Duality in Condensed Matter and High Energy Physics

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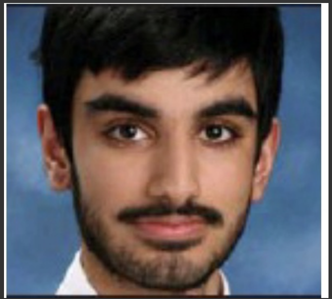

Electromagnetic Duality (1931)

$$\vec{\nabla}\cdot\vec{E} = \rho \,, \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \vec{J} \,, \qquad \vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

* EM Duality: $\vec{E} \longleftrightarrow \vec{B}$ (if $\rho = 0$ and $\vec{J} = 0$)
* Same as the duality of forms
* electric charge $e \longleftrightarrow$ magnetic monopole m
* Dirac quantization: $em = 2\pi$

Duality in Statistical Mechanics

* 2D: the Ising Model is self-dual (Kramers – Wannier) (1941)
* Low T: Z is an expansion in closed domain wall loops
* High T: Z is an expansion in closed loops
* maps high T $\longleftrightarrow$ low T
  disorder $\longleftrightarrow$ order

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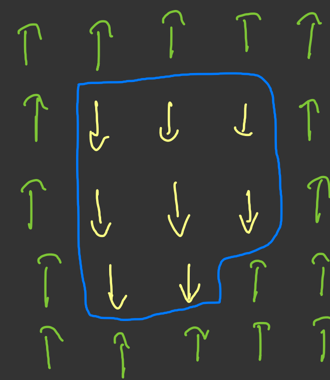
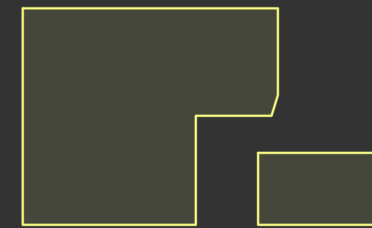
* 3D: Ising Model $\Longleftrightarrow$ $\mathbb{Z}_2$ gauge theory (Wegner) (1971)
* loops are dual to closed surfaces
* order $\longleftrightarrow$ confinement



domain walls (low T)



high T expansion diagrams

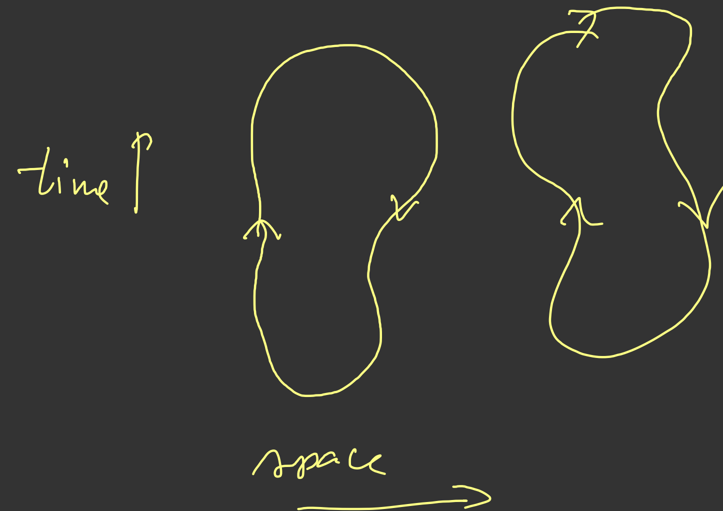
$$Z_{DW}\left[e^{-\frac{2}{T}}\right] = Z_{\text{loops}}\left[\tanh \frac{1}{T}\right]$$

$$Z = \sum_{[\sigma]} e^{\frac{1}{T}\sum_{n.n}\sigma\sigma}$$

3D: Particle-Vortex duality

(Peskin, Dasgupta, Halperin) (~ 1978 - 1981)

* Global U(1) symmetry
* 3D XY model (superfluid)
* High T: gas of closed loops with short-range interactions (i.e. particle worldlines)
* Low T: Closed vortex loops w/ Biot-Savart interactions
* Particle-vortex duality



Field Theory Picture of Particle-Vortex Duality

* Theory A

$$\mathcal{L} = |D_A \phi|^2 - m^2 |\phi|^2 - u|\phi|^4 , \quad D_A \equiv \partial - iA$$

background field

* Theory B

$$\mathcal{L} = |D_a \varphi|^2 + m^2 |\varphi|^2 - u|\varphi|^4 + \frac{1}{2\pi} \underbrace{A\, da}_{\varepsilon_{\mu\nu\lambda} A^\mu \partial^\nu a^\lambda} - \frac{1}{4e^2} f_{\mu\nu}^2$$

dynamical field

$$j_\mu \longleftrightarrow \frac{1}{2\pi} \varepsilon_{\mu\nu\lambda} \partial^\nu a^\lambda \qquad \text{(particle-vortex)}$$

Duality maps the unbroken phase of A to the Higgs phase of B
broken phase of A to the unbroken phase of B

* Wilson-Fisher Fixed Points are mapped into each other

Web of Dualities

① **Particle-Vortex duality** (Peskin, 1978; Dasgupta & Halperin, 1981)

$$\to \ |D_A\Phi|^2 - m^2|\Phi|^2 - u|\Phi|^4 \longleftrightarrow |D_b\varphi|^2 + m^2|\varphi|^2 - u|\varphi|^4 + \frac{1}{2\pi}Adb + \text{Maxwell}$$

external ; dynamical

$$\boxed{J_\mu \longleftrightarrow \frac{1}{2\pi}\varepsilon_{\mu\nu\lambda}\partial^\nu b^\lambda}$$

; vortices $\longleftrightarrow$ particles

② **Bosonization** (Fradkin & Schaposnik, 1994; Seiberg, Senthil, Wang & Witten, 2016)

$$\bar\Psi(i\not{D}_A - M)\Psi - \frac{1}{8\pi}AdA \longleftrightarrow |D_a\phi|^2 - m^2|\phi|^2 - u|\phi|^4 + \frac{1}{4\pi}ada + \frac{1}{2\pi}adA$$

Dirac fermion $\longleftrightarrow$ monopole ; $\boxed{\bar\Psi\gamma^\mu\Psi \longleftrightarrow \frac{1}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda}$

③ **Fermion Particle-Vortex duality** (Son, 2015; Metlitski & Vishwanath, 2016)

$$\bar\Psi(i\not{D}_A - M)\Psi - \frac{1}{8\pi}AdA \Leftrightarrow \bar\chi(i\not{D}_a + M)\chi + \frac{1}{8\pi}ada - \frac{1}{2\pi}adb + \frac{2}{4\pi}bdb - \frac{1}{2\pi}bdA$$

"QED_3"

* In general dimension duality often maps theories with ≠ character and symmetry
* In $D=4$ $\Rightarrow$ gauge theory $\Longleftrightarrow$ gauge theory
* There are many other dualities
* AdS / CFT $\Longleftrightarrow$ gauge/gravity duality
* S and T duality in String Theory

(S duality is related to particle-vortex duality)

* Conjectured web of dualities in 2+1 dimensions
* Fermion $\Longleftrightarrow$ Boson duality
* Can we "derive" these conjectures?

Strategy for a derivation

* We will use generalized loop models near criticality but still in the gapped phases
* Generalization of the particle-vortex duality
* We consider loop models in 2+1 dimensions
* Assume that the loops cannot intersect
* Include phase factors for linking numbers
* Frame the loops and include self-linking and Berry phase factors $\Rightarrow$ fractional spin

Loop Models in 2+1 Dimensions

$$Z[A] = \sum_{\{l_\mu\}}' \delta(\partial_\mu l_\mu)\, e^{-S[l] + i\pi\Phi[l]}$$

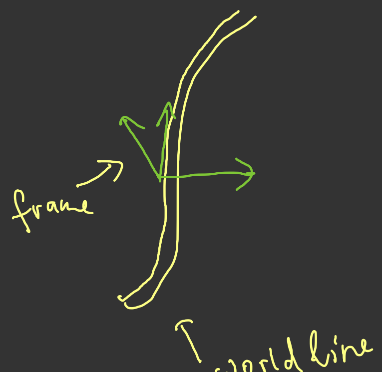
background field

loop configurations ("conserved currents")

weight per unit length + interactions

$\Phi[l]$ = linking number + self-linking number + Berry phase of the frame

Linking # of two loops l_1 and l_2



$$\Phi = 2 \times \text{linking \# of } l_1 \text{ with } l_2 + W[l_1] + W[l_2]$$

$$W[l] = \underbrace{SL[l]}_{\text{self-linking}} - \underbrace{T[l]}_{\text{twist}} = \text{"writhe"}$$

Twist $\quad T[\ell] = \frac{1}{2\pi} \int_0^1 ds \int_0^1 du \; \underset{\substack{\uparrow \\ \text{tangent}}}{\hat{e}} \cdot \partial_s \hat{e} \times \partial_u \hat{e}$

$T[\ell]$ in general is <u>not</u> quantized and depends on the metric $\Big\}$ Berry phase of the frame

Linking # of ℓ_1 and ℓ_2

$$\underbrace{\left\langle e^{i \oint_{\ell_1 \cup \ell_2} dz_\mu \cdot a_\mu} \right\rangle_{\text{Chern Simons}}}_{} \equiv e^{\frac{i\pi}{k} \underset{\substack{\uparrow \\ \text{linking \#}}}{N_\gamma}} \leftarrow \text{topological invariant}$$

wilson loop in Chern - Simons Theory (Witten 89')

$$S = \frac{k}{4\pi} \int d^3x \; a\, d\, a$$

$$k \in \mathbb{Z}$$

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Example (~ Polyakov 89')

* $Z_{fermion} = \mathrm{Det}(i\not{\partial} - M) \equiv \int \mathcal{D}j_\mu \, \delta(\partial_\mu j^\mu) \, e^{-|M| L[j] - i\pi \,\mathrm{sgn}(M)\, \Phi[j]}$

loop representation

linking # + spin factor

$k=1$

* $\mathcal{L}_{boson} = |D_a \phi|^2 - m^2 |\phi|^2 - u|\phi|^4 + \frac{1}{4\pi} a\,da + \underbrace{\frac{1}{2\pi} a\,dA}$

$Z[A] = \int \mathcal{D}j_\mu \mathcal{D}a_\mu \, \delta(\partial_\mu j_\mu) \, e^{-|m| L[j] + i S[j, a, A]}$

$S[j, a, A] = \int d^3x \left[j_\mu (a_\mu - A_\mu) + \frac{1}{4\pi} a\,da - \frac{1}{4\pi} A\,dA + \cdots \right]$

Integrating over $\underline{a_\mu}$ $\Rightarrow$ $-\pi\,\Phi[j] + \int d^3x \left(j_\mu A_\mu - \frac{1}{4\pi} A\,dA \right)$

$$\Rightarrow \quad \mathcal{L}_{\text{fermion}} = \bar{\Psi}\,(i \not{D}_A - M)\,\Psi - \frac{1}{8\pi} A\,dA \qquad \text{with} \quad M < 0$$

$\uparrow$ anomaly (η invariant)

$$Z_{\text{fermion}}[A, M<0]\, \underbrace{e^{-\frac{i}{2} S_{CS}[A]}}_{\uparrow} = \int \mathcal{D}j\, \delta(\partial \cdot j)\, e^{-|M| L[j]}\, e^{i S_{\text{fermion}}[j, A, M<0]}\, e^{-\frac{0}{2} S_{CS}[A]}$$

$$S_{\text{fermion}}[j, A, M<0] = \int d^3x \left[j \cdot A - \frac{1}{8\pi} A\,dA \right] - \pi\, \Phi[j] \quad \leftarrow$$

* To get the bosonization identity for $M>0$ one uses bosonic particle - vortex duality
* In the fermionic theory $M<0 \Leftrightarrow M>0 \Rightarrow \sigma_{xy} = 0 \Longleftrightarrow \frac{e^2}{h}$
* In the bosonic theory this is the transition from broken to the unbroken phase

Fermion Particle-Vortex Duality

* Duality from a free Dirac fermion $\longleftrightarrow$ QED$_3$ with a quantized CS term

$$\bar{\Psi}(i\not{D}_A + M)\Psi - \frac{1}{8\pi} AdA \longleftrightarrow \bar{\chi}(i\not{D}_a - M)\chi + \frac{1}{8\pi} ada - \frac{1}{2\pi} adb + \frac{2}{4\pi} bdb - \frac{1}{2\pi} bdA$$

loop model

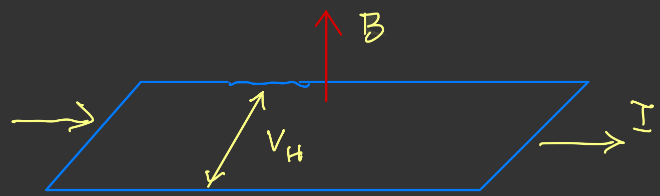
$$\rightarrow \int d^3x \; j_\mu A^\mu + \pi \Phi[j] \xleftarrow[\text{integrate out } b_\mu \text{ and } a_\mu]{} -\pi\Phi[j] + \int d^3x \left[j\cdot a - \frac{1}{2\pi} adb + \frac{2}{4\pi} bdb - \frac{1}{2\pi} bdA \right]$$

$$Z_{\text{fermion}}[A, M] = Z_{QED_3}[A, -M] \; ; \; Z_f[A, -M] = Z_{QED_3}[A, M]$$

Currents: $\bar{\Psi}\gamma^\mu\Psi \longleftrightarrow \frac{1}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$

Application: Fractional Quantum Hall States

In the beginning .... two-dimensional electron gases in large magnetic field



$\sigma_{xy} = \nu \dfrac{e^2}{h}$, $\sigma_{xx} \to 0$ $(T \to 0)$

no dissipation

Laughlin: $\Psi_m(z_1, \ldots, z_N) \sim \prod_{i<j} (z_i - z_j)^m \; e^{-\frac{1}{4 l_0^2} \sum_{j=1}^{N} |z_j|^2}$ (1983)

filling fraction $\nu = \dfrac{1}{m}$; $\{z_j\}$: electron coordinates $(z = x + iy)$

l_0: magnetic length

Jain: composite fermion : electron + $(m-1)$ fluxes (m odd)

FQH state : IQH state of composite fermions

$\to \; \nu_{\pm}(p, s) = \dfrac{p}{2sp \pm 1}$ $\quad p = 1, 2, \ldots$ $\quad s = 0, 1, 2 \ldots$ $\quad$ (Laughlin : $p = 1$, $+$) $\quad m = 2s + 1$

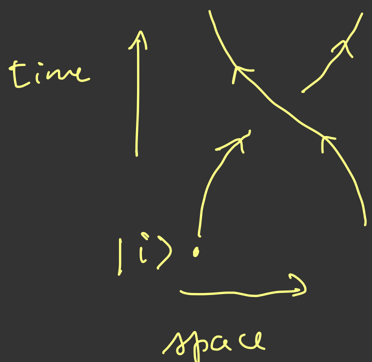
odd denominators

* The excitations of FQH fluids are vortices ("quasiholes") that

ⓐ carry fractional charge $q = \frac{1}{2sp \pm 1}$ ←

ⓑ fractional braiding statistics

ⓒ m degenerate ground states on a torus (topological protection)



amplitude $\sim e^{i\varphi}$

✓ $\varphi = \frac{\pi}{2sp \pm 1}$ ⟹ anyons labelled by one-dimensional representations of the Braid Group

✓ $e^{i\varphi_1} e^{i\varphi_2} \to e^{i(\varphi_1 + \varphi_2)}$ ("fusion")

Wen: ~1990 Effective Field Theory (Laughlin states)

$$\mathcal{L} = \underset{\text{Chern-Simons term}}{\frac{m}{4\pi} \varepsilon_{\mu\nu\lambda} a^{\mu} \partial^{\nu} a^{\lambda}} + \underbrace{\frac{e}{2\pi} \varepsilon_{\mu\nu\lambda} \partial^{\nu} a^{\lambda}}_{\text{current}} \underset{\text{e.m. field}}{A^{\mu}} + \underset{\text{vortex world lines}}{j_{\mu}^{\nu}} a^{\mu} \equiv$$

a_{μ}: hydrodynamic gauge field

Duality at the FQH Plateau Transition

* Limiting value of the Jain sequence

$$\lim_{p\to\infty} \frac{p}{2np \pm 1} = \frac{1}{2n}$$

* In this limit the average CS field cancels A_μ
* Halperin - Lee - Read: this is a "Fermi Liquid"
* Good phenomenology but...
* singular forward scattering interactions and violation of particle-hole symmetry at $\nu = 1/2$ $(\nu \leftrightarrow 1-\nu)$

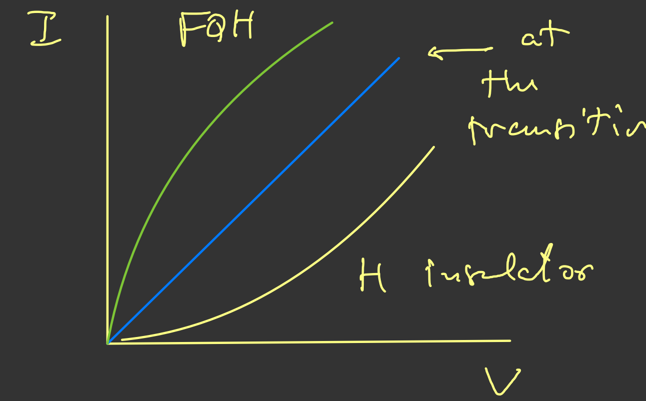
Symmetry of the I-V curves at the transition

* The I-V curves show a "mirror" symmetry at all transitions

* For general Jain states

$$\nu = \frac{p}{2np+1} \leftrightarrow \nu' = \frac{1+p}{2n(1+p)-1}$$

* For $\nu = 1/2 \Longleftrightarrow$ PH symmetry



I-V curves at the $0 \leftrightarrow \frac{1}{3}$ transition $\left(\nu \cong \frac{1}{4}\right)$

Hall insulator $\leftrightarrow$ FQH

$\nu = \frac{1}{2}$: Son's Conjecture

General case: $\nu = \frac{1}{2n}$

$$\mathcal{L}_{1/2n} = i\bar{\Psi}\not{D}_a\Psi - \frac{1}{4\pi}\left(\frac{1}{2} - \frac{1}{2n}\right) a\,da + \frac{1}{2\pi}\frac{1}{2n} a\,dA + \frac{1}{2n}\frac{1}{4\pi} A\,dA$$

$\partial \times A = B$

a: flux attachment

Electron filling $\quad \nu = \frac{2\pi}{B}\left\langle \frac{\delta \mathcal{L}_{\nu=1/2n}}{\delta A_0}\right\rangle = \frac{1}{2n}\left(1 + \frac{b_*}{B}\right)$

$$b^* = \partial \wedge a = 0 \Rightarrow \nu = \frac{1}{2n}$$

Composite fermion Ψ Fermi Surface set by a_0

$$\rho_\Psi = \frac{1}{2\pi}\left(\frac{1}{2} - \frac{1}{2n}\right) b_* - \frac{1}{2\pi}\frac{B}{2n}$$

$$\Rightarrow \quad \nu_\psi = 2\pi \frac{\rho_\psi}{b_*} = \frac{1}{2} + \frac{\nu}{1-2n\nu}$$

filling fraction of ψ

$\Rightarrow$ If $\nu_\psi = p + \frac{1}{2}$ $\Rightarrow$ $\nu = \dfrac{p}{2np+1}$

(Dirac)

But, if $\nu_\psi \to -\nu_\psi$ $\Rightarrow$ $\nu = \dfrac{p}{2np+1} \longrightarrow \dfrac{1+p}{2n(1+p)-1}$ ✓ !

(PH transf.)

$\Rightarrow$ PH transf. of the Dirac composite fermion is equivalent to the reflection symmetry!

Self-Duality at the Transition

* Use fermion - boson duality

$$\mathcal{L}_{1/2n} \longleftrightarrow |D_{g-A}\phi|^2 - |\phi|^4 + \frac{1}{4\pi}\frac{1}{2n-1} g\,dg \longleftarrow \nu_\phi$$

* Followed by a (boson) particle - vortex duality

$$\mathcal{L}_{1/2n} \longleftrightarrow |D_h \varphi|^2 - |\varphi|^4 - \frac{2n-1}{4\pi} h\,dh + \frac{1}{2\pi} h\,dA$$

$$-\frac{1}{\nu_\varphi}$$

* $\nu = \frac{1}{2n} \longleftrightarrow \nu_\phi = -\nu_\varphi = 1$

* Reflection symmetry $\nu_\phi(\nu) = -\nu_\varphi(\nu')$

* Reflection $\Longleftrightarrow$ boson - vortex symmetry !

* Reflection symmetry <u>at</u> $\nu = \frac{1}{2n} \longleftrightarrow$ boson self-duality !

Non-Abelian States: Moore-Read (1991)

Pfaffian

$$\Psi_{MR}(\{z_i\}) \sim \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \underbrace{\prod_{i<j} (z_i - z_j)^n}\, e^{-\frac{1}{4\ell_0^2}\sum_{i=1}^N |z_i|^2}$$

Pfaffian: expectation value of chiral Majorana fermions $\quad \chi(z) = \chi^\dagger(z)$

Propagator: $\langle \chi(z)\, \chi(w) \rangle = \dfrac{1}{z-w}$

$$\mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) = \langle \chi(z_1) \ldots \chi(z_N) \rangle$$ ⟵ "paired states" ($p_x + ip_y$ superconductor)

$\varphi(z)$: chiral boson $\quad \varphi(z) \sim \varphi(z) + 2\pi\sqrt{n}$

$$\Psi_{MR} \sim \langle \chi(z_1) \ldots \chi(z_N) \rangle \left\langle \left(\prod_{i=1}^N e^{i\sqrt{n}\,\varphi(z)}\right) e^{-\int d^2 z' \sqrt{n}\, \rho_0\, \varphi(z')} \right\rangle$$

Filling fraction: $\nu = \frac{1}{n}$

n even → fermions ; n odd ⟷ bosons ; e.g. $\nu = \frac{1}{2}$ fermions

$\nu = 1$ bosons

Generalization: Read-Rezayi states (RR) (1998)

Based on $\mathbb{Z}_k$ parafermions (and $SU(2)_k$)

$$\psi_n(z) * \psi_m(z') \sim \frac{1}{(z-z')^{\Delta_n + \Delta_m - \Delta_{n+m}}} \psi_{n,m}(z') + \dots$$

Fradkin & Kadanoff (1980) (!)

$$\Delta_n = \frac{n(k-n)}{k} \quad , \quad n, m = 1, \dots, k-1$$

RR states use the Parafermion CFT (Zamolodchikov & Fateev, 1985
Gepner & Qiu, 1987)

$$\Psi_{RR}(\{z_i\}) \sim \langle \psi_1(z_1) \dots \psi_1(z_N) \rangle \prod_{i<j} (z_i - z_j)^{M + \frac{2}{k}} \times \text{gaussians}$$

vanishes when $k+1$ particles come together.
clustering

$M \in \mathbb{Z}$ divisible by k; M even: bosons, M odd: fermions; $\nu = \frac{k}{Mk+2}$

The most interesting case is $k=3$ ($\mathbb{Z}_3$) ($\nu = \frac{3}{2}$ (B), $\frac{3}{5}$ (F))

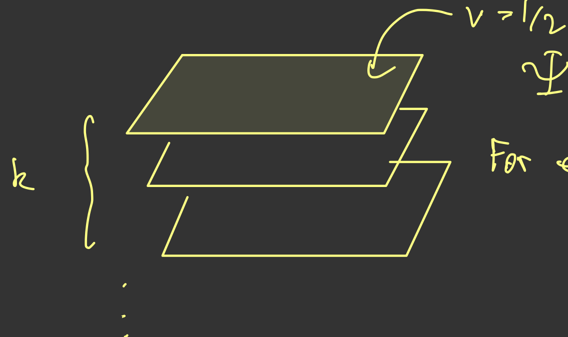
In addition to the $\mathbb{Z}_3$ parafermion, it has a Fibonacci anyon τ

Fusion rule: $\tau * \tau = I + \tau$ $\Rightarrow$ its unitary braiding matrices cover $SU(2)$
(Fibonacci sequence) $\Rightarrow$ universal quantum computer

Effective Field Theory Approaches (Fradkin, Nayak, Schoutens, 1999) (Goldman, Sohal, EF, 2019, 2020)

We will discuss bosons for simplicity, $\nu = \frac{k}{2}$

Consider k layers of bosons in a $\nu = \frac{1}{2}$ Laughlin state



$$\Psi_{1/2} \sim \prod_{i<j} (z_i - z_j)^2 \times \text{gaussians} \longleftarrow U(1)_2$$

For each layer $\mathcal{L} = \frac{2}{4\pi} \varepsilon_{\mu\nu\lambda} a^\mu \partial^\nu a^\lambda + \dots$

$$\equiv \frac{2}{4\pi} a\, d a + \frac{1}{2\pi} A\, d a + \dots$$

Symmetry $\underbrace{U(1)_2 \times \dots \times U(1)_2}_{k \text{ factors}}$

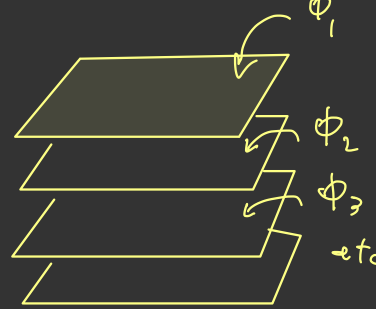
Chern-Simons level-rank duality $\quad U(1)_2 \longleftrightarrow SU(2)_1 \quad$ group is non-abelian

$I, \; e^{i\varphi/\sqrt{2}} \qquad j = 0, \frac{1}{2} \qquad$ the braids are abelian

Q: how to get to a state with non-abelian statistics?

Hint: somehow we need a theory on $SU(2)_k$

you need $U(1)_2 \times \dots \times U(1)_2 \longrightarrow SU(2)_k$



$\langle \phi_j \, \phi_{j+1} \rangle \neq 0$

(A) ① use the Chern-Simons level-rank duality

$SU(2)_1 \times \dots \times SU(2)_1$

② construct a condensate $\rightarrow SU(2)_k$

The 1999 paper did this by condensing pairs of excitations on two layers at a time

$\Rightarrow$ Higgs (Meisner) mechanism projects onto a state with symmetry $SU(2)_k$ (clustering)

1999 was basically right (but not completely)

$\Rightarrow$ Dualities solve the problem

Construction of a Fibonacci FQH state (Goldman, Sohal, EF, 2021)

* Want a FQH state with only Fibonacci anyons

$$\tau * \tau = \mathbb{1} + \tau \quad \text{(and no other anyons)}$$

$\Rightarrow$ universal quantum computing (3 τ's form a qubit)

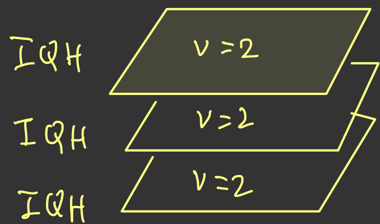
* Topological QFT?

$$(G_2)_1 \longleftrightarrow U(2)_{3,1} = \frac{SU(2)_3 \times U(1)_2}{\mathbb{Z}_2}$$

$$\mathcal{L}_{Fib} = \frac{3}{4\pi} \underbrace{\mathrm{Tr}\left[a\, d a - \frac{2}{3} i a^3\right]}_{SU(2)\text{ gauge field}} - \frac{1}{4\pi} \underbrace{\mathrm{Tr}[a]\, d\, \mathrm{Tr}[a]}_{U(1)_2} + \frac{1}{4\pi} \underbrace{A}_{\text{background}} d\, \mathrm{Tr}[a]$$

$$\Rightarrow \quad \nu = 2 \qquad \left(\sigma_{xy} = 2 \frac{e^2}{h} \right)$$

* Start with 3 layers of Diracs at $\nu = 2 \to 1$ transition (IQH)



$$\mathcal{L} = \sum_{n=1}^{3} \left[\bar{\Psi}_n (i \not{D}_A - M) \Psi_n - \frac{3}{2} \frac{1}{4\pi} A\, d A \right]$$

$$D_A = \partial - i A$$

Parity anomaly

Duality: Free Dirac $\psi \longleftrightarrow$ Wilson–Fisher boson ϕ + $U(N)_1$

OK since $U(N)_1 \longleftrightarrow \mathcal{L}_{eff} = -\frac{N}{4\pi} A\, d A$ (trivial)

* Set $N=2$

$$\mathcal{L} = \sum_n \left[|D_{a_n} \phi_n|^2 - r |\phi_n|^2 - |\phi|^4 + \mathcal{L}_{CS}[a_n] \right] + \frac{1}{2\pi} A\, d\, \mathrm{Tr}[a_1 - a_2 + a_3]$$

* Clustering: $\langle \Gamma_{mn} \rangle = \langle \phi_m^\dagger \phi_n \rangle \neq 0 \ (m \neq n)$, $\langle \phi_n \rangle = 0$

$\Rightarrow$ pins $a_1 = a_2 = a_3 \equiv a$ $\Rightarrow \frac{1}{2\pi} A\, d\, \mathrm{Tr}[a_1 - a_2 + a_3] \equiv \frac{1}{2\pi} A\, d\, \mathrm{Tr}[a]$

* The physical densities are pinned $\rho_1 = -\rho_2 = \rho_3$

$\Rightarrow$ layer exchange symmetry is broken

$$\Rightarrow \mathcal{L}_{U(2)_3} = 3\,\mathcal{L}_{CS}[a] + \frac{1}{2\pi} A\, d\, \mathrm{Tr}[a]$$

* To get Fibonacci $\Leftrightarrow$ attach a unit of flux to the fermions

$\Rightarrow$ fermions $\rightarrow$ bosons

flux attachment: $3\,\mathcal{L}_{CS}[a] + \frac{1}{2\pi} b\, d\, \mathrm{Tr}[a] + \frac{1}{4\pi}(b+A)\, d\,(b+A)$

fluctuating $U(1)$ gauge field

* Integrating out $b_\mu \Rightarrow$ obtain $\mathcal{L}_{Fib}$!

$\Rightarrow$ interpret $\underline{\phi^\dagger t^a \phi}$ as the Fibonacci anyon τ

* Alternatively we can attach (+) flux to layers 1, 3 and (−) to layer 2

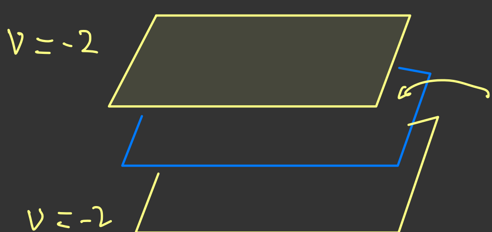
<u>before</u> clustering

$\Rightarrow$ layers 1, 3 become $|D_A \Phi|^2 + \frac{2}{4\pi} A\,dA$ (trivial)

layer 2 : $|D_\alpha \Phi|^2 + \frac{2}{4\pi} \alpha\, d\alpha + \frac{2}{4\pi} \beta\, d\beta + \frac{1}{2\pi} \alpha\, d\beta + \frac{1}{2\pi} \beta\, dA$

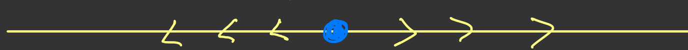
layer 2 $\Rightarrow$ Halperin (2,2,1) state

Transition (2,2,1) $\leftrightarrow$ Fibonacci



(2,2,1) $\Rightarrow$

Fibonacci $U(2)_{3,1}$ (2,2,1) Halperin



$\langle \Gamma_{mn} \rangle \neq 0$ $\langle \Gamma_{mn} \rangle = 0$

One can use this construction to derive the Fibonacci wave function!

Summary

* Non-Abelian dualities can be used to understand the landscape of non-abelian FQH states
* define physical parent states
* construct ideal wave functions using CFT methods
* hopefully to find simple Hamiltonians!
* Opens a window to universal TQC!

References: Goldman, Sohal, EF PRB 100, 115111 (2019)
102, 195151 (2020)
103, 235118 (2021)