Matching higher symmetries across Intriligator-Seiberg duality

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based on [arXiv:2108.05369]
with K. Ohmori and Y. Tachikawa

Summary

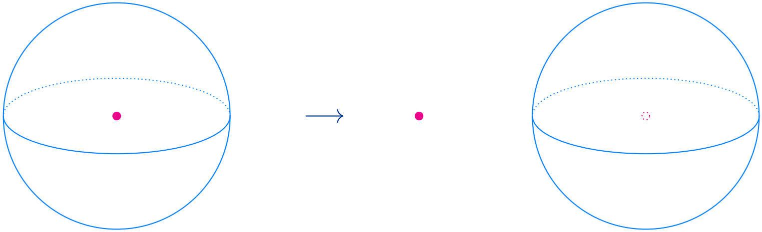
- SO/Spin QCDs in $(3+1)d$ have "higher symmetries"
- for $\mathcal{N}=1$ cases, they match across Intriligator-Seiberg duality

Outline

- Higher symmetries
  - p-form symmetry
  - 2-group
- Example : SO/Spin gauge theories
  - $\mathrm{SL}(2,\mathbb{Z}_2)$ group action
  - Intriligator-Seiberg duality
- Anomaly
  - bordism
  - invertible QFT

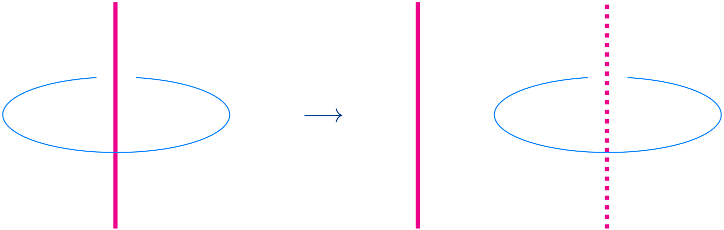
ordinary symmetry

operator $\mathcal{O} \to R(g)\,\mathcal{O}$



generalization

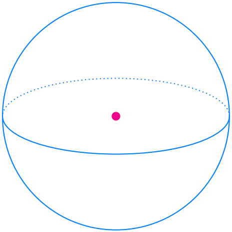
ex. line operator $\mathcal{L} \to R(g)\,\mathcal{L}$



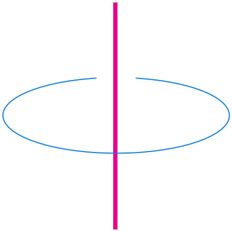
p-form symmetry

charged operator : p-dim.

symmetry operator : $(d-p-1)$-dim.



$p=0$



$p=1$

[Gaiotto-Kapustin-Seiberg-Willett (2014)]

mixed 't Hooft anomaly

QFT with both

$$\begin{cases} \text{0-form symmetry} & H\,[0] \\ \text{1-form symmetry} & \Gamma\,[1] \end{cases}$$

$\rightarrow$ might not be able to gauge them at the same time !

more "higher" symmetry

ordinary group extension

$$0 \longrightarrow \Gamma \longrightarrow G \longrightarrow H \longrightarrow 0$$

2-group

$$0 \longrightarrow \Gamma[1] \longrightarrow G \longrightarrow H[0] \longrightarrow 0$$

gauging

2-group $\quad 0 \longrightarrow \Gamma[1] \longrightarrow G \longrightarrow H[0] \longrightarrow 0$

gauge $\Gamma[1]$ $\downarrow$ $\uparrow$ gauge $\widehat{\Gamma}[1]$

mixed anomaly $\begin{cases} \text{0-form symmetry} & H[0] \\ \text{1-form symmetry} & \widehat{\Gamma}\,[1] \;(= d-p-2) \end{cases}$

[Tachikawa (2017)]

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Spin gauge theory

double cover of SO

$$0 \longrightarrow \mathbb{Z}_2^f \longrightarrow \mathrm{Spin} \longrightarrow \mathrm{SO} \longrightarrow 0$$

pure $\mathrm{Spin}(2n_c)$ gauge theory

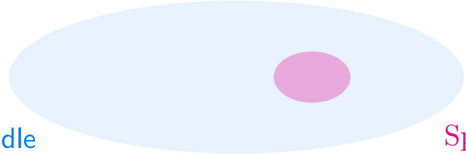
$$\text{center} = \begin{cases} \mathbb{Z}_2 \times \mathbb{Z}_2 & (n_c \text{ even}) \\ \mathbb{Z}_4 & (n_c \text{ odd}) \end{cases}$$

SO gauge theory

gauging $\mathbb{Z}_2[1]$ subgroup

gauge	center
G	C
G/Γ	C/Γ

pure $\mathrm{SO}(2n_c)$ gauge theory



SO-bundle

Spin-bundle $(w_2 = 0)$

bundles are characterized by $w_2 \in H^2(B\mathrm{SO};\mathbb{Z}_2)$ $\Leftrightarrow$ gauge field

discrete theta angle

SO$_-$ theory

$$\int \mathcal{D}A \quad \cdots \quad + \quad \theta F \wedge F$$

$$\sum_{w_2} \quad \cdots \quad + \quad \mathfrak{P}(w_2)$$

[Aharony-Seiberg-Tachikawa (2013)]

$$\mathrm{Spin} \xrightarrow{\text{gauge } \mathbb{Z}_2[1]} \mathrm{SO}_+$$

$$\mathrm{Spin} \xrightarrow{\text{add } \mathfrak{P}(w_2)} \xrightarrow{\text{gauge } \mathbb{Z}_2[1]} \mathrm{SO}_-$$

$\mathrm{SL}(2,\mathbb{Z}_2)$ group action

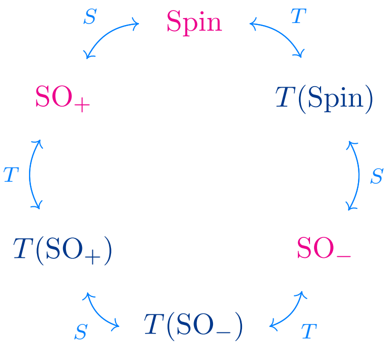
$$S \quad : \quad \text{gauge } \mathbb{Z}_2[1]$$
$$T \quad : \quad \text{add } \mathfrak{P}(w_2)$$

satisfy

$$S^2 = 1$$
$$T^2 = 1$$
$$(ST)^3 = 1$$

[Gaiotto-Kapustin-Seiberg-Willett (2014)]

$\mathrm{SL}(2, \mathbb{Z}_2)$ group action



Spin QCD

$2n_f$ massless fermions in vector rep.

$$\text{center} = \begin{cases} \mathbb{Z}_2 \times \mathbb{Z}_2 & (n_c \text{ even}) \\ \mathbb{Z}_4 & (n_c \text{ odd}) \end{cases} \longrightarrow \quad \mathbb{Z}_2[1]$$

Wilson line W

$$\text{spinor} + \text{spinor} = \begin{cases} \text{trivial} & (n_c \text{ even}) \\ \text{vector} \xrightarrow{\text{screen}} \text{odd under } (-1) \in \textbf{flavor} & (n_c \text{ odd}) \end{cases}$$

$\to$ for n_c odd, $\mathbb{Z}_2[1]$ and $\mathrm{SU}(2n_f)[0]$ are mixed = 2-group ! [Hsin-Lam (2020)]

SO QCD

magnetic $\mathbb{Z}_2[1]$ symmetry $\leftrightarrow$ 't Hooft line H

$$H + H \xrightarrow{\text{screened}} \text{~~trivial...?~~}$$

monopole : flavor charge from fermionic zero modes

Higgs $SO(2n_c) \to SO(2)^{n_c}$,

reduce to 't Hooft-Polyakov monopoles, ...

2-group when

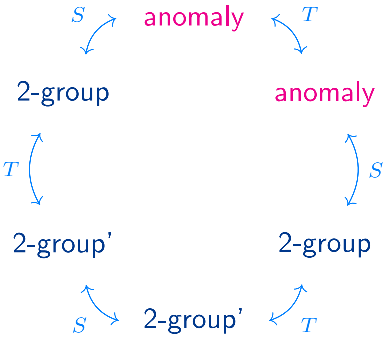
$$\begin{aligned} \mathrm{SO}_+ &: \quad n_f \quad \text{mod } 2 \\ \mathrm{SO}_- &: \quad n_c + n_f \quad \text{mod } 2 \end{aligned}$$

2-group structure

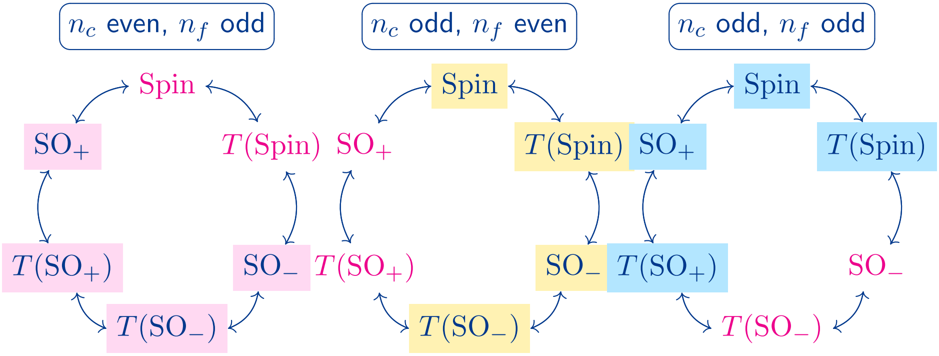
	Spin	SO_+	SO_-
n_c even, n_f even			
n_c even, n_f odd		2-group	2-group
n_c odd, n_f even	2-group		2-group
n_c odd, n_f odd	2-group	2-group	

$\mathrm{SL}(2, \mathbb{Z}_2)$ transformations

recall that gauging was $S \in \mathrm{SL}(2, \mathbb{Z}_2)$, ...



The Only Neat Thing to Do



Intriligator-Seiberg duality

$$\mathfrak{so}(N_c) \quad \leftrightarrow \quad \mathfrak{so}(N_f - N_c + 4)$$

N_f flavors $\qquad$ N_f flavors

[Intriligator-Seiberg (1995)]

more precisely

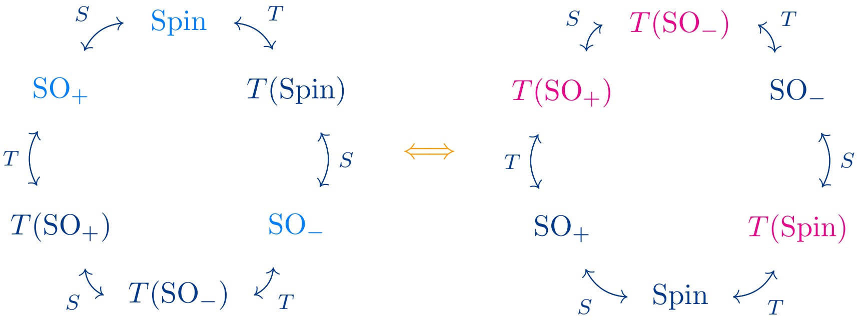
$$\begin{aligned} \mathrm{Spin} &\leftrightarrow T(\mathrm{SO}_-) \\ \mathrm{SO}_+ &\leftrightarrow T(\mathrm{SO}_+) \\ \mathrm{SO}_- &\leftrightarrow T(\mathrm{Spin}) \end{aligned}$$

[Strassler (1997)]

[Aharony-Seiberg-Tachikawa (2013)]

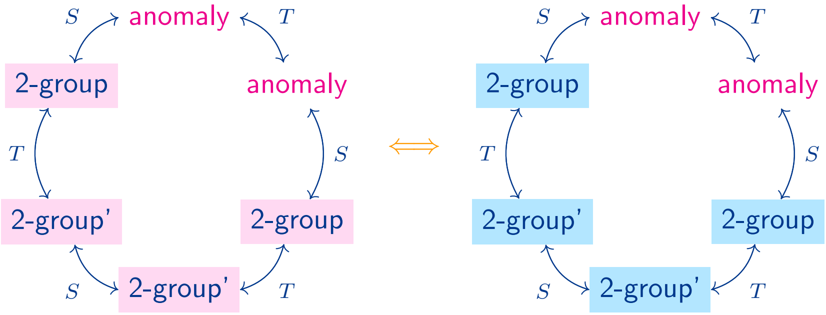
[Gaiotto-Kapustin-Seiberg-Willett (2014)]

Intriligator-Seiberg duality



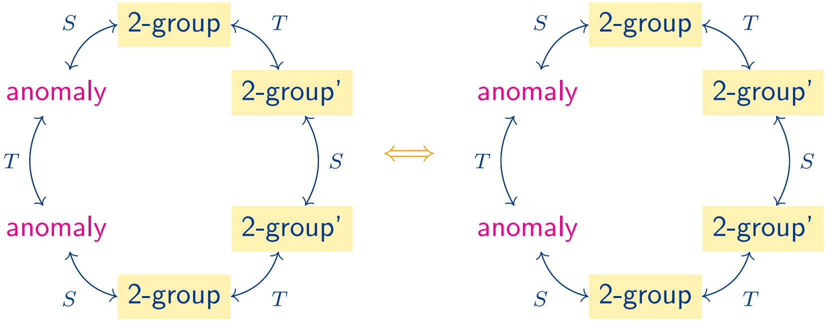
matching higher symmetries

n_c even, n_f odd : $n'_c = n_f - n_c + 2$ odd, n_f odd



matching higher symmetries

n_c odd, n_f even : $n_c' = n_f - n_c + 2$ odd, n_f even



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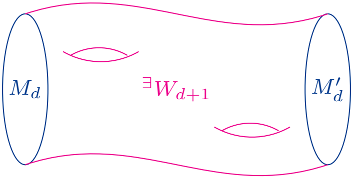
  - bordism
  - invertible QFT

bordism

bordism group Ω_d

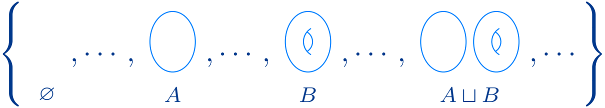
element	:	[closed manifold M_d]
operation	:	disjoint union $\sqcup$
identity	:	$[\varnothing]$

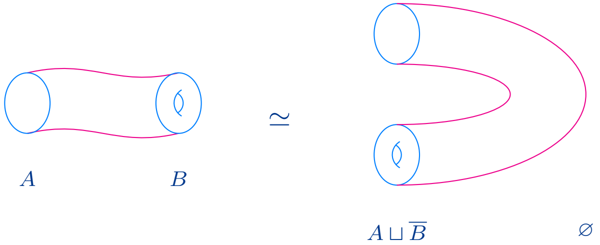
two manifolds are equivalent if



bordism

$\{\text{closed } d\text{-manifolds}\} =$



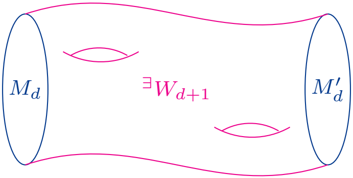


bordism

bordism group Ω_d^{spin}

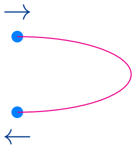
element	:	[closed spin manifold M_d]
operation	:	disjoint union $\sqcup$
identity	:	$[\varnothing]$

two manifolds are equivalent if

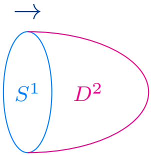


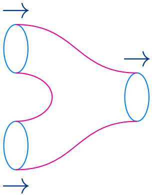
example

$\Omega_0^{\text{oriented}} = \mathbb{Z}$: generated by $[\text{pt.}]$



$\Omega_1^{\text{oriented}} = 0$





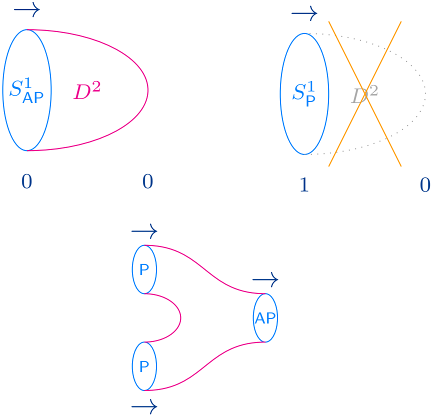
$\Omega_2^{\text{oriented}} = 0$

$\Omega_3^{\text{oriented}} = 0$

$\Omega_4^{\text{oriented}} = \mathbb{Z}$: generated by $[\mathbb{CP}^2]$

example

$\Omega_1^{\text{spin}} = \mathbb{Z}_2$ (vs. $\Omega_1^{\text{oriented}} = 0$)



#(zero modes) mod 2 — 0, 0 | 1, 0

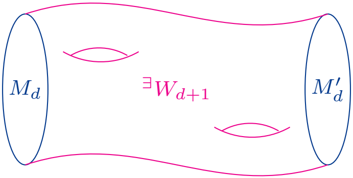
$2 = 0$ — 0

bordism

bordism group $\Omega_d^{\text{spin}}(X)$

element	:	$[\text{closed spin manifold } M_d,\ \text{map } \sigma : M_d \to X]$
operation	:	disjoint union $\sqcup$
identity	:	$[\varnothing, \sigma_{\text{trivial}}]$

two manifolds are equivalent if



invertible QFT

Definition

$\dim \mathcal{H} = 1$ on any closed spatial manifold

"invertible" under

$$Z_{Q \cdot Q'} = Z_Q \times Z_{Q'}$$

$$\mathcal{H}_{Q \cdot Q'} = \mathcal{H}_Q \otimes \mathcal{H}_{Q'}$$

where

$$Z_{Q_{\text{trivial}}} = 1$$

$$\dim \mathcal{H}_{Q_{\text{trivial}}} = 1$$

new light

short exact sequence

$$0 \to \mathrm{Ext}_{\mathbb{Z}}(\Omega_d^{\text{spin}}(X), \mathbb{Z}) \to \mathrm{Inv}^d_{\text{spin}}(X) \to \mathrm{Hom}_{\mathbb{Z}}(\Omega_{d+1}^{\text{spin}}(X), \mathbb{Z}) \to 0$$

$$\mathrm{Ext}_{\mathbb{Z}}(\Omega_d^{\text{spin}}(X), \mathbb{Z}) \cong \Omega_d^{\text{spin}}(X)_{\text{torsion}}, \qquad \mathrm{Hom}_{\mathbb{Z}}(\Omega_{d+1}^{\text{spin}}(X), \mathbb{Z}) \cong \Omega_{d+1}^{\text{spin}}(X)_{\text{free}}$$

where

$$\mathrm{Inv}^d_{\text{spin}}(X) = \{\text{invertible QFTs}\}/\sim$$

[Freed-Hopkins (2016)]

[Yamashita-Yonekura (2021)]

magical weapons

How do we calculate $\Omega_d^{\mathrm{spin}}(X)$ ...?

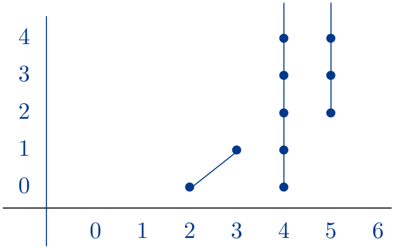
Atiyah-Hirzebruch SS

$$E^2_{p,q} = H_p(X;\Omega_q^{\mathrm{spin}}) \Rightarrow \Omega^{\mathrm{spin}}_{p+q}(X)$$

$q \backslash p$	0	1	2	3	4	5	6
5							
4	$\mathbb{Z}$		$*$		$*$	$*$	$*$
3							
2	$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$*$	$*$
1	$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$*$
0	$\mathbb{Z}$		$\mathbb{Z}_2$		$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$

Adams SS

$$E^2_{s,t} = \mathrm{Ext}^{s,t}_{\mathcal{A}(1)}(H^*(X;\mathbb{Z}_2),\mathbb{Z}_2) \Rightarrow \Omega^{\mathrm{spin}}_{t-s\,(\leq 7)}(X)$$



anomaly

belief

d-dim. anomalous QFT

||

boundary of $(d+1)$-dim. invertible QFT

example

ex. $d = 4$, $G = SU(n)$ gauge anomaly: $X = BSU(n)$

$$0 \to \mathrm{Ext}_{\mathbb{Z}}(\Omega_5^{\text{spin}}(X), \mathbb{Z}) \to \mathrm{Inv}^5_{\text{spin}}(X) \to \mathrm{Hom}_{\mathbb{Z}}(\Omega_6^{\text{spin}}(X), \mathbb{Z}) \to 0$$

$$\mathrm{Ext}_{\mathbb{Z}}(\Omega_5^{\text{spin}}(X), \mathbb{Z}) \cong \Omega_5^{\text{spin}}(X)_{\text{torsion}} = \begin{cases} \mathbb{Z}_2 & (n=2) \\ 0 & (n \geq 3) \end{cases}$$

global anomaly

$$\mathrm{Hom}_{\mathbb{Z}}(\Omega_6^{\text{spin}}(X), \mathbb{Z}) \cong \Omega_6^{\text{spin}}(X)_{\text{free}} = \begin{cases} 0 & (n=2) \\ \mathbb{Z} & (n \geq 3) \end{cases}$$

perturbative anomaly

global anomaly

should be captured by

$$\Omega_5^{\mathsf{spin}}\left(B\left(\frac{\mathrm{SO}(2n_c)\times\mathrm{SU}(2n_f)}{\mathbb{Z}_2}\right)\right) = ?$$

but in general this is very hard to compute...

for the easiest case

$$\Omega_5^{\mathsf{spin}}\left(B\left(\frac{\mathrm{SO}(4)\times\mathrm{SU}(2)}{\mathbb{Z}_2}\right)\right) = \mathbb{Z}_2$$

which gives

$$\int_{W_5} w_2 \beta a_2$$

Summary

- SO/Spin QCDs in $(3+1)d$ have "higher symmetries"
  - 2-group symmetry : line operators $\rightarrow$ flavor charge
  - mixed 't Hooft anomaly : arises from massless fermions
- for $\mathcal{N}=1$ cases, they match across Intriligator-Seiberg duality