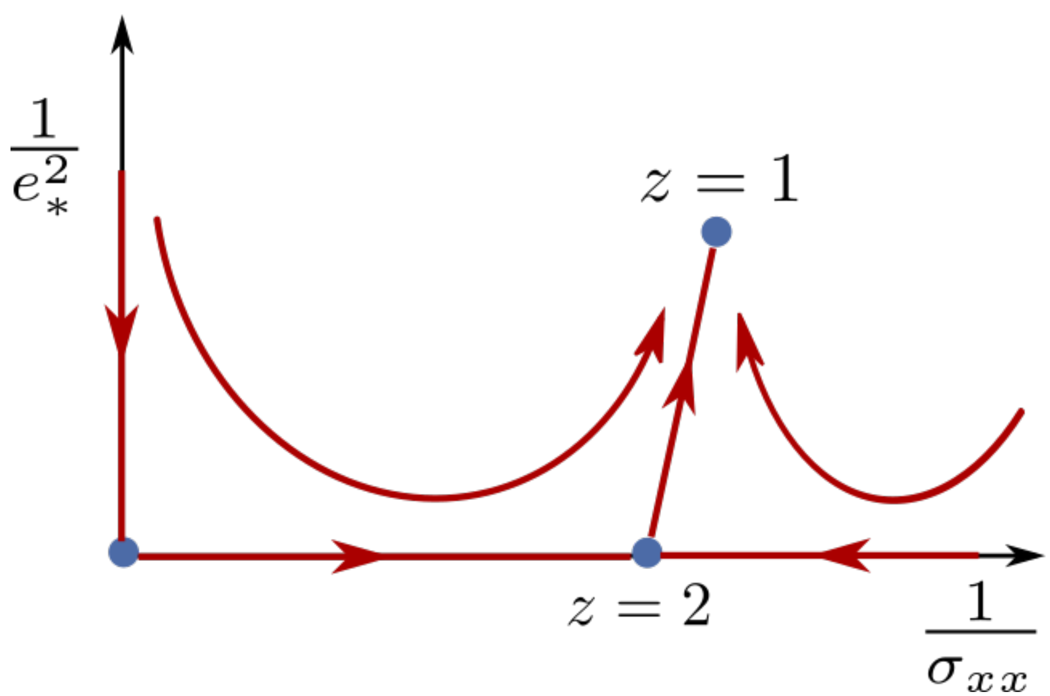
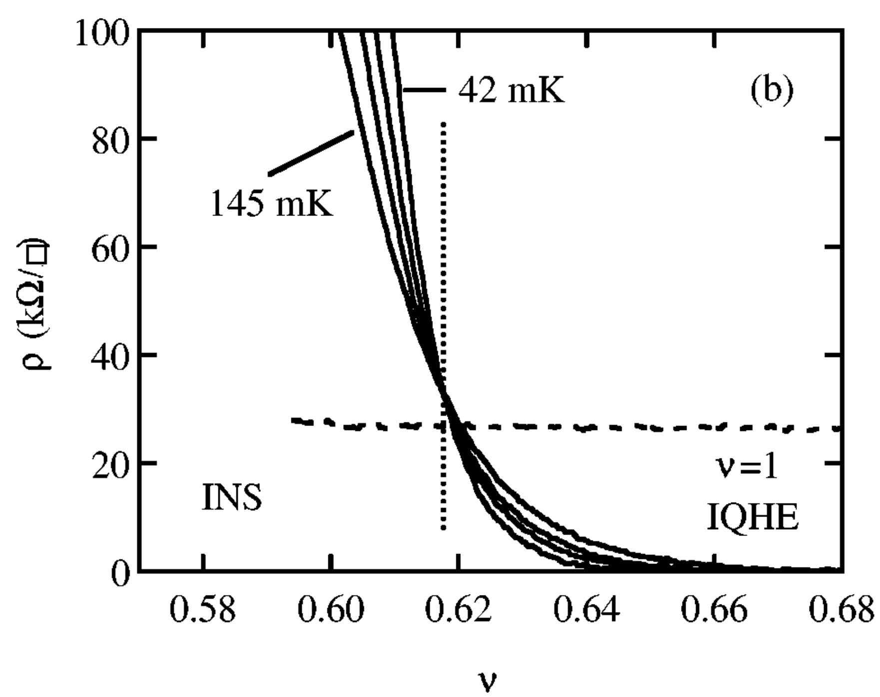
quantum Hall to insulator transitions with interactions and disorder

Srinivas Raghu (Stanford)





Based on: 1) P. Kumar, P. Nosov, SR, arXiv:2006.11862 -> PRB.
2) Kevin Huang, SR, P. Kumar PRL (2021)

Prashant Kumar
(Princeton U.)

Pavel Nosov
(Stanford GS)

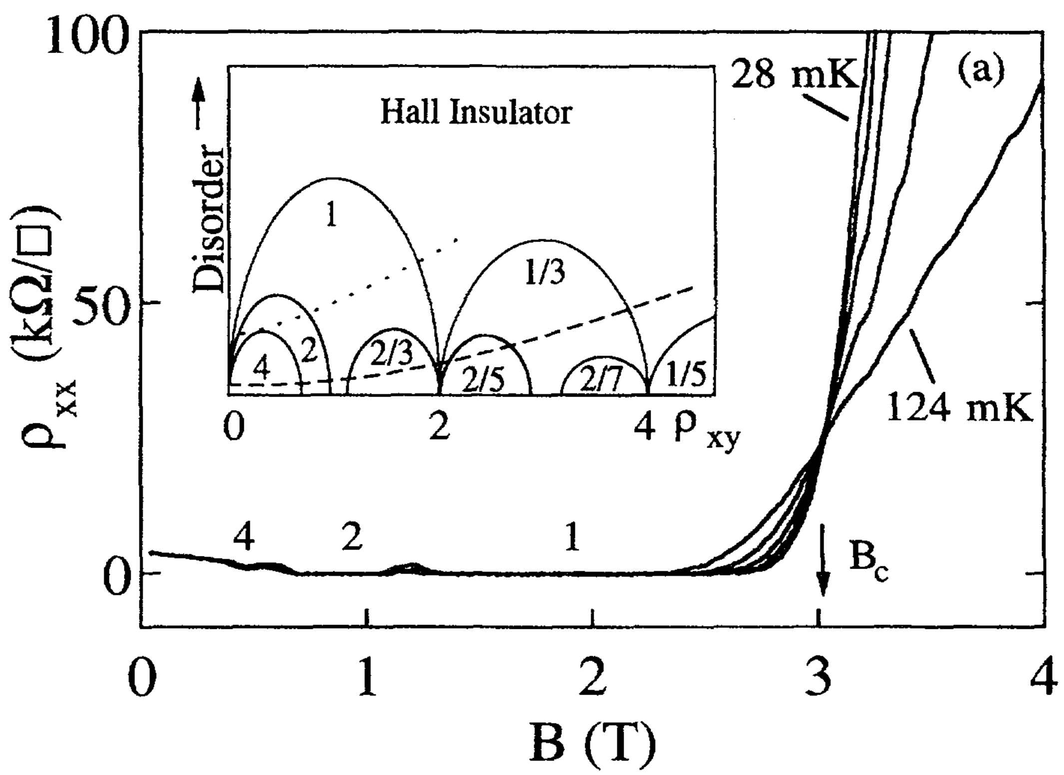
Kevin Huang
(Stanford ’21)

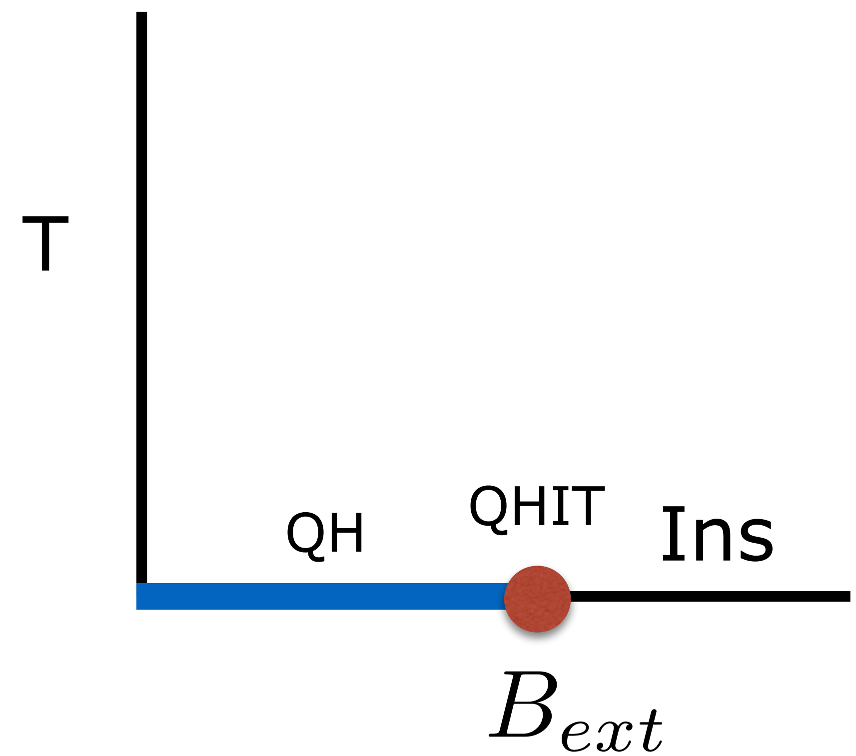
Older work: With Prashant Kumar, Yong-Baek Kim, Michael Mulligan.

arXiv:1803.07767,1805.06462, 1903.06297, 1907.13141

Quantum Hall transitions

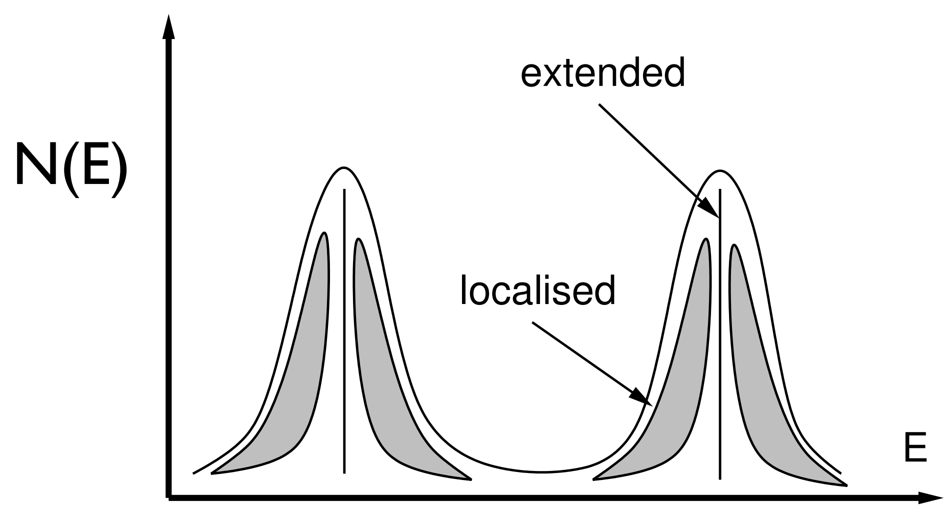
D. Shahar *et al.*, PRL **74**, 4511 (1995).

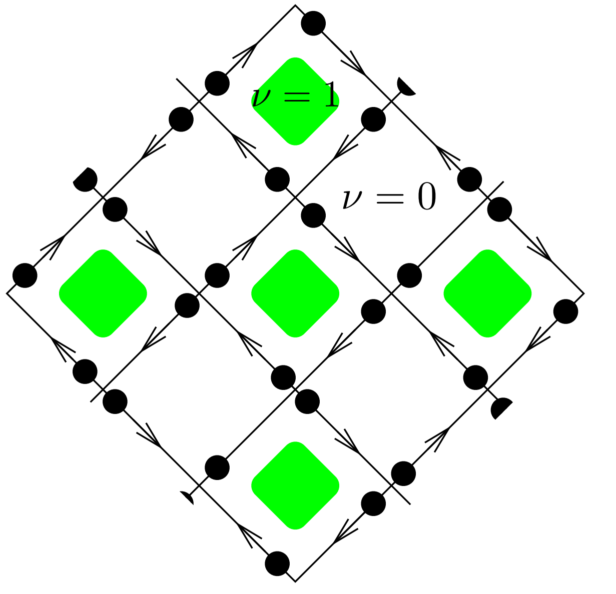




Integer vs fractional QH transitions

Integer QH:





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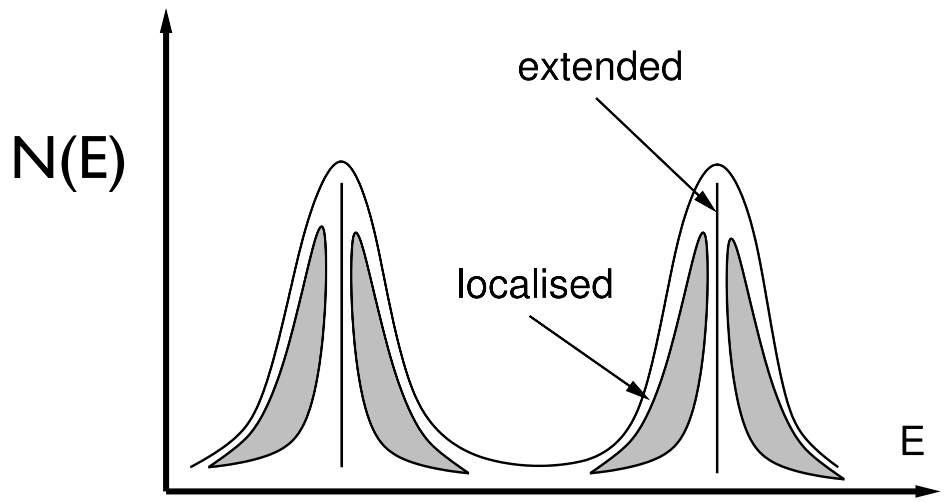
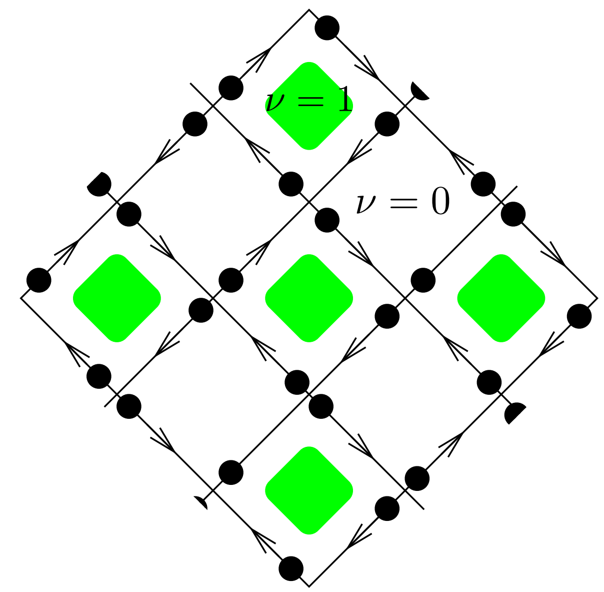


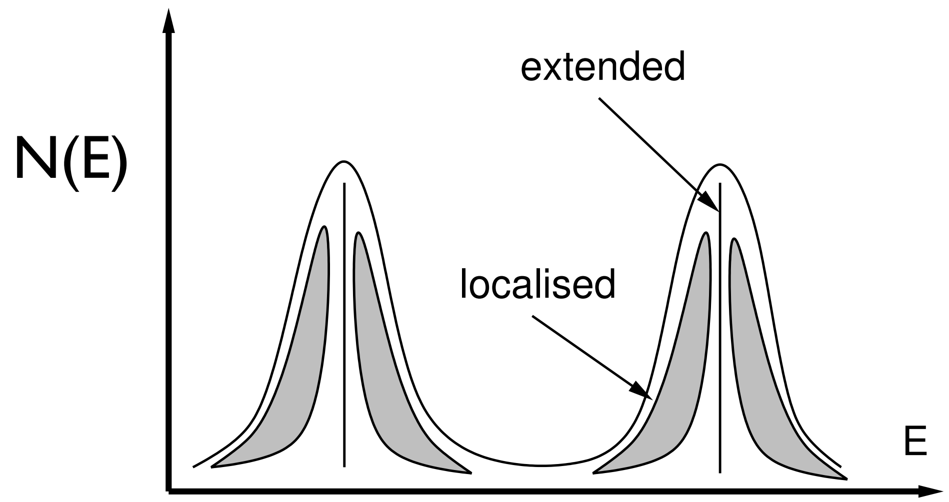


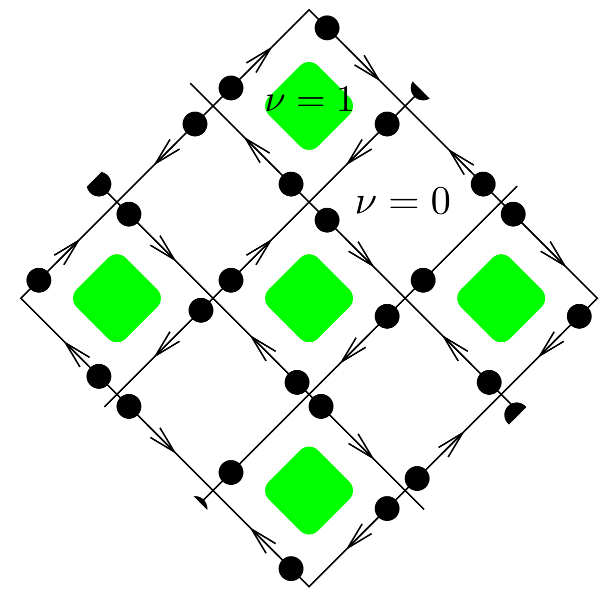
Fractional QH:

Naive expectation: Integer, fractional QH transitions are totally different.

Integer vs fractional QH transitions

Integer QH:





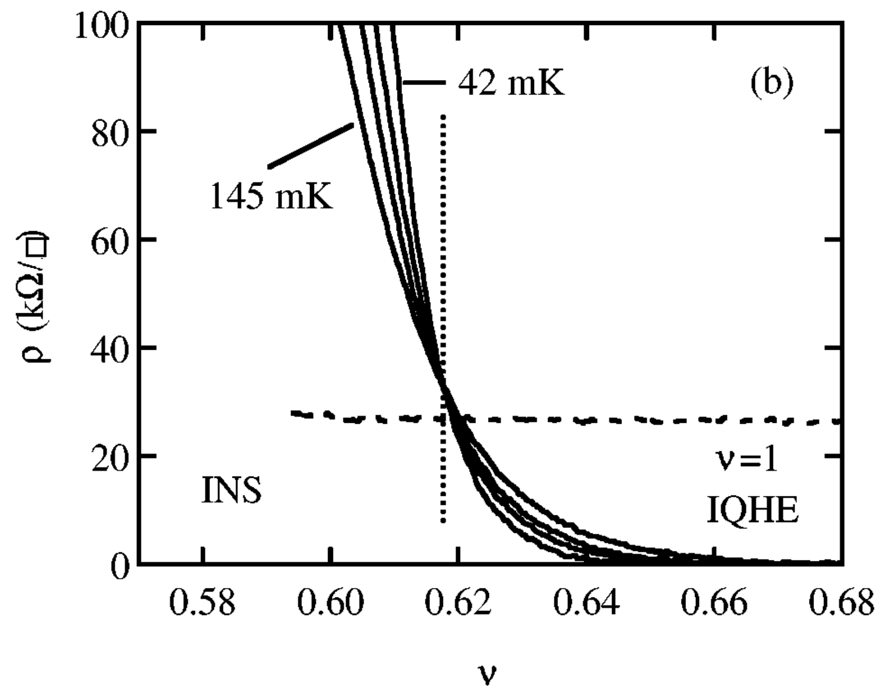
Fractional QH:

Naive expectation: Integer, fractional QH transitions are totally different.

Interactions: important for <u>both</u> I and FQH transitions.

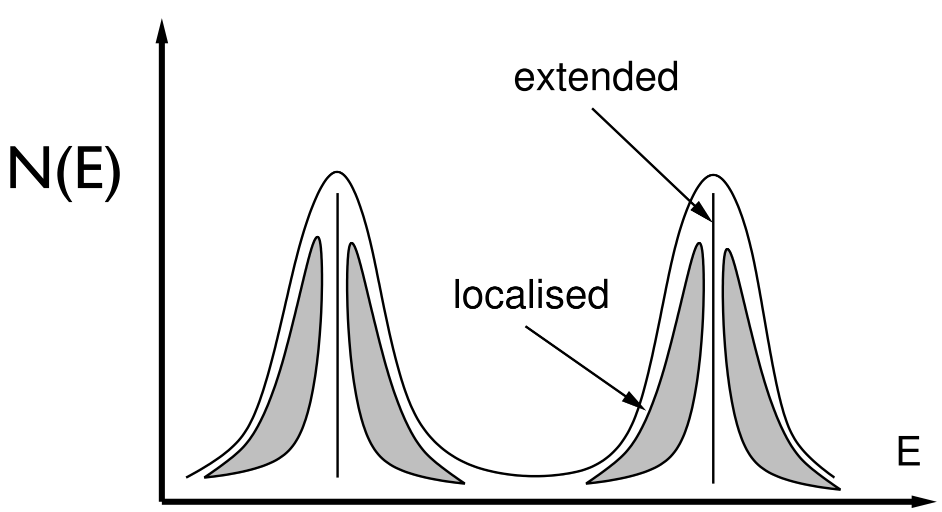
Why interactions are important

1) Finite critical conductivity   D. Shahar *et al.*, PRL **79**, 479 (1997).



$$\sigma_{dc} = \lim_{T\to 0}\lim_{\omega\to 0}\lim_{L\to\infty}\sigma(\omega, T, L)$$

= 0 without interactions.



Interactions (even if RG irrelevant) are important for finite dc resistance at T>0.

Z. Wang et al, PRB **61**, 8326 (2000).

Why interactions are important

2) Dynamical scaling laws. $\xi \sim \delta^{-\nu}$ $\delta = \frac{B - B_c}{B_c}$

$\xi_\tau \sim \delta^{-\nu z}$ $T \sim \frac{1}{\xi_\tau}$

Resistivity data near QCP: $$\rho(B,T) = \rho^* f\left(\frac{\delta}{T^{1/\nu z}}\right)$$

non-linear IV data: $$\rho(E,T) = \rho^* g\left(\frac{\delta}{E^{1/(1+z)\nu}}\right)$$ $\nu \approx 2.3, \quad z = 1$

Non-interacting problem: z=d=2 (from finite density of states).

z=1: natural from V(r) ~ 1/r.

Interactions are important even for IQH transitions.

RMP **69**, 315 (1997).

RMP Colloquia

Continuous quantum phase transitions

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Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544

"…z=1 strongly suggests that coulomb interactions are playing an important role…"

"..*a priori* validity [of superuniversality].. is still unclear."

"In summary, theorists have their work cut out for them!"

Main points of my talk

Theory of QH transitions: still in a primitive stage!

Most studies: neglect interactions.

Interactions needed for

1) Finite electrical resistance at criticality.
2) Correct dynamical scaling laws.
3) Comparing fractional vs Integer QH transitions.

Composite fermion (CF) representation readily addresses all 3!

What we don’t know yet: interaction effects on ν.

QH transitions in electron coordinates

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{dis} + \mathcal{L}_{int}$$

$$\mathcal{L}_0 = c^\dagger(r)\left[-i\partial_t + \mu - \frac{1}{2m}\left(\boldsymbol{\partial} - i\boldsymbol{A}\right)^2\right]c(r) \qquad\qquad B = \nabla \times A$$

$$\mathcal{L}_{dis} = c^\dagger(r)V(r)c(r)$$

$$\mathcal{L}_{int} = -\frac{1}{2}\int d^2r'\left[n(r) - \langle n\rangle\right]U(r-r')\left[n(r') - \langle n\rangle\right]$$

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QH transitions in CF coordinates

2 choices for CF theories:

1) Halperin, Lee, Read (HLR) - “flux attachment”. PRB **47**, 7312 (1993).

2) Dirac CF theory (Son) - CFs are “dual” vortices. PRX **5**, 031027 (2015).

Both motivate a traditional “mean field + fluctuations” approach.

Mean-field theory: identical predictions from HLR and Dirac CF theories.

Fluctuations: Dirac and HLR theories are distinct.

Dirac CFs

D.T. Son PRX **5**, 031027 (2015).

Idea: non-relativistic electrons in LLL behave similarly to massless Dirac electrons.

Dirac electron:

$$\mathcal{L}_{\text{Dirac el.}} = i\bar{c}\not{D}_A c + \frac{1}{8\pi} AdA$$

$$\Updownarrow \quad \textit{Particle-vortex duality}$$

Dirac CF:

$$\mathcal{L}_{cf} = i\bar{\psi}\not{D}_a \psi - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

A_μ : Background (EM) gauge field.

a_μ : Dynamical U(1) gauge field.

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Particle-vortex duality

Dirac CF:

$$\mathcal{L}_{cf} = i\bar{\psi}\not{D}_a \psi - \frac{1}{4\pi} Ada + \frac{1}{8\pi} AdA$$

A_μ : Background (EM) gauge field. $\langle c^\dagger c \rangle = \frac{B-b}{4\pi}$ $B = \nabla \times A$

a_μ : Dynamical U(1) gauge field. $\langle \psi^\dagger \psi \rangle = \frac{B}{4\pi}$ $b = \nabla \times a$

Disorder in Dirac CF theory

Dirac electron: $A_t \to A_t + V(r)$

$$\mathcal{L}_{\text{Dirac el.}} = i\bar{c}\not{D}_A c + \frac{1}{8\pi} AdA$$

$$\mathcal{L}_{\text{Dirac el.}} \to \mathcal{L}_{\text{Dirac el.}} + \mathcal{L}_{\text{dis}} \qquad \mathcal{L}_{dis} = V(r)c^\dagger(r)c(r)$$

Disorder in Dirac CF theory

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$$\mathcal{L}_{cf} \to \mathcal{L}_{cf} + \mathcal{L}'_{dis}$$

$$\mathcal{L}'_{dis} = -\frac{1}{4\pi} V(r)b(r)$$

$$b(r) = \nabla \times a(r)$$

Disorder in Dirac CF theory

Dirac CF: $$A_t \to A_t + V(r)$$

$$\mathcal{L}_{cf} \to \mathcal{L}_{cf} + \mathcal{L}'_{dis}$$

$$\mathcal{L}'_{dis} = -\frac{1}{4\pi} V(r) b(r)$$

$$b(r) = \nabla \times a(r)$$

V(r) sources a quenched random b(r).

$$a_j(r,t) \to a_j(r,t) + a'_j(r)$$

$$\mathcal{L}'_{dis} \to a'_j(r) \bar{\psi} \gamma_j \psi(r)$$

$$P[a'] = e^{-\pi N_F \tau \int d^2 r a'(r)^2}$$

N_F: density of states at E_F.

Interactions in Dirac CF theory

Dirac electron:

$$\mathcal{L}_{int} = -\frac{1}{2}\int d^2r' \, [n(r) - \langle n\rangle] \, U(r - r') \, [n(r') - \langle n\rangle]$$

Dirac CF:

$$\mathcal{L}'_{int} = -\frac{1}{2(4\pi)^2}\int d^2r' \; b(\boldsymbol{r}) U(|\boldsymbol{r} - \boldsymbol{r}'|) b(\boldsymbol{r}') \qquad b(r) = \nabla \times a(r)$$

Interactions: gauge fluctuations (gauge boson kinetic term).

Interactions in Dirac CF theory

Coulomb gauge: a_0, a_T

$$\mathcal{L}'_{int} = -\frac{1}{2(4\pi)^2}\int d^2r' \; b(\boldsymbol{r})U(|\boldsymbol{r}-\boldsymbol{r}'|)b(\boldsymbol{r}') \qquad b(r) = \nabla \times a(r)$$

$$U(r) = \frac{e_*^2}{r} \qquad S'_{int} = -\frac{e_*^2}{16\pi}\int_{\omega,q} |\boldsymbol{q}| a_T(q,\omega)a_T(-q,-\omega)$$

$$U(r) = U_0\delta(r) \qquad S'_{int} = -\frac{U_0}{16\pi}\int_{\omega,q} q^2 a_T(q,\omega)a_T(-q,-\omega)$$

Dirac CF + disorder + interactions

$$\mathcal{L}_{cf} = i\bar{\psi}\not{D}_a\psi - \frac{1}{4\pi}Ada + \frac{1}{8\pi}AdA + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

$$\mathcal{L}'_{dis} = -\frac{1}{4\pi}V(r)b(r) \rightarrow a'_j(r)\bar{\psi}\gamma_j\psi(r)$$

$$a_j(r,t) \rightarrow a_j(r,t) + a'_j(r)$$

$$\mathcal{L}'_{int} = -\frac{1}{2(4\pi)^2}\int d^2r'\ b(\boldsymbol{r})U(|\boldsymbol{r}-\boldsymbol{r}'|)b(\boldsymbol{r}')$$

CF Mean-field theory

CF mean-field theory

Mean-field theory: gauge dynamics only via eqs. of motion.

$$\mathcal{L}_{cf} = i\bar{\psi}\not{D}_a\psi - \frac{1}{4\pi}Ada + \frac{1}{8\pi}AdA + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

Include proper UV regularization in mean-field theory.

$$\mathcal{L}_{cf} = \mathcal{L}_\psi[a] + \mathcal{L}_{gauge}[a, A] + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

$$\mathcal{L}_\psi = i\bar{\psi}\not{D}_a\psi - \frac{ada}{\underbrace{8\pi}_{\text{Massive partner}}} \qquad \mathcal{L}_{gauge} = \frac{(a-A)d(a-A)}{8\pi}$$

CF mean-field theory

$$\mathcal{L}_{cf} = \mathcal{L}_\psi[a] + \mathcal{L}_{gauge}[a, A] + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

$$\mathcal{L}_\psi = i\bar{\psi}\not{D}_a\psi - \frac{ada}{8\pi} \qquad \mathcal{L}_{gauge} = \frac{(a-A)d(a-A)}{8\pi}$$

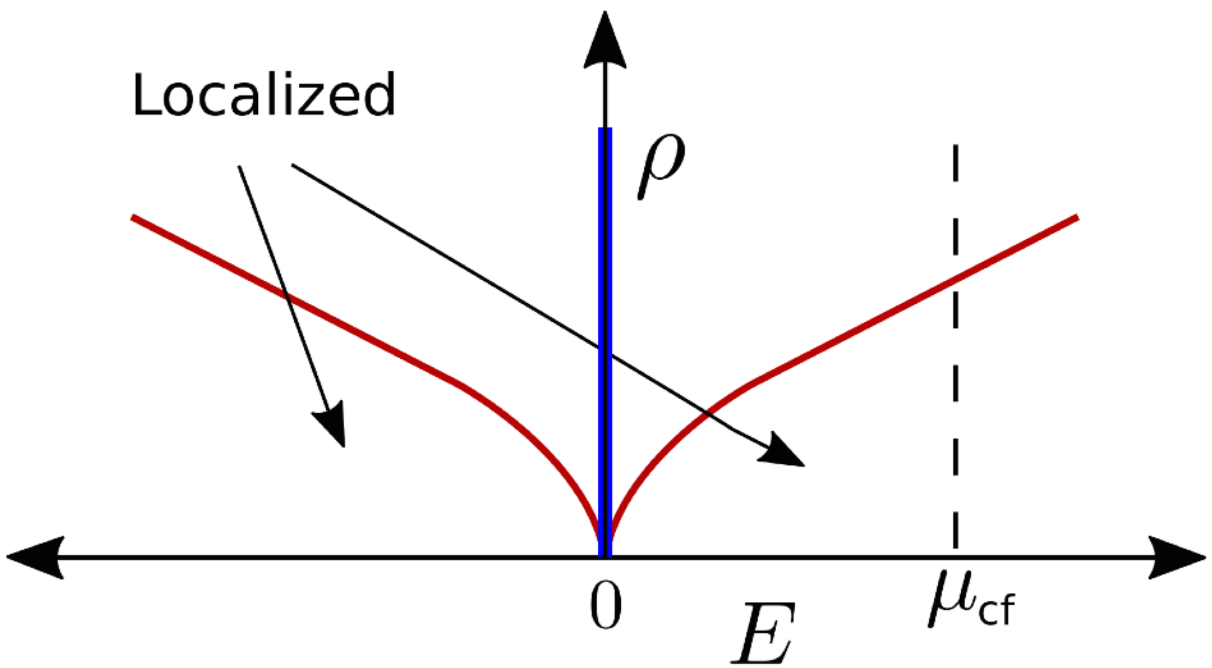
Associated Hamiltonian (also properly regularized): $\mathcal{H}_\psi = \mathcal{H}_1 + \mathcal{H}_2$

$$\mathcal{H}_1 = \boldsymbol{\sigma}.(\boldsymbol{p} - \boldsymbol{a}') - \mu_{\rm cf} \qquad \underbrace{\mathcal{H}_2}_{\text{Massive partner}} = \boldsymbol{\sigma}.(\boldsymbol{p} - \boldsymbol{a}') - m\sigma^z - \mu_{\rm cf}$$

$$m \gg \mu_{cf}$$

CF mean-field theory

Mean-field behavior of $\mathcal{H}_1 = \boldsymbol{\sigma}.(\boldsymbol{p} - \boldsymbol{a}') - \mu_{\rm cf}$

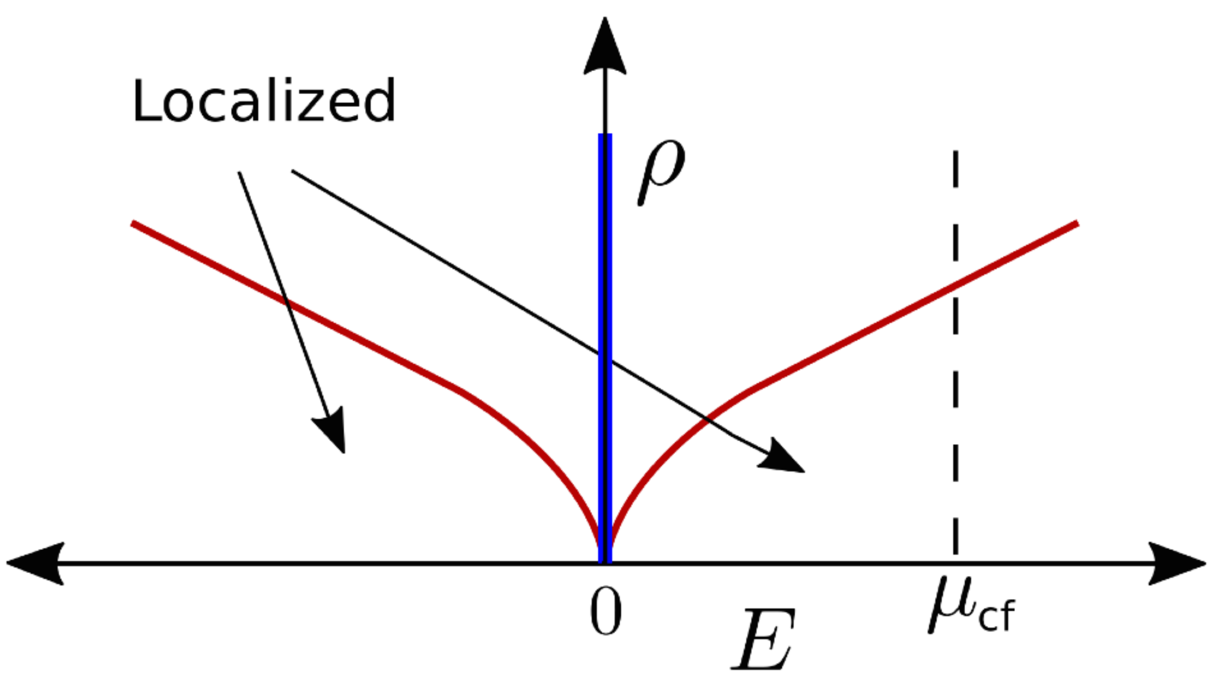


$$b_0 \neq 0$$

Tuning parameter for transition: average b: $\bar{b}(r) = b_0$

CF mean-field theory

Mean-field behavior of $\mathcal{H}_1 = \boldsymbol{\sigma}.(\boldsymbol{p} - \boldsymbol{a}') - \mu_{\text{cf}}$



$$b_0 \neq 0$$

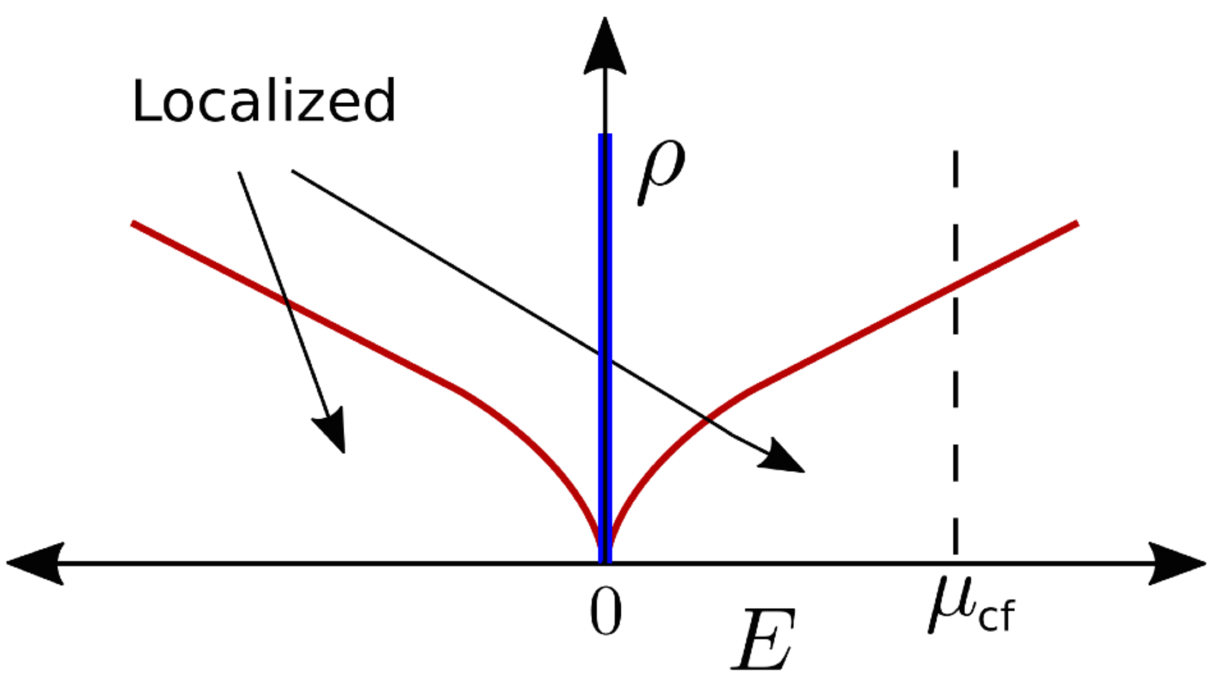
Contribution from light fermion:

$$\sigma_{xy}^{\text{cf}} = \begin{cases} \frac{\text{sgn}[b_0]}{4\pi}, & b_0 \neq 0 \\ 0 & b_0 = 0 \end{cases}$$

$b_0<0$: Integer QH. $b_0>0$: insulator.

CF mean-field theory

Mean-field behavior of $\mathcal{H}_1 = \boldsymbol{\sigma}.(\boldsymbol{p} - \boldsymbol{a}') - \mu_{\rm cf}$



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Contribution from light fermion:

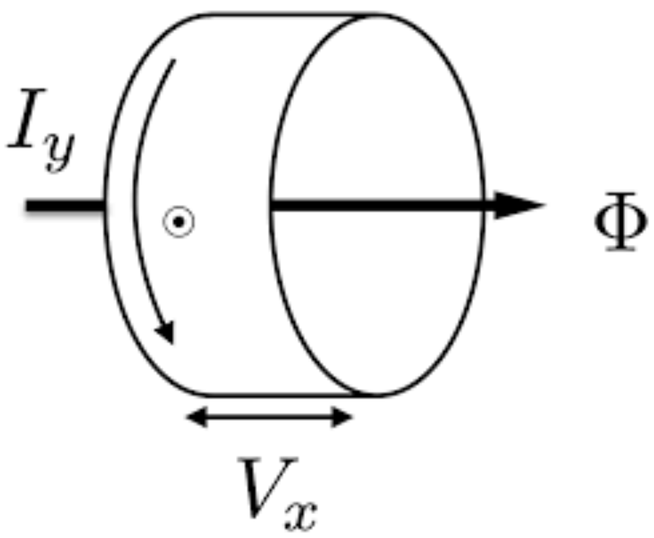
$$\sigma_{xy}^{\rm cf} = \begin{cases} \frac{\mathrm{sgn}[b_0]}{4\pi}, & b_0 \neq 0 \\ 0 & b_0 = 0 \end{cases}$$

Total contribution:

$$\sigma_{xy}^{\rm cf} = \begin{cases} \frac{\mathrm{sgn}[b_0]-1}{4\pi}, & b_0 \neq 0 \\ -\frac{1}{4\pi} & b_0 = 0 \end{cases}$$

$b_0<0$: Integer QH. $b_0>0$: insulator.

Delocalized states at criticality



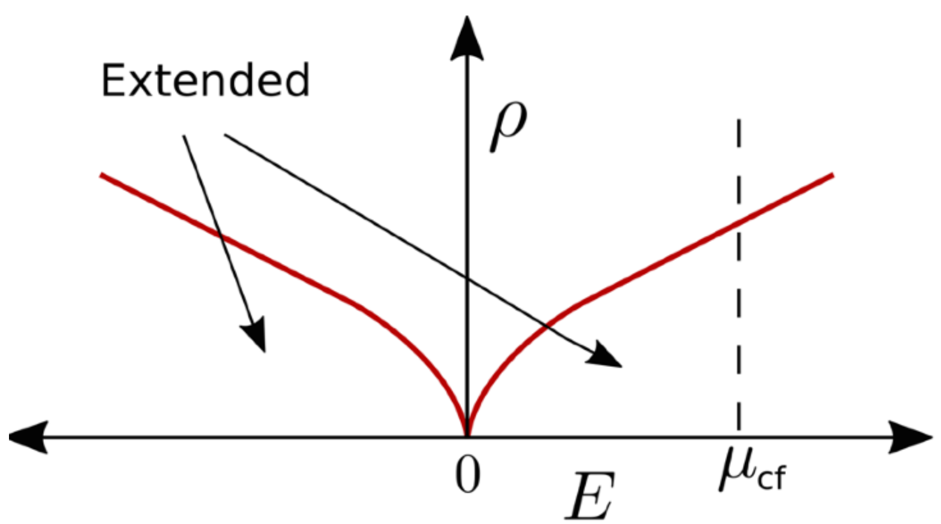
Laughlin gauge argument:

States at E_F localized $\Longrightarrow$ $\sigma_{xy} = \frac{1}{2\pi} \times \text{integer}$

$\sigma^{cf}_{xy} = -1/4\pi$ $\Longrightarrow$ States at E_F must be delocalized.

States are delocalized at all energies at the critical point!

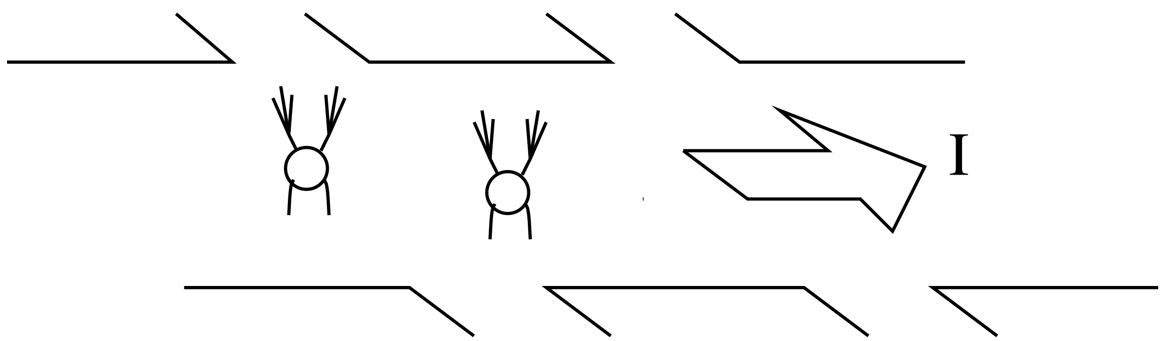
Implies a finite dc resistance of CFs at criticality.



$$b_0 = 0$$

Finite electrical resistance at criticality

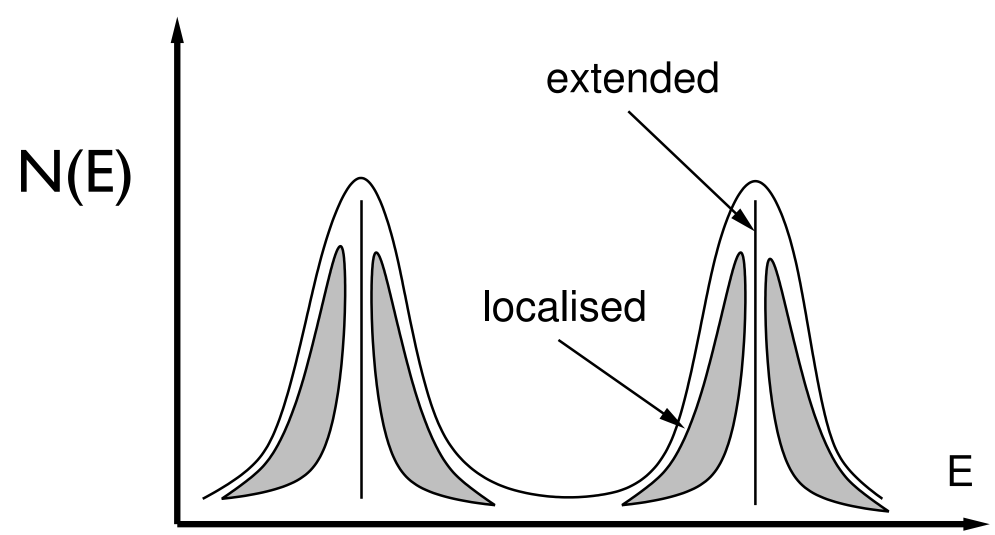
Exact relationship between CF and electrical linear response:



$$\rho_{ab}^{cf} = \rho_{ab} + 4\pi\epsilon_{ab}$$

Finite CF resistance implies finite electrical resistance at T>0.

Not obvious in electron coordinates!



Mean-field exponents

Numerical study of $\mathcal{H}_\psi = \mathcal{H}_1 + \mathcal{H}_2$

$$\xi \sim |b_0|^{-\nu} \quad \nu = 2.56 \pm 0.02$$

Previous work (Chalker-Coddington model): $\nu = 2.593 \pm 0.01$

Also: composite fermion multifractality identical to predictions of Chalker-Coddington model for electrons.

PHYSICAL REVIEW LETTERS **126,** 056802 (2021)

Editors' Suggestion

Numerical Study of a Dual Representation of the Integer Quantum Hall Transition

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[3]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

Abelian Fractional QH transitions (mean-field)

Transitions from filling fractions $\nu = \frac{1}{2m-1} \to 0$ $\quad m = 1, 2, 3, \cdots$

Mean-field theory almost identical to Integer case.

$$\mathcal{L}_{cf} = \mathcal{L}_{\psi}[a] + \mathcal{L}_{gauge}[a, A] + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

$$\mathcal{L}_{\psi} = i\bar{\psi}\not{D}_a\psi - \frac{ada}{8\pi} \qquad \mathcal{L}_{gauge} = \frac{(a-A)\,d\,(a-A)}{8\pi m}$$

H. Goldman, E. Fradkin PRB **98**, 165137 (2018).

Fermions again undergo an **integer** QHIT.

Finite electrical resistance at criticality

Exact relationship between CF and electrical linear response:

Transitions from filling fractions $\nu = \frac{1}{2m-1} \to 0 \quad m = 1, 2, 3, \cdots$

$$\rho_{ab}^{cf} = \rho_{ab} + 4\pi m \epsilon_{ab}$$

Critical resistance depends on m.

All critical exponents same as in the Integer QHIT.

Mean-field summary

Main message: at criticality, the CFs have a <u>finite</u> conductivity.

Transitions as viewed in CF mean-field theory are trivially superuniversal:

Conductivities are different, but z=2, nu ~ 2.6 for all transitions.

Next: gauge fluctuation effects.

II. Gauge Fluctuations

Basic roadmap

Key points:
1) non-zero dc conductivity at criticality of CFs.
2) finite static compresibility of CFs at criticality.

1) sets the key dynamical scaling relations.
2) establishes the superuniversality of the transitions with 1/r interactions.

To establish 1) and 2): disorder averaged theory: gauged NLSM.

Before disorder averaging

Start with the integer QH transition.

$$\mathcal{L}_{cf} = \mathcal{L}_{\psi}[a] + \mathcal{L}_{gauge}[a, A] + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

$$\mathcal{L}'_{int} = -\frac{1}{2(4\pi)^2}\int d^2r'\ b(\boldsymbol{r})U(|\boldsymbol{r}-\boldsymbol{r}'|)b(\boldsymbol{r}') \qquad \mathcal{L}_{gauge} = \frac{(a-A)d(a-A)}{8\pi}$$

$$\mathcal{L}_{\psi} + \mathcal{L}'_{dis} = i\bar{\psi}\not{D}_{a+a'}\psi - \frac{ada}{8\pi}$$

Disorder average free energy via replica trick:

$$\log\left[Z\right] = \lim_{n\to 0}\frac{Z^n - 1}{n}$$

Disorder averaging

Start with the integer QH transition.

$$\mathcal{L}_{cf} = \mathcal{L}_{\psi}[a] + \mathcal{L}_{gauge}[a, A] + \mathcal{L}'_{dis} + \mathcal{L}'_{int}$$

Replicate and integrate out disorder:

$$S = S_{gauge} + S'_{int} + \int d^2x \ \mathrm{Tr} Q^2 - \ \mathrm{Tr} \log \left[G_0^{-1} + \frac{i}{2\tau} Q + g \boldsymbol{v} \cdot \boldsymbol{a}_T \right]$$

g: gauge coupling.

$$Q^{ab}_{t,t'}(x) \rightarrow Q^{ab}_{\omega_n, \omega_m}(x)$$

EFT for Q: non-linear sigma model.

nonlinear sigma model

$$S = S_{gauge} + S'_{int} + \int d^2x \; \mathrm{Tr} Q^2 - \; \mathrm{Tr} \log \left[G_0^{-1} + \frac{i}{2\tau} Q + g \boldsymbol{v} \cdot \boldsymbol{a}_T \right]$$

Idea: double expansion of above:

1) saddle point for Q (NLSM).

2) expansion about g=0 (gauged NLSM).

1) saddle point for Q (NLSM) with g=0. $Q^2 = 1, \mathrm{Tr}\left[Q\right] = 0$

$Q \in U(2n)/U(n) \times U(n) \qquad n \to 0$

$$S = S_{gauge} + S_{int} + S[Q]$$

$$S[Q] = -i\pi N_F \mathrm{Tr}\left[\partial_\tau Q\right] + \frac{\pi \sigma_{xx}}{4} \mathrm{Tr}\left[\partial_i Q\right]^2 + i S_{top} \qquad S_{top} = \frac{\pi \sigma_{xy}}{4} \epsilon_{ij} \mathrm{Tr}\left[Q \partial_i Q \partial_j Q\right]$$

nonlinear sigma model

$$S = S_{gauge} + S'_{int} + \int d^2x \ \mathrm{Tr} Q^2 - \ \mathrm{Tr}\log\left[G_0^{-1} + \frac{i}{2\tau}Q + g\boldsymbol{v}\cdot\boldsymbol{a}_T\right]$$

Idea: double expansion of above:

1) saddle point for Q (NLSM).

2) expansion about g=0 (gauged NLSM).

2) expansion about g=0: (gauged NLSM).

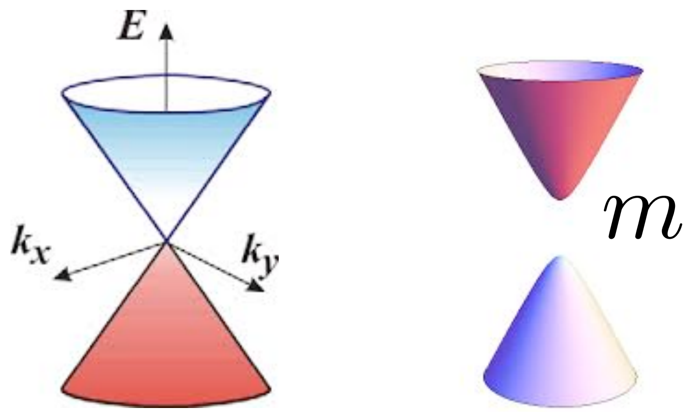
$$\delta S = ig\pi\sigma_{xx}\mathrm{Tr}\left[a_j \cdot Q\partial_j Q\right] - \frac{g^2\pi\sigma_{xx}}{2}\mathrm{Tr}\left[a_j Q a_j Q - a_j^2\right]$$

no corrections to topological term! Only σ_{xx} affected by gauge fluctuations at the critical point.

Why S_{top} is not gauged

Topological term is related to σ_{xy} : $S_{top} = \frac{\pi \sigma_{xy}}{4} \epsilon_{ij} \mathrm{Tr}\left[Q \partial_i Q \partial_j Q \right]$

Dirac CFs: massive modes generate σ_{xy}.



Massive modes unaffected by gauge fluctuations.

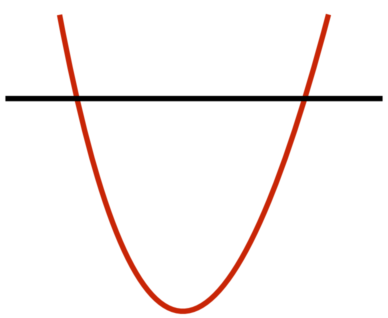
$$\mathcal{L}_\psi = i\bar{\psi} \not{D}_a \psi - \frac{ada}{8\pi}$$

Gauge fluctuations don't change dc Hall conductivity of Dirac CFs.

HLR theory: S_{top} is gauged

Topological term is related to σ_{xy} : $S_{top} = \frac{\pi \sigma_{xy}}{4} \epsilon_{ij} \mathrm{Tr}\left[Q \partial_i Q \partial_j Q\right]$

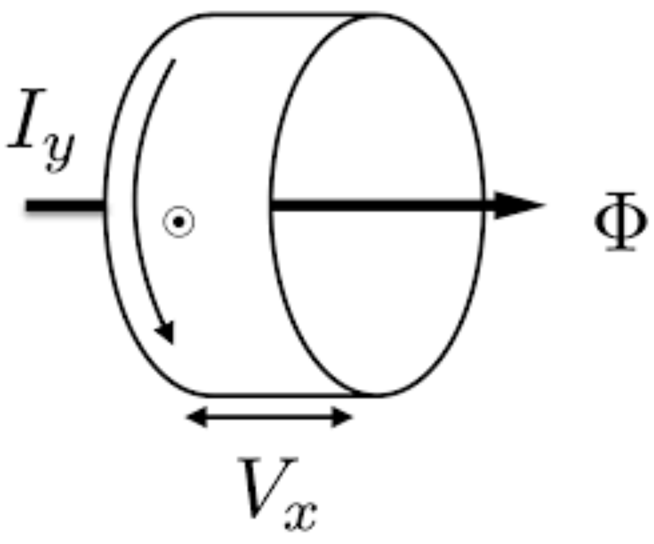
HLR theory: Hall conductance entirely from modes Fermi energy.



μ_{cf}

Topological term is gauged and dc Hall conductivity runs at criticality due to gauge fluctuations.

Back to delocalized states



Laughlin gauge argument:

States at E_F localized $\Longrightarrow$ $\sigma_{xy} = \frac{1}{2\pi} \times \text{integer}$

$\sigma_{xy}^{cf} = -1/4\pi$ $\Longrightarrow$ States at E_F must be delocalized.

Dirac CFs are delocalized at critical point even with gauge fluctuations.

Finite conductivity from delocalized states: Dissipation.

Finite density of delocalized states: Debye screening.

Implications

Superuniversality from Debye screening

Finite compressibility of CFs makes <u>fluctuation corrections</u> from CS terms irrelevant at criticality.

$\chi_0 = N_F$ (density of states at Fermi energy).

from $\mathcal{L}_{gauge}$ from $\mathcal{L}_{int}$

$$\mathcal{L}_{gauge} + \mathcal{L}_{int} = \frac{1}{2}\chi_0 a_0^2 + \frac{i\lambda}{4\pi} a_0 q a_T + a_T \left[q + \cdots\right] a_T$$

$$\mathcal{L}_{eff} \simeq a_T \left[q + \mathcal{O}(\lambda^2 q^2/\chi_0)\right] a_T$$

Coulomb effects + finite compressibility: RG <u>irrelevance</u> of CS.
IQH and FQH transitions are equivalent.

Screening and dissipation at criticality

1/r interactions:

$$S'_{int} = -\frac{e_*^2}{16\pi} \int_{\omega,q} |\boldsymbol{q}| a_T(q,\omega) a_T(-q,-\omega)$$

Transverse gauge boson inv. propagator:

$$D_{ret}^{-1} = -\frac{e_*^2}{8\pi}|q| + ig^2 \underbrace{\omega\sigma_{xx}^{cf}}$$

z=1 scaling

Kubo formula.

Dynamical scaling due to overdamped transverse gauge boson.

The result is self-consistent.

Screening and dissipation at criticality

short-range interactions:

$$S'_{int} = -\frac{U_0}{16\pi} \int_{\omega,q} q^2 a_T(q,\omega) a_T(-q,-\omega)$$

Transverse gauge boson inv. propagator:

$$D^{-1}_{ret} = -\frac{U_0}{8\pi} q^2 + \underbrace{i g^2 \omega \sigma^{cf}_{xx}}_{\text{Kubo formula.}}$$

z=2 scaling

Dynamical scaling due to overdamped transverse gauge boson.

Towards a scaling theory

Full theory will have 3 running couplings:

$$1)\sigma_{xx}$$

$$2)\sigma_{xy}$$

$$3)e_*^2$$

At the QHIT in the Dirac CF theory only 2 of them run:

$$1)\sigma_{xx}$$

$$3)e_*^2$$

Even though both couplings are order 1 at critical point, we can deduce z:

$$D_{ret}^{-1} = -\frac{e_*^2}{8\pi}|q| + ig^2\omega\sigma_{xx}^{cf}$$

We also know that transitions are superuniversal.

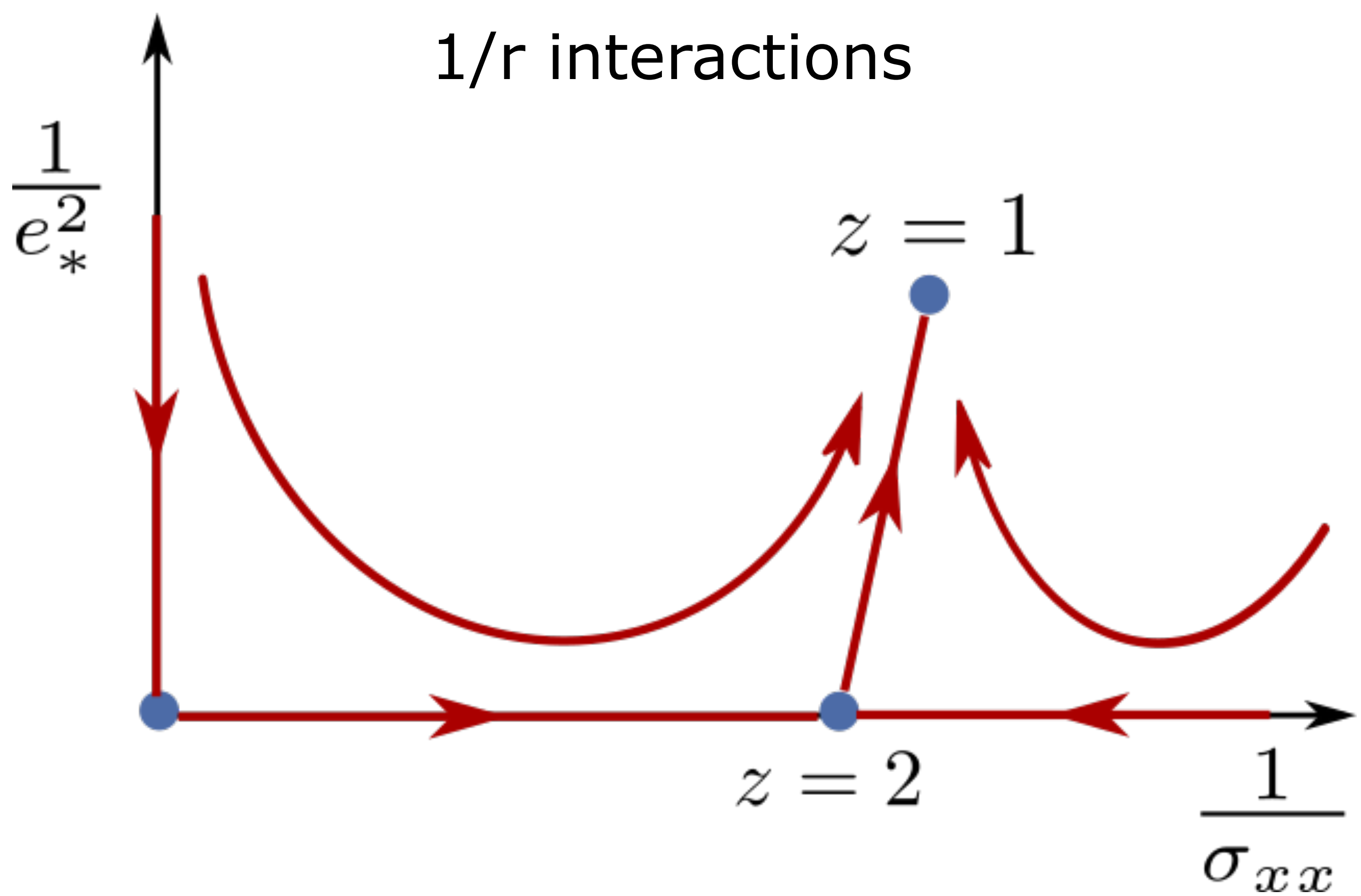
How to determine ν

Finite-size scaling of gauge boson spectral function.

$$\mathcal{L}[a_T] = a_T \left[|\boldsymbol{q}| + i g^2 \omega \sigma_{xx} \right] a_T$$

Finite-size scaling: $\sigma_{xx}(B, L) = \frac{e^2}{h} \mathcal{F}(\delta L^{1/\nu})$ $\delta = \frac{B - B_c}{B_c}$

Summary



Theme: Composite fermion viewpoint of QH critical points.