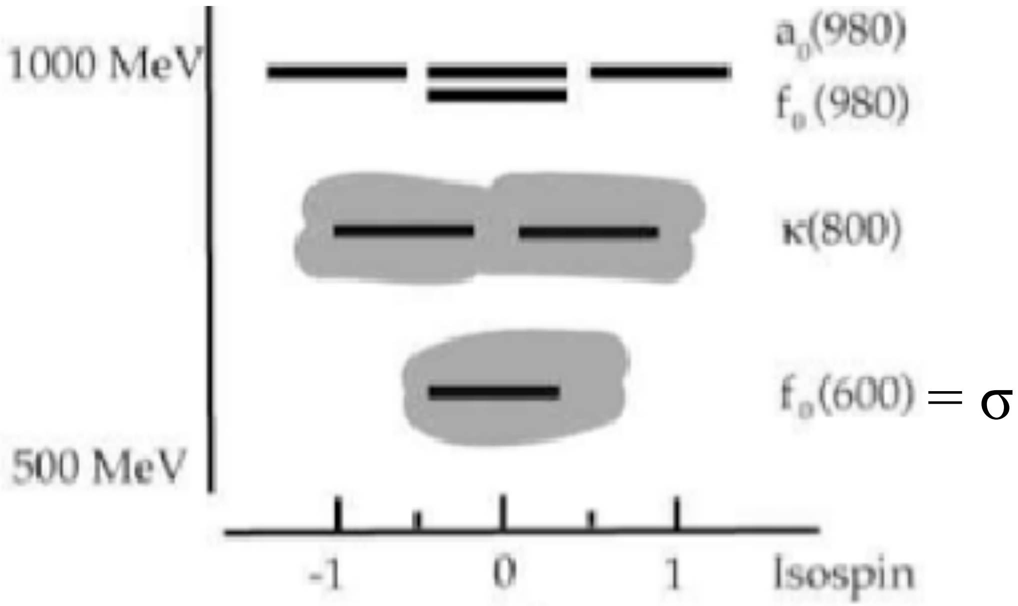
Lecture 3. Light and Heavy Tetraquarks

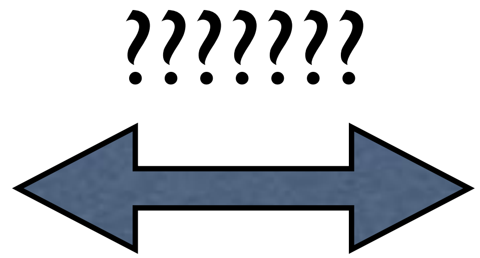
- ***Summary***

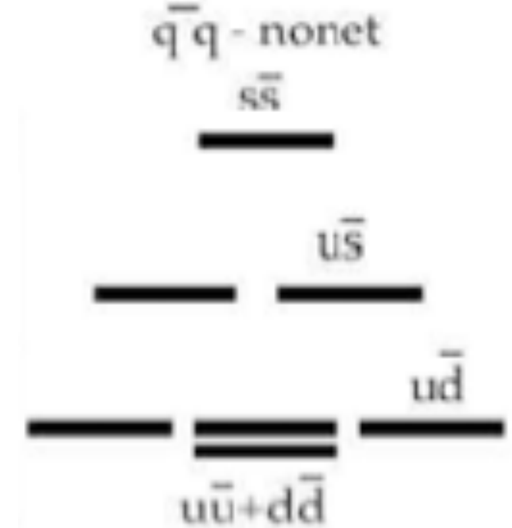
1. The overall panorama ✓
2. Constituent Quark Model and masses of conventional mesons and baryons ✓
3. Light and Heavy Tetraquarks. First comparison with hadron molecules ✓
4. Tetraquarks and the Eight Fold Way. Di-J/Ψ resonances.
5. $X(3872)$ and its missing partners
6. Born-Oppenheimer approximation for double charm baryons and tetraquarks
7. Multiquark states in N colours, in the $N \to \infty$ limit
8. Tetraquarks vs. molecules: the Weinberg criterium for $X(3872)$ and the double charm $\mathcal{T}^{+}_{cc}(3875)$

1. Light scalar mesons and tetraquarks



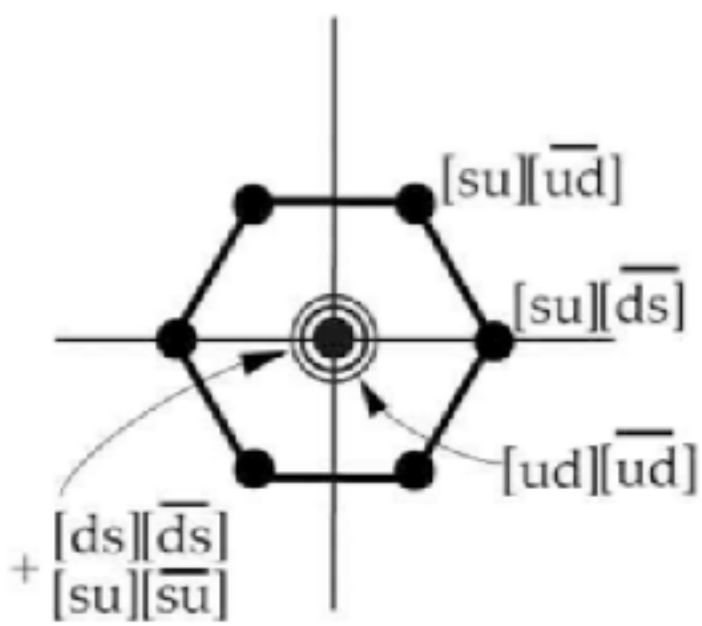
(c)



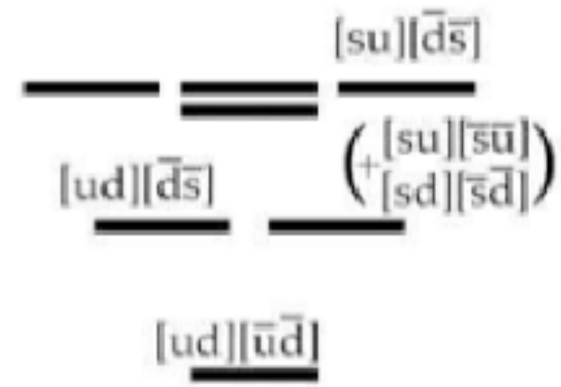


Antisymmetric tetraquarks work better

R.L. Jaffe, Phys. Rev. D15 (1977) 267; ibidem D15 (1977), 281



(a)

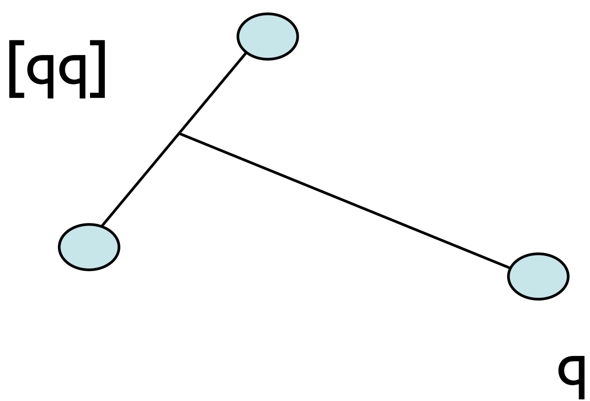


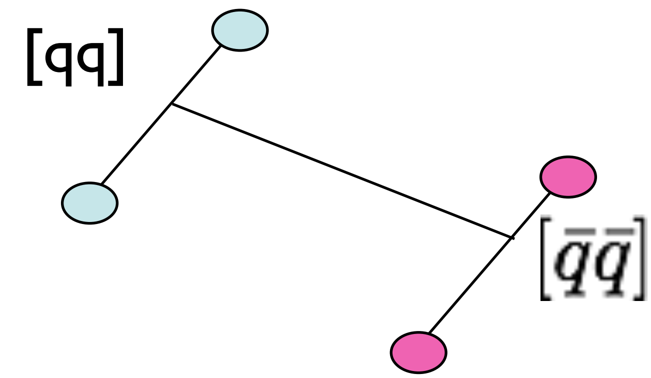
(b)

To avoid states with I=2, Jaffe assumes that *good diquarks* (colour $\bar{\mathbf{3}}$, $S_{qq} = 0$) do bind and that *bad diquarks* (colour $\bar{\mathbf{3}}$, $S_{qq} = 1$) do not

Diquarks: there should be many of them

- To form hadrons, good or bad diquarks need to combine with other colored objects:

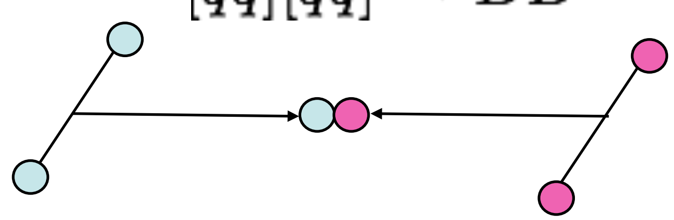




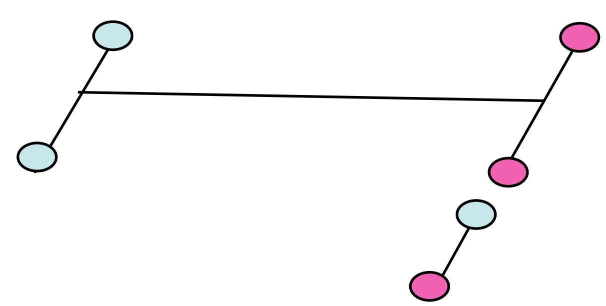
  - with q → baryon (e.g. Λ), Y-shape
  - with $[\bar{q}\bar{q}]$ → scalar meson, H-shape (Rossi &Veneziano,)

- We expect many states: the string joining diquarks may have radial and orbital excitations
- the string topology is more related to Baryon-antiBaryon: B-antiB decay

$[qq][\bar{q}\bar{q}] \to B\bar{B}$



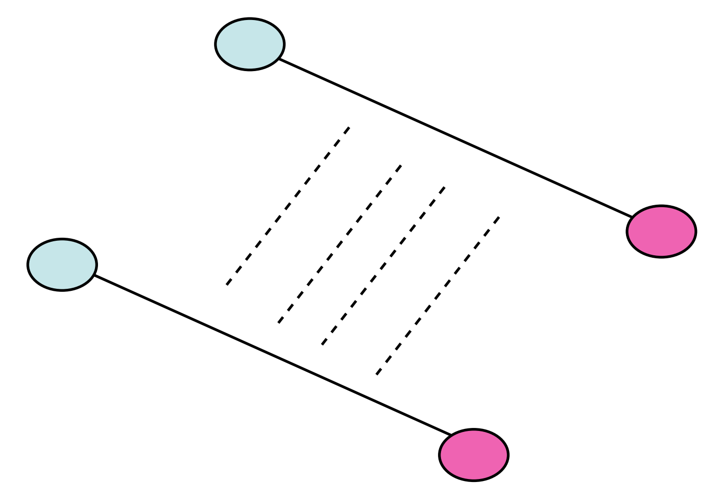
$[qq][\bar{q}\bar{q}] \to [qq][\bar{q}\bar{q}] + \mathrm{meson}$



2. Diquarks vs molecules

A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. Lett. 38 (1977) 317.

- The possibility of bound states of colourless hadrons was raised long ago by De Rujula, Georgi and Glashow;
- Meson-meson molecules have a different string topology:
  - are they bound?
  - very few states: no orbital or radial excitations expected



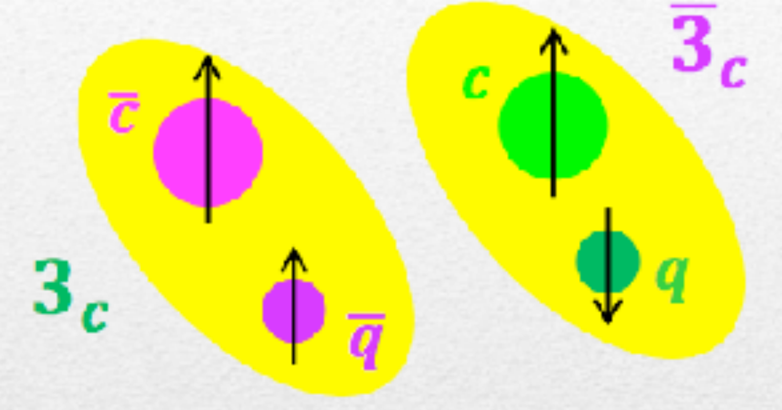
- Nuclei obviously 'are' hadron molecules, being 'made' by color singlet protons and neutrons
- Alice has measured the production of light nuclei, deuteron, He^3 and hypertriton, $H^3{}_\Lambda$ in relatively high p_T bins in Pb-Pb collisions, at s_{NN} = 2.76 TeV
- The cross section of these processes can be used as reference for a discrimination between tetra quarks (hadrons made by coloured subconstituents) and molecules (hadrons made by color singlet constituents), see later.

3. Tetraquark constituent picture of unexpected quarkonia

L.Maiani, F.Piccinini, A.D.Polosa and V.Riquer, Phys. Rev. D 71 (2005) 014028

$$[cq]_{S=0,1}[\bar{c}\bar{q}']_{\bar{S}=0,1}$$

- I=1, 0
- S-wave: positive parity
- total spin of each diquark: S=1,0
- heavy-light bad diquarks admitted!
- neutral states may be mixtures of isotriplet and isosinglet
- mass splitting described by the Hamiltonian:



$$\mathbf{H} = 2m_{(diquark)} + \sum_{i<j} 2\kappa_{ij}(\mathbf{s}_i \cdot \mathbf{s}_j)$$

The S-wave, $J^P=1^+$ charmonium tetraquarks

in the $|S, \bar{S}\rangle_J$ basis we have the following states

$$\mathrm{J}^{\mathrm{P}} = 0^+ \quad C = + \quad X_0 = |0,0\rangle_0,\ X_0' = |1,1\rangle_0$$

$$\mathrm{J}^{\mathrm{P}} = 1^+ \quad C = + \quad X_1 = \frac{1}{\sqrt{2}}\left(|1,0\rangle_1 + |0,1\rangle_1\right)$$

$$\mathrm{J}^{\mathrm{P}} = 1^+ \quad C = - \quad Z = \frac{1}{\sqrt{2}}\left(|1,0\rangle_1 - |0,1\rangle_1\right),\ Z' = |1,1\rangle_1$$

$$\mathrm{J}^{\mathrm{P}} = 2^+ \quad C = + \quad X_2 = |1,1\rangle_2$$

X(3872)=X_1; $Z^{\pm}$(3900), $Z^{\pm}$(4020)=lin. combs. of Z&Z' that diagonalize **H**;
$X^{\pm}(4250) = X_0'$??

Can we use the spin-spin couplings from mesons and baryons to guess the hidden charm tetraquark spectrum??

- Recall the constituent quark Hamiltonian:

$$\mathbf{H}_{ij} = 2\kappa_{ij}\,(\mathbf{s}_i \cdot \mathbf{s}_j) \qquad \kappa_{ij} = CF(R) \times \frac{g^2}{m_i m_j}\,|\psi(0)|^2$$

- Spin-spin interactions are proportional to the overlap probability $|\psi(0)|^2$ of the two quarks/antiquarks involved.
- No symmetry principle says that the overlap functions in tetraquarks have to be the same as in baryons or mesons
- spin-spin couplings in tetra quarks should be considered as free parameters to be determined from the mass spectrum.
- A better insight requires use of a trasformation analogous to the Fierz Transformation (introduced in the Fermi theory of weak interactions) to express the tetraquark $[cq][\bar{c}\bar{q}']$ as combination of tetraquark expressions in which we pair $\bar{q}'$ with q and $\bar{c}$ with c

a useful shortcut

- for tetraquark Fierz transformations, see e.g. A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)
- a simple shortcut is to use Charge Conjugation for fermion antifermion pairs in S-wave and total spin S, which is $C = (-1)^S$. One finds the correspondence
  - C=+1: $X_1 \leftrightarrow [\bar{q}q']_1[\bar{c}c]_1 \ (+1/2)$
  - C=-1: $Z, Z' \leftrightarrow [\bar{q}q']_1[\bar{c}c]_0 \ (+1/2), \ [\bar{q}q']_0[\bar{c}c]_1 \ (-3/2)$

  in parenthesis we have reported the corresponding eigenvalue of

  $2 < \frac{\sigma_q}{2} \cdot \frac{\sigma_{q'}}{2} >$, see Lect. 2
- neglecting charmed quarks spin-spin interaction (inversely proportional to mass squared), one concludes that,
  - dominant $q\bar{q}'$ interaction leads to *X_1 and Z degenerate, with a lighter Z'*
  - the observed pattern: X(3872)~Z(3900) < Z(4020) i.e. X and Z degenerate and a *heavier* Z' !

4. The right ansatz

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. **D 89** (2014), 114010

- Mass ordering is explained by the hypohesis that the ***dominant*** spin-spin interactions in tetraquarks are those ***inside the diquark or the antidiquark***.
- With this hypothesis, H is diagonal in the diquark spin basis used before and the Hamiltonian simply counts the number of diquarks with spin 1

- Assuming:

$$X(3872) = X_1 = \frac{1}{\sqrt{2}}\left(|1,0\rangle_1 + |0,1\rangle_1\right)$$

$$Z(3900) = Z = \frac{1}{\sqrt{2}}\left(|1,0\rangle_1 - |0,1\rangle_1\right), Z(4030) = Z' = |1,1\rangle_1$$

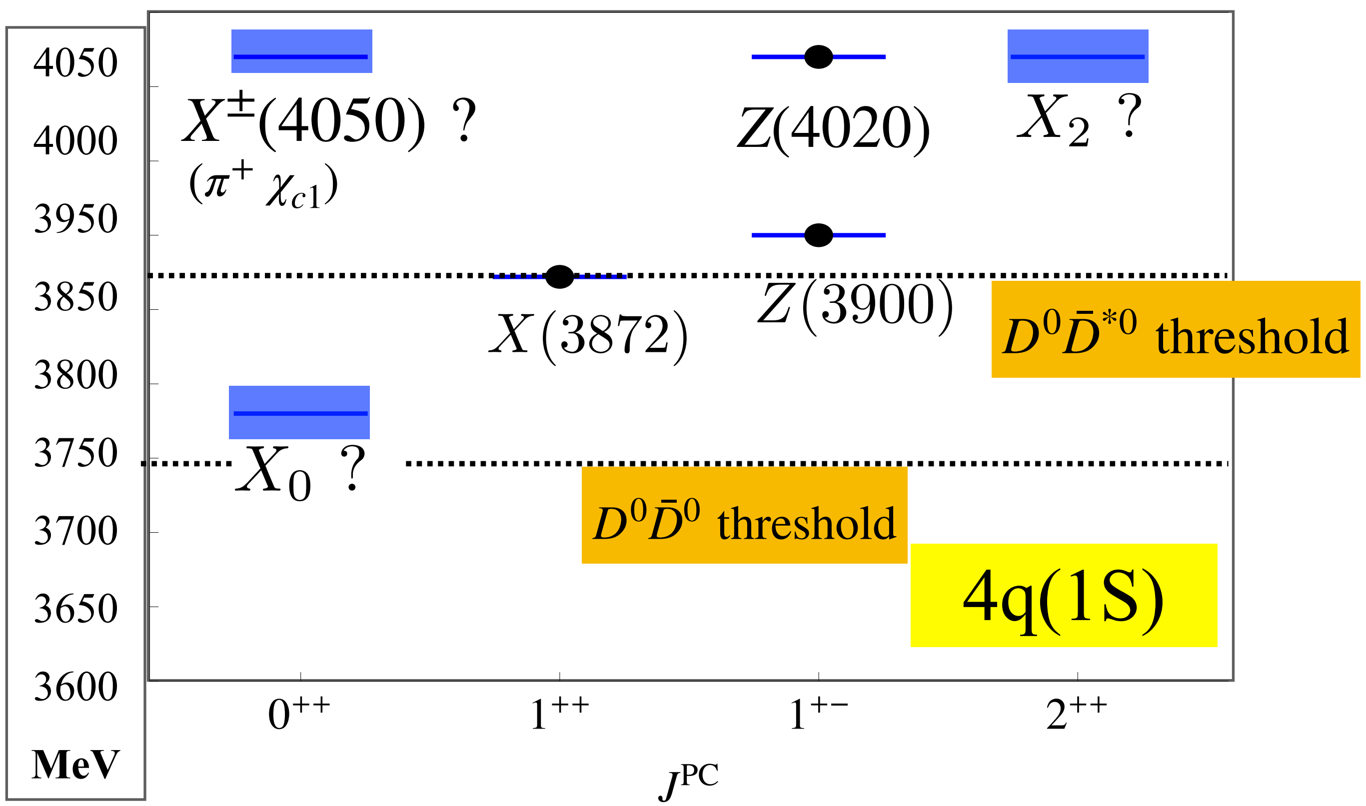
X(3872) and Z(3900) are ~degenerate with one unit of spin, Z(4030) has two units and therefore it is heavier.

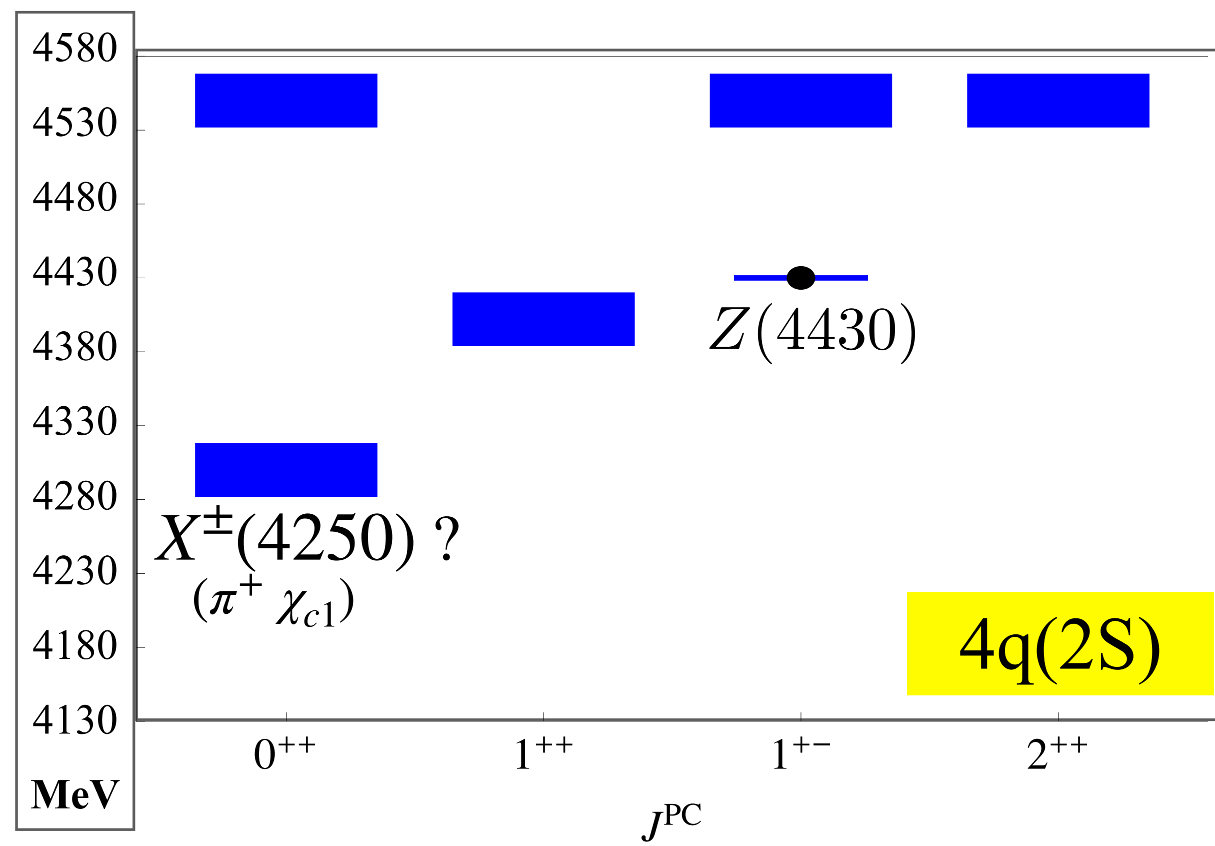
$X_0 = |0,0\rangle$ is one unit below X(3872), $X_0' = |1,1\rangle_0$ and $X_2 = |1,1\rangle_2$ at the same as level Z'.

- A simple explanation of the dominance of inter-diquark interaction could be that diquarks and antidiquarks are at relatively large distance in the hadron, as to suppress the overlap probability of constituents in different diquarks, unlike what happens, e.g., in the usual baryons.

Parameters

- The spectrum of 1S ground states is characterised by two quantities:
  - the diquark mass, $m_{[cq]}$
  - the spin-spin interaction inside the diquark or the antidiquark, κ_{cq}.
- The first radially excited, 2S, states are shifted up by a common quantity, the radial excitation energy, ΔE_r expected to be mildly dependent on the diquark mass: $E_r(cq) \sim E_r(cs)$



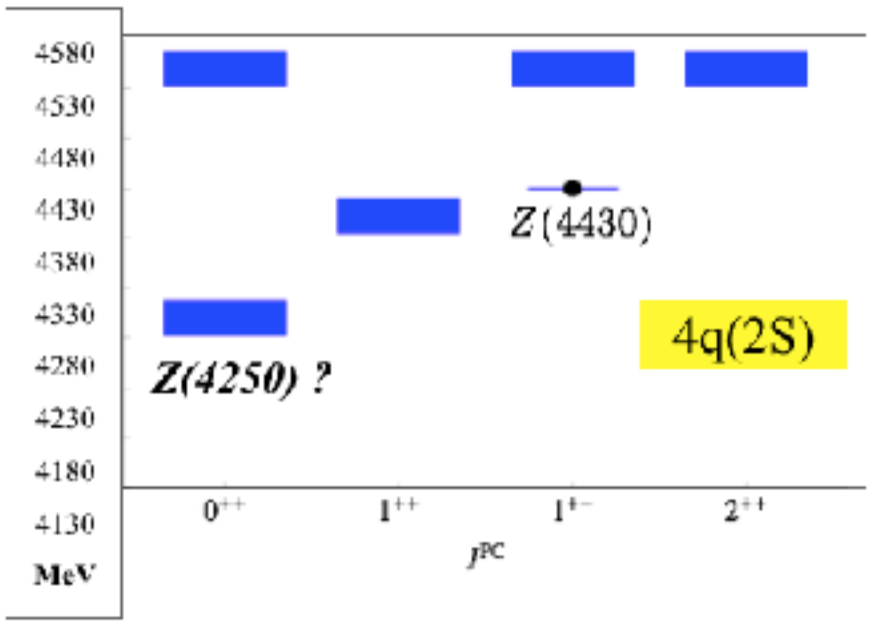


$$m_{[cq]} = 1980 \text{ MeV}$$

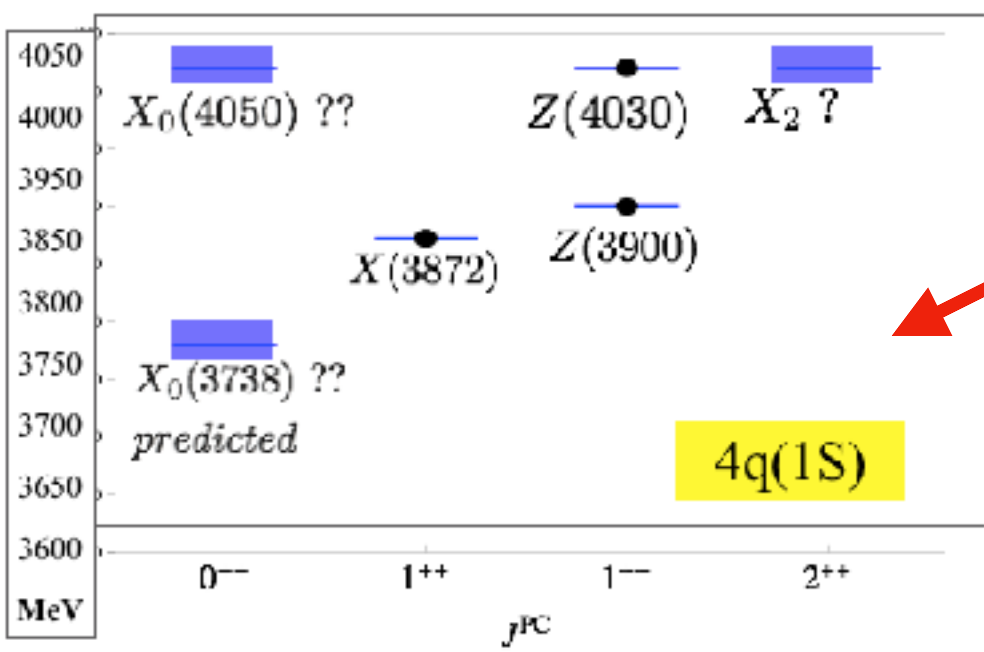
$$\kappa_{cq} = 67 \text{ MeV}$$

$$\Delta E_r(cq) = 530 \text{ MeV}$$

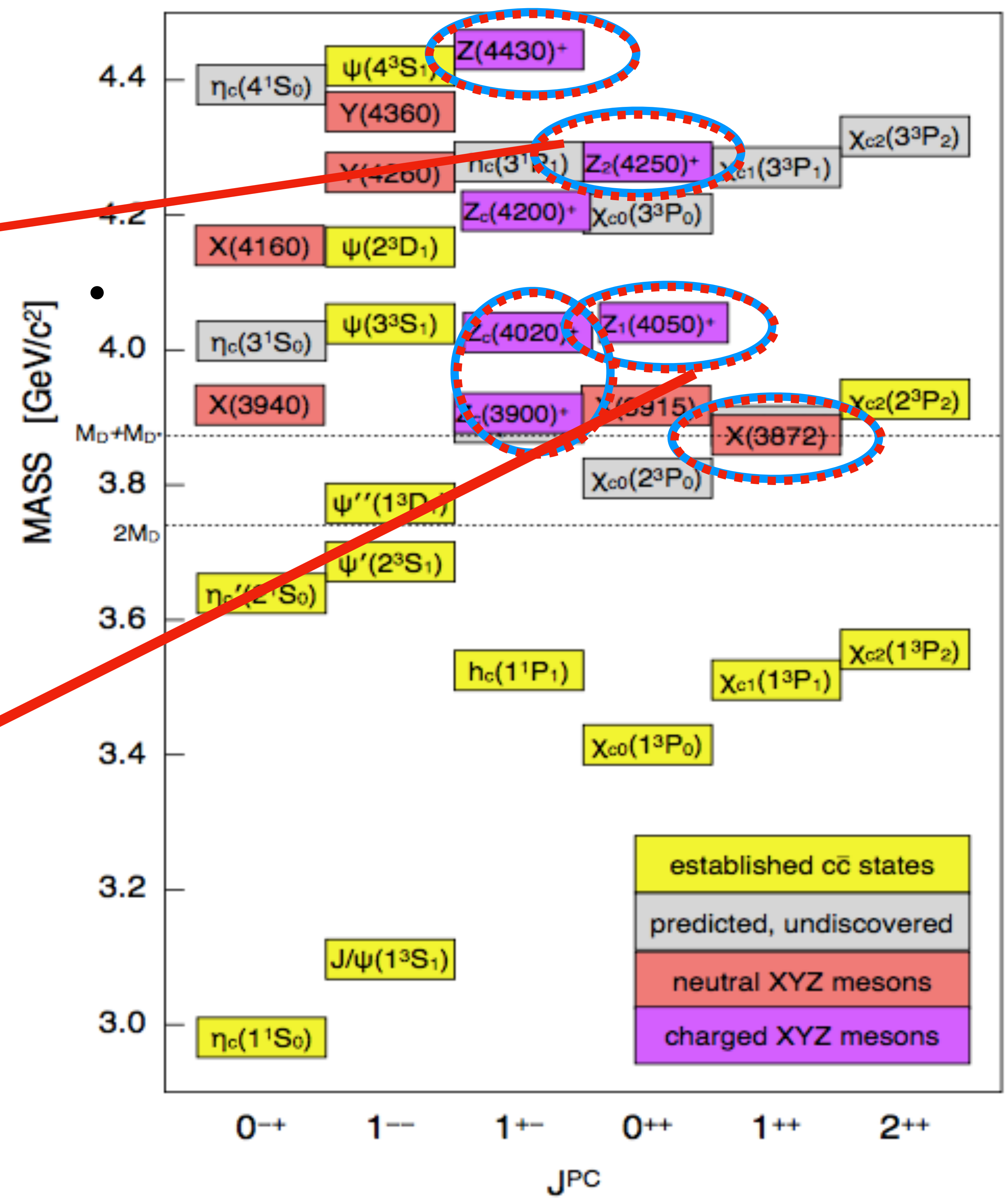
- A simple ansatz reproduces Z states ordering: spin-spin interaction is dominated by inter-diquark interaction
- constituents are ***not*** uniformely mixed in the bag, rather clump into two separate entities: diquarkonium
- The spectrum of 1S ground states is characterised by two quantities:
  - the diquark mass, $m_{[cq]}$
  - the spin-spin interaction inside the diquark or the antidiquark, κ_{cq}.
- The first radially excited, 2S, states are shifted up by a radial excitation energy, ΔE_r, mildly dependent on the diquark mass: $E_r(cq) \sim \sim E_r(cs)$



$$\Delta E_r(cq) = 530 \text{ MeV}$$



$$m_{[cq]} = 1980 \text{ MeV}; \ \kappa_{cq} = 67 \text{ MeV}$$

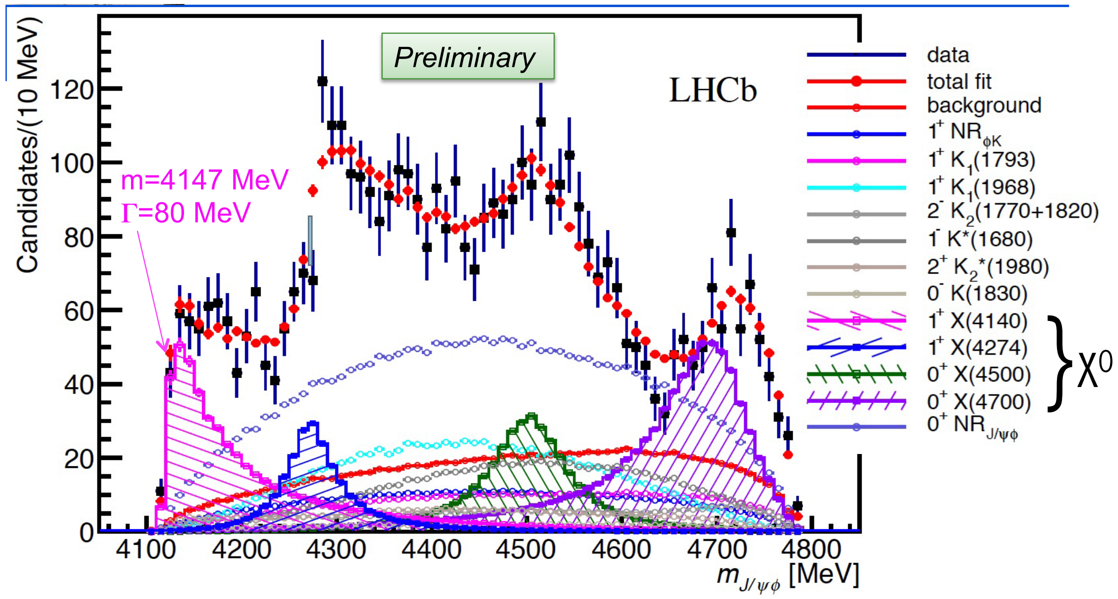


5. New J/Ψ-φ structures

- Four structures
- positive parity, J=0 and 1, positive charge conjugation
- X(4140) seen previously by CDF, D0, CMS and by BELLE

LHCb.

m(J/ψφ)



- 4 visible structures fit with BW amplitudes

28 Recontres de Blois, June 2, 2016

36

Results of fit

- J^P also measured all with >4σ significances

Particle	J^P	Signif-icance	Mass (MeV)	Γ (MeV)	Fit Fraction (%)
X(4140)	1^+	8.4 σ	$4146.5 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	$13.0 \pm 3.2^{+4.8}_{-2.0}$
X(4274)	1^+	6.0 σ	$4273.3 \pm 8.3^{+17.2}_{-3.6}$	$56 \pm 11^{+8}_{-11}$	$7.1 \pm 2.5^{+3.5}_{-2.4}$
X(4500)	0^+	6.1 σ	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$6.6 \pm 2.4^{+3.5}_{-2.3}$
X(4700)	0^+	5.6 σ	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$12 \pm 5^{+9}_{-5}$
NR	0^+	6.4 σ			$46 \pm 11^{+11}_{-21}$

28 Recontres de Blois, June 2, 2016

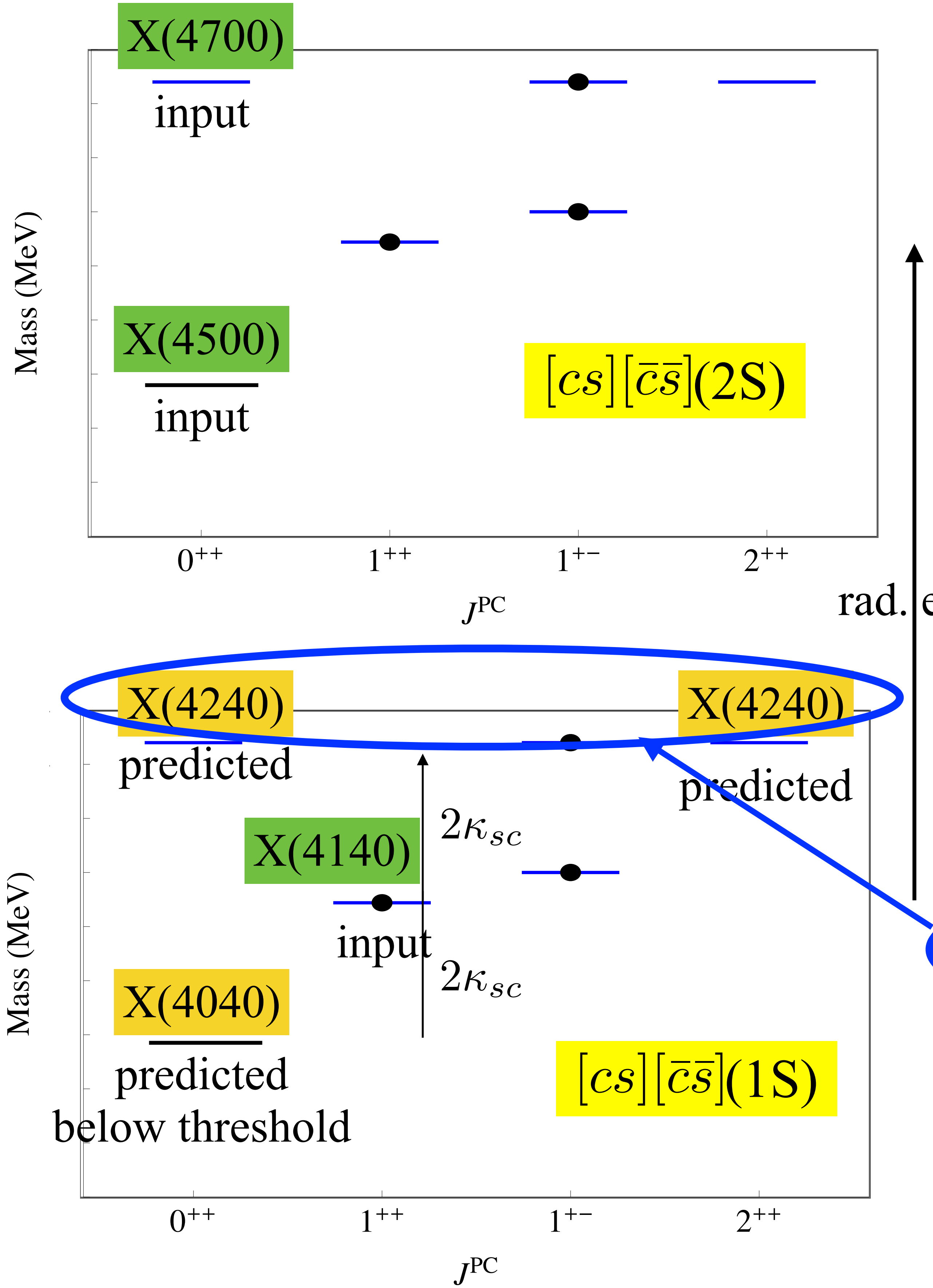
37

We suggest to fit the structures in two tetraquark multiplets, S-wave ground state and the first radial excitation, with composition $[cs][\bar{c}\bar{s}]$. L. Maiani, A. Polosa, V. Riquer, PRD **94** (2016) 054026

With the previously identified $[cq][\bar{c}\bar{q}]$ $(q = u,\ d)$ multiplet, the new resonances would make a step towards a full nonet of S-wave tetraquarks made by c c-bar with a pair of light (u, d, s) quarks.

A surprise and a proposal.

J/Ψ-φ structures and S-wave tetraquarks



$\Delta m = m_{cs} - m_{cq} = 129$ MeV;
$\kappa_{sc} = 50$ MeV $(\kappa_{qc} = 67$ MeV$)$
radial excit. $= 460$ MeV
$[Z(4430) - Z(3900) = 530$ MeV$]$

NOTE :
$X(4140) - X(3872) \sim 270$ MeV;
$\phi(1020) - \rho(770) \sim 244$ MeV

X(4274) cannot be 1^{++}

- **0^{++} ?**
- **2^{++} ?**
- **2 unresolved, almost degenerate lines with $0^{++} + 2^{++}$??**

Decay modes of $J^P=1^+$, C=-1:

$$s_{c\bar{c}} = 1 : J/\Psi + \eta, \chi_c + \eta \ (P - wave)$$
$$s_{c\bar{c}} = 0 : \eta_c + \phi, h_c + \phi \ (P - wave)$$

what about the strange members of the nonet?

- We expect strangeness= ±1 tetra quarks: $X_{\bar{s}} = [cq][\bar{c}\bar{s}]; \; X_s = [cs][\bar{c}\bar{q}]$
- partners of X(4140) should decay in: $J/\Psi + K^*/\bar{K}^* \to \mu^+\mu^- + \pi + K_S$
- or: $J/\Psi + K/\bar{K} \to \mu^+\mu^- + K_S$
- Mass can be estimated at: $M(X_s) = \dfrac{4147 + 3872}{2} = 4010$ (prediction)
- are they visible at LHCb/BELLE/BES III?

Variations on the theme

N.V. Drenska, R. Faccini and A. D. Polosa, Phys. Rev. **D** 79 (2009) 077502

- J/Ψ-φ spectrum obtained with meson&baryon spin-spin parameters does not fit with experiment
- QCD sum rules with tetra quark currents tried with some success and support X(4500) and X(4700) to be higher excitations, radial or D-wave

Z. G. Wang, arXiv:1607.00701 [hep-ph];

- flavour SU(3) nonet including J/Ψ-φ has been considered in:

R. Zhu, Phys Rev. **D** 94 (2016) 054009

- diquarks in color 6 have been considered by several authors

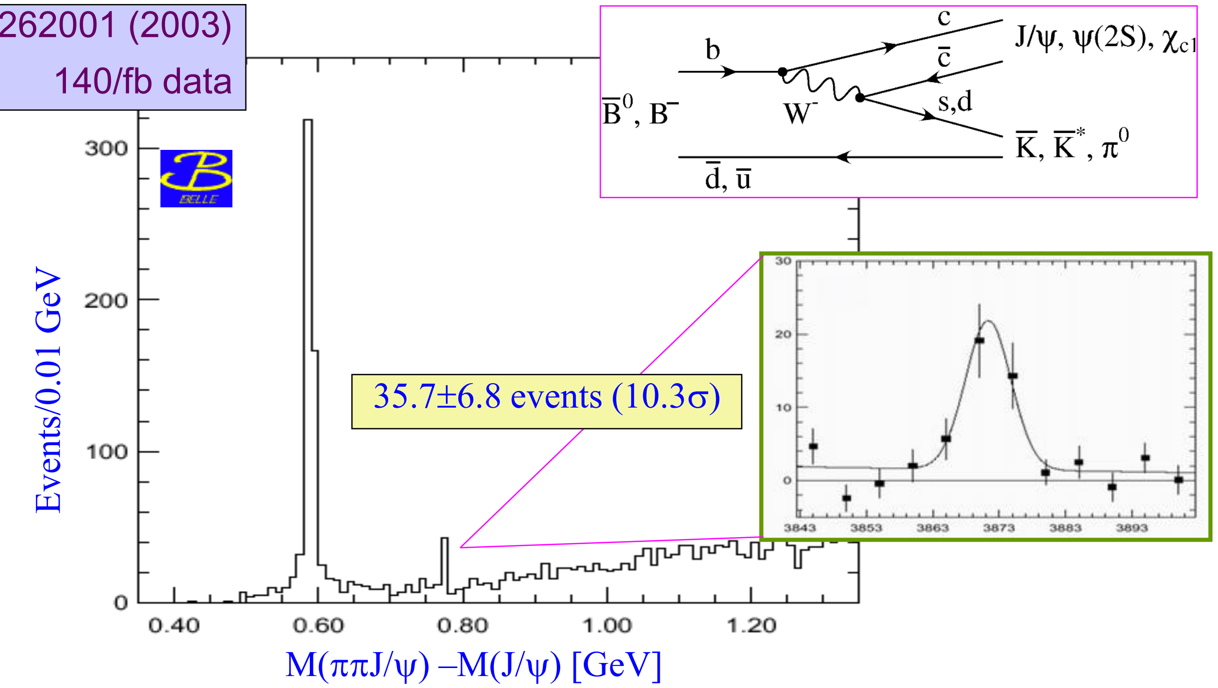
J. Wu *et al.*, arXiv:1608.07900 [hep-ph]

- if at all bound, tetraquarks made by color 6 diquarks would double the spectrum
- an option if X(4270) turns out to be a pure 1^{++} resonance?
- basic masses of diquark in color 3 and 6 must be different: X(4270)-X(4140) is not due only to spin-spin interactions and is essentially incalculable.

6. Few remarks about molecules

X(3872): observed 16 years ago, it is still very mysterious

PRL91 262001 (2003)
140/fb data



$M = 3872.0 \pm 0.6 \pm 0.5$ MeV, $\Gamma < 2.7$ MeV

$$\frac{B(B^\pm \to XK^\pm \to \pi^+\pi^- J/\psi K^\pm)}{B(B^\pm \to \psi' K^\pm \to \pi^+\pi^- J/\psi K^\pm)} = (6.3 \pm 1.2 \pm 0.7)\%$$

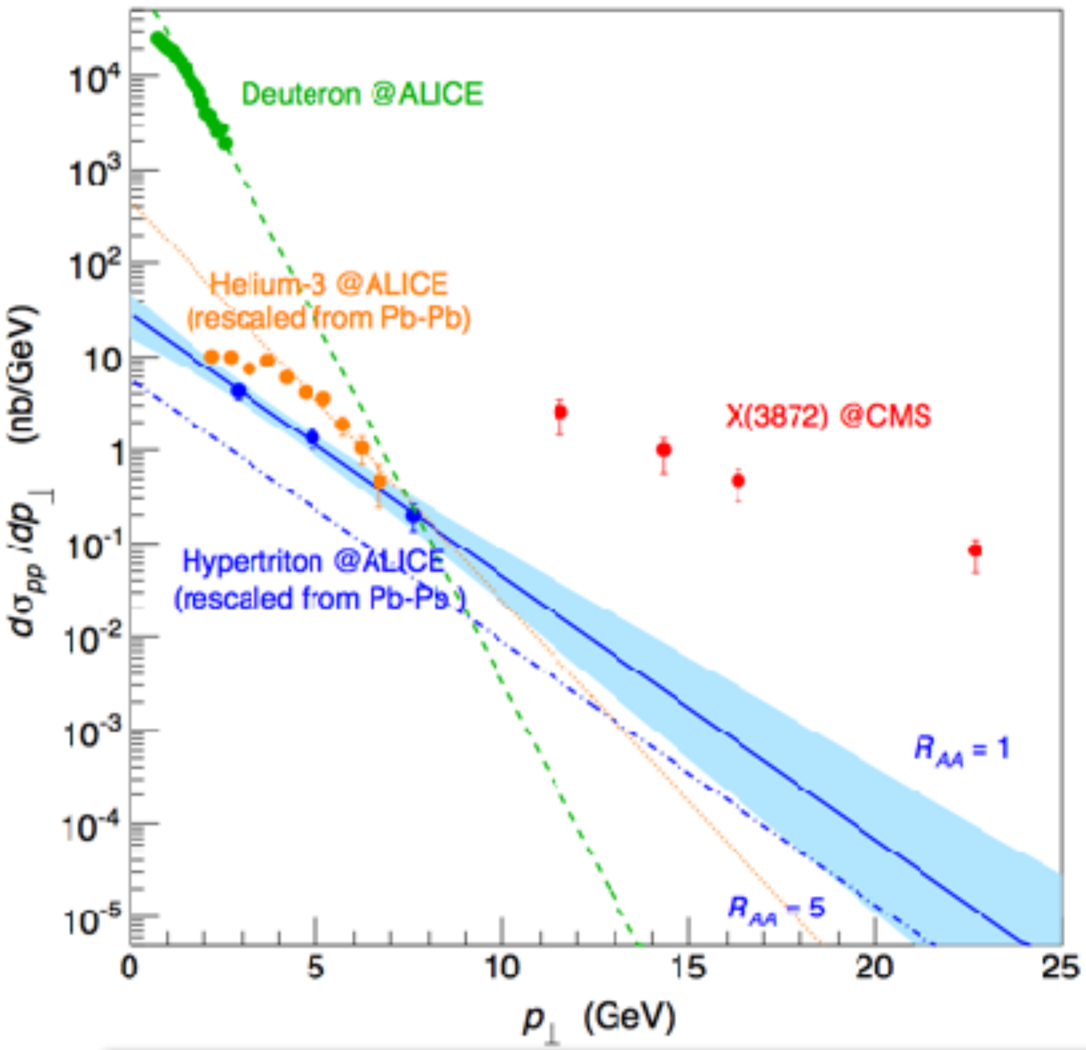
2

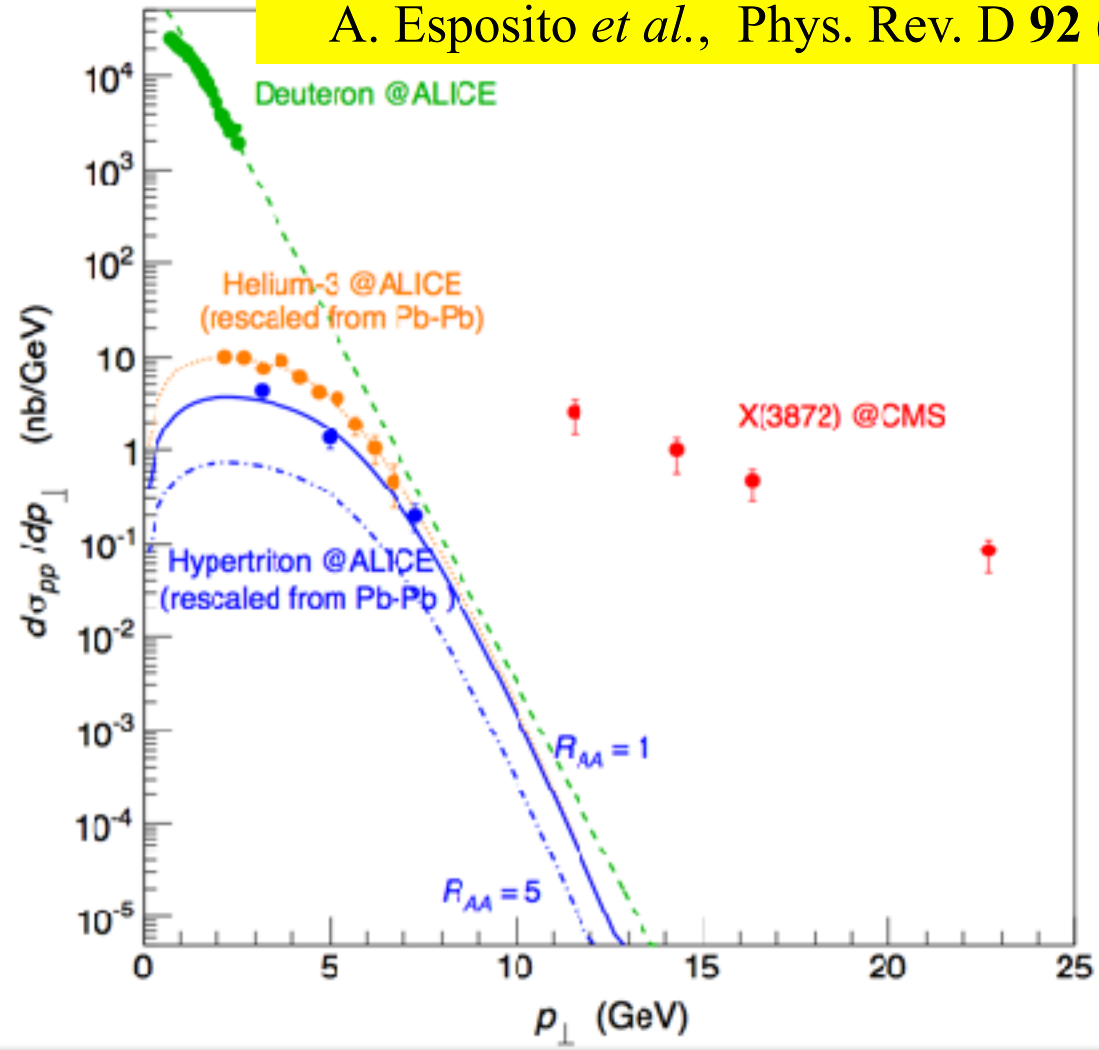
- present mass value: M=3871.69 ± 0.17 MeV
- very close to $M(D^0)+MD^{0*})$: a hadron molecule?
- if X is a D^0+D^{0*} molecule, |Binding Energy|<0.17 MeV
- i.e. radius R~ $(2\ M(D)/2\ {}^*BE)^{-1/2}$ ~ 10 fm ~ 15 · proton's radius
- how about production in collisions at large p_{perp} ?

$$M(D^0) + M(D^{*0}) = 3871.68$$
$$M(D^+) + M(D^{*-}) = 3979.91$$

Rescaling from Pb-Pb ALICE cross sections to p-p CMS cross section is done with: Glauber model (**left panel**) and blast-wave function (**right panel**}) (R_{AA} or R_{CP} =1)

A. Esposito *et al.*, Phys. Rev. D **92** (2015) 3, 034028





Collective effects in Pb-Pb (e.g.quark-gluon plasma) **enhance nuclear cross sections** and therefore **reduce the cross section rescaled to p-p**.

- There is a vast difference in the probability of producing X(3872) and that of producing light nuclei, true “hadronic molecules”, in high energy collisions
- high energy production of suspected exotic hadrons from quark-gluon plasma in HI collisions at colliders can be very effective tool to discriminate different models
- a long list of suspects: f_0(980), X(3872), $Z^\pm$(3900), $Z^\pm$(4020), $Z^\pm$(4430), X(4140)….

can molecule be saved by mixing with charmonium?

One-pion exchange vs X(3872), Z_c(3900) and Z_c(4020)

N.~A.~Tornqvist, Phys. Lett. B **590** (2004), 209-215
M. Karliner and J. L. Rosner, Phys. Rev. Lett. **115** (2015) 122001
Ali et al. Cambridge University Press (2019)

- The $DD^*\pi$ lagrangian is

$$\mathscr{L}_{\pi D^* D} = \frac{g}{f_\pi}\left[(D^*_\mu)^\dagger \mathbf{t} D + \bar{D}^\dagger \mathbf{t} \bar{D}^*_\mu\right] \cdot (\partial^\mu \pi) + \text{ h.c.}$$

- It produces a potential that can possibly make molecular bound states in S wave.

- Interaction: $H \propto I^a \otimes I^a$. Lowest energy states:

$D - \bar{D}^*, D^* - \bar{D}$, J=1^+, (C=+1, I=0) and (C=-1, I=1) (X(3872), Zc(3900))

- $D^* - \bar{D}^*$, J=1^+, (C=-1, I=0) yet to be seen? Z_c(4020) ?

- what exchange could bind a meson-meson molecular $X_{s\bar{s}}(4140)$? or Z_c

- exchange of $\eta, \phi \ldots J/\Psi \ldots$? ???