Bubble wall velocity and high-energy string scattering

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Based on:

F. Bigazzi, AC, T. Canneti, A. L. Cotrone 2104.12817
S. Bonansea, AC, G. D'Appollonio to appear

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Outline

- Gravitational waves from holography
- Bubble wall velocity
- High-energy string scattering
- Eikonal operator from string non-linear sigma model

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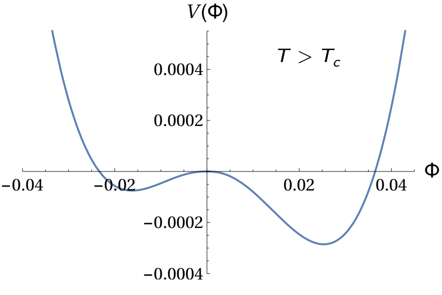
Context

- Cosmological first-order phase transitions (PT) source GWs
- No first-order phase transitions in the Standard Model
- Several Beyond-Standard Model scenarios involve strongly-coupled hidden gauge sectors where first-order phase transitions occur
- Holography can be employed to study the gravitational waves produced in these first-order PTs [Bigazzi, AC, Cotrone, Paredes '20, Ares, Hindmarsh, Hoyos, Jokela '20, Ares, Henriksson, Hindmarsh, Hoyos, Jokela '21]

GWs from first-order PTs



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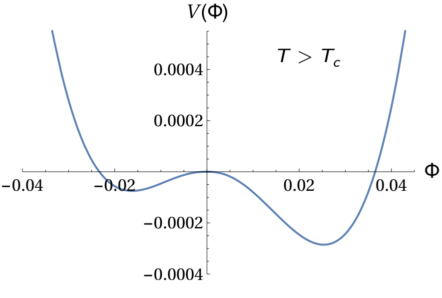


Decay through nucleation of true-vacuum bubbles

Nucleation rate computed through bounce action:

$$\Gamma \approx T^4 e^{-S_B(\Phi_B)} \qquad \text{[Coleman '77]}$$

Bounce solutions give access to the quantities needed to compute the GW spectrum.

Backreaction of N D4-branes wrapped on S^1 with inverse radius M_{KK}:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(dt^2 + dx^i dx^i + f(u)dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2}\frac{du^2}{f(u)} + R^{3/2}u^{1/2}d\Omega_4^2$$

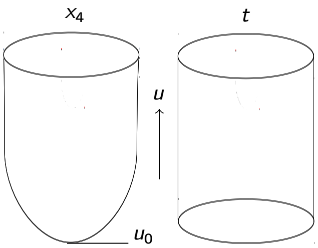
$$f(u) = 1 - \left(\frac{u_0}{u}\right)^3 \qquad\qquad u_0 = \frac{4}{9}R^3 M_{KK}^2$$

Physics encoded in the cigar geometry ($\lambda = g_{YM}^2 N$)

- Mass gap $\sim M_{KK}$
- Confinement: $g_{00}(u_0) \neq 0$
- The free energy density is

$$f_{conf} = -\frac{1}{3^7\pi^2}\lambda N^2 M_{KK}^4$$

- Dominant at low temperatures



High-temperature phase

At high temperatures the dominant background is the black hole one:

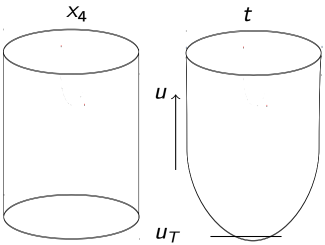
$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(f_T(u)dt^2 + dx^i dx^i + dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2}\frac{du^2}{f(u)} + R^{3/2}u^{1/2}d\Omega_4^2$$

$$f_T(u) = 1 - \left(\frac{u_T}{u}\right)^3 \qquad u_T = \frac{16}{9}\pi^2 R^3 T^2$$

- Deconfinement: $g_{00}(u_0) = 0$
- The free energy density is

$$f_{deconf} = -\frac{2^6\pi^4}{3^7}\lambda N^2 \frac{T^6}{M_{KK}^2}$$

- First-order phase transition at temperature $T_c = M_{KK}/2\pi$



Flavours [Sakai, Sugimoto '04, Antonyan et al. '06, Aharony et al. '07]:

Quark flavours introduced by N_f $D8/\overline{D8}$ pairs

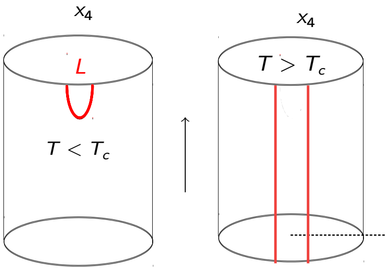
- If $N_f \ll N$, probe approximation: DBI action

$$S_{DBI} = -T_8 \int d^9x\, e^{-\phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}$$

- If $L < 0.97 M_{KK}^{-1}$, χSb PT at:

$$T_c^{\chi} = \frac{0.154}{L} > \frac{M_{KK}}{2\pi}$$

- First-order chiral symmetry PT
- Confinement PT at $T_c = M_{KK}/2\pi$

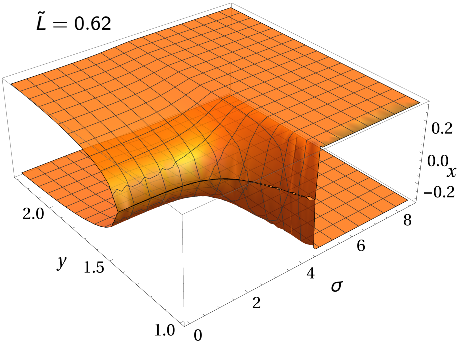


Chiral symmetry bubbles *[Bigazzi, AC, Cotrone, Paredes '20]*

In principle: to find bubble-like solution ($x = x(\sigma, y)$) from

$$S_{DBI} = \frac{NT^3\lambda^3}{486 M_{KK}^3} \int d\sigma dy\, \sigma^2 y^{5/2} \sqrt{1 + (y^3 - 1)(\partial_y x)^2 + (\partial_\sigma x)^2}$$

Bounce configuration from effective variational approach



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Bubble wall velocity [Bigazzi, AC, Canneti, Cotrone '21]

The GWs spectrum depends also on out-of-equilibrium quantities such as the asymptotic bubble wall velocity v.

Challenging also for weakly-coupled theories [Moore, Prokopec '95, Bodeker, Moore '17].

In the steady-state regime, the pressure gradient equates the friction force:

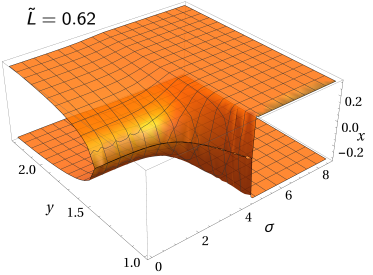
$$F = A\Delta p \qquad \text{zero-force condition}$$

We can thus determine the bubble wall velocity.

The friction force exerted on a probe quark by a strongly-coupled plasma has been holographically studied [Gubser '06, Herzog et al. '06]:

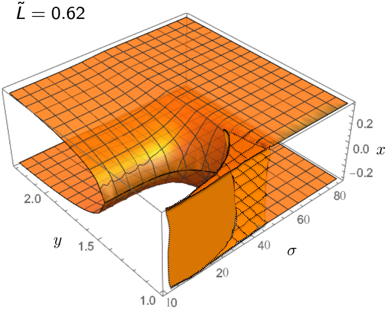
We analogously compute the drag force exerted on the bubble wall

Chiral symmetry bubbles *[Bigazzi, AC, Cotrone, Paredes '20]*



Configuration at nucleation time

The bubble wall is a $D8$-brane branch



Configuration at asymptotic times

The bubble wall trails towards the horizon

The problem

Let us use the ansatz $z = vt + \xi(u,x_4)$ (ansatz $z = vt$ not allowed!)

$$S = -\frac{k}{L}\int dt\, du\, dx_4 \left(\frac{u}{R}\right)^{-3/2} u^4 \sqrt{1 + (\partial_4\xi)^2 + f_T(u)\left(\frac{u}{R}\right)^3 (\partial_u\xi)^2 - f_T(u)^{-1} v^2}$$

$$\partial_u \pi_\xi^u + \partial_4 \pi_\xi^4 = 0 \qquad\qquad k \equiv \frac{T_8}{g_s} A\, L\, V(S^4)$$

where

$$\pi_\xi^u = k \frac{u^4 f_T(u)\left(\frac{u}{R}\right)^{3/2} \partial_u\xi}{\sqrt{1 + (\partial_4\xi)^2 + f_T(u)\left(\frac{u}{R}\right)^3 (\partial_u\xi)^2 - f_T(u)^{-1} v^2}}$$

$$\pi_\xi^4 = k \frac{u^4 \left(\frac{u}{R}\right)^{-3/2} \partial_4\xi}{\sqrt{1 + (\partial_4\xi)^2 + f_T(u)\left(\frac{u}{R}\right)^3 (\partial_u\xi)^2 - f_T(u)^{-1} v^2}}$$

Integration of e.o.m. gives zero-force condition. Difficult problem!

We employ a *rectangular* approximation

The drag force

Let us consider the bubble wall as a separate rectangular entity: $\pi^4_\xi = 0$.

$$\xi' = \pi^u_\xi \left(\frac{u}{R}\right)^{-3/2} f_T(u)^{-1/2} \sqrt{\frac{1 - f_T(u)^{-1} v^2}{k^2 f_T(u) u^8 - (\pi^u_\xi)^2}}$$

The momentum π^u_ξ is conserved.

$$u_c(v) = \frac{u_T}{(1-v^2)^{1/3}} \qquad \pi^u_\xi = k\, u_T^4 \frac{v}{(1-v^2)^{4/3}}$$

Drag force given by the momentum flow towards the horizon [Gubser '06, Herzog et al. '06]:

$$\frac{F_d}{A} = \frac{dp_z}{dt} = \pi^u_\xi = C_d \,\frac{T_{boost}}{T_c}\, w_f(T_{boost})\, v$$

where w_f is the enthalpy density of the false vacuum and

$$T_{boost} = \frac{T}{(1-v^2)^{1/6}} \qquad C_d = 2\pi \frac{p_{glue}}{w_{glue}} \kappa_c$$

The complete steady-state configuration

With wall embedded in the full brane configuration, we derive the zero-force condition

$$F = \frac{1}{L}\int dx_4\, \pi^u_\xi(u_T, x_4) = A\,\Delta p \qquad \Delta p = -\frac{T}{V_3}\Delta S_{DBI}$$

In the complete configuration π^u_ξ is not conserved. In rectangular approximation:

$$\partial_u \pi^u_\xi = -2\frac{k}{L}R^{3/2}u^{5/2}$$

After integration

$$u_*^{7/2} = u_T^{7/2} + \frac{7L}{4kR^{3/2}}A\,\Delta p \qquad u_* \geq u_T$$

The profile satisfies

$$\xi' = -\frac{4}{7}\frac{kR^{3/2}}{L}\left(u^{7/2} - u_*^{7/2}\right)\left(\frac{u}{R}\right)^{-3/2} f_T(u)^{-1/2}\sqrt{\frac{1 - f_T(u)^{-1}v^2}{k^2 f_T(u)u^8 - \frac{4}{7}\frac{kR^{3/2}}{L}\left(u^{7/2} - u_*^{7/2}\right)^2}}$$

Bubble wall velocity formula [Bigazzi, AC, Canneti, Cotrone '21]

At the induced horizon

$$k f_T^{1/2} u_c^4 = -\frac{4}{7}\frac{kR^{3/2}}{L}\left(u_c^{7/2} - u_*^{7/2}\right)$$

Explicitly,

$$\Delta p = \frac{2^5}{3^9}\pi^3\lambda^3 N N_f (LT)\frac{T^7}{M_{KK}^3}\left[\frac{v}{(1-v^2)^{4/3}} - \frac{4}{7}\frac{3}{4\pi LT}\left(1 - \frac{1}{(1-v^2)^{7/6}}\right)\right]$$

We can write

$$\Delta p = \frac{F}{A} \equiv \frac{F_d}{A} + p_f(T_{boost}) - p_f(T)$$

Using the result for F_d, we find the formula for the velocity:

$$\boxed{v = C_d^{-1}\frac{T_c}{T_{boost}}\frac{p_t(T) - p_f(T_{boost})}{w_f(T_{boost})}}$$

Formula valid also for analogous Dp-Dq-$\bar{D}q$-brane setups.

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- String Theory is a theory of gravity
- Usually formulated in flat-spacetime background

How do curved geometries emerge?

- High-energy string-string scattering [Amati, Ciafaloni, Veneziano '87]
- Unitarity retrieved in the eikonal approximation:

$$\mathcal{S}(s,\mathbf{b}) = e^{2i\hat{\delta}(s,\mathbf{b})}$$

$$2\hat{\delta}(s,\mathbf{b}) = \int \frac{d\sigma_u d\sigma_d}{(2\pi)^2} : \mathcal{A}_1\left(s,\mathbf{b}+\hat{\mathbf{X}}_{\mathbf{u}}(\sigma_u)-\hat{\mathbf{X}}_{\mathbf{d}}(\sigma_d)\right):$$

- Compactible with semiclassical propagation in Aichelburg-Sexl geometry

Emergent geometry very complicated at subleading orders

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String-brane scattering: an easier problem

Scattering of a string on stack of N D-branes [D'Appollonio, Di Vecchia, Russo, Veneziano '10].

Two descriptions:

- Open string sectors in flat-spacetimes
- p-brane solutions in supergravity

$$ds^2 = \frac{1}{\sqrt{H(r)}}\eta_{\alpha\beta}dx^\alpha dx^\beta + \sqrt{H(r)}\delta_{ij}dx^i dx^j$$

$$H(r) = 1 + \left(\frac{R_p}{r}\right)^{7-p} \,, \qquad R_p^{7-p} = \frac{g_s N(2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}}$$

We study string-brane scattering in the Regge limit

$$s \to \infty \,, \qquad \frac{t}{s} \to 0$$

The two complementary approaches are comparable in the limit

$$g_s \to 0 \qquad \lambda = g_s N \gg 1$$

Resumming many-boundaries ampl.ds $\leftrightarrow$ semicl. string prop. on p-brane geometry.

String-brane scattering: an easier problem

Sum of amplitudes with many boundaries:

$$\mathcal{S}(s,\mathbf{b}) = e^{2i\hat{\delta}(s,\mathbf{b})} \,, \qquad 2\hat{\delta}(s,\mathbf{b}) = \frac{1}{2E}\int \frac{d\sigma}{2\pi} : \mathcal{A}_1\left(s,\mathbf{b} + \hat{\mathbf{X}}(\sigma)\right) :$$

The eikonal operator $\hat{\delta}(s,\mathbf{b})$ can be expanded as

$$2\hat{\delta}(s,\mathbf{b}+\hat{\mathbf{X}}) \sim \frac{1}{2E}\left[\mathcal{A}_1(s,b) + \frac{1}{2}\frac{\partial^2 \mathcal{A}_1(s,b)}{\partial b^i \partial b^j}\,\overline{\hat{X}^i \hat{X}^j} + \ldots\right]$$

where

$$\mathcal{A}_1(s,b) \sim s\sqrt{\pi}\frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R_p^{7-p}}{b^{6-p}} + \text{imaginary part} + O\left(\frac{R_p}{b}\right)^{2(7-p)}$$

Studying the semiclassical string propagation in the Penrose limit of the p-brane solutions, the second term can be computed.

Agreement between the two approaches [D'Appollonio, Di Vecchia, Russo, Veneziano '10]

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Background field expansion of the Polyakov action

The propagation of a highly-energetic string is described by

$$S = -\frac{T}{2}\int d\sigma d\tau\, G_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu$$

The Penrose limit is the first order of a systematic expansion in Fermi coordinates around a null geodesic $X_{bg}(\tau)$ [Blau et al '06]

$$X(\sigma,\tau) = X_{bg}(\tau) + \xi(\sigma,\tau)$$

Recursive method to write the terms [Mukhi '85]

$$\begin{aligned}
S^{(0)}_{X=X_{bg}} &= \frac{T}{2}\int d\sigma d\tau\, G_{AB}(X_{bg})\dot{X}^A_{bg}\dot{X}^B_{bg} \\
S^{(2)}_{X=X_{bg}} &= -\frac{T}{2}\int d\sigma d\tau\, R_{ABCD}(X_{bg})\xi^B\xi^C\partial^\alpha X^A_{bg}\partial_\alpha X^D_{bg} + \\
&- \frac{T}{2}\int d\sigma d\tau\, G_{AB}(X_{bg})D^\alpha\xi^A D_\alpha\xi^B \\
S^{(3)}_{X=X_{bg}} &= -\frac{T}{6}\int d\sigma d\tau\, D_A R_{BCDE}(X_{bg})\partial_\alpha X^B_{bg}\partial^\alpha X^E_{bg}\xi^C\xi^D\xi^A + \\
&- \frac{2T}{3}\int d\sigma d\tau\, R_{ABCD}\partial^\alpha X^A_{bg} D_\alpha\xi^D\xi^B\xi^C
\end{aligned}$$

Background field expansion of the Polyakov action

The scaling of the quantities with energy is controlled by the vielbeins:

$$E_+ \sim E \ , \qquad E_- \sim E^{-1} \ , \qquad E_a \sim 1$$

Dominant terms with two "+" indices: plane wave → light-cone gauge.

We can resum the series. In the impulsive approximation:

$$S - S_0 = \frac{1}{2E} \int du E_+^\mu E_+^\nu h_{\mu\nu}(X_{bg}) + \sum_{n=2}^{\infty} \frac{1}{n!} C_{a_1 \dots a_{n-2} bc} \overline{\xi^{a_1}(0) \dots \xi^{a_{n-2}}(0) \xi^b(0) \xi^c(0)}$$

where S_0 is the string action in flat spacetime, $h_{\mu\nu} = G_{\mu\nu} - G_{\mu\nu}^{flat}$ and

$$C_{a_1 \dots a_{n-2} bc} = -\frac{1}{E} \int_{-\infty}^{\infty} du D_{a_1} \dots D_{a_{n-2}} R_{+b+c}(X_{bg}(u))$$

In the leading eikonal approximation:

$$S - S_0 = \frac{1}{2E} \int \frac{d\sigma}{2\pi} g(b + \xi(\sigma,0))$$

Comparison

We thus expect the correspondence

$$g(b+\xi(\sigma,0)) \leftrightarrow \mathcal{A}_1\left(s,\mathbf{b}+\hat{\mathbf{X}}(\sigma)\right)$$

As a check, we studied the shock-wave case for which the exact $\mathcal{S}$-matrix is known [Amati, Klimcic '87]. This case is eikonal exact and indeed g gives the exact $\mathcal{S}$-matrix.

For the p-brane case, at zero order we find

$$g(b) = E^2\sqrt{\pi}\frac{R_p^{7-p}}{b^{6-p}}\frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} + O\left(\frac{R_p}{b}\right)^{2(7-p)}$$

At quadratic order we (obviously) retrieve the results found with the Penrose limit.

Higher-order terms reconstruct the Taylor series such that

$$g(b) \to g(b+\xi(\sigma,0))$$

Eikonal operator from the curved-background approach!

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Thank you for your attention!