Condensing and factorizing in low dimensional gravity

joint work F. Benini and Lorenzo di Pietro (to appear)

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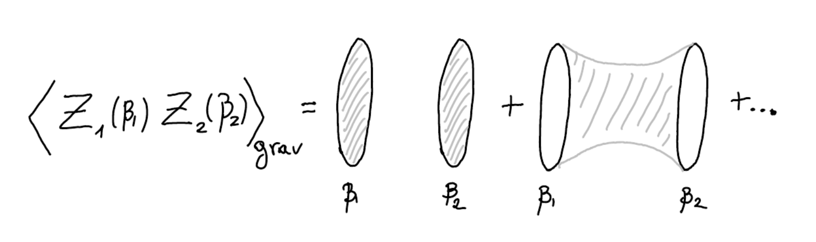
SISSA

Motivation and Introduction

Recent years have seen a resurgence of the study of gravitational PI in low dimensionality [Saad, Shenker, Stanford '19; ...]

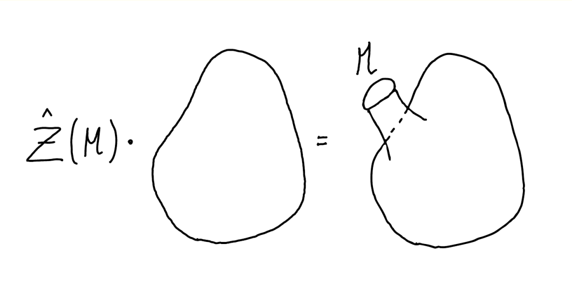
Workable example (JT gravity): spacetime wormholes give correlations between disjoint boundaries



Interpreted as statistical correlations that arise due to an underlying *ensemble* of dual QFTs.

Following [Marolf, Maxfield '20] and [Coleman '88]

Can (loosely) think of "third quantized" picture. A boundary creation operator $\hat{Z}(M)$ adds an "holographic" boundary M to the path integral



It can be argued that $[\hat{Z}(M)\,,\hat{Z}(M')] = 0 \rightarrow$ diagonalize!

$$\hat{Z}(M)|\alpha\rangle = Z_\alpha(M)|\alpha\rangle\,.$$

$|\alpha\rangle$ describe superselection sectors in which the gravitational Hilbert space splits. Expand

$$\langle Z_1 ... Z_n \rangle = \frac{\langle \Omega | \hat{Z}_1 ... \hat{Z}_n | \Omega \rangle}{\langle \Omega | \Omega \rangle} = \sum_\alpha p_\alpha \; Z_1^\alpha ... Z_n^\alpha$$

then $p_\alpha = \frac{|\langle \Omega | \alpha \rangle|^2}{\langle \Omega | \Omega \rangle}$ is a classical probability distribution $\leftrightarrow$ ensemble interpretation.

Understanding whether α-eigenstates may be given a geometric description is an important step towards understanding gravitational PIs / ways to restore factorization

Several works in 2d [Saad, Shenker, Stanford, Yau '21; Blommaert, Mertens, Vershelde '19, Blommaert, Kruthoff '21; ...] this talk 3d.

Three dimensions

[Maloney , Witten '20] computed explicit ensemble average over Narain lattices $M^{n,n}$:

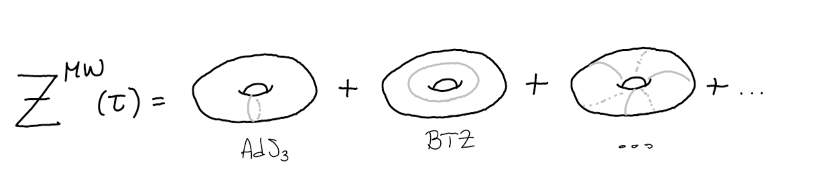
$$Z_{\text{Narain}}(\Omega) = \int_{M^{n,n}} du \sqrt{g_Z(u)} Z_u(\Omega)\,,$$

$u \sim \{G_{ab}\,, B_{ab}\}/$dualities, g_Z the Zamolodchikov metric. They have found a simple expression which, for T^2 ($\Omega = \tau$) reads:

$$Z_{\text{Narain}}(\tau) = \sum_{\gamma \in \Gamma'_n} \chi_0^{\mathbb{R}^n}(\gamma \cdot \tau) \bar{\chi}_0^{\mathbb{R}^n}(\gamma \cdot \bar{\tau})\,,$$

$\chi_0^{\mathbb{R}^n}(\tau) = \frac{1}{\eta(\tau)^n}$ a vacuum Virasoro character, $\Gamma' = SL(2,\mathbb{Z})/\Gamma_\infty$.

Looks like a saddle point expansion [Maloney, Witten '07] over inequivalent handlebodies (hence dividing Γ_∞):



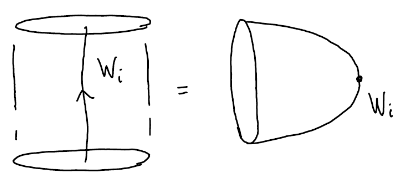
May interpret as bulk ($\mathbb{R}^{n,n}$?) Chern-Simons + prescribed "gravitational" sum.

Natural to consider $G_k \times G_{-k}$ instead of $\mathbb{R}^{n,n}$ instead.

$$S_k = i\frac{k}{4\pi}Tr\left[\int_Y AdA + \frac{2}{3}A^3\right] - i\frac{k}{4\pi}Tr\left[\int_Y \bar{A}d\bar{A} + \frac{2}{3}\bar{A}^3\right]$$

On a solid torus, each chiral half supports a set of (anti)holomorphic boundary conditions $\mathfrak{B}$: $A_{\bar{z}} = 0$. Depend on the modular parameter τ.

Overlap between these boundaries conditions and states:



Defines an RCFT character*:

$$\langle \mathfrak{B} | i \,, \gamma \rangle \equiv \chi_i(\gamma \cdot \tau)$$

The "gravitational" path integral:

$$Z_{grav} = \sum_{\gamma \in \Gamma'_k} \langle \mathfrak{B} | 0 \,, \bar{0} \,, \gamma \rangle = \sum_{\gamma \in \Gamma'_k} \chi_0(\gamma \cdot \tau) \bar{\chi}_0(\gamma \cdot \bar{\tau})$$

$\Gamma'_k = SL(2, \mathbb{Z})/\Gamma_k$ with Γ_k leaving the RCFT "seed" character invariant.

Reproduces the kind of Maloney-Witten sums we want to study. By construction Z_{grav} is modular invariant.

Physical modular invariants come from coupling left and right movers through mass matrix $Z^A_{i\bar{j}}$:

$$Z^A(\tau) = \sum_{i,\bar{j}} Z^A_{i\bar{j}} \chi_i(\tau) \bar{\chi}_{\bar{j}}(\bar{\tau})\,.$$

Such that:

$$Z^A_{i\bar{j}} \in \mathbb{Z} \geq 0 \quad \text{(unitarity)}$$
$$Z^A_{00} = 1 \quad \text{(uniqueness of the vacuum)}$$
$$\sum_{i\,,\bar{j}} S_{k\,i}\, Z^A_{i\,\bar{j}}\, \bar{S}_{\bar{j}\,\bar{l}} = Z^A_{k\,\bar{l}} \quad \text{(Modular invariance)}$$

Well known classification [Cappelli , Itzykson , Zuber '87; ...].

Generally ≥ 1 modular invariant for given $\mathcal{C}\,,\bar{\mathcal{C}}$.

Notation: $Z^A_{i\,\bar{j}} \to Z^A_a$. Modular: $\tilde{S}_a^b Z_b = Z_a$.

Expand Z_{grav} in the physical modular invariants [Castro, Gaberdiel, Hartman, Maloney, Volpato '11]:

$$Z^{MW}(\tau) = \sum_{A} p_A Z^A(\tau) + Z^{\text{non-phys}}(\tau)\,,$$

$Z^{\text{non-phys}}$ contains contributions from non physical mass matrices (i.e. with negative integer entries) See [Meruliya, Mukhi, Sing '21; Meruliya, Mukhi '21] for studies of these sums + [Benjamin, Keller, Ooguri, Zadeh '21] for studies in the non rational setting.

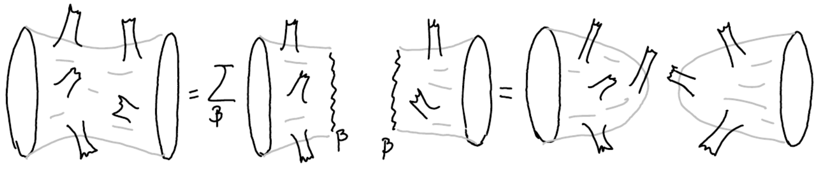
In certain cases the p_A are positive quantities Z^{MW} may be thought as an ensemble over physical RCFTs with the same chiral algebra.

In this talk we will expand on such an interpretation from the point of view of bulk AdS_3.

It seems natural to identify Z^A as giving rise to an $|\alpha\rangle$ eigenstate. We will also find that:

Summary

- An $|\alpha\rangle$ may be described by a certain bulk gas of defects (condensation).
- Wormholes in this background may be broken using a completeness relation and adsorbing the resulting states in the gas.
- Connected configurations (over $|\alpha\rangle$) become equivalent, they should be identified: no factorization problem.



Condensation and mass matrices

We need to understand the dual of a physical mass matrix $Z^A_{i\bar{j}}$:

A mass matrix $Z^A_{i\bar{j}}$ is isomorphic to the bulk **gauging** (condensation) of a **commutative Lagrangian** [a] **algebra** $\mathrm{A} \subset \mathcal{C} \boxtimes \bar{\mathcal{C}}$.

[a] special Frobenius

[Fuchs, Runkel, Schweigert; Kong; Kapustin, Saulina; Kaidi, Komargodski, Ohmori, Seifnashri, Shao; ...]

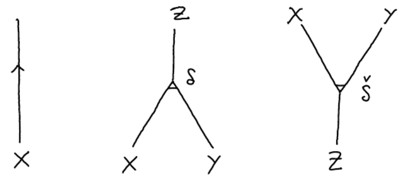
See also talks [**Ryu, Ohmori, Buican**] for more on related concepts.

We will now explain the boldface terms and their physical interpretation.

MTC primer

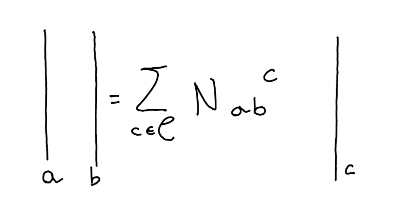
We briefly recap some fact about MTC and their relevance for our story:

Objects and morphisms



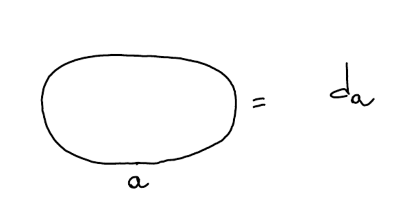
Can decompose $X = \oplus_a X_a a$. a simple lines: $Hom(a,b) = id_a\ \delta_{a,b}$.

Fusion:

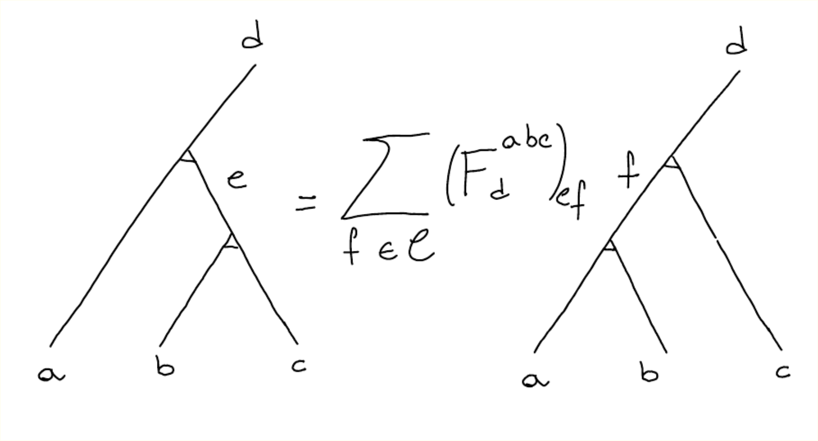


$N_{ab}{}^{c} = \dim Hom(a \otimes b, c) \in \mathbb{Z}_{+}$.

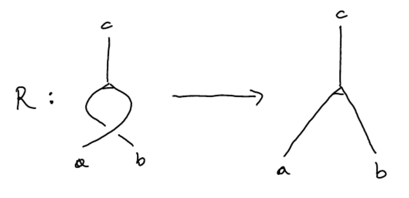
Quantum dimension

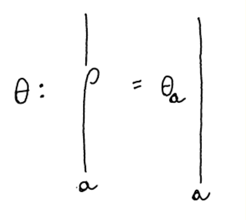


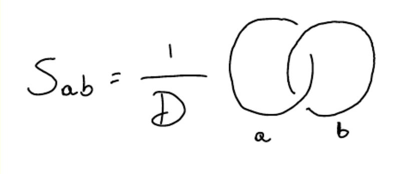
Associativity:



Modularity:







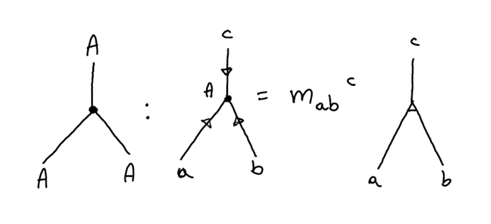
$\mathcal{D} = \left(\sum_a d_a^2\right)^{1/2}$. Relation with RCFT T matrix $T_{ab} = e^{2\pi i c/24}\theta_a\delta_{ab}$.

"Condensable" Algebras

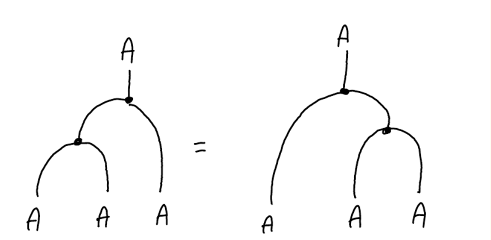
We decompose:

$$A = \oplus_b Z_b^A \ b \tag{1}$$

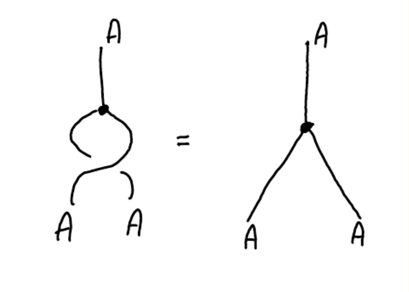
product $m : A \otimes A \to A$:



Associative:



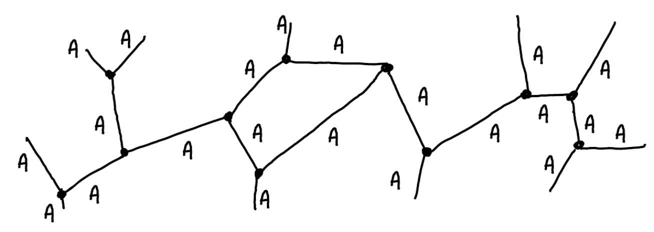
Commutative:



(Assures $\theta_a = 1$ for $a \in A$ in the case of our interest)

Gauging/Condensation

Gauging = fine enough mesh of A lines.

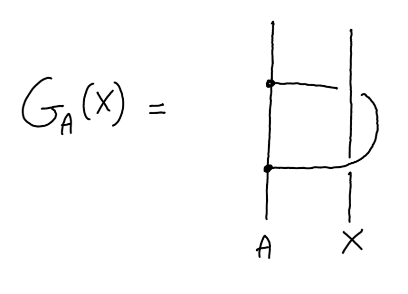


In practice triangulate manifold and put lines on edges.

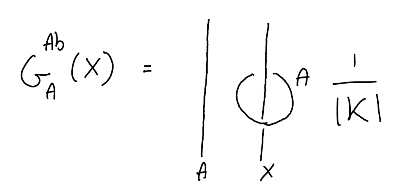
Properties of A ensure network is well defined (no 't Hooft anomalies) and allow simplifications.

For invertible lines becomes gauging of 1-form symmetry. Can be written as sum over bundles using Poincaré duality.

Useful analog of "Gauss law" [Fuchs, Runkel, Schweigert; Kong, Runkel, Kong; ....] $G_A(X) : A \otimes X \to A \otimes X$.



$G_A(X) \cdot G_A(X) = G_A(X)$ is a projector. Simple in Abelian case



Lagrangian algebras

An algebra A whose dimension satisfies:

$$d_A = \mathcal{D}\,,$$

Is called a Lagrangian algebra. For such algebras $G_A(X)$ has rank one (only A survives gauging). They describe maximal gaugings to "trivial" TQFTs.

It is a theorem [Muger, Kitaev; Kong, Runkel] that A is Lagrangian $\leftrightarrow$ Z_a^A is modular invariant.

Thus we conclude that one may generate physical RCFTs by condensing Lagrangian A.

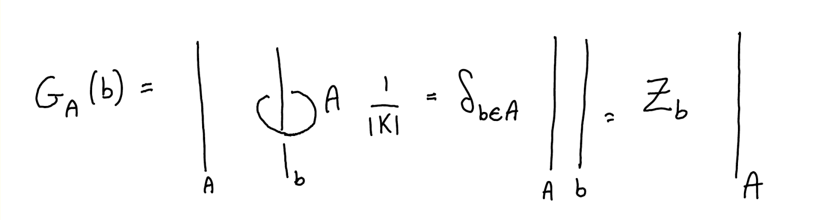
Abelian example

$U(1)_{2pq} \times U(1)_{-2pq}$ Chern-Simons. $\omega = r_0 p + s_0 q$, $r_0 p - s_0 q = 1 \rightarrow \omega^2 = 1 \mod 4pq$.
$\theta_l = \exp(\pi i l^2/2pq) = \theta_{\omega l}$. Define:

$$L_i = W_i \bar{W}_{\omega i}\,, \quad A = \oplus_j L_j$$

$$S_{A\{l\bar{l}\}} = \frac{1}{2pq} \sum_j e^{2\pi i \frac{j(l-\omega\bar{l})}{2pq}} = \delta_{\{l\bar{l}\},A}\,.$$

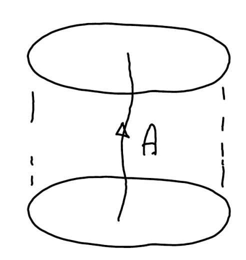
If $a \in A$ then $a \otimes A = A$ (A is a group for abelian TQFT).



Link with RCFT modular invariants

On a solid torus the condensed theory has only one state (to show need to evaluate Gauss law):

$$|A\rangle = |A, \gamma\rangle = \sum_a Z_a^A |a, \gamma\rangle \; =$$



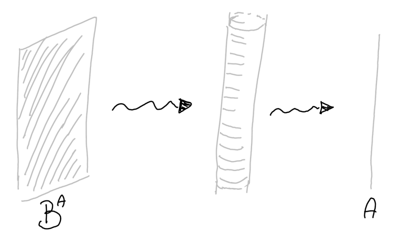
Independent of γ, indeed:

$$\langle \mathfrak{B}|A\rangle = \sum_{i\,\bar{j}} Z^A_{i\,\bar{j}} \; |i\,, \bar{j}\,, \gamma\rangle = Z^A(\tau)\,,$$

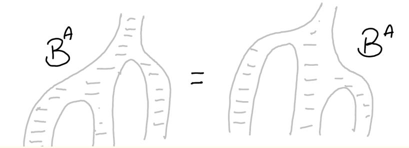
Is a modular invariant partition function.

Branes and algebras

There is a convenient way to characterize Lagrangian "condensable" algebras using topological branes (aka gapped boundary conditions).



$A =$ tube of $\mathcal{B}^A$ [Kapustin, Saulina; Kaidi, Komargodski, Ohmori, Seifnashri, Shao '21]

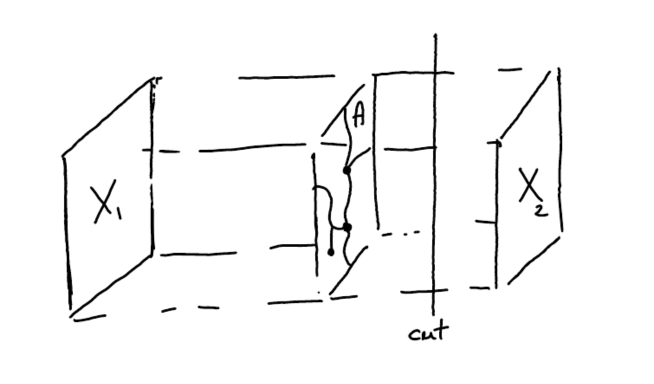


Factorization

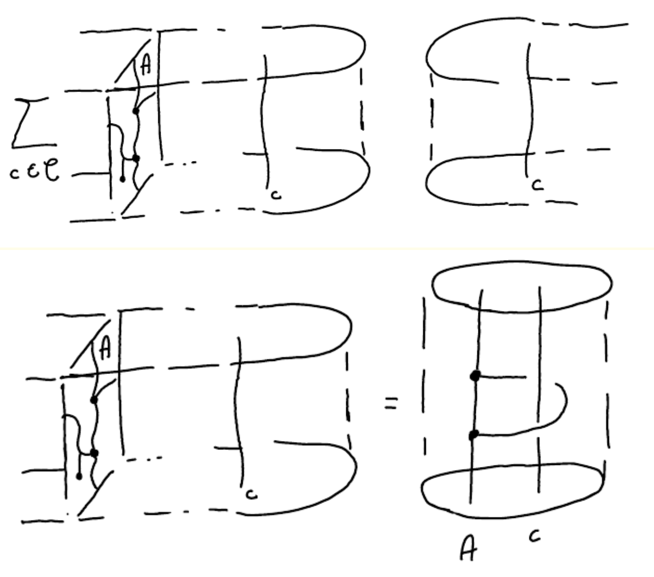
We will define a modified bulk path integral which includes the condensation of a certain Lagrangian anyon.

What is a wormhole-type configuration describing in this case?

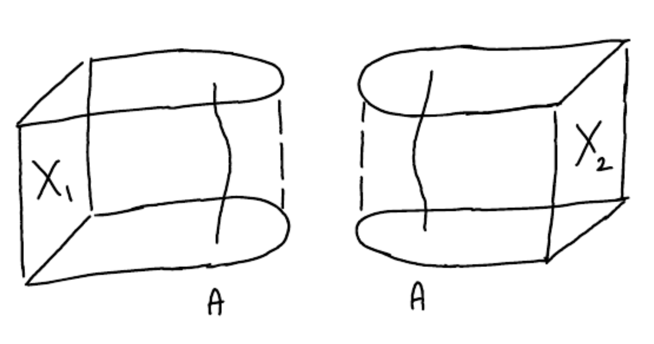
For a fixed geometry (say with T^2 boundaries) we can always simplify the condensation network in the vicinity of a boundary as follows



Cutting open the geometry and inserting a complete basis of states the l.h.s. can be rewritten in terms of the $G_A(X)$ map:



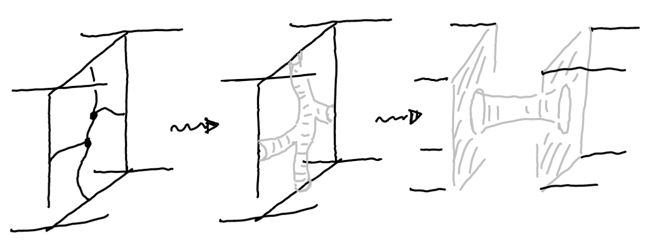
For a Lagrangian algebra the state created by $G_A(X)$ and $Z_X A$ are the same, thus:



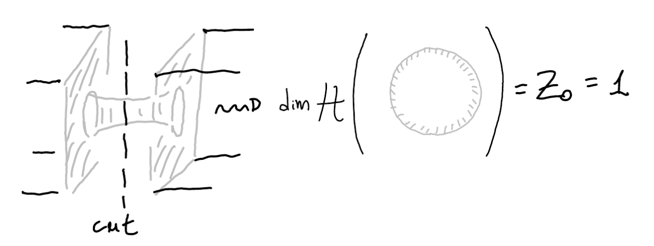
Abelian TQFT $\rightarrow$ simple ot extend to generic geometries / boundaries.

Main obstruction is working out the general form of the network.

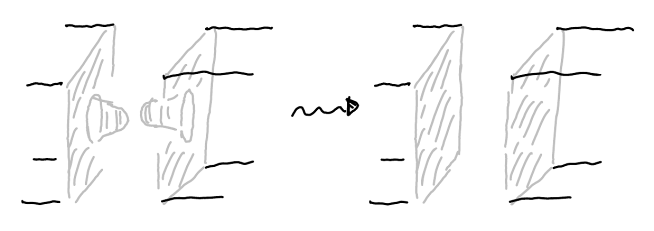
Alternative argument using $\mathcal{B}^A$ boundaries, use "fattening"



Now cut along the connecting tube:



Replace cut piece with (only) state and deform back



Final result = state with insertion of topological boundary $\mathcal{B}^A$.

Simpler to extend but less explicit.

Comments

- Above confirms the identification of Lagrangian algebras as α eigenstates for the bulk quantum gravity.
- At its core, factorization is a consequence of one dimensional Hilbert space on any closed manifold after condensation.
- All geometries give same contribution, reminds of background independence (e.g. [Eberhardt '21]).
- Plus: explicit path integral prescription to prepare $|\alpha\rangle$.

- Idea consistent with no global symmetry + gravity (though no proof for low dimensions)

- In our case no "quantum" symmetry after gauging (there are no charged operators under it), what about generic case?

- From QFT p.o.v. useful to streamline algebraic concepts into QFT language (various works in last years). Exciting developing field [Kapustin, Saulina '13; Bhardwaj, Tachikawa '17; Chang, Lin, Shao, Wang, Yin '18; Komargodski, Ohmori, Roumpedakis, Seifnashri '20;...]

Open questions

Many open questions:

- Extend this formalism to non-compact theories (e.g. $SL(2,\mathbb{R})$) and to two dimensional dilaton-gravity theories. We can already use our results to understand results of Maloney-Witten.
- What fixes which A we should condense? Maybe signal of UV completion?
- Positivity of p_A in MW sum not always clear (apart from special cases). Can give bulk interpretation. Any insight?
- Relation with recent story about "half wormholes" [Saad, Shenker, Stanford, Yao '21; Blommaert, Kruthoff '21; ...]

Thank you!