Non-linear Electrodynamics and its Applications

Dmitri Sorokin

INFN, Padova Section

GGI Tea Breaks' Seminar, December 15, 2021

Preface

- Models of non-linear electrodynamics (NED for short) have been extensively studied as possible guides to catch new physics: e.g. for tackling fundamental cosmological problems (such as physics of black holes, inflation and dark energy), and for an effective description of properties of certain condensed matter materials and optical media.
- **Notable examples**:
  - **Born-Infeld electrodynamics (1934)** - invented to have finite electric field self-energy of charged particles. An important ingredient of Modern String Theory.
  - **Euler-Heisenberg effective theory (1936)** of Quantum Electrodynamics
- **There exist a "zoo" of NED models** containing such species as logarithmic, double-logarithmic, exponential, rational, inverse, arcsin, power-law and others which have been created during decades for various purposes (*Soleng, Ayon-Beato & Garcia, Cataldo et al, Bronnikov, Hassaine & Martinez, Cembranos et al, Gullu &Mazharimousavi, Hendi, Gaete & Helayel-Neto, Kruglov,…..*)
- **The newest NED is ModMax (2020)** – the unique modification of Maxwell's theory that preserves all its symmetries including **electric-magnetic duality and conformal invariance** (*I. Bandos, K. Lechner, D.S. & P.K. Townsend* '20)

Main aim of the talk to overview properties of the most characteristic specimens of this "zoo", including Heisenberg-Euler, Born-Infeld, Bialynicki-Birula and ModMax, and to flash some effects via which non-linearities of these and other models may manifest themselves in physical phenomena, such as vacuum birefringence, and their role in the context of gravity and condensed matter theory.

Maxwell's electrodynamics and its symmetries

- Free Maxwell action

$$\mathcal{S} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\int d^4x\, (\mathbf{E}^2 - \mathbf{B}^2)$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x), \qquad E_i = \partial_0 A_i - \partial_i A_0, \qquad B^i = \frac{1}{2}\varepsilon^{ijk}F_{jk}$$

$$\mu, \nu = 0, 1, 2, 3; \qquad i, j, k = 1, 2, 3$$

- Free Maxwell equations

$$\partial_\mu F^{\mu\nu} = 0$$

Bianchi identities: $\partial_\mu \tilde{F}^{\mu\nu} \equiv \partial_\mu \left(\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\right) = 0$

$\Longrightarrow$

$$\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B}, \qquad \nabla \cdot \mathbf{E} = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \qquad \nabla \cdot \mathbf{B} = 0$$

- Conformal invariance (e.g. rescaling): $x^\mu \to a\, x^\mu, \quad A_\mu \to a^{-1}A_\mu, \quad F_{\mu\nu} \to a^{-2}F_{\mu\nu}$

- Invariance under SO(2) duality rotation:
  (only equations of motion)

$$\begin{pmatrix} F'^{\mu\nu} \\ \tilde{F}'^{\mu\nu} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} F^{\mu\nu} \\ \tilde{F}^{\mu\nu} \end{pmatrix}$$

Non-linear electrodynamics (NED). General structure

- Generic NED action: $$I_{NED} = \int d^4x\, \mathcal{L}\,(S, P)$$

(a gauge-invariant non-linear functional of two independent Lorentz invariants)

$$S = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right), \qquad P = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \mathbf{E}\cdot\mathbf{B}$$

No field-strength derivatives ∂F in the action to avoid ghosts

Common assumption: NED action reduces to Maxwell's one in the week field limit: $(FF)^2 \ll FF$

$$\mathcal{L}_{NED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + a\,(F_{\mu\nu}F^{\mu\nu})^2 + b\,(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \mathcal{O}\left((FF)^3\right)$$

(Some models, such as the conformal NEDs including ModMax, do not satisfy this assumption)

The structure of the higher order terms in the NED actions depend on their origin and purposes

e.g. in the Heisenberg-Euler theory for spinor QED: $$b = \frac{7}{4}\,a\,, \qquad a = \frac{1}{90\mu_0}\frac{\hbar^3\alpha^2}{m_e^4c^5} = 4\times 10^{-25}T^{-2}$$

for scalar QED: $$b = \frac{1}{7}\,a\,, \qquad a = \frac{7}{5760\mu_0}\frac{\hbar^3\alpha^2}{m_e^4c^5}$$

(Weisskopf '36, Schwinger '51)

NEDs. Consistency conditions and symmetries

$$I_{NED} = \int d^4x\, \mathcal{L}(S,P)\,, \qquad S = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \quad P = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Conditions of causality and unitarity (*Gibbons & Hereira ‘00, Schabad & Usov ’09 -’11*) require $\mathcal{L}(\mathbf{E},\mathbf{B})$ be a convex function of $\mathbf{E}$ i.e. that the following Hessian matrix must have only non-negative eigenvalues for all values of $\mathbf{E}$ and $\mathbf{B}$

$$\mathcal{L}_{ij}(\mathbf{E},\mathbf{B}) = \frac{\partial^2\mathcal{L}}{\partial E^i \partial E^j}$$

Convexity also ensures the existence of an involutive Legendre transform to the Hamiltonian formulation

Symmetries: Lorentz invariance by construction, but no conformal and duality invariance in general

- Requirement for conformal invariance: $\mathcal{L}(a\,S, a\,P) = a\,\mathcal{L}(S,P) \quad \to \quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,\mathcal{V}\left(\frac{P}{S}\right)$
  (homogeneous function of degree 1)

- Requirement for duality invariance (Gaillard & Zumino ‘81, Bialynicki-Birula ‘83,…):

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial F_{\mu\nu}}\right) = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \frac{\partial\mathcal{L}}{\partial F_{\mu\nu}} \\ \tilde{F}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} \frac{\partial\mathcal{L}(F')}{\partial F'_{\mu\nu}} \\ \tilde{F}'^{\mu\nu} \end{pmatrix} \Rightarrow \boxed{F_{\mu\nu}\tilde{F}^{\mu\nu} - 2\varepsilon^{\mu\nu\lambda\rho}\frac{\partial\mathcal{L}}{\partial F^{\mu\nu}}\frac{\partial\mathcal{L}}{\partial F^{\lambda\rho}} = 0}$$

Born-Infeld electrodynamics

$$\mathcal{L}_{BI} = T - T\sqrt{-\det(\eta_{\mu\nu} + T^{-\frac{1}{2}}F_{\mu\nu})} = T - T\sqrt{1 + \frac{1}{2T}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16T^2}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{32\,T}\left((F_{\mu\nu}F^{\mu\nu})^2 + (F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right) + \mathcal{O}\left((F^2)^3\right)$$

T – coupling parameter of dimension of energy density

BI theory is duality-invariant (*Bialynicki-Birula* '83), but not conformal

Week field limit: $T \to \infty \quad \mathcal{L}_{BI} \to \mathcal{L}_{Maxwell}$

Strong field limit: $T \to 0 \quad \mathcal{L}_{BI} \to \frac{\mathrm{i}}{4}|F_{\mu\nu}\tilde{F}^{\mu\nu}|$ - total derivative
(zero tension)

Strong field limit can be taken in the Hamiltonian formulation of BI theory (*Bialynicki-Birula* '83)

Bialynicki-Birula electrodynamics (‘83, ‘92)

Born-Infeld Hamiltonian $\mathcal{H}_{BI}(\mathbf{D},\mathbf{B}) = \mathbf{D}\cdot\mathbf{E} - \mathcal{L}_{BI}(\mathbf{E},\mathbf{B}) = \sqrt{T^2 + T(\mathbf{D}^2+\mathbf{B}^2) + |\mathbf{D}\times\mathbf{B}|^2} - T$

$\mathbf{D} = \frac{\partial\mathcal{L}}{\partial\mathbf{E}} \left(\leftrightarrow \mathbf{E} = \frac{\partial\mathcal{H}}{\partial\mathbf{D}}\right)$ electric displacement vector (the momentum conjugate to the vector potential $\mathbf{A}$)

Strong field limit (*zero tension*) $T \to 0:$ $\mathcal{H}_{BI} \to$ $\boxed{\mathcal{H}_{BB} = |\mathbf{D}\times\mathbf{B}|}$

Remarkable properties of Bialynicki-Birula theory

- describes all possible null EM fields: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) = 0, \quad -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \mathbf{E}\cdot\mathbf{B} = 0$
- conformal
- invariant under enhanced SL(2,$\mathbf{R}$) duality
- has an infinite number of conserved currents $Q^{i_1 i_2 \cdots i_k} = \int dx^3\, \mathcal{H} n^{i_1} n^{i_2} \cdots n^{i_k}, \quad \mathbf{n} = \frac{\mathbf{D}\times\mathbf{B}}{|\mathbf{D}\times\mathbf{B}|}$
  (integrability?)

Modified Maxwell theory (ModMax)

Bandos, Lechner, D.S and Townsend, 2020

- **Unique** non-linear electrodynamics which is simultaneously duality-invariant and conformal, and reduces to Maxwell's ED in the zero-coupling limit

$$\mathcal{L}_{ModMax} = -\frac{\cosh\gamma}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\sinh\gamma}{4}\sqrt{(F_{\mu\nu}F^{\mu\nu})^2 + (F_{\mu\nu}\tilde{F}^{\mu\nu})^2}$$

$$= \frac{\cosh\gamma}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{\sinh\gamma}{2}\sqrt{(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{E}\cdot\mathbf{B})^2}$$

γ - dimensionless coupling parameter, $\gamma \geq 0$ - causality and unitarity requirement

at $\gamma = 0$ $\mathcal{L}_{ModMax} = \mathcal{L}_{Maxwell}$

- ModMax energy-momentum tensor is traceless due to conformal invariance

$$T^{\mu\nu} = \underbrace{\left(F^{\mu}{}_{\rho}F^{\nu\rho} - \frac{1}{4}\eta^{\mu\nu}(F_{\rho\lambda}F^{\rho\lambda})\right)}_{\text{Maxwell energy-momentum}}\left(\cosh\gamma - \sinh\gamma\frac{FF}{\sqrt{(FF)^2 + (F\tilde{F})^2}}\right)$$

$$T^{\mu}{}_{\mu} = 0$$

$T^{00} \geq 0$

ModMax Hamiltonian

ModMax energy density: $T^{00} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)\left(\cosh\gamma + \sinh\gamma \frac{\mathbf{E}^2 - \mathbf{B}^2}{\sqrt{(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{E}\cdot\mathbf{B})^2}}\right) \geq 0$

not well defined for the null EM fields $\mathbf{E}^2 - \mathbf{B}^2 = \mathbf{E}\cdot\mathbf{B} = 0$ which are e.g. plane waves

- This ambiguity $\left(e^{-\gamma} \leq \frac{0}{|0|} \leq e^{\gamma}\right)$ gets resolved in the ModMax Hamiltonian formulation

$$T^{00}(\mathbf{E},\mathbf{B}) = \mathcal{H}(\mathbf{D},\mathbf{B}) = \mathbf{E}\cdot\mathbf{D} - \mathcal{L}(\mathbf{E},\mathbf{B}), \qquad \mathbf{D} = \frac{\partial\mathcal{L}(\mathbf{E},\mathbf{B})}{\partial\mathbf{E}}, \qquad \mathbf{E} = \frac{\partial\mathcal{H}(\mathbf{D},\mathbf{B})}{\partial\mathbf{D}}$$

$$\mathcal{H}_{ModMax} = \frac{1}{2}\left(\cosh\gamma\,(\mathbf{D}^2 + \mathbf{B}^2) - \sinh\gamma\sqrt{(\mathbf{D}^2 + \mathbf{B}^2)^2 - 4(\mathbf{D}\times\mathbf{B})^2}\right)$$

For the null EM fields $\mathbf{E}^2 - \mathbf{B}^2 = \mathbf{E}\cdot\mathbf{B} = 0 \quad \Rightarrow \quad \mathbf{D}^2 + \mathbf{B}^2 = 2\cosh\gamma|\mathbf{D}\times\mathbf{B}| \quad \Rightarrow \quad \mathcal{H}|_{null} = \mathcal{H}_{BB} = |\mathbf{D}\times\mathbf{B}|$

Exact solutions of ModMax Hamiltonian equations

- plane waves (*Bandos, Lechner, D.S. and Townsend ‘20*)
- topologically non-trivial EM knots (*Dassy and Govaerts ‘21*)

Other NEDs and their implications for gravity and CMT

In the last couple of decades, in addition to the Born-Infeld and Heisenberg-Euler theories, a variety of other NED models have been proposed, coupled to gravity and used

- to construct new black hole solutions and study their properties (*Demianski ' 86, Wiltshire '88, de Oliveira '94, Rasheed '97, Ayon-Beato & Garcia '98, Bronnikov '00, Yajima & Tamaki '00, Gibons & Herdeiro'01, Moreno & Sarbach '02, ..., Kruglov '16, ...,Nomura et al '20, Daghigh & Green '21, Bokulic, Juric, & Smolic '21*)

- to mimic dark energy and Universe acceleration, and to explore cosmological models, in particular those which might avoid initial Big-Bang singularities (*Garcia-Salcedo & Breton '00, De Lorenci et al.'02, Sami et al, Elizalde et al '03, Novello & Bergliaffa '08, Kruglov '15, Ovgun et al '17*, ...)

- in the context of AdS/CFT correspondence and gravity/CMT holography, for instance to describe properties of superconductors, planar Mott insulators and strange metals (*Jing & Chen'10, Jin, Pan & Chen'11, Gangopadhyay & Roychowdhury '12, Baggioli & Pujolas'16, Kiritsis & Li '16, Cremonini et. al.'17,..., Baggioli et al.'21, Lai & Pan '21, Bi & Tao'21*)

Common set-up for gravity related applications of NEDs

- In the case of General Relativity (with a cosmological constant Λ) one considers

Einstein's equations sourced by NED and matter fields

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} (R - 2\Lambda) = 8\pi \left(T^{\mu\nu}_{NED} + T^{\mu\nu}_{matter} \right)$$

NED equations sourced by electric and magnetic currents

$$\nabla_\mu \left(\frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \right) = j^\mu_e, \qquad \nabla_\mu \tilde{F}^{\mu\nu} = j^\mu_m$$

One then looks e.g. for static black-hole solutions with a metric

$$ds^2_{BH} = -f(r)\, dt^2 + \frac{1}{f(r)}\, dr^2 + u(r)\, d\Omega^2$$

spherically symmetric black holes: $u(r)\, d\Omega^2 = r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2 \right)$

black holes in gravity/CMT holography: $u(r)\, d\Omega^2 = r^2 \left(dx^2 + dy^2 \right)$

NEDs and exact regular electric and magnetic black holes

(Ayon-Beato & Garcia'98-'00, Bronnikov'00, Burinskii &Hildebrandt '00…)

- **Example:** Bardeen (1968) regular BH as a magnetic monopole (*Ayòn-Beato & Garcìa '00*)

f(r) function in the Bardeen BH metric

$$f(r) = 1 - \frac{Mr^2}{(r^2 + Q_m^2)^{3/2}}\,, \qquad M \;-\; \text{mass}, \quad Q_m \;-\; \text{magnetic charge}$$

NED's Lagrangian which produce the source for the Bardeen BH

$$\mathcal{L}_{ABG} = \frac{3M}{|Q_m|^3}\left(\frac{|Q_m|\sqrt{-2S}}{1+|Q_m|\sqrt{-2S}}\right)^{5/2}, \qquad S = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,.$$

Electro-magnetic field strength is that of the monopole

$$\frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu = Q_m \sin\theta\, d\theta \wedge d\varphi$$

ModMax effects on black holes
(Taub-NUT, Reissner-Nordstrom)

Flores-Alfonso et al.; Ballon Bordo, Kubiznak & Perche; Amirabi & Mazharimousavi '00; Bokulic, Juric & Smolic; Zhang & Jiang; Ali & Saifullah '21

$$\mathcal{L}_{ModMax} = -\frac{\cosh\gamma}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\sinh\gamma}{4}\sqrt{(F_{\mu\nu}F^{\mu\nu})^2 + (F_{\mu\nu}\tilde{F}^{\mu\nu})^2}$$

Example: electrically and magnetically charged Reissner-Nordstrom BH

$$A_\mu dx^\mu = -e^{-\gamma}\frac{Q_e}{r}\,dt + Q_m\,cos\theta\,d\varphi\,, \qquad f(r) = 1 - \frac{2M}{r} + \frac{(Q_e^2 + Q_m^2)\,e^{-\gamma}}{r^2}$$

RN-BH Horizons: $f(r) = 0 \quad \rightarrow \quad r_\pm = M \pm \sqrt{M^2 - (Q_e^2 + Q_m^2)e^{-\gamma}} \quad \rightarrow \quad M^2 \geq (Q_e^2 + Q_m^2)e^{-\gamma}$

(screening effect:
mass of extremal BH
can be less than its charge)

Cosmological solutions without primordial singularity (smoothed by electromagnetic non-linearities)

- Primordial singularities in cosmological models e.g. the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology, is an important issue that signals non-applicability of classical theory of gravity at early times of Universe's life

FLRW metric
$$ds^2 = -dt^2 + a(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right),$$

where k=1,0,-1 stands for closed, flat or open Universe

$a(t)$ depends on the matter content, and the singularity occurs if $a(t_0)=0$ at an initial time

- Electromagnetic non-linearities may lift this singularity (*De Lorenci et. al. '02*)

e.g. in the BI-like or HE-like models

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha^2(F_{\mu\nu}F^{\mu\nu})^2 + \beta(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \quad \Rightarrow \quad a^2(t) \propto \sqrt{t^2+\alpha^2}$$

never vanishes

Universe expansion accelerated by NED

- A number of papers studied possibilities of accelerating the expansion of the Universe by dominated electromagnetic non-linearities at early time (inflation) and later stages of its life (*Elizalde et al '03*; *Novello et al.'03, Kruglov '15, Ovgun et al '17*, …)

- Example of *Novello et al.'03* $\mathcal{L}_{NED} = S + \frac{\beta}{S}, \qquad S = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

- Example of *Kruglov '15* – "rational NED" $\mathcal{L}_{rNED} = \frac{S}{1 + 2\beta S}$

The accelerated expansion is caused by non-linear EM fields which behave as a fluid with negative pressure which induces a positive term on the r.h.s. of Freedmann's equation

$$\rho + 3p = 2\left(\mathcal{L} + |\mathbf{B}|^2 \frac{\partial \mathcal{L}}{\partial S}\right) < 0 \quad \Rightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p) > 0$$

NEDs and gravity/CMT holography

- This framework provides a toolkit for establishing a dual relation between a weakly coupled gravity theory in an asymptotically anti-de-Sitter space-time and a strongly coupled field theory leaving on the AdS boundary. It prescribes how to translate the physics of the boundary field theory into the gravitational bulk and vice versa.

NEDs in the gravitational bulk have been used to study

- holographic superconductors (*Jing & Chen'10, Jin, Pan & Chen'11, Gangopadhyay & Roychowdhury '12,..., Lai & Pan '21*)
- planar Mott insulators (*Baggioli & Pujolas'16,..., Bi & Tao '21*)
- strange metals (*Kiritsis & L. Li '16, Cremonini et. al.'17,..., Baggioli et. al. '21*)

- The holographic set-up in the gravity bulk includes (non-linear) gauge and matter fields (e.g. axion, dilaton and/or charged scalar fields) and is described by the action

$$I = \int d^4x \sqrt{-g}\,(R - 2\Lambda + \mathcal{L}_{NED}(S,P,\phi) + \mathcal{L}(\phi)\,) + I_{bd}$$

(axion-dilaton-like fields are usually introduced to break translational invariance in the bulk which in the boundary CMT corresponds to a momentum dissipation)

- An ansatz for the background EM field: $A_\mu\, dx^\mu = A_t(r)dt + \frac{h}{2}(xdy - ydx), \quad h - \text{magnetic field}$

NEDs and gravity/CMT holography

- The gravitational background is assumed to be an asymptotically AdS black hole

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + u(r)\left(dx^2 + dy^2\right)$$

where r is the radial "holographic" coordinate, x and y are associated with the AdS boundary (at $r \to \infty$), and $f(r)$ and $u(r)$ are functions of r whose explicit form depends on the structure of the matter and gauge sector of the model. $f(r_h) = 0$ determines the black hole horizon.

Black holes are an important ingredient of the AdS/CMT correspondence since they are thermal states with $T_{BH} = f'(r_h)/4\pi$ related to the temperature in the dual strongly coupled field theory

- electric current operator j^μ in the CMT is dual to the EM field A_μ in the bulk

The physical characteristics of the CMT system depend on the choice of the concrete model of NED. For a wide class of models (in a probe field limit) the components of the electrical conductivity matrix are

$$\sigma_{xx} = \sigma_{yy} = \frac{\partial \mathcal{L}(S,P,\phi)}{\partial S}\Big|_{r_h}, \quad \sigma_{xy} = -\sigma_{yx} = -\frac{\partial \mathcal{L}(S,P,\phi)}{\partial P}\Big|_{r_h}$$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{yy}^2}, \quad \cot\Theta_H = \frac{\sigma_{xx}}{\sigma_{yy}}$$ - elec. resistivity and Hall angle

a starting point for studying temperature dependence, phase transitions etc.

NEDs and Birefringence

- in uniform electric/magnetic backgrounds the most of NEDs exhibit the effect of birefringence - phenomenon of double refraction whereby a ray of light is split by polarization (with respect to the external EM background) into two rays taking slightly different geodesic paths.

To compute the birefringence effect, in the NED Lagrangian $\mathcal{L}(F_{\mu\nu})$ one splits $F_{\mu\nu} = F^{ext}_{\mu\nu} + f_{\mu\nu}$

and derives the dispersion relations for a linearly polarized light propagating , e.g. in a strong magnetic field $\mathbf{B}_{\text{ext}}$

$$(\|) \quad \omega^2 = \frac{|\mathbf{k}|^2}{n^2_{\|}(\mathbf{k}, \mathbf{B}_{ext})}\,, \qquad (\perp) \quad \omega^2 = \frac{|\mathbf{k}|^2}{n^2_{\perp}(\mathbf{k}, \mathbf{B}_{ext})}$$

refraction indices $n(\mathbf{k}, \mathbf{B}_{ext})$ depend on properties of NED

$$\Delta n = n_{\|} - n_{\perp}$$ measures birefringence

For Maxwell and Born-Infeld: $\Delta n_{Max} = 0 = \Delta n_{BI}$; Heisenberg –Euler: $\Delta n_{QED} = 3.96 \times 10^{-24} \frac{B^2_{ext}}{T^2}$

For ModMax: $(\perp) \quad \omega^2 = |\mathbf{k}|^{\mathbf{2}} \quad - \quad \text{standar light} - \text{cone}$

$$(\|) \quad \omega^2 = |\mathbf{k}|^2(\cos^2\varphi + e^{-2\gamma}\sin^2\varphi), \quad \varphi - \text{angle between } \mathbf{k} \text{ and } \mathbf{B}_{\mathbf{ext}}$$

$$\Delta n_{ModMax} = e^{\gamma} - 1 = \gamma + \mathcal{O}(\gamma^2) \quad (\text{for } \mathbf{k} \perp \mathbf{B}_{ext})$$

PVLAS experiment bounds: $\Delta n_{exp} \leq (12 \pm 17) \times 10^{-23}$ for $|\mathbf{B}_{ext}| = 2.5$ Tesla

(arXiv:2005.12913)

Outlook

- For almost 90 years non-linear electrodynamics has been an active area of research in theoretical physics with applications ranging from the theory of fundamental interactions and cosmology to condensed matter systems, non-linear optics and plasma physics.
- All (or most of) the models of non-linear electrodynamics should be regarded as effective field theories.
- Only for few of them, the Born-Infeld and Heisenberg-Euler electrodynamics, we know the corresponding fundamental theories, such as String Theory and QED.
- Some NEDs may arise upon Kaluza-Klein dimensional reduction of a multi-dimensional higher-curvature gravity related to String Theory (*Gibbons & Herdeiro '00*)
- It would be of interest to look for a fundamental origin of other NEDs, in particular, of ModMax whose structure is uniquely determined by two fundamental symmetries, electric-magnetic duality and conformal invariance.
- Can ModMax arises as a certain (conformal) limit of a theory in which electromagnetic fields interact with some matter fields (e.g. axion-dilaton-like) and the latter are integrated out in the effective action?
- Is there any interpretation of ModMax and its BI extension in the context of String Theory (*Nastase '21*) ?

Work on these and other issues of NEDs and their applications is going ahead

Thank you!