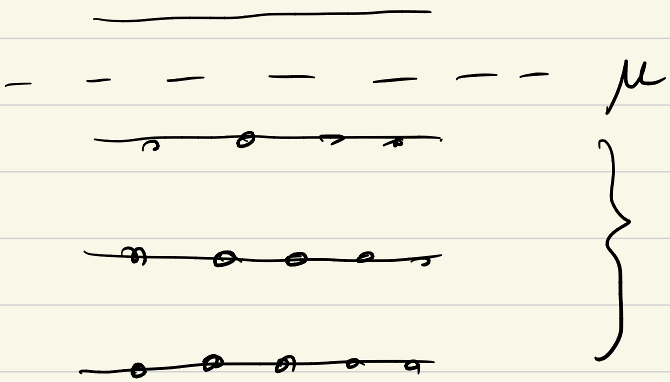
absorb e into B

$$\omega_c = \frac{B}{m}.$$



degeneracy $N_\phi = \frac{1}{2\pi} \int \vec{B} \cdot d\vec{S}$.

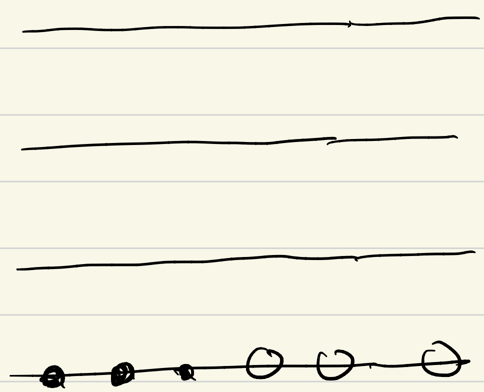
many electrons N

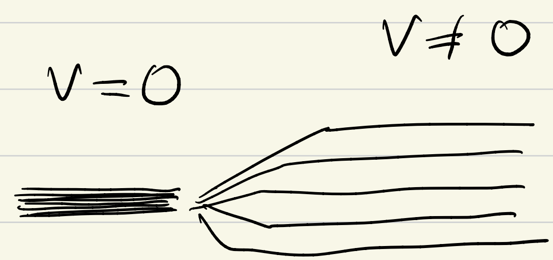


IQHE

FQHE: $\nu = \dfrac{N_e}{N_\phi} < 1$

interaction.





2 Energy scales

$$\hbar\omega_c \sim \frac{B}{m} \gg \underbrace{\Delta_c = \frac{e^2}{r}}_{\text{finite}}$$

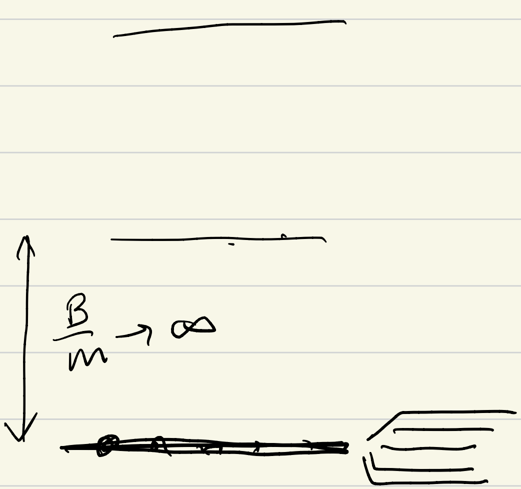
limit : LLL $\frac{B}{m} \to \infty$

$m \to 0$

(exp : $B \to \infty$)

$$\frac{N_\phi}{V} \sim B$$

$$\hbar\omega_c \sim \frac{B}{m} \gg \Delta_c \sim e^2\sqrt{B}$$



$H_{noninteraction} = 0$

Kinetic energy quenched.

nontrivial physics: potential energy.

$$H = \underbrace{\frac{1}{2m}\sum_{a=1}^{N} (\hat{p}_a - \hat{A}(x_a))^2}_{\text{puts particles on LLL.}} + \underbrace{\sum_{\langle ab \rangle} V(x_a - x_b)}_{\text{nontrivial part.}}$$

LLL $\sim$ noncommutative space.

classical mechanics

$$H = \frac{k^2}{2m} + V(x) \qquad \underbrace{\vec{k} = \vec{p} - \hat{A}(x).}$$

$$\{p, x\} = -1$$

$m \to 0 \qquad \vec{k} \approx 0 \qquad k_x \approx 0$

$k_y \approx 0.$

but $\{k_x, k_y\} = B$

<u>Dirac</u>: before $k_x = 0$, $k_y = 0$. $\qquad C_i \approx 0$

replace P.B. by D.B.

$$\{A, B\}_D = \{A, B\}_P - \{A, C_i\}\{C_i, C_j\}^{-1}\{C_j, B\}$$

$$\{x, y\}_D = \underbrace{\{x, y\}_P}_{=0} - \{x, k_x\}\{k_x, k_y\}^{-1}\{k_y, y\}$$

$$= \frac{1}{B} = l_B^2$$

LLL :

$$H = \sum_{ab}{}' \tilde{V}(\vec{x}_a - \vec{x}_b) \qquad [x_a^i, x_b^j] = i\varepsilon^{ij}\delta_{ab}\ell_B^2$$

$\tilde{V}$ related $\underbrace{V(x_a - x_b)}_{\Downarrow \; V(q)} = \dfrac{e^2}{|\hat{x}_a - \hat{x}_b|}$.

$$\tilde{V}(\hat{\vec{x}}_a - \hat{\vec{x}}_b) = \int dq \; V(q) \, e^{-\frac{q^2 \ell_B^2}{2}} \; e^{i\hat{q}\cdot(\hat{x}_a - \hat{x}_b)}.$$

The questions.

- Topological properties of gapped QH states.
- Low-energy physics.
  - $\nu = \frac{1}{2}$, $\nu = \frac{1}{4}$ states gapless
    - What is EFT
  - $\nu = \frac{N}{2N+1}$ $\frac{N+1}{2N+1}$ (Jain sequences).

gap $E \to 0$ as $N \to \infty$

$E \sim \frac{1}{N} \cdot \Delta_c$ ← dynamics

$\nu = \frac{N}{4N+1}$

$\frac{N+1}{4N+1}$

$E \ll \Delta_c$.

→ HLR candidate to EFT of $\nu = \frac{1}{2}$ state.

- Landau Fermi liquid theory.

  interacting ferimions. ^3He — interactions

  electrons in metals — $\sim$ Kinetic.

  - $E \sim T \ll \epsilon_F$
  - "proven" from perturbation theory.

HLR theory:
- guess.
- lack of symmetries. ← can be corrected.

HLR theory

idea : "flux attachment"

electron $\sim$ "composite fermion" + 2 fluxes.

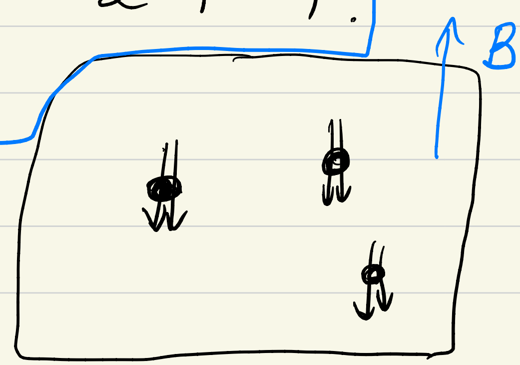
$\psi_e \neq \psi$

$$\mathcal{L} = \psi^\dagger i(\partial_0 - i A_0^{ext} - i a_0)\psi - \frac{1}{2m}\left|(\partial_i - iA_i^{ext} - i a_i)\psi\right|^2 + \frac{1}{2}\frac{1}{4\pi}\varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

$$\Longrightarrow \frac{1}{2\pi} a\,db + \frac{2}{4\pi} b\,db$$

imposes constraints

$$\boxed{-\frac{b}{2\pi} = 2\psi^\dagger\psi.}$$



MF : $B + \langle b \rangle = 0$ at $\nu = \frac{1}{2}$

$\Rightarrow \psi$ forms a Fermi liquid

"Maximalist claim":

- One can derive

$$\underbrace{\Psi(x_1 \ldots\ldots x_N)} = \prod_{\langle ij \rangle} \frac{(z_i - z_j)^2}{|z_i - z_j|^2} \underbrace{\tilde{\Psi}(x_1 \ldots\ldots x_N)}$$

$\Downarrow$ described by a theory of "CF"

each particles interacts with A-B. field of other particles

- Self-interaction.

- If HLR theory is a low-energy EFT ?

  - MF approximation.

Problems :

- "Energy scale problem".

$m_{CF} = m_e$ energy scale $\frac{B}{m_{CF}} \sim \frac{B}{m_e}$.

gap in $\nu = \frac{1}{3}$ state $\Delta \simeq \frac{1}{3} \frac{B}{m_e}$.

should be Δ_C.

should

$\nu = \frac{N}{2N+1}$

$\Delta \sim \frac{1}{2N+1} \frac{B}{m_e}$

E finite in $m \to 0$ at $\nu = \frac{1}{2}$

No LLL $m \to 0$. $\langle \rho \rho \rangle_{q,\omega} \sim q^4$

in $q \to 0$

ω = fixed

HLR : q^2.

- Lack of Particle-hole symmetry.

(at $\nu = \frac{1}{2}$ "Dirac CF")

- There must be an EFT $\nu = \frac{1}{2}$
- HLR theory has problems.
    - "no LLL projection" ?

as a lack of symmetries

- "Higher-rank" symmetries
  conservation laws. $\Big\}$ should be present.

Higher-rank conservation law.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

ρ = electron density.

$$\cancel{\frac{\partial}{\partial t} \vec{\pi}_i} + \boxed{\nabla_j T_{ji} = \rho E_i + \varepsilon_{ij} j_j B}$$

momentum density $= m\vec{j}$.

can be solved for $\vec{j}$

if $m \to 0$.

$$j_i = \rho v_i + \frac{1}{B}\left(\varepsilon_{ij} \text{"}\partial \cdot T\text{"}\right)$$

$$\begin{pmatrix} \rho \\ T_{ij} \end{pmatrix}$$

$$v_i = \frac{\varepsilon_{ij} E_j}{B} = \text{drift velocity.}$$

$E = 0$

$v = 0$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{1}{B} \partial_i \partial_j \left(\varepsilon_{ik} T_{kj} + \varepsilon_{jk} T_{ki}\right) = 0}$$

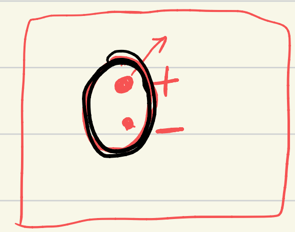
"fraction" (Pretko)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\vec{\nabla} T = 0$$

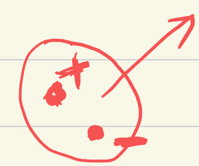
$$Q = \int d\vec{x}\, \rho \qquad \frac{dQ}{dt} = 0$$

$$= \int d\vec{x}\, \vec{x}\, \rho \qquad \boxed{\frac{d}{dt}\int dx\, \vec{x}\, \rho = 0}$$

$$\frac{d\rho}{dt} = \nabla^2 T$$



$$\frac{d}{dt}\int d\vec{x}\, x^2 \rho = 0 \leftarrow$$



In HLR: $\rho_e = \psi^\dagger \psi$ $\qquad \frac{d}{dt}\int dx\, \vec{x}\, \rho_e \neq 0.$

Consequence: $\quad \dfrac{\partial \rho}{\partial t} + \nabla \nabla T = 0.$

$$\langle \rho \, \rho \rangle (\omega, q) \sim q^4 \qquad \text{as} \quad q \to 0$$

$\omega =$ fixed.

$$\omega \rho + q^2 T = 0$$

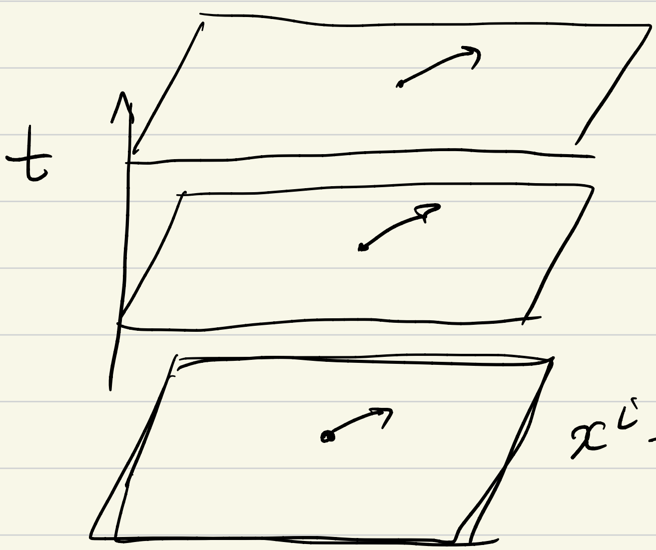
$$\rho \sim \frac{q^2 T}{\omega} \qquad \langle \rho \rho \rangle \sim \frac{q^4}{\omega^2} \langle T T \rangle$$

Conservation laws. $\leftarrow$ symmetry.

$$\mathcal{L}(\psi, A_0, g_{ij}) = \mathcal{L}(\psi) + A_0 \rho + g_{ij} T^{ij} + \cdots$$

$$\frac{\partial \rho}{\partial t} + \nabla^2 T = 0$$

time-dependent.
volume preserving diff.



$x^i \to x^i + \xi^i$

$\lambda = \lambda(t, \vec{x})$

$\vec{\nabla} \cdot \vec{\xi} = 0$

$\xi^i = \varepsilon^{ij} \partial_j \lambda$

Microscopic theory of e on LLL is invariant under:

$$\boxed{\begin{cases} g_{ij} \to g_{ij} - \nabla_i \xi_j - \nabla_j \xi_i \\ A_0 \to A_0 + \dot{\lambda} - l_B^2 \varepsilon^{ij} \partial_i A_0 \partial_j \lambda \end{cases}} \qquad \xi^i = l_B^2 \varepsilon^{ij} \partial_j \lambda$$

1 parameter

looks like U(1) symmetry

$S(\psi, A_0, g_{ij})$ $\qquad \delta S = 0$

$$\begin{cases} A_0 \to A_0 + \partial_0 \lambda \\ g_{ij} \to g_{ij} + \partial\partial \lambda \\ \Downarrow \\ \frac{\partial \rho}{\partial t} + D^2 T = 0. \end{cases}$$

Requirement on EFT.

What needed to ensure LLL

Take

$$S = \int_{t\,x} \psi^\dagger (\partial_t - A_0)\psi - \frac{1}{2m}|D\psi|^2 + \text{potential}.$$

invariant under $\begin{cases} \text{time-dependent spatial diff} & \xi^i \\ + U(1) \text{ gauge.} \end{cases}$

in limit $m \to 0$

Take comb keep A_i invariant.

$$\delta_{\lambda_1} \delta_{\lambda_2} - \delta_{\lambda_2} \delta_{\lambda_1} = \delta_{\{\lambda_1, \lambda_2\}}$$

$$\{\lambda_1, \lambda_2\} = l_B^2 \, \varepsilon^{ij} \partial_i \lambda_1 \, \partial_j \lambda_2 .$$

Charges not commute.

$$[\rho(x), \rho(y)] = l_B^2 \, \varepsilon^{ij} \, \partial_i \rho \; \partial_j \delta(x-y)$$

"GMP"

How to write down a theory invariant under ?

$$\begin{cases} \delta g_{ij} = \nabla_i \xi_j + \nabla_j \xi_i \\ \delta A_0 = \dot{\lambda} + \{ A_0 , \lambda \} \end{cases} \qquad \xi^i = \varepsilon^{ij} \partial_j \lambda .$$

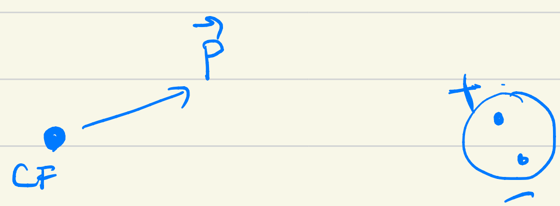
$$\mathcal{L} = i \psi^\dagger \partial_t \psi \Longrightarrow i \psi^\dagger \partial_t \psi + \boxed{\frac{i}{2} v^i \left(\psi^\dagger \partial_i \psi - \partial_i \psi^\dagger \psi \right)}$$

$+ \ldots$

$$v^i = \frac{\varepsilon^{ij} E_j}{B} \qquad E_i = \partial_i A_0 .$$

ψ = composite fermions has dipole moment $\vec{E} \cdot \vec{d}$

$$= \frac{1}{B} \left(\psi^\dagger \overleftrightarrow{\nabla} \psi \times \hat{z} \right)$$



$$\vec{d} \simeq \frac{\vec{p} \times \hat{z}}{B}$$

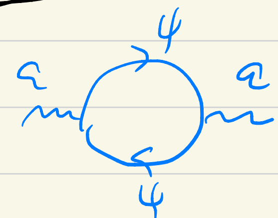
EFT of the $\nu = \frac{1}{2}$ state:

ψ_e.

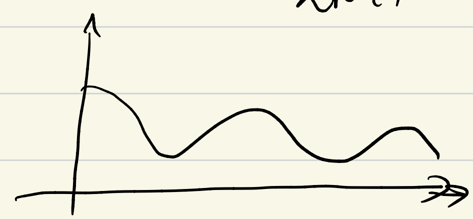
$$\boxed{\mathcal{L} = i\psi^\dagger(\partial_t - i a_0)\psi + i\psi^\dagger\hat{\sigma}\cdot(\vec{\nabla} - i\vec{a})\psi + \frac{1}{2}\frac{i\varepsilon^{ij}E_j}{B}(\psi^\dagger\overleftrightarrow{D}\psi) + m\bar{\psi}\psi + \frac{1}{4\pi}a\,dA + \text{other terms not fixed by symmetries.}}$$

$\cancel{a\,da}$

$\psi^\dagger\psi = \psi^\dagger\overleftrightarrow{\nabla}\psi$



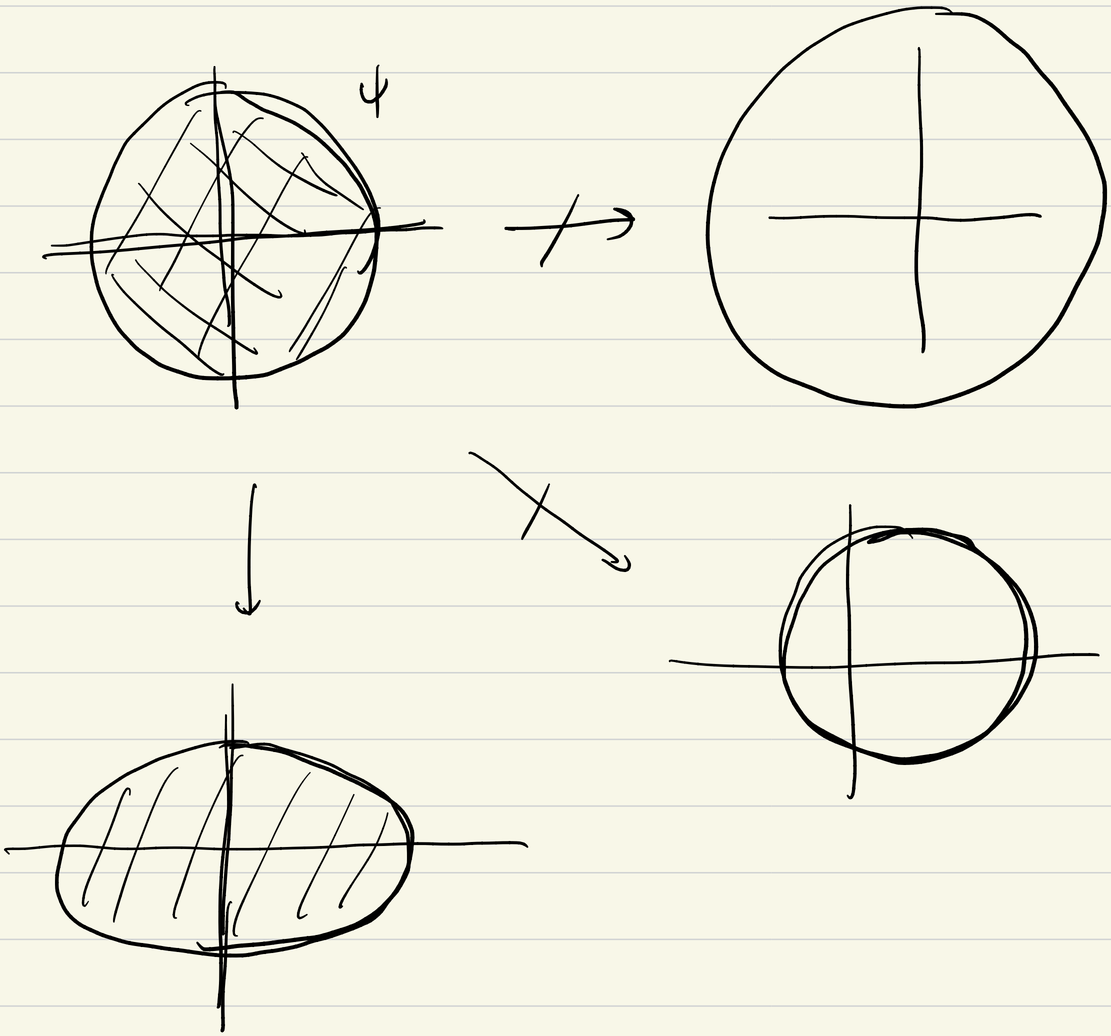
$\nu = \frac{N}{2N+1}$



- Compare to HLR 2 modification.
  - Dirac CF
  - Dipole terms (higher-rank conservation laws).

$\psi_e^\dagger\psi_e \Rightarrow \rho_e = \varepsilon^{ij}\partial_i\psi^\dagger\partial_j\psi$

Compute correlation functions.



$B = \psi^\dagger \psi$

operator. W_∞ :

Low energy

$$[\psi^\dagger \quad \psi] = \psi^\dagger |n\rangle\langle n| \psi - \psi |n\rangle\langle n| \psi^\dagger$$