From Inflation to the Physics of Sound

(One year at GGI)

Author: Rollo Rocco

17-th December 2021

Scientific activity

- **Publication**: Primordial Non-Gaussianity in Supersolid Inflation, arXiv:2103.10402
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- **Thesis Supervisor**: Giovanni Battista Carollo, student at the University of Padova.
  Thesis: Wandering in the cosmological consistency relations;

Works in progress

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  Argument: A controversial issue concerning NG in single-field Inflation;

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- **PART III**:

  Argument: The Physics of Sound;

  Group: Filipponi, Rollo, Trimarelli.

Part I: The long mode problem

- What can we measure in Cosmology?

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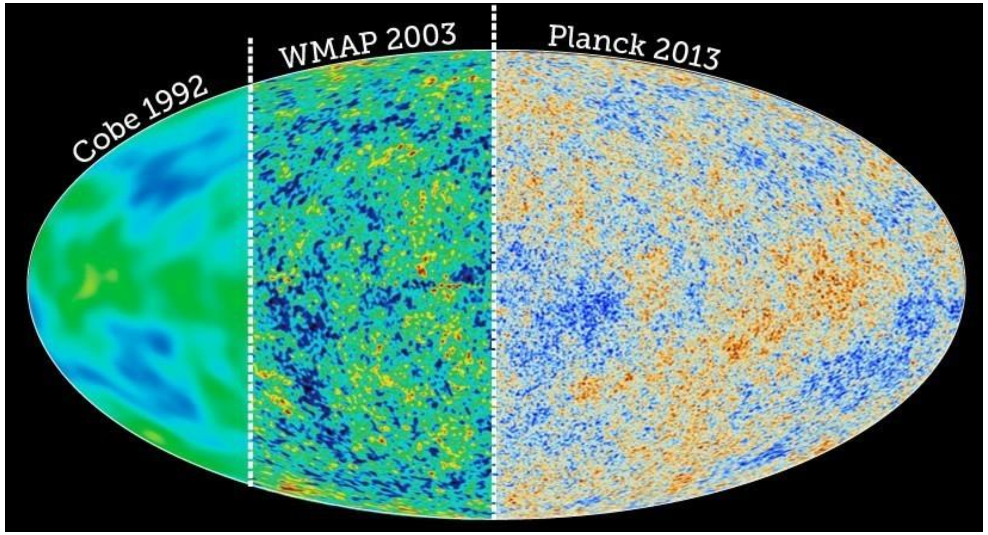

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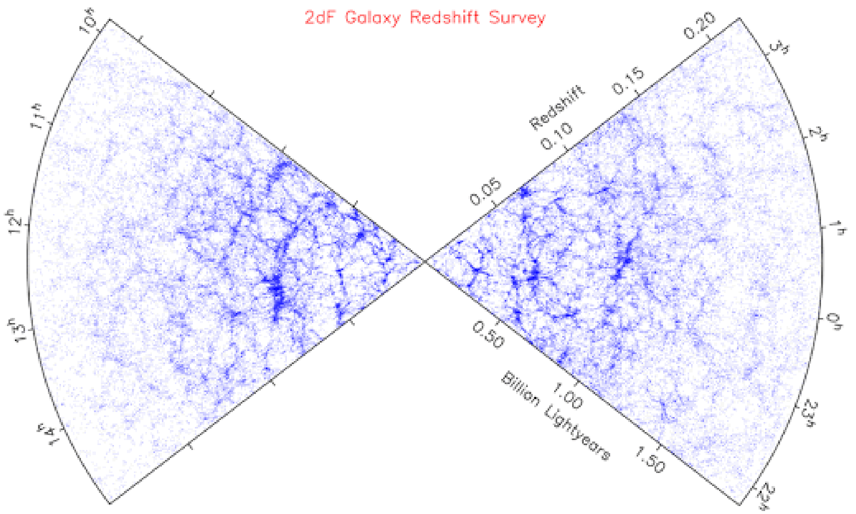

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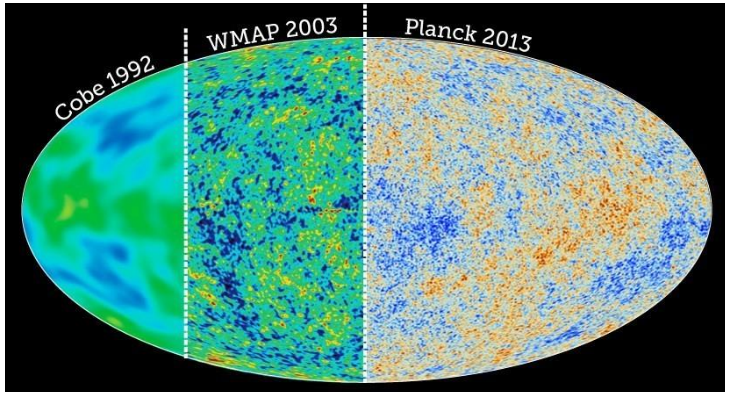

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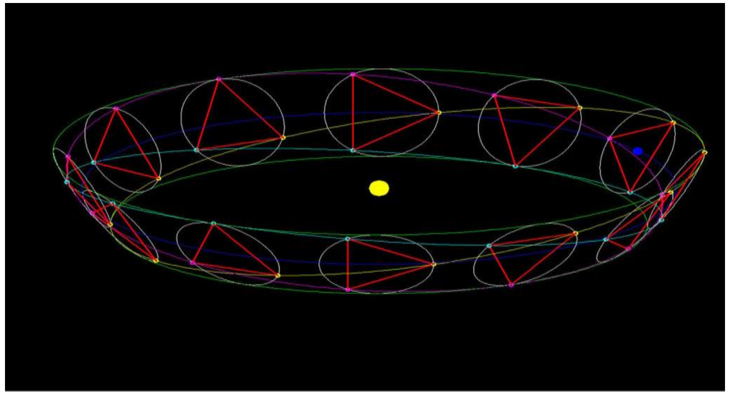

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Today

The Early Universe:
INFLATION

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- A particular focus on **N=3**: Non-Gaussianity

$$< \Phi_{k_1}\Phi_{k_2} > = (2\,\pi)^3 P(k_1)\,\delta^{(3)}(k_1 + k_2),$$

Φ: Newtonian Potential;

$$< \Phi_{k_1}\Phi_{k_2}\Phi_{k_3} > = (2\,\pi)^3 B(k_1, k_2, k_3)\,\delta^{(3)}(k_1 + k_2 + k_3)$$

$$B(k_1, k_2, k_3)\Big|_{k_1 \ll k_2 \approx k_3} \to \frac{5}{6} f_{NL}\, \underbrace{P(k_1)}_{\text{Long PS}}\, \underbrace{P(k_2)}_{\text{Short PS}}$$

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MODEL DEPENDENT!

Part I: The long mode problem

- Single field Inflation prediction: $f_{NL} = -\frac{5}{12}(n_s - 1);$ $P_\Phi(k)\ k^3 \propto k^{n_s-1}$

arXiv: 0209156
arXiv: 0210603

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- Gravitons, Graviton-Scalars correlators

TTT	$< h^{(s)}_{k_L}\ h^{(r)}_{k_S}\ h^{(o)}_{k_S} >$	$f_{TTT} = \frac{B_{TTT}}{P_h(k_L)P_h(k_S)}$	$\sim O(\epsilon^0)\varepsilon^s_{ij}(k_L)\ k^i_S k^j_S$
TTS	$< h^{(s)}_{k_S}\ h^{(r)}_{k_S}\Phi_{k_L} >$	$f_{TTS} = \frac{B_{TTS}}{P_h(k_S)P_\Phi(k_L)}$	$\sim O(\epsilon)\ \varepsilon^s_{ij}(k_S)\ \varepsilon^p_{ij}(k_S)$
TSS	$< h^{(s)}_{k_L}\Phi_{k_S}\ \Phi_{k_S} >$	$f_{TSS} = \frac{B_{TSS}}{P_h(k_L)P_\Phi(k_S)}$	$\sim O(\epsilon)\ \varepsilon^s_{ij}(k_L)\ k^i_S k^j_S$

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- Our first publication (07/2020) arXiv:2007.08877

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IN PROGRESS !

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Part II: EFT of Inflation

- The (spontaneus) breaking pattern:

$$\phi^0 = \varphi(t) + \pi_0$$

$$\phi^l = x^l + \partial_l \pi_L$$

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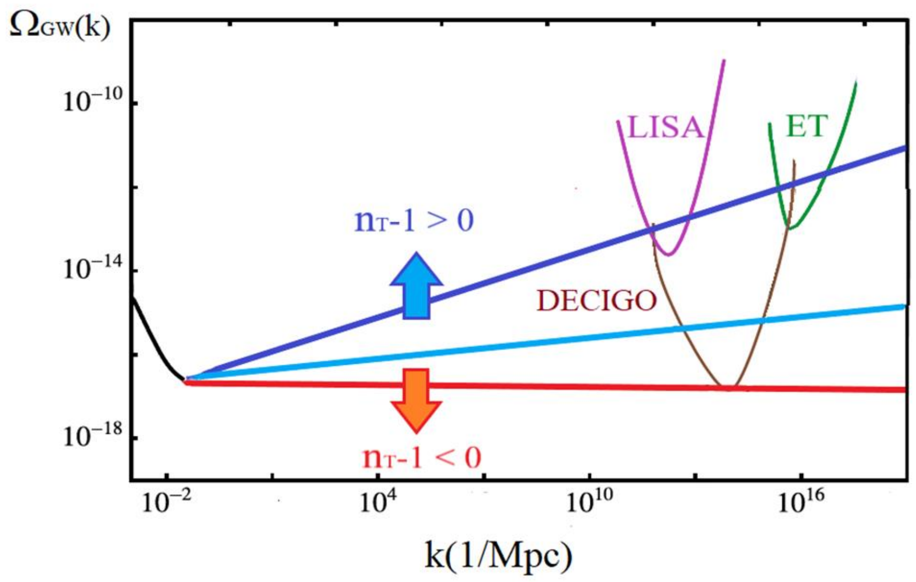
Both → **SUPERSOLID**

ArXiv: 1704.00322
ArXiv: 1605.05304

Part II: Beyond single-field

- Searching for a different signature:

-1) $\Omega_{GW} \propto P_h(k)\, k^3 \propto k^{n_T - 1}$



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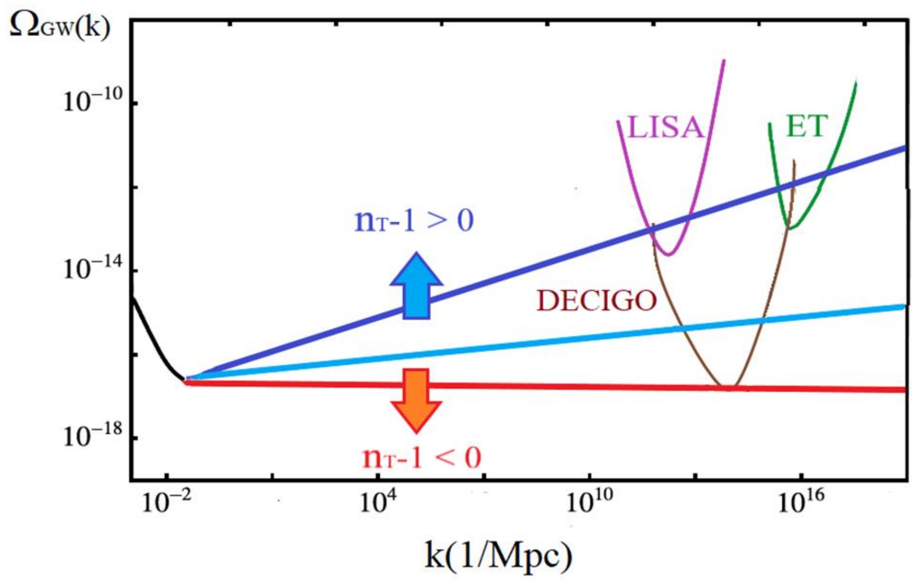

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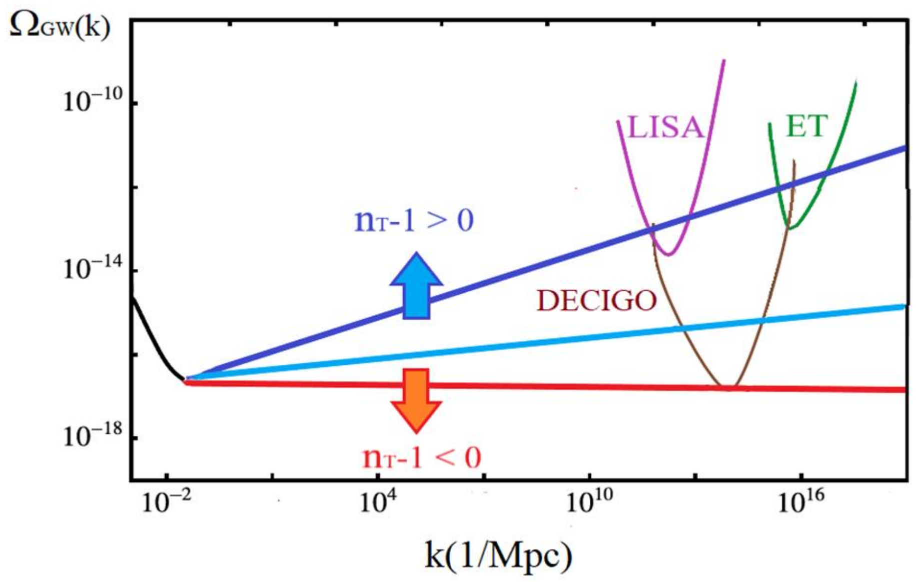

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These are the points we stressed in arXiv:2010.02023 and arXiv:2103.10402.

Part II: The next step

- Interactions that give a boosted f_T:

$$H_I = g\, a^n\, D(k_1, k_2, k_3) \begin{cases} h_{k_1}\pi_{k_2}\pi_{k_3} \\ h_{k_1}h_{k_2}\pi_{k_3} \end{cases}$$

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- A source of loop-corrections:

-Dyson formula expansion

$$< h(t)^2 >=< 0| \left(e^{-i\int_{t_0}^{t} dt'\, H_I(t')}\right)^{+} h(t)^2\, e^{-i\int_{t_0}^{t} dt'\, H_I(t')} |0 >$$

$$h''_{ij} - \frac{2}{t} h'_{ij} + k^2 h_{ij} = \int dq\, T_{ij}^{lm}\, g\, D(k, q, k-q)\, \pi_q\, \pi_{k-q}$$

-Results on ArXiv ASAP ;)

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(guitar, violin, viola, piano etc.)

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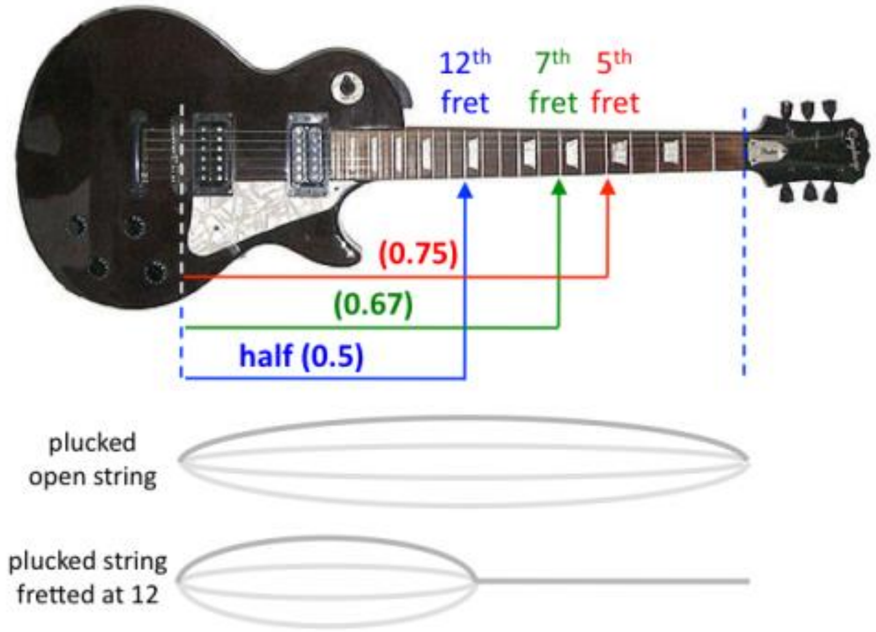
$$\omega_n = n\,\omega_0, \qquad \omega_0 \propto \frac{1}{L}$$



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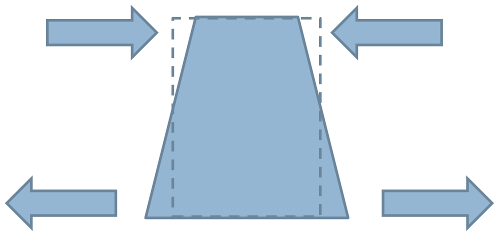

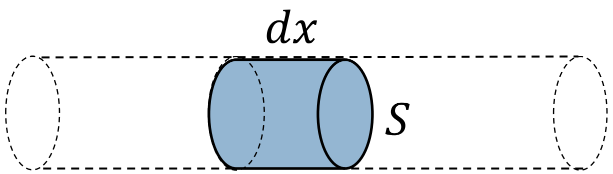

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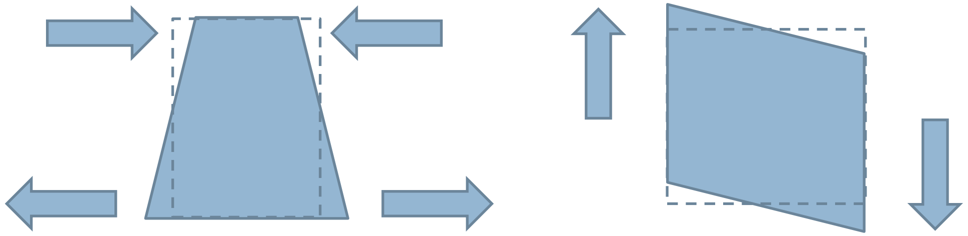

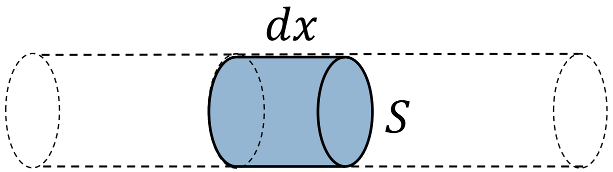

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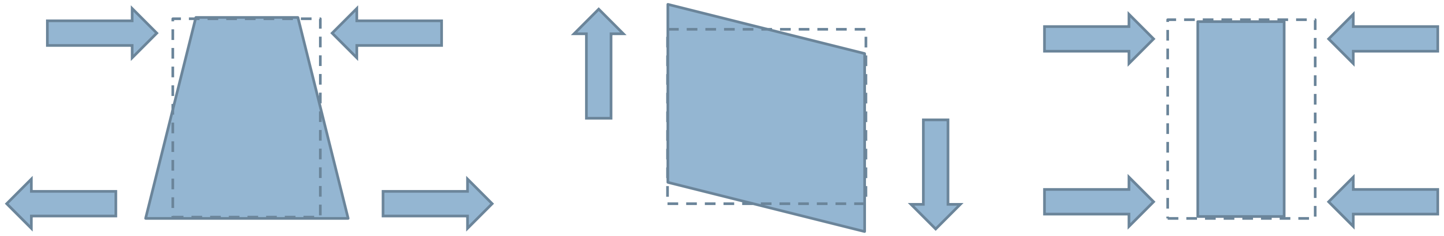

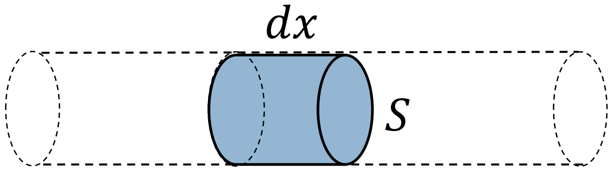

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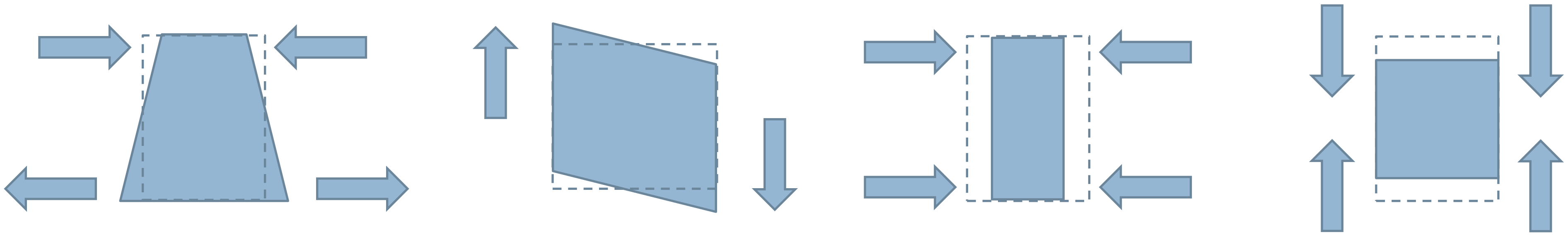

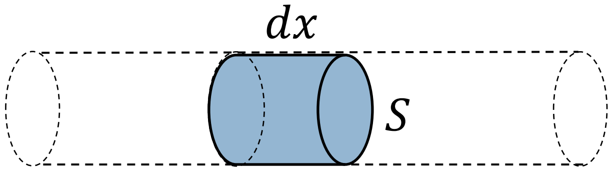

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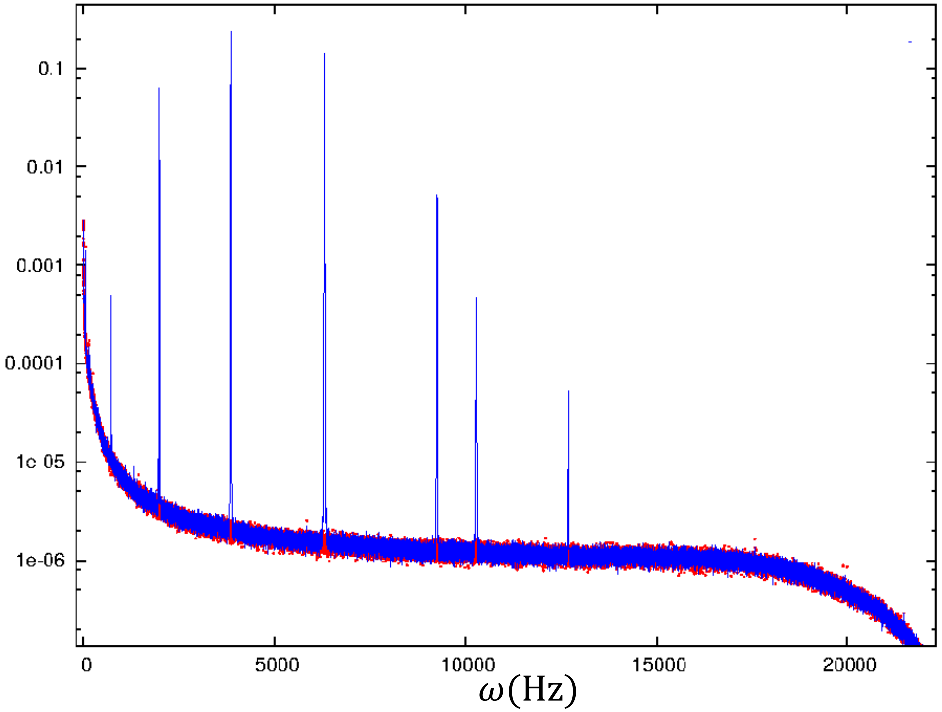

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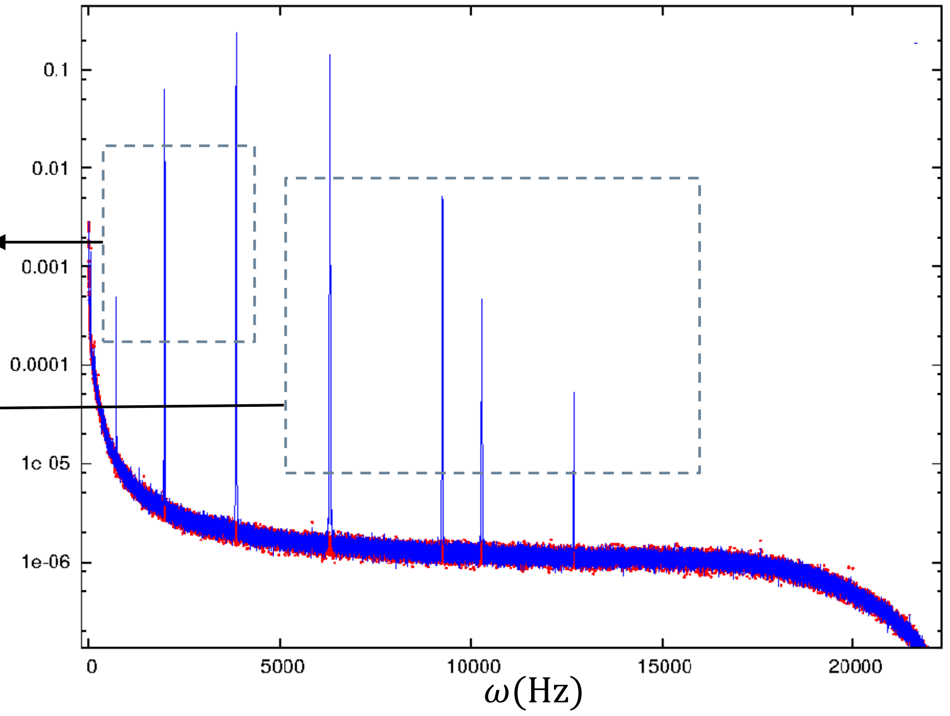
E-B: $\omega_n \approx n^2\omega_0,\ \ \omega_0(L, S, c_L^2, c_T^2)$

TIMOSHENKO: $\omega_n > n^2\omega_0$

NOT HARMONIC



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- Even more involved.. D=2:
  1) -Instruments under tension: the drum!

Part III: The Physics of Sound

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  1) -Instruments under tension: the drum!

$$z = R(r)\,\Phi(\theta)\,T(t)$$

$$R(r) \propto J_m\left(\frac{\omega_n}{v}\,r\right)$$

NOT HARMONIC !!!

n

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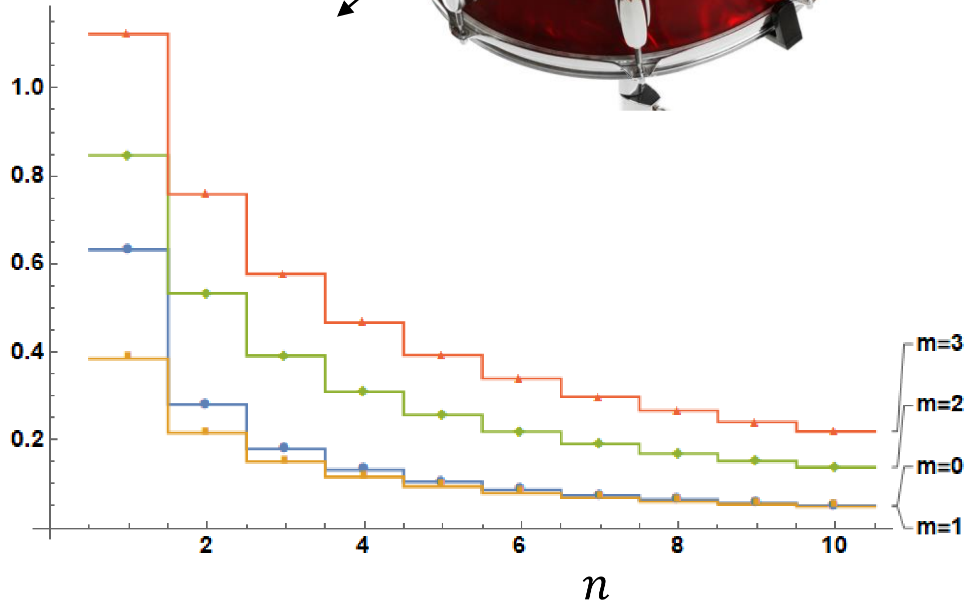
$$\epsilon = \frac{R}{\omega_n\,v}\left|\frac{\omega_n R}{v} - \left[n\,\pi + \frac{\pi}{2}\left(m - \frac{1}{2}\right)\right]\right|$$

High freq. Approx.



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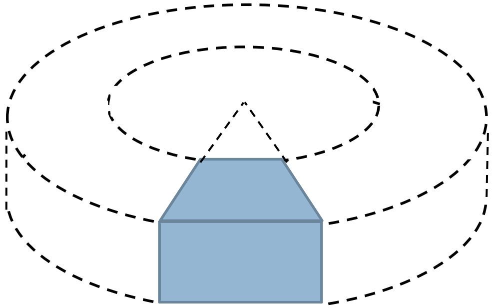
NOT HARMONIC !!!

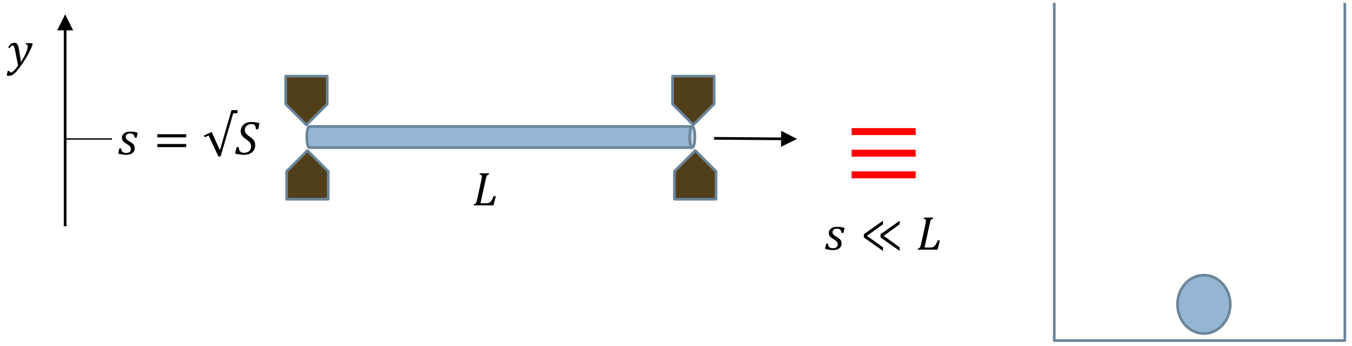
  2) -Elastic object: Metallic gong!

IN PROGRESS !!!



Part III: The Physics of Sound

- **Projects:**
  - A pedagogical paper: Teaching Quantum Mechanics through music



The same EOM

$$y^2 \rightarrow |\psi|^2$$
$$\omega_n \rightarrow E_n \propto n^2$$

  - A paper more focused on the physics of sound:
    - Solving some experimental discrepancies for the drum;
    - Solving the gong sound spectrum.

Thank you!!!

Backup slides

Recall of Cosmology

- ***Cosmological principle:*** Our Universe results to be homogeneous and Isotropic at sufficiently large scales.
- ***Idea:*** Homogeneous and isotropic background+ small perturbations

-Background $$ds^2 = \bar{g}_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + a(t)^2 \frac{\delta_{ij}}{(1 - x^2\chi)^2} dx^i dx^j$$

-Perturbations $$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + g^{(1)}_{\mu\nu}(t,x) + \frac{1}{2} g^{(2)}_{\mu\nu}(t,x)\ldots$$

$$g_{00} = -e^{2\Psi}, \qquad g_{0i} = a\, \partial_i F, \qquad g_{ij} = a^2[e^{2\Phi}\delta_{ij} + \partial_{ij}E + \partial_j C_i + \partial_i C_j + h_{ij}];$$

-Curvature perturbations:

$$R = -\Phi + H\, v, \qquad \zeta = -\Phi + H \frac{\delta\rho}{\partial_t \rho}, \ldots$$

Statistical properties of Cosmological Perturbations

- At the linear level we have no phase-correlation among different modes…

  Gaussianity

$$< R(x_1)..R(x_{2n}) >= \sum_{Perm.} \prod_{Pairs} < R(x_i)\, R(x_j) >,$$

$$< R(x_1)..R(x_{2n+1}) > = 0.$$

- At the non-linear level we get phase-correlation: ***Non-Gaussianity***

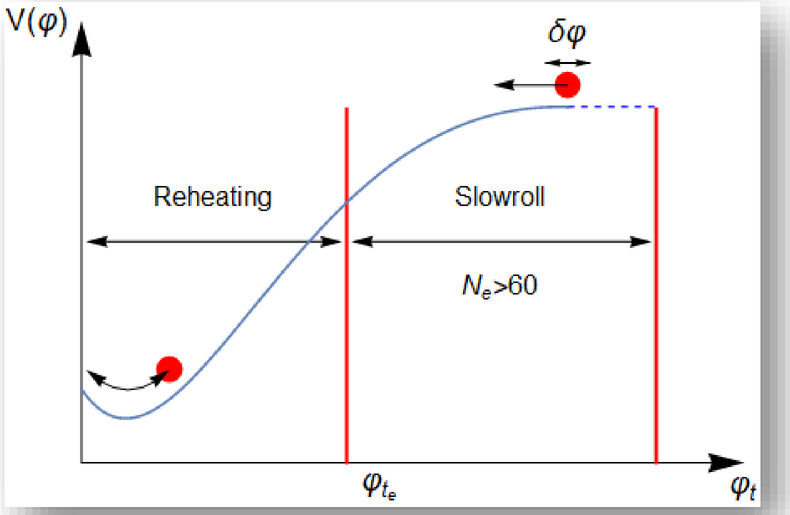
$$< R(x_1)..R(x_{2n+1}) > \neq 0.$$

$$< R(x_1)..R(x_{2n}) > \; - < R(x_1)..R(x_{2n}) > \Big|_G \neq 0.$$

Single-field Inflation

- Action: $S = M_{pl}^2 \int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\, g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + V(\phi)\right]$

- SR limit: $\epsilon = -\frac{\dot{H}}{H^2} \ll 1 \rightarrow p \approx -\rho, \qquad \eta = \frac{\dot{\epsilon}}{\epsilon H} \ll 1,$



- Scalar PS $\left(\frac{q}{H} \ll 1\right)$: $< \mathrm{R}_p \mathrm{R}_q > = 4\,\pi\, q^3 \boldsymbol{\mathcal{P}}_{\mathrm{R}}(q)\, \delta^{(3)}(q+p),$

$$\boldsymbol{\mathcal{P}}_R(q) \equiv \boldsymbol{\mathcal{P}}_\zeta(q) = \cdots; \quad \boldsymbol{\mathcal{P}}(\mathrm{q}) = \boldsymbol{\mathcal{P}}_{SF}\, q^{n_s-1} \begin{cases} \boldsymbol{\mathcal{P}}_{SF} = \dfrac{H^2}{8\pi^2 M_{pl}^2 \epsilon} & (2.4\ 10^{-9}), \\ n_s - 1 = -2\epsilon - \eta & (0.9652). \end{cases}$$

- Tensor PS $\left(\frac{q}{H} \ll 1\right)$: $h_{ij} = \sum_{s=-2}^{2} \varepsilon_{ij}^{(s)}\, h_k^{(s)}, \qquad < h_p^{(s)}\, h_q^{(r)} >= 4\, \delta_s^r\, \pi\, q^3 \boldsymbol{\mathcal{P}}_{\mathrm{h}}(q)\, \delta^{(3)}(q+p),$

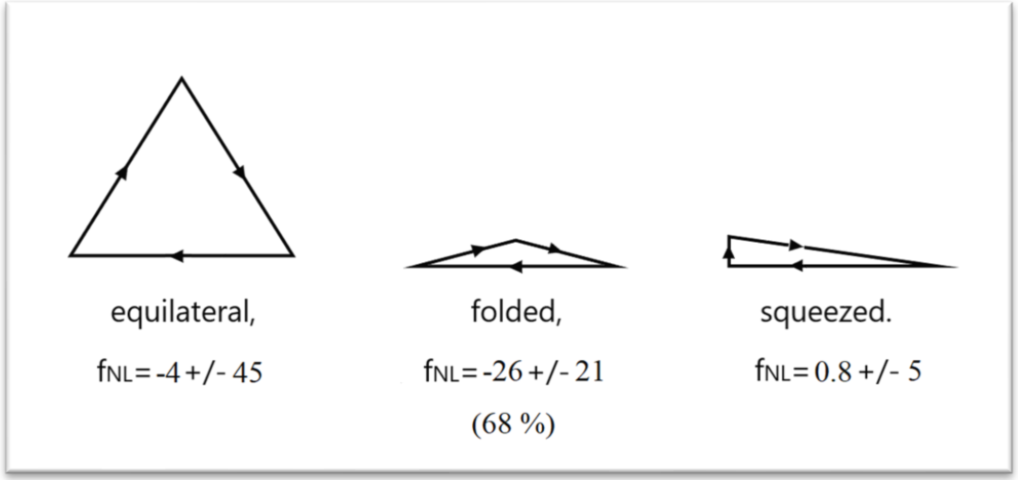
$$\boldsymbol{\mathcal{P}}_h(\mathrm{q}) = r\, \boldsymbol{\mathcal{P}}_{SF}\, q^{n_T-1} \begin{cases} r = 8\,\epsilon & (\leq 0.06), \\ n_T - 1 = -2\epsilon & (\pm 0.6). \end{cases}$$

Single-field Inflation NG

- Scalar 3-point functions:

$$< \mathrm{R}_k \mathrm{R}_p \mathrm{R}_q >= (2\pi)^3 B_{\mathrm{R}}(k,p,q)\ \delta^{(3)}(k+q+p),$$

$$B_{\mathrm{R}} \sim f_{NL} \frac{5}{6}\big(P_{\mathrm{R}}(k)P_{\mathrm{R}}(q) + P_{\mathrm{R}}(p)P_{\mathrm{R}}(q) + P_{\mathrm{R}}(k)P_{\mathrm{R}}(p)\big)$$



Validity of the consistency relation

$$B_R(k_1 = \mathrm{k_L} \ll k_2 \sim k_3) = -\frac{1}{2}(n_s - 1)P_R(k_L)P_R(k_S), \qquad f_{NL}^{sq} = -\frac{5}{12}(n_s - 1)$$

- Gravitons, Graviton-Scalars correlators:

TTT	$< h_p^{(s)}\ h_q^{(r)}\ h_k^{(o)} >$	$f_{TTT} = \frac{B_{TTT}}{P_h(p)P_h(k)}$	**SQ:** 300 ± 200
TTS	$< h_p^{(s)}\ h_q^{(r)} \Phi_k >$	$f_{TTS} = \frac{B_{TTS}}{P_h(p)P_\Phi(k)}$	??
TSS	$< h_p^{(s)} \Phi_q \Phi_k >$	$f_{TSS} = \frac{B_{TSS}}{P_h(p)P_\Phi(k)}$	**SQ:** 90 ± 40

f_{NL} detection...

- $< \frac{\delta T}{T}\frac{\delta T}{T}\frac{\delta T}{T} >$ Improving CMB (LiteBIRD) …

- Galaxy clustering.. The scale dependent halo bias

$$\delta g = \frac{n_g(x,z) - \bar{n}_g(z)}{\bar{n}_g(z)} \sim \mathrm{b}_0 \frac{\delta\rho}{\rho}$$

$$\delta g \sim \left(\mathrm{b}_0 + (\dots)\frac{f_\delta^{sq}}{k^2}[a] + (\dots) f_{TSS}^{sq}\, \varepsilon_{ij}^{(s)} \frac{k^i k^j}{k^2}[b,c]\right)\delta$$

- Detection of GWs background [d] and local non-linear corrections of tensor PS [e][f]

$$\Omega_{GW} \sim k^{n_T - 1} \qquad < h^2 > \Big|_{\mathrm{Non}-Lin.} \sim [1 + (\dots)\mathrm{R}_L\, f_{TTS}^{sq}] < \mathrm{h}^2 >$$

[a] Verde, Matarrese, 2009. Detectability of the effect of Inflationary non-Gaussianity on halo bias

[b] Jeong, Kamionkowski 2012. Clustering Fossils frome the Early-Universe.

[c] Akhshik, 2015. Clustering fossils in Solid Inflation

[d]: Bartolo et al., 2016. Probing inflation with gravitational waves.

[e]: Malhotra, Dimastrogiovani, Fasiello, Shiraishi, 2020. Cross-correlations as a Diagnostic Tool For PGWs.

[f]: Adshead et al., 2020. Multimessanger Cosmology: Correlating CMB and SGWB measurements.

k=0 world

- ***FUNDAMENTAL*** : Let us apply a dilatation, we can impose a gauge redundancy!

- For instance:

Initial gauge: comoving E=v=0

$\boldsymbol{x^i \to e^{\lambda} x^i}$

Final gauge: comoving

$$g_{00} = -e^{2\,\Psi}, \qquad g_{0i} = a\,\partial_i F, \qquad \boxed{g_{ij} = a^2 e^{-2R}\delta_{ij}.}$$

k=0 world

- Redundancy: $\Delta \mathrm{E} = \mathrm{E}'(\mathrm{x}) - E(x) = 0, \quad \Delta v = v'(\mathrm{x}) - v(x) = 0,$

  $$\Delta g_{ij} = (\mathrm{e}^{-2\lambda} - 1)\delta_{ij} \boxed{\approx -2\, a^2 \lambda\, \delta_{ij},} \qquad \boxed{\delta R_\lambda = \lambda.}$$

- Intuitive argument: $ds^{(3)} = 2\, a^2\, \mathrm{e}^{2\,\mathrm{R}} \mathrm{dx}^{\mathrm{i}} \mathrm{dx}^{\mathrm{j}} \rightarrow 2 \boxed{\mathrm{e}^{2\lambda} a^2} \mathrm{e}^{-2\,(\mathrm{R}+\lambda)} \mathrm{dx}^{\mathrm{i}} \mathrm{dx}^{\mathrm{j}},$

  Local scale factor a’

- Formally: Sym. Breaking pattern so(4,1) → rotations + translations.

- Applications: -The Weinberg Theorem;

  -The Consistency Relation.

The Consistency Relation

[g]: Creminelli, Norena, Simonovich. Conformal consistency relation for single-field inflation. 2012 .
[h]: Hui et al. An Infinite Set of Ward Identities for Adiabatic Modes in Cosmology. 2014 .
[i]: Hui et al. Conformal Symmetries Adiabatic Modes in Cosmology. 2012 .

- Single-field: Spontaneous breaking of so(4,1) global symmetries [g][h][i]

de Sitter: so(4,1) $\rightarrow$ rotations + translations.

1) Dilatation is a symmetry non-linearly realized. This implies a Norther current and charge: $Q = \int dx^3 \{P_R, \delta R_\lambda\};$

2) Using Ward identities, one can extract the consistency relation[q]:

$$\lim_{k \to 0} \langle R_k R_{k_1} R_{k_2} \rangle = \frac{5}{12} P(k) \left[3 + \sum_{a=1}^{2} k_a \, \partial_{k_a} \right] \langle R_{k_1} R_{k_2} \rangle$$

$0???$

3) Applying a second dilatation:

$$R_k \rightarrow R_k - \lambda = 0$$

Deformed dilatation

- A standard gauge transformation: $\Delta g_{ij} = -a^2[2\lambda\delta_{ij} + x^j\partial_i\lambda + x^i\partial_j\lambda]$

  Instead of $\Delta g_{ij} = -a^2[2\lambda\,\delta_{ij}]$ ...

- Basic element $x^i\partial_j\lambda = \frac{-1}{(2\pi)^{3/2}}\int dk^3\, e^{ikx}\partial_{k^i}(k^j\lambda_k) + BT.$ → $\sim \lambda_k$

- Final result

$$\Delta g_{ij} = 2a^2\frac{k^i k^j}{k}\partial_k\lambda_k$$

$$\Delta R_k = 0, \qquad \Delta E_k = \frac{-2}{k}\partial_k\lambda_k$$

→ $A\ gauge\ change!$

A DISCONTINUITY IN THE GRADIENT EXPANSION!

Non-linear deformed dilatation: $\Delta B_R = 0??$

- We give two independent demonstrations:

  -1) Using the in-in formalism;

  -2) Using field redefinitions.

$$\Rightarrow \Delta B = \langle R'(x)^3 \rangle - \langle R(x)^3 \rangle \equiv \text{BT} = 0.$$

Goal: We solved the halo bias scale dependence [l][m]

$$\Delta b(k)\ k^2 \propto f_\delta^{sq} = -\frac{5}{3} + \mathrm{f_{NL}} \rightarrow \mathbf{0}$$

Such effects are physical and observable in principle by future high-sensitivity experiments!

[l]: Cabass, Pajer, Schmidt, 2018. Imprints of oscillatory Bispectra on Galaxy Clustering.

[m]: de Putter, Dorè, Green, 2015. Is there scale-dependent bias in single-field inflation?

Non-linear deformed dilatation: $\Delta B_R = 0??$

- In-In Formalism demonstration:

$$< O_{k_1} O_{k_2} O_{k_3} >=< 0| \left(e^{-i \int_{t_{0(1+i\epsilon)}}^{t} dt'\, H_I(t')} \right)^{+} O_{k_1} O_{k_2} O_{k_3} e^{-i \int_{t_{0(1+i\epsilon)}}^{t} dt'\, H_I(t')} |0 >$$

$$\approx -i \int_{t_0}^{t} dt_1 < 0| \left[H_I(t_1), O_{k_1} O_{k_2} O_{k_3} \Big|_t \right] |0 >$$

1.) O_{k_j} operators are first order in perturbation theory, they are gauge invariant (we have the extension for GWS)!

2.) After boring manipulations $H' = H + \int d^3 x\, \partial_i \left[S(x) \left(\lambda\, x^i + \omega^i_j\, x^j \right) \right]$

$$\downarrow$$

$$< O_{k_1} O_{k_2} O_{k_3} >' =< O_{k_1} O_{k_2} O_{k_3} > + \mathrm{BT}.$$

EFT of Inflation

- Single-field inflationary models: time diff. Breaking $\tau \to \tau + \pi^0(x,\tau),$

- Solid inflation: spatial diff. Breaking $x^i \to x^i + \pi^i(x,\tau),$

- Let us break both : Supersolid inflation

$$\Phi^0 = \varphi^0(\tau) + \boxed{\pi^0(x,\tau),}$$
$$\Phi^{\mathrm{i}} = x^i + \partial^i \boxed{\pi_L(x,\tau)} + \boxed{\pi_T^i(x,\tau),}$$

Two scalar DoF

Vector

Global symmetries: $1.)\ \Phi^\mu \to \Phi^\mu + C^\mu$

$2.)\ \Phi^{\mathrm{l}} \to R_j^l \Phi^j$

EFT of Inflation

- Basic operators: $C^{AB} = g^{\mu\nu}\partial_\mu \Phi^A \partial_\nu \Phi^A, \quad B^{lm} = g^{\mu\nu}\partial_\mu \Phi^l \partial_\nu \Phi^m,$

$$W^{lm} = B^{lm} + \frac{C^{0l}C^{0m}}{C^{00}}$$

- Lagrangian: $S = \frac{M_{pl}^2}{2}\int d^4x \sqrt{-g}\, U(b, y, \chi, \tau_Y, \tau_Z, w_Y, w_Z),$

$$b = \sqrt{Det[B]}, \qquad \tau_Y = \frac{Tr[B^2]}{Tr[B]^2}, \qquad \tau_Z = \frac{Tr[B^3]}{Tr[B]^3},$$

→ π^i-dependent

$$y = u^\mu \partial_\mu \Phi^0, \qquad \chi = \sqrt{-C^{00}},$$

→ π_0-dependent

$$w_Y = \frac{Tr[W^2]}{Tr[W]^2}, \qquad w_Z = \frac{Tr[W^3]}{Tr[W]^3},$$

→ (π_0, π^i)-dependent

Effective media and symmetries

- Symmetries imply a medium-classification

Fluids: $U(b, y)$, invariance under VsDiff. and $\Phi^0 \to \Phi^0 + f(\Phi^a)$,

S-Fluids: $U(b, y, \chi)$, invariance under VsDiff. (entropy prop.)

Solids: $U(b, y, \tau_Y, \tau_Z)$, the most general Lag. with only Φ^l present

S-Solids: $U(b, y, \chi, \tau_Y, \tau_Z, w_Y, w_Z)$, the most general Lag. with 2 sclar DoF (entropy prop.)

$$T_{ij} = p\, g_{ij} + (\rho + p)\, u_i\, u_j + \alpha\, \partial_{ij}\, \pi_L$$

In-In formalism and Gws backreaction

- In-In formalism and De Sitter:

$$< h(t)^2 >= < 0| \left(e^{-i \int_{t_{0(1+i\epsilon)}}^{t} dt' H_I(t')}\right)^{+} h(t)^2 \, e^{-i \int_{t_{0(1+i\epsilon)}}^{t} dt' H_I(t')} |0 >$$

$$\sim i^2 \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \, [H_I(t_1), [H_I(t_2), h(t)^2]\,]$$

$$\sim i^2 \int_{t_0}^{t} dt_2 \, e^{-\epsilon\, t_2} \int_{t_0}^{t_2} dt_1 \, e^{-\epsilon\, t_1} [H_I(t_1), [H_I(t_2), h(t)^2]\,]$$

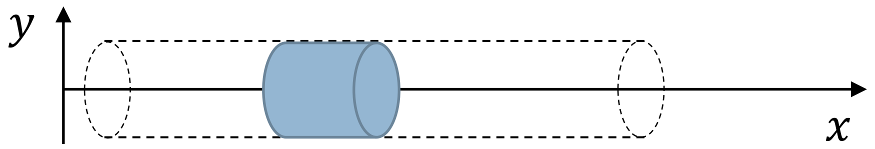
Finite terms at infinity!!

- Gws backreaction: -1) You need the commutators structure for the matching;
  -2) Finite terms at infinity never taken into account;
  -3) A pure quantum term is missing;
  -4) Spurious divergencies.

Euler and Timoshenko rod

- String equation: $[\partial_t^2 + c^2\,\partial_x^2\,]y(x,t) \equiv 0$

- Normal modes profile equation:



$$[D_{EB} + D_T\,]y(x,t) \equiv 0$$

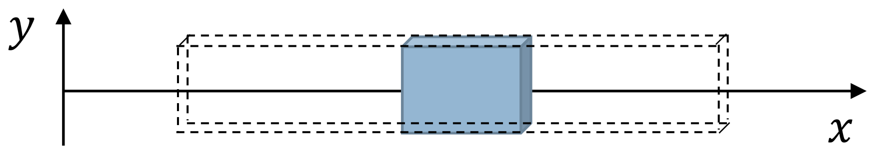
$$D_{EB} \equiv \partial_t^2 + \kappa^2 c_L^2 \partial_x^4 \longrightarrow \sum_i M_i \approx 0$$

$$D_T \equiv \frac{\kappa^2}{c_T^2\,S_h}\partial_t^4 + \kappa^2\left(1 + \frac{c_L^2}{c_T^2\,S_h}\right)\partial_t^2\partial_x^2 \longrightarrow \sum_i M_i \approx \rho\,I\,dx\,\partial_t^2\theta \sim \omega^2\rho\,dx\,S\,y + \frac{\omega^4}{c_L^2}\rho\,dx\,S^2\,y$$

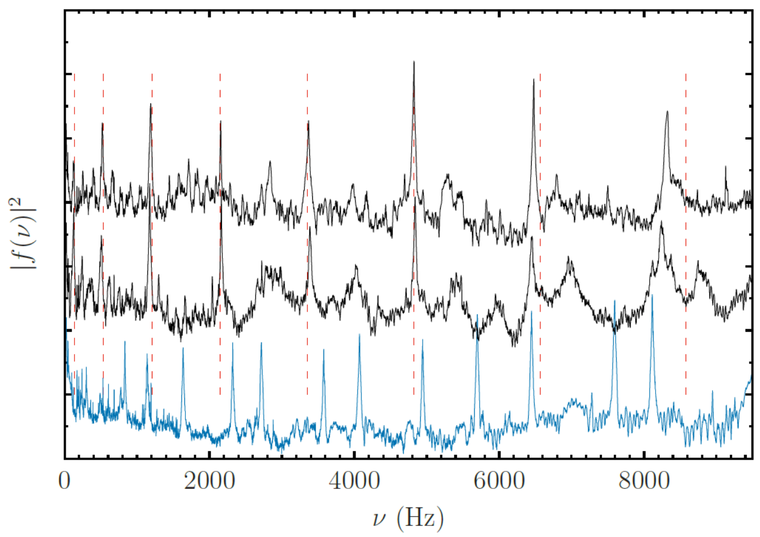
BOUNDARIES	X=0	X=L
FREE	$\partial_x^3 \boldsymbol{y} = \partial_x^2 \boldsymbol{y} = \boldsymbol{0}$	
PINNED	$\boldsymbol{y} = \partial_x^2 \boldsymbol{y} = \boldsymbol{0}$	

Euler-Bernoulli and the quantum analogy

- Normal modes profile equation:



$$D_{EB}\, y(x,t) \equiv 0 \qquad D_{EB} \equiv \partial_t^2 + \kappa^2 c_L^2 \partial_x^4$$



- Schrödinger Equation

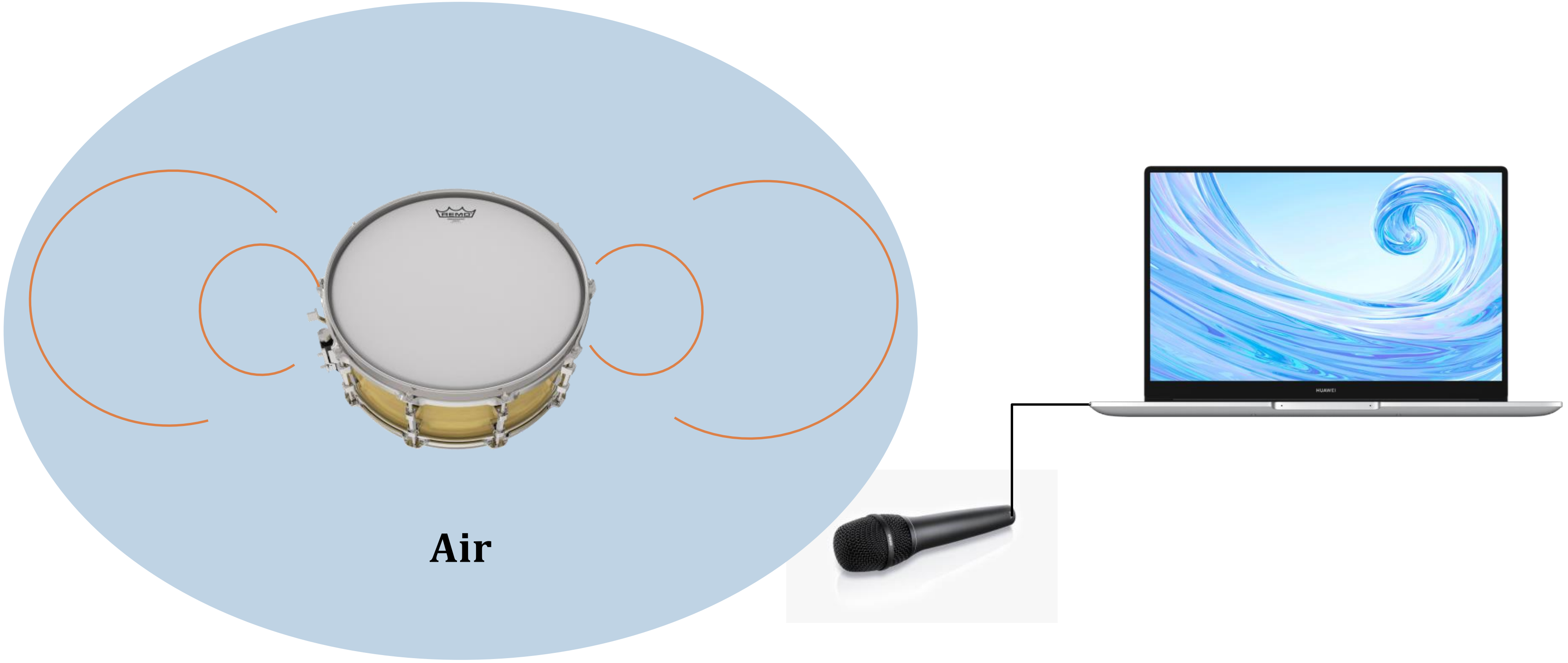
$$D_S\, \Phi(x,t) \equiv 0 \qquad D_S \equiv i\,\partial_t + \frac{1}{2\,m}\partial_x^2$$

$$D_S^* D_S\, \Phi(x,t) \equiv 0 \qquad D_S D_S^* \equiv \partial_t^2 + \frac{1}{4\,m^2}\partial_x^4$$

The drum problem

- Modes damping:

$m_{Air} \sim m_{Brane}$!!!



- Improvement: Using Helium

$$Z_{He} = 2 < Z_{Air} \in [7,8]$$