Lecture 5. $X(3872)$ and its missing partners

- ***Summary***

1. The overall panorama ✓
2. Constituent Quark Model and masses of conventional mesons and baryons ✓
3. Light and Heavy Tetraquarks. First comparison with hadron molecules ✓
4. Tetraquarks and the EightFold Way ✓
5. $X(3872)$ and its missing partners ✓
6. Born-Oppenheimer approximation for double charm baryons and tetraquarks
7. Multiquark states in N colours, in the $N \to \infty$ limit
8. Tetraquarks vs. molecules: the Weinberg criterium for $X(3872)$ and the double charm $\mathcal{T}^{+}_{cc}(3875)$

1. Isospin breaking

- The electromagnetic (e.m.) interaction violates Isospin symmetry, since proton and neutron or, equivalently, u and d quarks, have different charges.
- A current-algebra calculation of the purely e.m. $\pi^+ - \pi^0$ mass difference gives indeed:

$$m_{\pi^+} - m_{\pi^0} \simeq 5.0 \text{ MeV}, \; (\text{expt}.\; 4.5936 \pm 0.0005)$$

T. Das et al., PRL 18 (1967) 759

- This is not the whole story, however. Calculations of the purely e.m. neutron-proton mass difference gave systematically the wrong result: $m_p - m_n > 0$ (has had to be expected)
- On top of the e.m. corrections, S. Coleman and S. Glashow

S. Coleman, S. L. Glashow, PR 134 (1964) 1367.

made the hypotehesis that there is a part of the strong-interaction lagrangian (the tadpole) responsible for an additional violation of Isospin. For baryons, the tadpole dominates the p-n mass difference, to give the observed negative value $m_p - m_n \simeq -1.4$ MeV,

see e.g. J. Gasser, H.Leutwyler, NP B94, (1975) 269

- in QCD, the tadpole hamiltonian of Coleman and Glashow is what we called $\mathscr{L}_3$ (Lect. 4)

$$\mathscr{L}_3 = m_3(\bar{u}u - \bar{d}d),\; m_3 = \frac{m_u - m_d}{2}; \quad (\Delta I = 1)$$

u, d, and s current quark masses

- Current quark masses measure the degree of non-conservation of the Axial currents, e.g. $A^1_\mu(x)$, in terms of the Pseudoscalar current $P^1(x)$

$$A^1_\mu = \frac{1}{2}\left(\bar{u}\gamma_\mu\gamma_5 d + \bar{d}\gamma_\mu\gamma_5 u\right);\ P^1 = (\bar{u}\gamma_5 d + \bar{d}\gamma_5 u)$$

$$\partial^\mu A^1_\mu = \frac{m_u + m_d}{2} \cdot P_1,$$

etc., for $A^2_\mu \dots A^8_\mu$

- We introduce the full Scalar and Pseudoscalar octets:

$$S_i = \bar{q}\lambda_i q;\quad P_i = i\bar{q}\lambda_i\gamma_5 q\ (i, j = 0,1,\cdots,8)$$

- with commutation relations:
- $\left[V^i, S_j\right] = if_{ijk}\ S_k;\quad \left[V^i, P_j\right] = if_{ijk}\ P_k;\quad \left[A^i, S_j\right] = id_{ijk}\ P_k;\quad \left[A^i, P_j\right] = id_{ijk}\ S_k;$

the fully antisymmetric coefficients f_{ijk} are the $SU(3)_f$ structure functions; the fully symmetric coefficients d_{ijk} are defined in terms of the Gell-Mann's matrices: $\mathrm{Tr}[\lambda_i\{\lambda_j, \lambda_k\}] = 2d_{ijk}$

- The basic Ward identities ($J^\mu = V^\mu$, A^μ):

$$q_\mu \int d^4x\ e^{iqx} < 0|\,T\left[J^\mu(x)P_i(0)\right]|0> =$$

$$= i\int d^4x\ e^{iqx} < 0|\,T\left[\partial_\mu J^\mu(x)P_i(0)\right]|0> + i\int d^3x\ e^{-i\mathbf{q}\mathbf{x}} < 0|\left[J^0(\mathbf{x},0), P_i(0)\right]|0>$$

u, d, and s current quark masses (cont'd)

- In the limit of quark masses =0, Chiral symmetry $SU(3) \otimes SU(3)$ is exact
- Long ago,G. Jona-Lasinio and Y. Nambu proposed chiral symmetry to be spontaneusly broken, with pions the massless Goldston Bosons. Quark mass would then give a non vanishing pion mass.
- If spontaneous breaking respects (vector) flavour symmetry, $SU(3)_f$, pion and kaons would be in an octet, splitted by quark masses, mainly the octet $\mathscr{L}_8$, which is what we see.
- This situation obtains if scalar densities take a vacuum-expectation-value, with $< 0|S_0|0 > \neq 0, \;\; < 0|S_{3,8}|0 > = 0$
- relations between quark current masses and π, K masses has been worked out in 1968 by Gell-Mann, Oakes and Renner and by Glashow and Weinberg, using the basic Ward identities given before.

M. Gell-Mann, R. Oakes and J. Renner, PR **175**, 2195 (1968);
S. L. Glashow and S. Weinberg, PRL **20**, 224 (1968).

- The basic Ward identity for ($J^\mu = A^\mu$):

$$q^\mu \int d^4x \; e^{iqx} < 0 | T\left[A^1_\mu(x) P_i(0)\right] | 0 > =$$

$$= i \int d^4x \; e^{iqx} < 0 | T\left[\partial^\mu A^1_\mu(x) P_i(0)\right] | 0 > + \, i \int d^3x < 0 | \left[A^1_0(\mathbf{x},0) P_i(0)\right] | 0 >$$

- For $m_q \neq 0$ there are no massless Goldstone bosons, the correlation functions have no pole for $q \to 0$ and in this limit the first line tends to zero. We find

$$\frac{m_u + m_d}{2} \int d^4x \; e^{iqx} < 0 | T\left(P^1(x) P^1(0)\right) | 0 > = -\, i d_{110} < 0 | S_0 | 0 > = -\, iA$$

- Approximate: $\int d^4x \; e^{iqx} < 0 | T\left(P^1(x) P^1(0)\right) | 0 > = Z_\pi^2 \dfrac{i}{q^2 - m^2_{\pi^+}}$, to find:

$$m_u + m_d = C m^2_{\pi^+}$$

and, similarly

$$m_u + m_d = C m^2_{\pi^0}$$

$$m_u + m_s = C m^2_{K^+}$$

$$m_d + m_s = C m^2_{K^0}$$

$$\frac{m_u + m_d}{m_s + \frac{m_u + m_d}{2}} = \frac{m^2_{\pi^+} + m^2_{\pi^0}}{m^2_{K^+} + m^2_{K^0}} = R \simeq 0.077 \;\to\; \frac{m_u + m_d}{2} \simeq 6 \text{ MeV}, \; (m_s = 150 \text{ MeV})$$

• R. Dashen showed that the combination:

R. Dashen, R. Physical Review **183** (1969) 1245

$$[m^2_{K^+} - m^2_{K^0}] - [m^2_{\pi^+} - m^2_{\pi^0}]$$

is not affected by e.m. corrections. Using the previous results, we find

$$\frac{m_u - m_d}{m_s + \frac{m_u + m_d}{2}} = 2\frac{[m^2_{K^+} - m^2_{K^0}] - [m^2_{\pi^+} - m^2_{\pi^0}]}{m^2_{K^+} + m^2_{K^0}} \simeq -0.021$$

- in total:

$$m_s = 150 \text{ MeV} : \ m_u \simeq 4.3 \text{ MeV}; \ m_d \simeq 7.6 \text{ MeV} \quad m_d - m_u = 3.3 \text{ MeV}$$

$$m_s = 180 \text{ MeV} : m_u \simeq 5.0 \text{ MeV}; \ m_d \simeq 8.8 \text{ MeV} \ , m_d - m_u = 3.9 \text{ MeV}$$

The mass difference of up and down quarks is not small compared to their masses.
The real world is close to isotopic spin symmetry not because the quark masses are very similar, like the masses of the neutron and proton, but because both are roughly equal to zero!

N. Cabibbo, L. Maiani, in M. Conversi (editor) *Evolution of Particle Physics. A Volume Dedicated to Edoardo Amaldi on his Sixtieth Birthday*. Academic Press (1970)

Current vs Constituents quark masses ?

- In Lect.2 we found much larger constituent quark masses then the current masses just found
- the difference is believed to be due to the mass associated to the QCD field that surround quarks in the bound states.

MESONS	q	s	c	b
mass(MeV)	308	484	1664	5005

BARYONS	q	s	c	b
mass(MeV)	362	540	1710	5044

$m_s - m_q \sim 180$

- Indeed constituent quark masses from baryon and meson spectrum are different
- but the q-s mass difference, which is determined by the Ward identity valid to all orders in QCD turn out to be the same for baryon or meson constituent masses
- 150 MeV is an estimate from the equal spacing rule for baryon spin 3/2 resonances.
- If we take our value 180 MeV, in which the breaking due to spin-spin interaction has been removed, we would find the slightly larger value of Isospin breaking: $m_d - m_u = 4$ MeV.

Isospin breaking effects have two components:

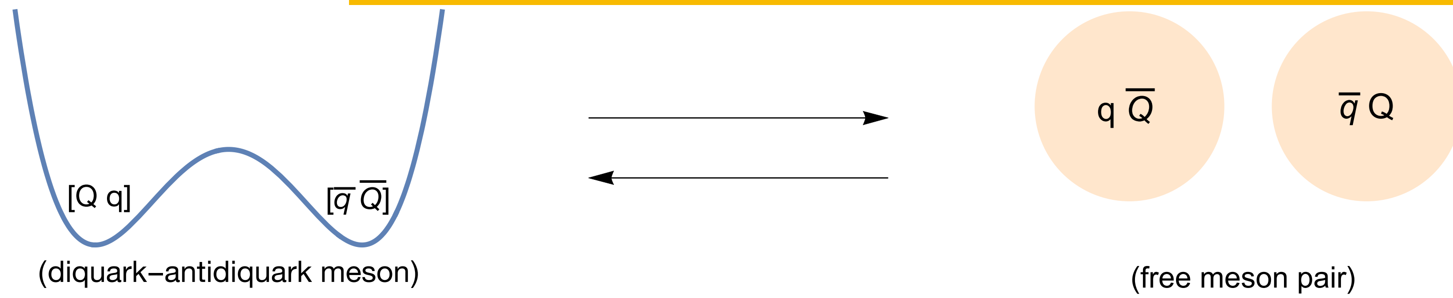
- $u - d$ quark mass difference, including the one arising from e.m. hyperfine interactions
- electromagnetic effects, in particular the electrostatic repulsion between quarks
- Karliner & Rosner: fit isospin breaking differences in baryons to determine the relevant parameters
- in particular the electrostatic corrections, which scale with the radius of the diquark in the baryon

$$a = \frac{1}{137} < \frac{1}{R} >^{\text{Baryon}}_{cq} \sim 2.83 \text{ MeV} \rightarrow R^B_{(qq')_{\bar{3}}} \sim 0.5 \text{ fm}$$

M. Karliner and J. L. Rosner, PR **D96**, 033004 (2017)

2. Going inside the tetraquark: the X_u, X_d and $X^{\pm}$ puzzles

L. Maiani, A. D. Polosa and V. Riquer, Phys. Lett. B **778** (2018) 247



- The attraction that generates the diquark implies that diquarks, or antidiquarks, are segregated in two different potential wells, separated in space.
- QCD confining forces prevail at large distances, where the diquarks see each other as QCD point charges.
- At shorter distances the internal structure is felt and the competing interactions that tend to dissociate the diquark, e.g, attraction between quarks and antiquarks, produce repulsive forces between diquark and antidiquark and a rise in the potential.

A. Selem and F. Wilczek, Hadron systematics and emergent diquarks, hep-ph/0602128.

- A phenomenological basis is provided by the mass ordering in Z(3900) vs Z(4020):
  - spin-spin interactions between light quark and antiquark located in different diquarks are definitely smaller than one would guess from the same interactions within mesons;
  - spin-spin interaction inside the diquark is about four times larger that the same interaction in the diquarks inside charmed baryon states.

Isospin breaking in tetraquarks

- If quark mass difference dominates: $M_{X_u} - M_{X_d} = 2(m_u - m_d) \sim -6$ MeV $= \Delta_m$
- but we do not see two distinct lines, X_u and X_d, around 3872 MeV ????
- However, one must account (mainly) for electrostatic interactions inside diquarks and between diquark and antidiquark
- K&R give the parameter that determines the electrostatic forces among quarks:

$$a = \frac{1}{137} < \frac{1}{R} >^{\rm Baryon}_{cq} \sim 2.83 \text{ MeV} \to R^B_{(qq')_{\bar{3}}} \sim 0.5 \text{ fm}$$

M. Karliner and J. L. Rosner, PR **D96**, 033004 (2017)

- For tetraquarks there are two radii, $R_{cq} \sim R^B_{(qq')_{\bar{3}}} \left(\frac{\kappa^B_{cq}}{\kappa^X_{cq}}\right)^{1/3} \sim 0.3$ fm
- and the X radius, $R_X = \lambda\ R^X_{cq}$ with λ = free parameter,

- We find:

u-c vs *d-c*

diquark-antidiquark repulsion /attraction

$$M(X_u) - M(X_d) = \Delta_m + \frac{4}{3}a' + \frac{5}{3}\frac{a'}{\lambda} + \dots$$

–	$\lambda = 1$	$\lambda = 3$
$M(X_u) - M(X_d)$	-6.1 ± 0.1	-1.2 ± 0.3
$M(X_u) - M(X^+)$	-5.31 ± 0.05	-1.34 ± 0.12

- $\lambda = 3$ gives reasonable values for radii: $R_{cq} \sim 0.3$ fm, $R_X \sim 1$ fm .

X_u and X_d may be degenerate within the observed X line
Is experimental resolution the key to the puzzle ?

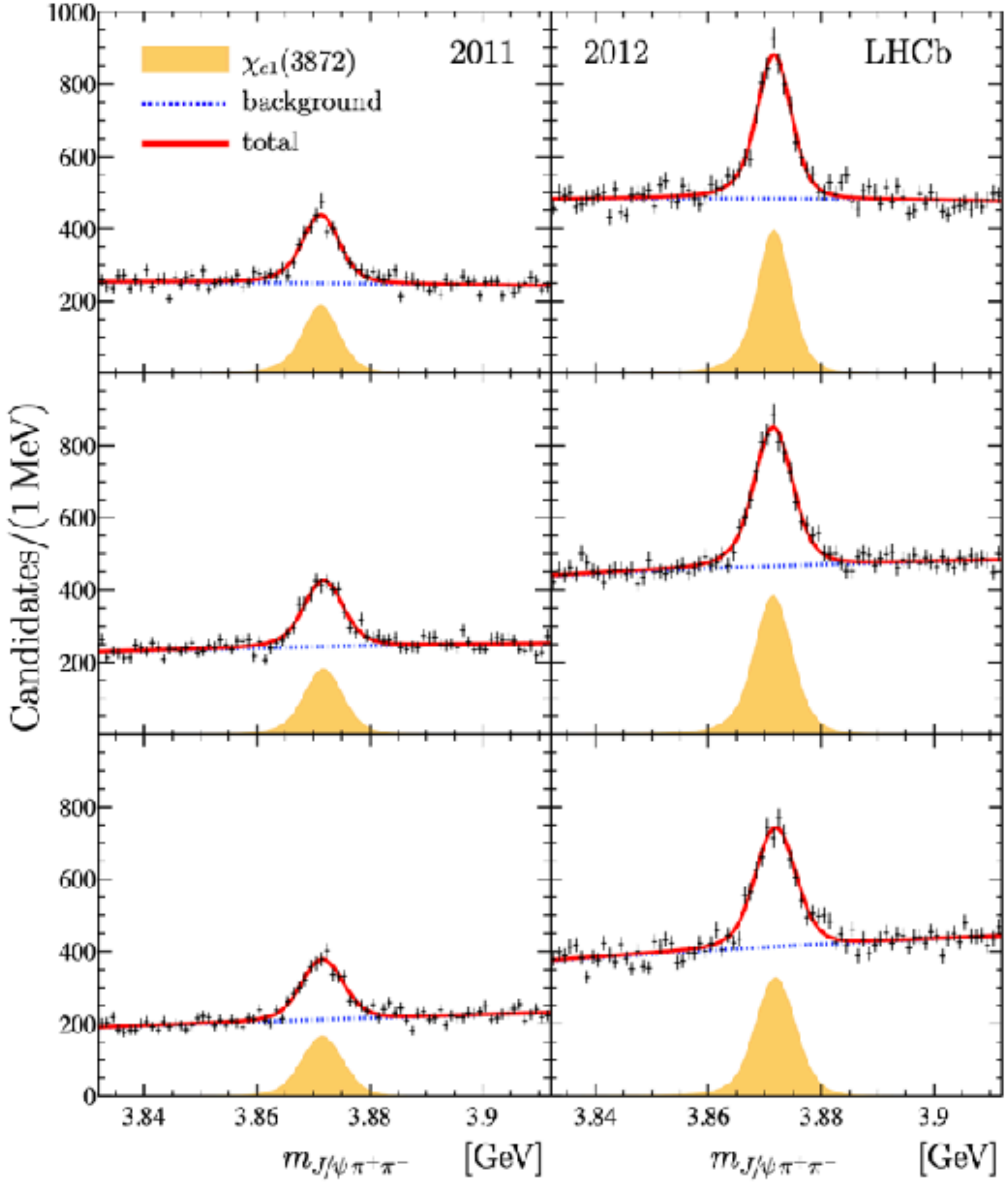


Figure 2: Mass distributions for $J/\psi\pi^+\pi^-$ candidates in the $\chi_{c1}(3872)$ region for (top) the low, (middle) mid and (bottom) high $p_{\pi^+\pi^-}$ bins. The left (right)-hand plot is for 2011 (2012) data. The projection of the fit described in the text is superimposed.

Where is it the $X^+ = [cu][\bar{c}\bar{d}]$?

- The charged X has been searched in B^0 and B^+ decays: $B \to K\, X^+$.
- Present experimental upper limits are inconsistent wth the hypothesis that X(3872) has isospin=1, which however is a very restrictive hypothesis
- To test consistency of data with the tetraquark picture, we have analysed the observed four ratios of decay rates of $B^{0,+}, B_s \to \text{meson} + X(3872)$:

L. Maiani, A. D. Polosa and V. Riquer, PRD **102** (2020) 034017

$$R^{(++)} = \frac{\Gamma(B^+ \to K^+ + X(3872) \to K^+ + 3\pi\ \psi)}{\Gamma(B^+ \to K^+ + X(3872) \to K^+ + 2\pi\ \psi)}$$

$$R^{(00)} = \frac{\Gamma(B^0 \to K^0 + X(3872) \to K^+ + 3\pi\ \psi)}{\Gamma(B^0 \to K^0 + X(3872) \to K^+ + 2\pi\ \psi)}$$

$$R_{2\pi}^{(++,00)} = \frac{\Gamma(B^+ \to K^+ + X(3872) \to K^+ + 2\pi\ \psi)}{\Gamma(B^0 \to K^0 + X(3872) \to K^+ + 2\pi\ \psi)}$$

$$R_{2\pi}^{(s\phi,00)} = \frac{\Gamma(B_s \to \phi + X(3872) \to \phi + 2\pi\ \psi)}{\Gamma(B^0 \to K^0\, X(3872) \to K^+ 2\pi\ \psi)}$$

- At the 2021 Lepton Photon Conference LHCb has presented the new result:

$$\frac{g_{X(3872)\to\rho^0\,\psi}}{g_{X(3872)\to\omega\,\psi}} = 0.29 \pm 0.0$$

- With a significantly better precision than previous data, the result indicates a considerable violation of Isospin symmetry, in particular compared to the analogous ratio of a typical charmonium:

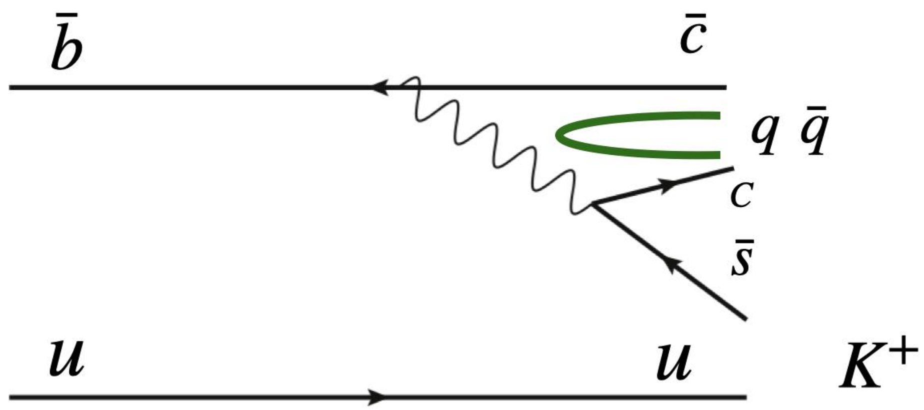
$$\frac{g_{\psi(2S)\to\pi^0\,\psi}}{g_{\psi(2S)\to\eta\,\psi}} = 0.045 \pm 0.001,$$

(against the tendence of the recent by Pdg renaming $X(3872) \to \chi_{c1}$).

- the new result leads to the re-evaluation $R^{(++)} = 0.70 \pm 0.4 \to 2.0 \pm 0.5$, with respect to the previous analysis
- leaving unchanged the other values:

$$R^{(00)} = 1.4 \pm 0.6,\ R_{2\pi}^{(++,00)} = 2.0 \pm 0.6,\ R^{(s\phi,00)} \sim 1$$

The model



- Diagram for the decay $B^+ \to K^+ +$ tetraquark. Restricting to non-strange tetraquarks X_u and X_d, the K meson can be formed by the $\bar{s}$ from weak decay and either the spectator quark (A1) or the quark from the sea (A2). One has:

$$\mathscr{A}(B^+ \to X_d K^+) = A_1,\ \mathscr{A}(B^+ \to X_u K^+) = A_1 + A_2;\quad \mathscr{A}(B^+ \to X^+ K^0) = A_2$$

$$\mathscr{A}(B^0 \to X_d K^0) = A_1 + A_2,\ \mathscr{A}(B^0 \to X_u K^0) = A_1;\quad \mathscr{A}(B^0 \to X^- K^+) = A_2$$

- Physical tetraquarks $X_{1,2}$ are combinations of $X_{u,d}$ with a mixing angle ϕ:

$$X_1 = \cos\phi \frac{X_u + X_d}{\sqrt{2}} + \sin\phi \frac{X_u - X_d}{\sqrt{2}};\ X_2 = -\sin\phi \frac{X_u + X_d}{\sqrt{2}} + \cos\phi \frac{X_u - X_d}{\sqrt{2}}$$

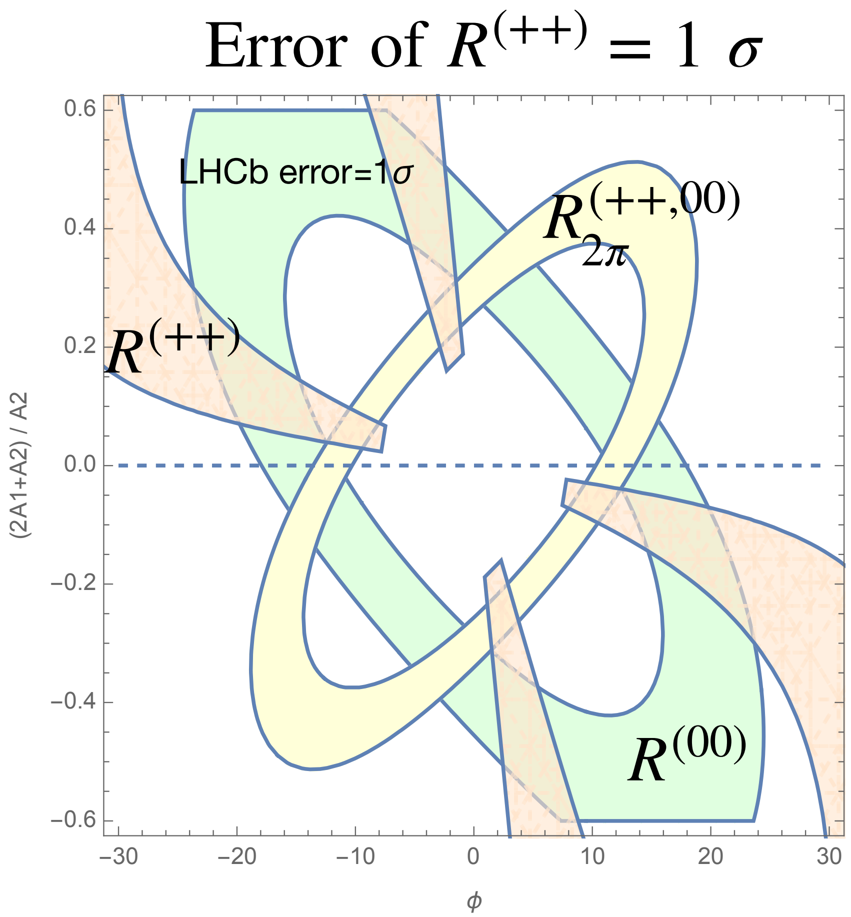
- so that, e.g.

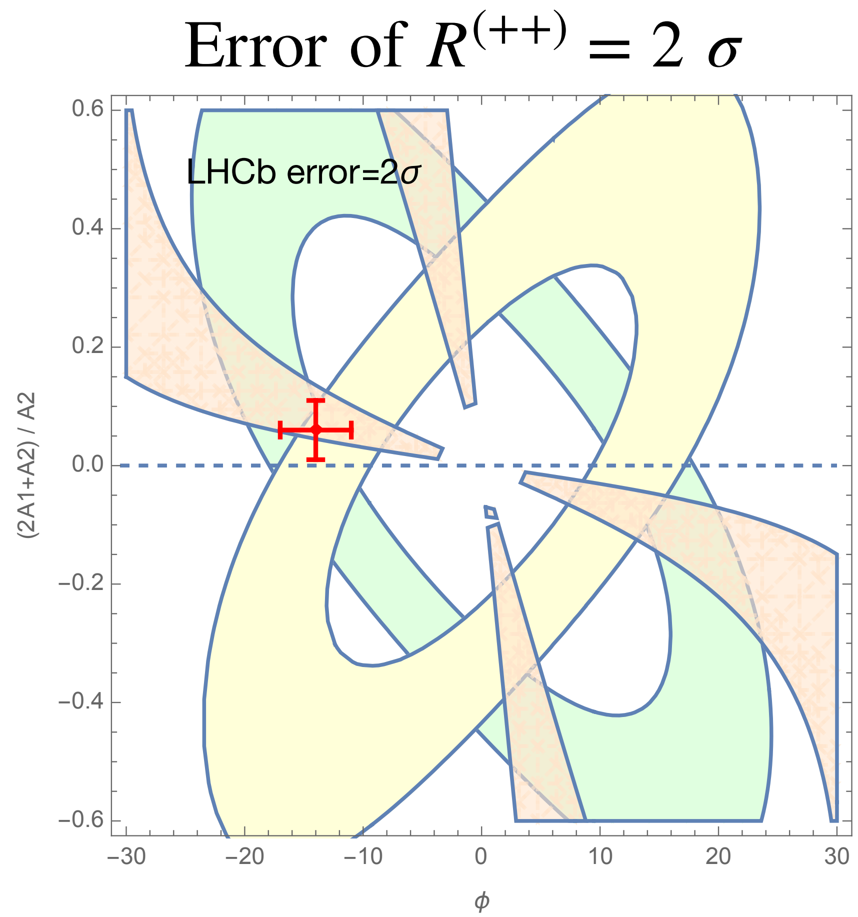
$$A(B^+ \to K^+ X_1 \to K^+ \rho^0 + \psi) = A(B^+ \to K^+ X_1) \cdot A(X_1 \to K^+ \rho^0 + \psi) \propto$$

$$\propto \Big[(\cos\phi + \sin\phi)(A_1 + A_2) + (\cos\phi - \sin\phi)A_1\Big]\sin\phi = \Big((2A_1 + A_2)\ \cos\phi + A_2\ \sin\phi\Big)\sin\phi$$

- the observed rates are obtained by summing incoherently the amplitudes squared of the two unresolved lines inside X(3827), i.e. summing the rates into X_1 and X_2;
- apart from phase space, ratios of rates are functions of ϕ and $z = A_2/(2A_1 + A_2)$

Results





The red cross in the figure on the right indicates a solution with errors estimated from the extension of the overlap.

Parameters of this solution:

$$\phi = -14^0 \pm 3^0;\ z = +0.06 \pm 0.005$$

(rather similar to those obtained previously)

Limits to $X^\pm$ production

$$R_{2\pi}^{(0+,00)} = \frac{\Gamma(B^0 \to K^+ X^- \to K^+ \psi\ \pi^0 \pi^-)}{\Gamma(B^0 \to K^0 X(3872) \to K^0 \psi\ \pi^+ \pi^-)}$$

$PdG:\ R_{2\pi}^{(0+,00)} < 1$

$0.05 < R_{2\pi}^{(0+,00)} < 0.57$ (previous analysis)

$0.0 < R_{2\pi}^{(0+,00)} < 0.26$, (present analysis)

SUMMARY

- Chiral Symmetry spontaneous breaking fits well in the Standard Theory and current quark masses are well understood
- Attemps to understand constituent quark masses and their differences between Meson Baryons are still ongoing

see e.g. M.Karliner, J. L. Rosner, PRD **90** (2014) 094007, arXiv:1408.5877

- Lattice QCD calculations support Standard Theory Spectroscopy for classical hadrons and are taking momentum for Exotic Hadrons

- Under the hypothesis of the two separated wells, the key to the problems of X(3872) are:
  - resolution (are there two lines under X(3872) ?)
  - and statistics (can we go to branching fractions <1 to see the $X^{\pm}$?).

Will time tell ??