Strong Interactions, Color Confinement, and Strings

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Colloquium

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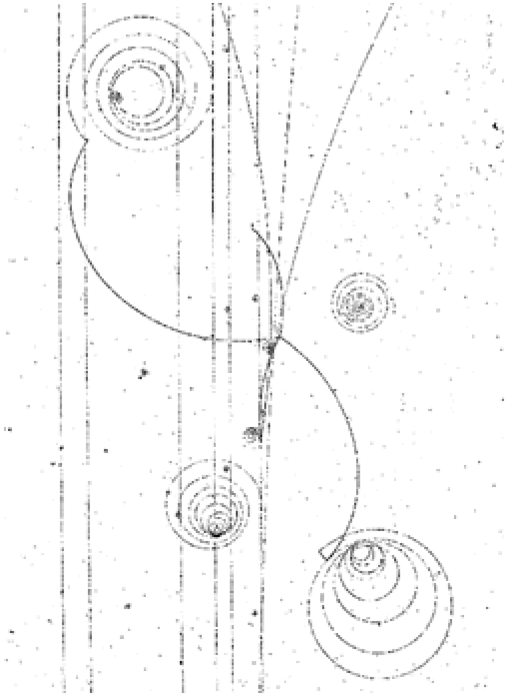
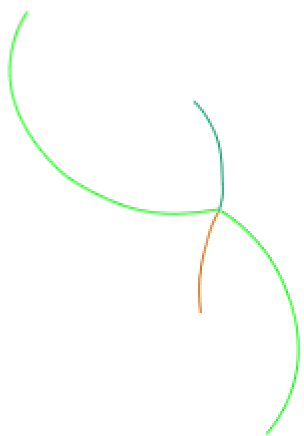
Introduction

- One often hears of string theory as the leading hope for unifying all known interactions-- strong (nuclear), electromagnetic, weak (β-decay) and gravitational-- into a consistent quantum theory. Some have even dubbed it `The Theory of Everything.'
- Actually, string theory has had more humble beginnings. It was invented in the late 60's to model `just' the strong (nuclear) interactions.

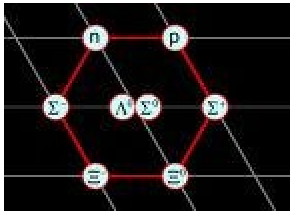
- In the 1950's and 60's technological progress led to discovery of many \`strongly interacting' sub-nuclear particles (hadrons).
- Hadron (heavy) vs. lepton (light).
- Hadrons come in two types: baryons (protons, neutrons, Λ, etc.) and mesons (π, ρ, K, etc.).
- Here is a bubble chamber picture of a charged K-meson decaying into three charged π-mesons.

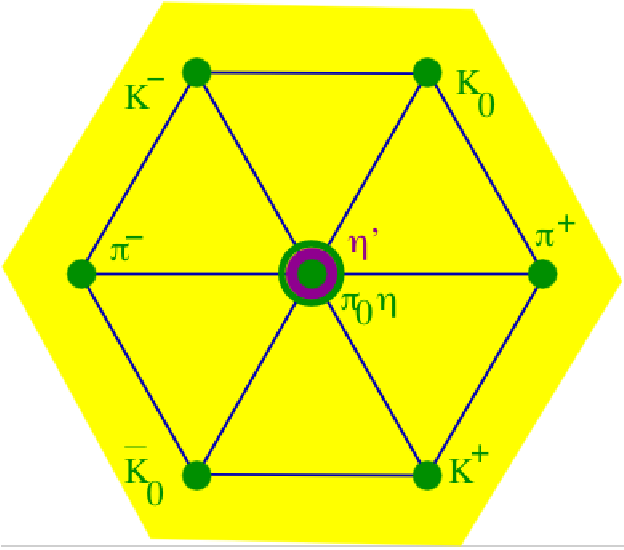




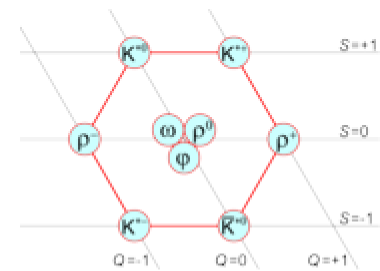
- The strong interactions are short range (~ 1 fm = 10 $^{-15}$ m), but much stronger than the electromagnetic.
- The strong interaction analogue $\alpha_s = g_{YM}{}^2/(4\,\pi)$ of the electromagnetic `fine structure constant' (α=1/137) is about 100 times bigger.
- As more hadrons were discovered, they were grouped into multiplets. For example, the lightest spin-0 mesons form an `octet.' This gave impetus to the Quark Model.

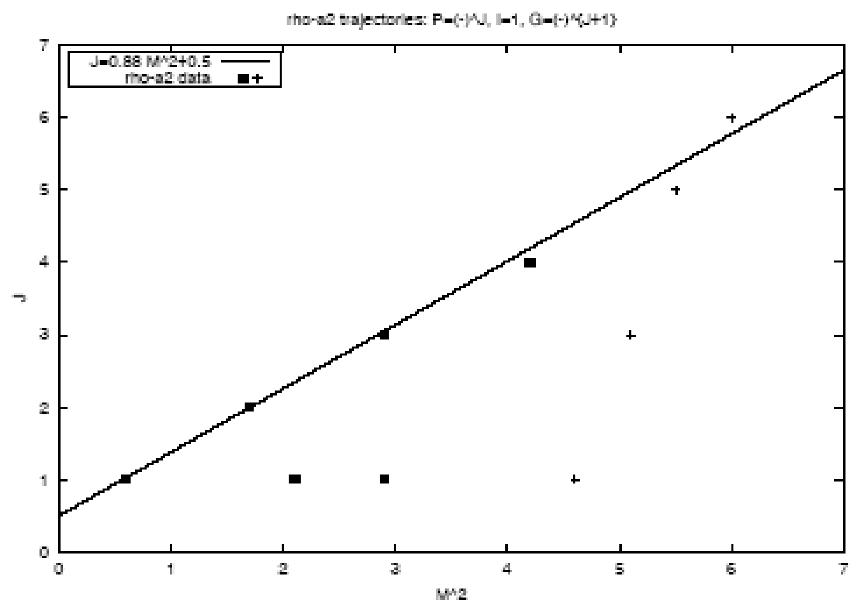
Gell-Mann, Zweig





- Lots of heavier mesons, with higher intrinsic spin, have also been discovered.
- Early empirical evidence for the string-like structure of hadrons comes from arranging mesons and baryons into `Regge trajectories’ on plots of angular momentum vs. mass-squared.
- A leading `Regge trajectory’ of mesons is shown (ρ, a_2 …)





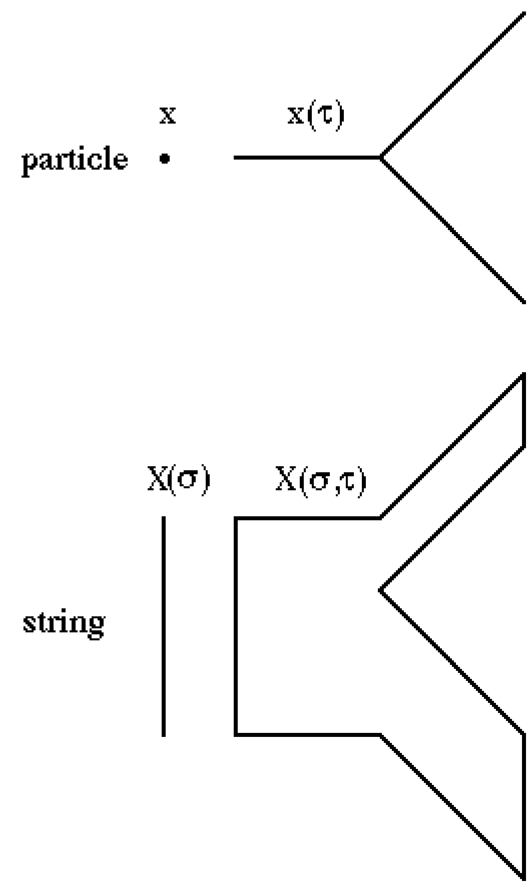
- In 1968 Veneziano proposed a scattering amplitude for elastic pion scattering:

$$A(s,t) \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

with the linear Regge trajectory

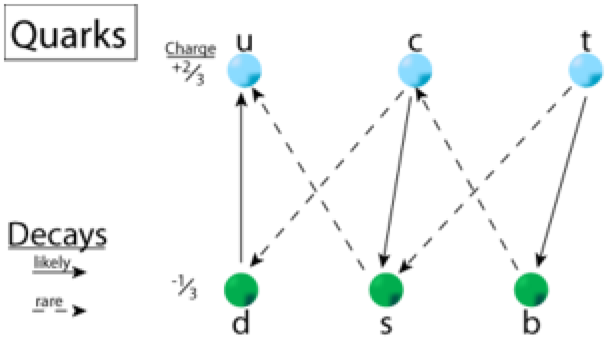
$$\alpha(s) = \alpha(0) + \alpha' s$$

- Nambu, Nielsen and Susskind independently proposed its open string interpretation



QCD

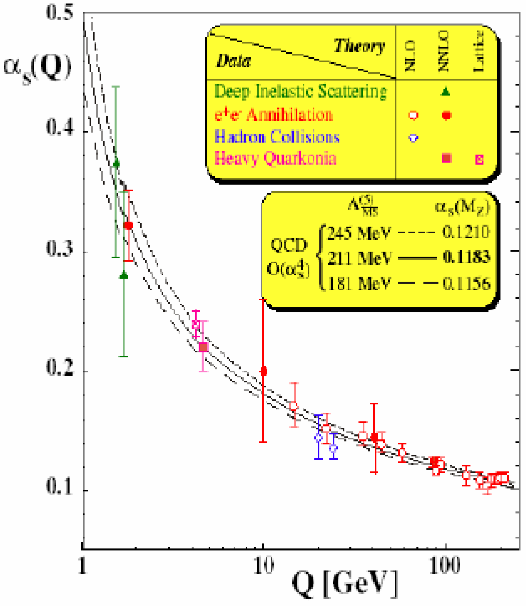
- In 1973 a point particle theory of strong interactions, inspired by the quark model, was proposed: the Quantum Chromodynamics – a Yang-Mills theory with gauge group SU(3).
- The hadrons are made of spin 1/2 constituents called quarks and spin 1 ones called gluons. Quarks come in 6 known flavors (up, down, strange, charm, bottom, top), and each flavor comes in 3 different color states (red, green, blue).
- The adjoint gluons come in $3^2 - 1=8$ different color states:



$$S = -\int d^4x \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu}^2$$

Asymptotic Freedom

- As the energy increases, the interactions between the quarks and gluons, though still quite strong, gradually weaken. This provided a theoretical explanation of such effects observed in the late 60's at energies above a few GeV.



Gross & Wilczek; Politzer

Clay Millenium Problems

Yang-Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Status: Unsolved

STRINGS MICHIGAN THEORY

July 10-15, 2000 University of Michigan Ann Arbor

"Millennium Madness"

Physics Problems for the Next Millennium

The best 10 problems were selected at the end of the conference by a selection panel consisting of:

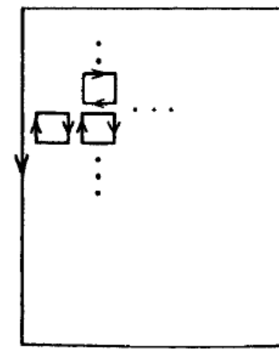
- Michael Duff (University of Michigan)
- David Gross (Institute for Theoretical Physics, Santa Barbara)
- Edward Witten (Caltech & Institute for Advanced Studies)

10. Can we quantitatively understand quark and gluon confinement in Quantum Chromodynamics and the existence of a mass gap?
    Igor Klebanov, Princeton University
    Oyvind Tafjord, McGill University

Lattice SU(N) Gauge Theory

- The gauge field kinetic term is encoded in the plaquette terms.

$$S = -(1/2g^2) \sum_{n,\,\mu\nu} \mathrm{tr}\, U_\mu(n) U_\nu(n+\mu)\, U_{-\mu}(n+\mu+\nu) U_{-\nu}(n+\nu) + \mathrm{h.c.}$$



- In the strong coupling expansion where these terms are treated as perturbations, the Area Law of the Wilson loop is obvious.

$$\left\langle \prod_C \exp[iB_\mu(n)] \right\rangle \approx \exp[-F(g^{-2})A]$$

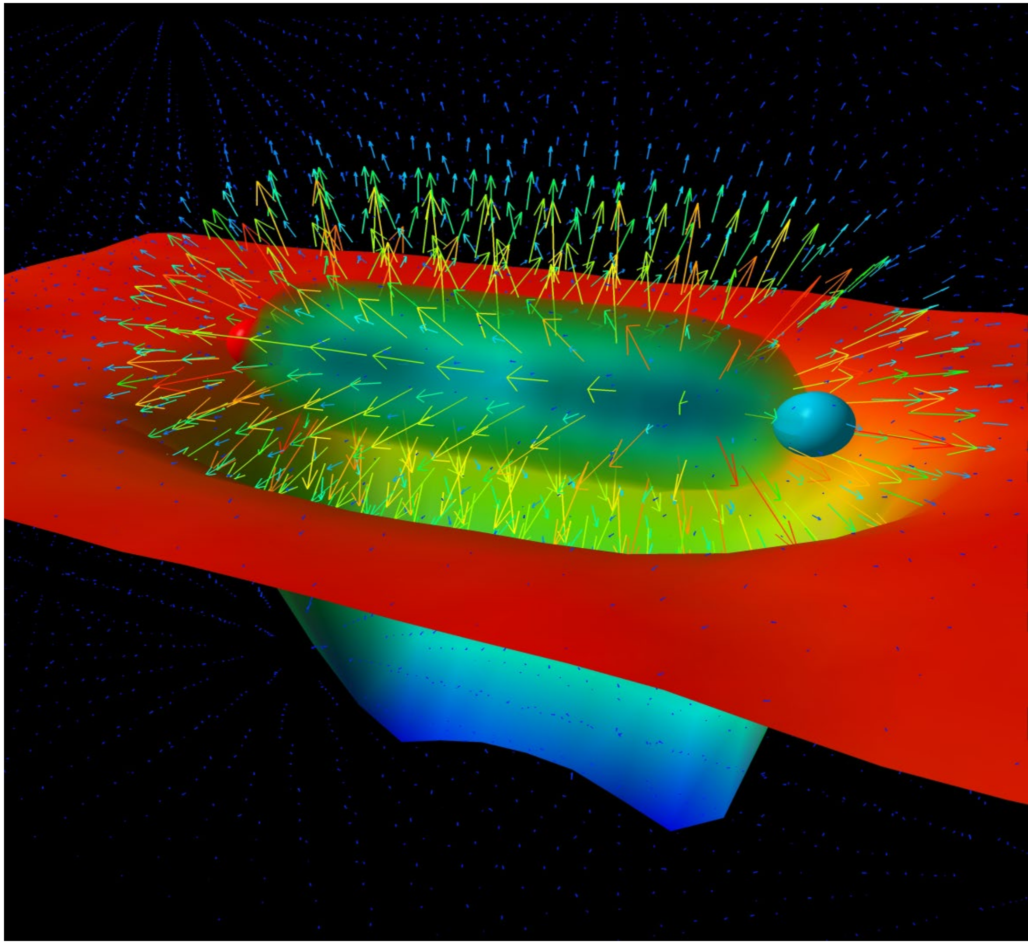
- To obtain the continuum limit, one needs to interpolate to the weak coupling limit on lattice scale due to Asymptotic Freedom

$$g^2(a) = \frac{g_0^2}{1 + (Cg_0^2/2\pi)\ln(a_0/a)}$$

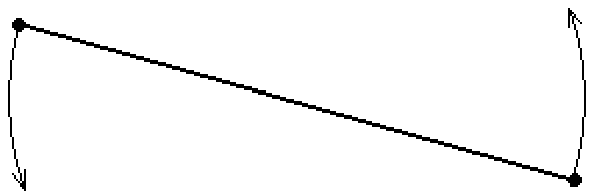
- Can color confinement disappear in this limit? Numerical simulations strongly suggest that the answer is “No.” Lattice sizes up to ~100^4 now.

QCD and Strings

- At distances much smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
- At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is described by the Nambu-Goto area action with corrections.
- So, strings have been observed in numerical simulations of Yang-Mills theory!

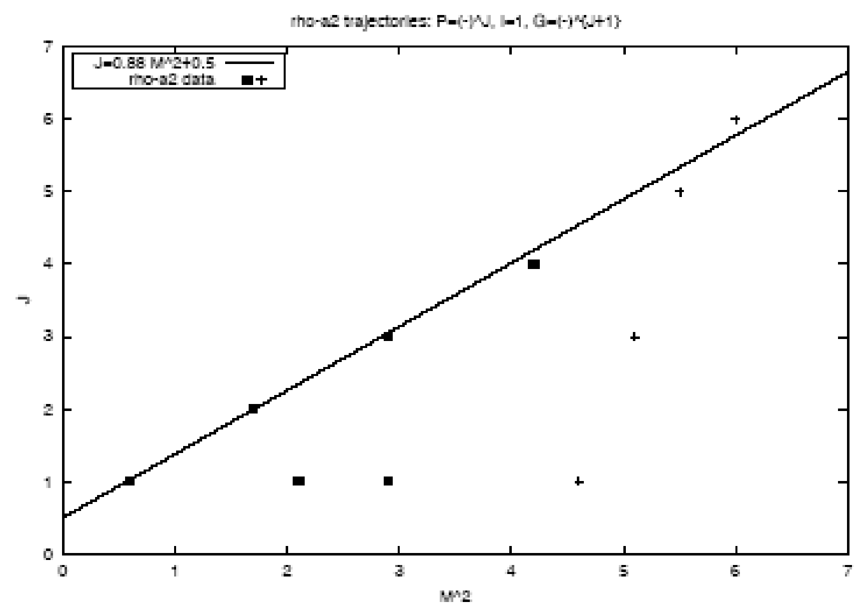


Open String Picture of Mesons



$$J = \alpha' m^2 + \alpha(0)$$

- Mesons are identified with excitations (rotational and vibrational) of a relativistic string of energy density ~ 1 GeV/fm, which is around 1.6 kiloJoules/cm.
- Regge trajectory starting with J=1 ρ meson.



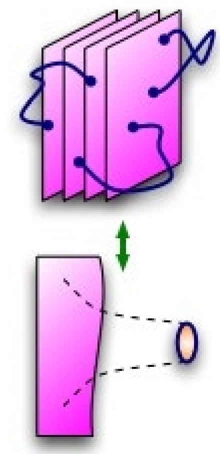
Large N Yang-Mills Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling fixed:

$$\lambda = g_{\mathrm{YM}}^2 N$$

- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.

D-Branes vs. Geometry



- Dirichlet branes led string theory back to gauge theory in the mid-90's. Polchinski
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

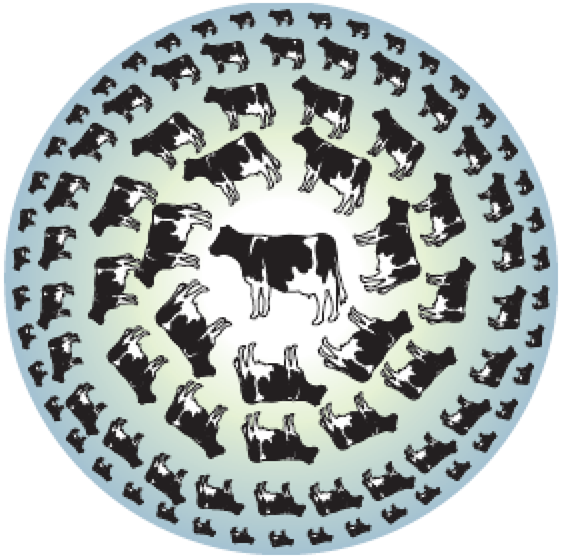
which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by $L^4 = g_{\rm YM}^2 N \alpha'^2$

The AdS/CFT Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS space-time is a generalized hyperboloid. It has negative curvature.



- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\mathrm{YM}}^2 N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of
$$\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

Entanglement Entropy

- Divide d-dimensional space into two complementary regions, A and B. Their quantum entanglement entropy is the entropy seen by an observer in A who does not have access to the degrees of freedom in B:

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

The reduced density matrix is

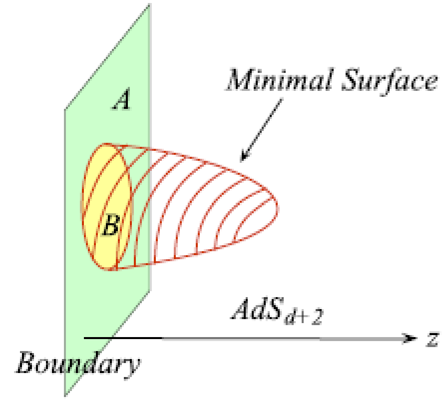
$$\rho_A = \mathrm{Tr}_B \rho_0 \qquad \rho_0 = |0\rangle\langle 0|$$

- In a QFT, the entanglement entropy is UV divergent and proportional to the volume of the boundary

$$S_A \simeq \frac{V_{d-1}}{a^{d-1}}$$

- In a d+2 dimensional gravity dual, the entanglement entropy is the area of the minimal d dimensional manifold γ which at the AdS boundary approaches the boundary between A and B

  Ryu, Takayanagi



$$S_A = \frac{1}{4G_N^{(d+2)}} \int_\gamma d^d\sigma \sqrt{G_{\mathrm{ind}}^{(d)}}$$

- For d=1 this gives the expected result $S_A = \frac{c}{3} \log \frac{l}{a}$

  Holzhey, Larsen, Wilczek; Cardy, Calabrese

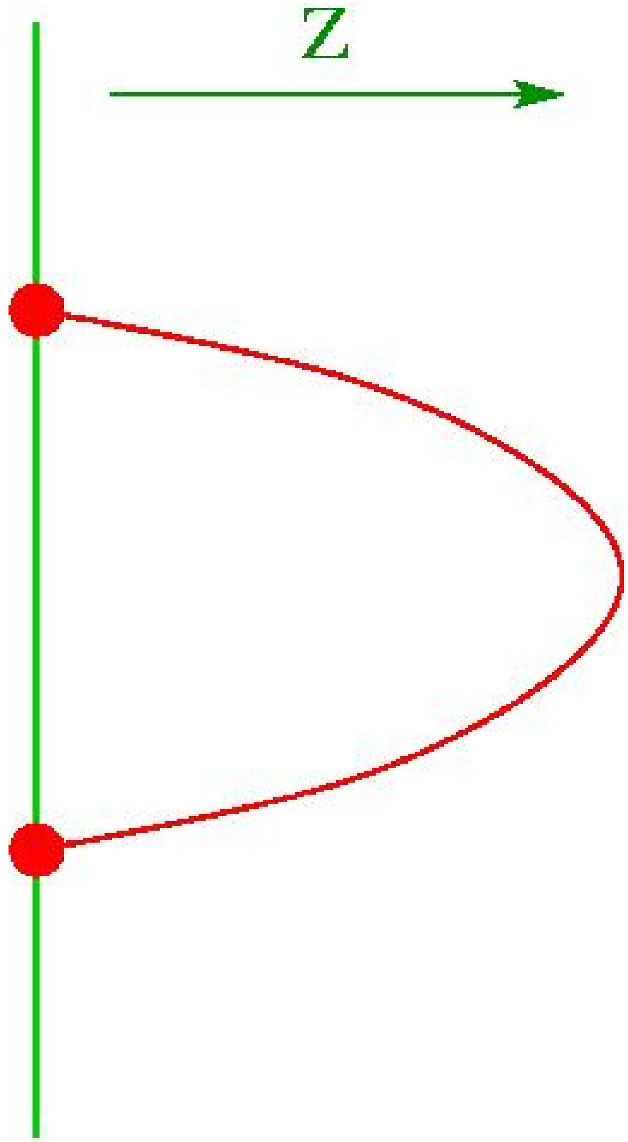
- For d>1 we get predictions about EE of strongly coupled field theories.

The quark anti-quark potential

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.

- The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential Maldacena; Rey, Yee

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



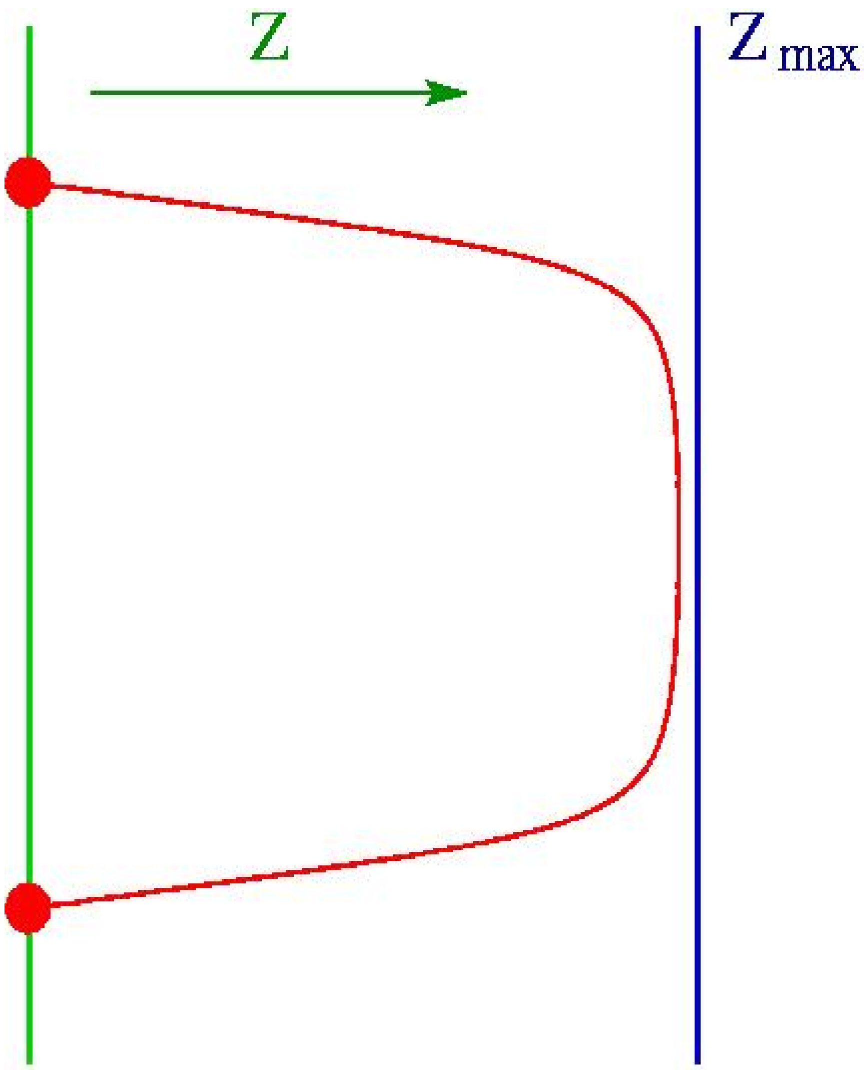
Color Confinement

- The quark anti-quark potential is linear at large distance but nearly Coulombic at small distance.
- The 5-d metric should have a warped form   Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

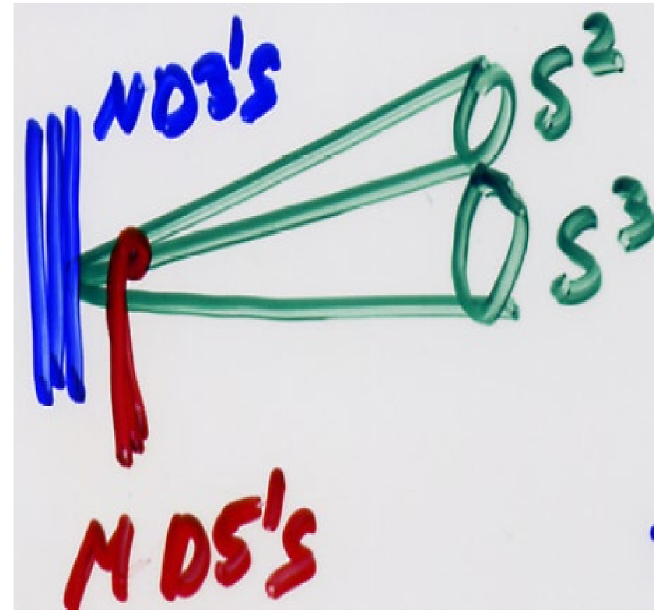
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Confinement and Warped Throat

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)



$$ds_{10}^2 = h^{-1/2}(y)\left(-(dx^0)^2 + (dx^i)^2\right) + h^{1/2}(y)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).
- The dual gravity provides a \`hyperbolic cow' approximation, i.e. a toy model, for QCD.

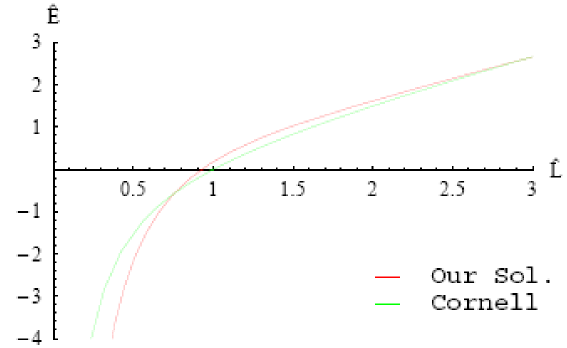
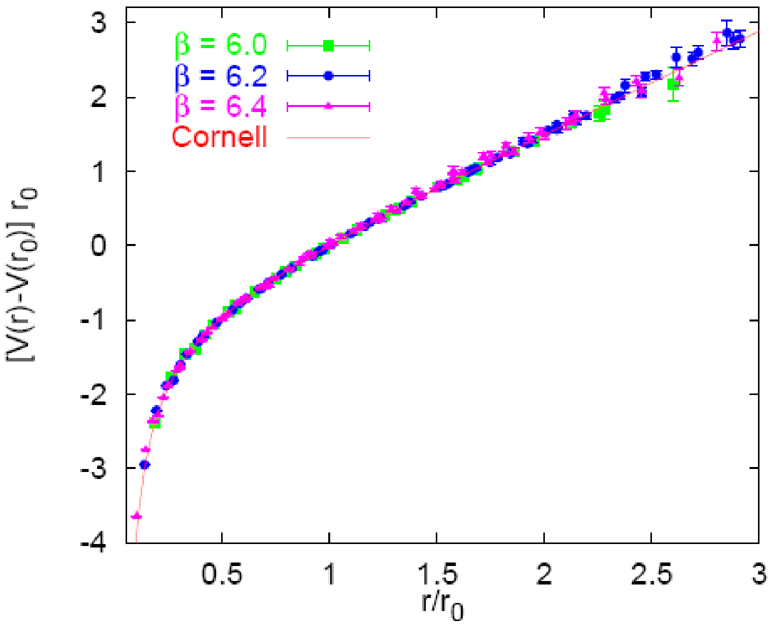
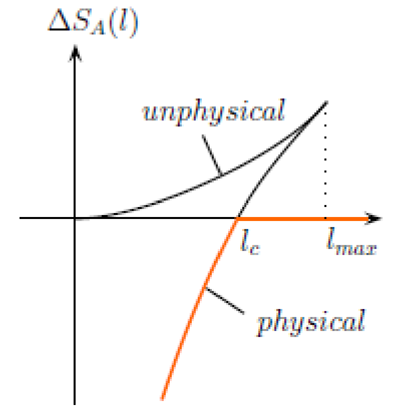


Figure 11: Comparison to the Cornell model



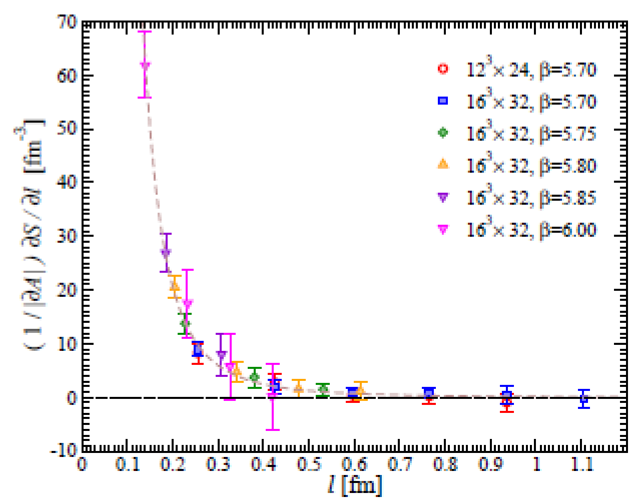
Confinement and Entanglement

- Due to the confinement, there is a phase transition in the behavior of the entanglement entropy as function of the strip width.
  IK, Kutasov, Murugan; Nishioka, Takayanagi



- There is evidence for a similar transition or crossover in lattice gauge theory. Velitsky; Buividovich, Polikarpov;

  Nakagawa, Nakamura, Motoki, Zakharov



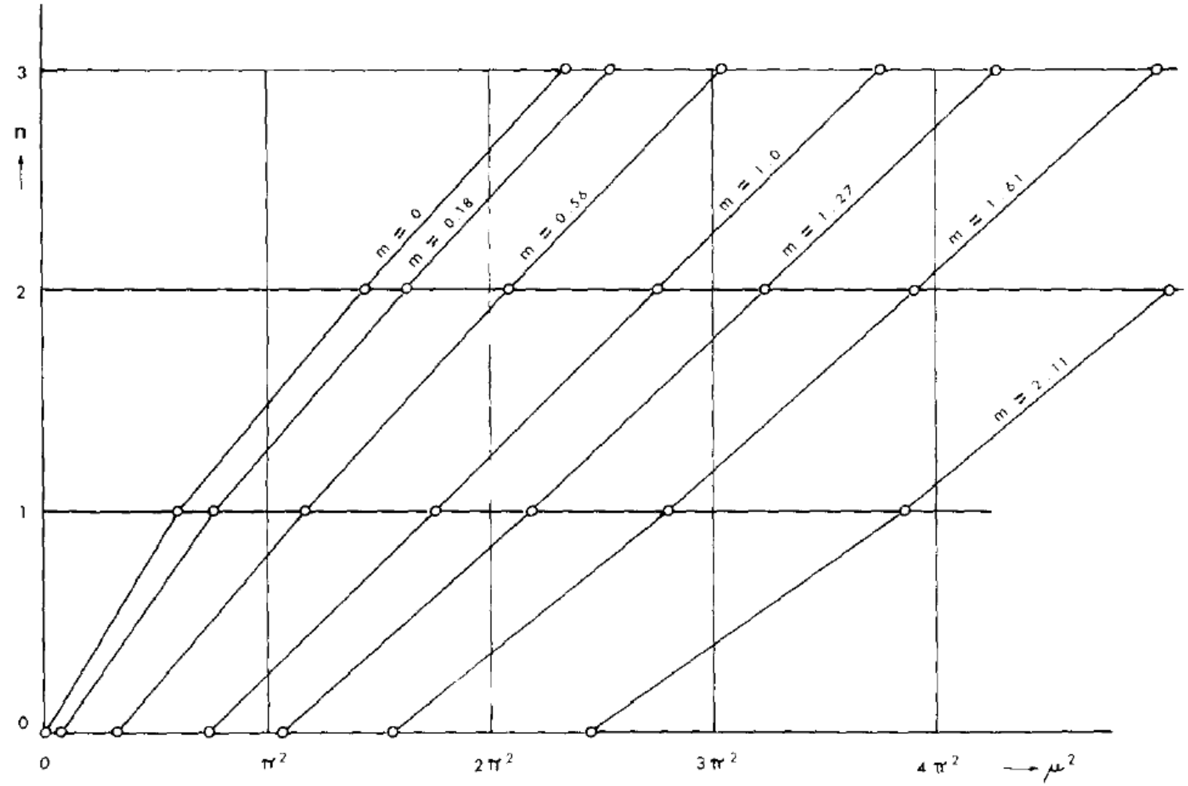
Back to Basics?

- The gauge/string duality has provided us with a "physicist's proof of confinement" in some exotic gauge theories like the one described by the warped deformed conifold.
- Yet, we still don't have a quantitative handle on the Asymptotically Free theories in 3+1 dimensions.
- As a modest step, "drop back" to 1+1 dimensional gauge theories, which can provide some intuition about aspects of the higher dimensional dynamics.

The ‘t Hooft Model

- 2d SU(N) gauge theory coupled to N_f fermions in the fundamental representation.
- Exactly solvable in the large N limit using the light-cone gauge: $A_- = 0$
- Find a single Regge trajectory of mesons whose masses are obtained by solving an integral equation.

$$\mu^2 \varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right) \varphi(x) - \mathrm{P}\int_0^1 \frac{\varphi(y)}{(y-x)^2}\, \mathrm{d}y$$



- Meson trajectories for different quark masses from ‘t Hooft’s 1974 paper.
- This beautiful toy model does not have any local adjoint degrees of freedom, which are crucial for higher-dimensional QCD.

2D QCD with Adjoint Matter

- Not exactly solvable at large N, but numerically tractable using Discretized Light-Cone Quantization (DLCQ). Dalley, IRK; Gross, Hashimoto, IRK
- The model with an adjoint Majorana fermion (a toy gluino) has particularly nice properties.
- Super-renormalizable, has non-trivial vacua and a mass gap. Witten; Kutasov; Demeterfi, IRK, Bhanot; Smilga

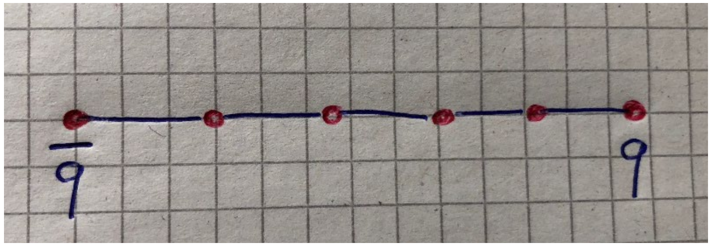
$$S_{\mathrm{f}} = \int d^2x \, \mathrm{Tr} \left[i\Psi^T \gamma^0 \gamma^\alpha D_\alpha \Psi - m\Psi^T \gamma^0 \Psi - \frac{1}{4g^2} F_{\alpha\beta} F^{\alpha\beta} \right]$$

A New 2D Model for Mesons

- If we add N_f fundamental Dirac fermions to the adjoint Majorana, we find a model which contains both gluinoballs and mesons: Dempsey, IRK, Pufu

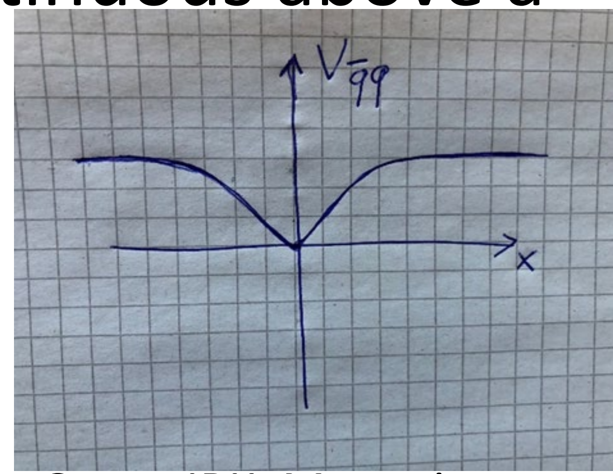
$$S = \int d^2x \left[\mathrm{tr} \left(-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \overline{\Psi} \not{D} \Psi - \frac{m_{\mathrm{adj}}}{2} \overline{\Psi} \Psi \right) + i \sum_{\alpha=1}^{N_f} \left(\overline{q}_\alpha \not{D} q_\alpha - m_{\mathrm{fund}} \overline{q}_\alpha q_\alpha \right) \right]$$

- The mesons are more complicated than in the ‘t Hooft model, since they also contain the adjoint quanta. There are now multiple Regge trajectories of mesons, which can be bosonic or fermionic.



Liberation

- For heavy fundamentals and massless adjoints the spectrum of mesons is continuous above a certain threshold. Dempsey, IRK, Pufu
- This implies that the q-qbar potential flattens at infinity!
- The massless adjoints renormalize string tension to 0. Gross, IRK, Matytsin, Smilga; Komargodski, Ohmori, Roumpedakis, Seifnashri
- For a small mass, it becomes non-zero giving a model of weak confinement.



Discussion

- Don’t take confinement for granted, even in 1+1 dimensions where it seems obvious.
- Proof of Color Confinement in 2+1 and 3+1 dimensions would be very important.
- Improve connections between QCD Strings and Fundamental Strings.
- Need new analytical insights and approximations to the spectra of hadrons and multi-string tensions.
- Guided by the improving numerical results on SU(N) gauge theories for a range of N. Athenodorou, Teper; …