Planckian metals and black holes

GGI Tea Break
The Galileo Galilei Institute for Theoretical Physics
Florence, Italy
October 20, 2021

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS

HARVARD

1. Introduction to Planckian metals

2. Introduction to black holes

3. The SYK model

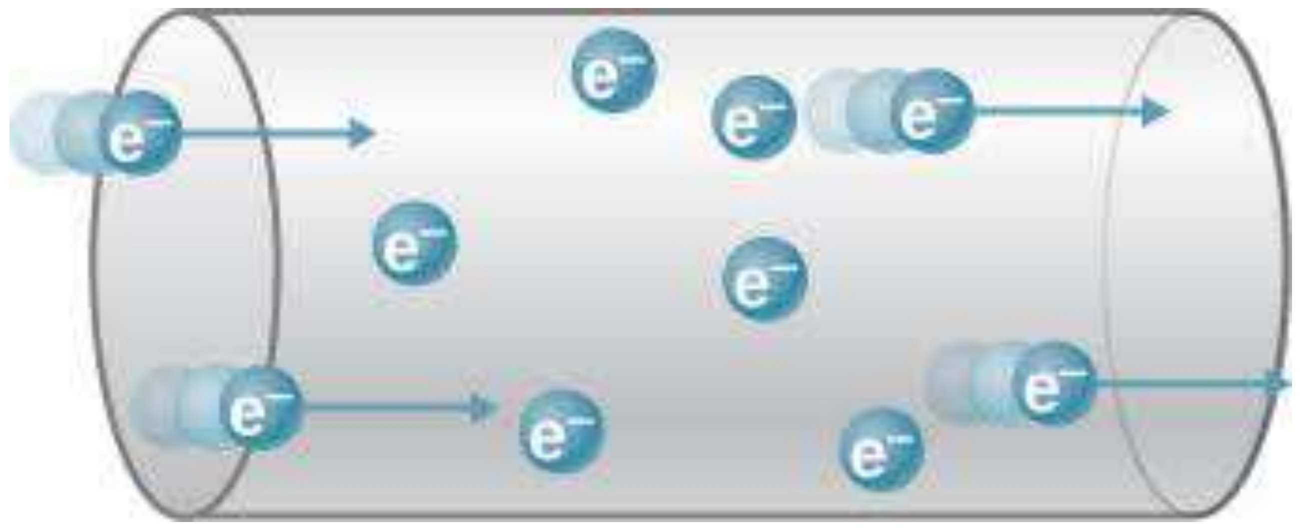
4. Progress on the theory of black holes

5. Progress on the theory of Planckian metals

Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Current flow with quasiparticles in Copper

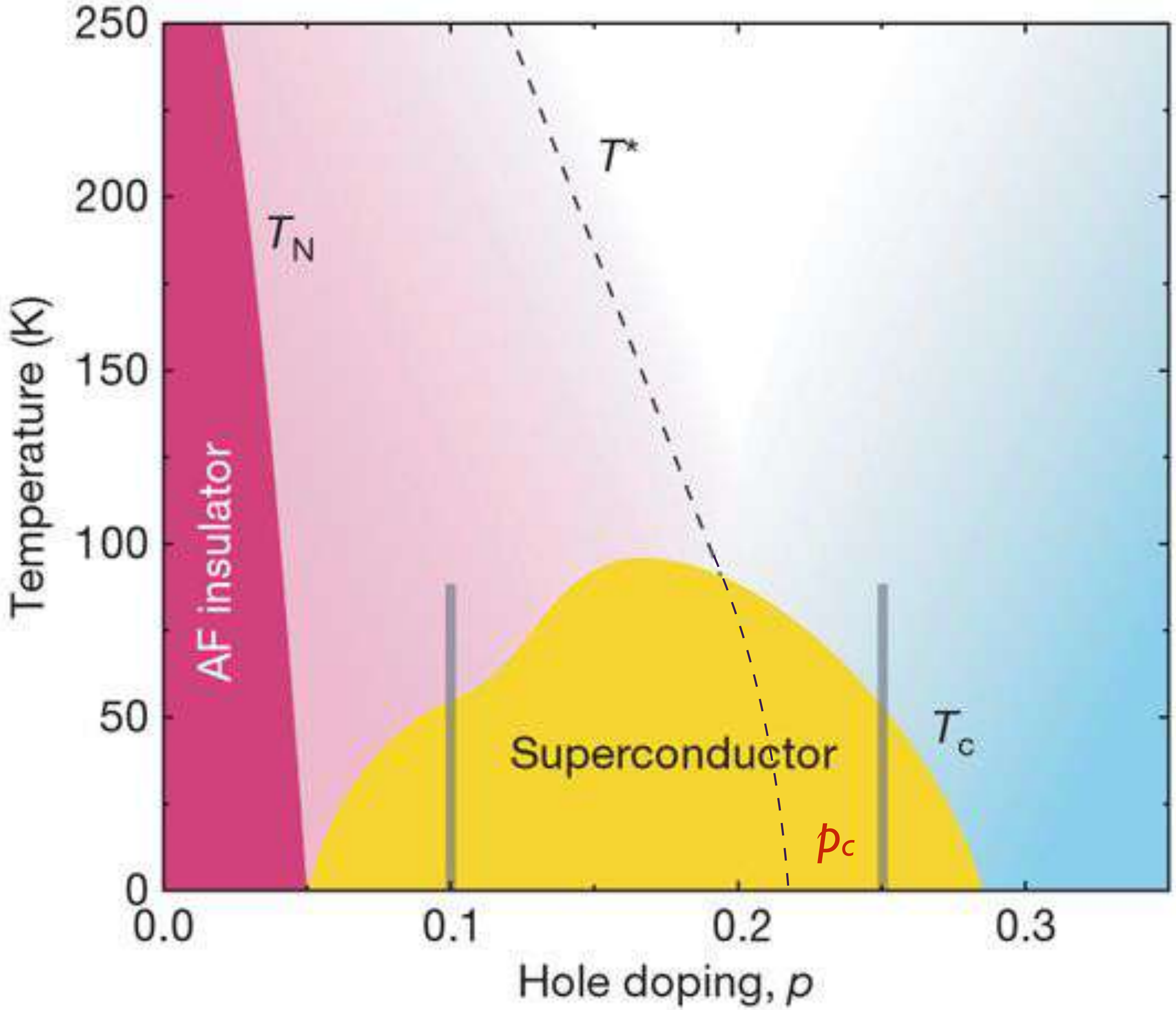


Flowing quasiparticles scatter off each other in a typical scattering time τ

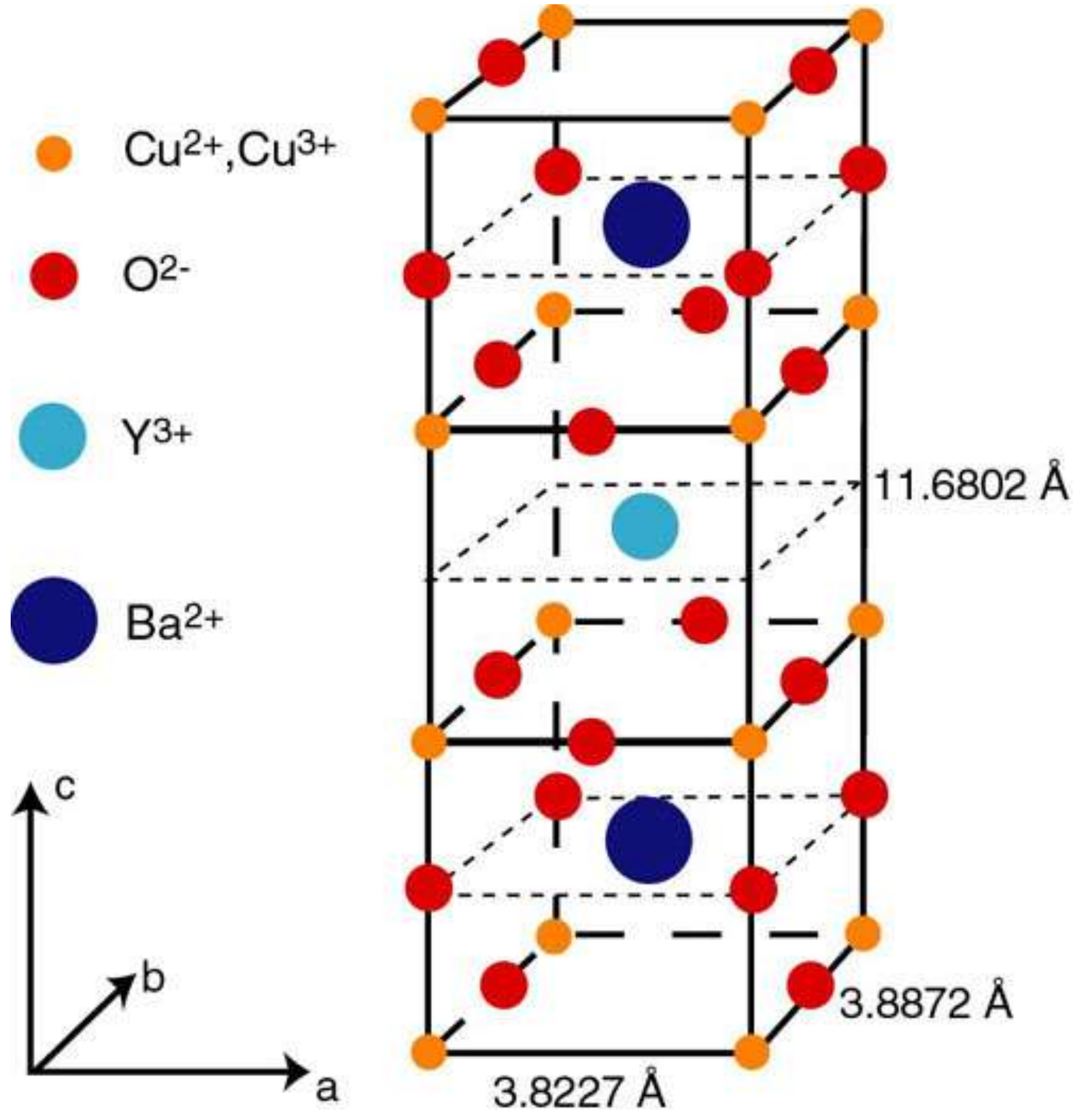
This time is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

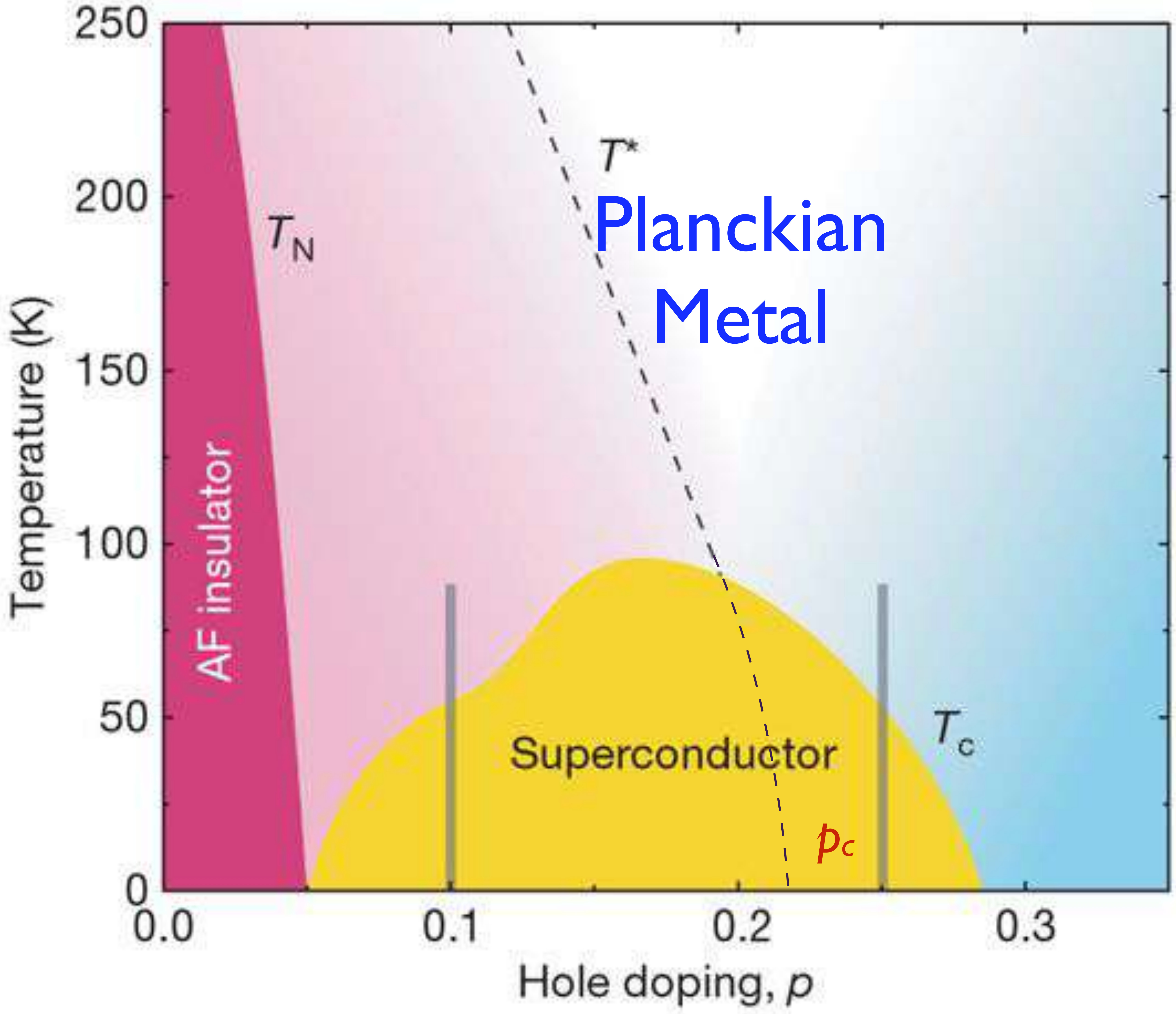
The motion of quasiparticles is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



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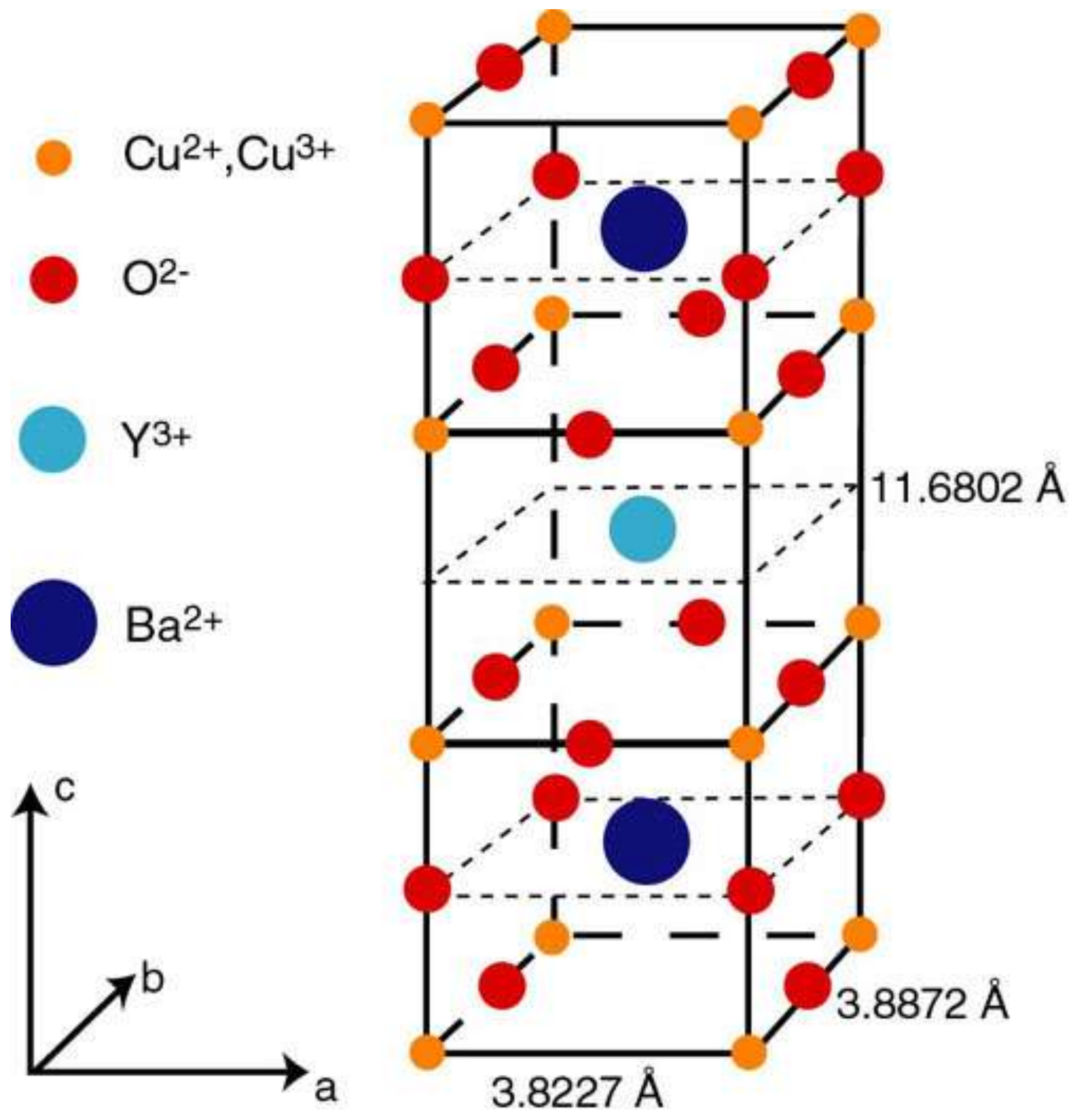

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Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Electron scattering time τ in 7 different Planckian metals

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

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Electron scattering time τ in 7 different Planckian metals

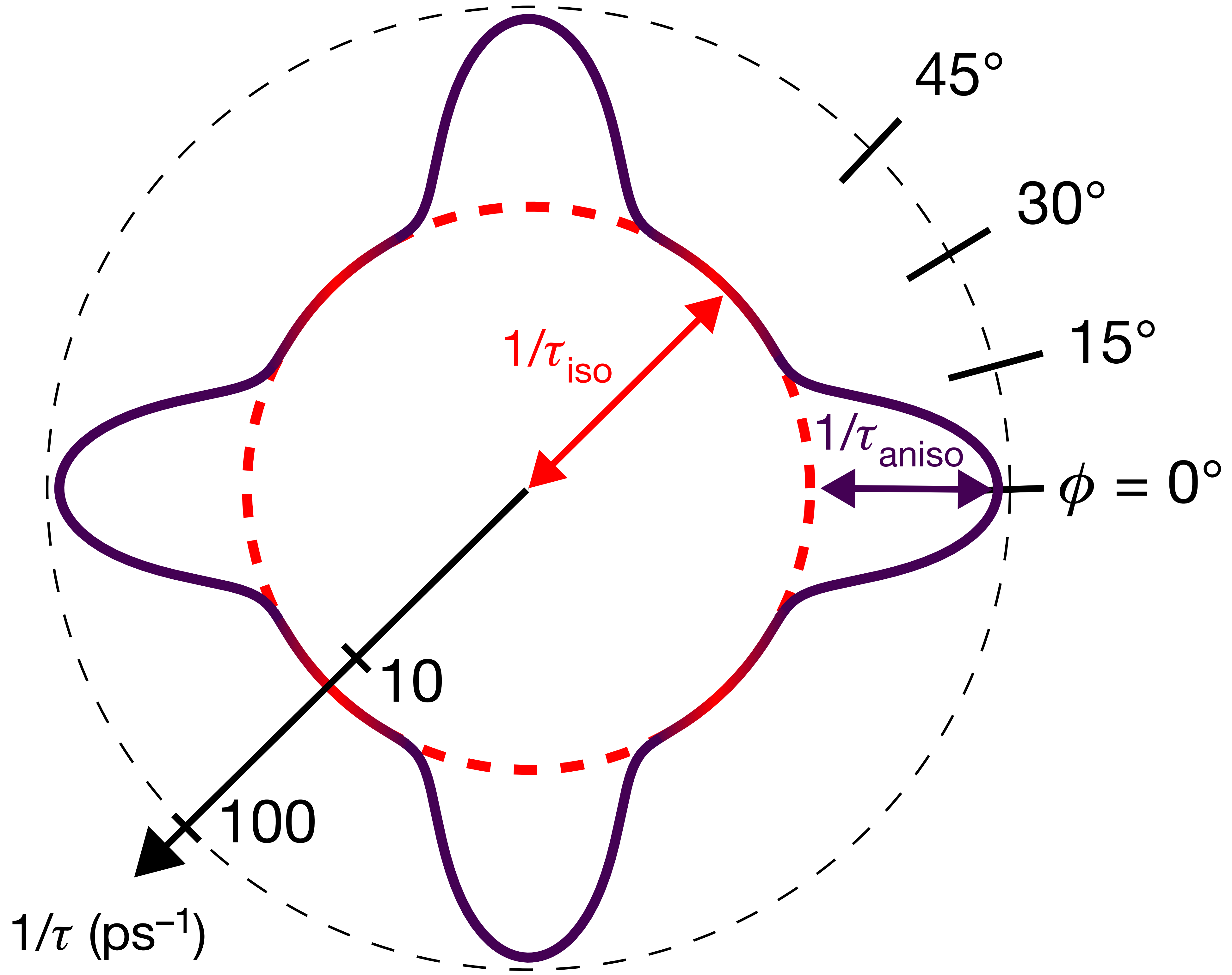
Current flow without quasiparticles

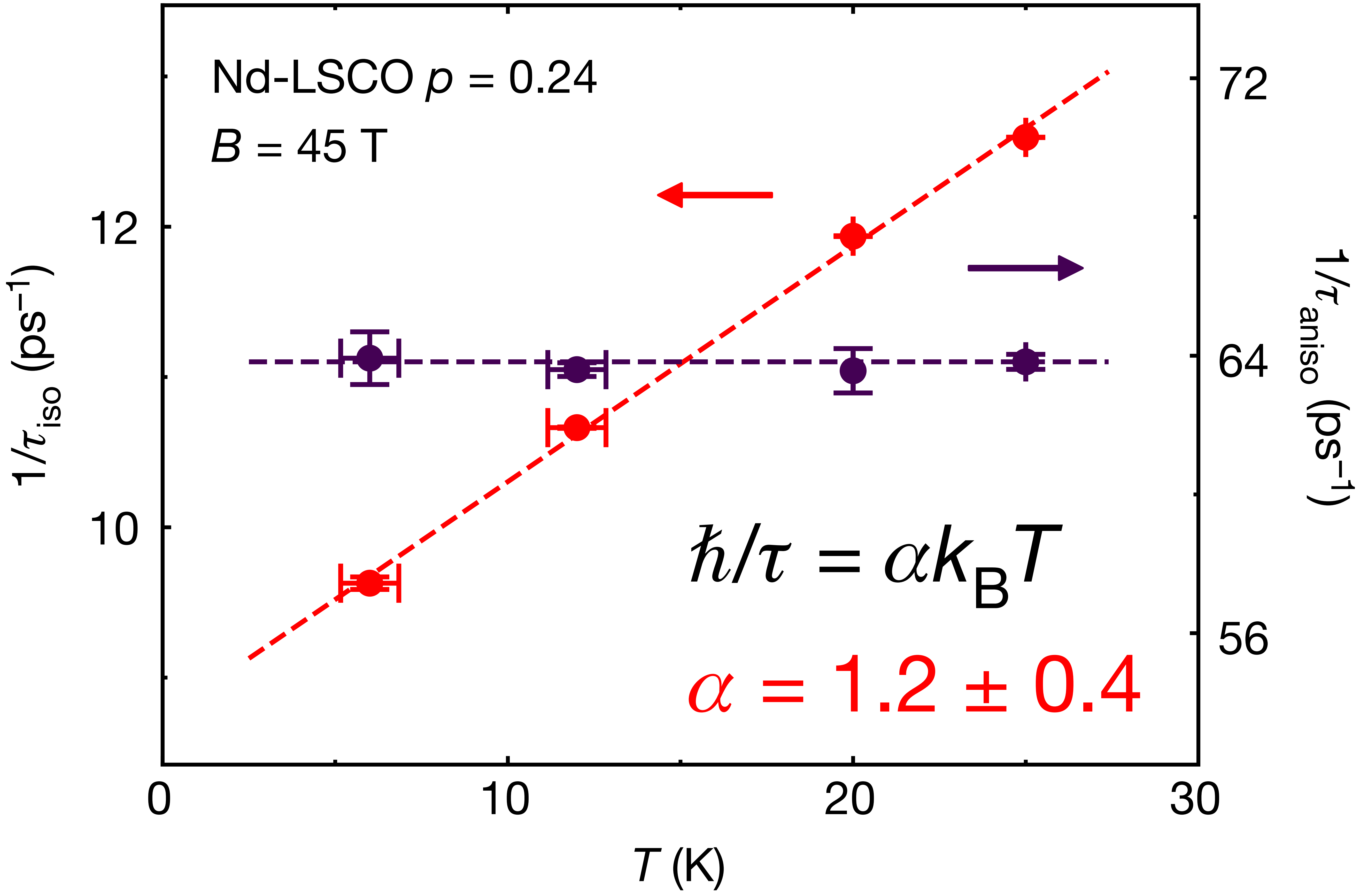
$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





Questions

- Theory for a fermion system with variable density with quasiparticles, and relaxation time $\sim \hbar/(k_B T)$.
- Needed: theory for collision time in resistivity $\sim \hbar/(k_B T)$.
- Needed: theory for the appearance of superconductivity (and other broken symmetries) in such a 'Planckian metal'.

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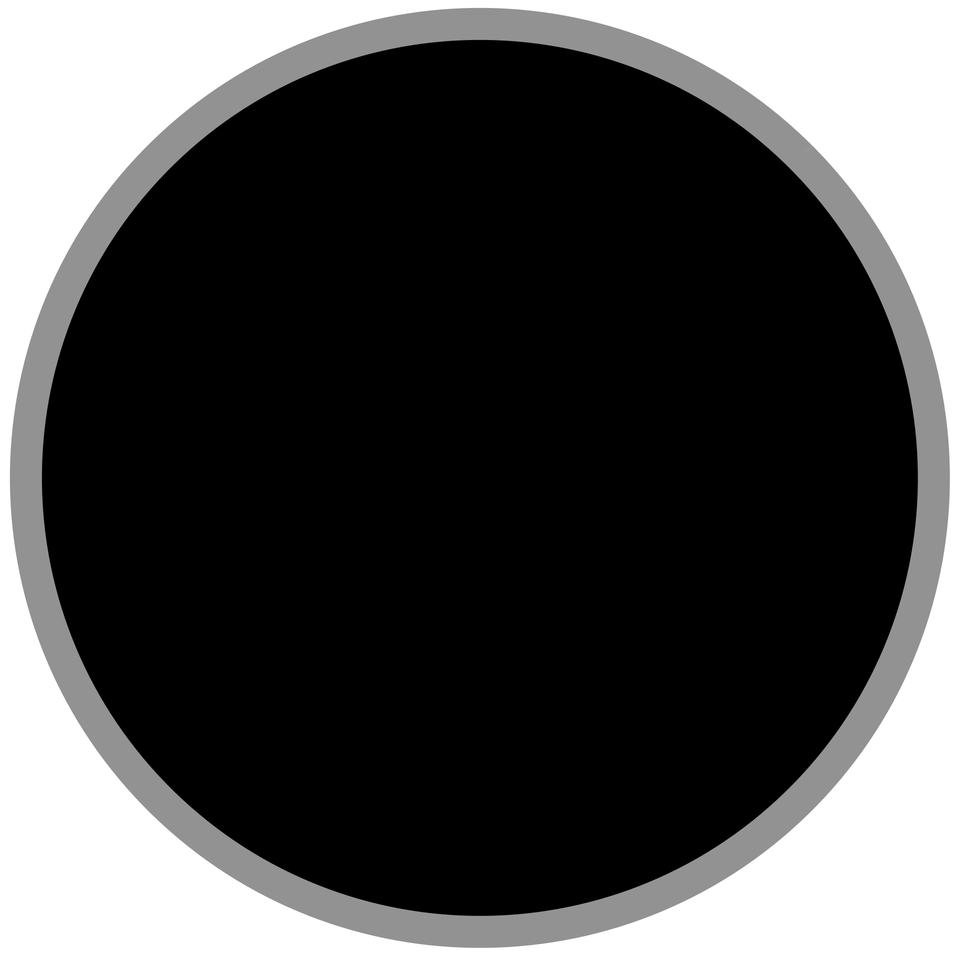
Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \dfrac{2GM}{c^2}$

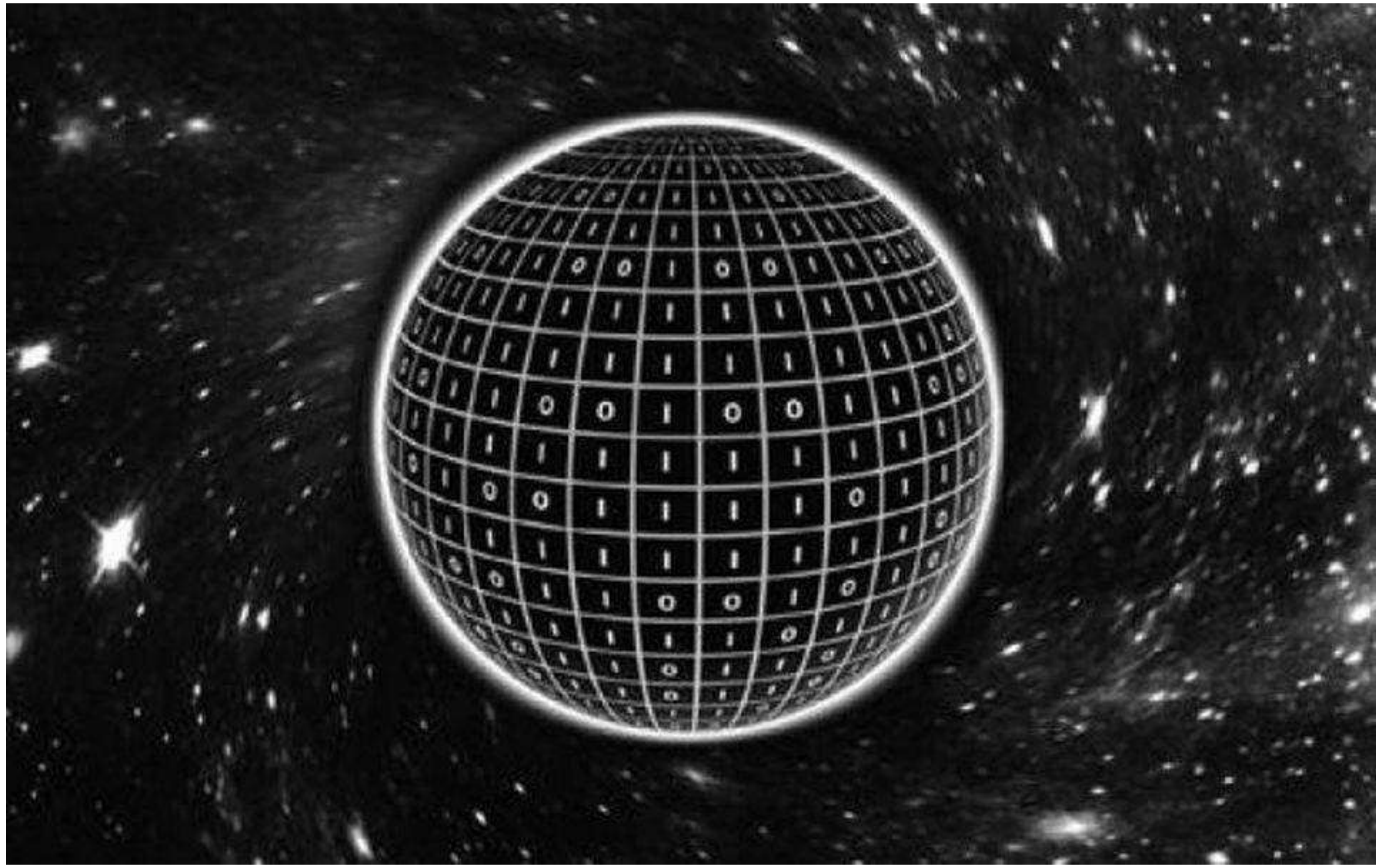


G Newton's constant, c velocity of light, M mass of black hole

For M = earth's mass, $R \approx 9\,mm$!

Quantum Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

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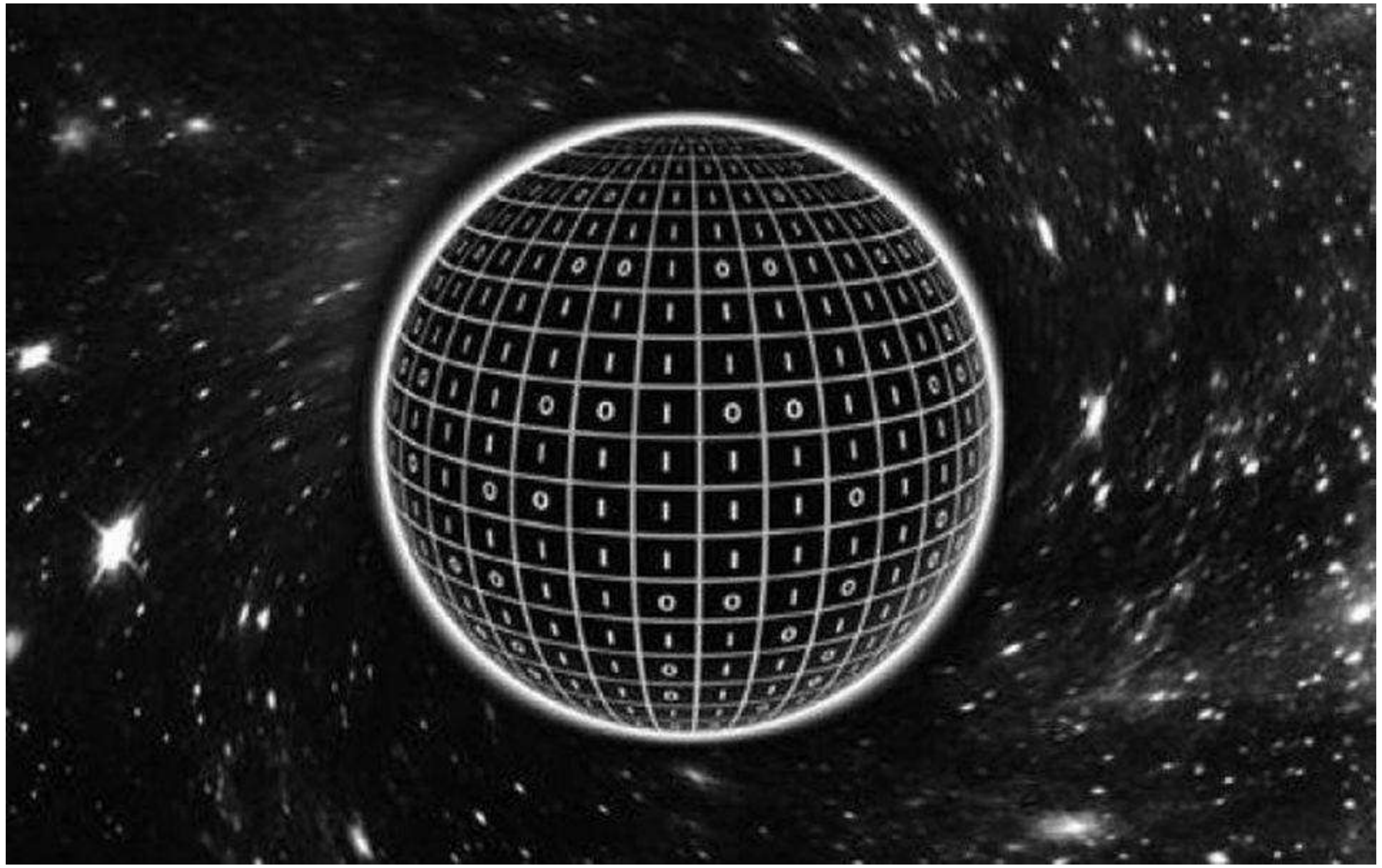

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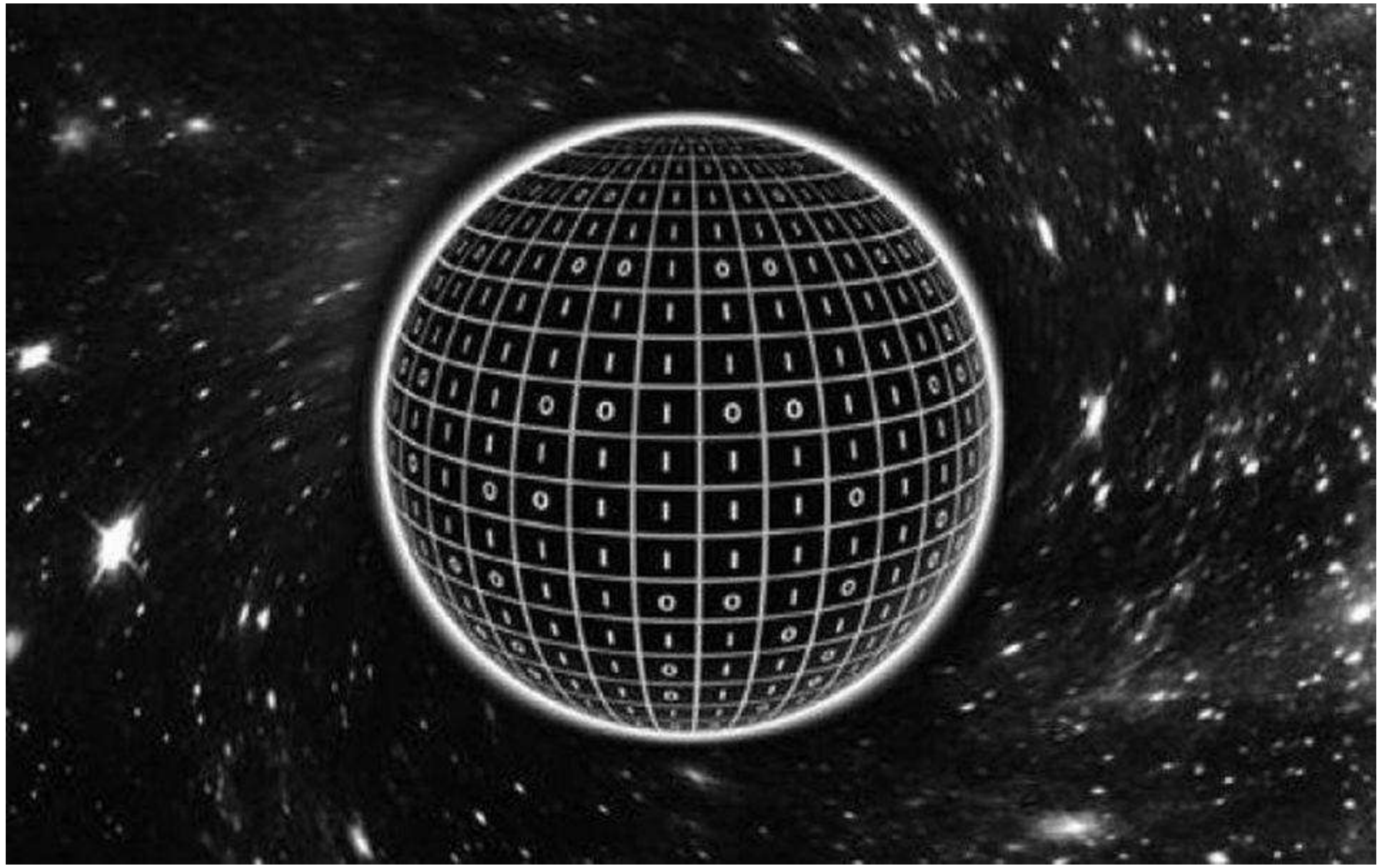

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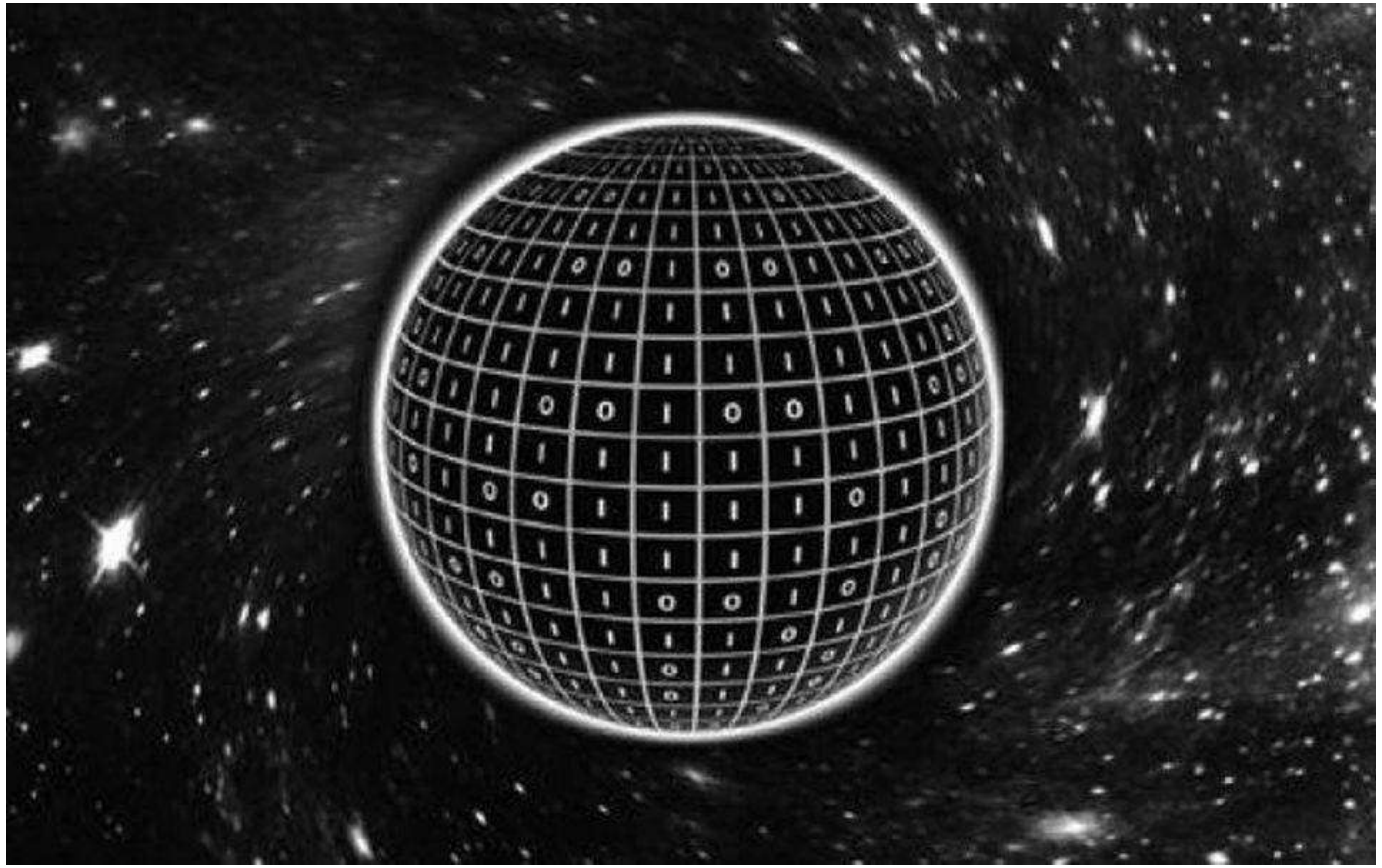

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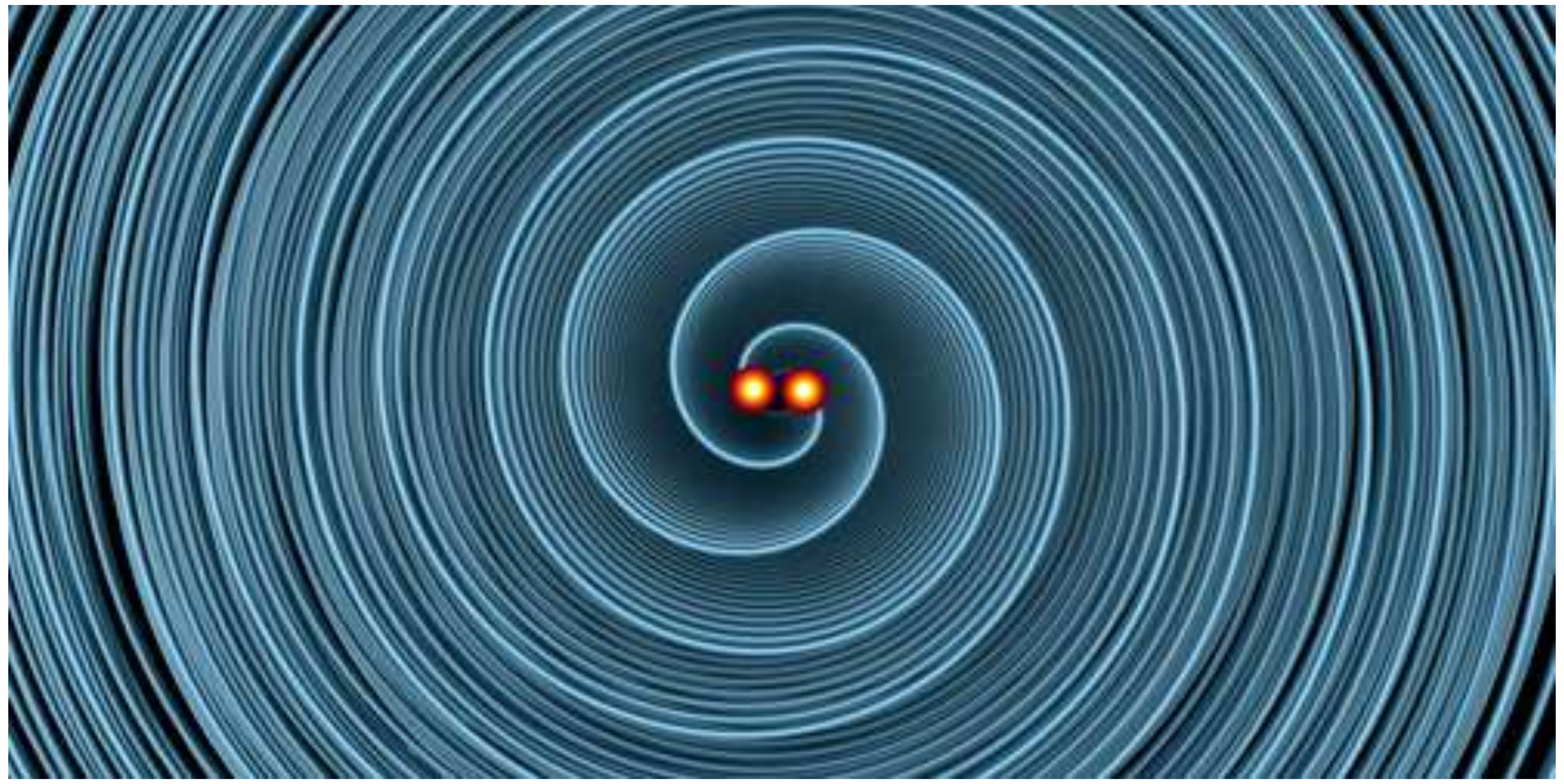
Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

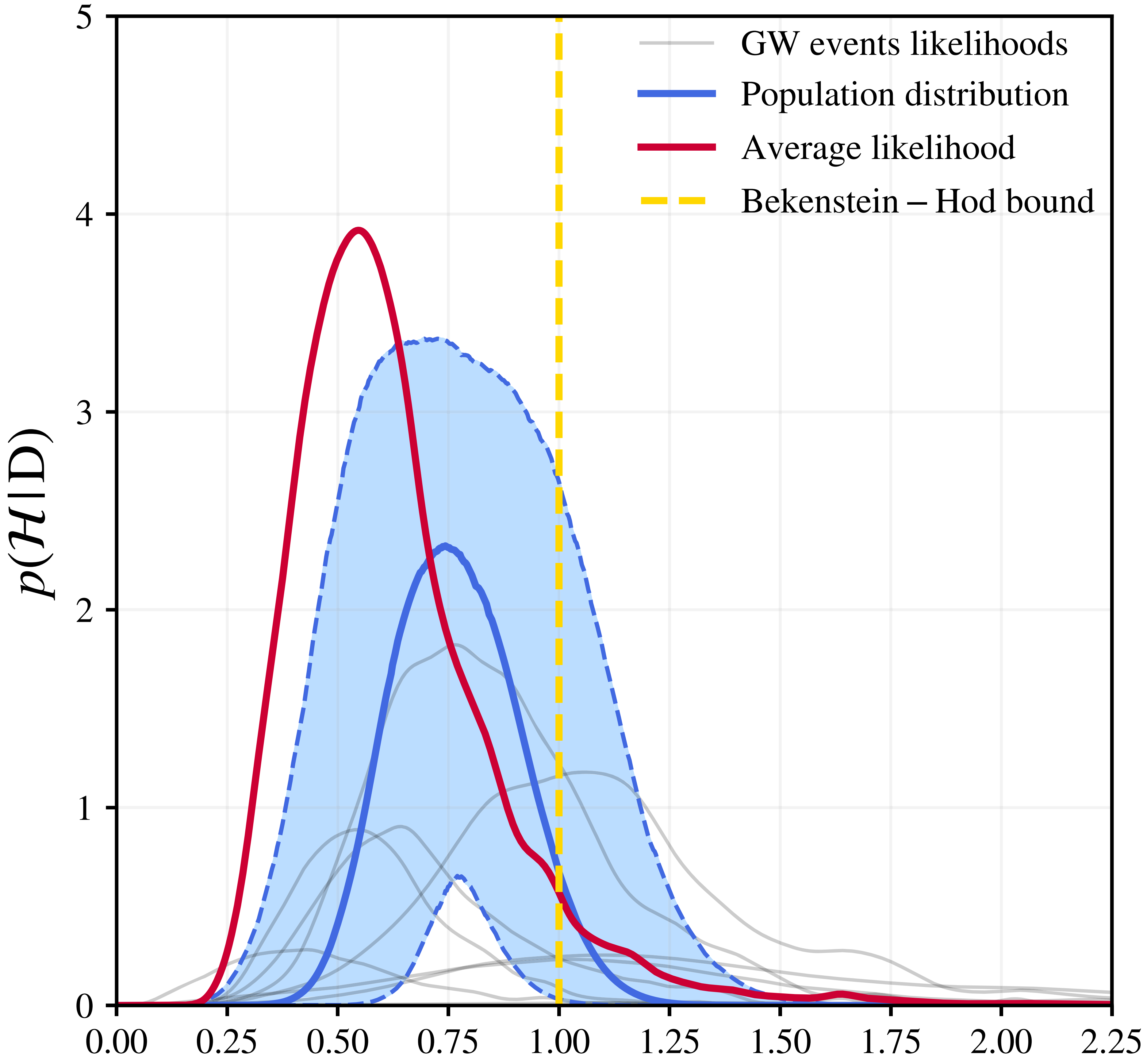
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

Metric of spacetime

Electromagnetic gauge field

In general, this integral is not well defined, because of an uncontrollably large number of spacetime configurations.

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$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$

$$= \exp\Big(S_{BH}\Big) \times \left(\quad ....????... \quad \right)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

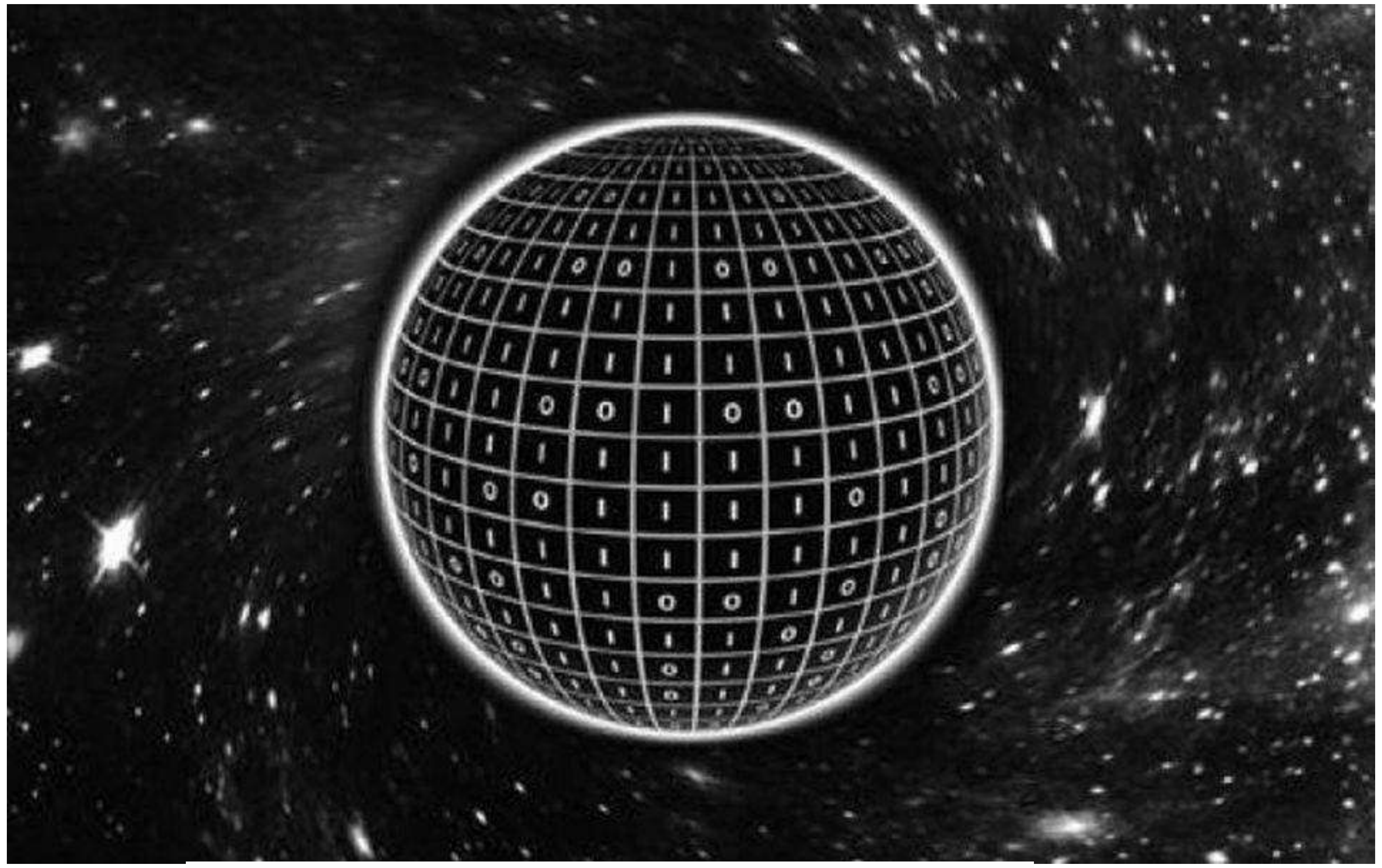
A_0 is the area of the charged black hole horizon at $T=0$.
$\mathcal{Q}$ is the black hole charge.
A_0 is a function of $\mathcal{Q}$.

Questions

- Can we compute corrections to S_{BH} in semiclassical Einstein-Maxwell theory?

- Can the resulting entropy be understood as that of a unitary quantum system with a discrete spectrum ?

- Can we compute the evolution of the entropy as the black hole evaporates? Is it that of an evaporating unitary quantum system?



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

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A simple model of a metal with quasiparticles

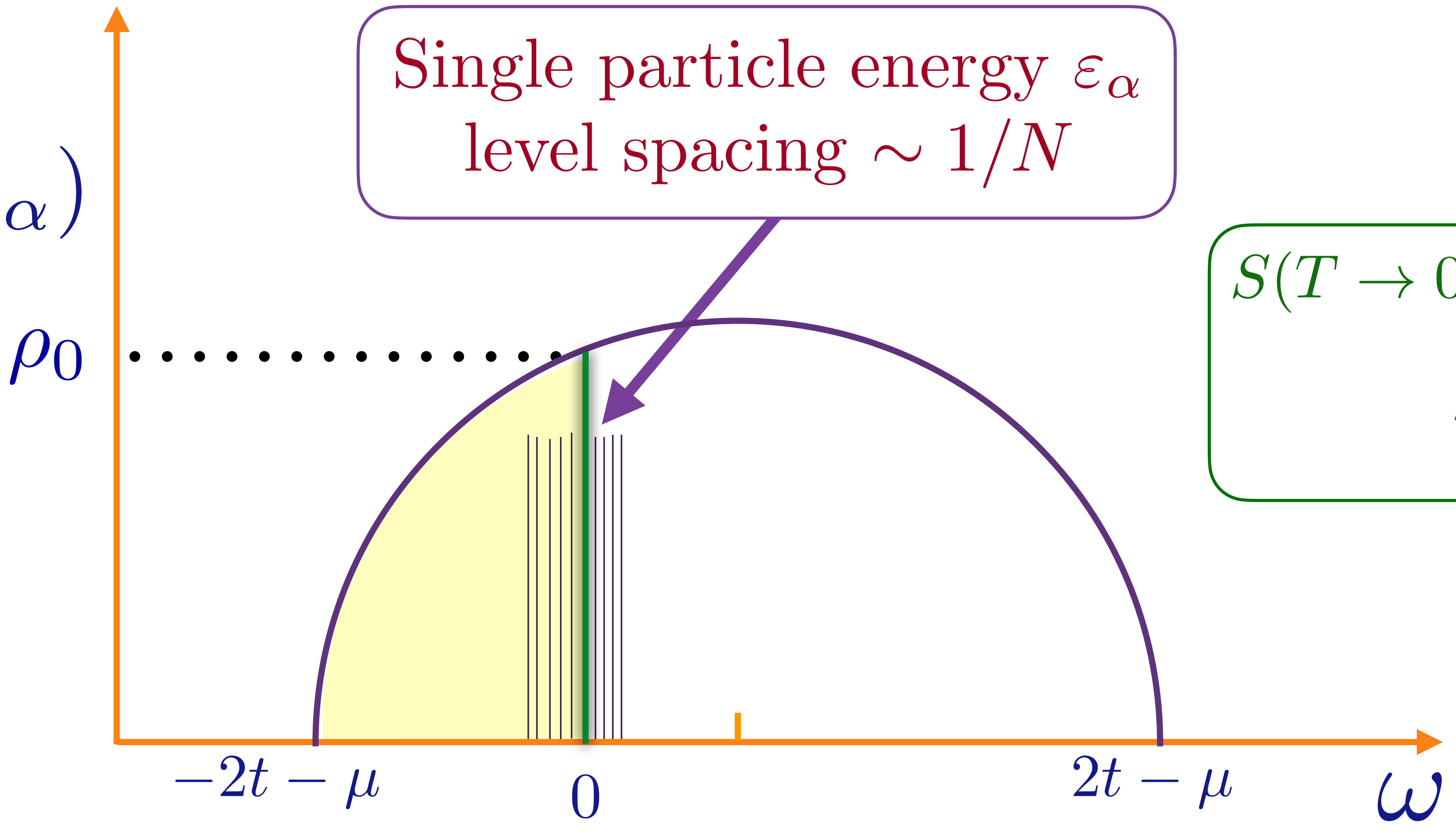
$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

$\rho(\omega) = \frac{1}{N} \sum_\alpha \delta(\omega - \varepsilon_\alpha)$

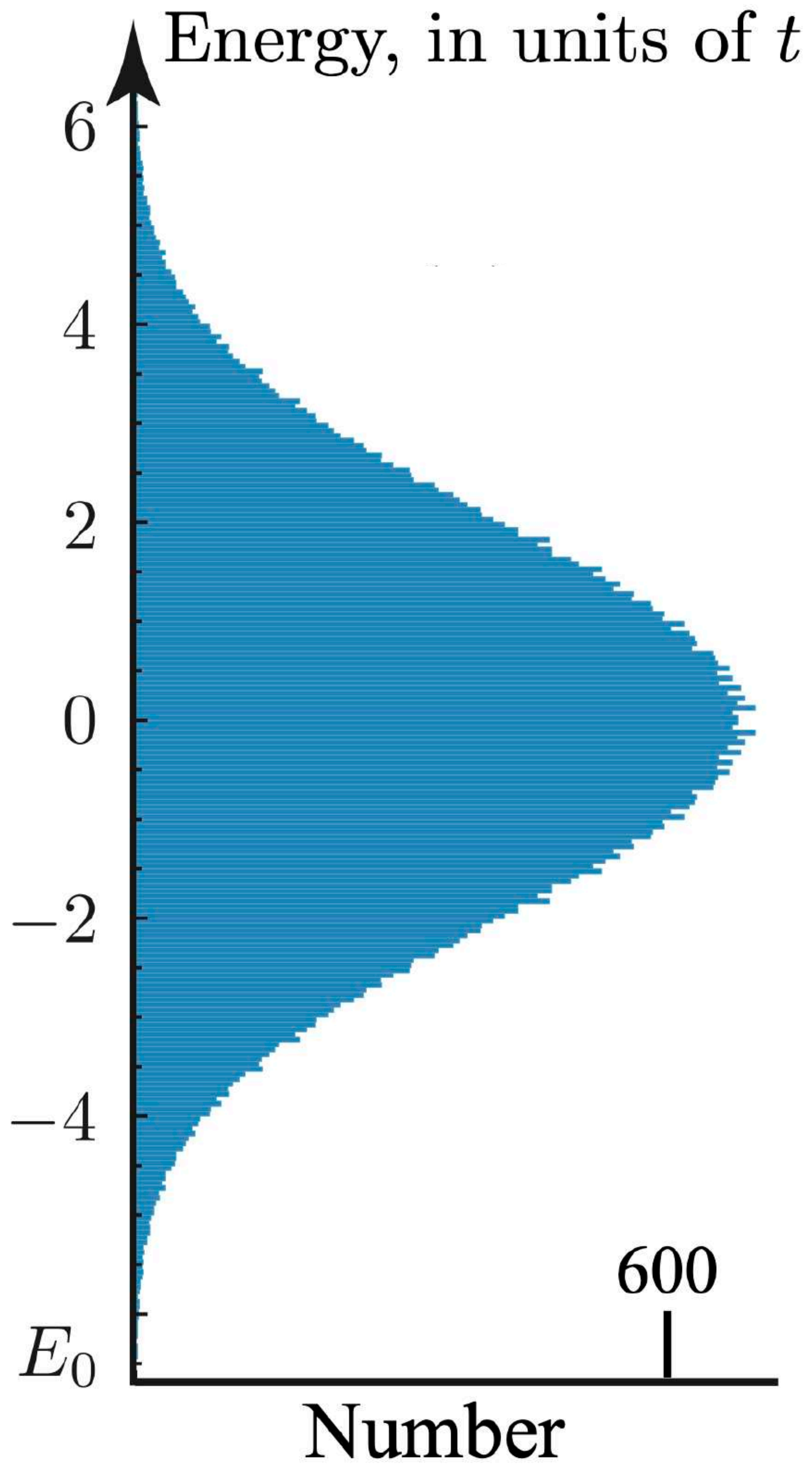


$S(T \to 0) = N\gamma T$

$\gamma = \frac{\pi^2}{3}\rho_0$

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number

Random matrix model

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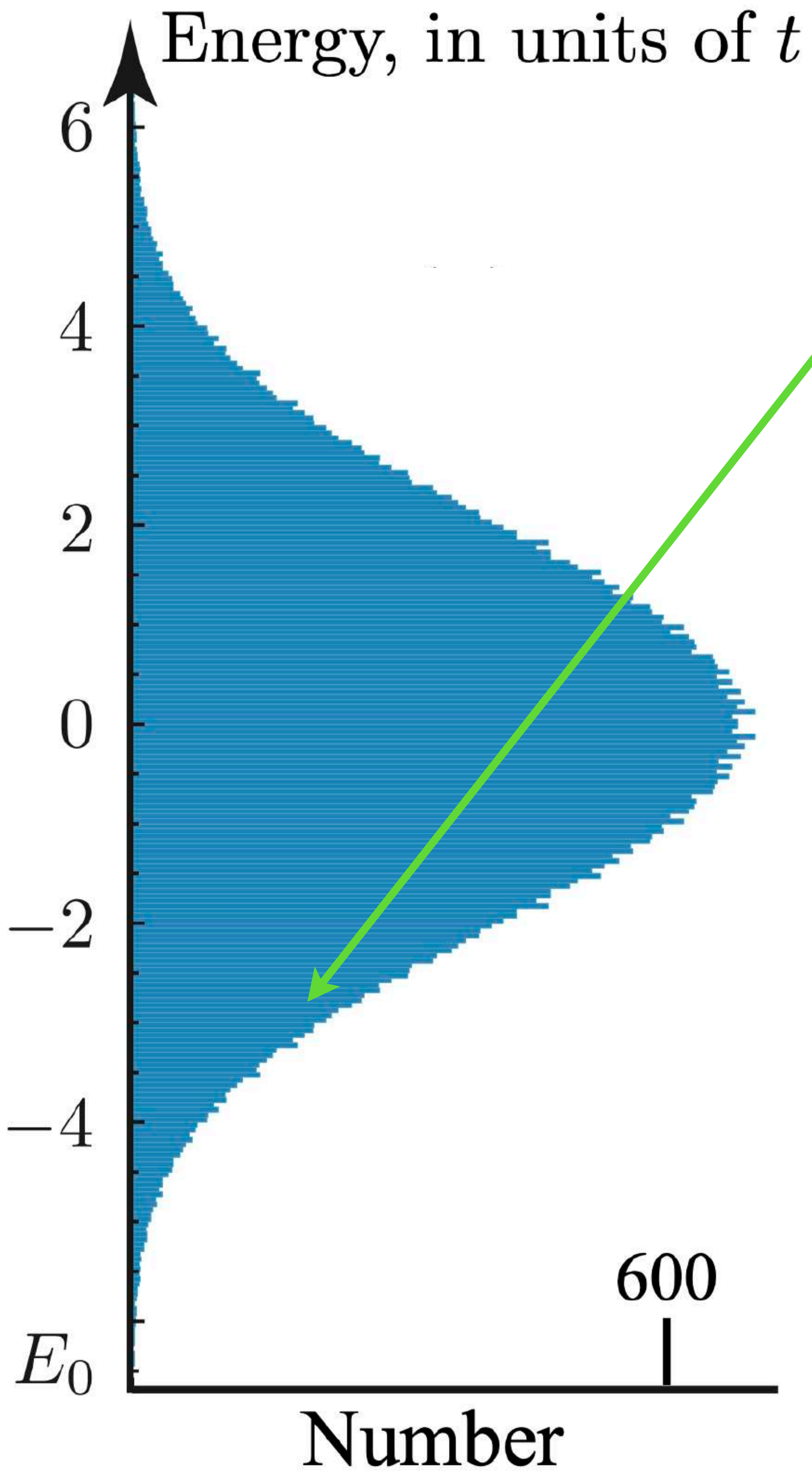

$$D(E) \sim e^{S(E)} = e^{\sqrt{2N\gamma E}}$$

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$D(E) \sim e^{S(E)}$

$= e^{\sqrt{2N\gamma E}}$

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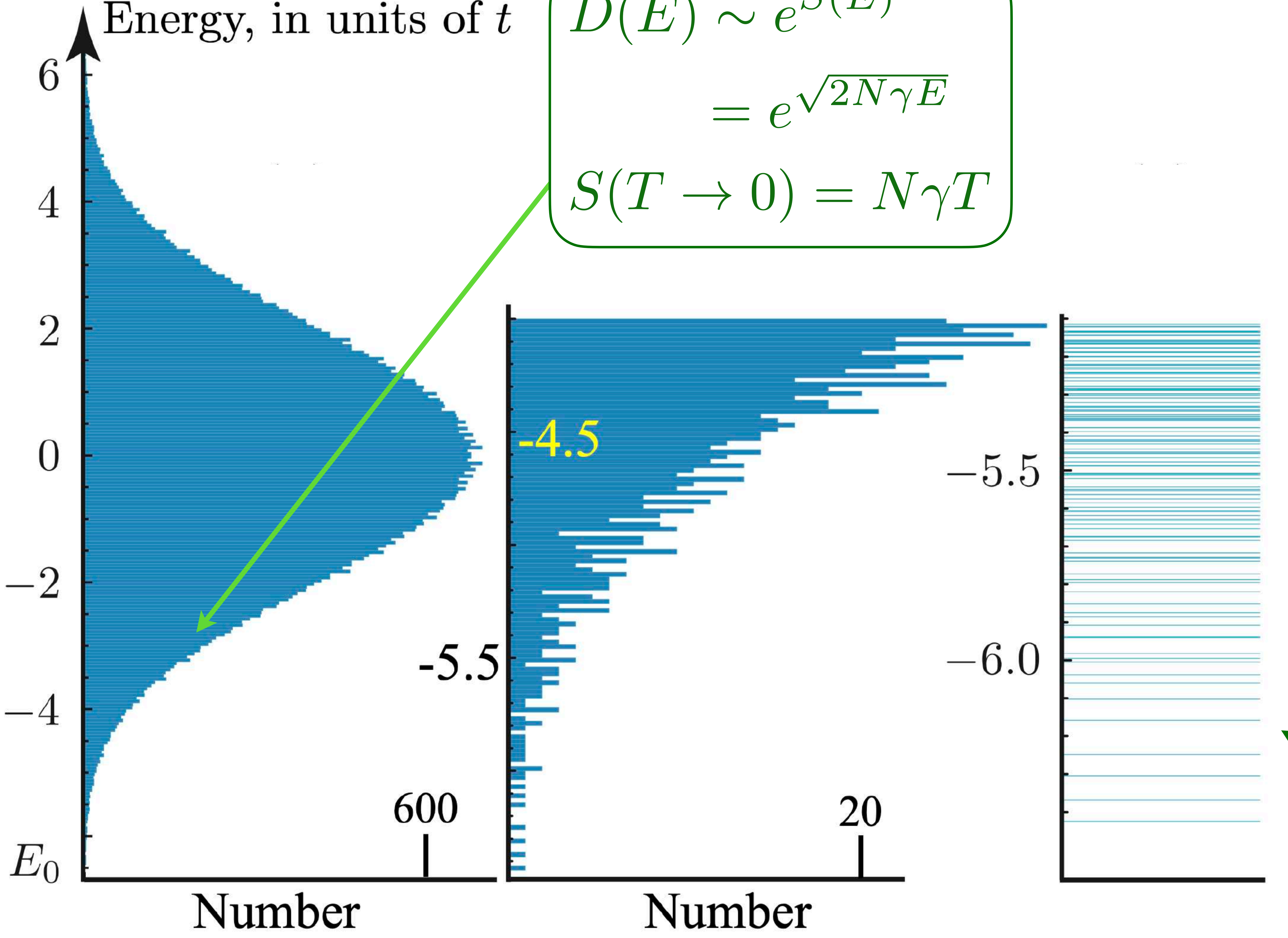


$D(E) \sim N$

Random matrix model

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

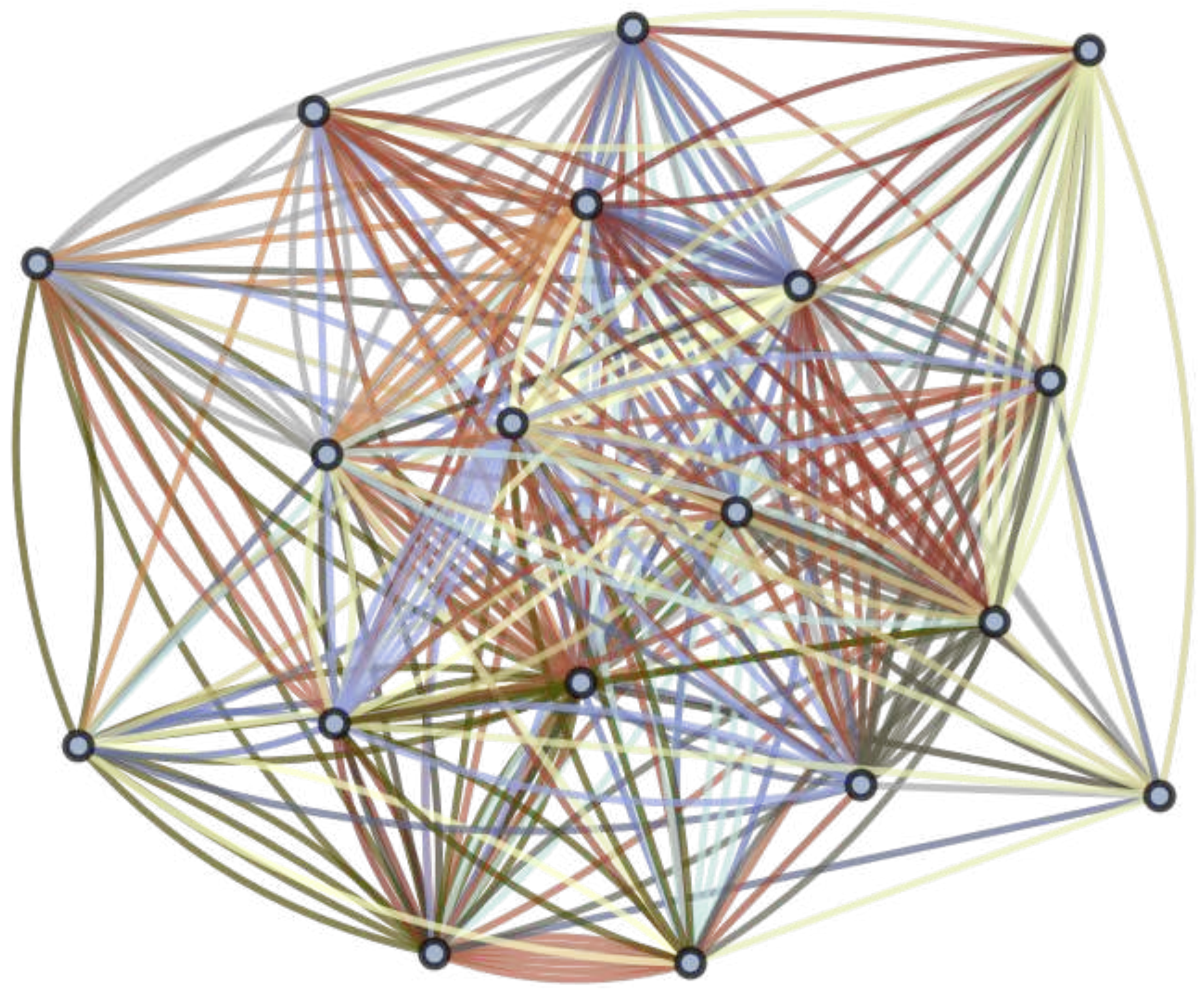
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

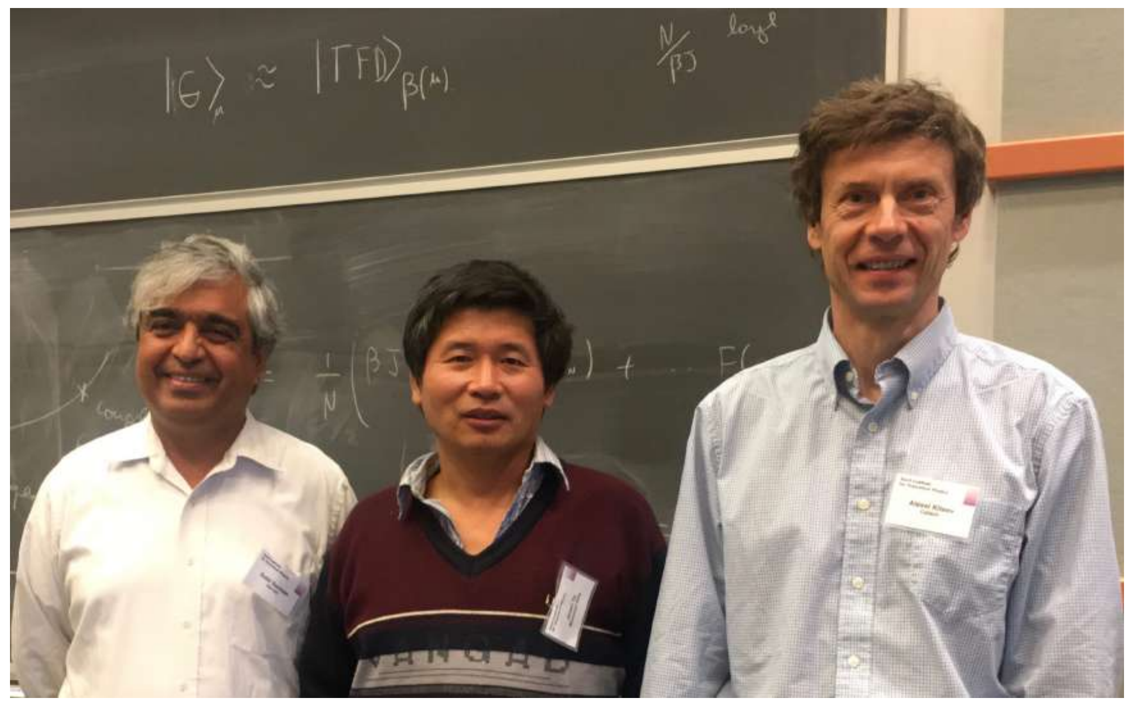
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

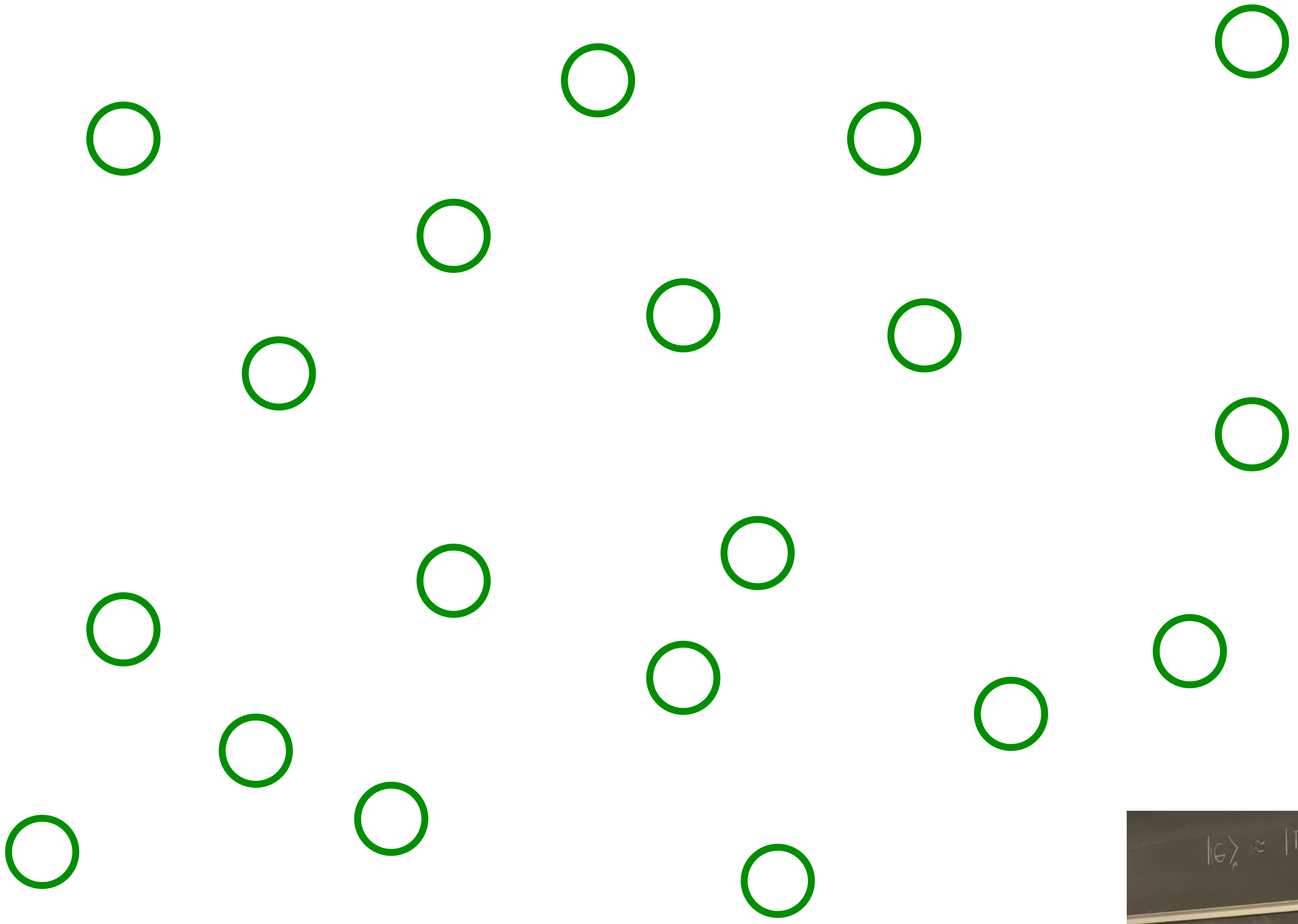
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



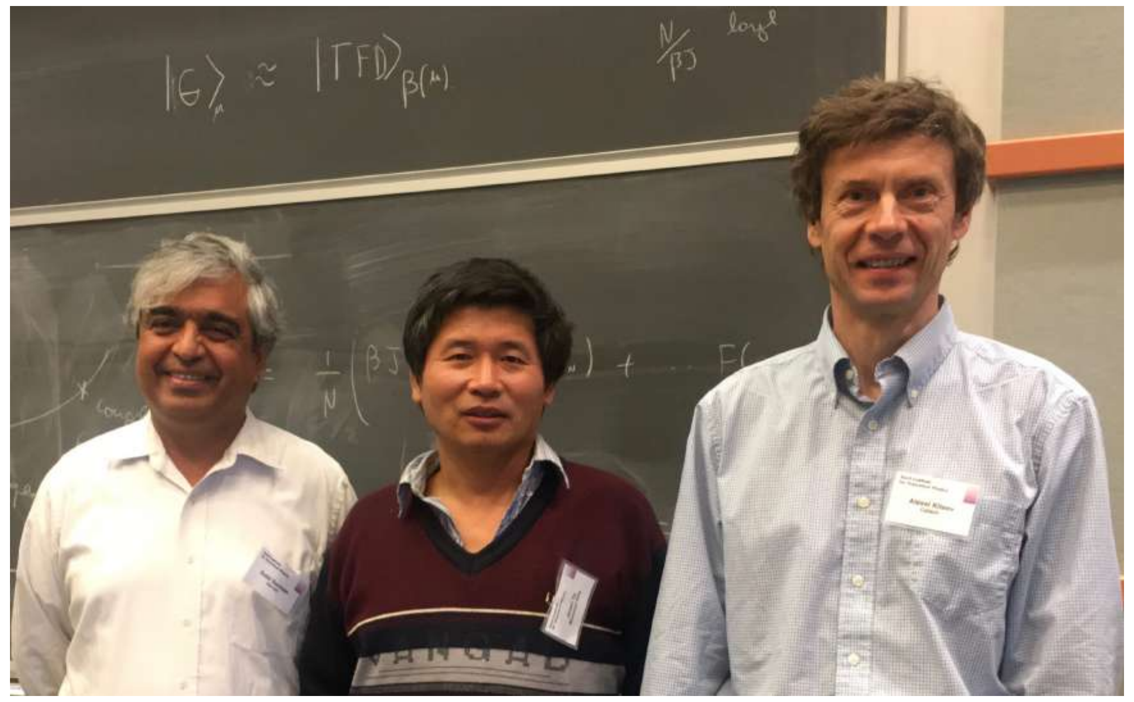


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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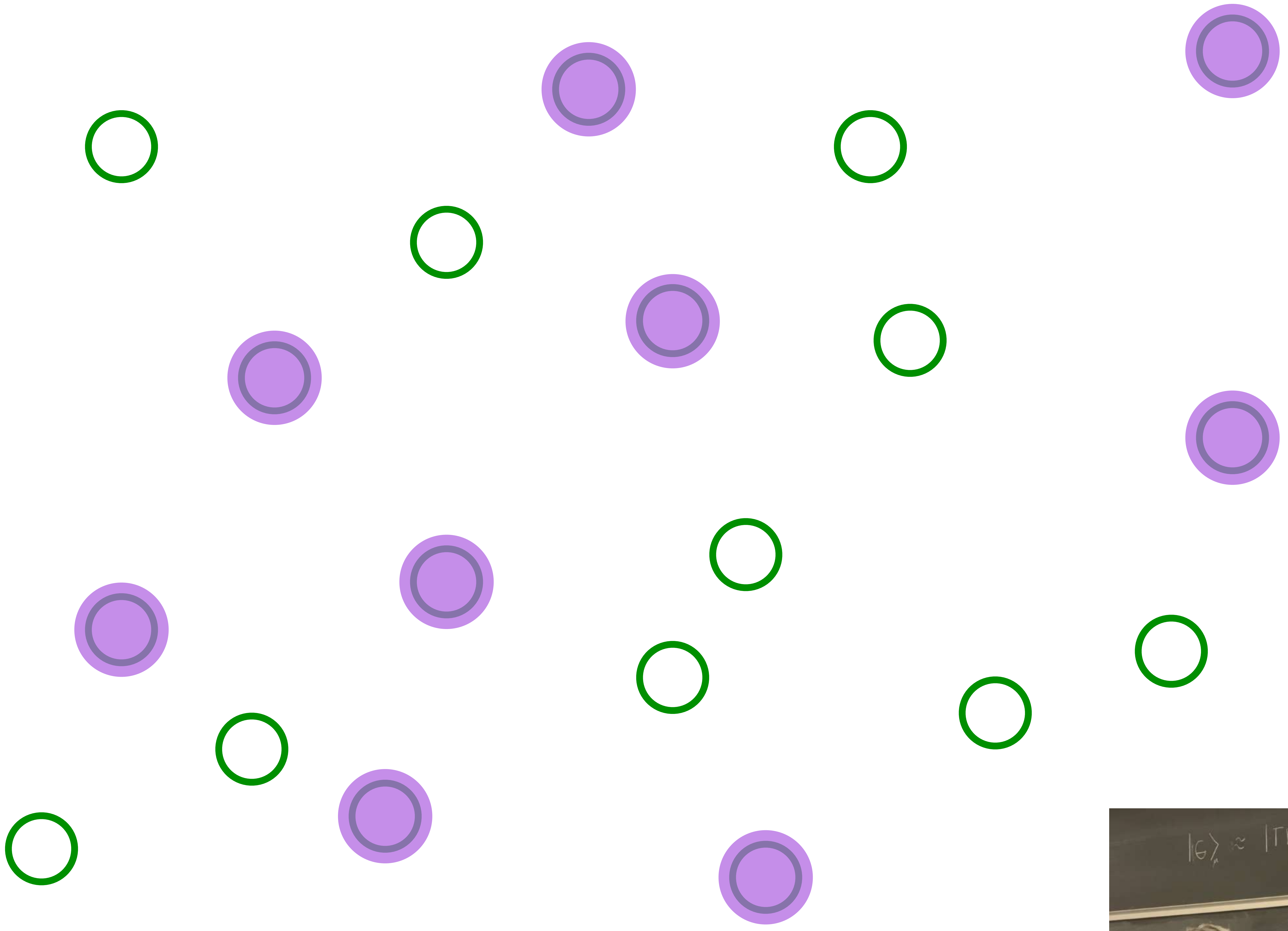
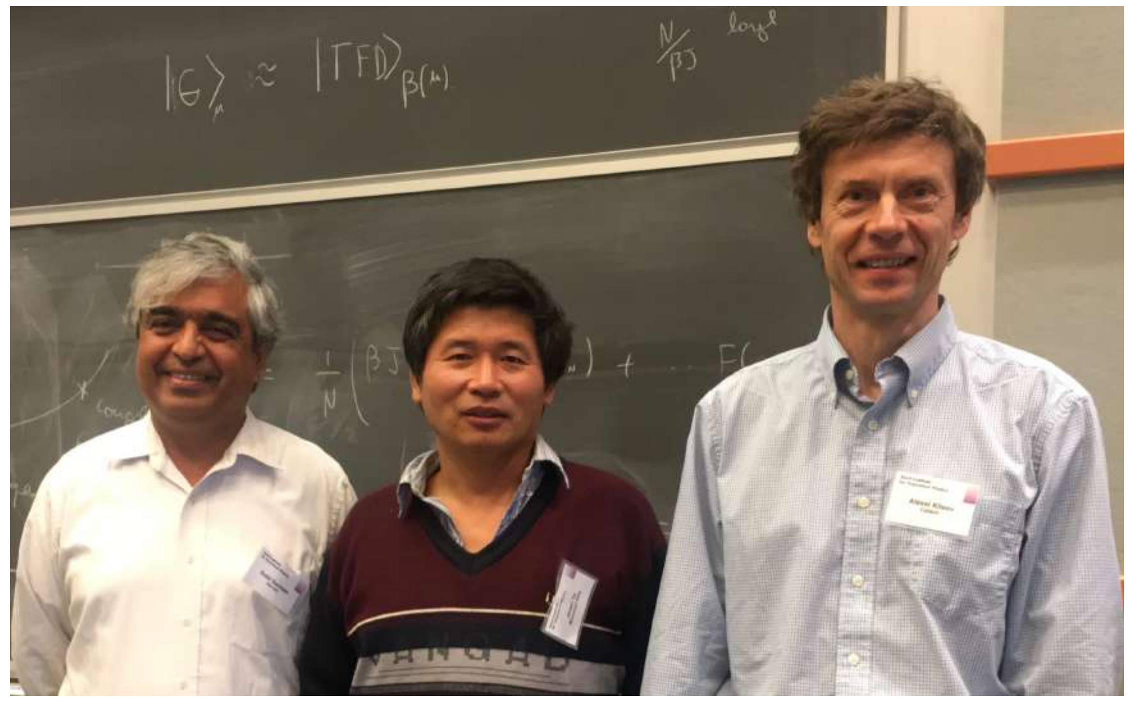


Place electrons randomly on some sites



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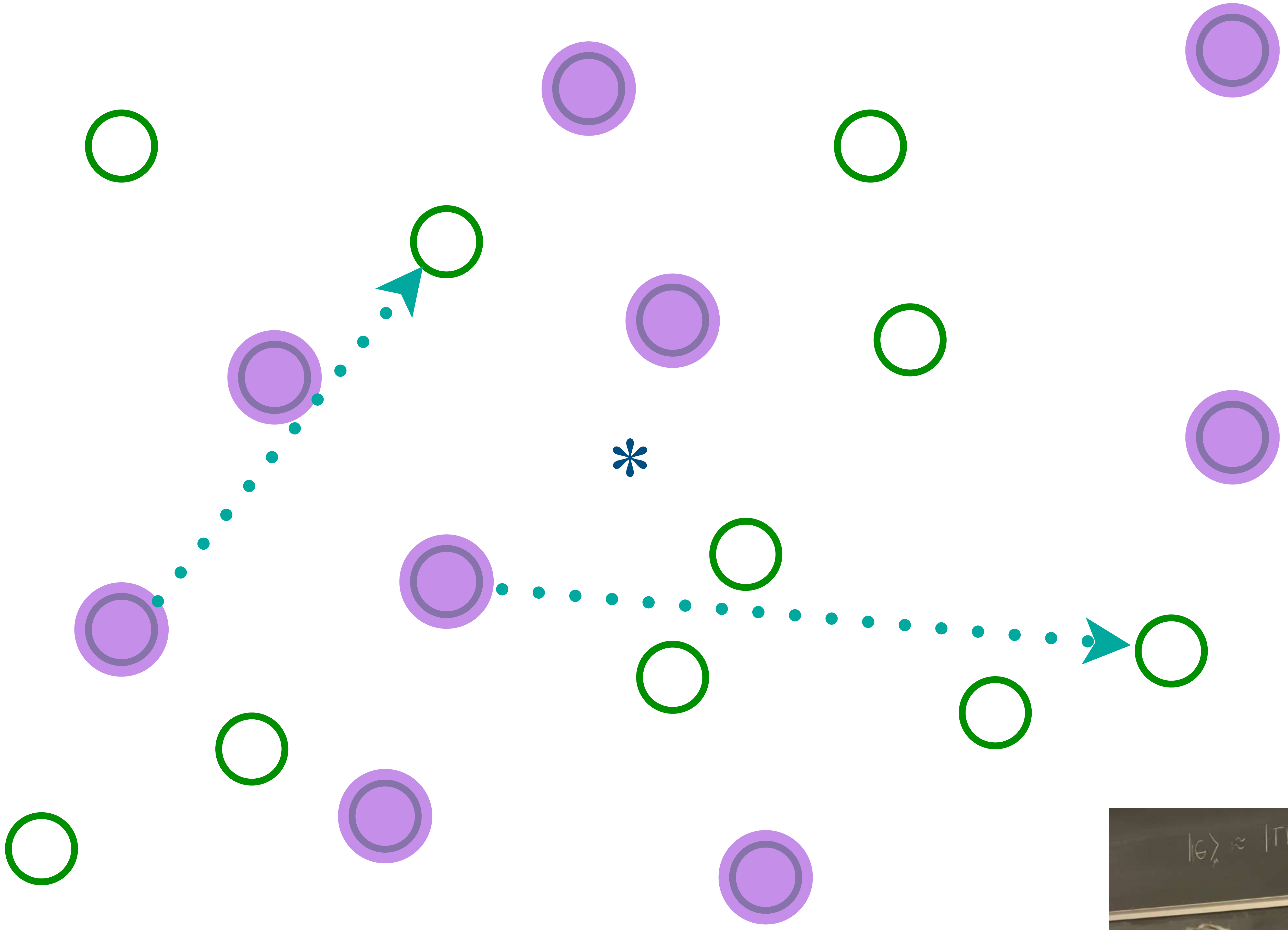

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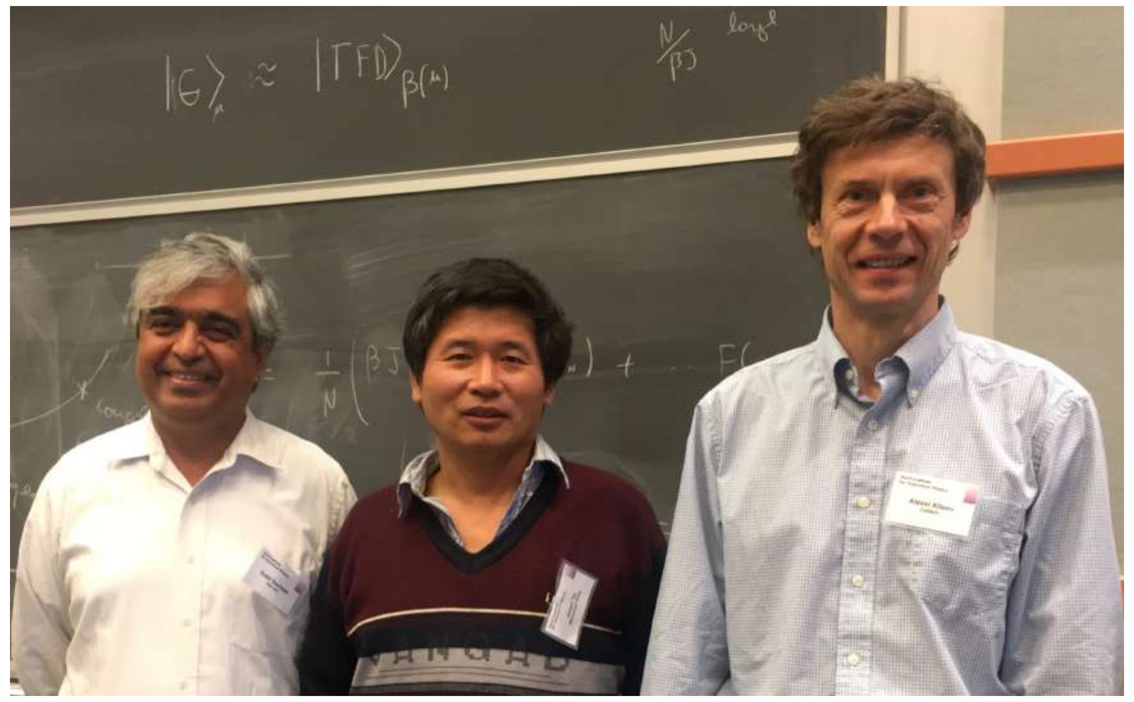

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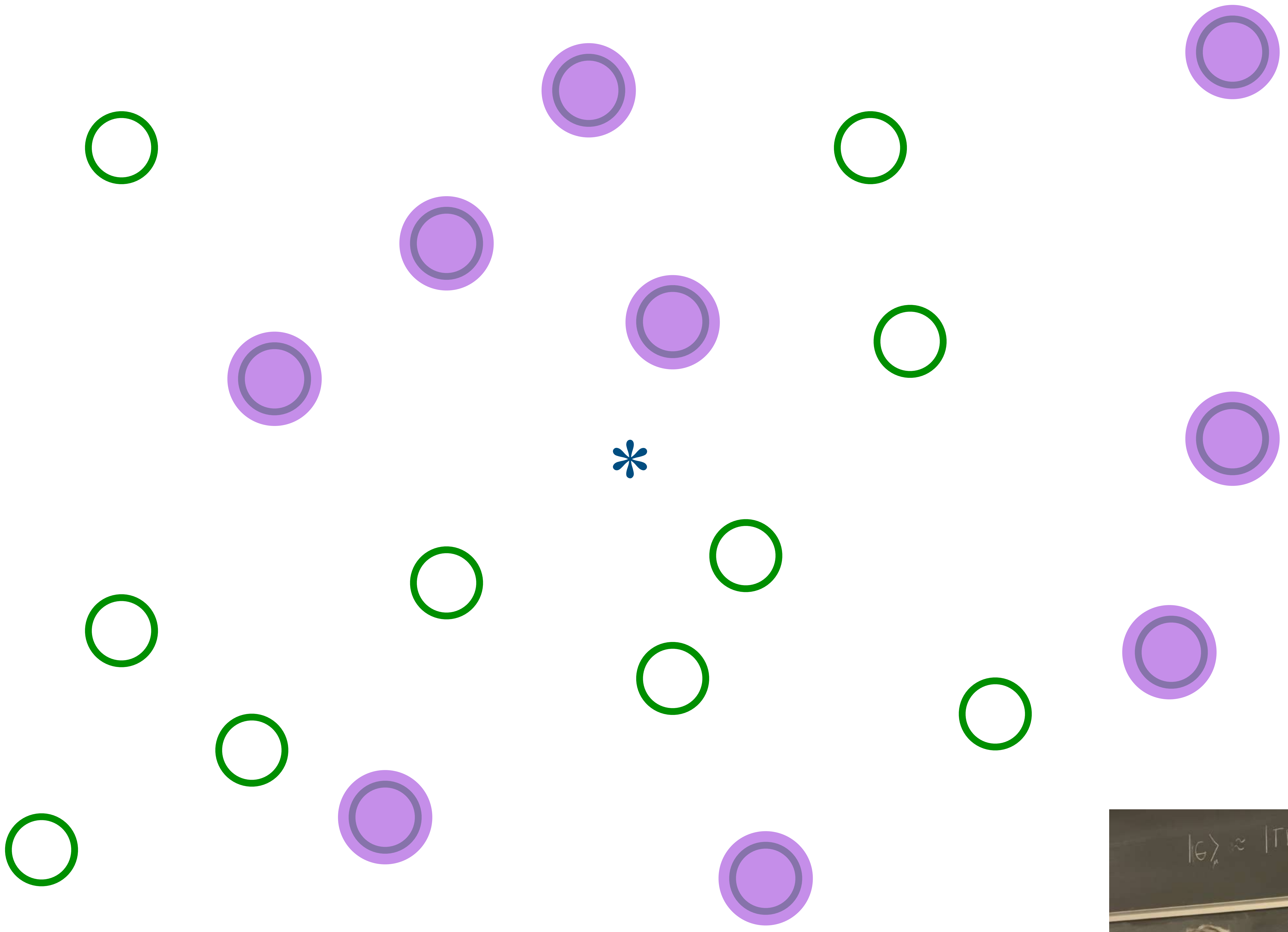
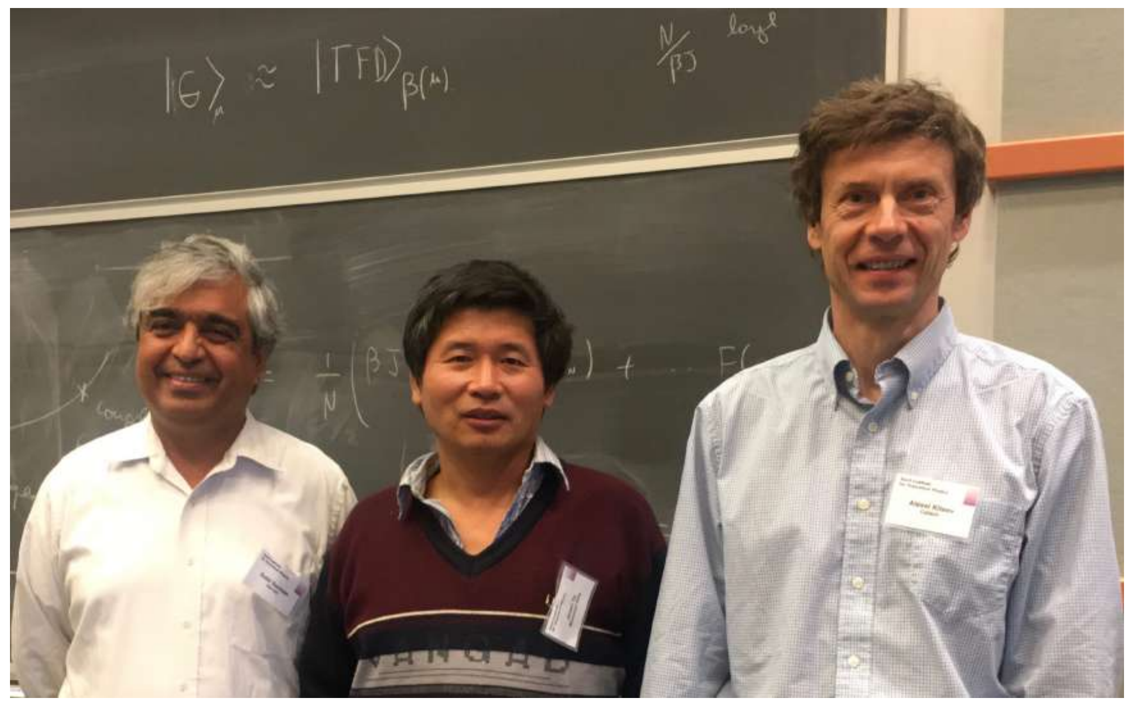


Entangle electrons pairwise randomly



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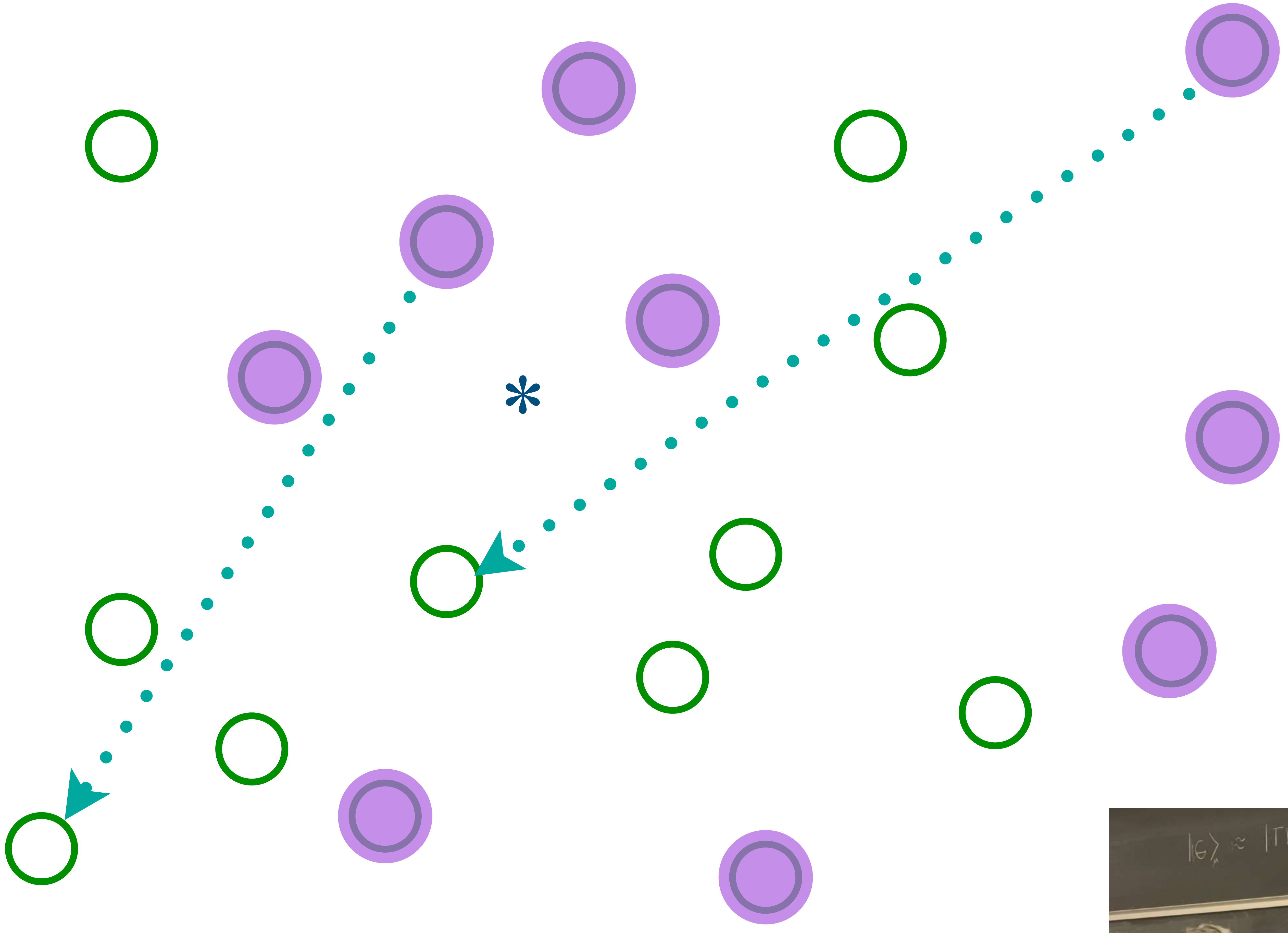

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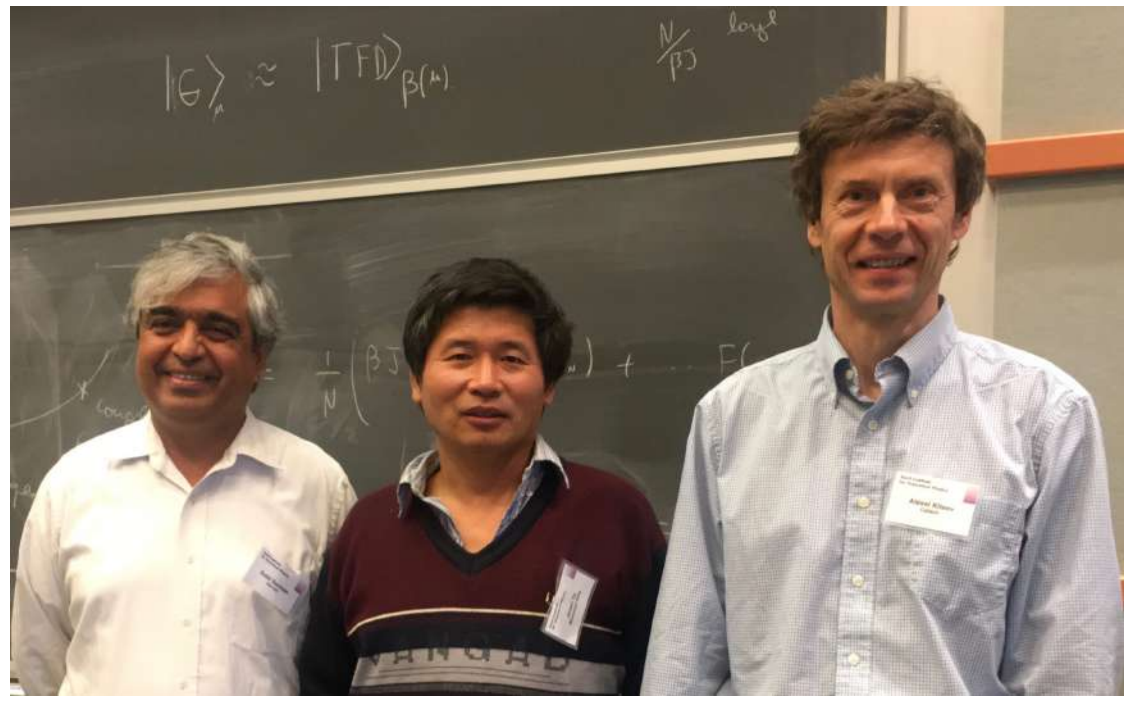

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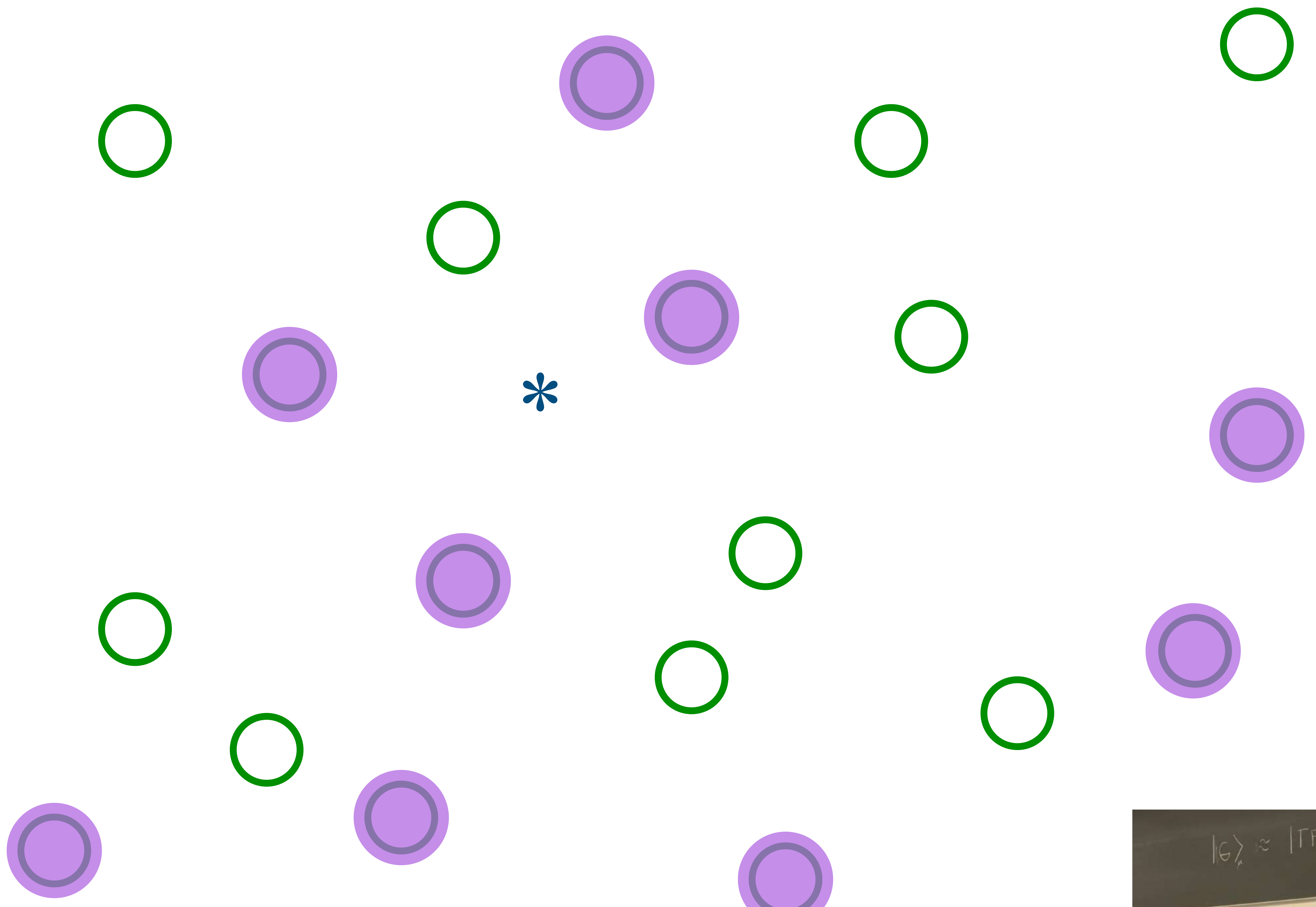

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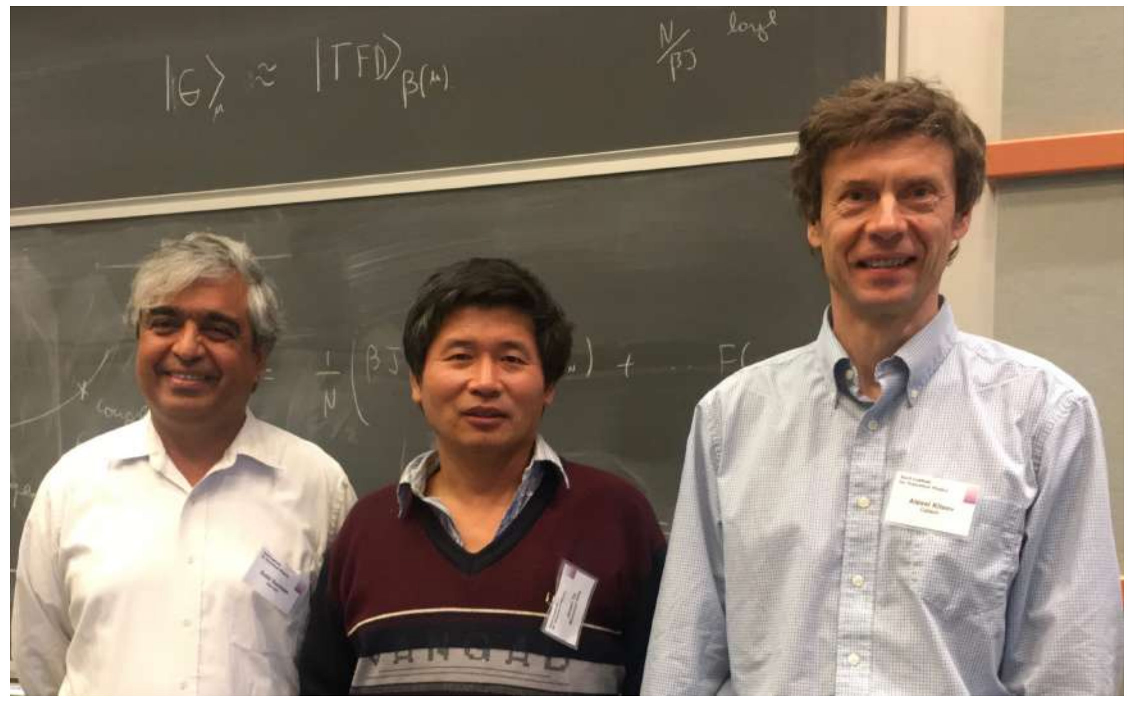

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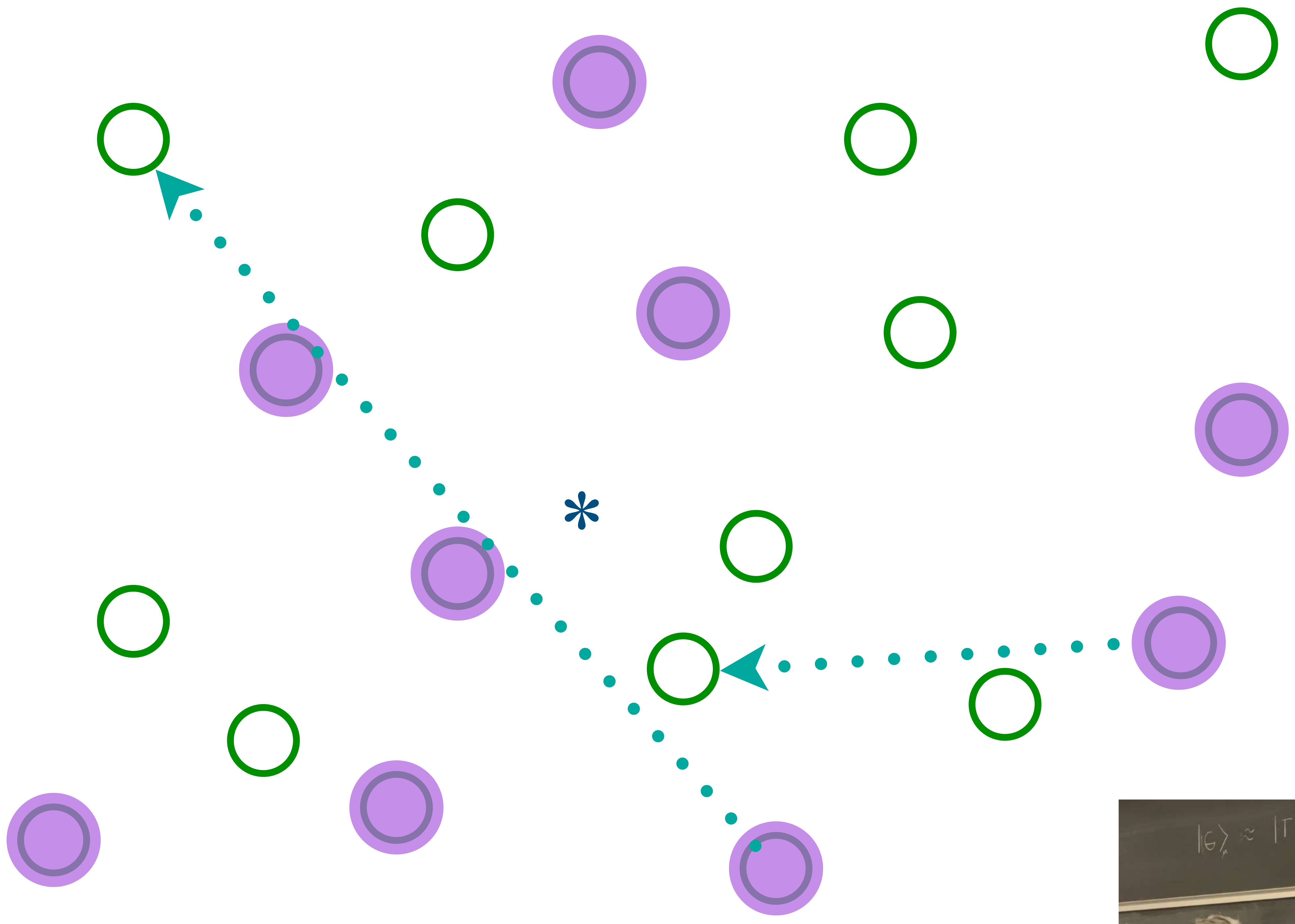

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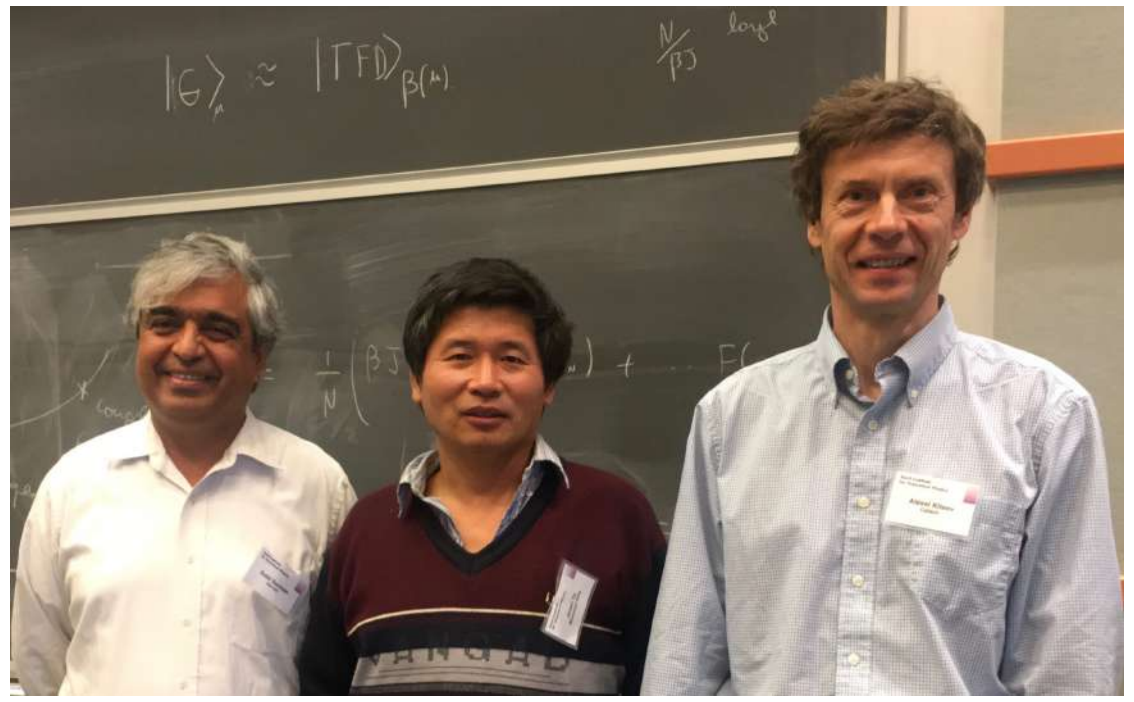

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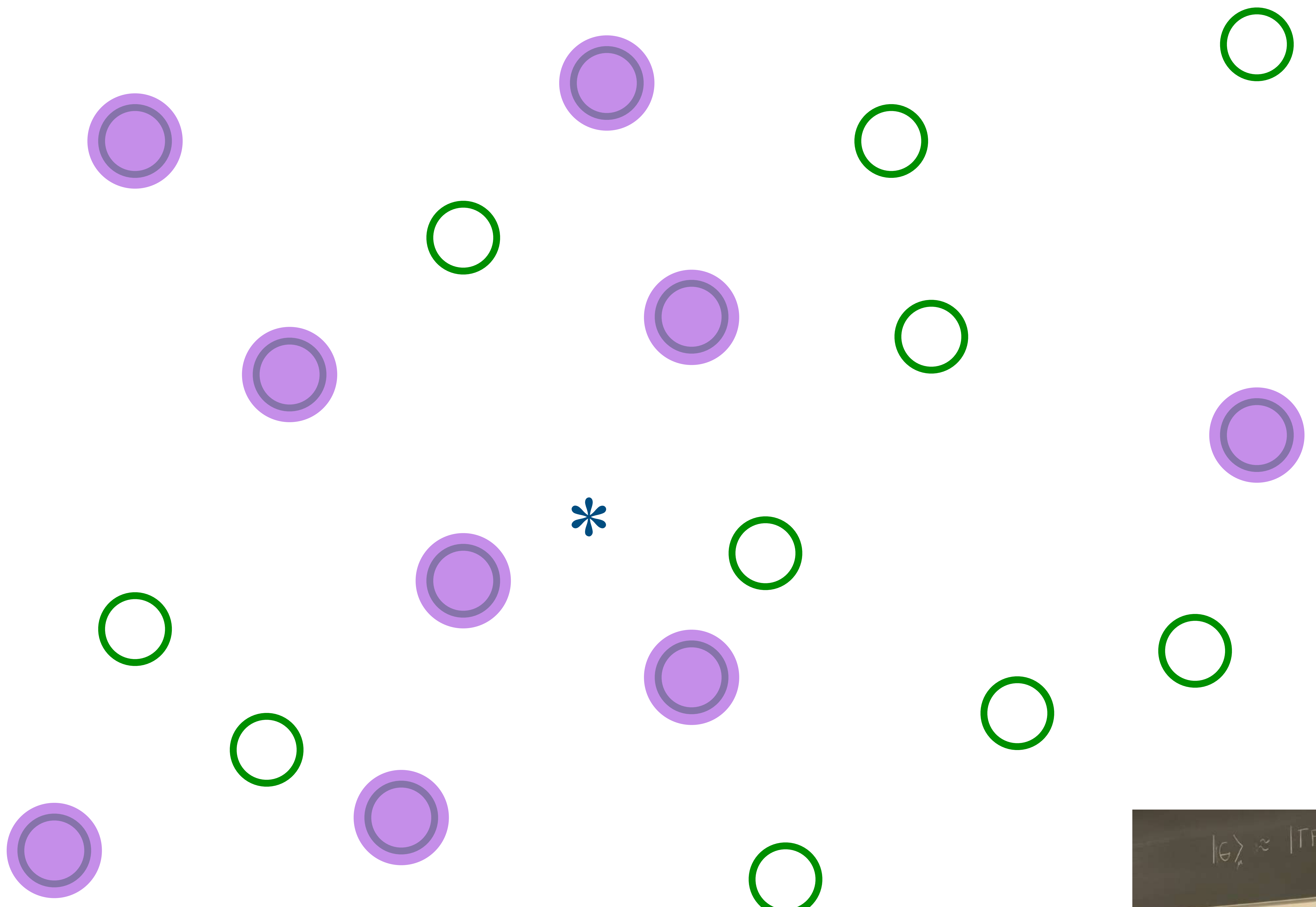

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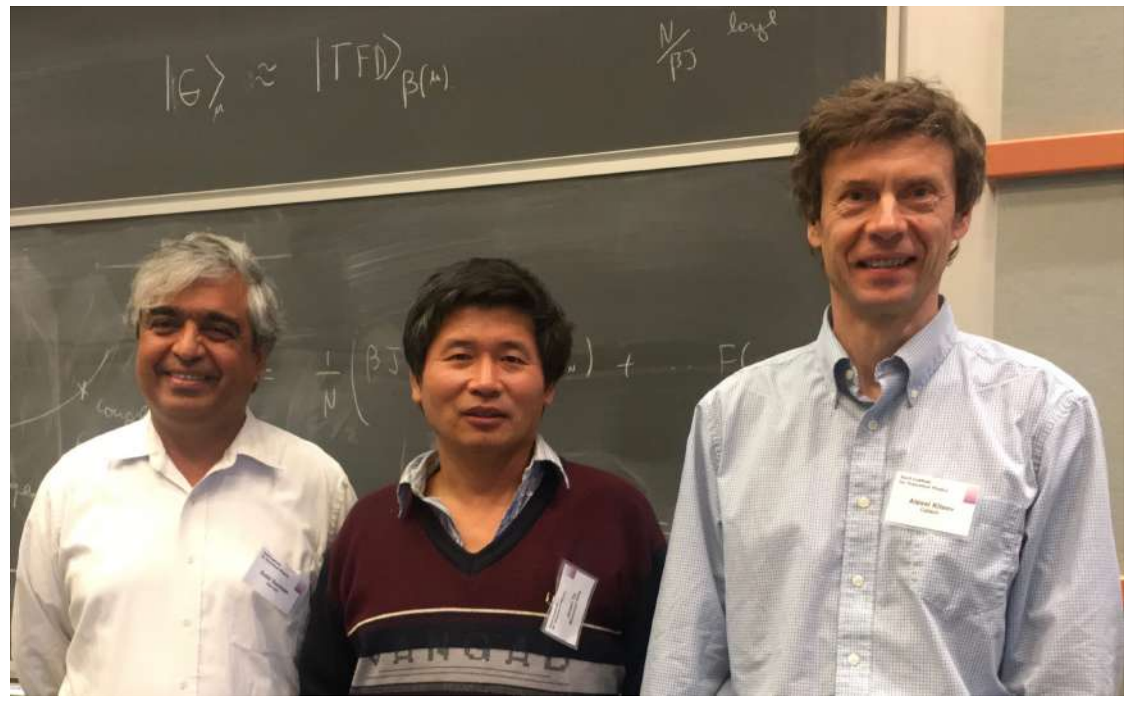
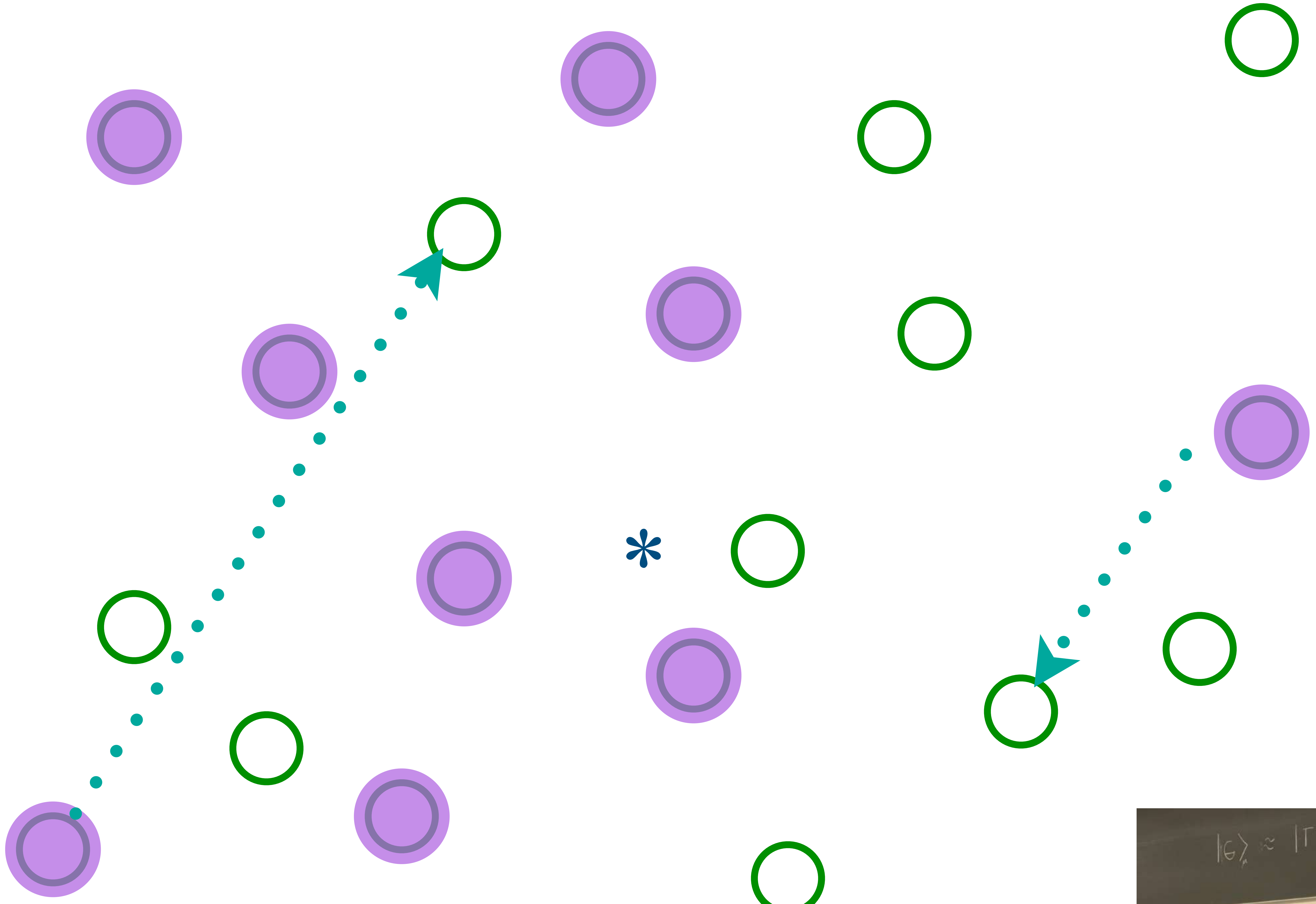


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



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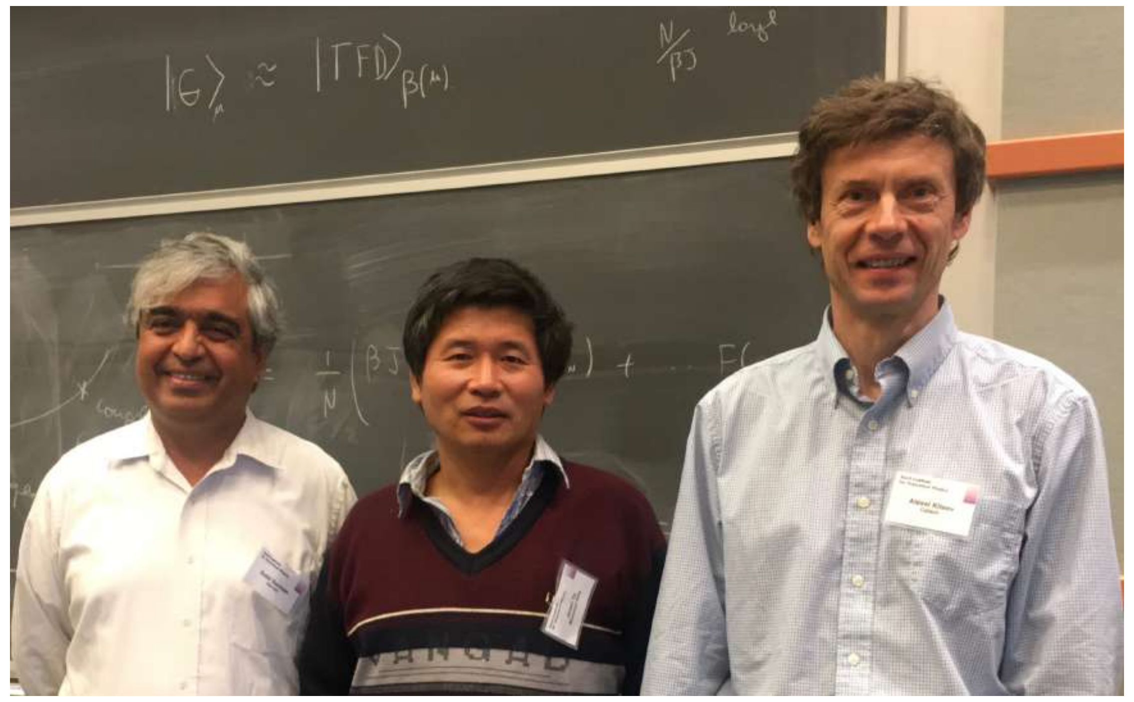

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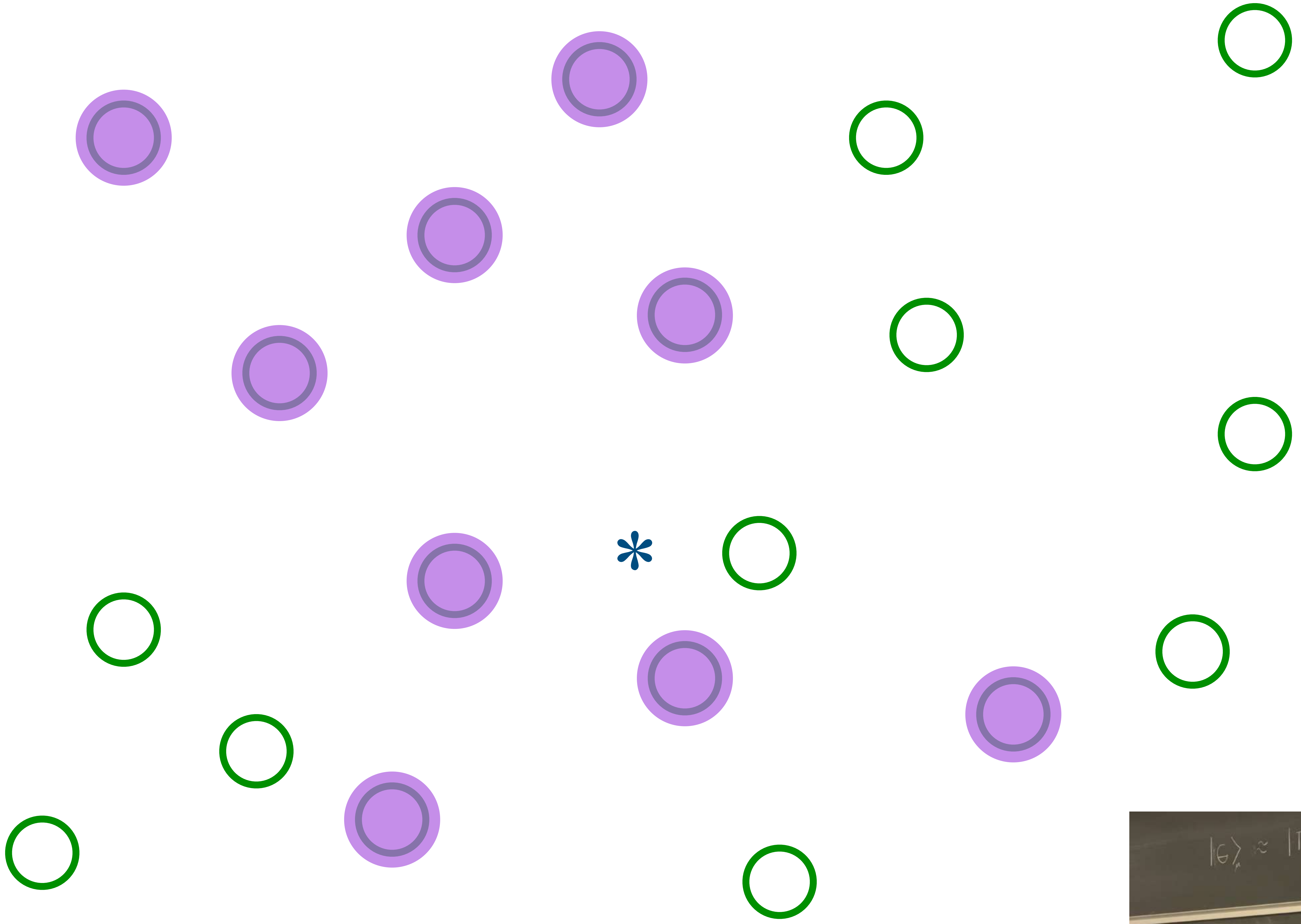

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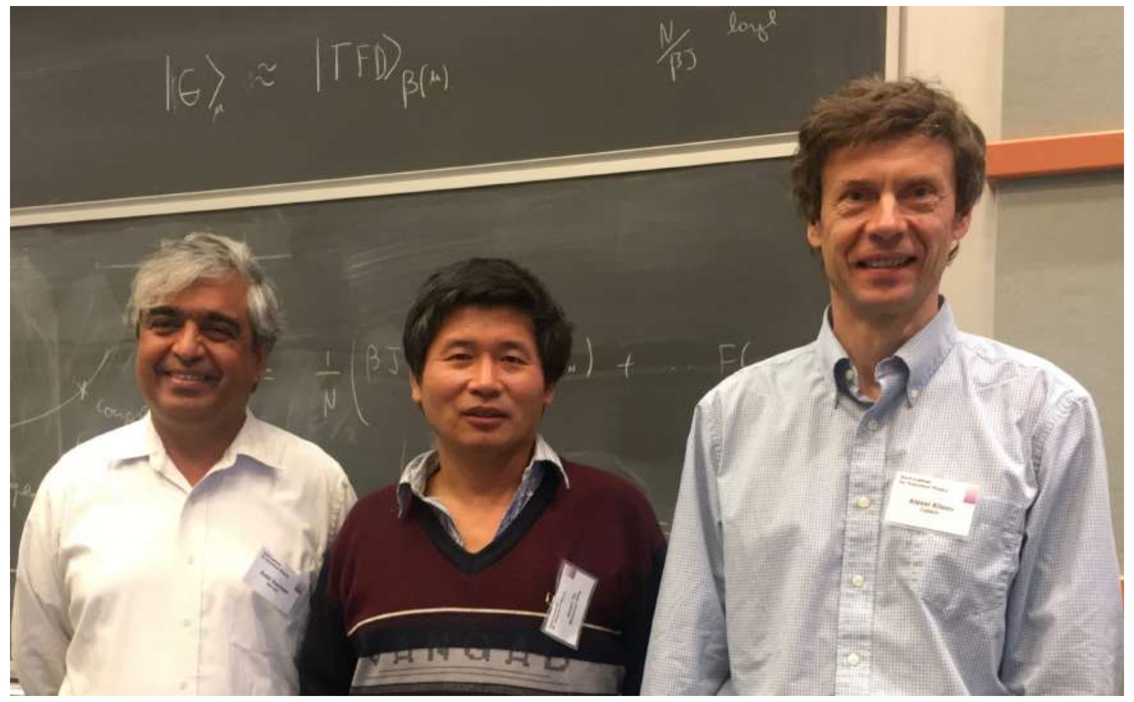

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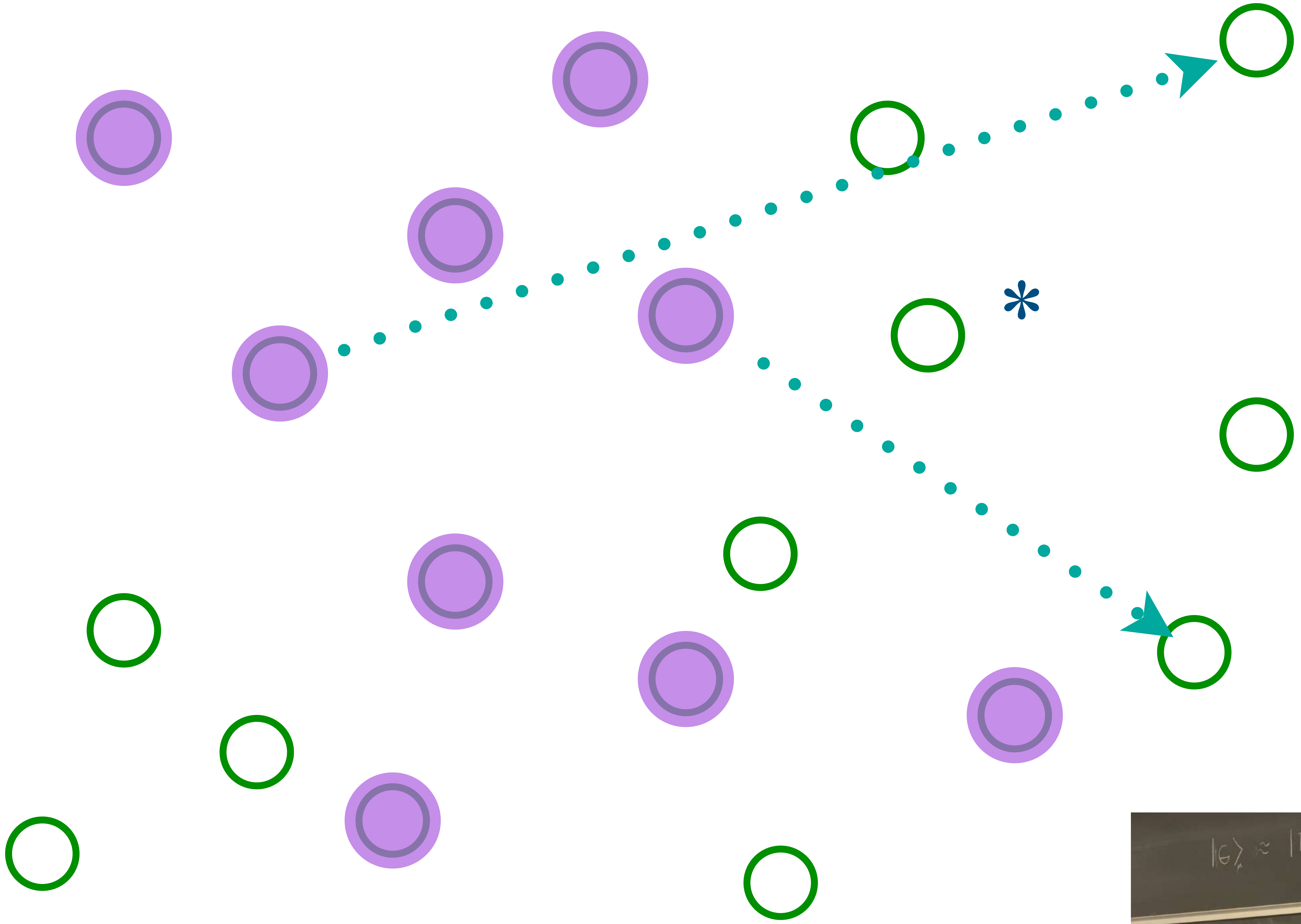

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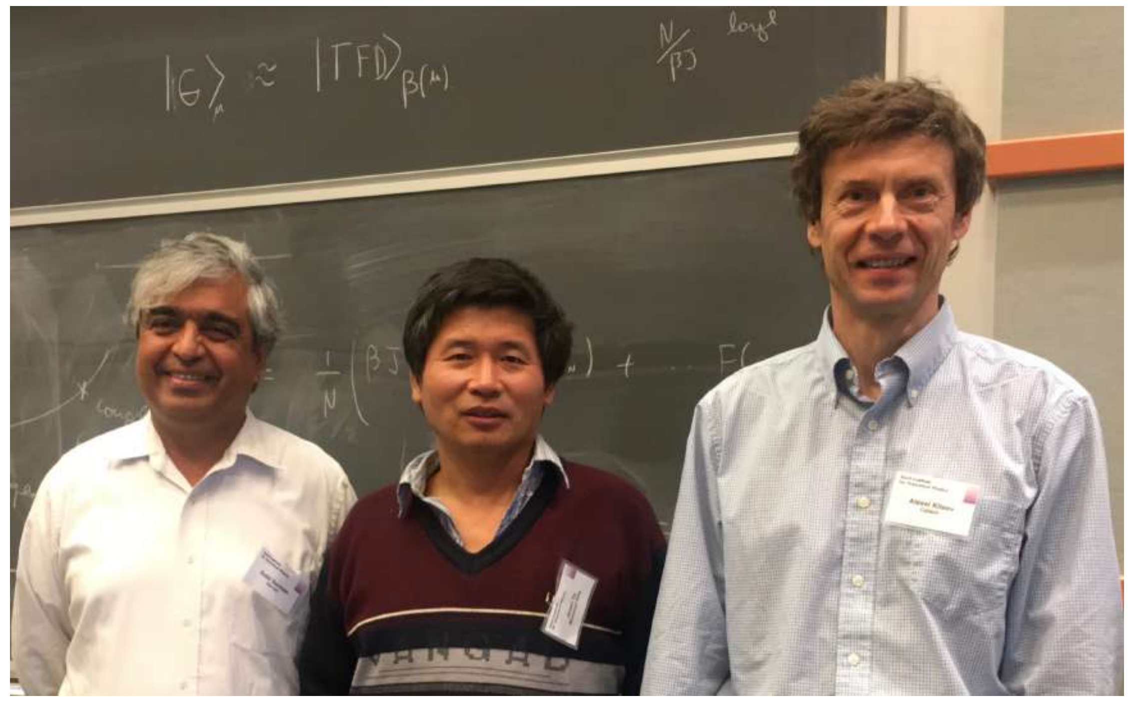

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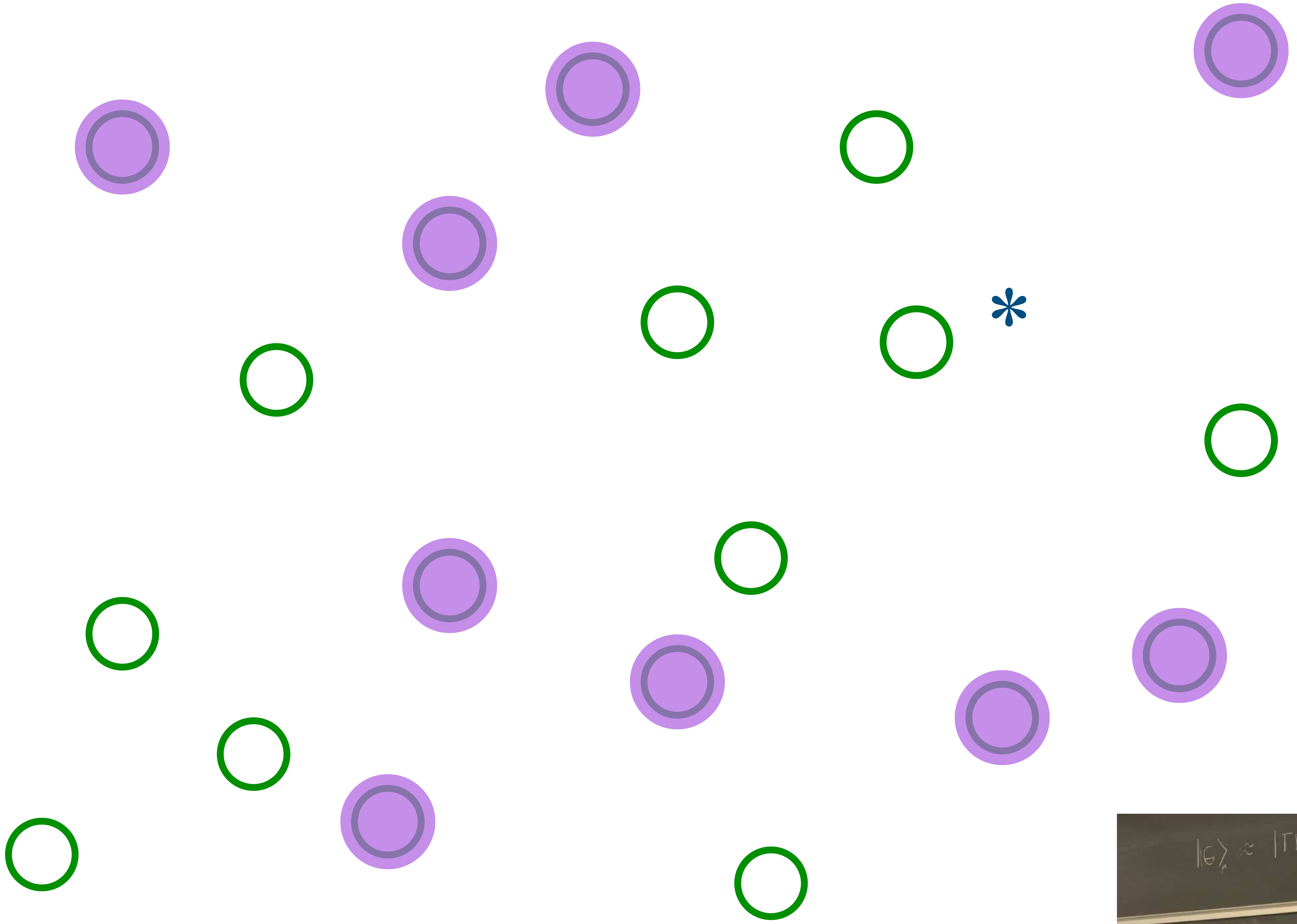

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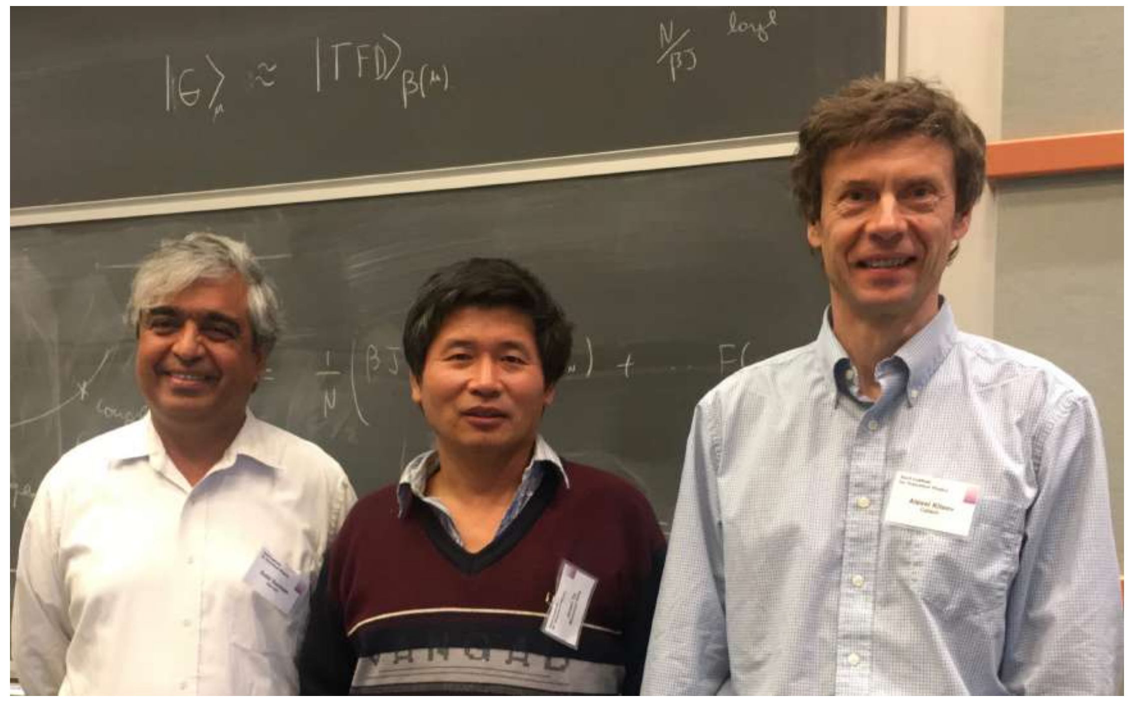

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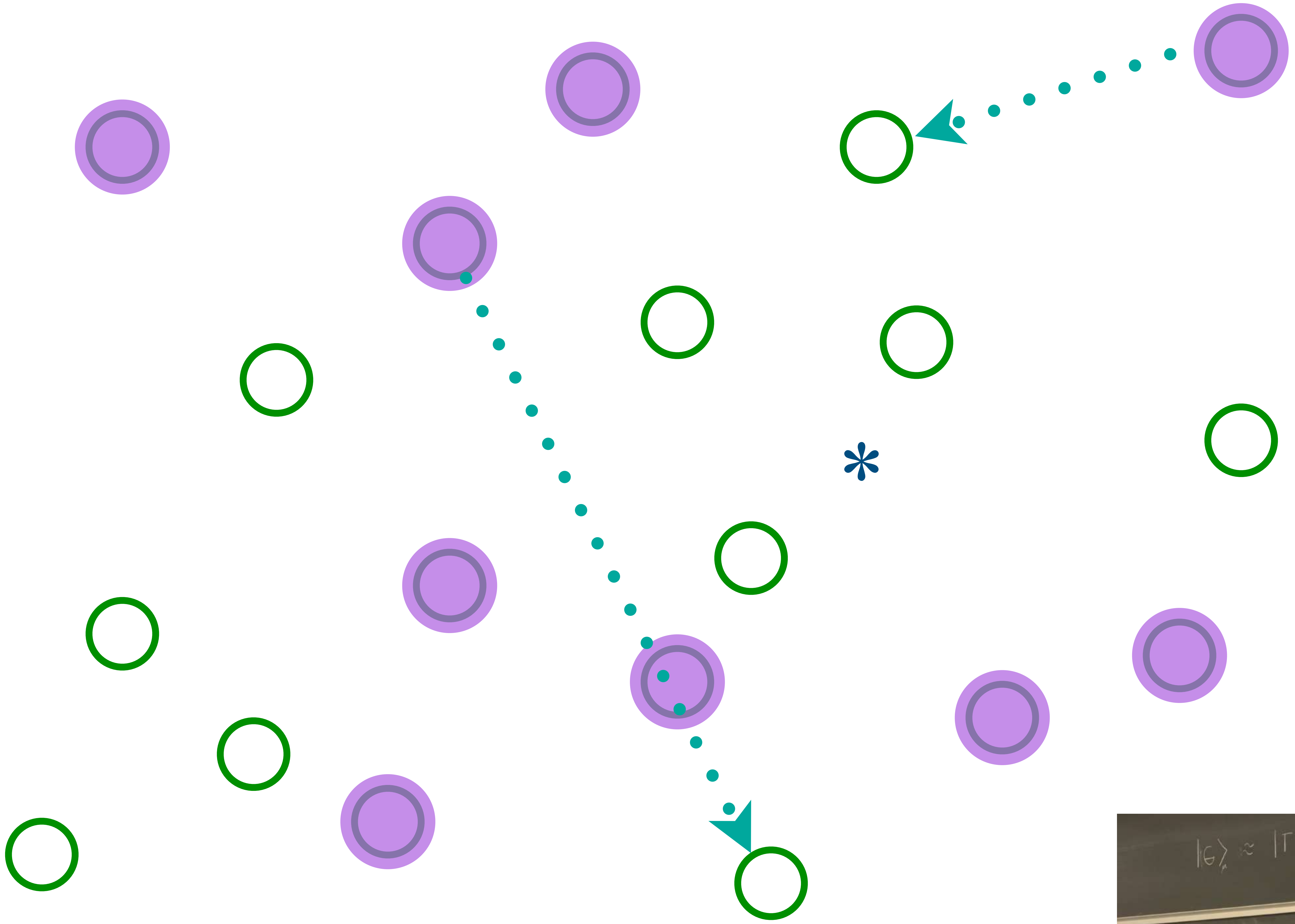

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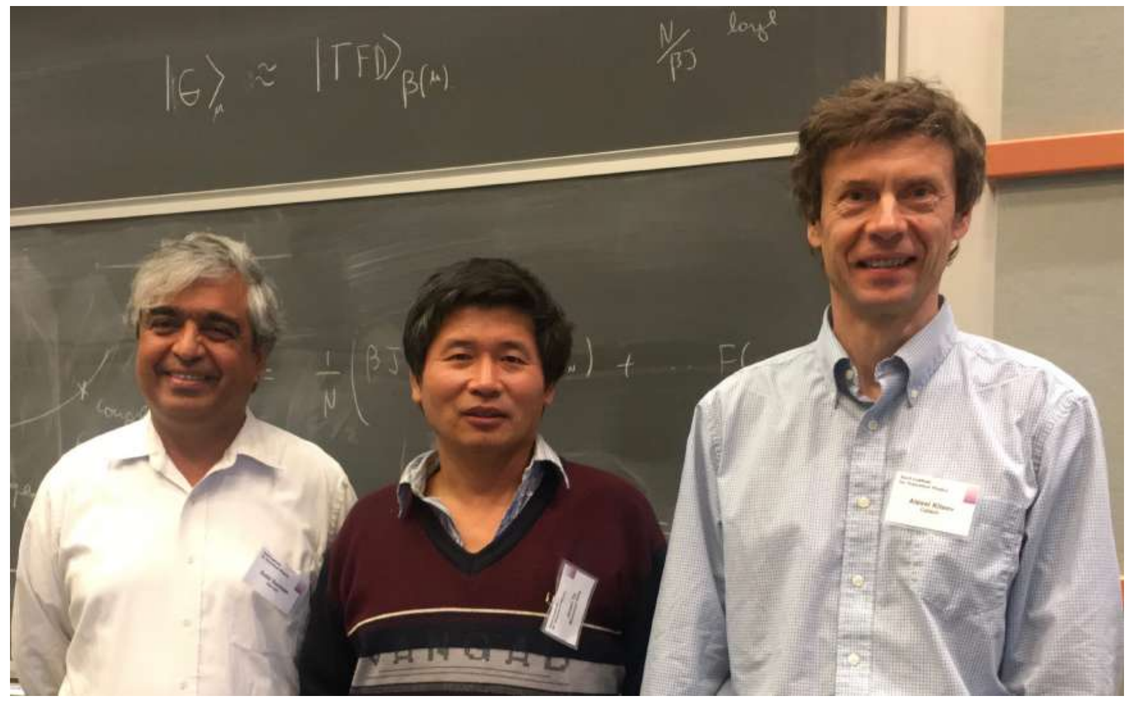

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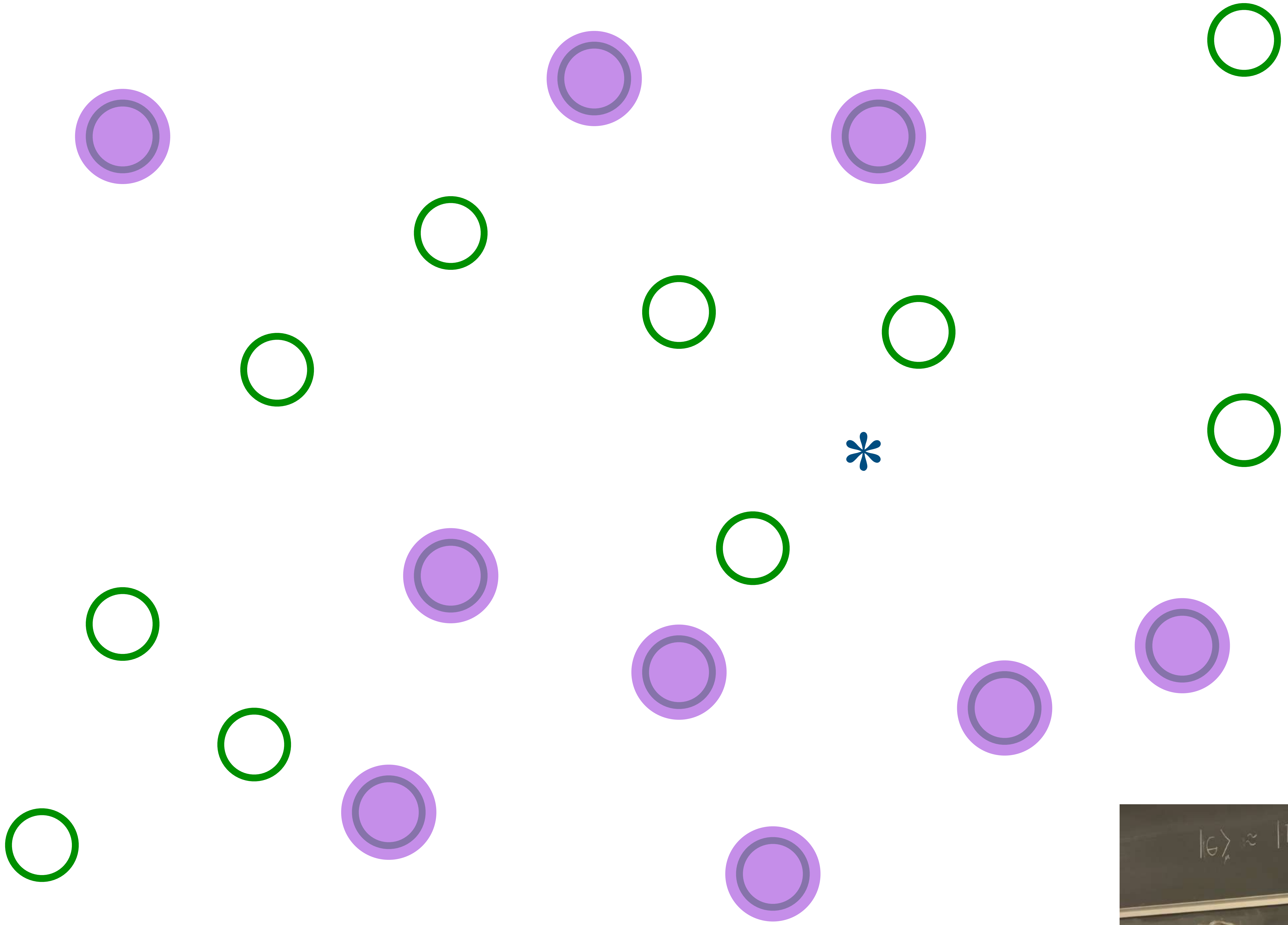

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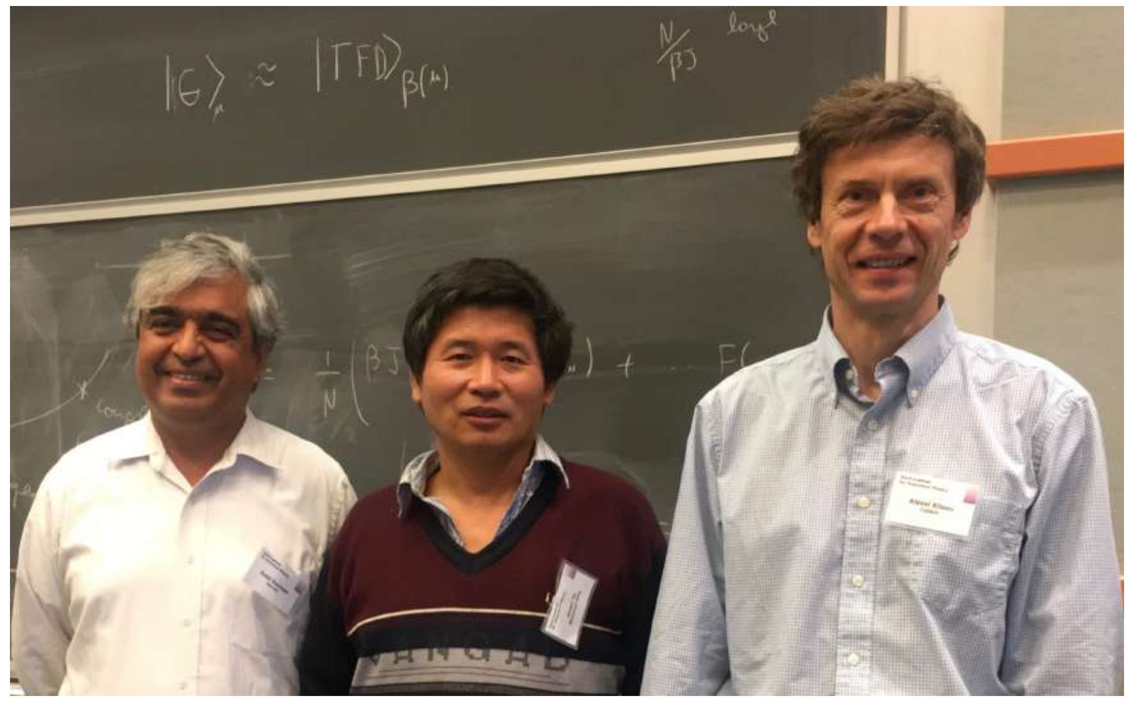
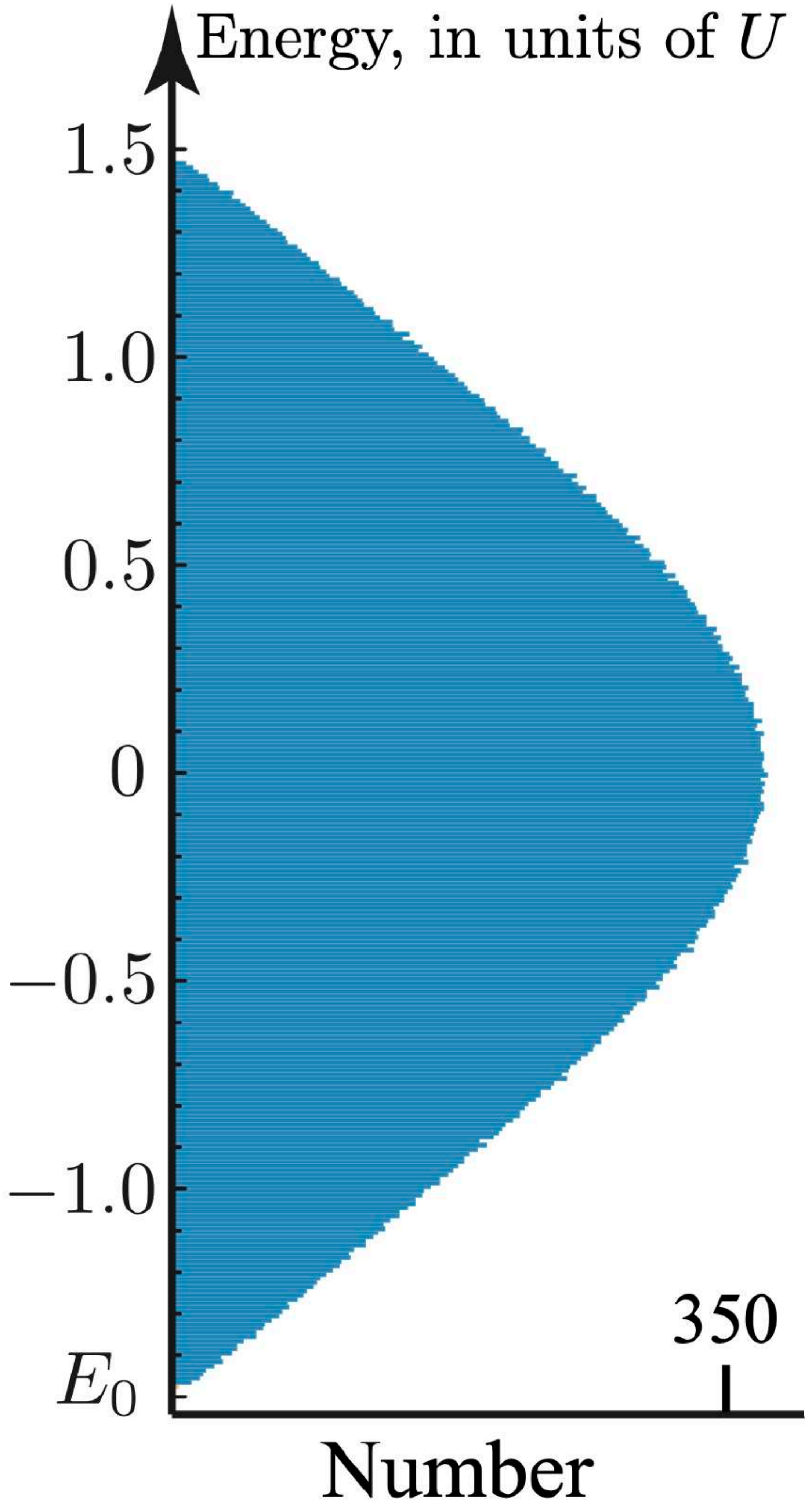


Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

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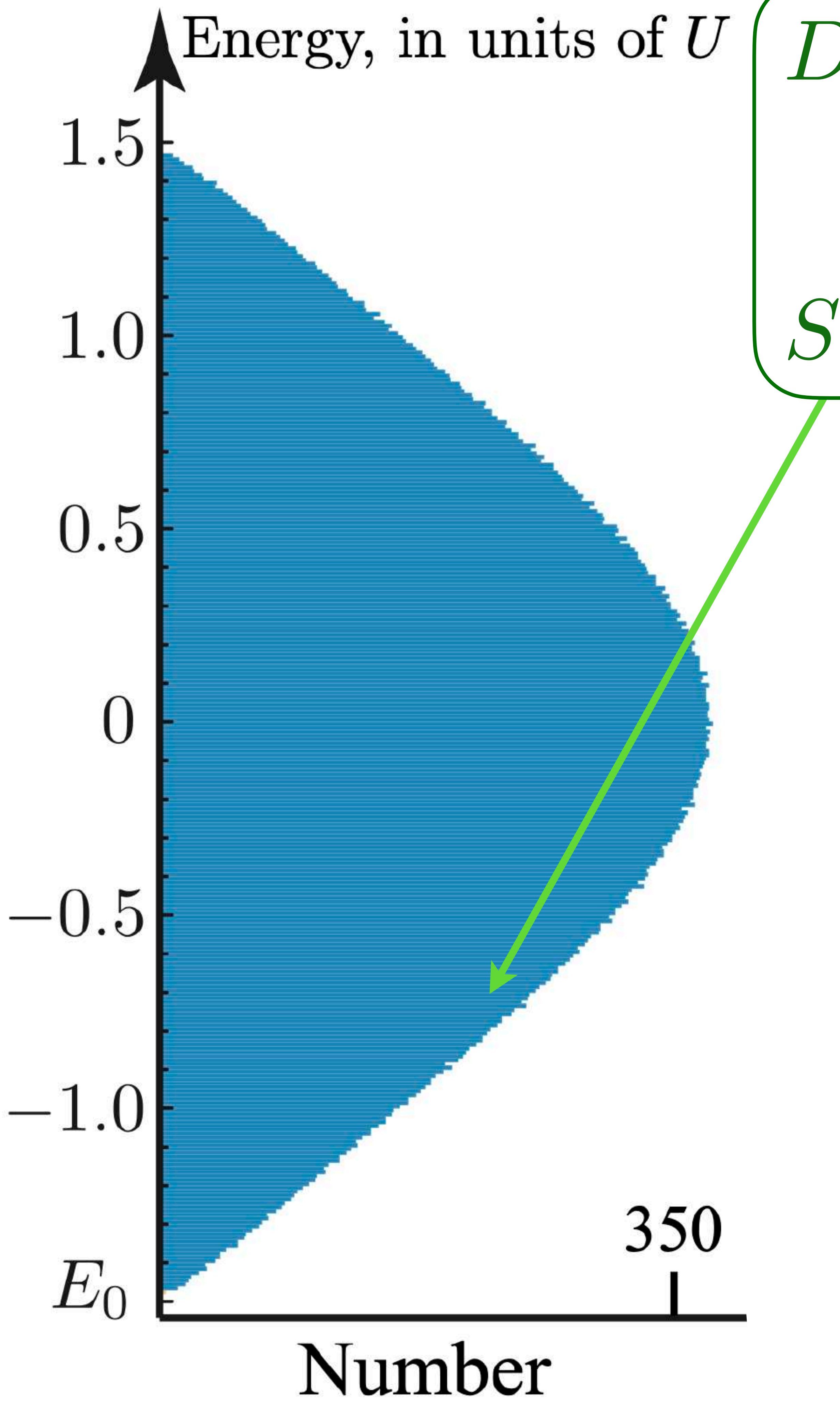

$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2 N \gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$s_0 = 0.464848\ldots$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Complex SYK model

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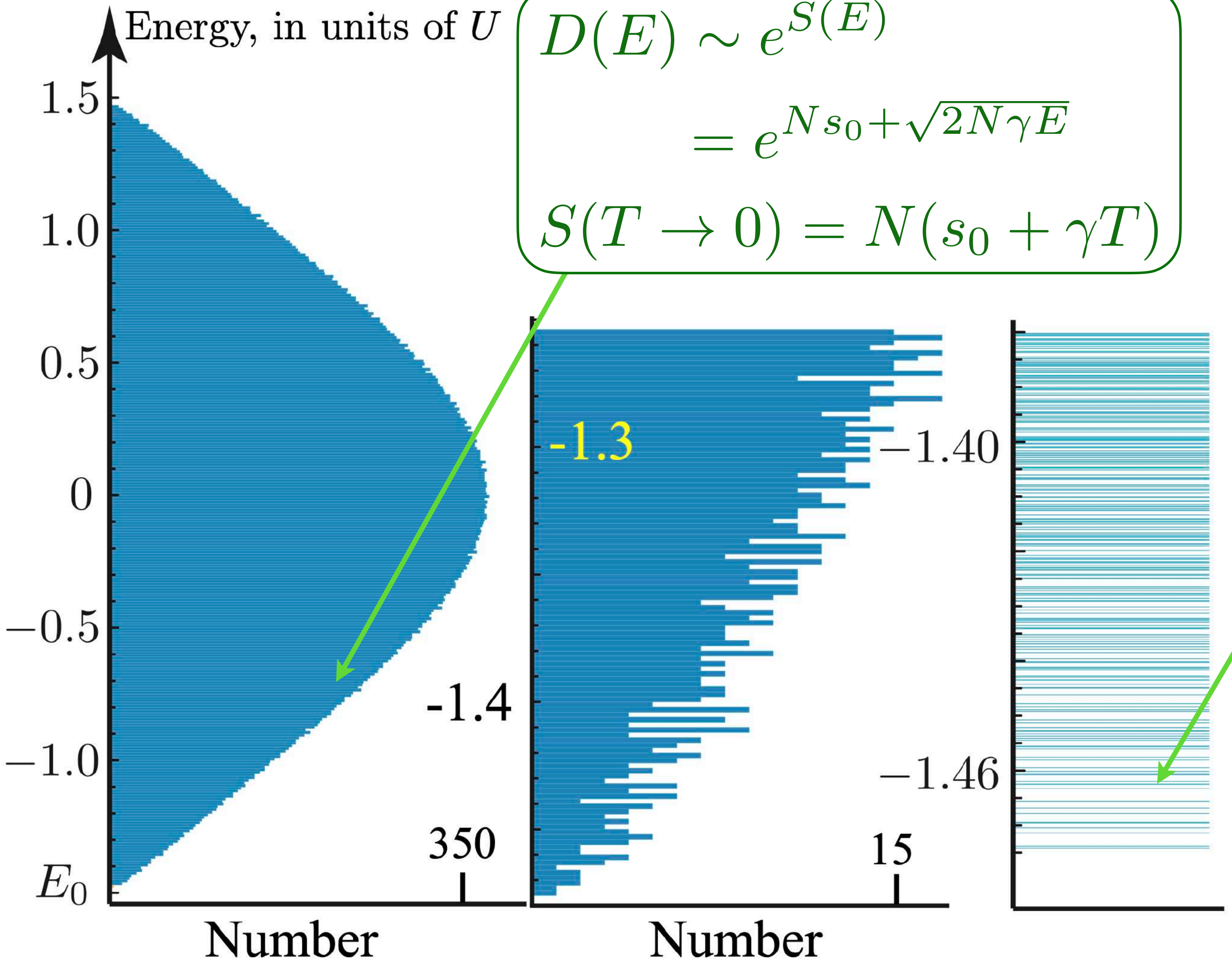

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No quasiparticle decomposition of many-body states

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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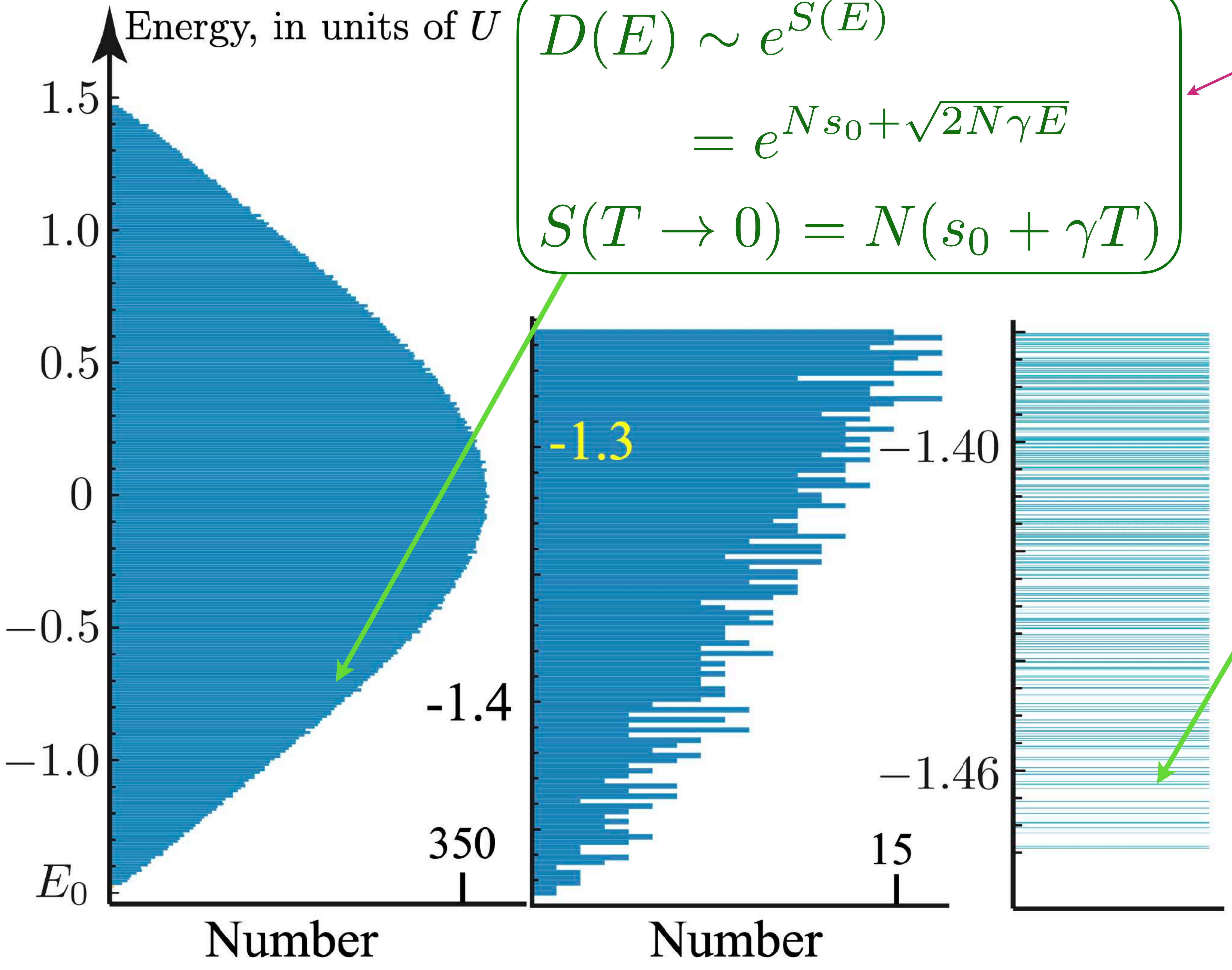

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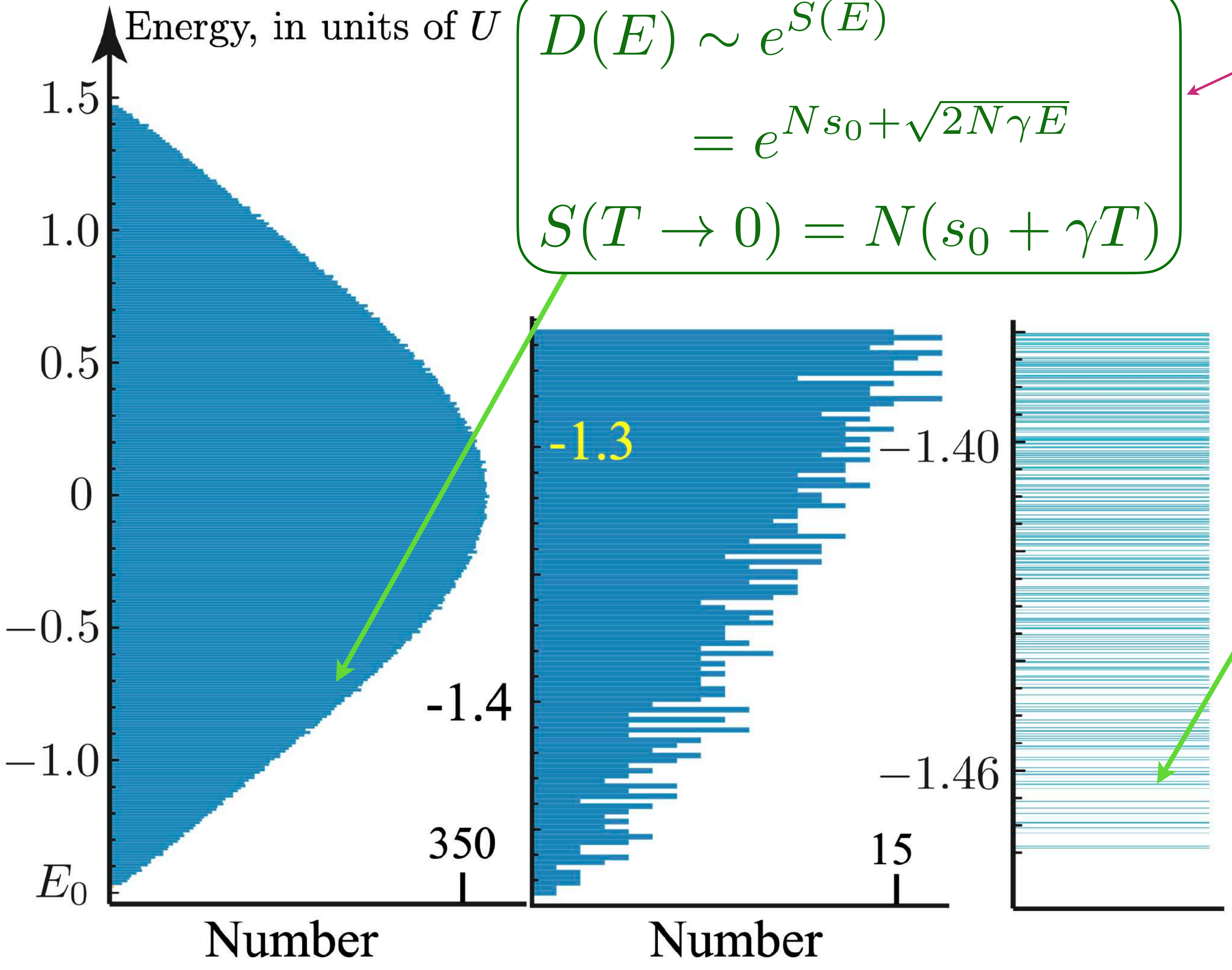

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$$D(E) \sim 2\, e^{Ns_0} \sinh(\sqrt{2N\gamma E})$$

$$e^{-F(T)/T} = \int_0^\infty dE D(E) e^{-E/T}$$
$$S(T) = -\partial F/\partial T$$

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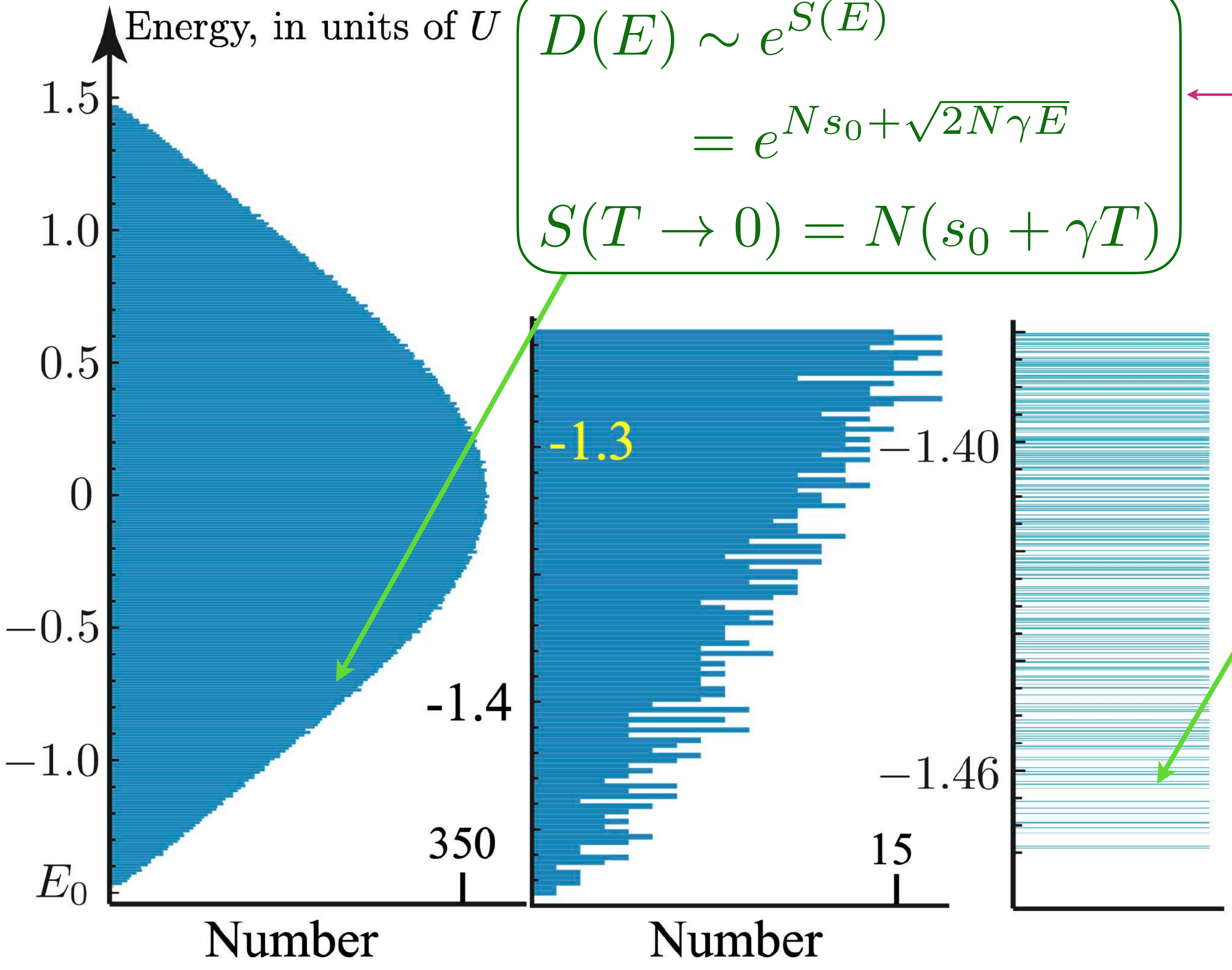

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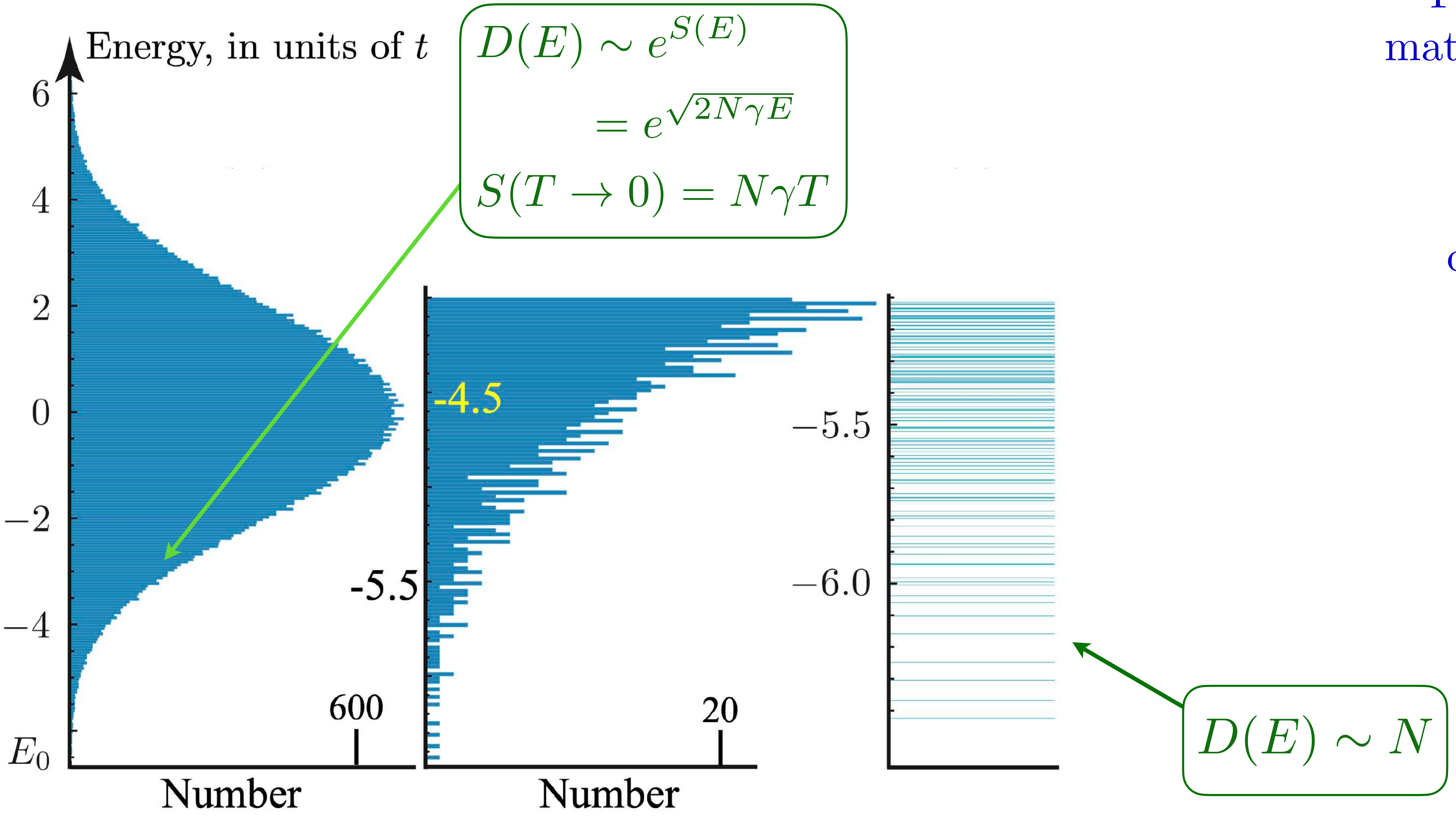
For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number



Random matrix model

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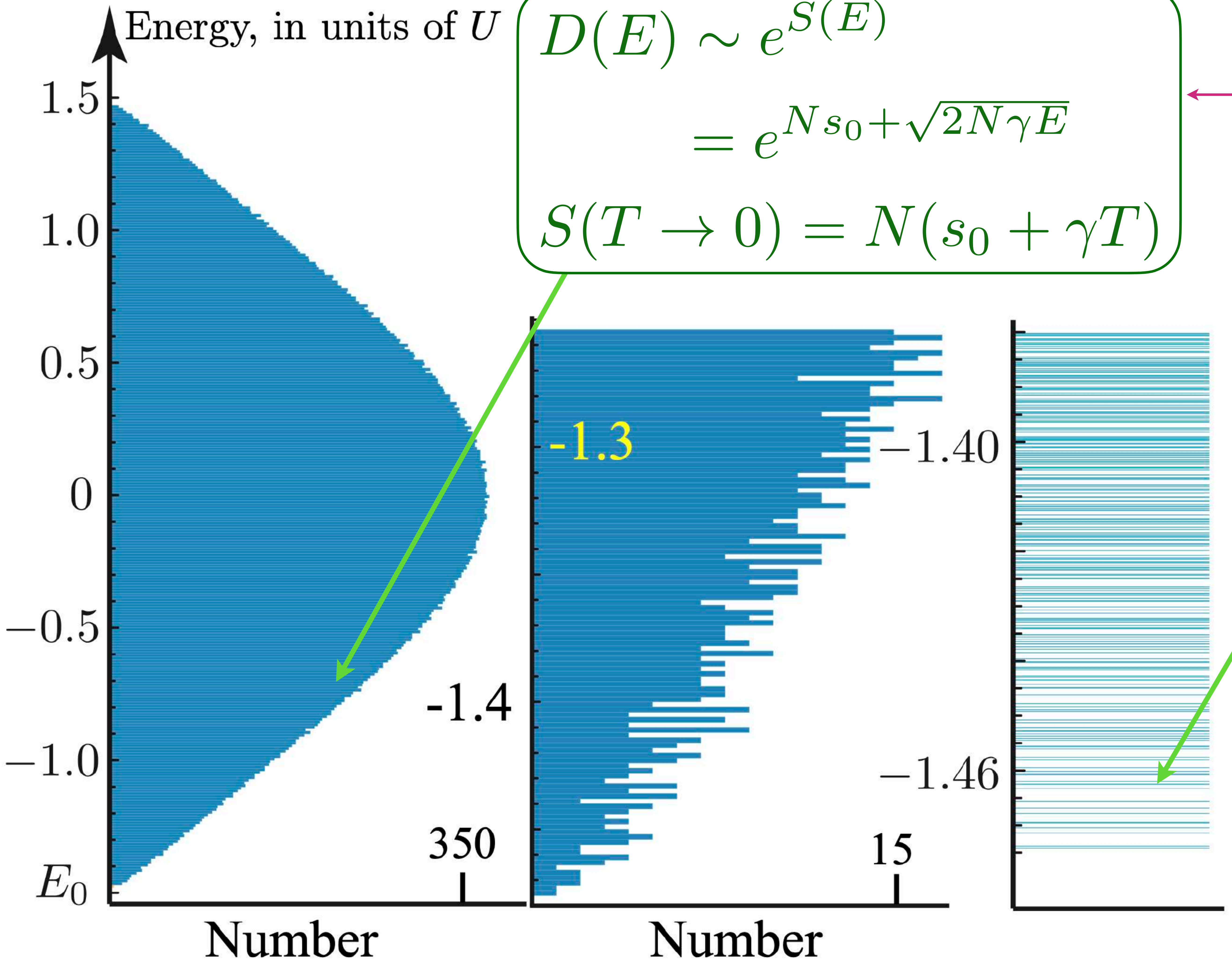

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Complex SYK model

1. Introduction to Planckian metals

2. Introduction to black holes

3. The SYK model

4. Progress on the theory of black holes

5. Progress on the theory of Planckian metals

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

$$= \exp\left(S_{BH}\right) \times \left(\quad ....????... \quad \right)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T = 0$.
$\mathcal{Q}$ is the black hole charge.
A_0 is a function of $\mathcal{Q}$.

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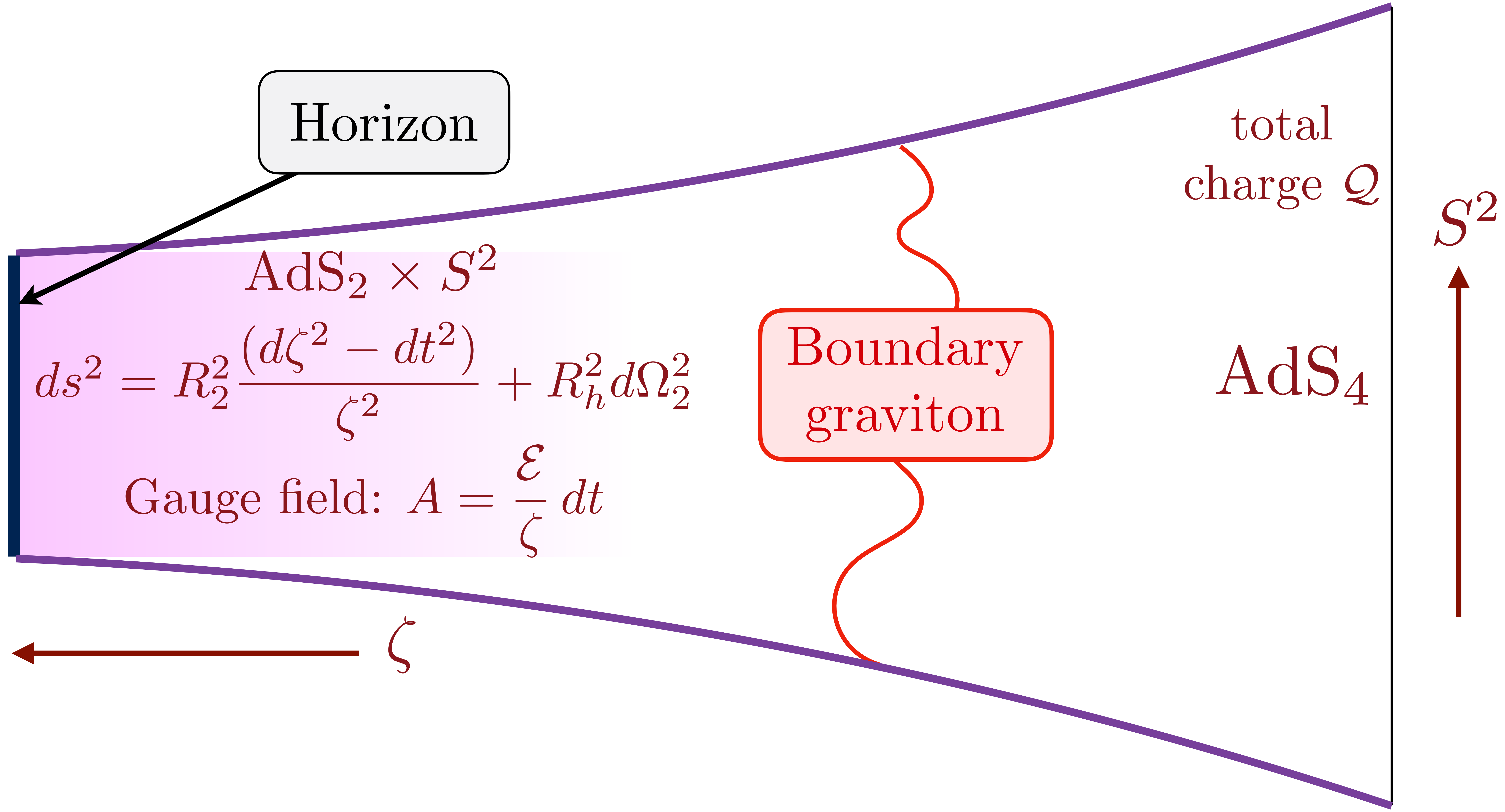
Note the similarity to the large *N* entropy of the SYK model !
(along with other similarities) Sachdev PRL 2010

Reissner-Nordstrom black hole of Einstein-Maxwell theory



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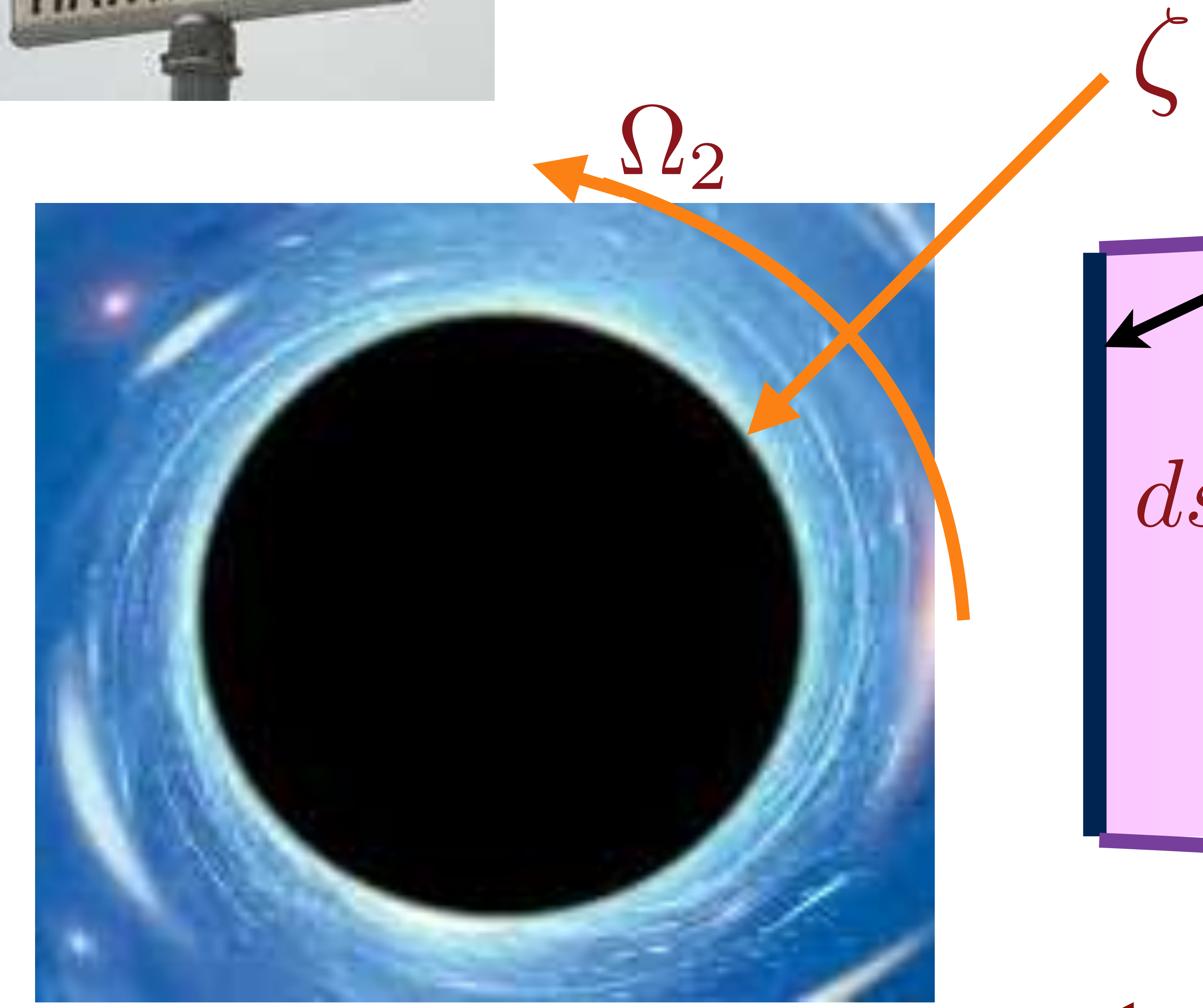

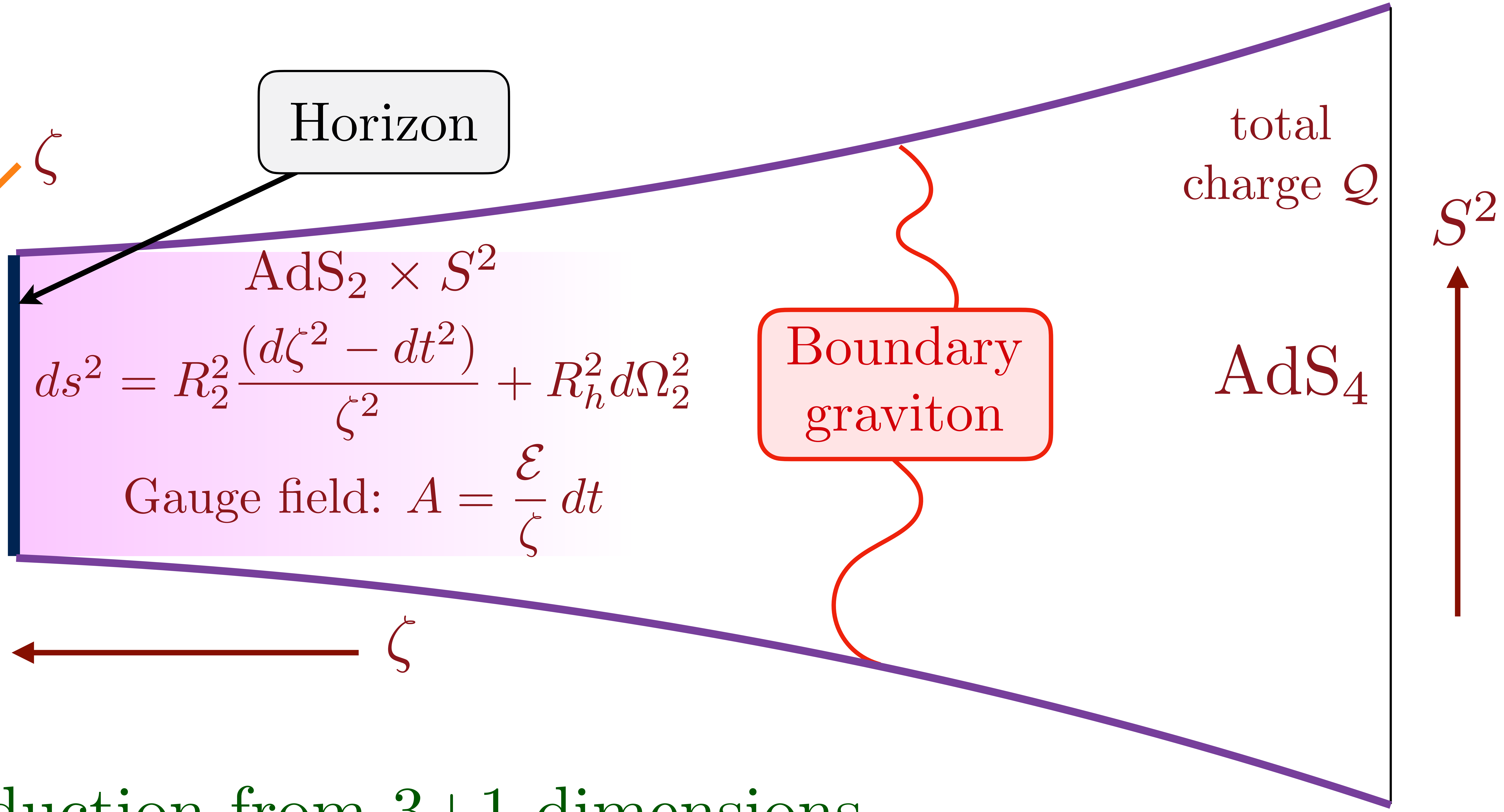

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

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$$\approx \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\text{JT gravity of AdS}_2\text{ and boundary}}[g_{\mu\nu}, A_\mu]\right)$$

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

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Maldacena, Stanford, Yang (2016)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) \exp\left(-\text{Schwarzian boundary graviton + rotor action}[f,\phi]\right)$$

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$$S(T\to 0,\mathcal{Q}) = S_{BH} - \frac{3}{4}\ln\left(\frac{\hbar c^5}{GT^2}\right)$$

$$S_{BH} = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T=0$, $\mathcal{Q}$ is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

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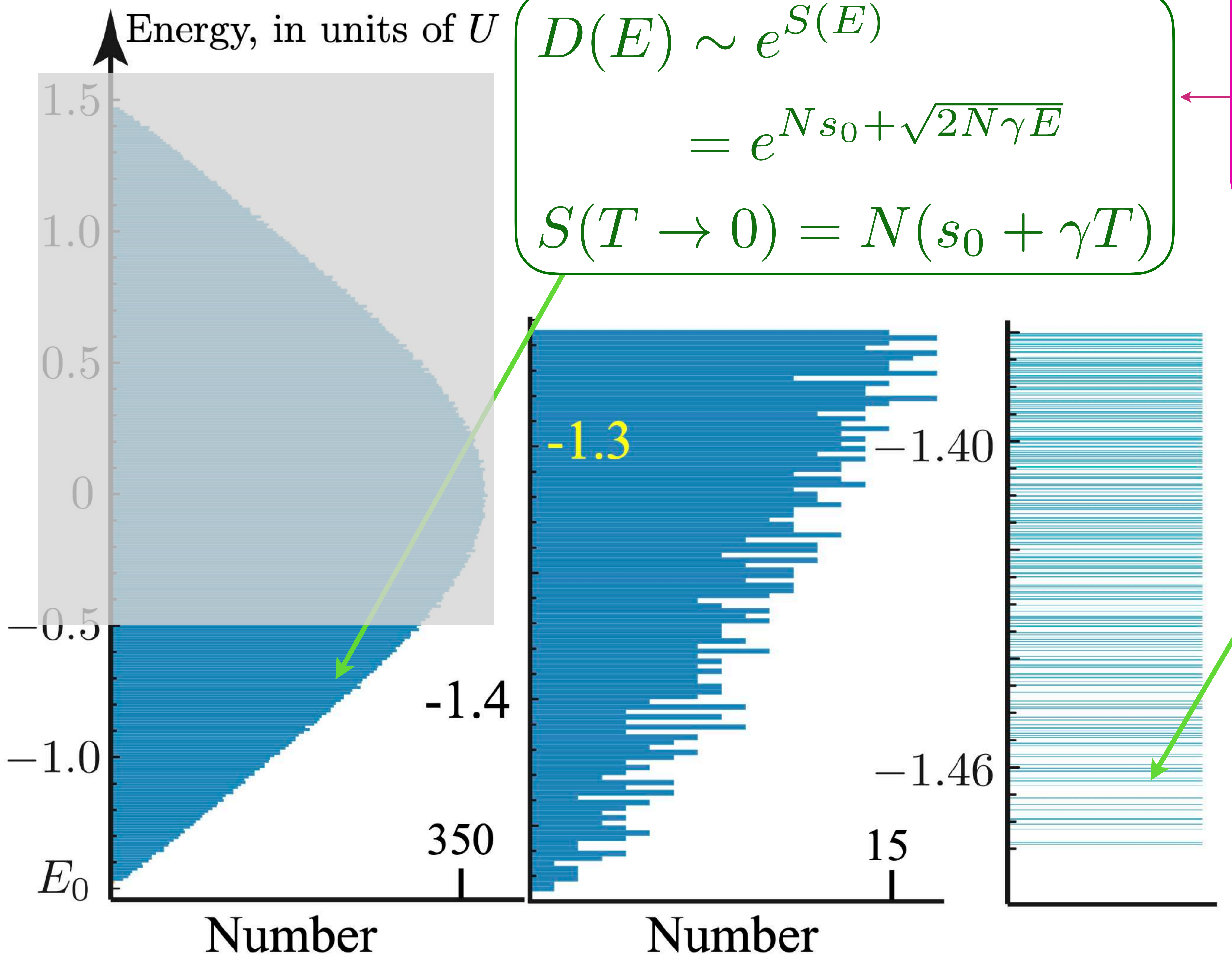
$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



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No quasiparticle decomposition of many-body states

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Complex SYK model

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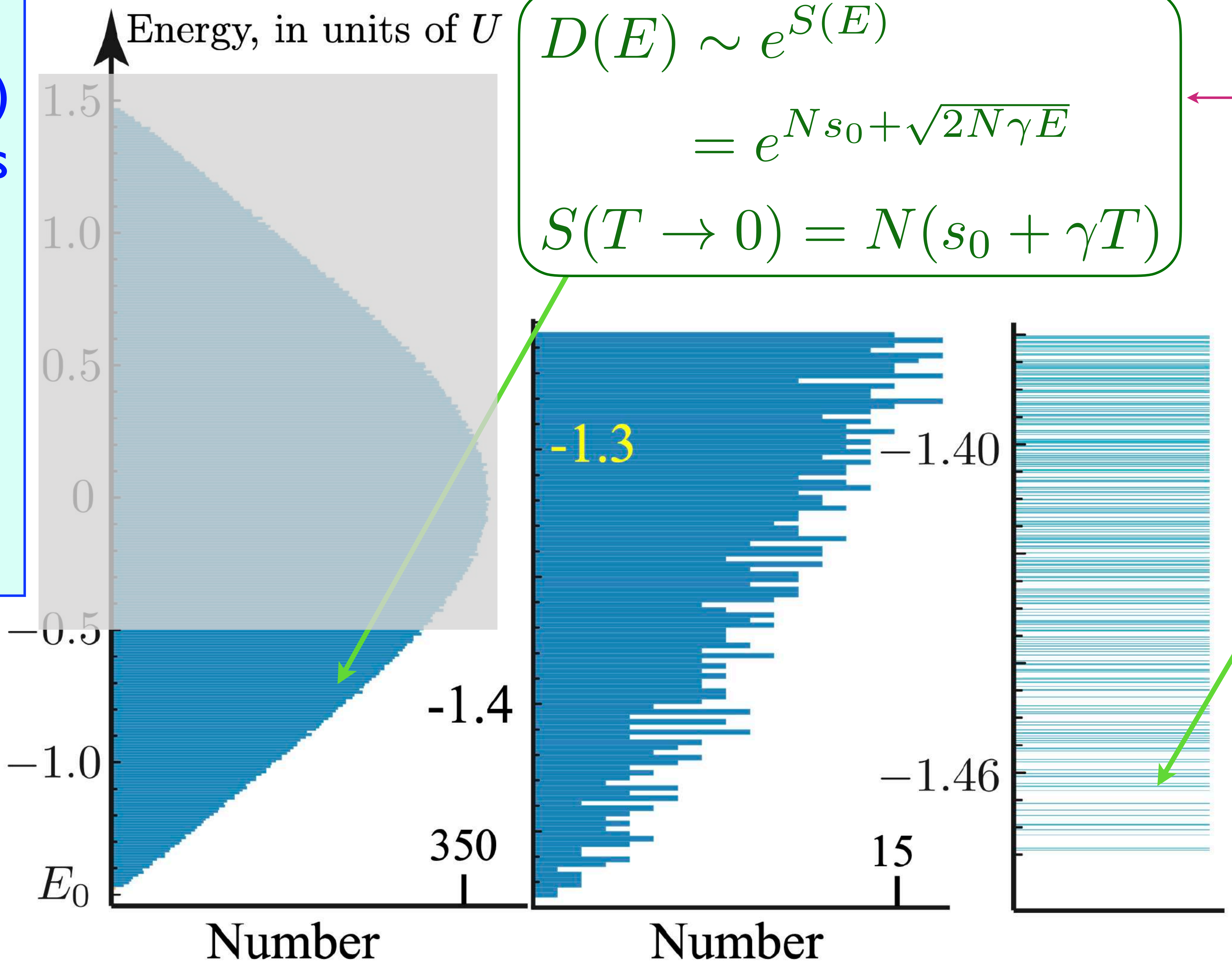
Same entropy and (coarse-grained) density of states in a model of interacting (fermionic) qubits with a discrete spectrum!



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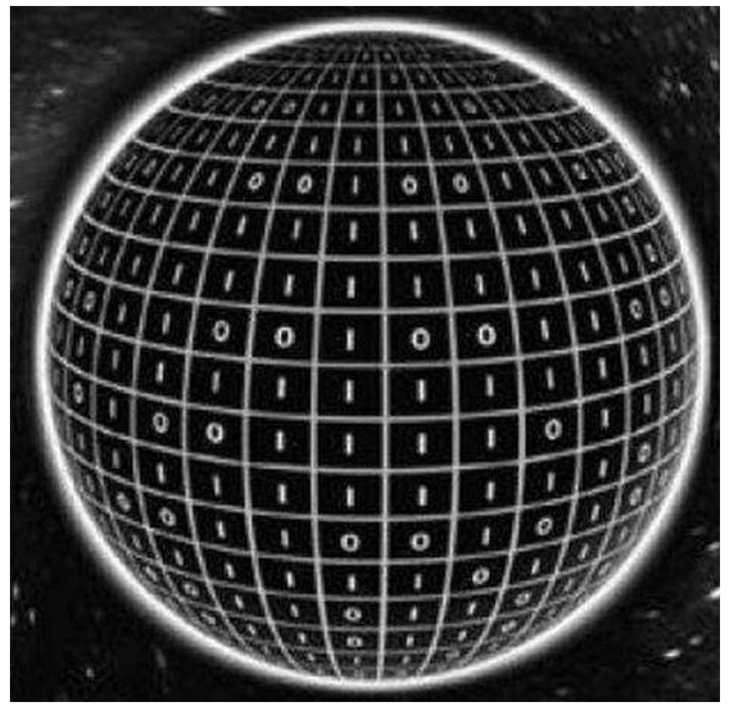

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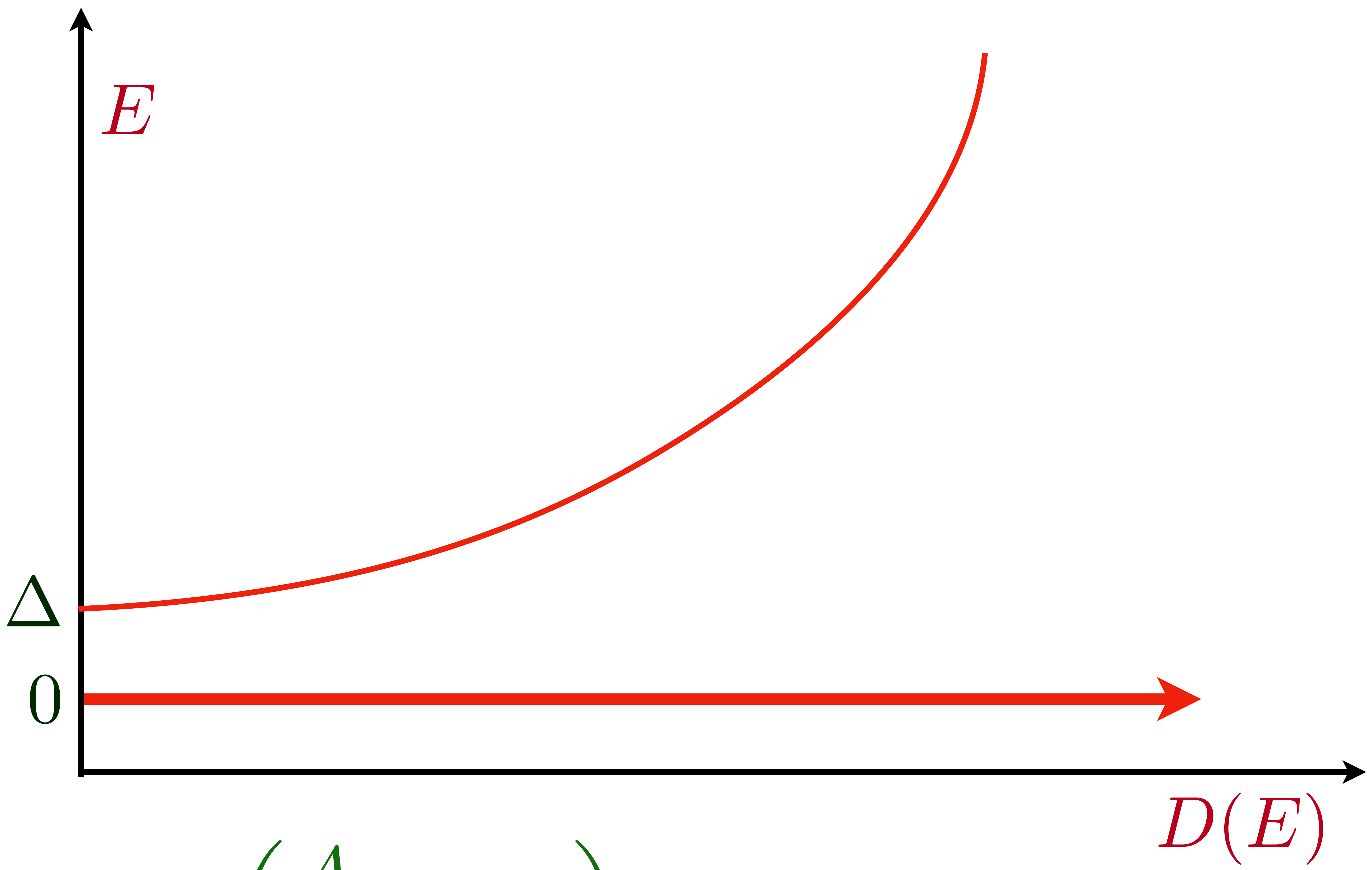

$$D(E) \sim \exp\left(\frac{A_0}{4G} + \ldots\right) \delta(E) + f_{\rm reg}(E - \Delta), \quad \Delta \sim R_h^{-1}$$

Supersymmetric black holes and SYK models

Fu, Gaiotto, Maldacena, Sachdev (2017); Stanford, Witten (2017); Heydeman, Iliesiu, Turiaci, Zhao (2020)

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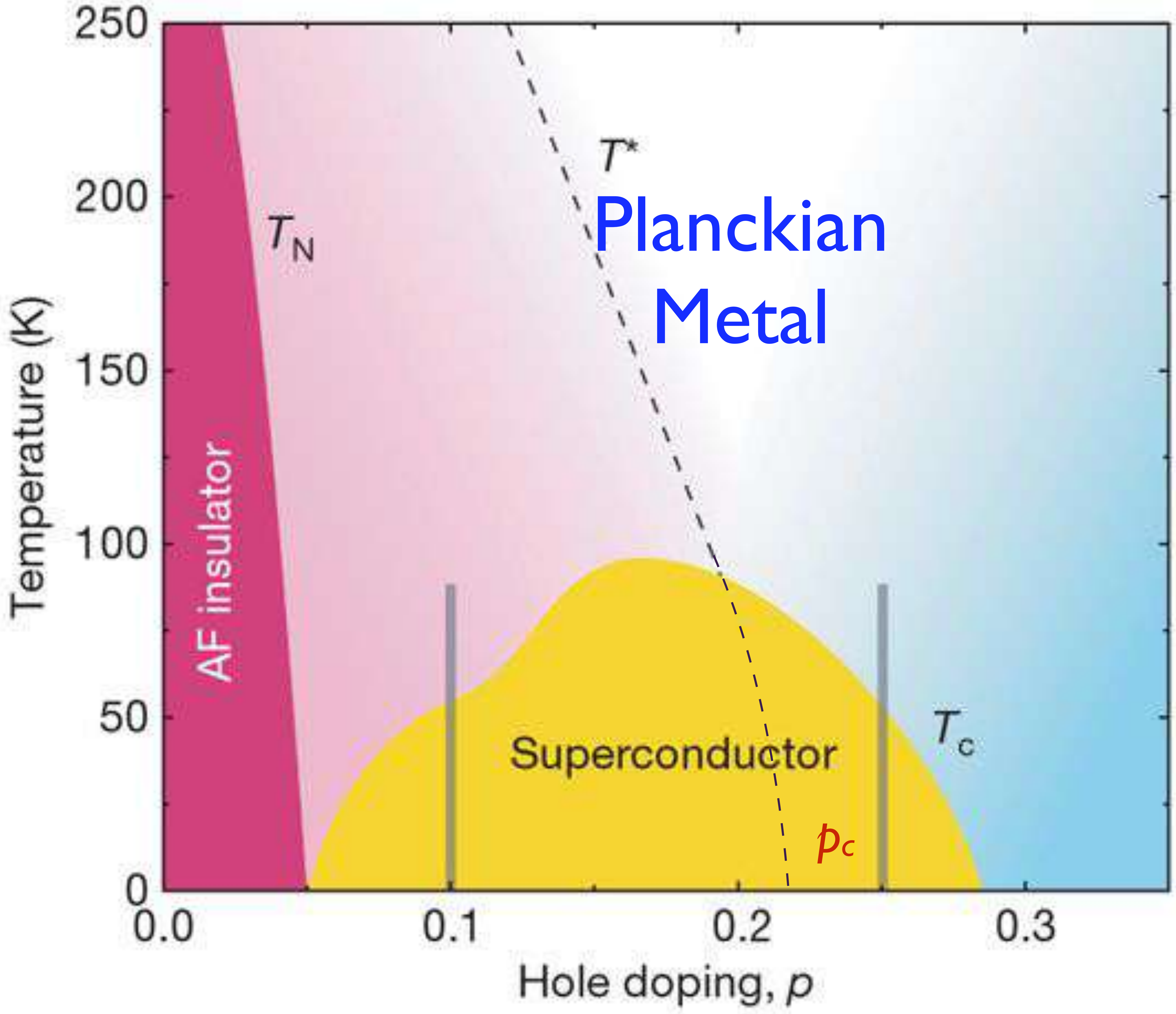
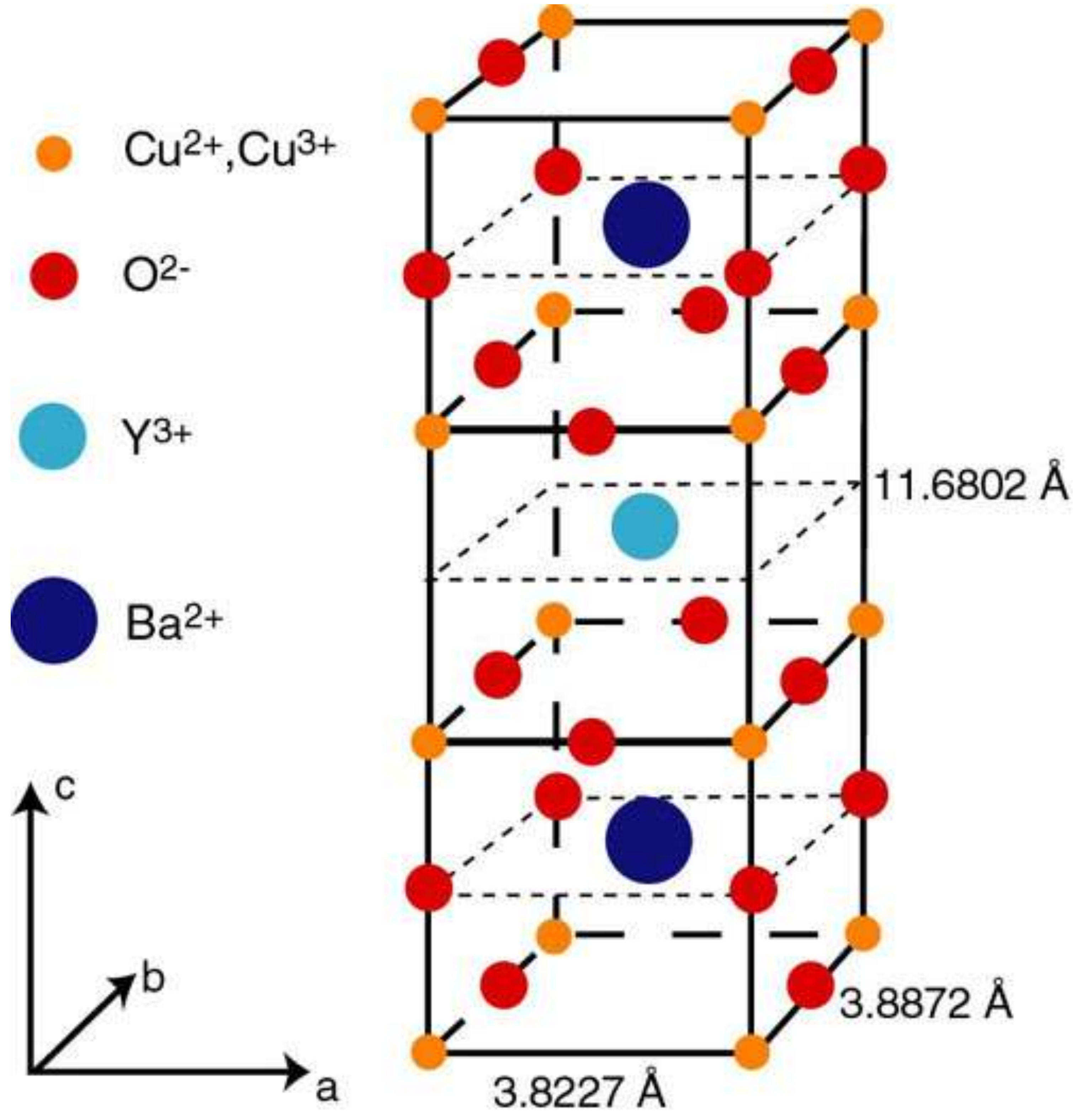
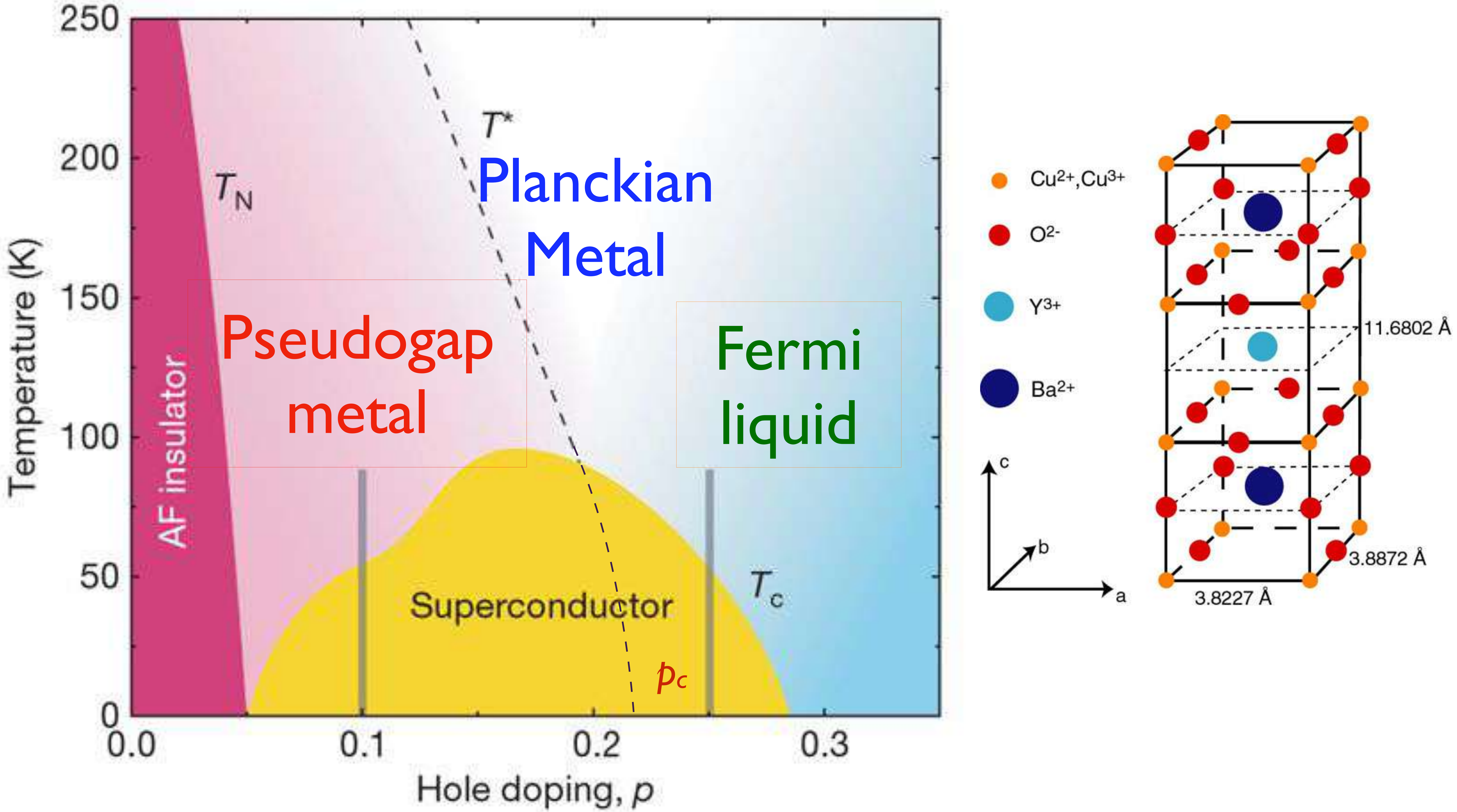
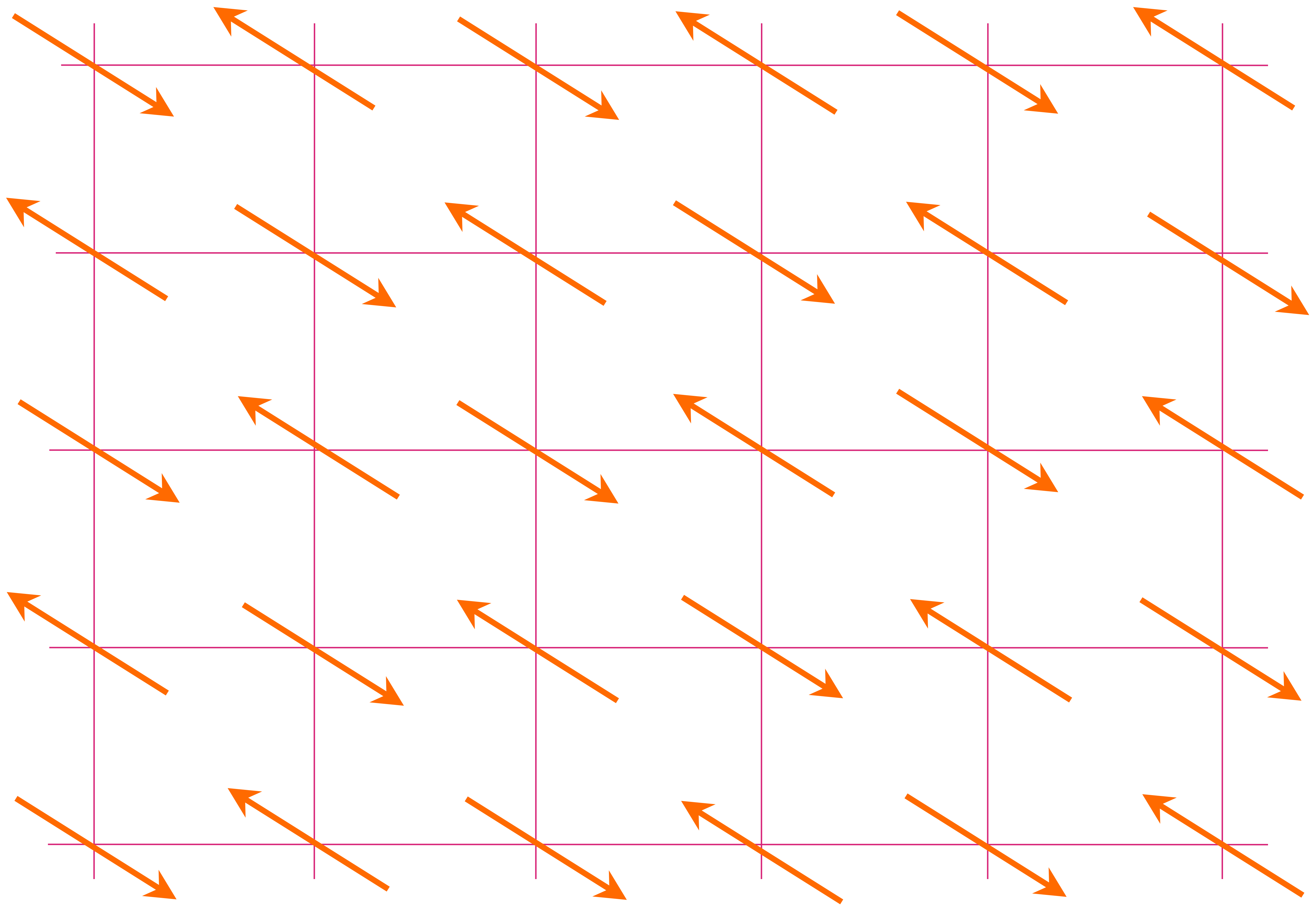

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Planckian
Metal
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



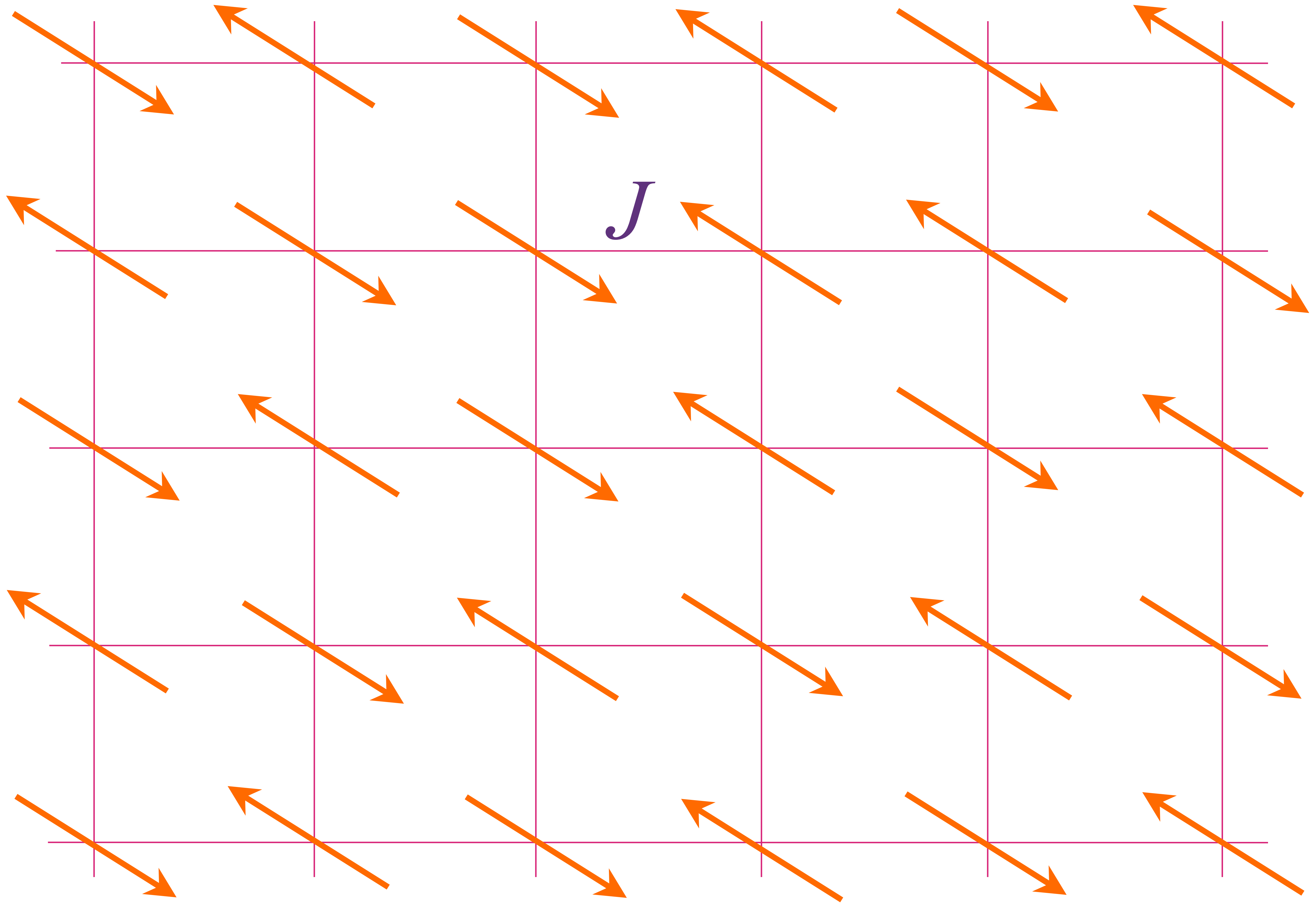
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Planckian
Metal
Pseudogap
metal
Fermi
liquid
Superconductor
T_c
p_c
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Insulating antiferromagnet



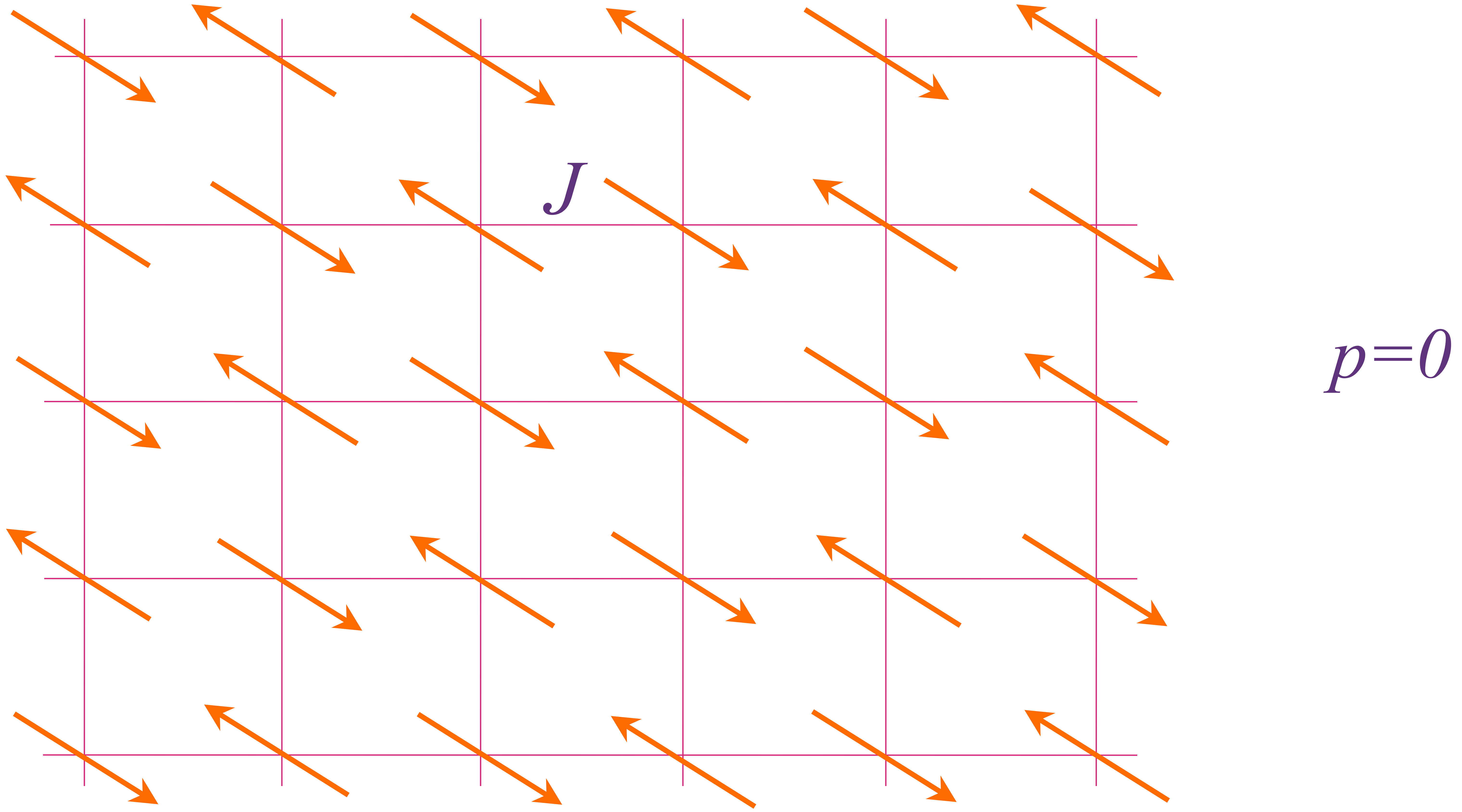
$p=0$

Insulating antiferromagnet

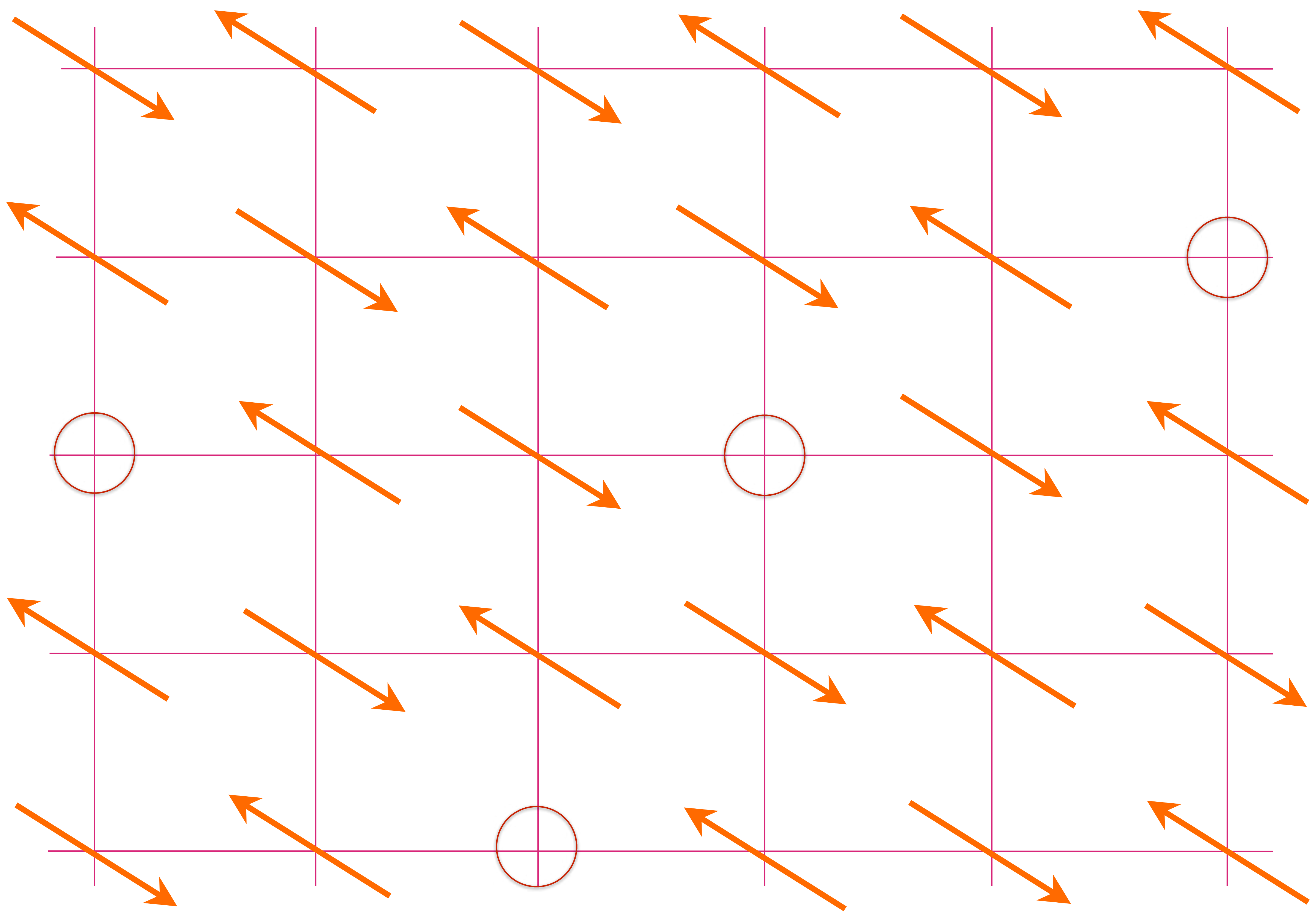


$p=0$

Insulating antiferromagnet



Antiferromagnet doped with hole density p



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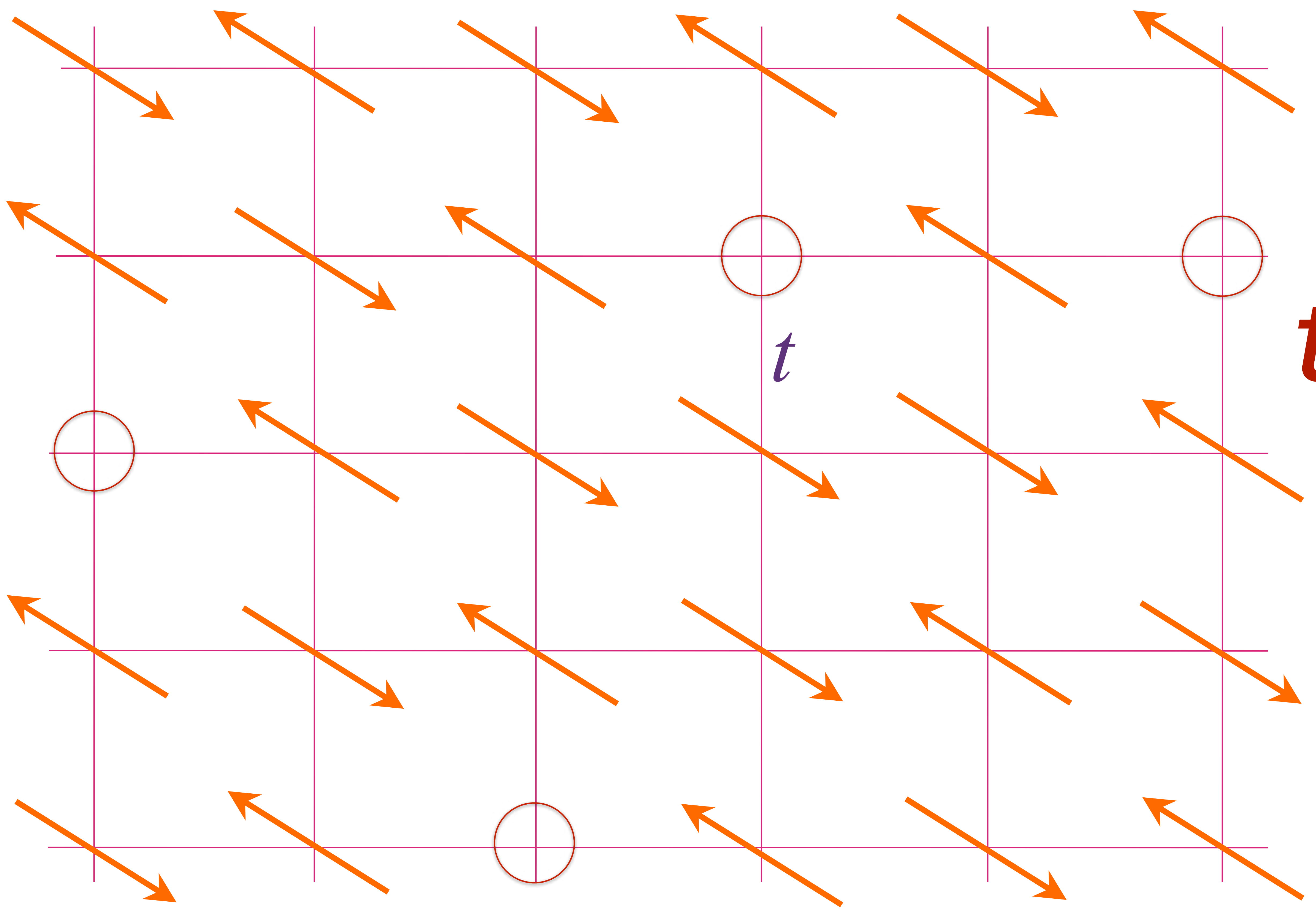

***t*-*J* model**

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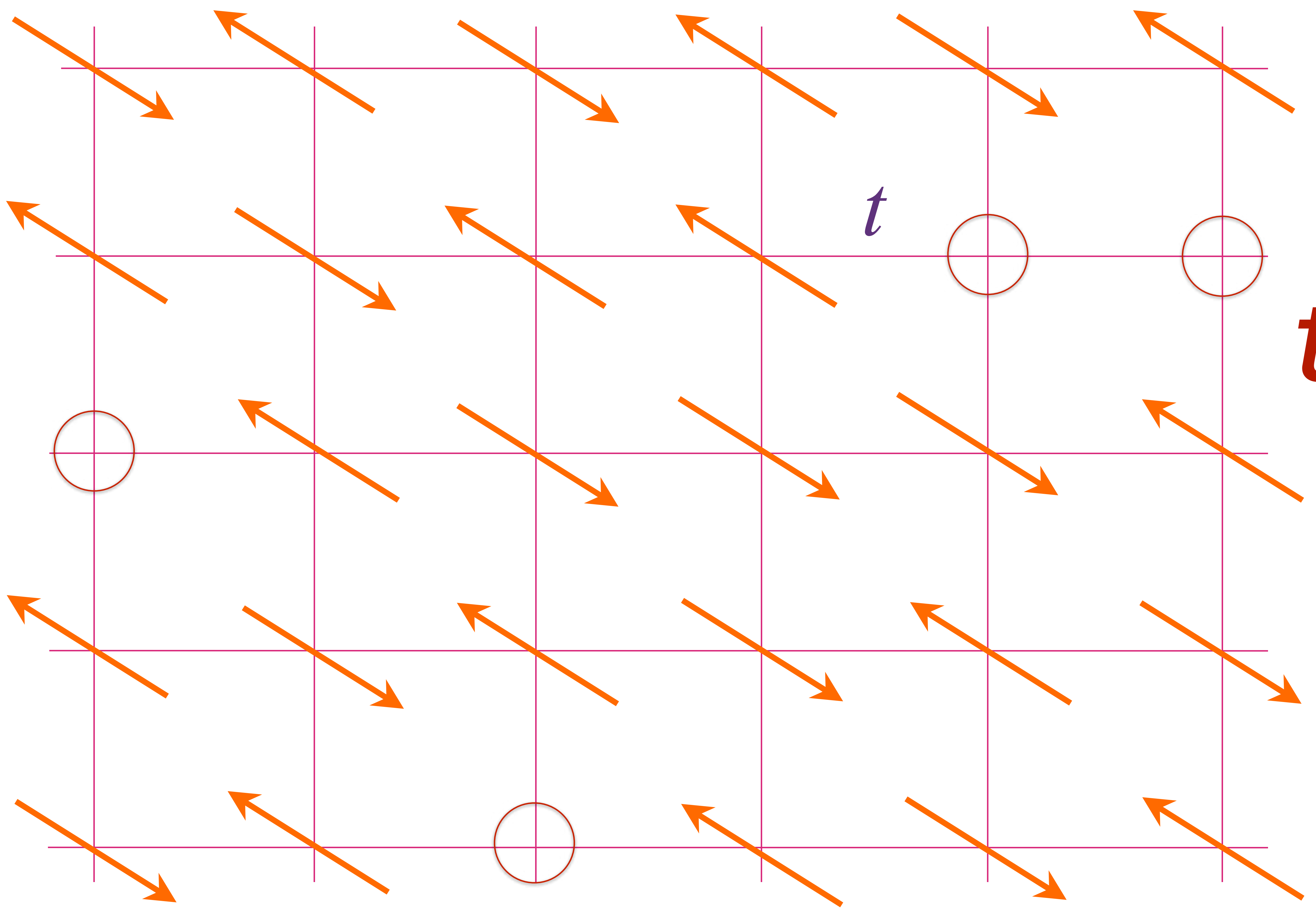

t-*J* model

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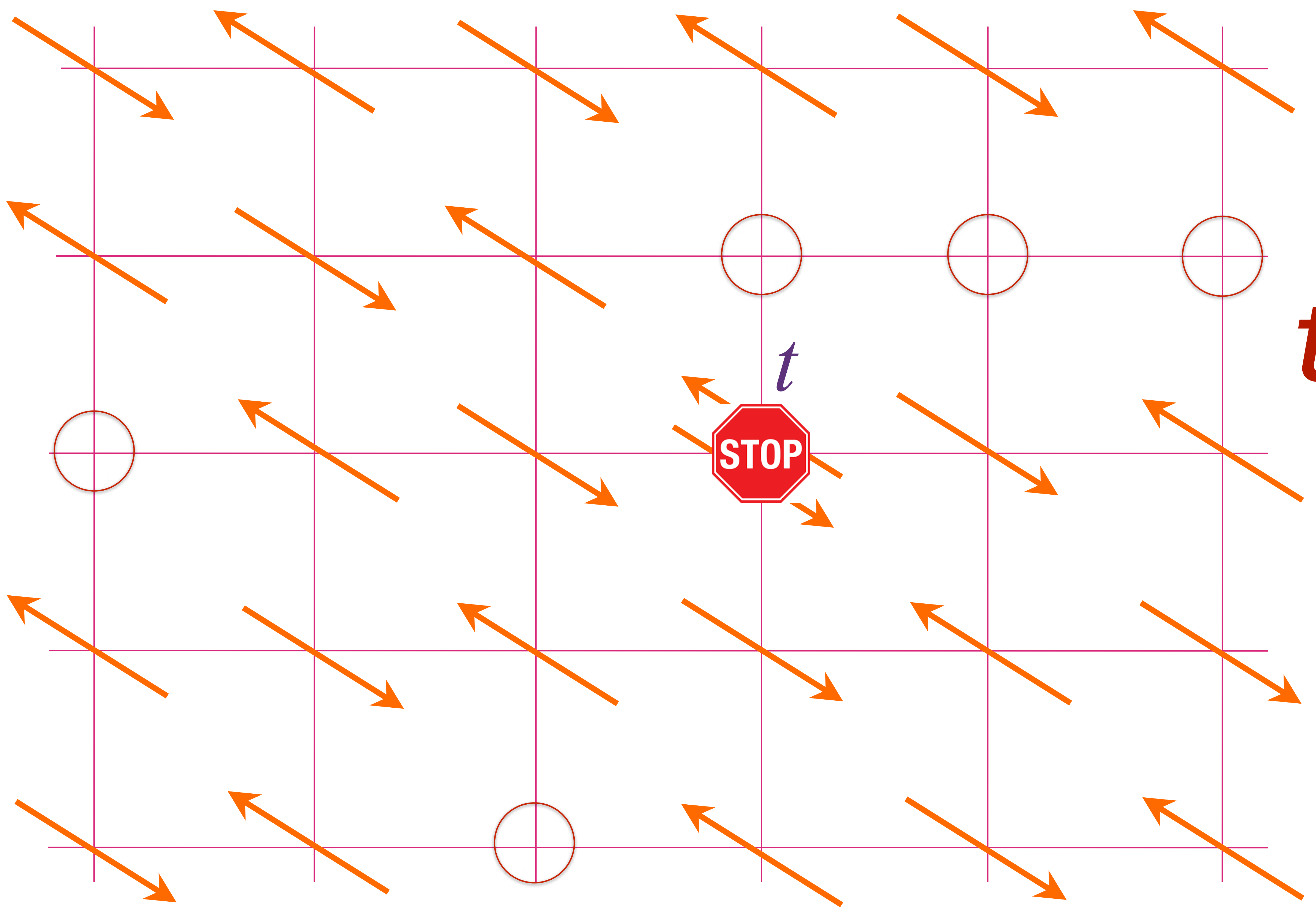
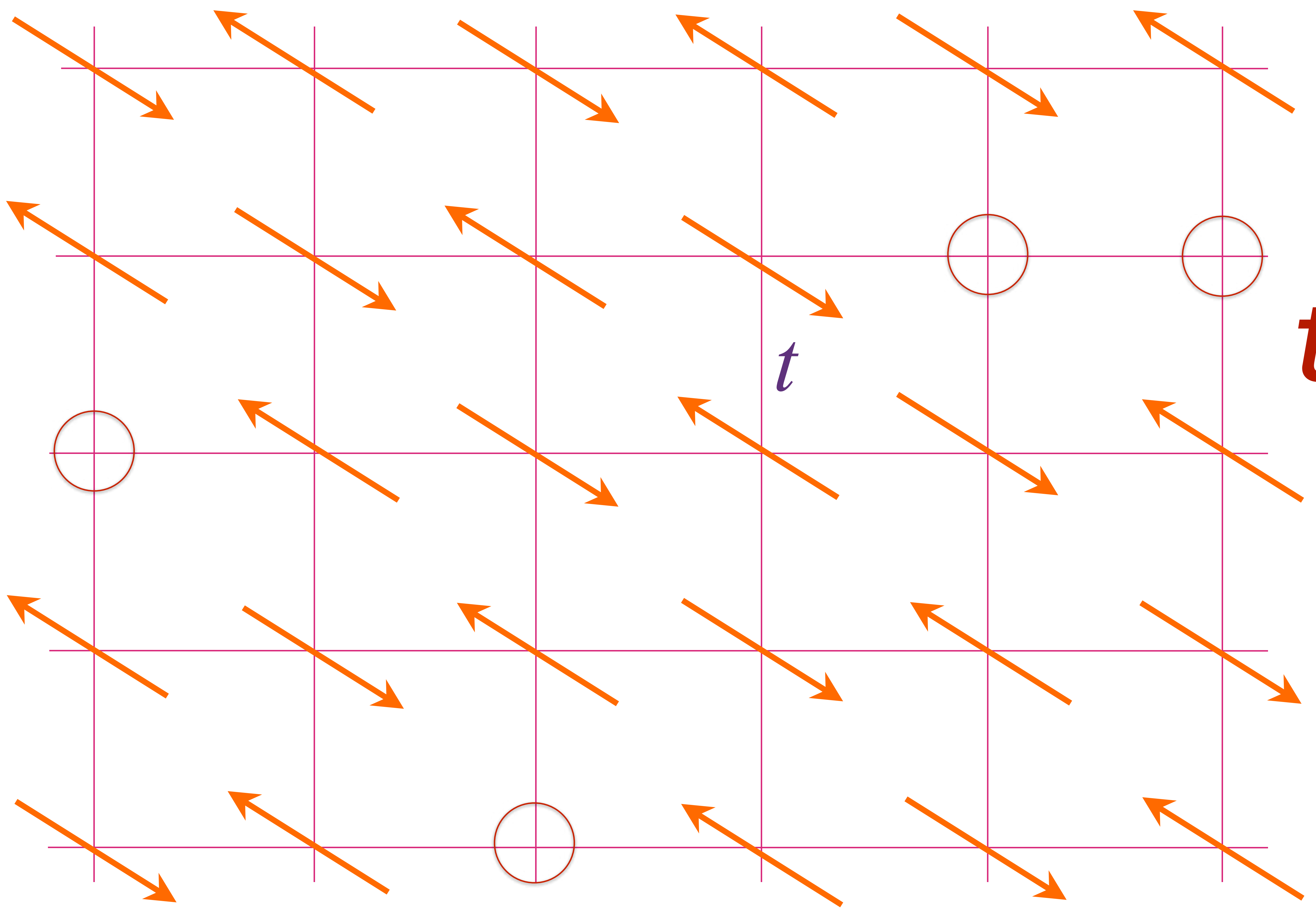


t-*J* model

Antiferromagnet doped with hole density p



t-*J* model

Antiferromagnet doped with hole density p

$$H = -t \sum_{\langle ij \rangle} \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

t-J model

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

Random *t*-*J* model doped with hole density p

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

J_{ij} random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$

t_{ij} random, $\overline{t_{ij}} = 0$, $\overline{t_{ij}^2} = t^2$

$J \quad \Rightarrow \quad$ two-particle interaction, as in SYK

$t \quad \Rightarrow \quad$ one-particle hopping, as in random matrices

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Parisi solved the model with this term only with $\vec{S}_i$ replaced by classical Ising spins $\sigma_i = \pm 1$

$$\vec{S}_i = \tfrac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

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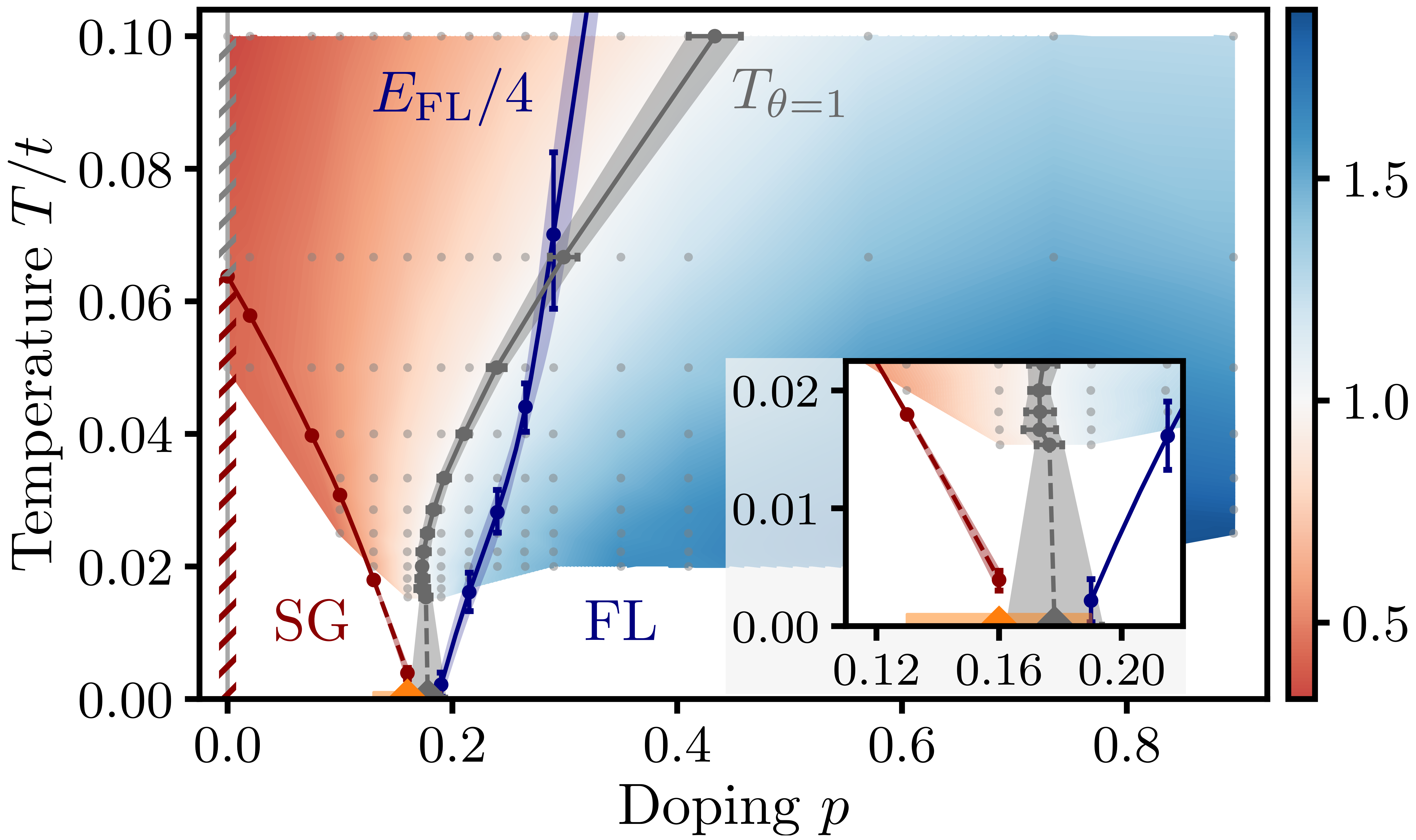
Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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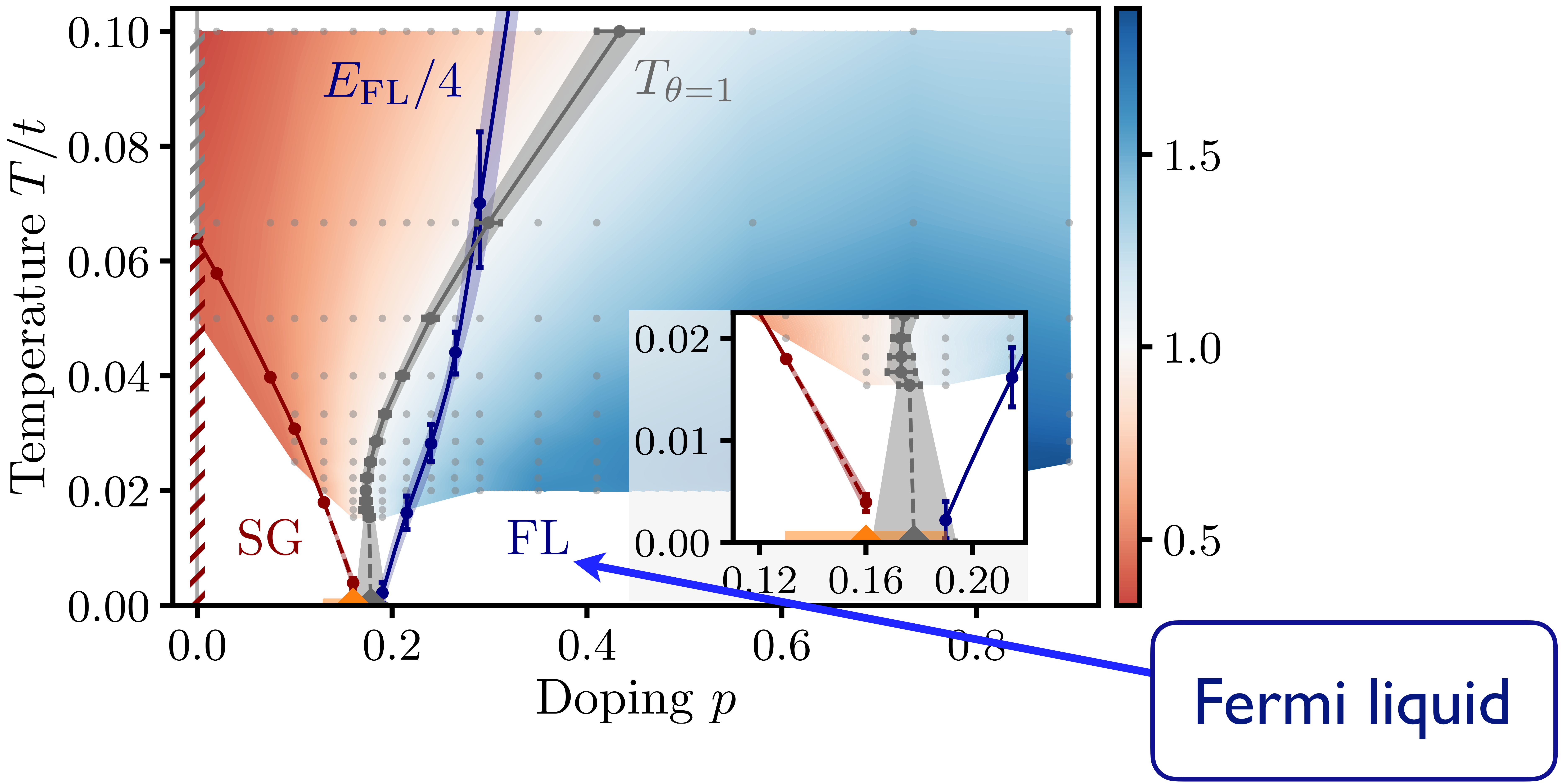

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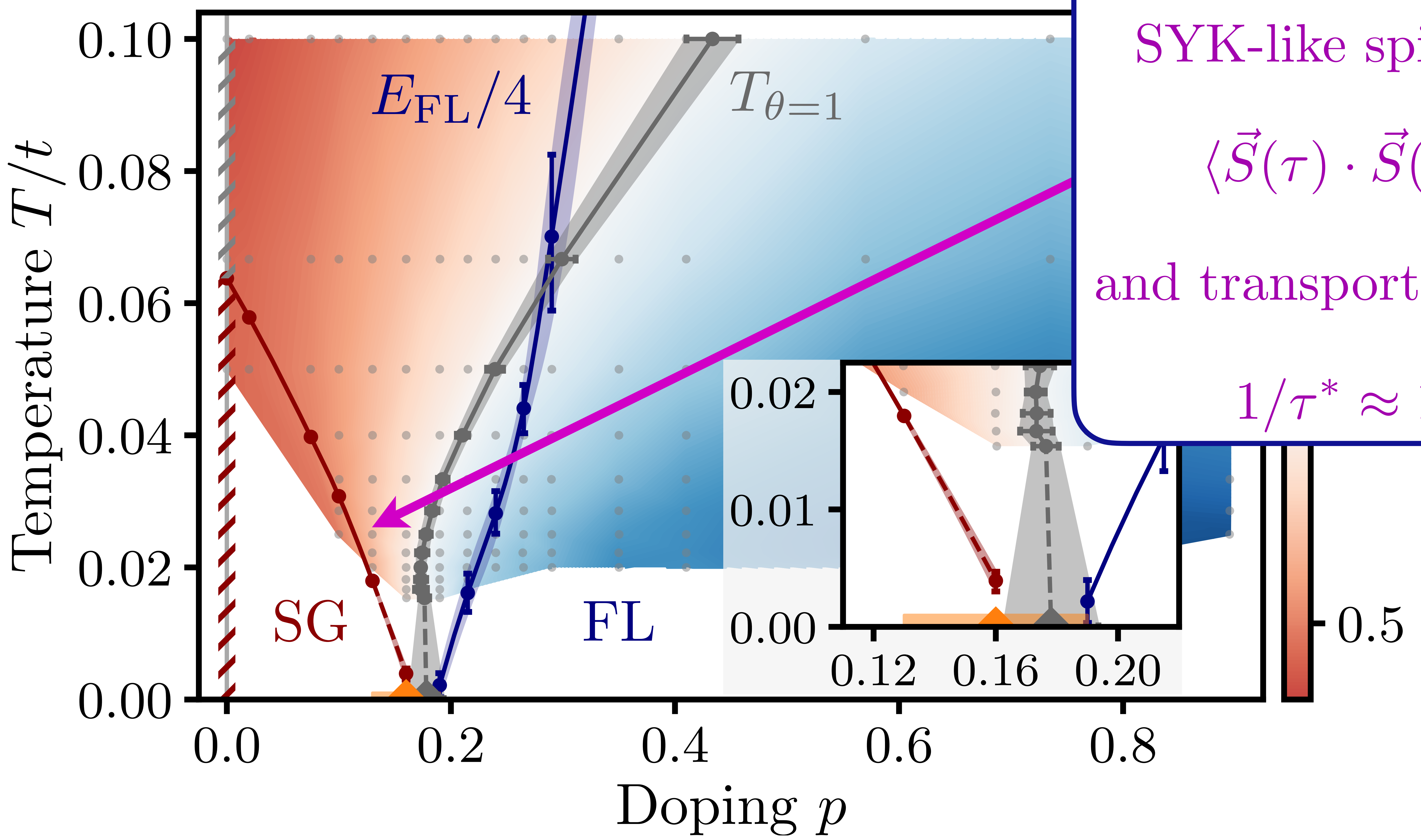


Planckian metal with SYK-like spin correlations

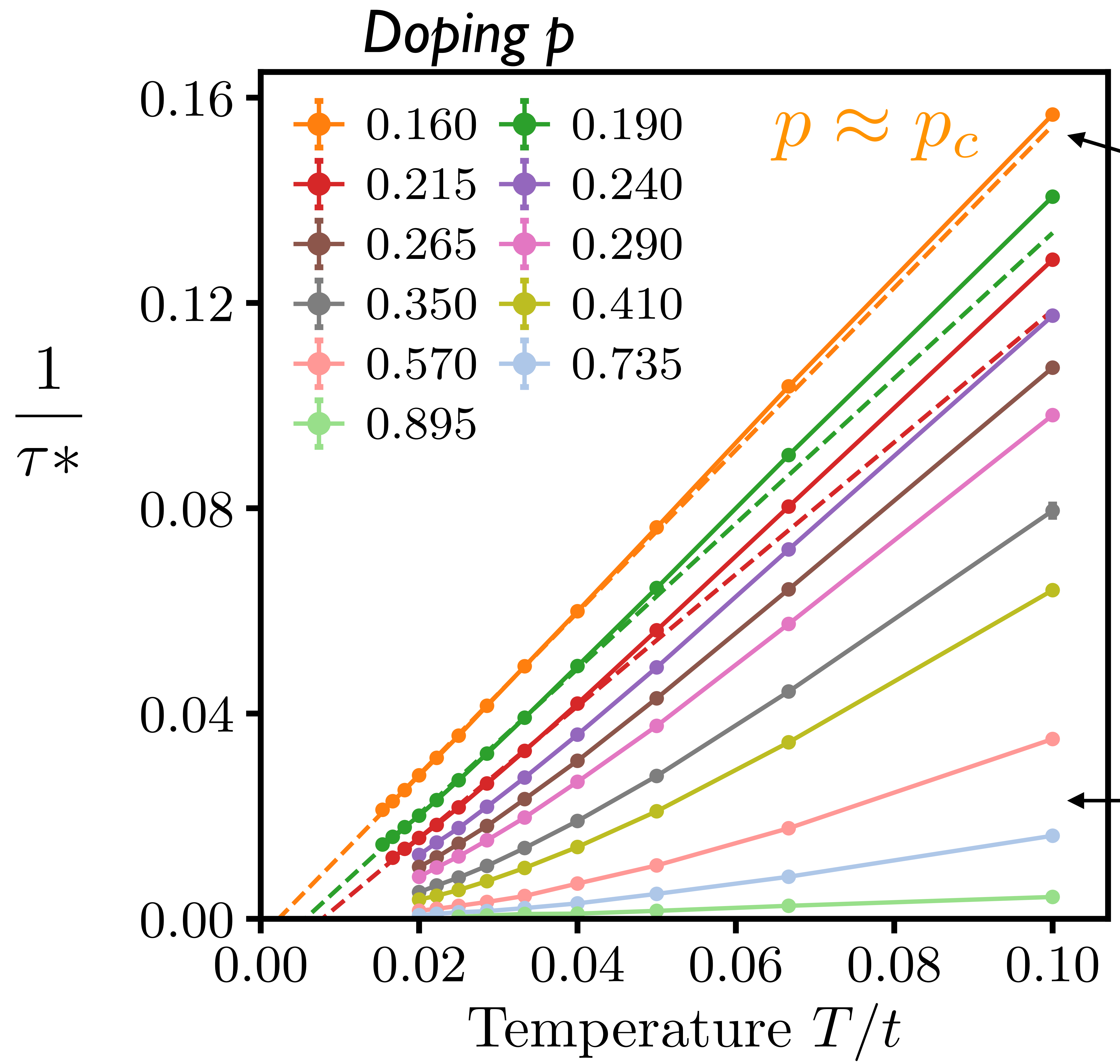
$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim |\tau|^{-1}$$

and transport scattering time

$$1/\tau^* \approx 1.4\, k_B T/\hbar$$

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

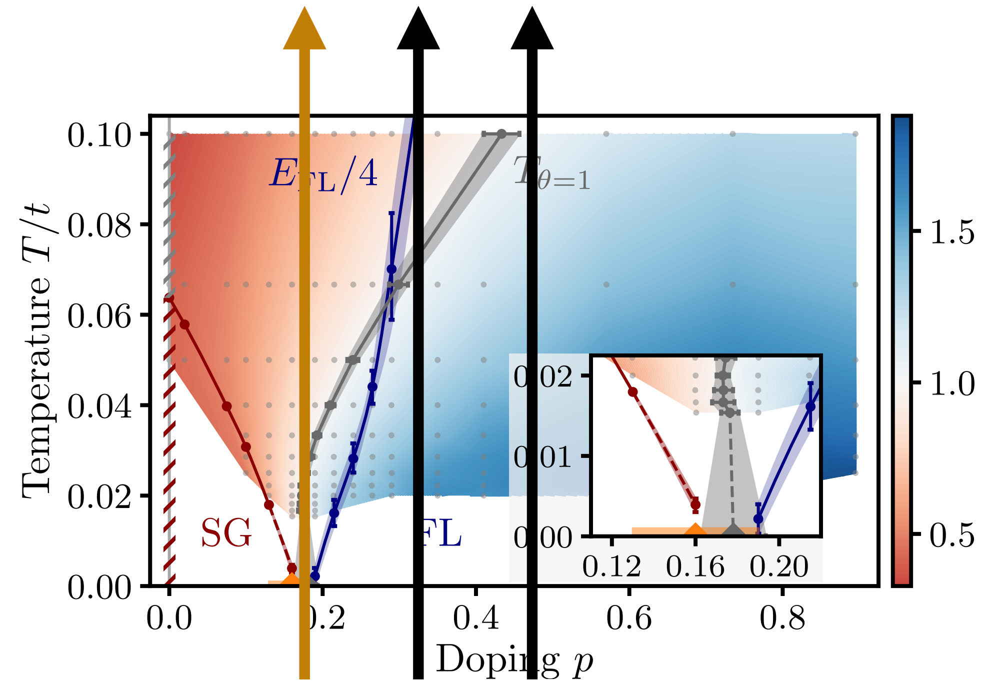
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)



$$\frac{1}{\tau *} \simeq c\,\frac{k_B T}{\hbar}$$

Planckian metal
for $p \approx p_c$

$$\frac{1}{\tau *} \propto T^2$$



P. T. Dumitrescu, N. Wentzell, A. Georges,
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- Why should the 2+1 dimensions t-J model be described by a 0+1 dimensional SYK-like theory over a significant temperature range ?

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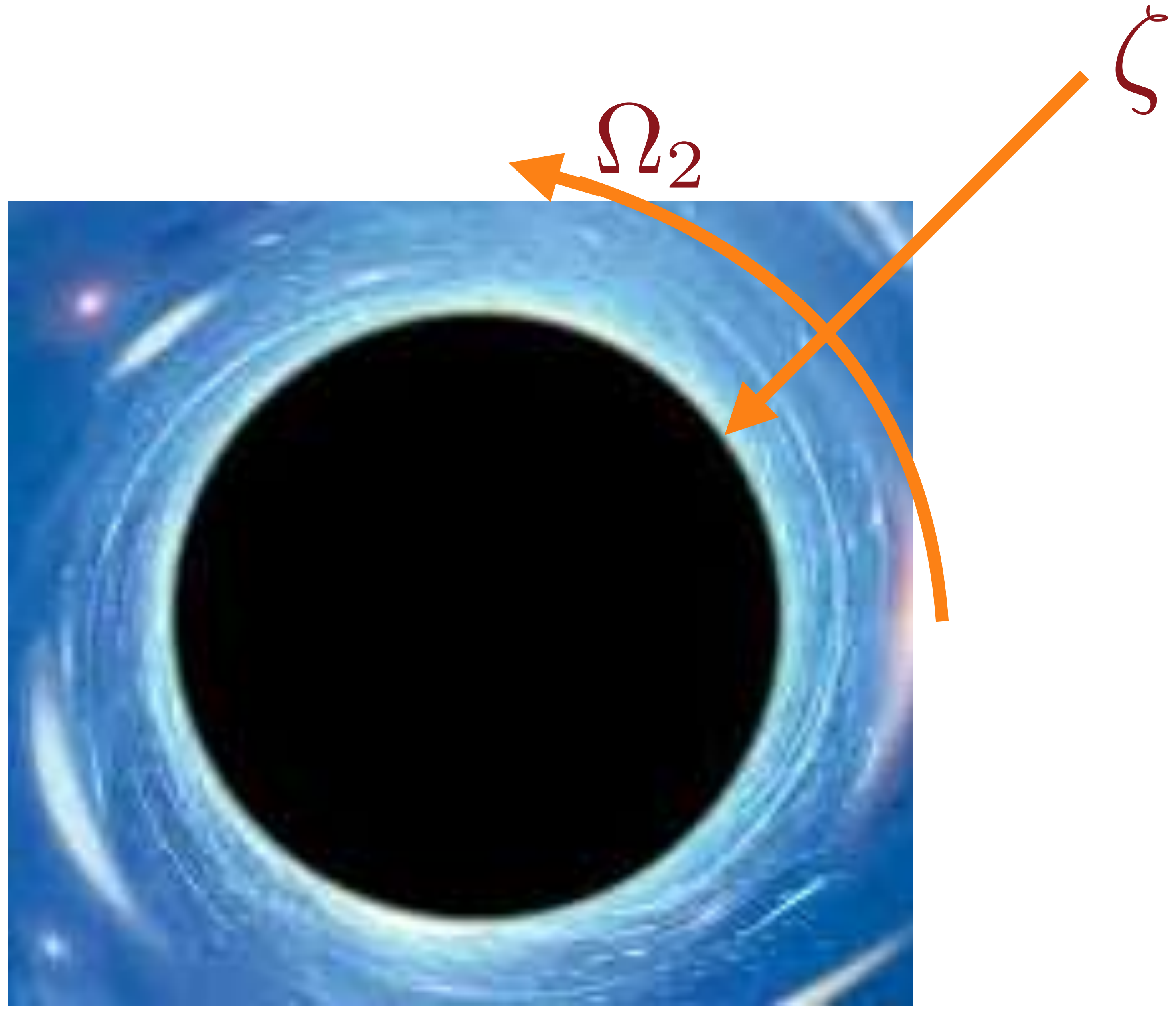

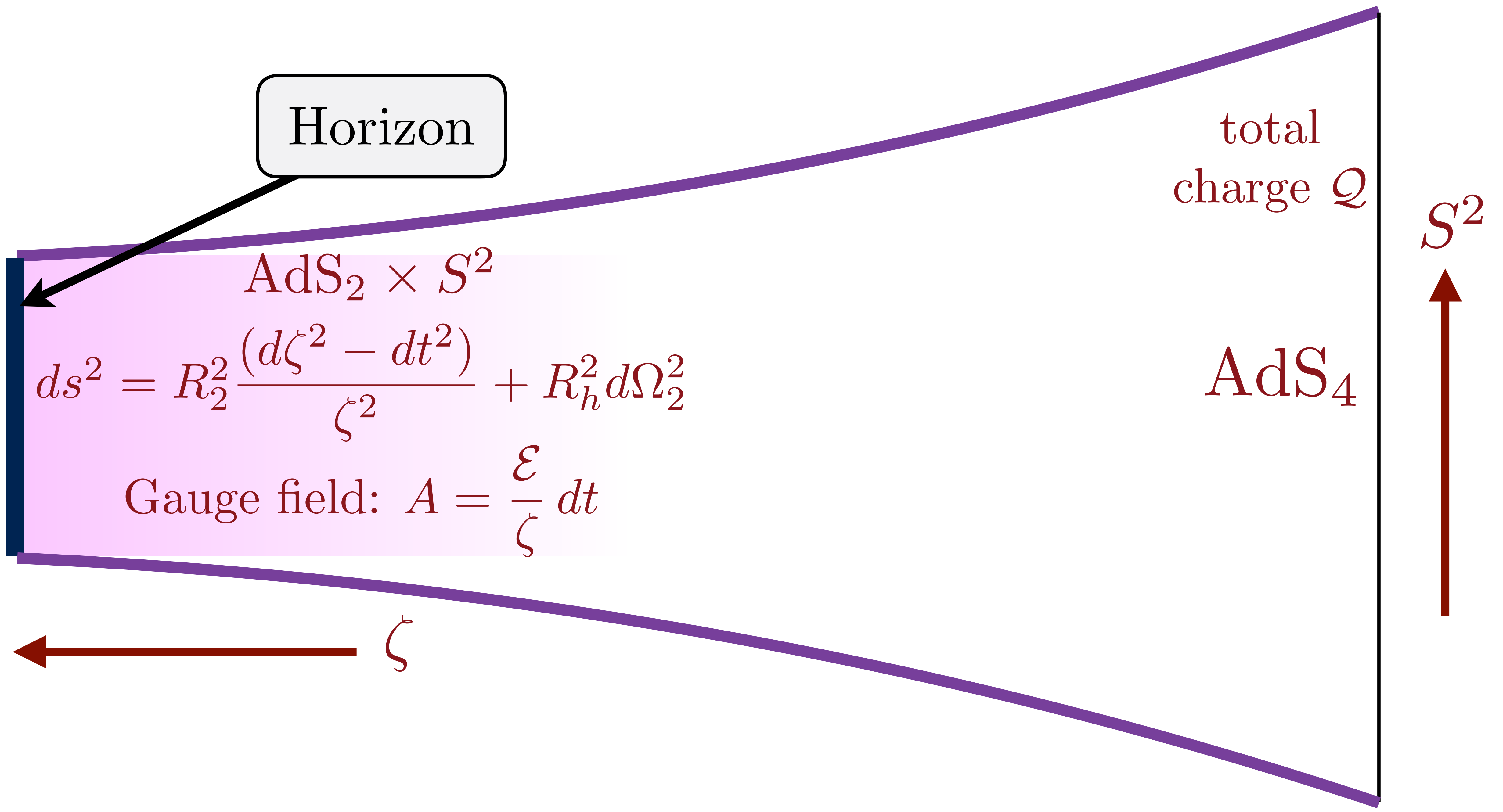

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- Recall the dimensional reduction of 3+1 dimensional gravity of a charged black hole to a 1+1 dimensional theory on AdS_2.

- By the holographic mapping, this implies that certain large N models of Fermi surfaces coupled to gauge fields in 2+1 dimensions display a dimensional reduction to a 0+1 dimensional theory. The t-J model can be written as a similar theory, but at small N.

Tom Faulkner, Nabil Iqbal, Hong Liu, John McGreevy, Mark Mezei, David Vegh, 2011

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037

Summary

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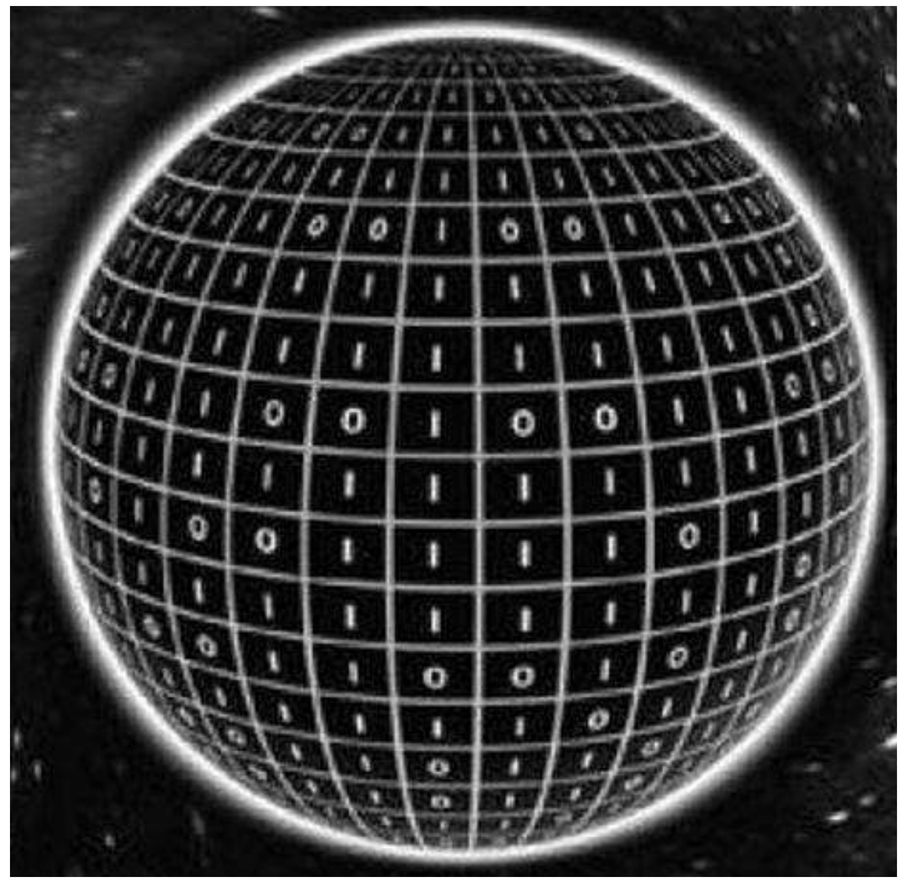
- SYK: a solvable model without quasiparticle excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Low energy theory of time reparameterizations is the theory of the boundary graviton in 2D quantum gravity on AdS_2.

Summary

- Boundary graviton leads to universal $-3/2\ln(1/T)$ correction to Bekenstein-Hawking entropy of low T charged black holes in Einstein gravity, and to the SYK model. So the semiclassical entropy of Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



Summary

- SYK-like random t-J model captures many aspects of the cuprates over a wide intermediate temperature range, including the Planckian metal behavior.