Conformal boundary conditions for free fields

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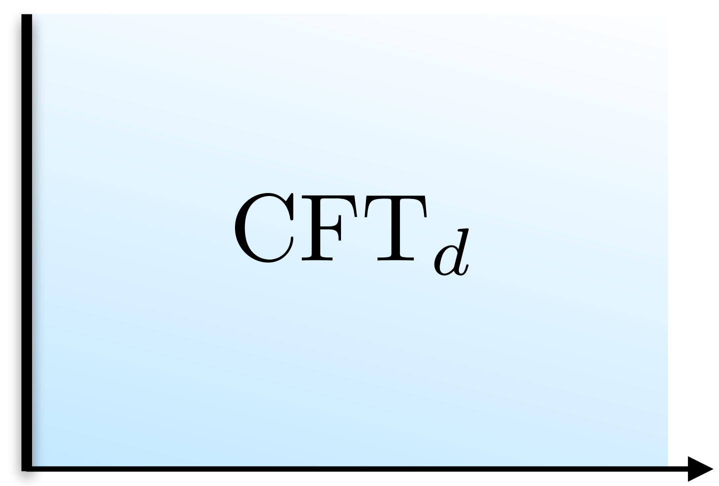
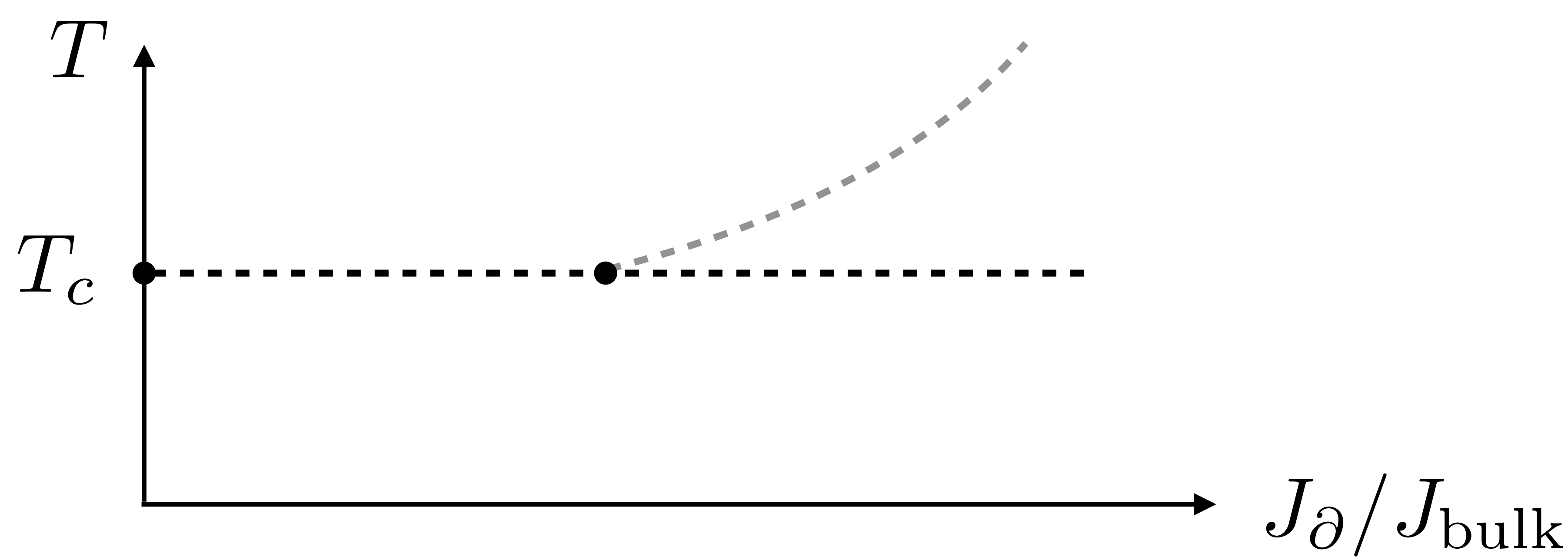
Conformal boundary conditions



Preserving $d-1$ dim. conformal symmetry.

Describe “surface critical behavior”



Observables: correlation functions of boundary operators. Critical exponents different from bulk.

Question: Given a bulk CFT, what are the possible conformal boundary conditions?

Not much known besides rational 2d CFTs.

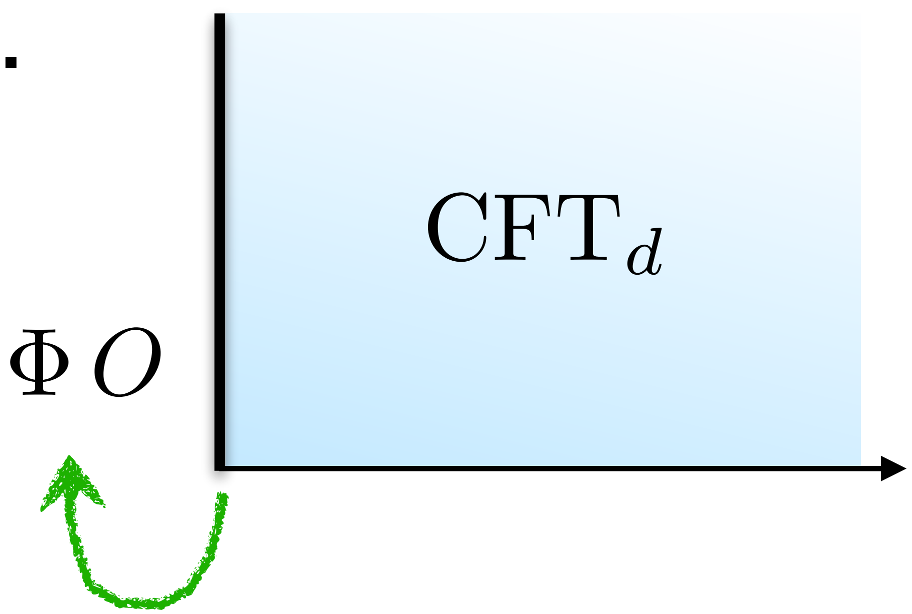
Exploring the set of possible boundary conditions

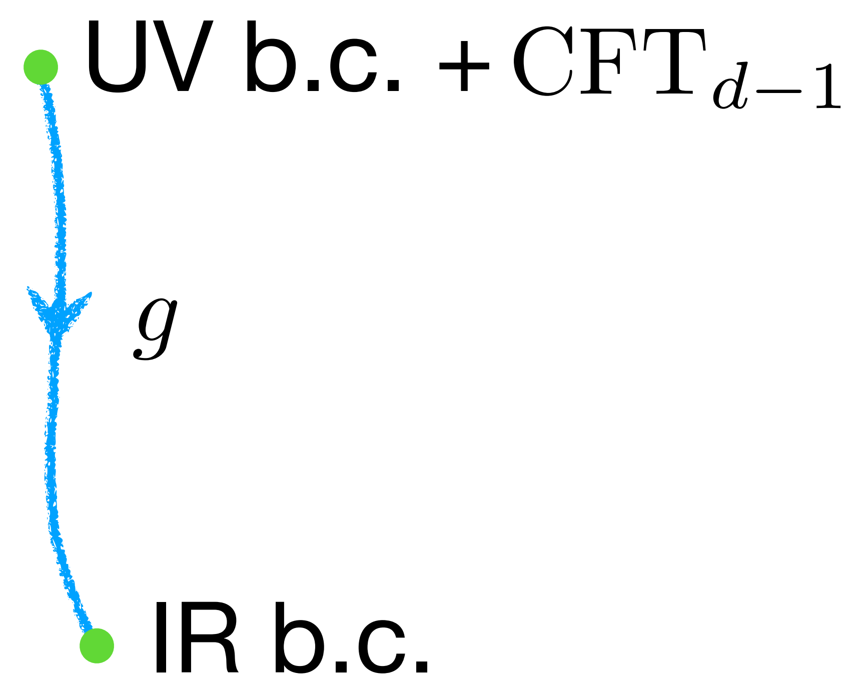
- RG intuition → hopeless problem, the answer is ~ as rich as the space of one-lower dimensional CFTs

$\Phi \in$ UV b.c.

$O \in \mathrm{CFT}_{\text{d-1}}$

$$\int d^{d-1}x \; g \, \Phi \, O$$





- Possible loophole: the IR fixed point might generically decouple from the bulk

Conformal boundary conditions for free conformal field theories: free bulk, interaction with bdry d.o.f.

- Boundary phase transitions in systems described by free fields in the bulk;

- Rich enough: interesting classification problem;

→ Examples

- Simple enough: worth trying to attack;

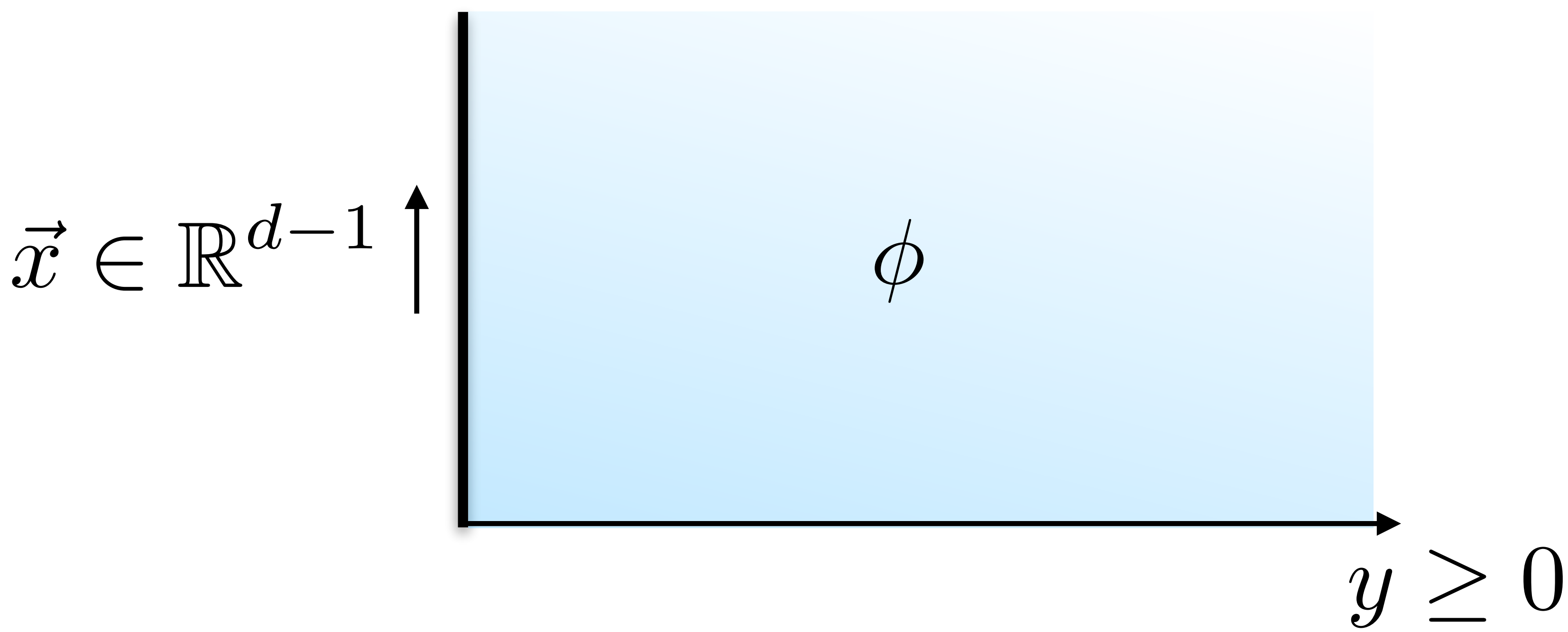
→ Make it amenable to numerical bootstrap

Applications:

- Free scalar: goldstone phase, $\mathbb{Z}_2$ breaking transition in 4d bulk;

- Maxwell: new way to construct & study 3d abelian gauge theories

Introduction: Free Bulk & Free Boundary



In the bulk: **free scalar CFT**

$$\Box \phi = 0 \quad \Rightarrow \quad \phi(y, \vec{x}) \underset{y \to 0}{\sim} (\hat{\phi}(\vec{x}) + \ldots) + y(\widehat{\partial_y \phi}(\vec{x}) + \ldots)$$

Stationary action (w/out interactions on the boundary):

$\hat{\phi}(\vec{x}) = 0$	or	$\widehat{\partial_y \phi}(\vec{x}) = 0$
Dirichlet		**Neumann**

Introduction: Free Bulk & Free Boundary

Similar for **free Dirac CFT** $\gamma^\mu \partial_\mu \Psi(y, \vec{x}) = 0$

$$\widehat{\psi_\pm}(\vec{x}) = \frac{\mathbb{1} \pm \gamma^y}{2} \Psi(y, \vec{x}) \Big|_{y=0}$$

$\widehat{\psi_+}(\vec{x}) = 0$	or	$\widehat{\psi_-}(\vec{x}) = 0$

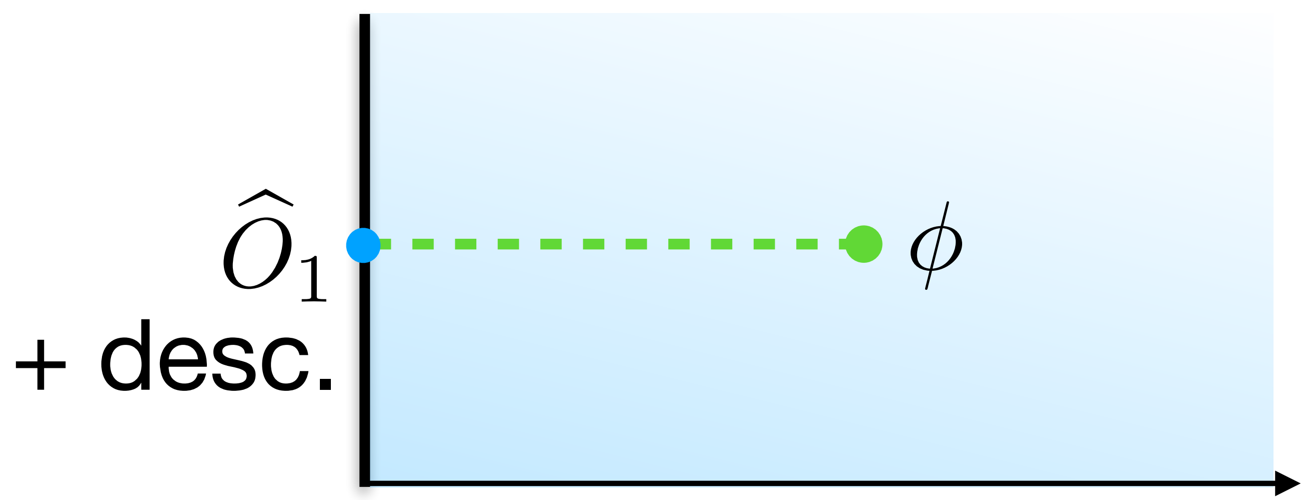
or **free higher-form CFT**, e.g. in 4d $\partial^\mu F_{\mu\nu} = 0 = \partial^\mu \tilde{F}_{\mu\nu}$

$$\hat{J}_a(\vec{x}) = \frac{1}{g^2} F_{ya}(y, \vec{x})\big|_{y=0} \;, \; \hat{I}_a(\vec{x}) = \frac{1}{4\pi i} \epsilon_{abc} F^{bc}(y, \vec{x})\Big|_{y=0}$$

$\hat{J}_a(\vec{x}) = 0$	or	$\hat{I}_a(\vec{x}) = 0$

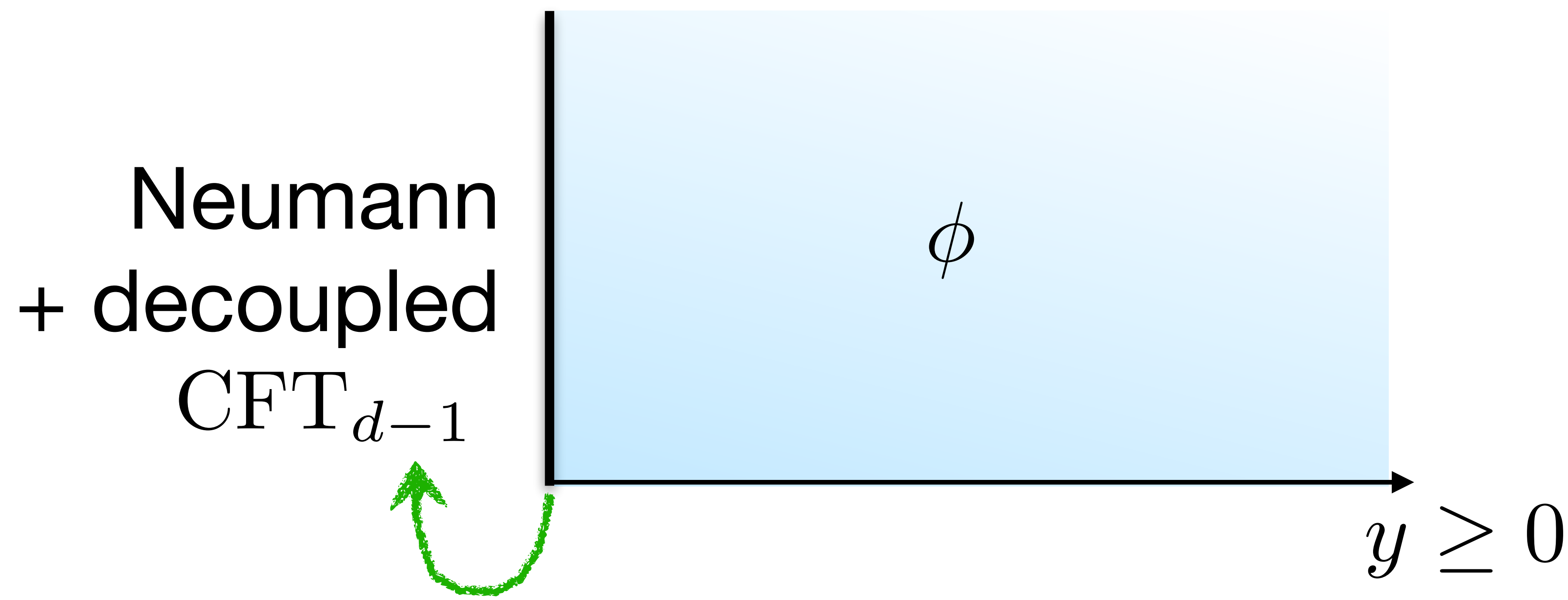
Introduction: Free Bulk & Free Boundary

In all these cases: $\widehat{O}_1$ and $\widehat{O}_2$, one set to zero; only 1 boundary primary in the **boundary OPE**.



Boundary theory: Mean Field Theory for the remaining operator. (MFT = GFF = Wick contractions)

Adding boundary interactions: Scalar Example



$$g \int d^{d-1}\vec{x}\, \hat{\phi}(\vec{x})\, \mathcal{O}(\vec{x}) \qquad \mathcal{O} \in \mathrm{CFT}_{d-1}$$

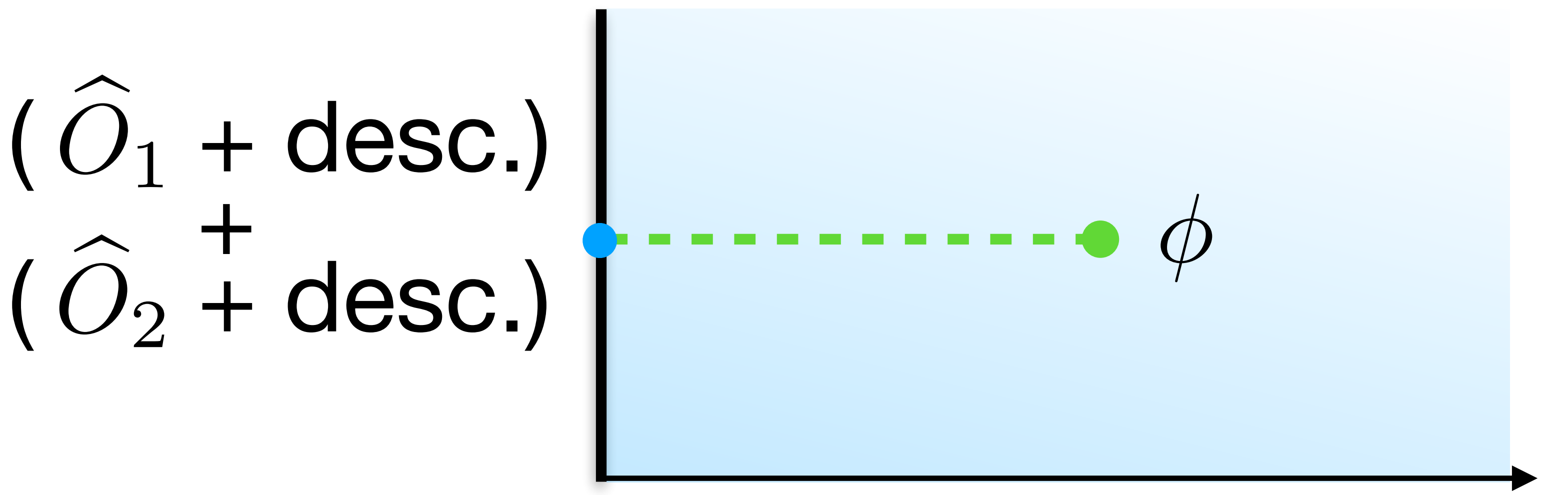
$\Rightarrow$ $\widehat{\partial_y \phi} = g\mathcal{O}$ **Modified Neumann**

Starts an RG, endpoint: conformal b.c. with both

$$\hat{\phi} \neq 0 \qquad \widehat{\partial_y \phi} \neq 0$$

<u>Interacting boundary condition</u> for free fields:

- Both $\widehat{O}_1$ and $\widehat{O}_2$ in the set of boundary operators, and they appear in the boundary OPE of the bulk free field



Scaling dimensions $\hat{\Delta}_1$ and $\hat{\Delta}_2$ fixed by e.o.m.

Boundary theory is **<u>not MFT</u>**

Example (1): [D, Gaiotto, Lauria, Wu]

interacting conformal b.c. for 4d Maxwell CFT

$$S_{\text{bulk}} = -\frac{i}{8\pi}\int \left[\tau (F^-)^2 - \bar{\tau}(F^+)^2\right] \ , \quad \tau \equiv \frac{\theta}{2\pi} + i\frac{2\pi}{g^2}$$

$$\hat{J}_a(\vec{x}) = \frac{1}{g^2}\, F_{ya}(y,\vec{x})\big|_{y=0} \ , \ \hat{I}_a(\vec{x}) = \frac{1}{4\pi i}\epsilon_{abc}\, F^{bc}(y,\vec{x})\big|_{y=0}$$

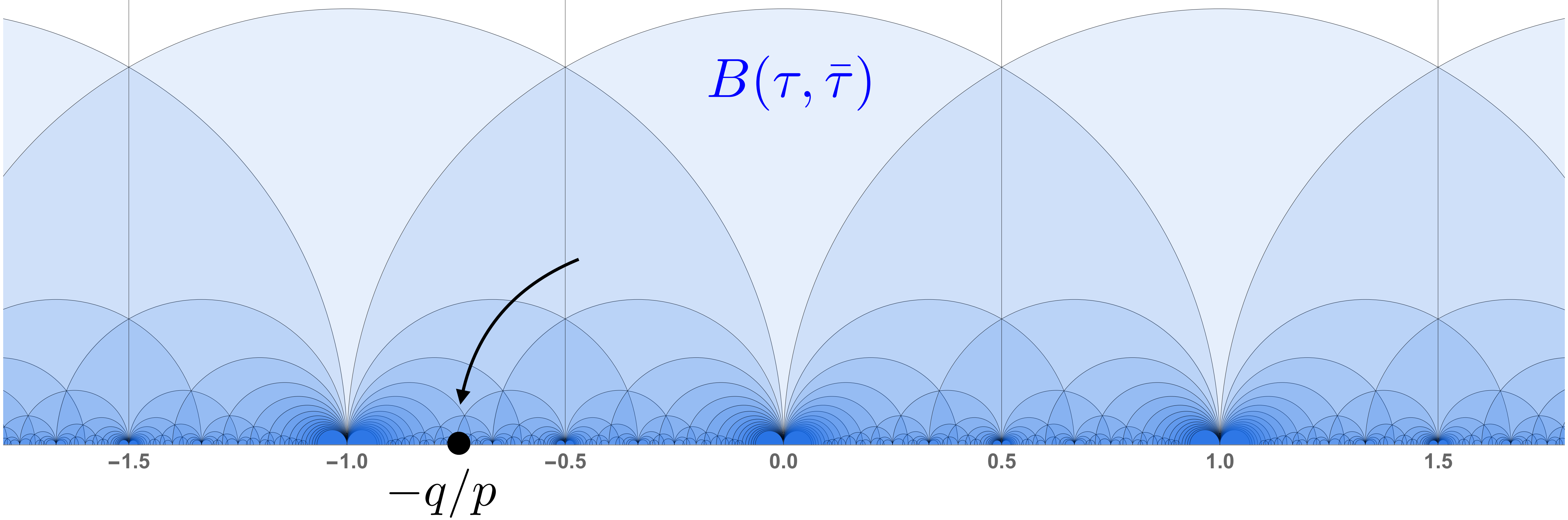
Start with Neumann + 3d CFT with $U(1)$ symmetry

$$\int d^{d-1}\vec{x}\, A_a(\vec{x})\, J^a_{\text{CFT}}(\vec{x})$$

$\Rightarrow$ $\hat{J}_a = J_{\text{CFT}\,a}$ **Modified Neumann**

τ coefficient of a bulk operator: **exactly marginal**

$B(\tau, \bar{\tau}) \to \hat{I}_a$ MFT + 3d CFT $T_{0,1}$



$B(\tau, \bar{\tau}) \to p\hat{J}_a + q\hat{I}_a$ MFT + 3d CFT $T_{p,q}$

$T_{p,q}$ from $T_{0,1}$ through $SL(2, \mathbb{Z})$ [Witten]

Example (2):
attempts with 4d free scalar

[D, Lauria, Niro]

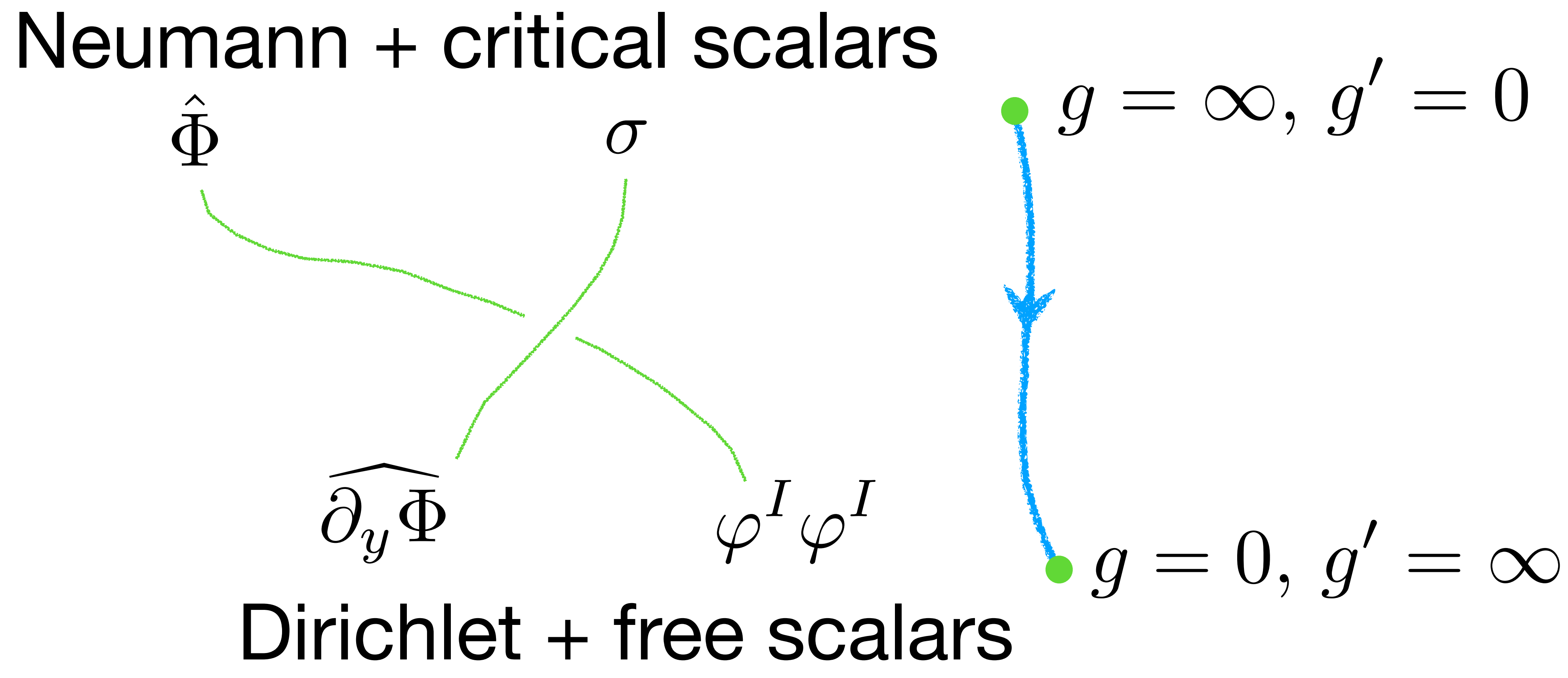
Calculable coupling to 3d CFT → use large N

- Large N vector models: φ^I N free fields;
  with O(N)-invariant quartic interaction: interacting CFT, solvable in 1/N expansion

- Singlet scalar operator of dimension 1 (free scalar) or 2 (critical scalar): $\varphi^I \varphi^I$, σ .

Dirichlet + free: $g\, \widehat{\partial_y \Phi}\, \varphi^I \varphi^I$

Neumann + critical: $g'\, \widehat{\Phi}\, \sigma$



No fixed point with bulk-boundary interactions.

Boundary RG with two dual descriptions: $g' = 1/g$

New connection between critical/free theory.

Example (3):

[Behan, D, Lauria, Van Rees]

examples with 3d free scalar

Coupling to **Minimal Models** on the 2d boundary

Dirichlet + $\mathcal{M}_{m+1,m}$ in the large m limit

$$g\mathcal{O}_{(1,3)} + g'\mathcal{O}_{(1,2)}\widehat{\partial_y \phi}$$

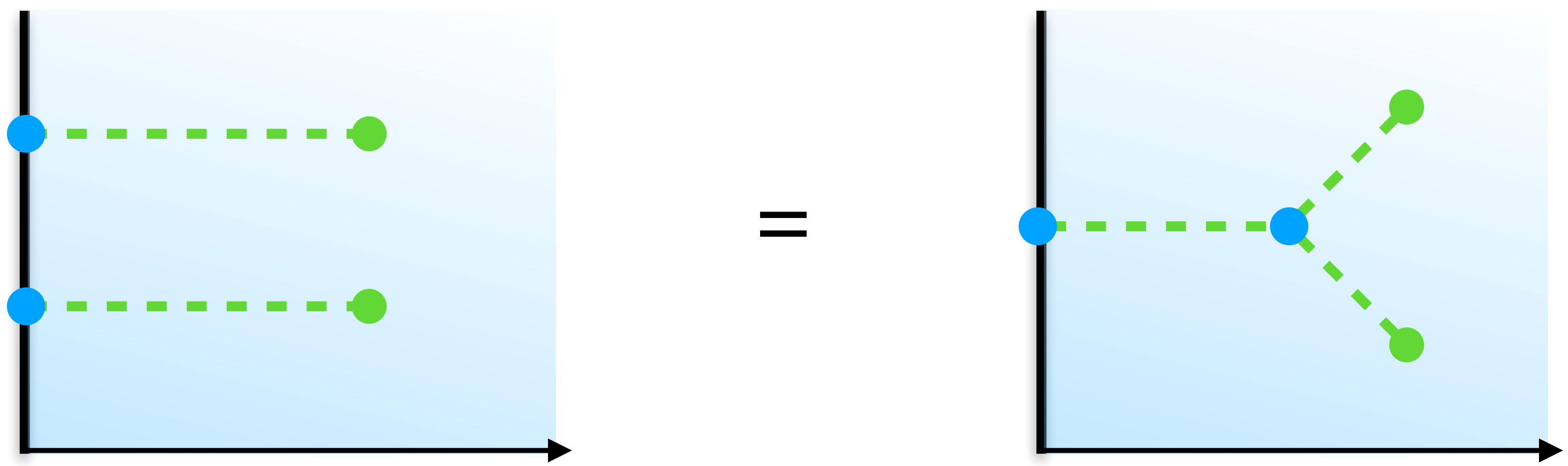
$$\Delta_{(1,3)} = 2 + O(m^{-1}) \ , \quad \Delta_{(1,2)} = 1/2 + O(m^{-1})$$

→ Perturbative fixed point with $g_*, g'_* \sim 1/m$

Beyond perturbation theory?

→ **Numerical Conformal Bootstrap**

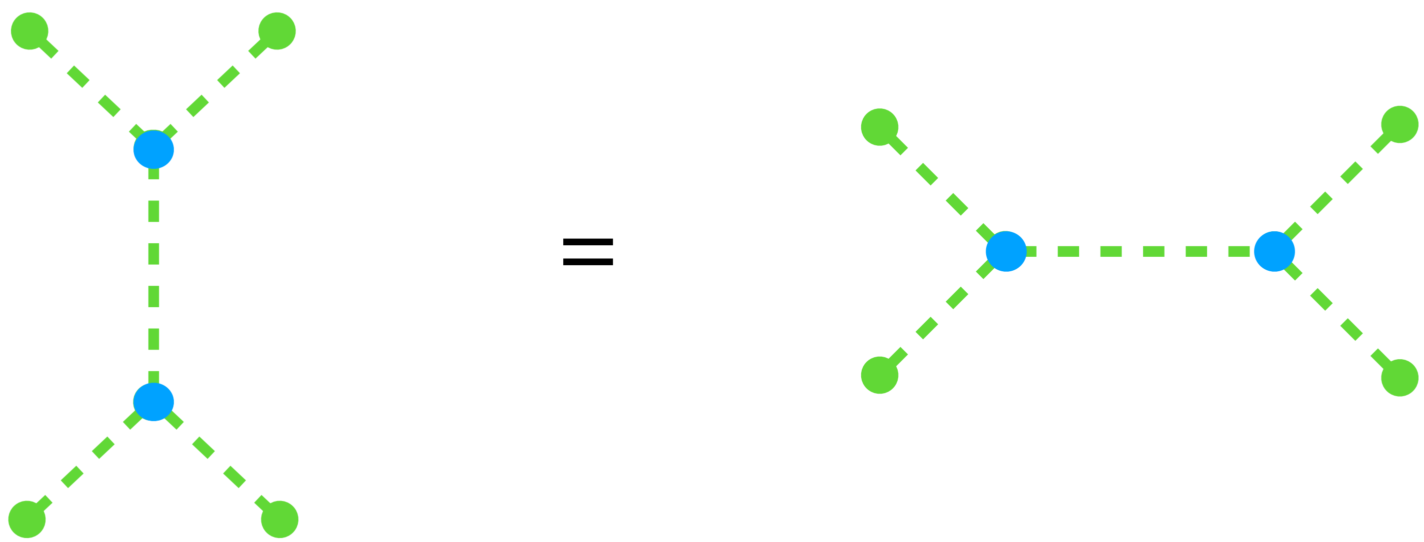
Conformal Bootstrap to look for interacting boundary conditions.



General approach [Liendo, Rastelli, Van Rees]

In our setup: simplicity in the bulk allows us to concentrate on the boundary correlators. Akin to bootstrapping a non-local conformal theory. [Behan]

Idea: numerical bootstrap applied to boundary 4pt functions of $\widehat{O}_1$ and $\widehat{O}_2$.

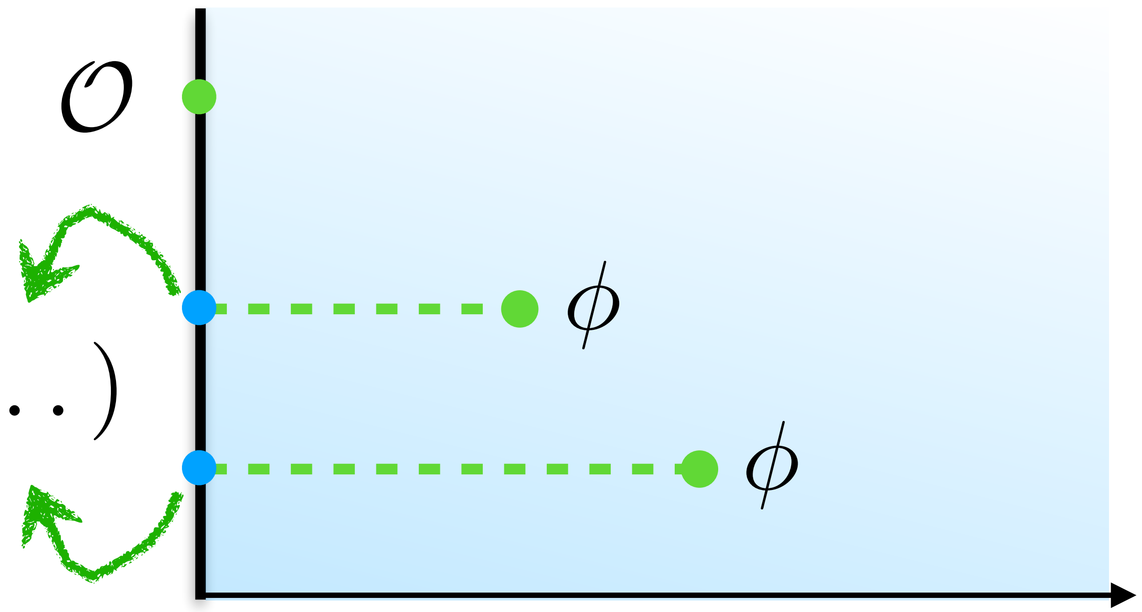


Conformal Data that enters: $\Delta_{\mathcal{O}},\ \lambda_{11\mathcal{O}},\ \lambda_{22\mathcal{O}},\ \lambda_{12\mathcal{O}}$

Input from the free bulk theory: **constraints among the OPE coefficients**.

E.g. free scalar theory: $\widehat{O}_1 \equiv \hat{\phi}$, $\widehat{O}_2 \equiv \widehat{\partial_y \phi}$

$$b_1(\widehat{O}_1 + \ldots) + y\, b_2(\widehat{O}_2 + \ldots)$$



3pt function from resummation of boundary OPE.

Compatibility with bulk OPE limit:

$$\lambda_{11\mathcal{O}} = F_1(\lambda_{12\mathcal{O}}, \Delta_{\mathcal{O}}, b_1/b_2)$$
$$\lambda_{22\mathcal{O}} = F_2(\lambda_{12\mathcal{O}}, \Delta_{\mathcal{O}}, b_1/b_2)$$

b_1/b_2 constrained by unitarity in a known interval.

Convenient parametrization:

- $\hat{\tau}_{ab}$ lowest spin 2

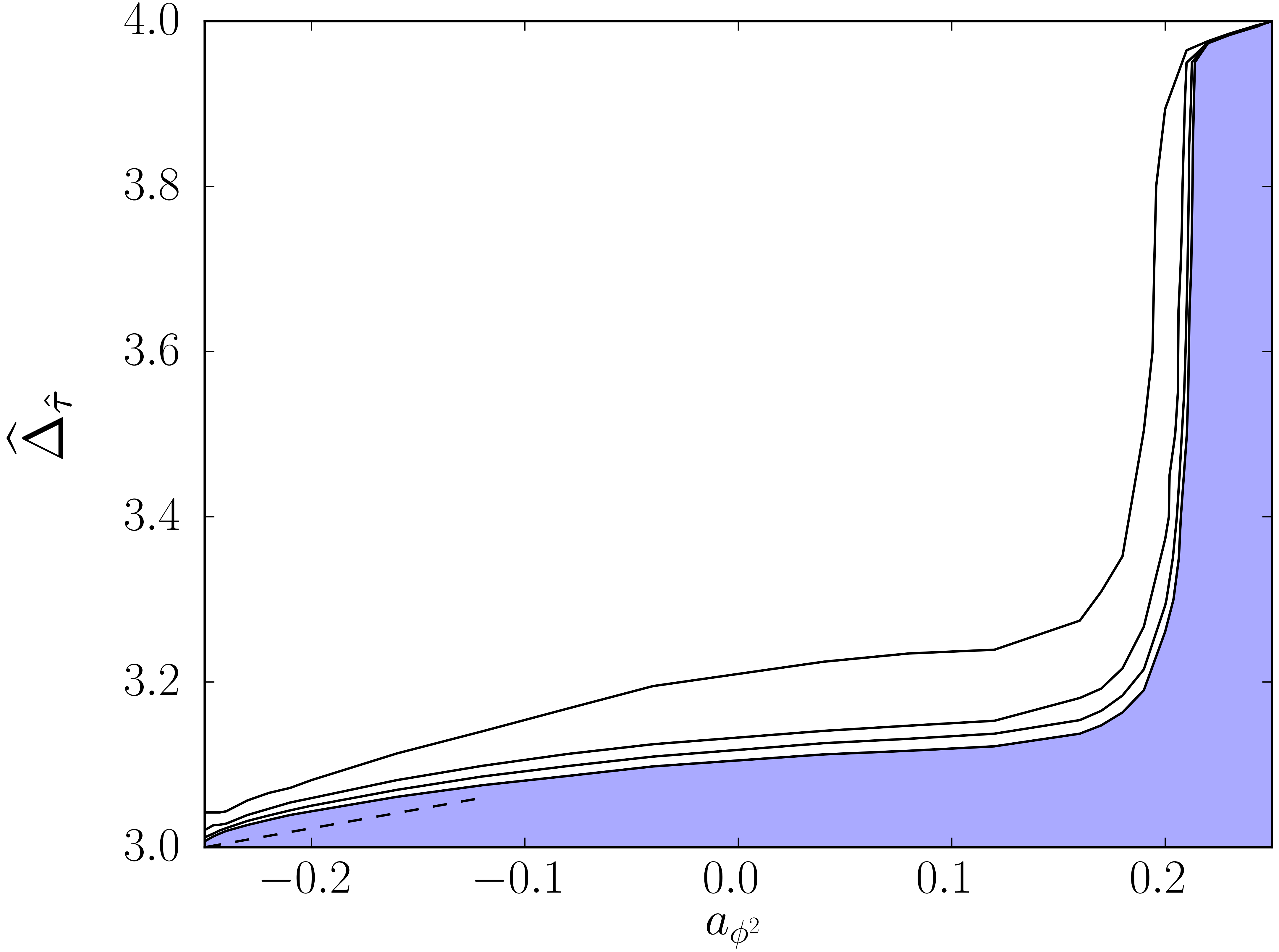
$\Rightarrow$ $\widehat{\Delta}_{\hat{\tau}} \geq d-1$ measures “non-locality”

- b_1/b_2 can be traded with a_{ϕ^2} : $\langle \phi^2 \rangle = \dfrac{a_{\phi^2}}{y^{d-2}}$

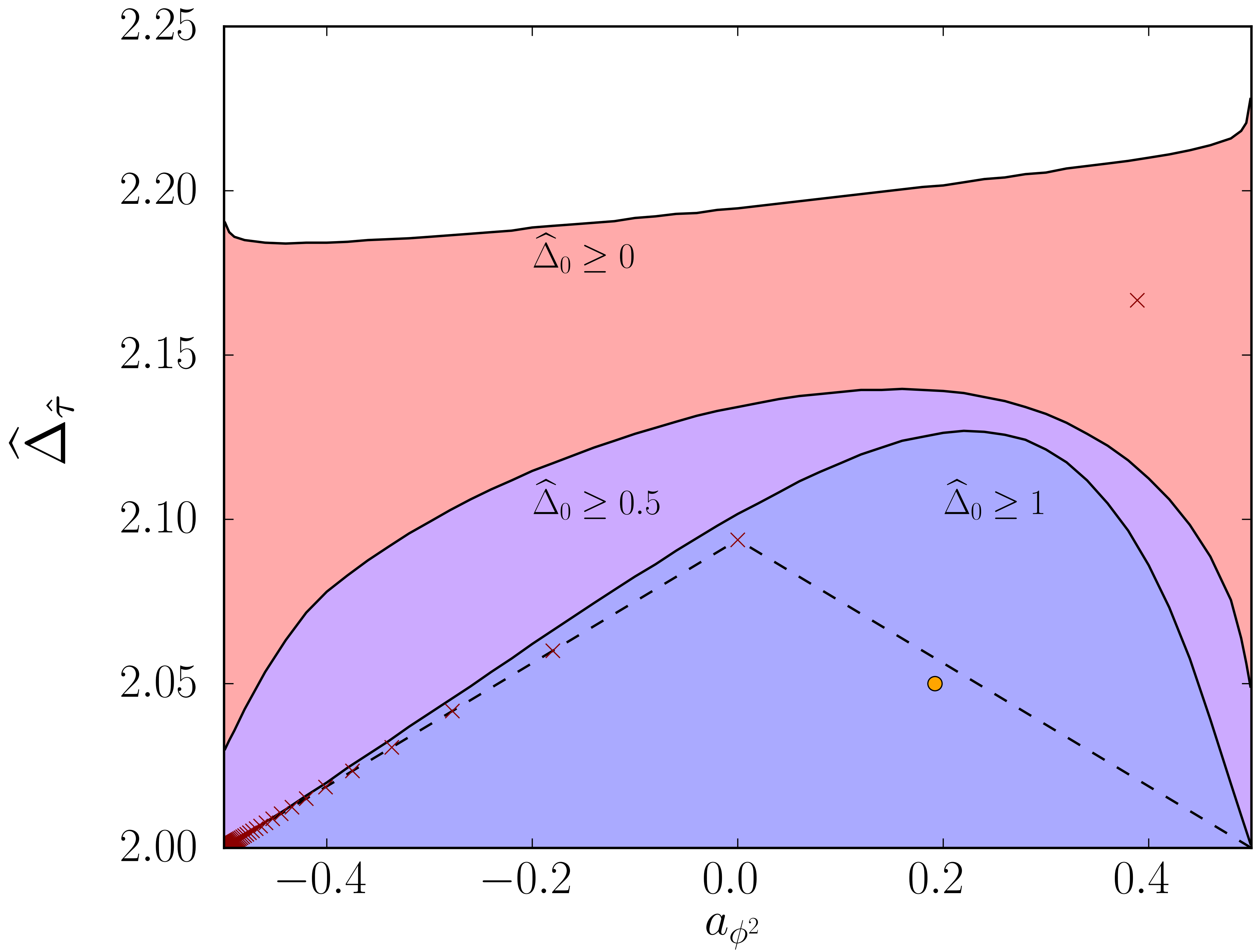
$$-2^{2-d} \leq a_{\phi^2} \leq 2^{2-d}$$

unitarity interval to scan

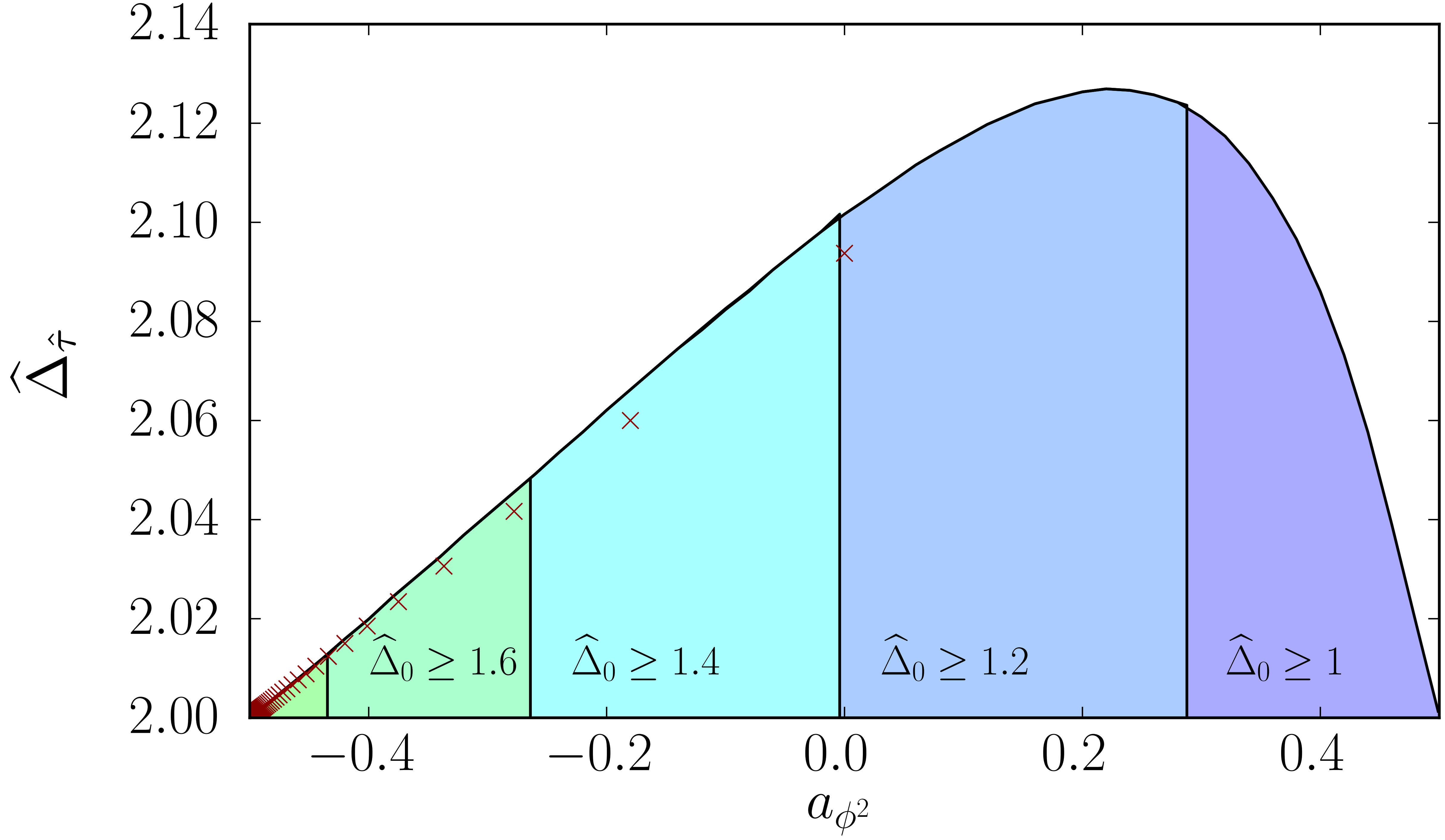
Results for 4d scalar [Behan, D, Lauria, Van Rees]



Results for 3d scalar [Behan, D, Lauria, Van Rees]



Results for 3d scalar



"Minimal Model" b.c. up to $m = 4$

Summary:

- Interacting boundary conditions for free fields;
- Examples: Maxwell theory with BCFT manifold; 4d scalar and large N duality; 3d scalars and mm's.
- Bootstrap approach: rigorous results for free scalar in 4d and 3d;

To do list:

- Apply bootstrap approach to Maxwell case;
- Free fermions in the bulk;
- Compare with boundary criticality in experiments/ simulations of systems described by free scalars (e.g. superfluids).

Thank You