Callan Rubakov Effect and Higher Charge Monopoles

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upcoming work and [2106.13820]

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Callan Rubakov Effect

What is Callan Rubakov Effect?

- A curious feature of interaction between smooth non-abelian monopoles and massless fermions
- Process by which smooth non-abelian monopoles can catalyze proton decay in $SU(N)$ GUT models [Callan,Rubakov]

$$p + M \longrightarrow e^- + M + \ldots$$

- Effect is due to the fact that massless fermions have non-trivial interaction with GUT scale degrees of freedom trapped in monopole core

Introduction

Why is the Callan Rubakov Effect interesting?

- Important for phenomenology
- Provides a concrete example of UV-IR mixing
  - Fermion-monopole interaction is sensitive to UV physics
- Monopole scattering probes anomalies
  - Inherently non-perturbative
  - In GUT models B,L symmetry violated, but B-L symmetry preserved
- Exhibits 2D physics in 4D system
  - States that do not have good particle interpretation

Introduction

What is already known about Callan Rubakov Effect?

- Only base case: $SU(2)$ gauge theory with minimal monopole and fermions in fundamental representation
  - Can be embedded in higher rank gauge theory
  - $SU(N)$ gauge theory with minimal monopole and fermions in representation with unit charge pairing

[Callan,Rubakov,Polchinski,Maldacena-Ludwig,Affleck-Sagi,etc]

Introduction

What will we discuss today?

- Derive general low energy monopole-fermion interaction from $SU(N)$ gauge theories for general spherically symmetric monopole and fermion representations
  - Semiclassical analysis, heavily rely on fermion zero-modes
- Clarify the interpretation of the Callan Rubakov effect
  - The interpretation is a little unclear when considering the scattering approach

$$e + M \to d_3^c + M - \frac{1}{2}(u_1 + u_2 + e^c + d_3^c)$$

much confusion in literature on interpretation of out-going state.
- Classify continuous symmetries preserved by interaction

Outline

Setup

Here we will consider a UV complete theory in a vacuum that has monopoles and massless fermions in IR description.

- $SU(N)$ gauge theory with adjoint Higgs Φ and fermions ψ_R in representations R
  - Want to consider vacuum where Φ has vev $\Phi_\infty \neq 0$ that breaks

$$SU(N) \longrightarrow \widetilde{G}_{IR} = U(1)^r \times \prod_a SU(N_a)$$

- Fermion ψ_R decomposes into $U(1)^r \subset G_{IR}$ representations ψ_a

$$h^I \cdot \psi_a = Q^I{}_a \, \psi_a$$

where $\{h^I\}$ generate $U(1)^r$

Setup

Breaking pattern allows for a collection of monopoles which are labeled by asymptotic magnetic charge

$$B_r \sim \frac{\gamma_m}{2r^2} + \dots \qquad \gamma_m = \sum_I n_I h^I \quad n_I \in \mathbb{Z}_{\geq 0}$$

- UV theory: smooth monopoles (spherically symmetric)
- IR theory: monopole operators
  - Defined by excising infinitesimal S^2 and imposing boundary conditions on gauge field

$$A = \frac{\gamma_m}{2}(1 - \cos\theta)d\phi \quad \to \quad B_r \sim \frac{\gamma_m}{2r^2}$$

- Problem: Hamiltonian is not Hermitian without additional, fermionic boundary conditions

IR Fermions

To demonstrate the issue with the Hamiltonian, consider the spectrum of the Dirac operator. In Dirac monopole background the time-independent solutions go like

$$\psi_a^{(0)} = \frac{1}{r}\begin{cases} \mathcal{D}^{(j_a)}_{m,j_a}(\theta,\phi)\,\chi_+ & p_a > 0 \\ \mathcal{D}^{(j_a)}_{m,-j_a}(\theta,\phi)\,\chi_- & p_a < 0 \end{cases}$$

where p_a is magnetic charge coupling of ψ_a:

$$\gamma_m \cdot \psi_a = p_a\,\psi_a = \sum_I n_I\,Q_a^I\,\psi_a$$

Here $\chi_\pm$ are radially polarized spinors and $\mathcal{D}^{(j_a)}_{m,q}(\theta,\phi)$ are irreps of the rotation group of spin $j_a = |p_a| - \frac{1}{2} \geq 0$: multiplicity $2|p_a|$.

IR Fermion Modes

On-top of each fermion zero-mode there exists a continuum of scattering states

$$\psi_a^{(k)} = \frac{c_{a,m}^{(k)}}{r}\begin{cases} e^{ik(t+r)}\mathcal{D}^{(j_a)}_{m,j_a}(\theta,\phi)\,\chi_+ & p_a>0 \\ e^{ik(t-r)}\mathcal{D}^{(j_a)}_{m,-j_a}(\theta,\phi)\,\chi_- & p_a<0 \end{cases}$$

Fixed angular dependence allows us to spherically reduce to effective 2D theory on t-r half-plane:

- Charge $p_a>0$ fermion has $2p_a$ left-moving 2D fermions

$$\chi_A(t,r) = \int dk\, c_A^{(k)} e^{ik(t+r)}\chi_+$$

- Charge $p_a<0$ fermion has $-2p_a$ right-moving 2D

$$\widetilde{\chi}_A(t,r) = \int dk\, c_A^{(k)} e^{ik(t-r)}\chi_-$$

where $A=(a,m)$ is a unified index.

Spherically Symmetric Monopoles

Now we are forced to impose boundary conditions at $r = 0$:

- There is a collection of purely left- and right-moving fermions in 2D on a half space (i.e. with boundary).
- Since fermions can reach boundary ($1/r$ dependence in 4D) Hamiltonian not Hermitian without boundary conditions at $r = 0$ on monopole world-volume.
- The only form this can take is to relate left-moving to right-moving fermions even though come from different 4D fermions (at least with respect to $U(1)^r \subset G_{IR}$ representations).

2D Boundary Conditions

Boundary conditions for fermions in 2D have recently been discussed by series of papers by [Smith-Tong]:

- Key to writing down most general boundary conditions is to bosonize the 2D fermions

$$\partial H_A = \chi_A^\dagger \chi_A \quad , \quad \bar{\partial}\tilde{H}_A = \widetilde{\chi}_A^\dagger \widetilde{\chi}_A$$

- General boundary condition given in terms of $H_A, \widetilde{H}_A$:

$$H_A - \mathcal{R}_{AB}\widetilde{H}_B\big|_{r=0} = 0$$

where $\mathcal{R}_{AB}$ is an invertible matrix.

2D Boundary Conditions

For example, the simplest boundary conditions I.) Neumann and II.) Dirichlet for the total scalar field $h_A(z,\bar{z}) = H_A(z) + \widetilde{H}_A(\bar{z})$ correspond to

$$\text{I.)}\ H_A - \widetilde{H}_A\big|_{r=0} = 0 \qquad \text{II.)}\ H_A + \widetilde{H}_A\big|_{r=0} = 0$$

Using bosonization map, can write boundary conditions in terms of fermions:

$$\text{I.)}\ \chi_A = \widetilde{\chi}_A\big|_{r=0} \qquad \text{II.)}\ \chi_A = \widetilde{\chi}_A^\dagger\big|_{r=0}$$

However, for general $\mathcal{R}_{AB}$:

$$H_A - \mathcal{R}_{AB}\widetilde{H}_B\big|_{r=0} = 0$$

there does not exist a general fermionic interpretation. This is standard feature of 2D physics.

2D Boundary Conditions: Symmetries

An important feature of the bosonization map is that $\partial H_A, \bar{\partial}\widetilde{H}_A$ are the number currents of the fermions $\chi_A, \widetilde{\chi}_A$.

$$J_A = \partial H_A \qquad \widetilde{J}_A = \bar{\partial}\widetilde{H}_A$$

The boundary conditions can then be interpreted as

$$J_A - \mathcal{R}_{AB}\tilde{J}_B\big|_{r=0} = 0$$

which implies that the conserved symmetries are the unit eigenvectors.

- In the case of orthogonal matrix, the boundary condition is completely specified by conserved symmetries

2D Boundary Conditions

Conversely, we can define a boundary condition that preserves a given set of symmetries.

Consider N_f $\chi_A, \widetilde{\chi}_A$ with $N_f - U(1)$ global symmetries $(\mathcal{J}_I, \widetilde{\mathcal{J}}_I)$:

	$U(1)_I$
χ_A	Q_{IA}
$\widetilde{\chi}_A$	$\widetilde{Q}_{IA}$

$$\mathcal{J}_I = Q_{IA} J_A \quad \widetilde{\mathcal{J}}_I = \widetilde{Q}_{IA} \widetilde{J}_A$$

where we assume $Q, \tilde{Q}$ are invertible. The boundary conditions that preserve these symmetries

$$\mathcal{J}_I - \widetilde{\mathcal{J}}_I \big|_{r=0} = 0$$

which can be rewritten

$$J_A - \mathcal{R}_{AB} \widetilde{J}_B \big|_{r=0} = 0 \qquad \mathcal{R}_{AB} = (Q^{-1} \widetilde{Q})_{AB}$$

Callan Rubakov Effect

The Callan Rubakov effect corresponds to a particular choice of $\mathcal{R}_{AB}$ where there are N_f 2D fermions $\chi_A, \widetilde{\chi}_A$. The boundary condition is given

$$\mathcal{R}_{AB} = \begin{cases} 1 - \frac{2}{N_f} & A = B \\ -\frac{2}{N_f} & A \neq B \end{cases}$$

Correspond to the symmetries

$$\begin{aligned} (N_f\text{-}1)\ \#\ \text{sym:}\quad & \mathcal{J}_I = J_I - J_{I+1} & \widetilde{\mathcal{J}}_I = \widetilde{J}_I - \widetilde{J}_{I+1} \\ \text{Gauge sym:}\quad & \mathcal{J}_N = \sum_A J_A & \widetilde{\mathcal{J}}_N = -\sum_A \widetilde{J}_A \end{aligned}$$

$\mathcal{R}_{AB}$ is fractional: no good interpretation in terms of pure fermionic fields.

Callan Rubakov Effect

The fact that $\mathcal{R}_{AB}$ has no good interpretation in terms of the fermions has lead to much confusion about the interpretation of the Callan Rubakov effect.

- Contrary to semiclassical/perturbative intution, fermion species number is not a good quantum number
  - It has an ABJ anomaly
  - This is usually fine, but monopole is inherently non-perturbative and sources magnetic flux to $r = \infty$ necessary to activate anomaly
- Massless (effectively 2D) charged fields also lead to somewhat strange effects
  - Electric sources can be screened or have charge radiated away with arbitrarily low energy

UV Fermion-Monopole Interactions

The boundary condition encodes the interaction of the fermions with UV degrees of freedom confined to the core of a smooth monopole in the full UV theory. To derive this interaction we need to study the smooth spherically symmetric monopole solution and quantize the fermions in this background.

Our plan will be:

- Derive low energy degrees of freedom for monopole
- Derive interaction with low energy fermion degrees of freedom
    - Derive effective fermion boundary condition

Spherically Symmetric Field Configuration

Consider $SU(N)$ gauge theory with adjoint Higgs field Φ with vev Φ_∞ that breaks

$$SU(N) \longrightarrow \widetilde{G}_{IR} = U(1)^r \times \prod_a SU(N_a)$$

Field configuration sphericaly symmetric if invariant under standard rotation generators up to gauge transformation. Equivalent to invariance under

$$\vec{K} = -i\vec{r} \times \vec{\nabla} + \vec{T}$$

for $\vec{T}$ generators of $SU(2)_T \subset SU(N)$.

Spherically Symmetric Monopoles

In this vacuum, there are a collection of spherically symmetric monopoles:

$$B_a = D_a\Phi \qquad \lim_{r\to\infty} B_a = \frac{\gamma_m}{r^2}\hat{r} \qquad \lim_{r\to\infty}\Phi = \Phi_\infty - \frac{\gamma_m}{r}$$

Solution specified by

- $SU(2)_T \subset SU(N)$ generated by T_i: defines rotational symmetry
- $SU(2)_I \subset SU(N)$ generated by I_i

$$\gamma_m = T_3 - I_3 \qquad [I_i, \gamma_m] = 0$$

Analytic solution:

$$A = T_3 A_{Dirac} \pm \frac{i}{2} M_\pm(r)\, e^{\mp i\phi}(d\theta \mp i\sin\theta d\phi)$$

where

$$M_\pm(r) = diag_{\pm 1}(a_1(r), a_2(r), \ldots, a_{N-1}(r))$$

Spherically Symmetric Monopole

The asymptotic form is given by

$$\lim_{r\to\infty} M_\pm(r) = I_1 \pm i\, I_2 \qquad \lim_{r\to 0} M_\pm(r) = T_1 \pm i\, T_2$$

where the $a_I(r)$ along broken directions in $SU(N)$ fall off exponentially

$$a_I(r) \sim e^{-\Delta_I m_W r} \qquad \Delta_I > 0$$

The asymptotic form of A is gauge equivalent to

$$\lim_{r\to\infty} A \sim \gamma_m A_{Dirac} + \ldots$$

IR Degrees of Freedom

We now want to quantize the gauge field around the smooth monopole solution.

In the low energy limit, long range gauge symmetry is G_{IR} due to the Higgs vev.

However, in the presence of the monopole $\lim_{r\to 0}\Phi = 0$. Thus, in the core of the monopole the $SU(N)$ gauge symmetry is restored.

- Confines W-bosons to monopole core. Their phases φ_I give rise to gauge degrees of freedom that are localized to the monopole core that carry charge under $U(1)^r \subset G_{IR}$

IR Gauge Field

Can compute the contribution of φ_I to the action by parametrizing

$$A_{SU(N)} = A_{IR} + T_3 A_{Dirac} + W_+ + W_-$$

where $W_\pm$ correspond to the broken directions

$$W_\pm = w_\pm + e^{-i\varphi_I h^I}\left(\frac{i}{2}M_\pm(r)e^{\mp i\phi}(d\theta \mp i\sin\theta d\phi)\right)e^{i\varphi_I h^I}$$

where $w_\pm$ encode long range broken degrees of freedom and h^I are generators of $U(1)_I \subset G_{IR}$.

Heuristically, φ_I parametrize the phases of the $a_I(r)$ of $M_\pm(r)$.

IR Degrees of Freedom

Plugging in our ansatz into the action, we find in the low energy theory:

- $w_\pm$-degrees of freedom are completely gapped by 1.) the Higgs field outside of the core and 2.) the monopole gauge field inside the core
- φ_I degrees of freedom do not get a mass, but $a_I(r) \sim e^{-m_W r}$ in broken directions so φ_I are massless and exponentially confined to monopole core.
  - φ_I are charged under $U(1)^r \subset G_{IR}$
  - Excitations make monopole into dyon: φ_I are dyon degrees of freedom

Fermion Interactions

We now want to describe the interaction of the φ_I with the low energy fermion sector.

- This is difficult
  - Requires completely solving for the spectrum of the Dirac operator in the non-abelian monopole background
  - Actually doable, but results are very messy and hard to manipulate.
  - Easier to solve for the fermion zero-modes which we can match explicitly onto continua of low energy fermions.

Fermion Zero-Modes

Given a fermion ψ_R in representation R, we can expand in irreps of the rotation group

$$\psi_R = \sum_a \psi_a v_a = \sum_a \begin{pmatrix} f_a(r)\, \mathcal{D}^{(j)}_{m,q_a+\frac{1}{2}}(\theta,\phi) \\ g_a(r)\, \mathcal{D}^{(j)}_{m,q_a-\frac{1}{2}}(\theta,\phi) \end{pmatrix} v_a$$

where j is the total angular momentum and q_a is the charge under T_3. Only solutions for spin-$j \leq |q_{max}| - \frac{1}{2}$

$$F(r) = \sum_a \begin{pmatrix} f_a(r) \\ g_a(r) \end{pmatrix} v_a = P\,\exp\left[\int \mathcal{M}(r)dr\right] F_0$$

where $\mathcal{M}$ is an explicitly known r-dependent matrix.

Asymptotic Zero-Modes

The asymptotic form of the spin-j zero-modes is given by

$$\psi_R^{(j)} = \frac{1}{r}\left(\mathcal{D}^{(j)}_{m,j}(\theta,\phi)\chi_+ v_a - \mathcal{D}^{(j)}_{m,-j}(\theta,\phi)\chi_- v_{a^*}\right)$$

where only fermions $q_a = j + \frac{1}{2}$ contributes and a^* is the complex conjugate fermion w.r.t. $SU(2)_T \subset SU(N)$.

Zero-modes imply that boundary condition relates each IR fermion mode with the charge conjugate:

$$\chi_A \sim \widetilde{\chi}_A\big|_{r=0}$$

Fermion Boundary?

The explicit form of the fermion zero-modes do not give us the IR fermion boundary conditions

- The obstensible boundary condition corresponds to

$$\chi_A = \widetilde{\chi}_A\big|_{r=0}$$

  is not gauge invariant.

For this relation to make sense, we need to use the dyon degree of freedom.

- Claim: the boundary condition is actually

$$\chi_A = e^{ic_A^I \varphi_I}\widetilde{\chi}_A$$

  where $c_A^I \in \mathbb{Z}$ so boundary condition is gauge invariant.

Fermion-Dyon Coupling

To derive this boundary condition, we can expand the 4D fermions in terms of 2D fields

$$\psi_R = \frac{1}{r}\sum_a \Big(\mathcal{D}^{(j_a)}_{m,q_a+\frac{1}{2}}(\theta,\phi)\chi^+_{a,m}(t,r) + \mathcal{D}^{(j_a)}_{m,q_a-\frac{1}{2}}(\theta,\phi)\chi^-_{a,m}(t,r)\Big)v_a$$

Plug this ansatz into the action with the gauge degrees of freedom, find effective interactions of the spin-j fermions of the form

$$S_{int} = \int g_a(r)e^{i\varphi_I}(\chi^+_{a,m})^\dagger\chi^-_{a',m}\, d^2x + c.c.$$

where $g_a(r) \sim (m_W r)^\ell e^{-m_W r}$ and

$$v_{a'} = E^-_{\alpha_I} v_a$$

Effective Boundary Condition

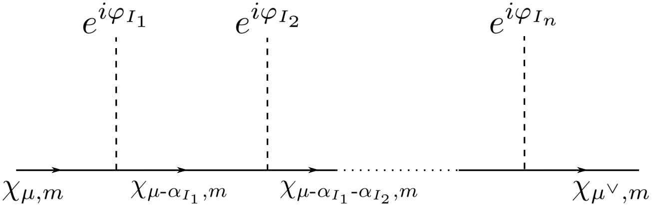
At fixed spin-j, only $\chi^{\pm}_{a,m}$ with $|q_a| = j + \frac{1}{2}$ are long range fields (top/bottom components)

- This means we should integrate out all other $\chi_{a,m}$ fields
- Interating out is in some sense encoded in explicit structure of fermion zero-modes

$$\lim_{r\to\infty} r\psi_R^{(j)} = \psi_{top}^{(j)} + \psi_{bot}^{(j)} + \underbrace{\sum_{a\neq\mu_{top/bot}} \frac{1}{(m_W r)^{\delta_a}} \psi_a^{(j)}}_{virtual}$$

for $\delta_a > 0$.

Effective Boundary Condition



This leads to proposed boundary term

$$S_{int} = \int d^2x\, \delta(r)\, e^{ic_A^I \varphi_I} \chi_A^\dagger \widetilde{\chi}_A + c.c.$$

where

$$\prod_I (E_I^-)^{c_A^I} v_A = v_{A^*}$$

The c_A^I are known explicitly.

Unfolding Trick

Following an analysis similar to [Polchinski '84], we can rewrite the theory of coupled fermions and the dyon degree of freedom.

We can "unfold" the theory onto the full plane while combining $\chi_A, \widetilde{\chi}_A$ into a single right-moving fermion Ψ_A. Then by performing a phase rotation, we find that the low energy effective theory can be written

$$S = \sum_A \int d^2x \left(i\Psi_A^\dagger(\partial_+ - i\Theta(x)c_A^I\dot{\varphi}_I)\Psi_A \right) + \sum_I \int dt \frac{1}{2} m_W \dot{\varphi}_I^2$$

Exchanges boundary term for localized interaction at $r = 0$.

Effective Boundary Condition

To get the effective fermionic boundary condition, we need to integrate out the φ_I-degrees of freedom

- φ_I excitations are not long lived because sources long range electric field has energy $E \sim m_W$.

Can analyze the effective boundary conditions using symmetries following [Polchinski '84]:

$$\partial_+ J_A = \dot{\varphi}_I c^I_A \delta(x) \qquad \partial_t \Pi_I = \sum_A c^I_A \int dx\, \delta(x) J_A(x)$$

Leads to current conservation

$$J_A^{(out)} = J_A^{(in)} - c^I_A \dot{\varphi}_I$$

Effective Boundary Condition

Integrating out φ_I leads to decay

$$\dot{\varphi}_I \to \mathcal{J}_I = \frac{1}{N_I} \sum_A c_A^I J_A$$

where N_I is normalization of the gauge current. The full boundary condition is given by:

$$\boxed{\mathcal{R}_{AB} = \delta_{AB} - \sum_I \frac{2}{\sum_A c_A^I \mu_{N-I}^{(A)}} c_A^I c_B^I}$$

where χ_A has highest weight $\mu^{(A)} = \sum_I \mu_I^{(A)} \lambda^I$. Note that $\mathcal{R}_{AB}$ is always symmetric, rational, non-diagonal, and generally non-integral

Physics of Boundary Condition

Boundary condition:

$$\mathcal{R}_{AB} = \delta_{AB} - \sum_I \frac{2}{\sum_A c_A^I \mu_{N-I}^{(A)}} c_A^I c_B^I$$

In terms of UV degrees of freedom, these boundary conditions mean that an in-going fermion ψ_a scatters off of the monopole, turning into its complex conjugate while virtually exciting dyon degrees of freedom:

$$\psi_a + M \to \psi_{a^*} + M + c_a^I \varphi_I$$

The dyon degrees of freedom then radiate via soft, charged, fermionic modes:

$$\psi_a + M \to \psi_{a^*} + M + c_a^I \varphi_I \to \psi_{a^*} + M + \sum_I c_a^I \mathcal{J}_I$$

This is the general version of the Callan Rubakov effect.

Preserved Symmetries

Now that we have the matrix defining the effective boundary conditions:

$$J_A = \mathcal{R}_{AB}\widetilde{J}_B\big|_{r=0} ~,$$

we can ask what global symmetries are preserved.

Boundary condition preserves continuous global symmetries of UV theory that have no ABJ-type anomaly

- $\psi_a \to \psi_{a^*}$ preserves any global symmetry
  - ψ_a, ψ_{a^*} part of same $SU(N)$ multiplet
  - Global symmetry only violated if

$$\varphi_I \to \frac{1}{N_I}\sum_A c_A^I J_A$$

violates global symmetry.

Anomalies

Violation of the global symmetry generated by Q_f is given by:

$$\Delta Q_f \sim \sum_A Q_f[\psi_A] c_A^I$$

c_A^I encodes the difference in $U(1)_I$ gauge charge between ψ_A and ψ_{A^*}

$$c_A^I = Q_I[\psi_A] - Q_I[\psi_{A^*}]$$

We now see that the violation of the global charge is controlled by a 2D ABJ anomaly

$$\Delta Q_f \sim \sum_A Q_f[\psi_A] \times (Q_I[\psi_A] - Q_I[\psi_{A^*}])$$

which descends from a 4D ABJ anomaly:

$$\Delta Q_f \sim \sum_\mu Q_f[\psi_\mu] \times Q_I[\psi_\mu] \times q_\mu \sim Tr_R[Q_f\, h^I\, T_3]$$

Preserved IR Symmetries

An IR approach to determine what symmetries are conserved is to note that a symmetry is preserved if $\mathcal{J}_i = \widetilde{\mathcal{J}}_i\big|_{r=0}$. Using

$$\mathcal{J}_i = Q_{aA} J_A \qquad \widetilde{\mathcal{J}}_i = \widetilde{Q}_{iA} \widetilde{J}_A$$

we see that a symmetry will only be preserved if

$$\boxed{\mathcal{R}_{AB} Q_{iB} = \widetilde{Q}_{iA}}$$

for fixed i. In the case of a global symmetry that does not mix with UV $SU(N)$ gauge symmetry (i.e. $Q_{iA} = \widetilde{Q}_{iA}$), the preserved global symmetries are the unit-eigenvectors of $\mathcal{R}_{AB}$.

$SU(5)$ Examples

Original setting for Callan Rubakov effect was $SU(5)$ Georgi-Glashow model.

- Adjoint Higgs Φ with vev that breaks

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

- $\mathbb{Z}_6$ classification of monopoles
- Fermions in $\bar{5}$ and 10 representation:

$$\psi_{\bar{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu \end{pmatrix} \qquad \chi_{10} = \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^c \\ & & & & 0 \end{pmatrix}$$

$SU(5)$ Examples

Charge 1 Monopole

Monopole with $\gamma_m = diag(0,0,1,-1,0)$

- Single dyon d.o.f., 4 IR fermions:

in-going	out-going	spin-j
e	d_3^c	0
d_3	e^c	0
u_1^c	u_2	0
u_2^c	u_1	0

Boundary condition given by [Callan]

$$\mathcal{R}_{AB} = \frac{1}{2}\begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$

$SU(5)$ Examples

Charge 1 Monopole

Global symmetries of standard model: $U(1)_L, U(1)_B$: non-anomalous combination is $U(1)_{B-L}$

$$\mathcal{R}_{AB} = \frac{1}{2}\begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$

Charge vectors for $U(1)_{B,L}$:

$$v_B^{(in)} = (0, 1, -1, -1) \qquad v_B^{(out)} = (-1, 0, 1, 1)$$
$$v_L^{(in)} = (1, 0, 0, 0) \qquad v_L^{(out)} = (0, -1, 0, 0)$$

B, L violated but $B - L$ not violated.

$SU(5)$ Examples

Charge 2 Monopole

Monopole with $\gamma_m = (1, 1, 0, -1, -1)$.

- 2 dyon d.o.f., 6 IR fermions

in-going	out-going	spin-j
e	d_1^c	0
u_1^c	d_3	0
ν	d_2^c	0
u_2^c	u_3	0
u_3^c	e^c	$\frac{1}{2}$

$$\mathcal{R}_{AB} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$SU(5)$ Examples

Charge 2 Monopole

Baryon, Lepton number symmetries:

$$\mathcal{R}_{AB} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$$v_B^{(in)} = (0,-1,0,-1,-1,-1) \qquad v_B^{(out)} = (-1,1,-1,1,0,0)$$

$$v_L^{(in)} = (1,0,0,0,0,0) \qquad v_L^{(out)} = (0,0,0,0,-1,-1)$$

B, L violated but $B - L$ not violated.

$SU(5)$ Examples

Charge 3 Monopole

Monopole with $\gamma_m = diag(1,1,1,-1,-2)$:

$$T_3 = diag(2,1,0,-1,-2) \qquad I_3 = diag(1,0,-1,0,0)$$

Simplest spherically symmetric monopole that is not simply embedded $SU(2)$ monopole.

Has 2 dyon d.o.f. and 9 IR fermions

in-going	out-going	spin-j
e	d_2^c	0
$u_1^c + u_3^c$	e^c	1
ν	$d_1^c + d_3^c$	$\frac{1}{2}$
u_1^c	d_2	0
u_2^c	$d_1 + d_3$	$\frac{1}{2}$

$SU(5)$ Examples

Charge 3 Monopole

$$\mathcal{R}_{AB} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & \frac{11}{20} & -\frac{9}{20} & -\frac{9}{20} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{4} & -\frac{9}{20} & \frac{11}{20} & -\frac{9}{20} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{4} & -\frac{9}{20} & -\frac{9}{20} & \frac{11}{20} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{4}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{4}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{4}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{4}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

Again B, L symmetries are violated while $B - L$ symmetry is preserved.

Overview

Here we derived the general boundary conditions for massless fermions on monopole operator coming from (simple) UV $SU(N)$ gauge theory

$$\mathcal{R}_{AB} = \delta_{AB} - \sum_I \frac{2}{\sum_A c_A^I \mu_{N-I}^{(A)}} c_A^I c_B^I$$

Clarify some confusions in literature/lore

- "fractional fermion" states really enocode soft fermionic raditiation

$$\psi_a + M \to \psi_{a^*} + M + c_a^I \varphi_I \to \psi_{a^*} + M + \sum_I c_a^I \mathcal{J}_I$$

Boundary conditions preserve symmetry if no ABJ anomaly

- Gives probe of anomalies in scattering experiments

Open Problems

- I believe there is a similar statement about when ABJ anomaly preserves discrete subgroup. Something like when $U(1) \to \mathbb{Z}_M$ that

$$\mathcal{R}_{AB} Q_B = \tilde{Q}_A \text{ mod}_M$$

- May be able to adapt this formalism to detect anomalies in higher form global symmetries involving magnetic 1-form global symmetries
- It would be interesting to further explore what possible boundary conditions can exist. E.g. simple to construct boundary conditions that preserve only discrete groups by introducing symmetry breaking couplings in UV that are confined to monopole core by Higgs profile.

Open Problems

- When fermions have a mass, scattering at threshold does not allow for the Callan Rubakov effect simply by energetic arguments. I believe in this case that the monopole captures a fermion and permanently becomes a dyon.
- Should be similar effect for cosmic strings in theories where $SU(N)$ is broken by condensation of Higgs fields in fundamental representations.

End.

The End.

Questions?