Bubble wall velocity and high-energy string scattering

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Outline

- Gravitational waves from holography
- Bubble wall velocity
- High-energy string scattering
- Eikonal operator from string non-linear sigma model
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Cosmological first-order phase transitions (PT) source GWs

No first-order phase transitions in the Standard Model

Several Beyond-Standard Model scenarios involve strongly-coupled hidden gauge sectors where first-order phase transitions occur

Holography can be employed to study the gravitational waves produced in these first-order PTs [Bigazzi, AC, Cotrone, Paredes ’20, Ares, Hindmarsh, Hoyos, Jokela ’20, Ares, Henriksson, Hindmarsh, Hoyos, Jokela ’21]
GWs from first-order PTs

\begin{align*}
V(\Phi) &< 0.04 < 0.02 \quad \text{for } T > T_c \\
V(\Phi) &< 0.0004 < 0.0002 \quad \text{for } T < T_c
\end{align*}
GWs from first-order PTs

Decay through nucleation of true-vacuum bubbles

Nucleation rate computed through bounce action:

\[ \Gamma \approx T^4 e^{-S_B(\Phi_B)} \]

[Coleman '77]

Bounce solutions give access to the quantities needed to compute the GW spectrum.
Holographic Yang-Mills-like theory [Witten '98]

Backreaction of $N$ D4-branes wrapped on $S^1$ with inverse radius $M_{KK}$:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left(dt^2 + dx^i dx^i + f(u) dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2$$

$$f(u) = 1 - \left(\frac{u_0}{u}\right)^3$$

$$u_0 = \frac{4}{9} R^3 M_{KK}^2$$

Physics encoded in the cigar geometry ($\lambda = g_{YM}^2 N$)

- Mass gap $\sim M_{KK}$
- Confinement: $g_{00}(u_0) \neq 0$
- The free energy density is

$$f_{conf} = -\frac{1}{37 \pi^2} \lambda N^2 M_{KK}^4$$

- Dominant at low temperatures
High-temperature phase

At high temperatures the dominant background is the black hole one:

\[
ds^2 = \left(\frac{u}{R}\right)^{3/2} \left( f_T(u)dt^2 + dx^i dx^i + dx_4^2 \right) + \left( \frac{R}{u} \right)^{3/2} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2
\]

\[
f_T(u) = 1 - \left( \frac{u_T}{u} \right)^3
\]

- Deconfinement: \( g_{00}(u_0) = 0 \)
- The free energy density is

\[
f_{\text{deconf}} = - \frac{2^6 \pi^4}{3^7} \frac{\lambda N^2 T^6}{M_{KK}^2}
\]

- First-order phase transition at temperature \( T_c = M_{KK}/2\pi \)
Flavours [Sakai, Sugimoto '04, Antonyan et al. '06, Aharony et al. '07]:

Quark flavours introduced by $N_f$ D8/$\overline{D}8$ pairs

- If $N_f \ll N$, probe approximation: DBI action
  \[ S_{DBI} = - T_8 \int d^9 x e^{-\phi} \sqrt{- \det(g_{ab} + 2\pi \alpha' F_{ab})} \]

- If $L < 0.97 M_{KK}^{-1}$, $\chi$Sb PT at:
  \[ T_c^\chi = \frac{0.154}{L} > \frac{M_{KK}}{2\pi} \]

- First-order chiral symmetry PT
- Confinement PT at $T_c = M_{KK}/2\pi$
Chiral symmetry bubbles \cite{Bigazzi, AC, Cotrone, Paredes '20}

In principle: to find bubble-like solution \((x = x(\sigma, y))\) from

\[
S_{DBI} = \frac{NT^3 \lambda^3}{486 M^3_{KK}} \int d\sigma dy \, \sigma^2 y^{5/2} \sqrt{1 + (y^3 - 1)(\partial_y x)^2 + (\partial_\sigma x)^2}
\]

Bounce configuration from effective variational approach
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The GWs spectrum depends also on out-of-equilibrium quantities such as the asymptotic bubble wall velocity $v$.

Challenging also for weakly-coupled theories [Moore, Prokopec '95, Bodeker, Moore '17].

In the steady-state regime, the pressure gradient equates the friction force:

$$ F = A \Delta p $$

zero-force condition

We can thus determine the bubble wall velocity.

The friction force exerted on a probe quark by a strongly-coupled plasma has been holographically studied [Gubser '06, Herzog et al. '06]:

We analogously compute the drag force exerted on the bubble wall
Chiral symmetry bubbles [Bigazzi, AC, Cotrone, Paredes ’20]

Configuration at nucleation time

The bubble wall is a $D8$-brane branch

Configuration at asymptotic times

The bubble wall trails towards the horizon
The problem

Let us use the ansatz \( z = vt + \xi(u,x_4) \) (ansatz \( z = vt \) not allowed!)

\[
S = -\frac{k}{L} \int dt \, du \, dx_4 \left( \frac{u}{R} \right)^{-3/2} u^4 \sqrt{1 + (\partial_4 \xi)^2 + f_T(u) \left( \frac{u}{R} \right)^3 (\partial_u \xi)^2} - f_T(u)^{-1} v^2
\]

\[
\partial_u \pi^u_\xi + \partial_4 \pi^4_\xi = 0 \quad k \equiv \frac{T_8}{g_s} A L V(S^4)
\]

where

\[
\pi^u_\xi = k \frac{u^4 f_T(u) \left( \frac{u}{R} \right)^{3/2} \partial_u \xi}{\sqrt{1 + (\partial_4 \xi)^2 + f_T(u) \left( \frac{u}{R} \right)^3 (\partial_u \xi)^2} - f_T(u)^{-1} v^2}
\]

\[
\pi^4_\xi = k \frac{u^4 \left( \frac{u}{R} \right)^{-3/2} \partial_4 \xi}{\sqrt{1 + (\partial_4 \xi)^2 + f_T(u) \left( \frac{u}{R} \right)^3 (\partial_u \xi)^2} - f_T(u)^{-1} v^2}
\]

Integration of e.o.m. gives zero-force condition. Difficult problem!

We employ a rectangular approximation
The drag force

Let us consider the bubble wall as a separate rectangular entity: \( \pi_\xi^4 = 0 \).

\[
\xi' = \pi_\xi^u \left( \frac{u}{R} \right)^{-3/2} f_T(u)^{-1/2} \sqrt{\frac{1 - f_T(u)^{-1} v^2}{k^2 f_T(u) u^8 - (\pi_\xi^u)^2}}
\]

The momentum \( \pi_\xi^u \) is conserved.

\[
u_c(v) = \frac{u_T}{(1 - v^2)^{1/3}} \quad \quad \pi_\xi^u = k \frac{u_T^4}{(1 - v^2)^{4/3}} \frac{v}{w_f(1 - v^2)^{1/6}}
\]

Drag force given by the momentum flow towards the horizon [Gubser '06, Herzog et al. '06]:

\[
\frac{F_d}{A} = \frac{dp_z}{dt} = \pi_\xi^u = C_d \frac{T_{boost}}{T_c} w_f(T_{boost}) v
\]

where \( w_f \) is the enthalpy density of the false vacuum and

\[
T_{boost} = \frac{T}{(1 - v^2)^{1/6}} \quad \quad C_d = 2\pi \frac{p_{\text{glue}}}{w_{\text{glue}}} \kappa_c
\]
The complete steady-state configuration

With wall embedded in the full brane configuration, we derive the zero-force condition

\[ F = \frac{1}{L} \int d x_4 \pi_\xi^u(u_T, x_4) = A \Delta p \quad \Delta p = -\frac{T}{V_3} \Delta S_{DBI} \]

In the complete configuration \( \pi_\xi^u \) is not conserved. In rectangular approximation:

\[ \partial_u \pi_\xi^u = -2 k R^{3/2} u^{5/2} / L \]

After integration

\[ u_*^{7/2} = u_T^{7/2} + \frac{7L}{4kR^{3/2}} A \Delta p \quad u_* \geq u_T \]

The profile satisfies

\[
\xi' = -\frac{4 k R^{3/2}}{7} \left( u^{7/2} - u_*^{7/2} \right) \left( \frac{u}{R} \right)^{-3/2} f_T(u)^{-1/2} \sqrt{\frac{1 - f_T(u)^{-1} \nu^2}{k^2 f_T(u) u^8 - \frac{4}{7} \frac{k R^{3/2}}{L} \left( u^{7/2} - u_*^{7/2} \right)^2}}
\]
**Bubble wall velocity formula** [Bigazzi, AC, Canneti, Cotrone '21]

At the induced horizon

\[ kf_{T}^{1/2} u_{c}^{4} = -\frac{4}{7} \frac{k R^{3/2}}{L} (u_{c}^{7/2} - u_{*}^{7/2}) \]

Explicitly,

\[ \Delta p = \frac{2^{5}}{3^{9}} \pi^{3} \lambda^{3} N N_{f} (LT) \frac{T^{7}}{M_{KK}^{3}} \left[ \frac{v}{(1 - v^{2})^{4/3}} - \frac{4}{7} \frac{3}{4\pi L T} \left( 1 - \frac{1}{(1 - v^{2})^{7/6}} \right) \right] \]

We can write

\[ \Delta p = \frac{F}{A} \equiv \frac{F_{d}}{A} + p_{f}(T_{\text{boost}}) - p_{f}(T) \]

Using the result for \( F_{d} \), we find the formula for the velocity:

\[
\nu = C^{-1}_{d} \frac{T_{c}}{T_{\text{boost}}} \frac{p_{t}(T) - p_{f}(T_{\text{boost}})}{w_{f}(T_{\text{boost}})}
\]

Formula valid also for analogous \textit{Dp-Dq-}\textit{Dq}-brane setups.
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String Theory is a theory of gravity
Usually formulated in flat-spacetime background

How do curved geometries emerge?

- High-energy string-string scattering [Amati, Ciafaloni, Veneziano '87]
- Unitarity retrieved in the eikonal approximation:

\[ S(s,b) = e^{2i\hat{\delta}(s,b)} \]

\[ 2\hat{\delta}(s,b) = \int \frac{d\sigma_u d\sigma_d}{(2\pi)^2} : A_1 \left( s,b + \hat{X}_u(\sigma_u) - \hat{X}_d(\sigma_d) \right) : \]

- Compactible with semiclassical propagation in Aichelburg-Sexl geometry

Emergent geometry very complicated at subleading orders
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  \]
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Emergent geometry very complicated at subleading orders
String-brane scattering: an easier problem

Scattering of a string on stack of $N$ $D$-branes [D’Appollonio, Di Vecchia, Russo, Veneziano ’10].

Two descriptions:
- Open string sectors in flat-spacetimes
- $p$-brane solutions in supergravity

\[ ds^2 = \frac{1}{\sqrt{H(r)}} \eta_{\alpha\beta} dx^\alpha dx^\beta + \sqrt{H(r)} \delta_{ij} dx^i dx^j \]

\[ H(r) = 1 + \left( \frac{R_p}{r} \right)^{7-p}, \quad R_p^{7-p} = \frac{g_s N (2\pi \sqrt{\alpha'})^{7-p}}{(7-p) \Omega_{8-p}} \]

We study string-brane scattering in the Regge limit

\[ s \to \infty, \quad \frac{t}{s} \to 0 \]

The two complementary approaches are comparable in the limit

\[ g_s \to 0 \quad \lambda = g_s N \gg 1 \]

Resumming many-boundaries ampl. $\leftrightarrow$ semi-cl. string prop. on $p$-brane geometry.
String-brane scattering: an easier problem

Sum of amplitudes with many boundaries:

\[ S(s, b) = e^{2i\hat{\delta}(s, b)}, \quad 2\hat{\delta}(s, b) = \frac{1}{2E} \int \frac{d\sigma}{2\pi} : A_1 \left( s, b + \hat{X}(\sigma) \right) : \]

The eikonal operator \( \hat{\delta}(s, b) \) can be expanded as

\[ 2\hat{\delta}(s, b + \hat{X}) \sim \frac{1}{2E} \left[ A_1(s, b) + \frac{1}{2} \frac{\partial^2 A_1(s, b)}{\partial b_i \partial b_j} \hat{X}^i \hat{X}^j + ... \right] \]

where

\[ A_1(s, b) \sim s \sqrt{\pi} \frac{\Gamma \left( \frac{6-p}{2} \right) R_p^{7-p}}{\Gamma \left( \frac{7-p}{2} \right) b^{6-p}} + \text{imaginary part} + O \left( \frac{R_p}{b} \right)^{2(7-p)} \]

Studying the semiclassical string propagation in the Penrose limit of the \( p \)-brane solutions, the second term can be computed.

Agreement between the two approaches [D'Appollonio, Di Vecchia, Russo, Veneziano '10]
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The propagation of a highly-energetic string is described by

\[ S = -\frac{T}{2} \int d\sigma d\tau G_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu \]

The Penrose limit is the first order of a systematic expansion in Fermi coordinates around a null geodesic \( X_{bg}(\tau) \) [Blau et al '06]

\[ X(\sigma, \tau) = X_{bg}(\tau) + \xi(\sigma, \tau) \]

Recursive method to write the terms [Mukhi '85]

\[
S_{X=X_{bg}}^{(0)} = \frac{T}{2} \int d\sigma d\tau \ G_{AB}(X_{bg}) \dot{X}^A_{bg} \dot{X}^B_{bg} \\
S_{X=X_{bg}}^{(2)} = -\frac{T}{2} \int d\sigma d\tau \ R_{ABCD}(X_{bg}) \xi^B \xi^C \partial_\alpha X^A_{bg} \partial_\beta X^D_{bg} + \\
-\frac{T}{2} \int d\sigma d\tau \ G_{AB}(X_{bg}) D_\alpha \xi^A D_\alpha \xi^B \\
S_{X=X_{bg}}^{(3)} = -\frac{T}{6} \int d\sigma d\tau \ D_A R_{ABCDE}(X_{bg}) \partial_\alpha X^B_{bg} \partial_\alpha X^E_{bg} \xi^C \xi^D \xi^A + \\
-\frac{2T}{3} \int d\sigma d\tau \ R_{ABCD} \partial_\alpha X^A_{bg} D_\alpha \xi^D \xi^B \xi^C
\]
Background field expansion of the Polyakov action

The scaling of the quantities with energy is controlled by the vielbeins:

\[ E_+ \sim E, \quad E_- \sim E^{-1}, \quad E_a \sim 1 \]

Dominant terms with two "+" indices: plane wave \(\rightarrow\) light-cone gauge.

We can resum the series. In the impulsive approximation:

\[
S - S_0 = \frac{1}{2E} \int d\mu d\nu E_+^\mu E_+^\nu h_{\mu\nu}(X_{bg}) + \sum_{n=2}^{\infty} \frac{1}{n!} C_{a_1...a_{n-2}bc} \xi_{a_1}(0) ... \xi_{a_{n-2}}(0) \xi_{b}(0) \xi_{c}(0)
\]

where \(S_0\) is the string action in flat spacetime, \(h_{\mu\nu} = G_{\mu\nu} - G_{\mu\nu}^{\text{flat}}\) and

\[
C_{a_1...a_{n-2}bc} = -\frac{1}{E} \int_{-\infty}^{\infty} du D_{a_1} ... D_{a_{n-2}} R_{+b+c}(X_{bg}(u))
\]

In the leading eikonal approximation:

\[
S - S_0 = \frac{1}{2E} \int \frac{d\sigma}{2\pi} g(b + \xi(\sigma,0))
\]
Comparison

We thus expect the correspondence

\[ g(b + \xi(\sigma,0)) \leftrightarrow A_1 \left( s, b + \hat{X}(\sigma) \right) \]

As a check, we studied the shock-wave case for which the exact $S$-matrix is known [Amati, Klimcic '87]. This case is eikonal exact and indeed $g$ gives the exact $S$-matrix.

For the $p$-brane case, at zero order we find

\[
g(b) = E^2 \sqrt{\pi} \frac{R_p^{7-p} \Gamma \left( \frac{6-p}{2} \right)}{b^{6-p} \Gamma \left( \frac{7-p}{2} \right)} + O \left( \frac{R_p}{b} \right)^{2(7-p)}
\]

At quadratic order we (obviously) retrieve the results found with the Penrose limit. Higher-order terms reconstruct the Taylor series such that

\[ g(b) \to g(b + \xi(\sigma,0)) \]

Eikonal operator from the curved-background approach!
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Higher-order terms reconstruct the Taylor series such that

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Eikonal operator from the curved-background approach!
Thank you for your attention!