Non-Fermi Liquid Quantum Critical Points in 2+1 d

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Quantum Critical Points and Quantum Critical Regions

[*Quantum Phase Transitions*, S. Sachdev, 2011, Cambridge Univ. Press]

- Quantum Critical Points (QCP) are associated to transitions between phases of matter at **zero temperature**.
- Occur when one (or several) coupling takes a particular **critical value**, which can in turn be tuned by varying certain external parameters (e.g. magnetic field, doping, pressure, etc…)
- At this particular locus, the Ground State dramatically changes (usually associated to some **Symmetry Breaking Pattern**).
- Hence, they depend exclusively on the properties of the GS, and **thermal fluctuations don't play any role** !

(For instance, don't confuse with magnetic orders destroyed by thermal fluctuations upon heating up the system (Curie transition at finite T))

- Prototypical example: Quantum Ising model

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - Jg \sum_i \sigma_i^x$$

$g \ll 1$ $\quad H \approx -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z$

$$|GS\rangle \in \{\prod_i |\uparrow\rangle_i, \prod_i |\downarrow\rangle_i\}$$

$$M_i \equiv \langle \sigma_i^z \rangle = \pm N_0$$

$$M_{ij} \equiv \langle \sigma_i^z \sigma_j^z \rangle = N_0^2$$

Quantum Ferromagnet

QCP

$g \gg 1$ $\quad H \approx -Jg \sum_i \sigma_i^x$

$$|GS\rangle = \prod |\rightarrow\rangle_i$$

$$M_i \equiv \langle \sigma_i^z \rangle = 0$$

$$M_{ij} \equiv \langle \sigma_i^z \sigma_j^z \rangle = \delta_{ij}$$

Quantum Paramagnet

$$g = g_c$$

- What are the relevant degrees of freedom at the QCP?

Interested on **2nd order phase transitions**:

Typical energy scale vanish
(gap, scale of fluctuations, stiffness, masses, etc)

$$\Delta \sim |g - g_c|^{z\nu}$$

Divergent correlation length
(exponential to power law)

$$\xi^{-1} \sim |g - g_c|^{\nu}$$

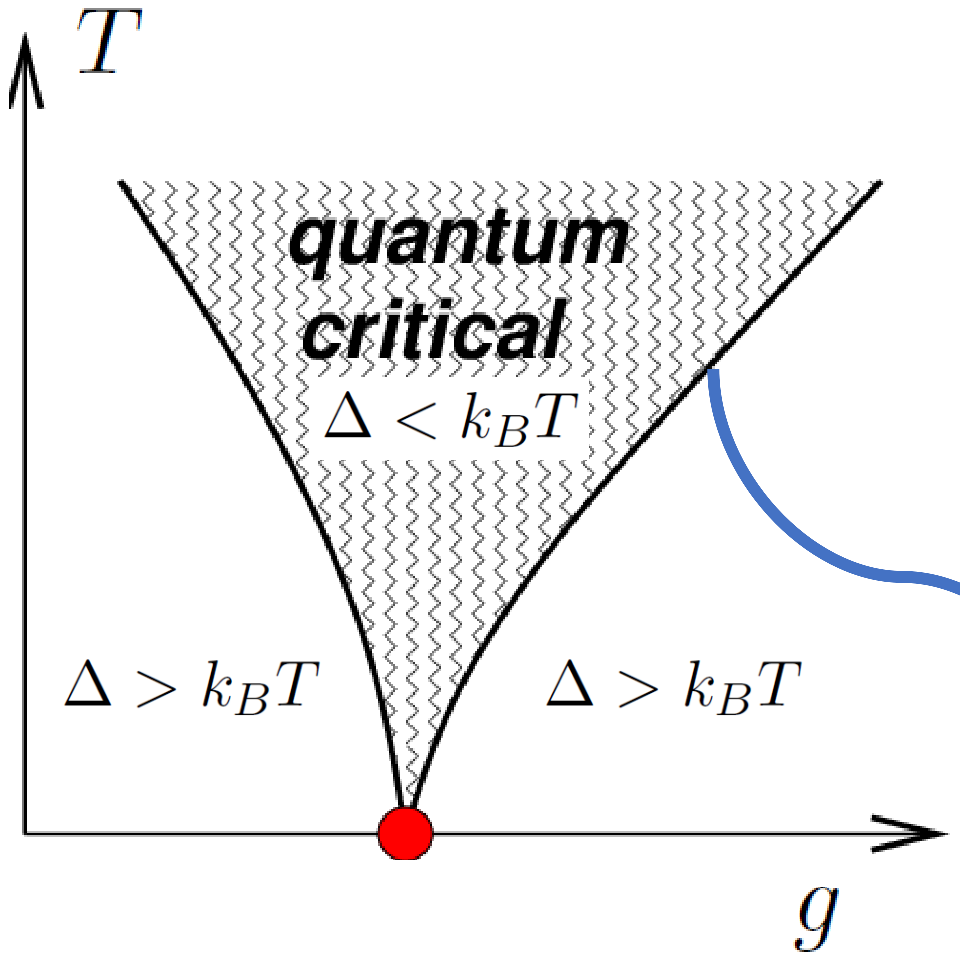
Scale invariant theory of gapless degrees of freedom

Dispersion relation determined by the critical exponent

$$\Delta \sim \xi^{-z} \sim p^{z} \qquad z \equiv \text{Dynamical Critical Exponent}$$

But experiments are at finite temperature so... is this useful at all?

- Yes! At finite T, there's a **Quantum Critical Region (QCR)**

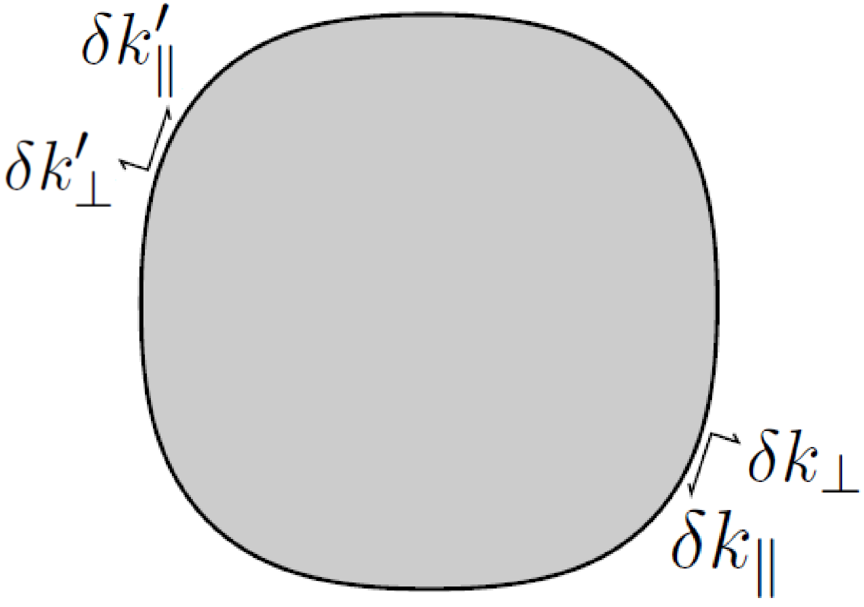


- **Quantum fluctuations still dominate** over thermal fluctuations on the QCR.
- **Scaling behavior dictated by the QCP extends over the QCR**, up to a given (dynamically generated) scale
- This affect thermodynamic properties, transport coefficients, etc. **Experiments detect the QCR !**

Crossover : $T \sim \xi^{-z} \sim p^{z}$

So far we haven't talked about **FERMIONS**…

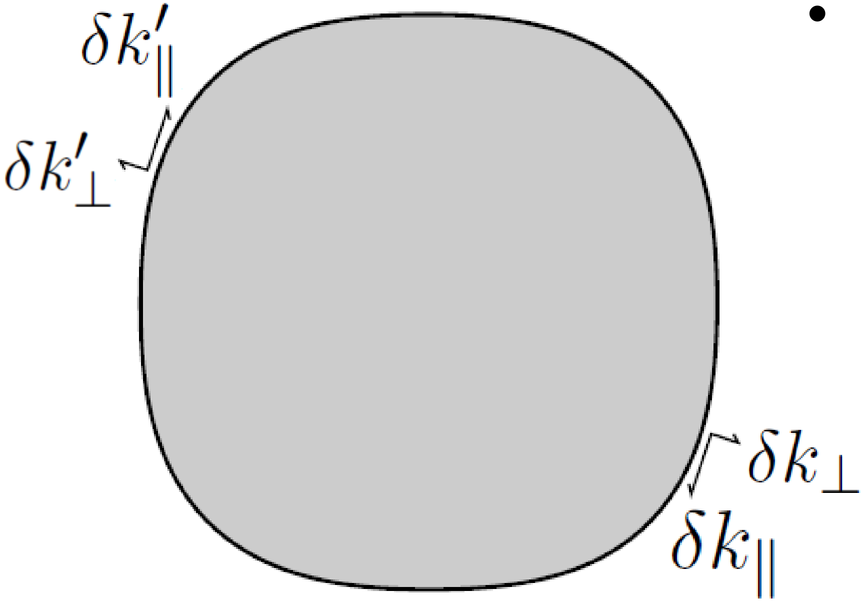
- In metallic systems, there are FERMIONIC degrees of freedom, whose ground state is characterized by the presence of a FERMI SURFACE (FS)
- Strong constraints associated to the FS lead to a dramatic simplification of the dynamics of fermions at low energies: **Fermi Liquid Theory** [Landau]



- Pauli's exclusion principle **strongly suppress the phase space** of allowed states participating on scattering processes at low energies
- It may involve large renormalizations, but they do not change drastically the analytic structure of the physical observables:

$$\text{Decay rate} \sim \left(\frac{\epsilon}{E_F}\right)^2 \qquad \epsilon = E - E_F$$

- Adiabatic Continuity: most metallic systems are described in the IR by a theory of **weakly coupled fermionic quasiparticles**.



- In a more "modern" approach **[Polchinski, Shankar]** :

ϵ scales with $k_{\perp}$ $\longrightarrow$ Renders 4-fermi (and beyond) interactions **IRRELEVANT**

- Transport functions determined completely by the (almost free) effective theory:

$$\rho \sim T^2$$

- There's a **MARGINALLY RELEVANT** interaction, usually induced by phonons : **BCS**

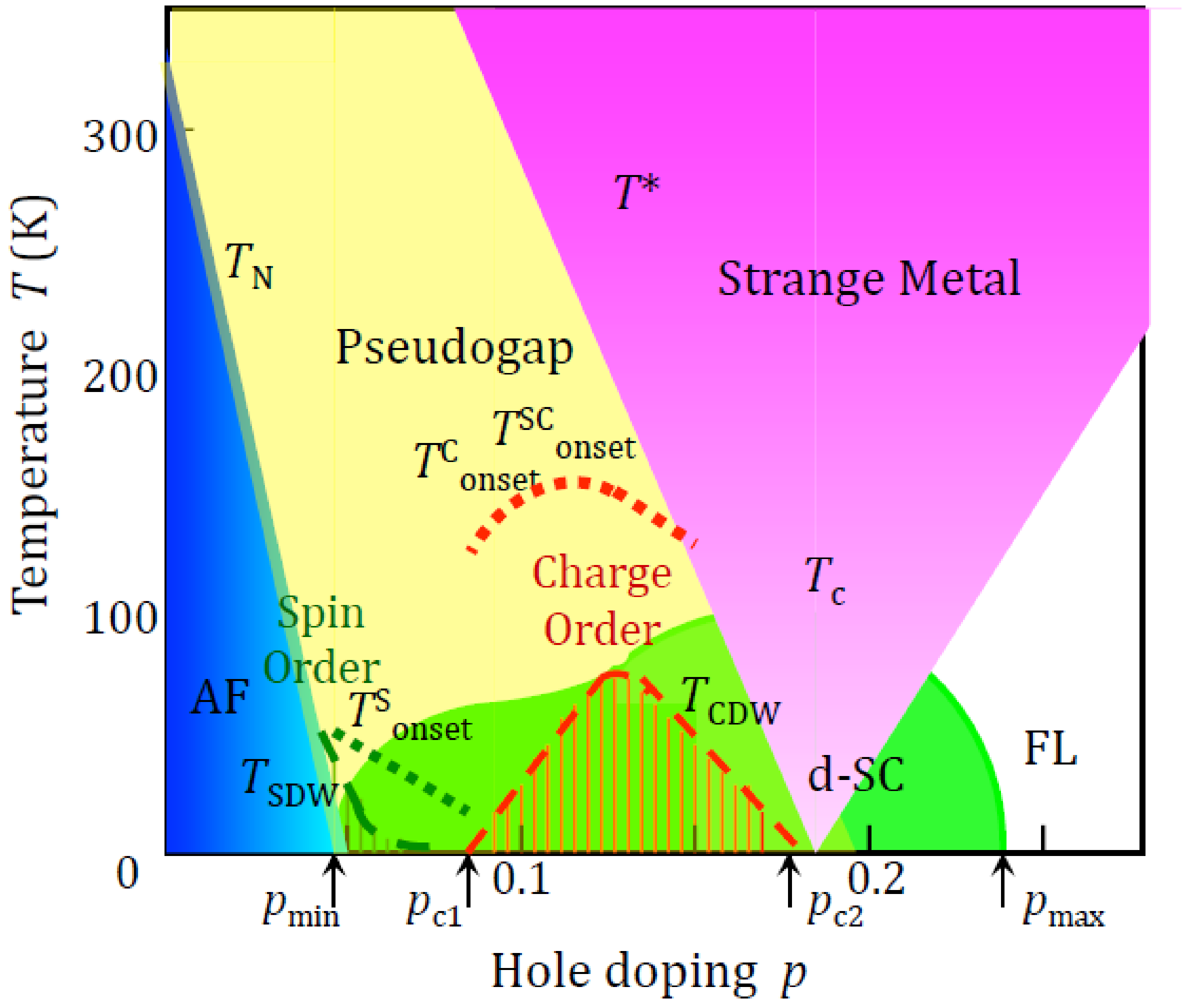
Pairing instability to a SC state $\xrightarrow[\text{BCS , Eliashberg}]{\text{Dimensional transmutation}}$ $\Delta_{sc} \sim T_c$

Exponentially suppressed

- Fermi Liquid Th + BCS mechanism : successful description of many observed metallic samples. One of the biggest achievements of the Wilsonian RG!

  - How does it breaks?
  - Is it possible for a metallic system to present different scaling behavior?
  - Alternative dissipation dynamics, scaling for the resistivity?
  - Are there alternatives to the BCS mechanism for superconductivity? Non-exponentially suppressed critical temperatures?

But, first of all, do we actually need to find an alternative?
Why should we look for Non-Fermi Liquids?



- Usual phenomenology observed in **High Tc Superconductors**, e.g. Cuprates at optimal doping.
- Strange Metal behavior above the SC dome signals the presence of a QCP at T=0. **Planckian dissipation!**
- Generically, a superconducting phase takes on before reaching T=0
- Underlying metallic state is a key ingredient for the onset of SC at **high critical temperatures**.

- Moreover, in some cases the SC dome can be shrunk by applying external magnetic fields, so uncovering more of the QCR. (**Observation of a naked QCP?**)

- A recipe to cook a Non-Fermi Liquid:
  - Add singular interactions, such that quantum corrections modify the analytic structure of the fermionic quasiparticles
  - Find a way to get singular (at low energies) contributions to pairing, which may enhance the critical scales

A QCP provides the key ingredient to get both effects: **GAPLESS BOSONS**!

QCP + FS $\longrightarrow$ Non-Fermi Liquid Critical Point

- Presence of a given QCR stands as a plausible explanation for the anomalous scaling observed in Strange Metallic States (e.g. anomalous scaling of the resistivity with T)

Our Task:

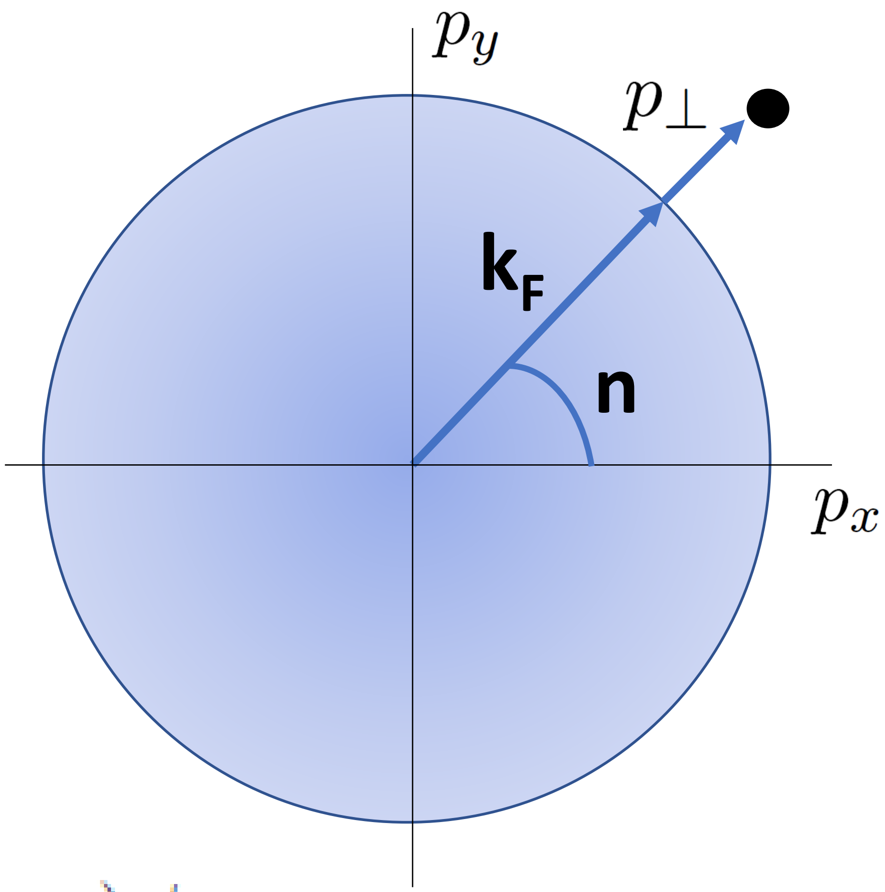
- Find a NFL critical point by coupling the FS to gapless bosons in the vicinity of a QCP. [JAD, Kachru, Raghu, Torroba]

- Characterize the QCR (finite T). In particular, it will demand to develop a mechanism for solving certain IR divergencies associated with the presence of gapless bosonic modes [JAD, Torroba, Solis]

- Explore the implications for Superconductivity? [JAD, Torroba, Solis]

The model at T=0: The QCP

- We will concern about models in **d=2** spatial dimensions
- Spherical Fermi surface of radius $\mathbf{k_F}$ (Fermi momentum) . This is usually the largest scale in the game.
- Near the FS, the quasiparticle energy only depends on the perpendicular component of the momentum



$$\mathbf{p} = (k_F + p_\perp)\mathbf{n}$$

$$\varepsilon_p = \frac{p^2}{2m} - \mu_F \approx v p_\perp \Rightarrow \int_{\omega,p} \psi^\dagger (i\omega - v p_\perp) \psi$$

$$\mu_F = \frac{k_F^2}{2m} \quad v = \frac{k_F}{m}$$

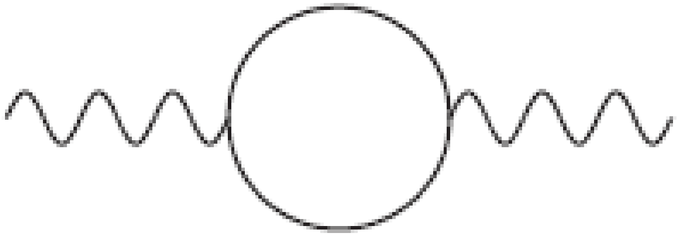
- Gapless boson:

$$\int_{\Omega,q} \phi(\Omega^2 + q^2)\phi$$

- Yukawa Interaction:

$$\int \phi\, \psi^\dagger\, \psi$$

Is the relativistic boson dispersion relation robust at low energies?



$$\approx \frac{|\Omega|}{\sqrt{\Omega^2 + (vq)^2}} \approx \begin{cases} \text{cnst} & , \quad |\Omega| \gg q \quad \textbf{massive} \\ \frac{|\Omega|}{q} & , \quad |\Omega| \ll q \quad \textbf{Overdamped} \end{cases}$$

Moral:

At low energies, dynamics of the relevant modes is governed by Landau Damping

Consider overdamped bosons right from the outset of the problem

- Introduce the so-called **Debye Mass**

M_D = Debye Mass $\qquad \int_{\Omega,q} \phi \left(q^2 + M_D^2 \frac{\Omega}{q} \right) \phi$

- This scale will act as the natural UV cutoff in our Effective Field Theory
- As the model is **tuned to criticality**, there is no mass term

$\Omega \sim q^z$

z = Dynamical exponent

Relativistic boson : $\int_{\Omega,q} \phi(\Omega^2 + q^2)\phi \;\Rightarrow\; z = 1$

Overdamped boson (Landau damping) : $\int_{\Omega,q} \phi \left(\frac{\Omega}{q} + q^2 \right) \phi \;\Rightarrow\; z = 3$

<u>Our Model:</u> FS interacting with a (gapless) **z=3** boson

(Quite common! Bosons associated to order parameters tend to display non-relativistic dispersions)

- Yukawa interaction is **RELEVANT** in d=2. The model is **STRONGLY COUPLED**
- Final ingredient: allow the fields to transform in a non-trivial representation of a flavor **SU(N)**, in order to solve the theory within a large **N** expansion

$$S_f = -\int_{\omega,p} \psi_i^\dagger \left(i\omega - v p_\perp\right) \psi^i$$

Fermionic excitations, transforming in the fundamental of the SU(N)

$$S_b = \frac{1}{2}\int_{\Omega,q} \phi_i^j \left(q^2 + M_D^2 \frac{|\Omega|}{q}\right) \phi_j^i$$

Overdamped adjoint scalars with **z=3** dynamical exponent

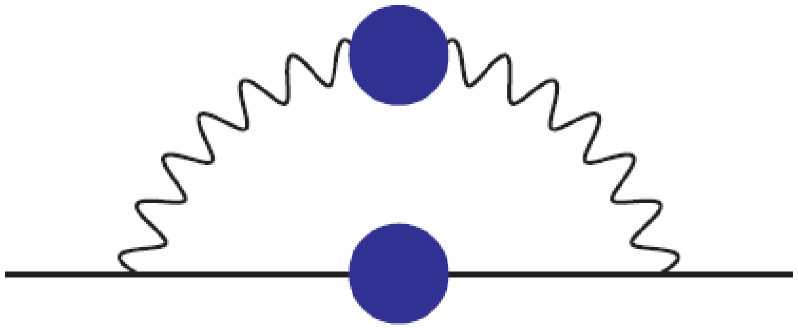
Relevant coupling **g** is kept fixed as **N** goes large

$$S_Y = \frac{g}{\sqrt{N}} \int_{\omega,p} \int_{\Omega,q} \phi_j^i(\Omega,q)\psi_i^\dagger(\omega,p)\psi^j(\omega-\Omega,p-q)$$

Solve the associated **large-N Schwinger-Dyson** equations

- At leading order in **N** , the (1-loop exact) solution of the Schwinger-Dyson equations is

Σ =



$$\Sigma(\omega) = \Lambda^{1/3}\mathrm{sgn}(\omega)|\omega|^{2/3}$$

$$\boxed{\Lambda = \frac{g^6}{(2\pi\sqrt{3}\,v)^3 M_D^2}}$$

- Dynamically generated scale below which the system flows to a non-trivial fixed point with NFL scaling

$$\omega < \Lambda \;\Rightarrow\; \Sigma(\omega) > \omega$$

$$\boxed{\int_{\omega,p} \psi^\dagger \left(i\Lambda^{1/3}|\omega|^{2/3} - v p_\perp\right) \psi \;\Rightarrow\; z_f = 3/2}$$

- The Yukawa coupling flows to a finite (O(1)) value given by the zero of the beta function. Therefore the model flows to an **interacting fixed point with NFL dynamics**

- The QCP just described is scale invariant. In particular, for the bosonic variables, the scale transformation is

$$\Omega \to e^{s}\Omega \ , \ q \to e^{\frac{s}{3}} q \ , \ \phi \to e^{-\frac{7}{6}s}\phi$$

- Note we haven't considered boson self-interactions. In particular, a quartic interaction is allowed by symmetries, so there is no reason to not include it in our effective action.

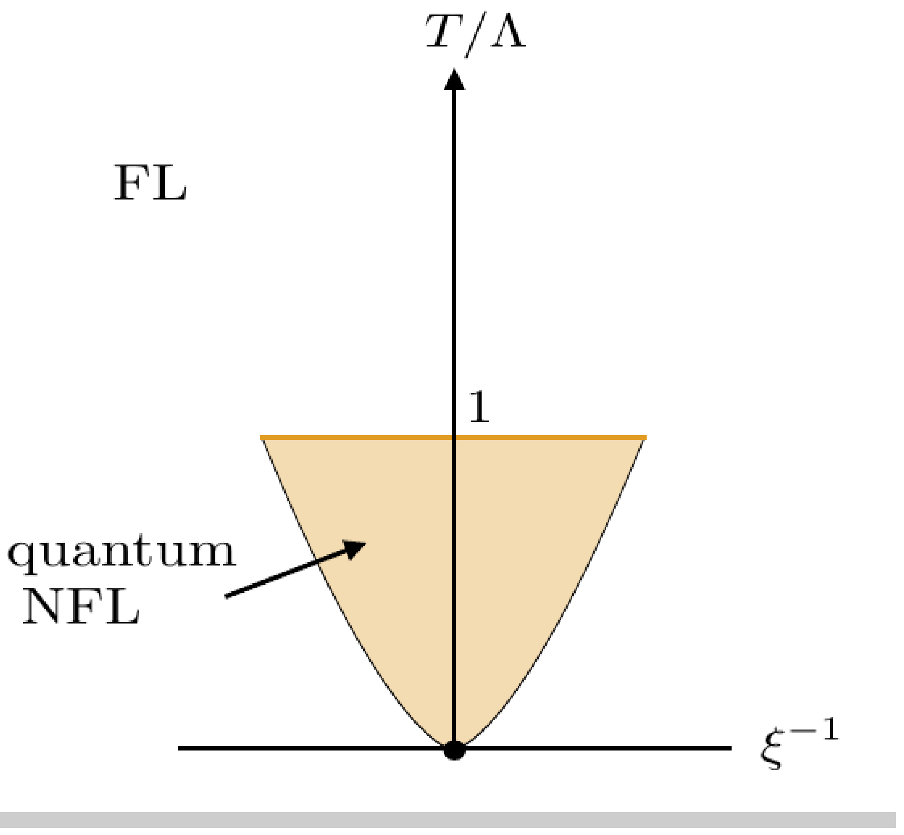
$$\lambda_\phi \int_{\{\Omega_i, q_i\}} \phi_i^4$$

- However, under the scale transformation, $\lambda_\phi \to e^{-\frac{s}{3}} \lambda_\phi$
- The interaction is **IRRELEVANT** at the fixed point thus being consistently ignored in the description of the QCP. **We will come back to it at finite temperatures!**

Heating up the QCP

- So far we have found the fixed point at T=0, describing the QCP. It displays NFL scaling by means of a fermionic dynamical exponent $\mathbf{z_f = 3/2}$
- This occurs below a dynamically generated scale Λ
- The **naïve intuition** about the finite T physics is that a QCR displaying the same anomalous scaling should extend above the QCP. As there are **no additional scales** in play, one would expect the crossover to occur at temperatures of the same order

Crossover : $T \sim \Lambda$



Q: Is the naïve intuition actually accurate?

A: NO

Q: Why is that? You said there's no additional scales in the game!

A: Because, once turning on T, the model is plagued of IR divergencies.

- Of course, these **divergencies compromise the perturbative expansion of the theory**

- So one has to **find a mechanism to resolve them**. On doing that, **a new scale is generated**, thus drastically changing the naïve picture just described

$$\int_{\Omega,q} \phi \left(\frac{\Omega}{q} + q^2 \right) \phi \qquad \Omega = 2\pi n T$$

$$\Omega = 0$$

- No damping
- Eff. 2d : IR divergent
- Destroy z_f **= 3/2**
- Dominate at high T

COMPETENCE ⟷

$$\Omega \neq 0$$

- Landau damping
- Regular
- Induce z_f **= 3/2**
- Dominate at low T

This will lead to a richer structure for the QCR!

IR divergencies and its resolution

- In these sort of models, IR divergent amplitudes arise from the exchange of **static (zero frequency) bosonic modes**

$$i\Sigma(\omega_n) = i\frac{g^2}{2\pi}T\sum_m \int qdq\,\frac{\mathrm{sgn}(\omega_m)}{\sqrt{\omega_m^2+(vq)^2}}\,D(i\omega_n - i\omega_m, q) = \underbrace{\boxed{\Sigma_T(\omega_n)}}_{m=n} + \underbrace{\Sigma_{\mathrm{NFL}}(\omega_n)}_{m\neq n}$$

$$D(q, i\Omega) = \frac{1}{q^2 + M_D^2\frac{|\Omega|}{q}}$$

$$D(q,0) = \frac{1}{q^2}$$

$$\Sigma(\omega_n) = \boxed{\frac{g^2}{2\pi}T\left\{\int\frac{dq}{q}\,\frac{\mathrm{sgn}(\omega_n)}{\sqrt{\omega_n^2+(vq)^2}}\right.} + \left.\sum_{m\neq n}\int qdq\,\frac{\mathrm{sgn}(\omega_m)}{\sqrt{\omega_m^2+(vq)^2}}\,D(i\omega_n - i\omega_m, q)\right\}$$

LOG DIVERGENT AS

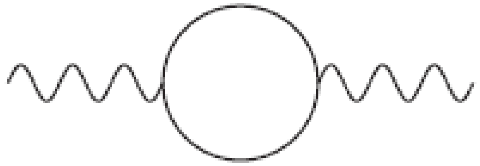
$q \to 0$

- IR divergences of this kind appear also at any higher loop diagram involving running static modes, so plaguing the whole perturbative series.
- For the model to be sensible, we need to resolve them:

<u>1st attempt : A perturbative mass (FAILED)</u>

It is a possibility for a mass term to be generated by fermions running in a loop, that would solve our problem already at 1-loop. We know it does not happen at T=0, but maybe there's a chance at finite T…

However…



$$\Pi(\Omega_n, q) = k_F \frac{g^2}{2\pi v N} \frac{|\Omega_n|}{\sqrt{\Omega_n^2 + (vq)^2}} \Longrightarrow \boxed{\Pi(q,0) = 0}$$

It is just a correction to the Landau damping (already present at tree level) , so it doesn't do the job

2nd attempt : Non-perturbative resolution (FAILED)

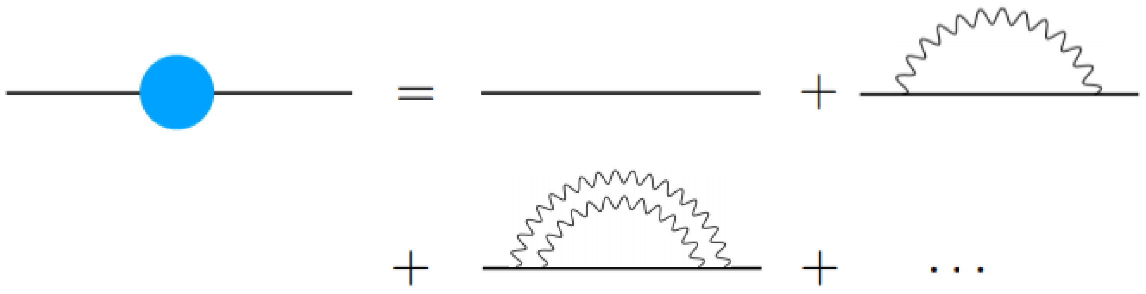
- It is not new that, in many cases, one could get rid of the divergencies by resuming an infinite set of diagrams. It usually leads to a finite result with non-analytic dependence on the coupling
- This mechanism proved to be useful for similar models in higher dimensions. So it is a natural choice here.

RECIPE

- Regulate the integrals, for instance by working in dimensional regularization

$$d = 2 + \epsilon$$

- Write down the SD equation by summing the rainbow diagrams and solve it
- Cross fingers and take the limit $\epsilon \to 0$
- Is the result finite???



$$\Sigma_T(\omega_n) \approx \mathrm{sgn}(\omega_n) \left(\frac{v^\epsilon}{2\pi\epsilon} g^2 T \right)^{\frac{1}{2-\epsilon}} \sim \frac{1}{\epsilon^{1/2}} \quad !!!$$

This method is not suitable . IR divergencies on **d=2** are too strong to be cured this way…

3rd attempt : Thermal mass generated by self-interactions

- So far we've been neglecting the 4-boson self-interaction. That is just fine as long as the bosonic dynamics are governed by the **z=3** scaling, for which this is an irrelevant term in the EFT.
- However, at finite T , one has to be more careful, as the effective action for the static mode does not have this property!
- At low energies and momenta, there's a large gap between the zero frequency mode and the remaining ones. So we can focus on the effective theory for this mode, now including the quartic interaction

$$E \ll T, q \ll (2\pi T M_D^2)^{1/3} \qquad \tilde{\phi}(q) \equiv T^{1/2}\phi(\Omega_n = 0, q)$$

$$S_{\text{eff}} = \int \frac{d^2q}{(2\pi)^2}\frac{1}{2}\,\text{tr}\,(\tilde{\phi}_q q^2 \tilde{\phi}_{-q}) + \frac{\lambda_\phi T}{8N}\int \prod_{i=1}^{3} \frac{d^2 q_i}{(2\pi)^2}\,\text{tr}(\tilde{\phi}_{q_1}\tilde{\phi}_{q_2}\tilde{\phi}_{q_3}\tilde{\phi}_{-q_1-q_2-q_3})$$

- Now, under a scale transformation which leaves the kinetic term invariant

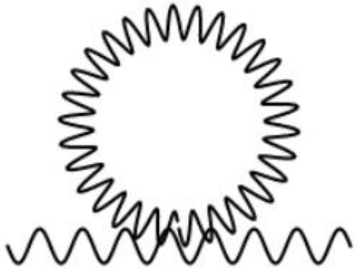
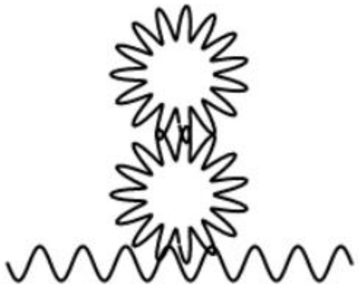
$$q \to e^{\frac{s}{3}} q$$
$$\tilde{\phi} \to e^{-\frac{2}{3}s} \tilde{\phi}$$

$$\Rightarrow \quad \boxed{\lambda_\phi T \to e^{\frac{2}{3}s} \lambda_\phi T \qquad \textbf{RELEVEANT !!!}}$$

- This is the case of “dangerously irrelevant” interaction becoming relevant in a particular situation. Physically, one could understand this as a consequence of the effective theory for the first mode being on one less dimension.

- Being now relevant, we can no longer ignore its effects. Moreover, **this interactions generate a mass term which is enough to render the model finite!**

- **The self-consistent Thermal Mass:**

$m_b^2 =$  $+$  $+ \quad \cdots \quad = \lambda_\phi T \int \frac{d^2 q}{(2\pi)^2} \frac{1}{q^2 + m_b^2}$

$$\boxed{m_b^2 \approx \frac{\lambda_\phi T}{4\pi} \log\left(4\pi \frac{(2\pi T M_D^2)^{2/3}}{\lambda_\phi T}\right)}$$

$$\Rightarrow \quad \boxed{D(q,0) = \frac{1}{q^2 + m_b^2}} \quad \Rightarrow$$

Problem Solved!

Solving the model at finite T

- By including the thermal mass, the integral SD equations are now finite
- We are thus in place to solve the model, that is, computing the fermionic self-energy

$$\Sigma(\omega_n) = \Sigma_T(\omega_n) + \Sigma_{\rm NFL}(\omega_n)$$

Thermal component:

- Contribution of static modes only
- Resolved by the thermal mass
- **This is the new feature** which arises at finite T
- **Introduces a new scale together with new scaling behavior!**
- Vanishes at T = 0

"Quantum" NFL component:

- Comprises quantum effects inherited from the QCP
- Preserves the QCP scaling **(z_f = 3/2)**
- Limits continuously to its T=0 counterpart

The “Quantum” Self-Energy:

$$\frac{\Sigma_{\mathrm{NFL}}(\omega_n)}{\mathrm{sgn}(\omega_n)} \approx \Lambda^{1/3}(2\pi T)^{2/3}\frac{2}{3}\left(\zeta(\frac{1}{3}) - \zeta(\frac{1}{3}, |n+\frac{1}{2}|+\frac{1}{2})\right)$$

- Goes continuously to the QCP at T=0

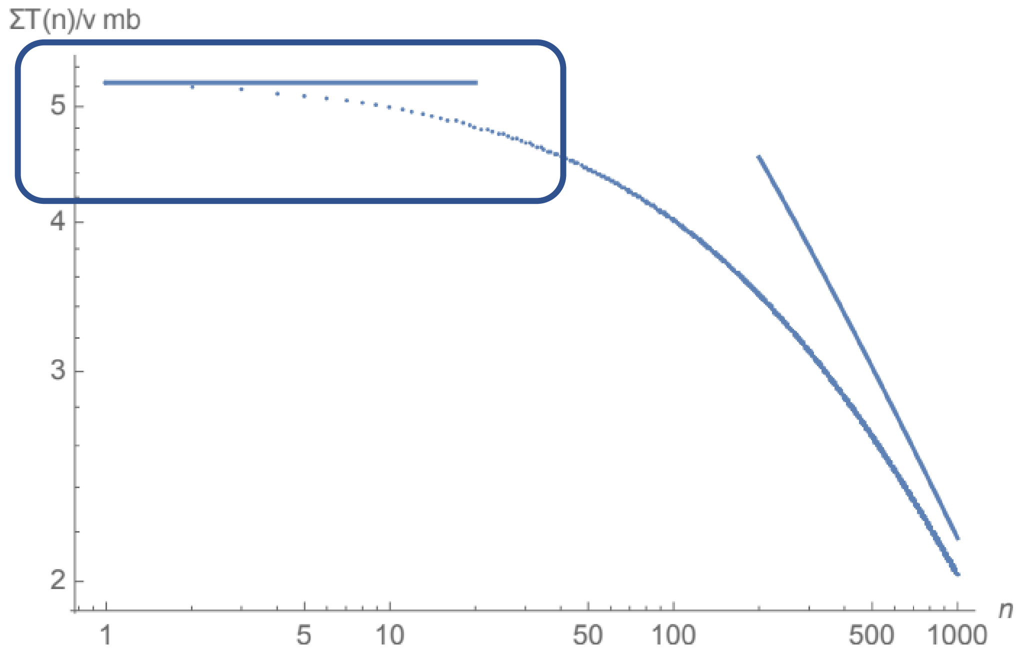
$$T \to 0 \;\; n \to \infty \quad nT \sim \omega \qquad \to \Sigma(\omega) = \Lambda^{1/3}|\omega|^{2/3}$$

- Anomalous scaling with Temperature, dictated by the QCP

$$\Sigma_{\mathrm{NFL}}(\omega_n) \sim \omega_n^{2/3} \sim T^{2/3}$$

The Thermal Self-Energy:

$$\Sigma_T(\omega_n) \approx \begin{cases} \boxed{\sqrt{g^2 T} \quad , \quad \omega_n < \Lambda_T} \\ \dfrac{g^2 T}{\Sigma_{\mathrm{NFL}}(\omega_n)} \quad , \quad \Lambda_T < \omega_n < \Lambda \\ \dfrac{g^2 T}{\omega_n} \quad , \quad \Lambda > \omega_n \end{cases}$$



- A new (T-dependent) scale:

$$\Lambda_T(T) \approx \frac{g^{3/2} T^{3/4}}{\Lambda^{1/2}}$$

- Violates the QCP scaling!

$$\Sigma_T(\omega_n) \sim T^{1/2} \ , \ (\omega_n - \mathrm{indep.})$$

This is the main point in this story!

- With the appearance of a **new dynamically generated scale**, one would expect a **new crossover** in the scaling behavior as a function of the temperature

- That might **affect considerably the structure of the QCR**, breaking our naïve intuition. In fact that's what happens!

- Some regions on parameter space will be governed by the **new NFL scaling**. It is no more caused by quantum effects, but thermal ones. So it is called **Thermal NFL**

- Lets establish some hierarchy of the relevant parameters

$$\Lambda \ll g^2 \ll M_D$$

- Then we get three well defined regimes as T increases

1) $T = 0$:

$$\omega + \Sigma(\omega) \approx \begin{cases} \Lambda^{1/3}\omega^{2/3} & , \quad \omega < \Lambda \\ \omega & , \quad \Lambda > \omega \end{cases}$$

QC NFL
+
FL

2) $0 < T < (\Lambda/g^2)\Lambda$:
$(\Lambda_T \ll \Lambda)$

$$\omega_n + \Sigma(\omega_n) \approx \begin{cases} \sqrt{g^2 T} & , \quad \omega_n < \Lambda_T \\ \Lambda^{1/3}\omega_n^{2/3} & , \quad \Lambda_T < \omega_n < \Lambda \\ \omega_n & , \quad \Lambda > \omega_n \end{cases}$$

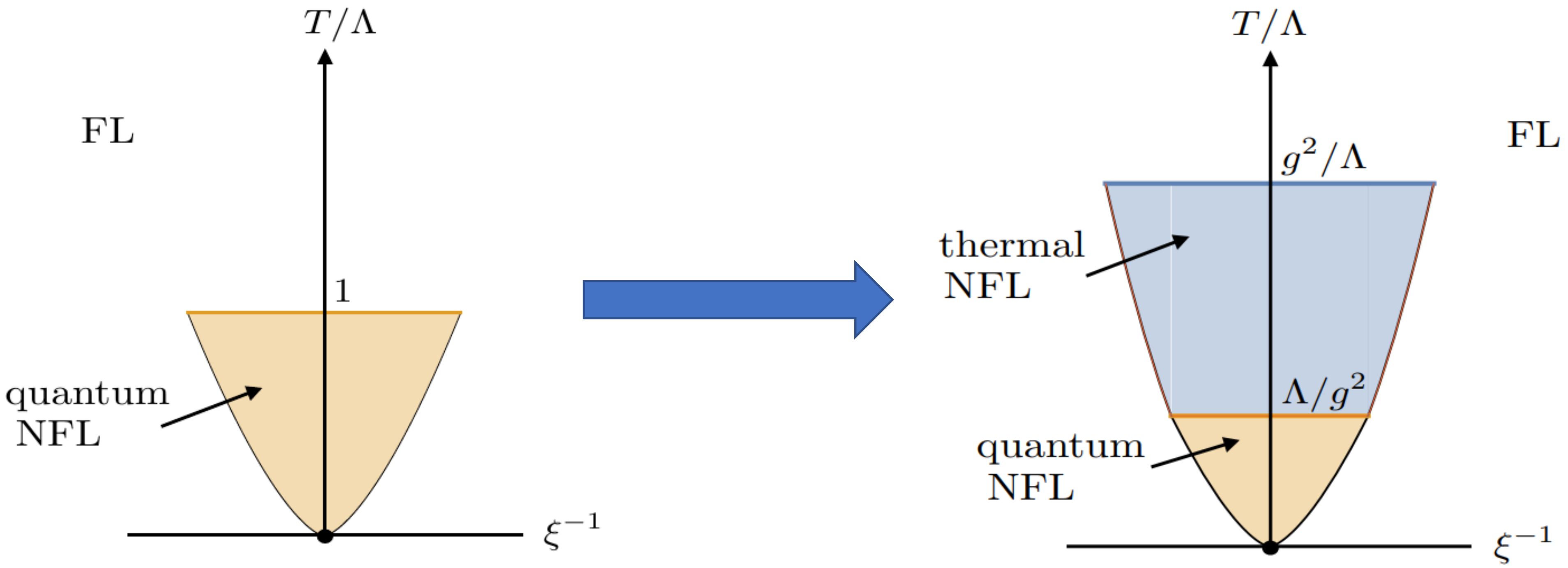
Th NFL
+
QC NFL
+
FL

3) $(\Lambda/g^2)\Lambda < T < g^2$
$(\Lambda_T = \Lambda)$

$$\omega_n + \Sigma(\omega_n) \approx \begin{cases} \sqrt{g^2 T} & , \quad \omega_n < \sqrt{g^2 T} \\ \omega_n & , \quad \sqrt{g^2 T} > \omega_n \end{cases}$$

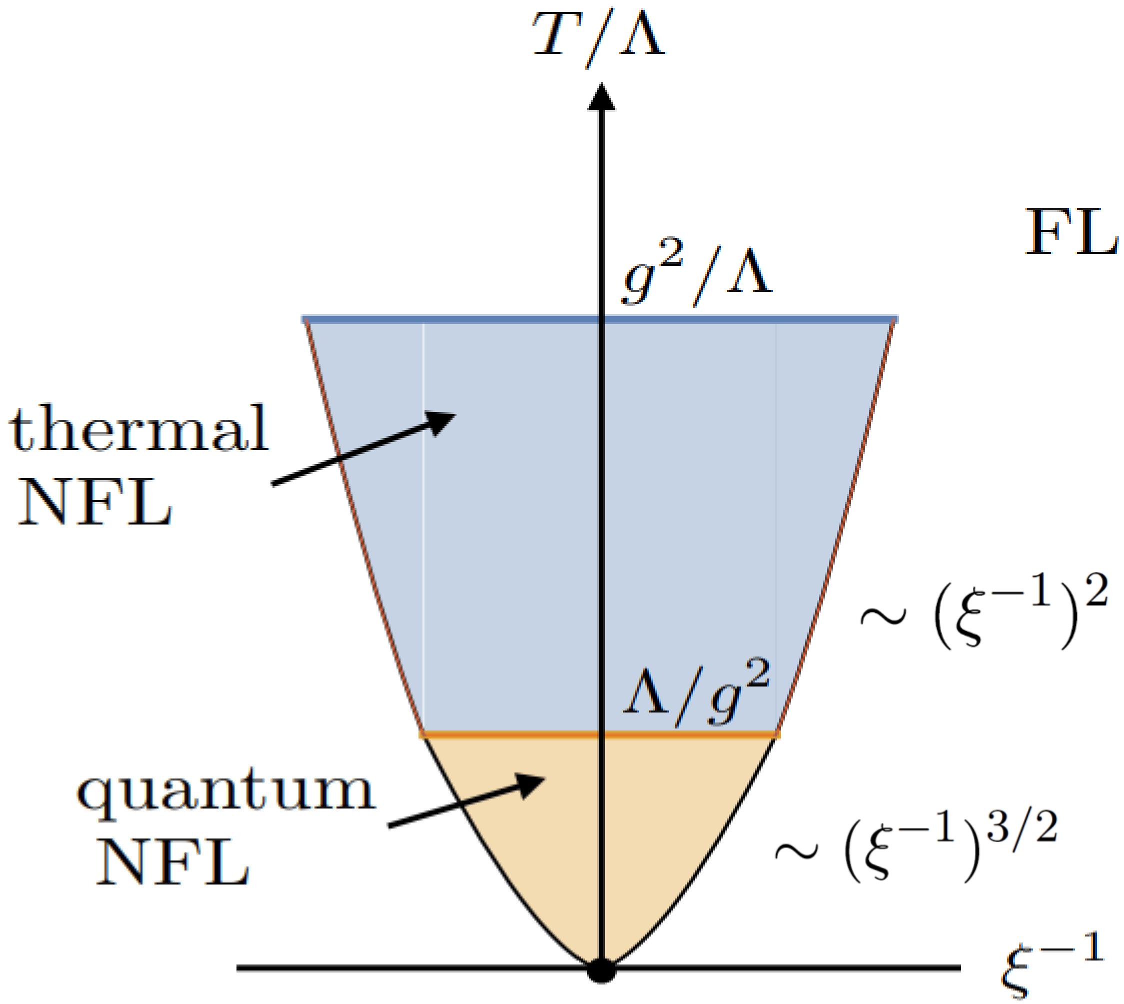
Th. NFL
+
FL

- Now we can develop a better intuition about the QCR



NAÏVE **TRUE**

- Recall our hierarchy $\Lambda \ll g^2 \ll M_D$



- The QCR is smaller in comparison with the naïve intuition
- This occurs because IR singularities (or its consequences after being resolved) are strong enough to dominate over the QCP dynamics above a certain scale smaller than Λ
- A new “phase” dominated by static boson exchange extends over the QCR
- It presents anomalous scaling with a different exponent

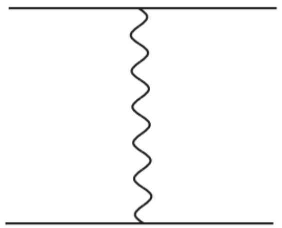
What about Superconductivity?

- NFL superconductivity displays new features caused by both quantum and thermal effects
- **Quantum**: Dual effect of interactions mediated by gapless bosons

Gapless boson fluctuations

Destroy quasiparticles

Boson Exchange



QC NFL scaling = BCS irrelevant

COMPETENCE

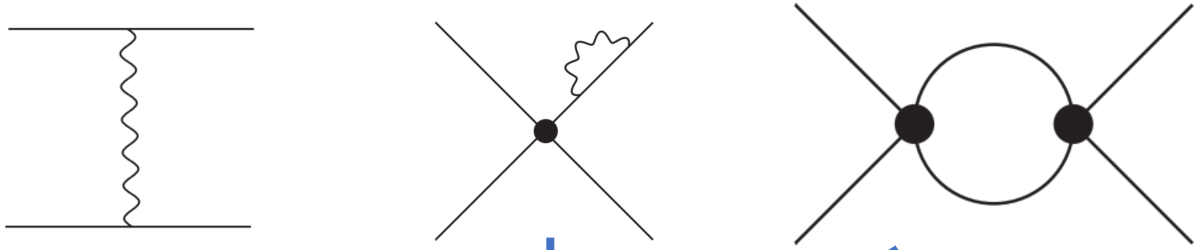
Singular contribution to BCS vertex (enhancement)

Already considered in color SC [D.T. Son]

CRITICAL N

- Perturbative RG at zero temperature:

$$S_{BCS} = -\frac{\lambda}{4k_F N}\int \psi_i^\dagger \psi^j \psi_j^\dagger \psi^i$$

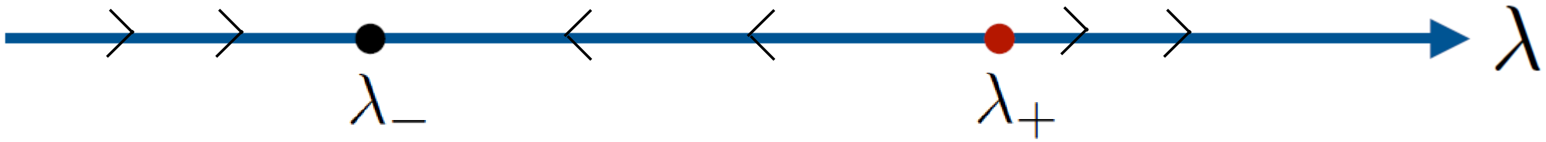


$$\frac{d\lambda}{d\mu} = \boxed{-\frac{8\pi}{9} + \frac{\lambda}{3}} - \frac{\lambda^2}{4\pi N}$$

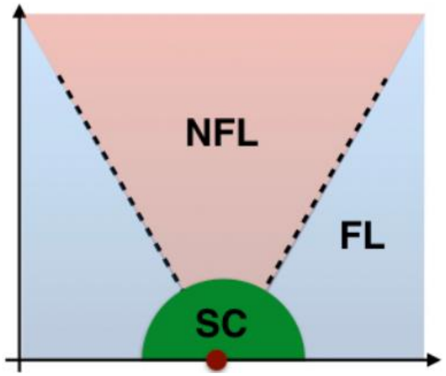
$$\lambda_\pm = \frac{2\pi}{3} N \left(1 \pm \sqrt{1 - 8/N}\right)$$

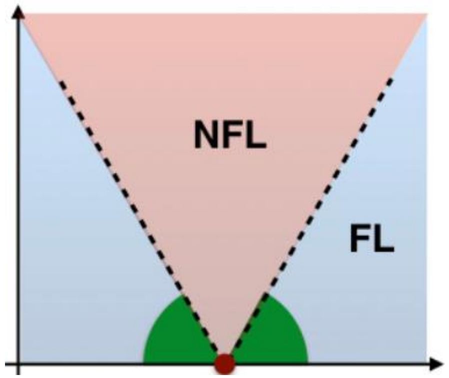
Real for

$$N > N_c = 8$$



SC phase takes over before reaching T=0





Quantum Critical metallic phase with finite BCS coupling

$$N_c = 8 \qquad\qquad N$$

- Fixed points collide at **N=8**
- Correlation length diverges non-analytically $\xi^{-1} \sim e^{-\frac{1}{\sqrt{8-N}}}$

Berezinskii-Kosterlitz-Thouless Transition

Thermal effects:

- Static boson contributions are **usually excluded** by appealing to Anderson's theorem (disorder doesn't affect superconductivity)
- Not possible to do that in models with N>1
- Lead to **additional suppression** of SC, together with **scale separation**

$$\frac{T_c}{\Lambda} \approx \boxed{\frac{4\pi}{\log\frac{g^2}{\lambda_\phi}}}\,\boxed{\frac{\Lambda}{g^2}}\,\boxed{\exp\left(-\frac{8\pi}{\sqrt{N_{cr}/N-1}}\right)}$$

Remnant of IR divergencies (log suppression)

Overall scale suppression $\Lambda \to \Lambda/g^2$

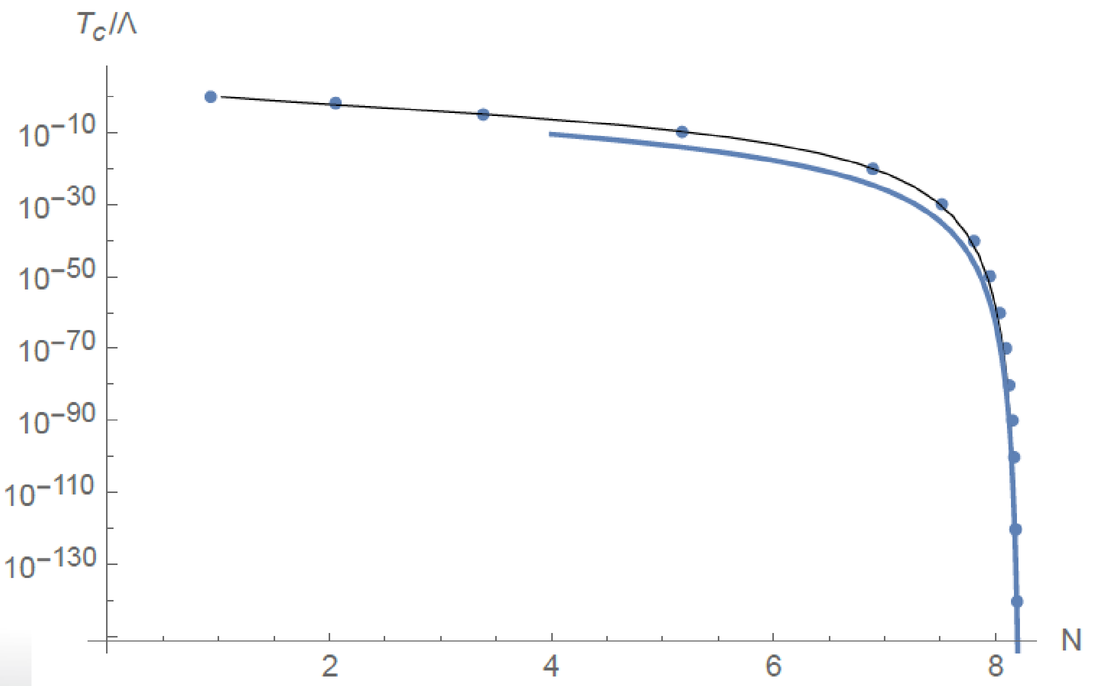
Incoherence due to Quantum Critical Fluctuations

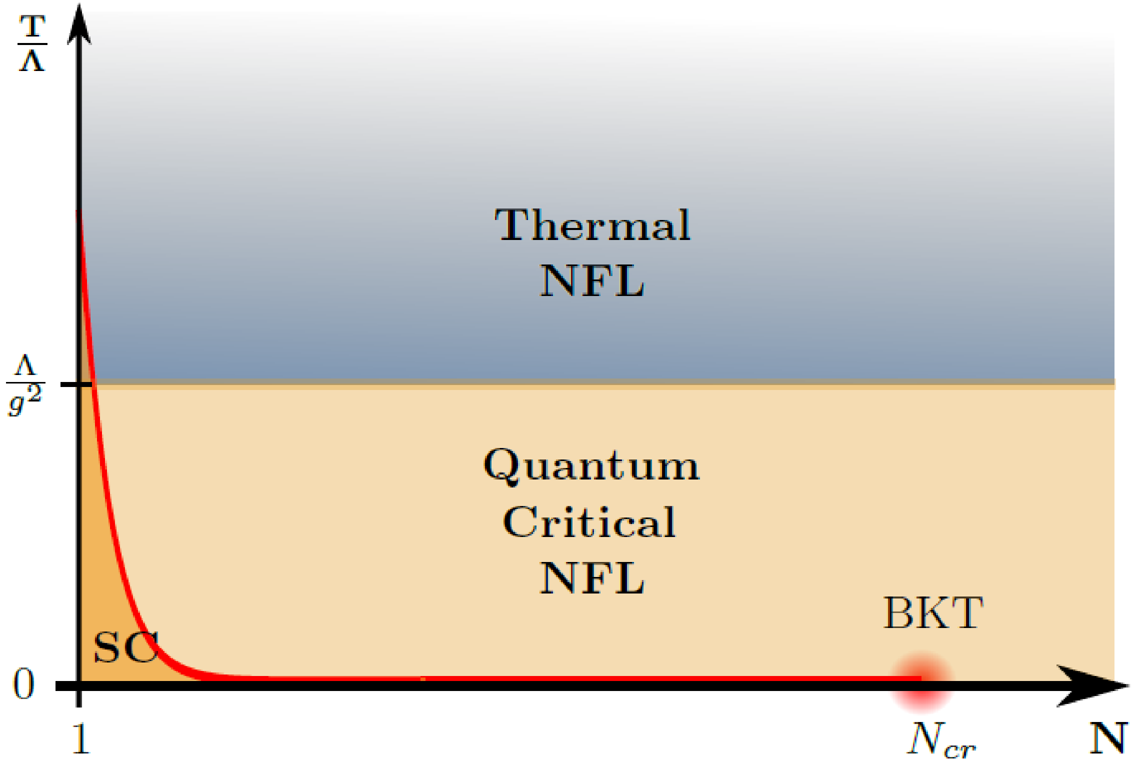
$$\frac{T_c}{\Delta_{sc}} \ll 1$$

Benchmark of NFL superconductivity!

(Recall the BCS case: $\frac{T_c}{\Delta_{sc}} \approx \mathcal{O}(1)$)

- Numerical calculations confirmed the analytical expectations





Final T-N phase diagram

Summary and conclusions:

- The theory of Non Fermi Liquid metals stands as a plausible theoretical approach to the phenomenology of both Strange Metallic normal states and the on-set of Superconductivity beyond BCS.
- We worked out a particular example under analytical control, so amenable to extract precise predictions for the anomalous scaling.
- Look for further examples of this kind may lead to a better understanding of the underlying physics and competing effects (maybe more realistic ones? Models with **z=2** order parameter?)
- Besides SC: Strong incoherence may suppress SC completely. Engineering of materials with large N? Naked QCP’s in the lab?

THANK YOU VERY MUCH!