Axions as Dark Matter Candidates

Andreas Ringwald
FNHP 2022
Frontiers in Nuclear and Hadronic Physics
Galileo Galilei Institute School
Online Event
Feb 21, 2022 - Mar 04, 2022

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

DESY.

Overall Plan

Overall Plan

- Vacuum Structure in Quantum Chromodynamics

Overall Plan

- Vacuum Structure in Quantum Chromodynamics
- Strong CP Puzzle

Overall Plan

- Vacuum Structure in Quantum Chromodynamics
- Strong CP Puzzle
- The Axion

Overall Plan

- Vacuum Structure in Quantum Chromodynamics
- Strong CP Puzzle
- The Axion
- Axion Dark Matter

Overall Plan

- Vacuum Structure in Quantum Chromodynamics
- Strong CP Puzzle
- The Axion
- Axion Dark Matter
- Axion Experiments

Today's Plan

- **Vacuum Structure in Quantum Chromodynamics**
- **Strong CP Puzzle**
- **The Axion**

Vacuum Structure in Quantum Chromodynamics

Action of Quantum Chromodynamics (QCD)

Vacuum Structure in Quantum Chromodynamics

Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation

Vacuum Structure in Quantum Chromodynamics

Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation

- Most general gauge- and Lorentz-invariant renormalizable action:

$$S_{\rm QCD} = \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q}\,(i\gamma^\mu D_\mu - \mathcal{M}_q)\,q + \theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \right\}$$

[Gross,Wilczek 73;Politzer 73; Fritzsch,Gell-Mann,Leutwyler 73] [Belavin et al. `75;´t Hooft 76;Callan et al. `76;Jackiw,Rebbi `76]

Vacuum Structure in Quantum Chromodynamics

Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation
- Most general gauge- and Lorentz-invariant renormalizable action:

$$S_{\rm QCD} = \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q}\,(i\gamma^\mu D_\mu - \mathcal{M}_q)\, q + \theta\, \frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\}$$

[Gross,Wilczek 73;Politzer 73; Fritzsch,Gell-Mann,Leutwyler 73] [Belavin et al. \`75;´t Hooft 76;Callan et al. \`76;Jackiw,Rebbi \`76]

  - Covariant gauge kinetic term, $-\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$, build from gluonic field strength tensors, $G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, in terms of gluonic gauge fields, G^a_μ

Vacuum Structure in Quantum Chromodynamics

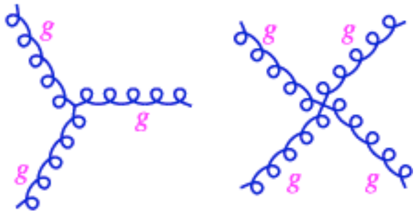
Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation
- Most general gauge- and Lorentz-invariant renormalizable action:

$$S_{\rm QCD} = \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q}\,(i\gamma^\mu D_\mu - \mathcal{M}_q)\, q + \theta\, \frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\}$$

[Gross,Wilczek 73;Politzer 73; Fritzsch,Gell-Mann,Leutwyler 73] [Belavin et al. \`75;´t Hooft 76;Callan et al. \`76;Jackiw,Rebbi \`76]

  - Covariant gauge kinetic term, $-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$, build from gluonic field strength tensors, $G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, in terms of gluonic gauge fields, G^a_μ , describes gluons and their interactions



Vacuum Structure in Quantum Chromodynamics

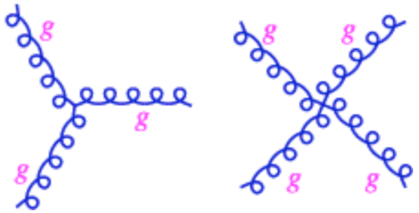
Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation
- Most general gauge- and Lorentz-invariant renormalizable action:

$$S_{\rm QCD} = \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q}\,(i\gamma^\mu D_\mu - \mathcal{M}_q)\, q + \theta\, \frac{g_s^2}{32\pi^2}\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\}$$

[Gross,Wilczek 73;Politzer 73; Fritzsch,Gell-Mann,Leutwyler 73] [Belavin et al. \`75;´t Hooft 76;Callan et al. \`76;Jackiw,Rebbi \`76]

  - Covariant gauge kinetic term, $-\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$, build from gluonic field strength tensors, $G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, in terms of gluonic gauge fields, G^a_μ , describes gluons and their interactions



  - Covariant quark kinetic term, $\overline{q} i \gamma_\mu D^\mu q$, involving covariant derivative $D_\mu = \partial_\mu - i g_s T^a G^a_\mu$

Vacuum Structure in Quantum Chromodynamics

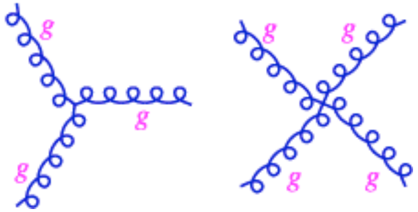
Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation
- Most general gauge- and Lorentz-invariant renormalizable action:

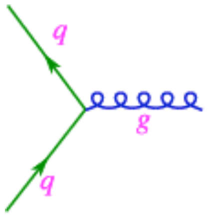
$$S_{\rm QCD} = \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q}\left(i\gamma^\mu D_\mu - \mathcal{M}_q\right) q + \theta \, \frac{g_s^2}{32\pi^2} \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\}$$

[Gross,Wilczek 73;Politzer 73; Fritzsch,Gell-Mann,Leutwyler 73] [Belavin et al. `75;´t Hooft 76;Callan et al. `76;Jackiw,Rebbi `76]

  - Covariant gauge kinetic term, $-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$, build from gluonic field strength tensors, $G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, in terms of gluonic gauge fields, G^a_μ , describes gluons and their interactions



  - Covariant quark kinetic term, $\overline{q} i \gamma_\mu D^\mu q$, involving covariant derivative $D_\mu = \partial_\mu - i g_s T^a G^a_\mu$, describes quarks and their interactions



Vacuum Structure in Quantum Chromodynamics

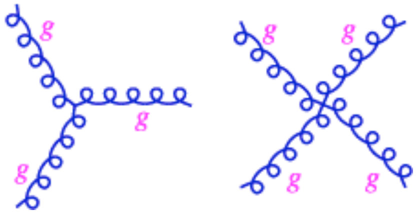
Action of Quantum Chromodynamics (QCD)

- **QCD:** SU(3) gauge theory with 6 flavours of quarks (u,d,s,c,b,t) in fundamental representation
- Most general gauge- and Lorentz-invariant renormalizable action:

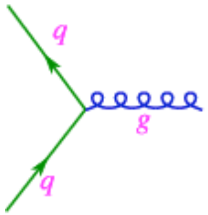
$$S_{\rm QCD} = \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q} \left(i\gamma^\mu D_\mu - \mathcal{M}_q \right) q + \theta \, \frac{g_s^2}{32\pi^2} \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\}$$

[Gross,Wilczek 73;Politzer 73; Fritzsch,Gell-Mann,Leutwyler 73] [Belavin et al. `75;´t Hooft 76;Callan et al. `76;Jackiw,Rebbi `76]

  - Covariant gauge kinetic term, $-\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$, build from gluonic field strength tensors, $G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, in terms of gluonic gauge fields, G^a_μ , describes gluons and their interactions



  - Covariant quark kinetic term, $\overline{q} i \gamma_\mu D^\mu q$, involving covariant derivative $D_\mu = \partial_\mu - i g_s T^a G^a_\mu$, describes quarks and their interactions



  - What about last term?

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\mathrm{QCD}} \supset \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\rm QCD} \supset \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\rm QCD} \supset \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$
- Integrand is total derivative:

$$\theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, \epsilon^{\mu\alpha\beta\gamma} \partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3} f^{abc} G^a_\alpha G^b_\beta G^c_\gamma \right)$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\mathrm{QCD}} \supset \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$
- Integrand is total derivative:

$$\theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, \epsilon^{\mu\alpha\beta\gamma} \partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3} f^{abc} G^a_\alpha G^b_\beta G^c_\gamma \right)$$

  - Divergence of Chern-Simons current

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\rm QCD} \supset \theta\, \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{a\,\rho\sigma}$
- Integrand is total derivative:

$$\theta\, \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} = \theta\, \frac{g_s^2}{32\pi^2} \int d^4x\, \epsilon^{\mu\alpha\beta\gamma}\partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3} f^{abc} G^a_\alpha G^b_\beta G^c_\gamma\right)$$

  - Divergence of Chern-Simons current
- Theta-term does not contribute at the classical level and in perturbation theory

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\rm QCD} \supset \theta\, \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$
- Integrand is total derivative:

$$\theta\, \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta\, \frac{g_s^2}{32\pi^2} \int d^4x\, \epsilon^{\mu\alpha\beta\gamma} \partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3} f^{abc} G^a_\alpha G^b_\beta G^c_\gamma \right)$$

  - Divergence of Chern-Simons current
- Theta-term does not contribute **at the classical level** and in perturbation theory
  - Depends only on boundary information (`topological´)

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\rm QCD} \supset \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$
- Integrand is total derivative:

$$\theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, \epsilon^{\mu\alpha\beta\gamma} \partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3} f^{abc} G^a_\alpha G^b_\beta G^c_\gamma \right)$$

  - Divergence of Chern-Simons current
- Theta-term does not contribute at the classical level and **in perturbation theory**
  - Depends only on boundary information (`topological´)
  - Does not give rise to new vertices in Feynman rules

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$S_{\text{QCD}} \supset \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

- Involving Hodge dual of gluonic field tensor, $\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$
- Integrand is total derivative:

$$\theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \, \frac{g_s^2}{32\pi^2} \int d^4x \, \epsilon^{\mu\alpha\beta\gamma} \partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3} f^{abc} G^a_\alpha G^b_\beta G^c_\gamma \right)$$

  - Divergence of Chern-Simons current
- Theta-term does not contribute at the classical level and in perturbation theory
  - Depends only on boundary information (`topological´)
  - Does not give rise to new vertices in Feynman rules
- But it gives non-perturbative contribution in quantum theory because of the topologically non-trivial vacuum structure of QCD

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+ | 0_- \rangle = \int \mathcal{D}G\, \mathcal{D}\overline{q}\, \mathcal{D}q\, e^{-\int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{q}(i\gamma_\mu D_\mu - \mathcal{M})q \right\}} e^{+i\theta \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}}$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+|0_-\rangle = \int \mathcal{D}G\,\mathcal{D}\overline{q}\,\mathcal{D}q\, e^{-\int d^4x \left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\bar{q}(i\gamma_\mu D_\mu-\mathcal{M})q\right\}} e^{+i\theta\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})} \to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})} \to iG_4^{a(\mathrm{E})}$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+|0_-\rangle = \int \mathcal{D}G\,\mathcal{D}\overline{q}\,\mathcal{D}q\, e^{-\int d^4x\,\left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\bar{q}(i\gamma_\mu D_\mu-\mathcal{M})q\right\}}\, e^{+i\theta\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})}\to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})}\to iG_4^{a(\mathrm{E})}$, such that

$$i\int d^4x\left\{-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}+\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\}\Rightarrow -\int d^4x\left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-i\,\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\}$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+ | 0_- \rangle = \int \mathcal{D}G\, \mathcal{D}\overline{q}\, \mathcal{D}q\, e^{-\int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{q}(i\gamma_\mu D_\mu - \mathcal{M})q \right\}} e^{+i\theta \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})} \to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})} \to iG_4^{a(\mathrm{E})}$, such that

$$i \int d^4x \left\{ -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\} \Rightarrow - \int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - i\,\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right\}$$

  - Finite Euclidean action requires that the classical gauge fields contributing to the path integral should be such that the field strength tensor vanishes at $|x| \to \infty$.

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+|0_-\rangle = \int \mathcal{D}G\,\mathcal{D}\overline{q}\,\mathcal{D}q\, e^{-\int d^4x\,\left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\bar{q}(i\gamma_\mu D_\mu-\mathcal{M})q\right\}}\, e^{+i\theta\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})} \to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})} \to iG_4^{a(\mathrm{E})}$, such that

$$i\int d^4x \left\{-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\} \Rightarrow -\int d^4x \left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - i\,\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\}$$

  - Finite Euclidean action requires that the classical gauge fields contributing to the path integral should be such that the field strength tensor vanishes at $|x| \to \infty$. This means that the gauge fields should approach a pure gauge form,

$$G^a_\mu(x)T^a \to \frac{i}{g_s}U(x)\partial_\mu U^{-1}(x),\ \text{for } |x| \to \infty,\ \text{where } U(x) \in SU(3)$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+|0_-\rangle = \int \mathcal{D}G\,\mathcal{D}\overline{q}\,\mathcal{D}q\, e^{-\int d^4x\,\left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\bar{q}(i\gamma_\mu D_\mu-\mathcal{M})q\right\}}\, e^{+i\theta\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})} \to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})} \to iG_4^{a(\mathrm{E})}$, such that

$$i\int d^4x\left\{-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}+\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\} \Rightarrow -\int d^4x\left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-i\,\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\}$$

  - Finite Euclidean action requires that the classical gauge fields contributing to the path integral should be such that the field strength tensor vanishes at $|x|\to\infty$. This means that the gauge fields should approach a pure gauge form,

$$G^a_\mu(x)T^a \to \frac{i}{g_s}U(x)\partial_\mu U^{-1}(x),\ \text{for } |x|\to\infty,\ \text{where } U(x)\in SU(3)$$

  - The theta-term can then be evaluated as

$$\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = \frac{g_s^2}{32\pi^2}\int d^4x\,\epsilon_{\mu\alpha\beta\gamma}\partial_\mu\left(G^a_\alpha G^a_{\beta\gamma}-\frac{g_s}{3}f^{abc}G^a_\alpha G^b_\beta G^c_\gamma\right)$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+|0_-\rangle = \int \mathcal{D}G\,\mathcal{D}\overline{q}\,\mathcal{D}q\, e^{-\int d^4x \left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \bar{q}(i\gamma_\mu D_\mu - \mathcal{M})q\right\}} e^{+i\theta \frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})} \to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})} \to iG_4^{a(\mathrm{E})}$, such that

$$i\int d^4x \left\{-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\} \Rightarrow -\int d^4x \left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - i\,\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\}$$

  - Finite Euclidean action requires that the classical gauge fields contributing to the path integral should be such that the field strength tensor vanishes at $|x| \to \infty$. This means that the gauge fields should approach a pure gauge form,

$$G^a_\mu(x)T^a \to \frac{i}{g_s}U(x)\partial_\mu U^{-1}(x),\ \text{for } |x| \to \infty,\ \text{where } U(x) \in SU(3)$$

  - The theta-term can then be evaluated as

$$\begin{aligned}\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} &= \frac{g_s^2}{32\pi^2}\int d^4x\, \epsilon_{\mu\alpha\beta\gamma}\partial_\mu \left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3}f^{abc}G^a_\alpha G^b_\beta G^c_\gamma\right) \\ &= \frac{1}{24\pi^2}\int_{S_3^\infty} d\sigma_\mu\, \epsilon_{\mu\alpha\beta\gamma}\mathrm{tr}\left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}]\right)\end{aligned}$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- To see this, we will use the Euclidean path integral representation of the vacuum-to-vacuum amplitude,

$$\langle 0_+|0_-\rangle = \int \mathcal{D}G\,\mathcal{D}\overline{q}\,\mathcal{D}q\, e^{-\int d^4x\,\left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\bar{q}(i\gamma_\mu D_\mu-\mathcal{M})q\right\}}\, e^{+i\theta\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

  - Obtained from real time path integral representation by Wick rotation, $x_0^{(\mathrm{M})} \to -ix_4^{(\mathrm{E})}, G_0^{a(\mathrm{M})} \to iG_4^{a(\mathrm{E})}$, such that

$$i\int d^4x \left\{-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\} \Rightarrow -\int d^4x \left\{\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - i\,\theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}\right\}$$

  - Finite Euclidean action requires that the classical gauge fields contributing to the path integral should be such that the field strength tensor vanishes at $|x| \to \infty$. This means that the gauge fields should approach a pure gauge form,

$$G^a_\mu(x)T^a \to \frac{i}{g_s}U(x)\partial_\mu U^{-1}(x),\ \text{ for } |x| \to \infty,\ \text{ where } U(x) \in SU(3)$$

  - The theta-term can then be evaluated as

$$\begin{aligned}\frac{g_s^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} &= \frac{g_s^2}{32\pi^2}\int d^4x\,\epsilon_{\mu\alpha\beta\gamma}\partial_\mu\left(G^a_\alpha G^a_{\beta\gamma} - \frac{g_s}{3}f^{abc}G^a_\alpha G^b_\beta G^c_\gamma\right)\\ &= \frac{1}{24\pi^2}\int_{S_3^\infty} d\sigma_\mu\,\epsilon_{\mu\alpha\beta\gamma}\mathrm{tr}\left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}]\right)\\ &= Q \in \mathbb{Z}\end{aligned}$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr} \left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}] \right) \in \mathbb{Z}$$

The Axion

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr}\left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}] \right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr} \left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}] \right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2}\int_{S_3^\infty} d\sigma_\mu\, \epsilon_{\mu\alpha\beta\gamma}\mathrm{tr}\left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}]\right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$
  - Pontryagin index Q counts the number of times S_3^∞ is wrapped around the group manifold $SU(2) \sim S_3$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr}\left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}]\right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$
  - Pontryagin index Q counts the number of times S_3^∞ is wrapped around the group manifold $SU(2) \sim S_3$
  - Pontryagin index aka winding number aka topological charge

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr} \left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}] \right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$
  - Pontryagin index Q counts the number of times S_3^∞ is wrapped around the group manifold $SU(2) \sim S_3$
  - Pontryagin index aka winding number aka topological charge
- In Euclidean space, SU(2) field configurations contributing to the vacuum-to-vacuum amplitude fall in homotopy classes of different topological charge

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr} \left([U \partial_\alpha U^{-1}][U \partial_\beta U^{-1}][U \partial_\gamma U^{-1}] \right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$
  - Pontryagin index Q counts the number of times S_3^∞ is wrapped around the group manifold $SU(2) \sim S_3$
  - Pontryagin index aka winding number aka topological charge
- In Euclidean space, SU(2) field configurations contributing to the vacuum-to-vacuum amplitude fall in homotopy classes of different topological charge
- It is not possible to continuously deform a field configuration into another of different winding number while maintaining the action finite

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr}\left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}]\right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$
  - Pontryagin index Q counts the number of times S_3^∞ is wrapped around the group manifold $SU(2) \sim S_3$
  - Pontryagin index aka winding number aka topological charge
- In Euclidean space, SU(2) field configurations contributing to the vacuum-to-vacuum amplitude fall in homotopy classes of different topological charge
- It is not possible to continuously deform a field configuration into another of different winding number while maintaining the action finite
- General SU(3) gauge field configurations can be classified in the same SU(2) homotopy classes

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

$$Q \equiv \frac{1}{24\pi^2} \int_{S_3^\infty} d\sigma_\mu \, \epsilon_{\mu\alpha\beta\gamma} \mathrm{tr} \left([U\partial_\alpha U^{-1}][U\partial_\beta U^{-1}][U\partial_\gamma U^{-1}] \right) \in \mathbb{Z}$$

- First consider $U \in SU(2) \subset SU(3)$
  - Provides a mapping $S_3^\infty \to SU(2) \simeq S_3$
  - Pontryagin index Q counts the number of times S_3^∞ is wrapped around the group manifold $SU(2) \sim S_3$
  - Pontryagin index aka winding number aka topological charge
- In Euclidean space, SU(2) field configurations contributing to the vacuum-to-vacuum amplitude fall in homotopy classes of different topological charge
- It is not possible to continuously deform a field configuration into another of different winding number while maintaining the action finite
- General SU(3) gauge field configurations can be classified in the same SU(2) homotopy classes
  - The reason being that any mapping from S3 into any simple Lie group G can be deformed into a mapping to a SU(2) subgroup of G in a continuous way, hence with no change of homotopy class.

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- Vacuum-to-vacuum amplitude can be written in terms of Fourier series

$$\langle 0_+ | 0_- \rangle = \sum_{Q=-\infty}^{+\infty} Z_Q \exp[i\theta Q] \equiv Z(\theta)$$

of Euclidean path integrals over gauge fields with fixed topological charge,

$$Z_Q = \int_Q \mathcal{D}G\, \mathcal{D}\bar{q}\, \mathcal{D}q\, e^{-\int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{q}(i\gamma_\mu D_\mu - \mathcal{M})q \right\}}$$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- Vacuum-to-vacuum amplitude can be written in terms of Fourier series

$$\langle 0_+ | 0_- \rangle = \sum_{Q=-\infty}^{+\infty} Z_Q \exp[i\theta Q] \equiv Z(\theta)$$

of Euclidean path integrals over gauge fields with fixed topological charge,

$$Z_Q = \int_Q \mathcal{D}G\, \mathcal{D}\overline{q}\, \mathcal{D}q\, e^{-\int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \overline{q}(i\gamma_\mu D_\mu - \mathcal{M}) q \right\}}$$

- Theta is an angular parameter, $-\pi \leq \theta \leq \pi$

Vacuum Structure in Quantum Chromodynamics

Theta-term in action of QCD

- Vacuum-to-vacuum amplitude can be written in terms of Fourier series

$$\langle 0_+ | 0_- \rangle = \sum_{Q=-\infty}^{+\infty} Z_Q \exp[i\theta Q] \equiv Z(\theta)$$

of Euclidean path integrals over gauge fields with fixed topological charge,

$$Z_Q = \int_Q \mathcal{D}G\, \mathcal{D}\overline{q}\, \mathcal{D}q\, e^{-\int d^4x \left\{\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{q}(i\gamma_\mu D_\mu - \mathcal{M})q\right\}}$$

- Theta is an angular parameter, $-\pi \leq \theta \leq \pi$
- Ordinary perturbative QCD resides in topologically trivial sector

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Vacuum-to-vacuum amplitude related to vacuum energy density via

$$\langle 0_+ | 0_- \rangle = \lim_{\mathcal{V}_4 \to \infty} e^{-\epsilon_0\, \mathcal{V}_4}$$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Vacuum-to-vacuum amplitude related to vacuum energy density via

$$\langle 0_+ | 0_- \rangle = \lim_{\mathcal{V}_4 \to \infty} e^{-\epsilon_0\, \mathcal{V}_4}$$

- Theta-dependence of vacuum energy density:

$$\epsilon_0(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln Z(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln \left[\sum_{Q=-\infty}^{+\infty} Z_Q\, e^{i\theta Q} \right]$$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Vacuum-to-vacuum amplitude related to vacuum energy density via

$$\langle 0_+ | 0_- \rangle = \lim_{\mathcal{V}_4 \to \infty} e^{-\epsilon_0\, \mathcal{V}_4}$$

- Theta-dependence of vacuum energy density:

$$\epsilon_0(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln Z(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln \left[\sum_{Q=-\infty}^{+\infty} Z_Q\, e^{i\theta Q} \right]$$

- Vacuum energy density is periodic: $\epsilon_0(\theta) = \epsilon_0(\theta + 2\pi)$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Vacuum-to-vacuum amplitude related to vacuum energy density via

$$\langle 0_+ | 0_- \rangle = \lim_{\mathcal{V}_4 \to \infty} e^{-\epsilon_0\, \mathcal{V}_4}$$

- Theta-dependence of vacuum energy density:

$$\epsilon_0(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln Z(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln \left[\sum_{Q=-\infty}^{+\infty} Z_Q\, e^{i\theta Q} \right]$$

- Vacuum energy density is periodic: $\epsilon_0(\theta) = \epsilon_0(\theta + 2\pi)$
- Vacuum energy density has a minimum for vanishing theta: $\epsilon_0(0) \leq \epsilon_0(\theta)$

[Vafa,Witten `84]

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Vacuum-to-vacuum amplitude related to vacuum energy density via

$$\langle 0_+ | 0_- \rangle = \lim_{\mathcal{V}_4 \to \infty} e^{-\epsilon_0 \, \mathcal{V}_4}$$

- Theta-dependence of vacuum energy density:

$$\epsilon_0(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln Z(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln \left[\sum_{Q=-\infty}^{+\infty} Z_Q \, e^{i\theta Q} \right]$$

- Vacuum energy density is periodic: $\epsilon_0(\theta) = \epsilon_0(\theta + 2\pi)$
- Vacuum energy density has a minimum for vanishing theta: $\epsilon_0(0) \leq \epsilon_0(\theta)$ [Vafa,Witten `84]
  - Relies on the fact that Z_Q is positive definite for a vector-like theory as QCD, which implies $Z(\theta) \leq Z(0)$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Vacuum-to-vacuum amplitude related to vacuum energy density via

$$\langle 0_+ | 0_- \rangle = \lim_{\mathcal{V}_4 \to \infty} e^{-\epsilon_0 \, \mathcal{V}_4}$$

- Theta-dependence of vacuum energy density:

$$\epsilon_0(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln Z(\theta) = - \lim_{\mathcal{V}_4 \to \infty} \frac{1}{\mathcal{V}_4} \ln \left[\sum_{Q=-\infty}^{+\infty} Z_Q \, e^{i\theta Q} \right]$$

- Vacuum energy density is periodic: $\epsilon_0(\theta) = \epsilon_0(\theta + 2\pi)$
- Vacuum energy density has a minimum for vanishing theta: $\epsilon_0(0) \leq \epsilon_0(\theta)$ [Vafa,Witten \`84]
  - Relies on the fact that Z_Q is positive definite for a vector-like theory as QCD, which implies $Z(\theta) \leq Z(0)$
  - Important for axion physics: axion field corresponds to space-time dependent theta parameter, field-dependence of axion potential is then given by theta-dependence of vacuum energy density. Result implies, axion has vanishing vacuum expectation value.

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - Consider first unrealistic case of one light flavour called u.

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - Consider first unrealistic case of one light flavour called u.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for a phase of the quark mass

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - Consider first unrealistic case of one light flavour called u.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for a phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product $m_u e^{i\theta}$:

$$\epsilon_0(\theta) = f(m_u e^{i\theta})$$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - Consider first unrealistic case of one light flavour called u.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for a phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product $m_u e^{i\theta}$:

$$\epsilon_0(\theta) = f(m_u e^{i\theta})$$

    - The spectrum of the theory does not contain massless particles in the chiral limit. Expansion in powers of mass does therefore not give rise to infrared divergencies, i.e. is an ordinary Taylor expansion, [Leutwyler,Smilga `92]

$$\epsilon_0(\theta) = \epsilon_0(0) - \Sigma\, m_u \cos\theta + \mathcal{O}(m_u^2)$$

where $\Sigma = \sqrt{\langle \bar{q}q \rangle^2 + \langle \bar{q} i\gamma_5 q \rangle^2}|_{m_u=0}$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - Consider first unrealistic case of one light flavour called u.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for a phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product $m_u e^{i\theta}$:

$$\epsilon_0(\theta) = f(m_u e^{i\theta})$$

    - The spectrum of the theory does not contain massless particles in the chiral limit. Expansion in powers of mass does therefore not give rise to infrared divergencies, i.e. is an ordinary Taylor expansion, [Leutwyler,Smilga `92]

$$\epsilon_0(\theta) = \epsilon_0(0) - \Sigma\, m_u \cos\theta + \mathcal{O}(m_u^2)$$

    where $\Sigma = \sqrt{\langle \bar{q}q \rangle^2 + \langle \bar{q} i\gamma_5 q \rangle^2}|_{m_u=0}$
      - Vacuum energy has absolute minimum at vanishing theta angle

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - Consider first unrealistic case of one light flavour called u.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for a phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product $m_u e^{i\theta}$:

$$\epsilon_0(\theta) = f(m_u e^{i\theta})$$

    - The spectrum of the theory does not contain massless particles in the chiral limit. Expansion in powers of mass does therefore not give rise to infrared divergencies, i.e. is an ordinary Taylor expansion, [Leutwyler,Smilga `92]

$$\epsilon_0(\theta) = \epsilon_0(0) - \Sigma\, m_u \cos\theta + \mathcal{O}(m_u^2)$$

    where $\Sigma = \sqrt{\langle \bar{q}q\rangle^2 + \langle \bar{q} i\gamma_5 q\rangle^2}|_{m_u=0}$
      - Vacuum energy has absolute minimum at vanishing theta angle
      - Vacuum energy independent of theta if quark mass zero

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - In realistic case of two light flavours, say u and d.

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - In realistic case of two light flavours, say u and d.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a common chiral transformation, $q \to e^{i\gamma_5 \alpha} q \,,\; \mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for an overall phase of the quark mass

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - In realistic case of two light flavours, say u and d.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a common chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for an overall phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product:

$$\epsilon_0(\theta) = f(\mathcal{M} e^{i\theta/2})$$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - In realistic case of two light flavours, say u and d.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a common chiral transformation, $q \to e^{i\gamma_5 \alpha} q$, $\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for an overall phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product:

$$\epsilon_0(\theta) = f(\mathcal{M} e^{i\theta/2})$$

    - However, in this case the spectrum of the theory does contain massless particles in the chiral limit, namely three pseudo-scalar Nambu-Goldstone bosons, the pions, originating from spontaneous breaking of the chiral symmetry by light quarks.

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - In realistic case of two light flavours, say u and d.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a common chiral transformation, $q \to e^{i\gamma_5 \alpha} q\,,\ \mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha \frac{g_s^2}{16\pi^2} \int d^4x\, G\tilde{G}} \right) \mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for an overall phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product:

$$\epsilon_0(\theta) = f(\mathcal{M} e^{i\theta/2})$$

    - However, in this case the spectrum of the theory does contain massless particles in the chiral limit, namely three pseudo-scalar Nambu-Goldstone bosons, the pions, originating from spontaneous breaking of the chiral symmetry by light quarks.
    - Vacuum-to-vacuum amplitude for realistic QCD at large four volume and small quark masses should be well described by the effective field theory (EFT) of pions

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

- Further progress can be made in the chiral limit, $\mathcal{M} \to 0$
  - In realistic case of two light flavours, say u and d.
    - Exploiting the chiral anomaly, that is the non-invariance of the fermionic measure in the path integral under a common chiral transformation, $q \to e^{i\gamma_5\alpha} q\,,\ \mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha\frac{g_s^2}{16\pi^2}\int d^4x\, G\tilde{G}}\right)\mathcal{D}q\mathcal{D}\bar{q}$, one may trade the theta parameter for an overall phase of the quark mass
    - Therefore, the vacuum-to-vacuum amplitude and thus the energy density depends on the quark mass and the theta parameter only through the product:
$$\epsilon_0(\theta) = f(\mathcal{M}e^{i\theta/2})$$
    - However, in this case the spectrum of the theory does contain massless particles in the chiral limit, namely three pseudo-scalar Nambu-Goldstone bosons, the pions, originating from spontaneous breaking of the chiral symmetry by light quarks.
    - Vacuum-to-vacuum amplitude for realistic QCD at large four volume and small quark masses should be well described by the effective field theory (EFT) of pions
    - Exploiting chiral EFT, one finds

[Di Vecchia,Veneziano `80; Leutwyler,Smilga `92]

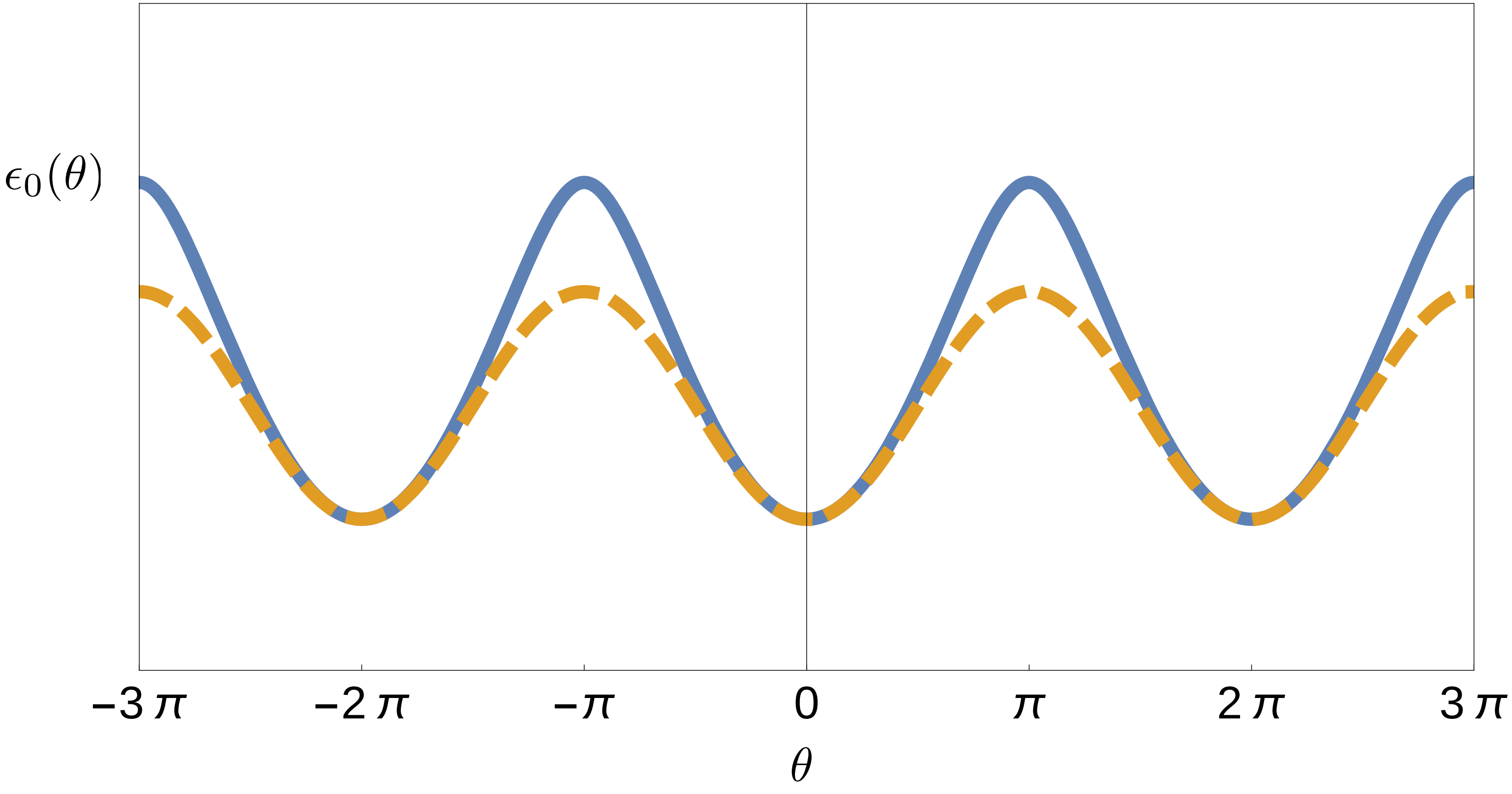
$$\epsilon_0(\theta) = \epsilon_0(0) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\theta}}{1+z}$$

$$z \equiv m_u/m_d \approx 1/2$$

Vacuum Structure in Quantum Chromodynamics

Theta-dependence of vacuum energy density in QCD

$$\epsilon_0(\theta) = \epsilon_0(0) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\theta}}{1+z}$$



Strong CP Puzzle

Neutron electric dipole moment

- Theta-term violates both T and P, and thus CP

Strong CP Puzzle

Neutron electric dipole moment

- Theta-term violates both T and P, and thus CP
  - This can be understood by expressing theta term in terms of color electric and color magnetic fields,

$$\theta \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \frac{g_s^2}{8\pi^2} \int d^4x \, \mathbf{E}^a \cdot \mathbf{B}^a$$

Strong CP Puzzle

Neutron electric dipole moment

- Theta-term violates both T and P, and thus CP
  - This can be understood by expressing theta term in terms of color electric and color magnetic fields,

$$\theta \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \frac{g_s^2}{8\pi^2} \int d^4x \, \mathbf{E}^a \cdot \mathbf{B}^a$$

    - Electric fields are even under T and odd under P: $\mathbf{E}^a(\mathbf{x},t) = \mathbf{E}^a(\mathbf{x},-t), \quad \mathbf{E}^a(\mathbf{x},t) = -\mathbf{E}^a(-\mathbf{x},t)$

Strong CP Puzzle

Neutron electric dipole moment

- Theta-term violates both T and P, and thus CP
  - This can be understood by expressing theta term in terms of color electric and color magnetic fields,

$$\theta \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \frac{g_s^2}{8\pi^2} \int d^4x \, \mathbf{E}^a \cdot \mathbf{B}^a$$

    - Electric fields are even under T and odd under P: $\mathbf{E}^a(\mathbf{x},t) = \mathbf{E}^a(\mathbf{x},-t), \quad \mathbf{E}^a(\mathbf{x},t) = -\mathbf{E}^a(-\mathbf{x},t)$
    - Magnetic fields are odd under T and even under P: $\mathbf{B}^a(\mathbf{x},t) = -\mathbf{B}^a(\mathbf{x},-t), \quad \mathbf{B}^a(\mathbf{x},t) = \mathbf{B}^a(-\mathbf{x},t)$

Strong CP Puzzle

Neutron electric dipole moment

- Theta-term violates both T and P, and thus CP
  - This can be understood by expressing theta term in terms of color electric and color magnetic fields,

$$\theta \frac{g_s^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \frac{g_s^2}{8\pi^2} \int d^4x\, \mathbf{E}^a \cdot \mathbf{B}^a$$

    - Electric fields are even under T and odd under P: $\mathbf{E}^a(\mathbf{x},t) = \mathbf{E}^a(\mathbf{x},-t), \quad \mathbf{E}^a(\mathbf{x},t) = -\mathbf{E}^a(-\mathbf{x},t)$
    - Magnetic fields are odd under T and even under P: $\mathbf{B}^a(\mathbf{x},t) = -\mathbf{B}^a(\mathbf{x},-t), \quad \mathbf{B}^a(\mathbf{x},t) = \mathbf{B}^a(-\mathbf{x},t)$
    - Electric fields are odd under C, with the consequence that they are even under CP

Strong CP Puzzle

Neutron electric dipole moment

- Theta-term violates both T and P, and thus CP
  - This can be understood by expressing theta term in terms of color electric and color magnetic fields,

$$\theta \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} = \theta \frac{g_s^2}{8\pi^2} \int d^4x \, \mathbf{E}^a \cdot \mathbf{B}^a$$

    - Electric fields are even under T and odd under P: $\mathbf{E}^a(\mathbf{x},t) = \mathbf{E}^a(\mathbf{x},-t), \quad \mathbf{E}^a(\mathbf{x},t) = -\mathbf{E}^a(-\mathbf{x},t)$
    - Magnetic fields are odd under T and even under P: $\mathbf{B}^a(\mathbf{x},t) = -\mathbf{B}^a(\mathbf{x},-t), \quad \mathbf{B}^a(\mathbf{x},t) = \mathbf{B}^a(-\mathbf{x},t)$
    - Electric fields are odd under C, with the consequence that they are even under CP
    - Magnetic fields are odd under C, with the consequence that they are odd under CP

Strong CP Puzzle

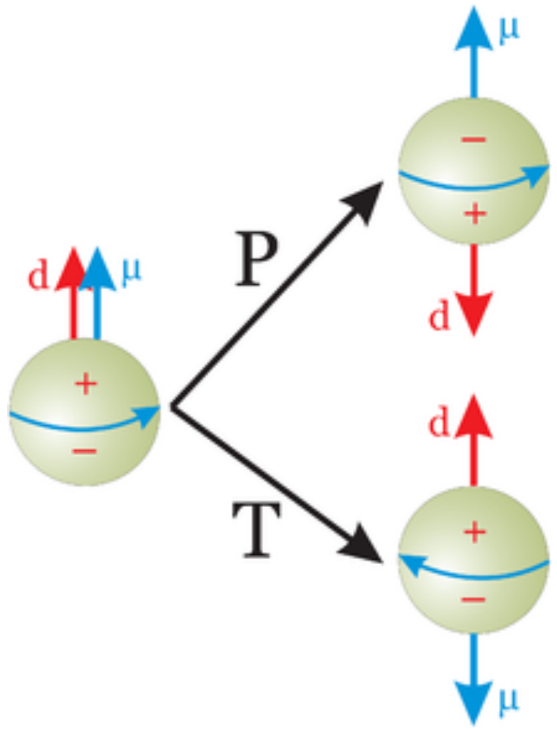
Neutron electric dipole moment

- Most sensitive probe of T and P violation in flavor conserving interactions: electric dipole moment d_n of neutron, corresponding to effective electric dipole interaction:

$$\mathcal{L}_{\rm eff} \supset -\frac{i}{2} d_n \, \overline{\Psi}_n \sigma_{\mu\nu} \gamma_5 \Psi_n F^{\mu\nu}$$

$$d_n = C_{\rm nEDM}\, e\, \theta$$

[Crewther,Di Vecchia,Veneziano,Witten 79]



Strong CP Puzzle

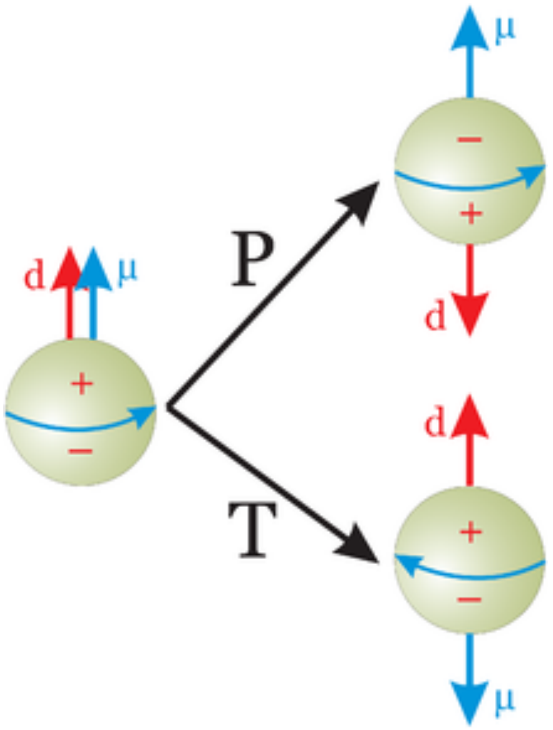
Neutron electric dipole moment

- Most sensitive probe of T and P violation in flavor conserving interactions: electric dipole moment d_n of neutron, corresponding to effective electric dipole interaction:

$$\mathcal{L}_{\rm eff} \supset -\frac{i}{2} d_n \, \overline{\Psi}_n \sigma_{\mu\nu} \gamma_5 \Psi_n F^{\mu\nu}$$

$$d_n = C_{\rm nEDM} \, e \, \theta$$

[Crewther,Di Vecchia,Veneziano,Witten 79]



- Educated guess:

$$C_{\rm nEDM} \sim \frac{m_*}{m_n^2} \sim 10^{-16}\,{\rm cm}$$

$$m_* = \frac{m_u m_d}{m_u + m_d}$$

Strong CP Puzzle

Neutron electric dipole moment

- Most sensitive probe of T and P violation in flavor conserving interactions: electric dipole moment d_n of neutron, corresponding to effective electric dipole interaction:

$$\mathcal{L}_{\rm eff} \supset -\frac{i}{2} d_n\, \overline{\Psi}_n \sigma_{\mu\nu}\gamma_5 \Psi_n F^{\mu\nu}$$

$$d_n = C_{\rm nEDM}\, e\, \theta$$

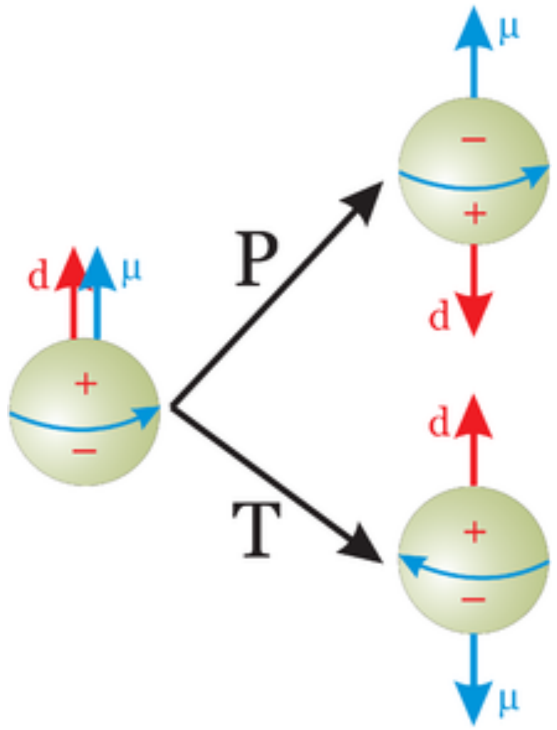
[Crewther,Di Vecchia,Veneziano,Witten 79]

- Educated guess: $$C_{\rm nEDM} \sim \frac{m_*}{m_n^2} \sim 10^{-16}\,{\rm cm}$$ $$m_* = \frac{m_u m_d}{m_u + m_d}$$

- Currently best theoretical determination:

$$C_{\rm nEDM} = 2.4(1.0) \times 10^{-16}\,{\rm cm}$$

[Pospelov,Ritz 00]



Strong CP Puzzle

Neutron electric dipole moment

- Most sensitive probe of T and P violation in flavor conserving interactions: electric dipole moment d_n of neutron, corresponding to effective electric dipole interaction:

$$\mathcal{L}_{\rm eff} \supset -\frac{i}{2} d_n\, \overline{\Psi}_n \sigma_{\mu\nu}\gamma_5 \Psi_n F^{\mu\nu}$$

$$d_n = C_{\rm nEDM}\, e\, \theta$$

[Crewther,Di Vecchia,Veneziano,Witten 79]

- Educated guess:

$$C_{\rm nEDM} \sim \frac{m_*}{m_n^2} \sim 10^{-16}\,{\rm cm}$$

$$m_* = \frac{m_u m_d}{m_u + m_d}$$

- Currently best theoretical determination:

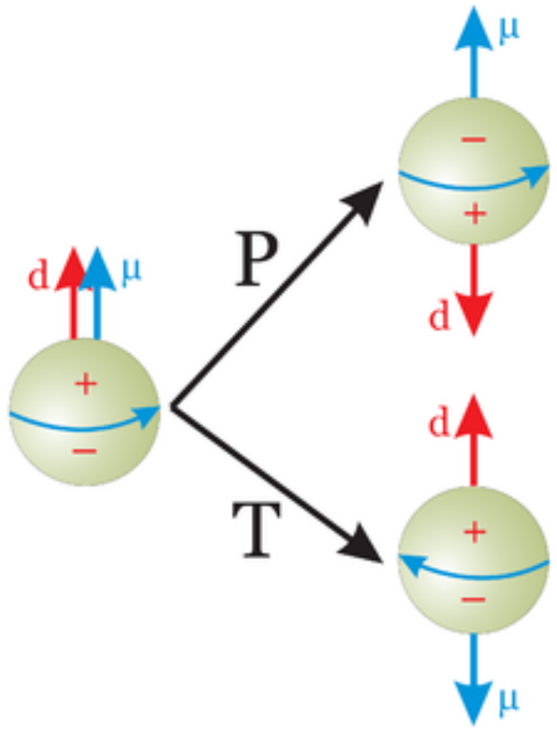
$$C_{\rm nEDM} = 2.4(1.0) \times 10^{-16}\,{\rm cm}$$

[Pospelov,Ritz 00]

- Experiment:

$$|d_n| < 1.8 \times 10^{-26}\, e\,{\rm cm}$$

[Abel et al. 20]



Strong CP Puzzle

Neutron electric dipole moment

- Most sensitive probe of T and P violation in flavor conserving interactions: electric dipole moment d_n of neutron, corresponding to effective electric dipole interaction:

$$\mathcal{L}_{\rm eff} \supset -\frac{i}{2} d_n \, \overline{\Psi}_n \sigma_{\mu\nu}\gamma_5 \Psi_n F^{\mu\nu}$$

$$d_n = C_{\rm nEDM}\, e\, \theta$$

[Crewther,Di Vecchia,Veneziano,Witten 79]

- Educated guess:

$$C_{\rm nEDM} \sim \frac{m_*}{m_n^2} \sim 10^{-16}\,{\rm cm}$$

$$m_* = \frac{m_u m_d}{m_u + m_d}$$

- Currently best theoretical determination:

$$C_{\rm nEDM} = 2.4(1.0) \times 10^{-16}\,{\rm cm}$$

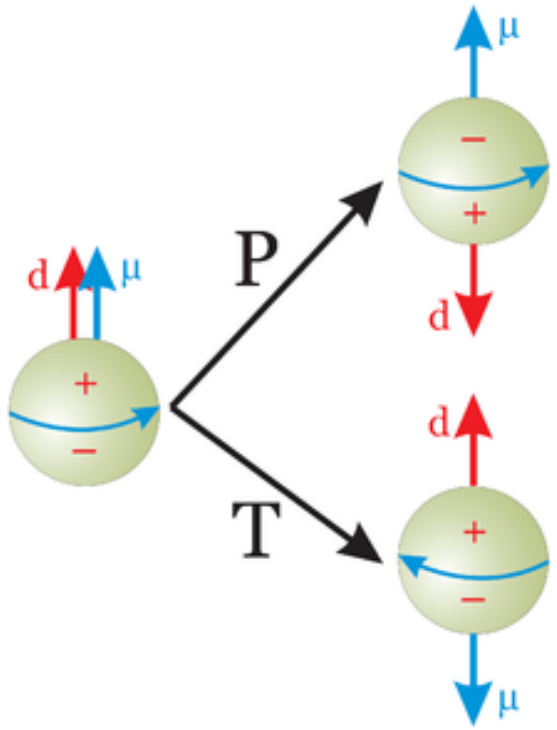
[Pospelov,Ritz 00]

- Experiment:

$$|d_n| < 1.8 \times 10^{-26}\, e\,{\rm cm}$$

[Abel et al. 20]

$$\Rightarrow |\theta| \lesssim 10^{-10}$$



The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + {\rm const.}$).

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

  a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + \text{const.}$).

  - In analogy to pion physics, f_a dubbed `decay constant´

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + {\rm const.}$).

  - In analogy to pion physics, f_a dubbed `decay constant´
  - f_a of order the breaking scale of the global $U(1)_{\rm PQ}$ symmetry

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

a non-linearly realized $U(1)_{\mathrm{PQ}}$ symmetry ($\theta(x) \to \theta(x) + \mathrm{const.}$).

- In analogy to pion physics, f_a dubbed `decay constant´
- f_a of order the breaking scale of the global $U(1)_{\mathrm{PQ}}$ symmetry
- Dimension 5 interaction; theory breaks down at scales of order f_a; needs UV completion at scales above this scale

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

  a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + \mathrm{const.}$).

  - In analogy to pion physics, f_a dubbed \`decay constant´
  - f_a of order the breaking scale of the global $U(1)_{\rm PQ}$ symmetry
  - Dimension 5 interaction; theory breaks down at scales of order f_a ; needs UV completion at scales above this scale
- Exploiting the shift symmetry and this effective interaction allows us to eliminate the QCD $\overline{\theta}$-parameter by the shift: $\theta(x) \to \theta(x) - \overline{\theta}$

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

  a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + {\rm const.}$).

  - In analogy to pion physics, f_a dubbed \`decay constant´
  - f_a of order the breaking scale of the global $U(1)_{\rm PQ}$ symmetry
  - Dimension 5 interaction; theory breaks down at scales of order f_a ; needs UV completion at scales above this scale

- Exploiting the shift symmetry and this effective interaction allows us to eliminate the QCD $\overline{\theta}$-parameter by the shift: $\theta(x) \to \theta(x) - \overline{\theta}$

$$\mathcal{L} \supset \overline{\theta}\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) + \theta(x) \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

  a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + {\rm const.}$).

  - In analogy to pion physics, f_a dubbed `decay constant´
  - f_a of order the breaking scale of the global $U(1)_{\rm PQ}$ symmetry
  - Dimension 5 interaction; theory breaks down at scales of order f_a ; needs UV completion at scales above this scale

- Exploiting the shift symmetry and this effective interaction allows us to eliminate the QCD $\overline{\theta}$-parameter by the shift: $\theta(x) \to \theta(x) - \overline{\theta}$

$$\begin{aligned} \mathcal{L} &\supset \overline{\theta}\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) + \theta(x)\frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) \\ &\to \theta(x)\frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) \end{aligned}$$

The Axion

Promote theta parameter to a dynamical field

- Add to the Standard Model a Nambu-Goldstone field, $\theta(x) \equiv a(x)/f_a \in [-\pi, \pi]$, respecting, apart from an effective interaction with the gluonic topological charge density

$$\mathcal{L} \supset \theta(x)\, q(x) \equiv \theta(x)\, \frac{\alpha_s}{8\pi}\, G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

a non-linearly realized $U(1)_{\rm PQ}$ symmetry ($\theta(x) \to \theta(x) + \text{const.}$).

  - In analogy to pion physics, f_a dubbed `decay constant´
  - f_a of order the breaking scale of the global $U(1)_{\rm PQ}$ symmetry
  - Dimension 5 interaction; theory breaks down at scales of order f_a ; needs UV completion at scales above this scale

- Exploiting the shift symmetry and this effective interaction allows us to eliminate the QCD $\overline{\theta}$-parameter by the shift: $\theta(x) \to \theta(x) - \overline{\theta}$

$$\mathcal{L} \supset \overline{\theta}\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x) + \theta(x) \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$
$$\to \theta(x) \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

- QCD theta parameter promoted to dynamical field

The Axion

Peccei-Quinn mechanism

[Peccei,Quinn 77]

- Dynamics of $\theta(x) \equiv a(x)/f_a$, at energy scales below f_a, but above $\Lambda_{\rm QCD}$, described by

$$\mathcal{L} \supset \frac{f_a^2}{2} \partial_\mu \theta(x)\, \partial^\mu \theta(x) + \theta(x)\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x) \tilde{G}^{b,\mu\nu}(x)$$

The Axion

Peccei-Quinn mechanism

[Peccei,Quinn 77]

- Dynamics of $\theta(x) \equiv a(x)/f_a$, at energy scales below f_a, but above $\Lambda_{\rm QCD}$, described by

$$\mathcal{L} \supset \frac{f_a^2}{2} \partial_\mu \theta(x)\, \partial^\mu \theta(x) + \theta(x)\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x) \tilde{G}^{b,\mu\nu}(x)$$

- At energies below $\Lambda_{\rm QCD}$,

$$\mathcal{L} \supset \frac{f_a^2}{2} \partial_\mu \theta(x) \partial^\mu \theta(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\theta}}{1+z}$$

$$z \equiv m_u/m_d \approx 1/2$$

The Axion

Peccei-Quinn mechanism

[Peccei,Quinn 77]

- Dynamics of $\theta(x) \equiv a(x)/f_a$, at energy scales below f_a, but above $\Lambda_{\rm QCD}$, described by

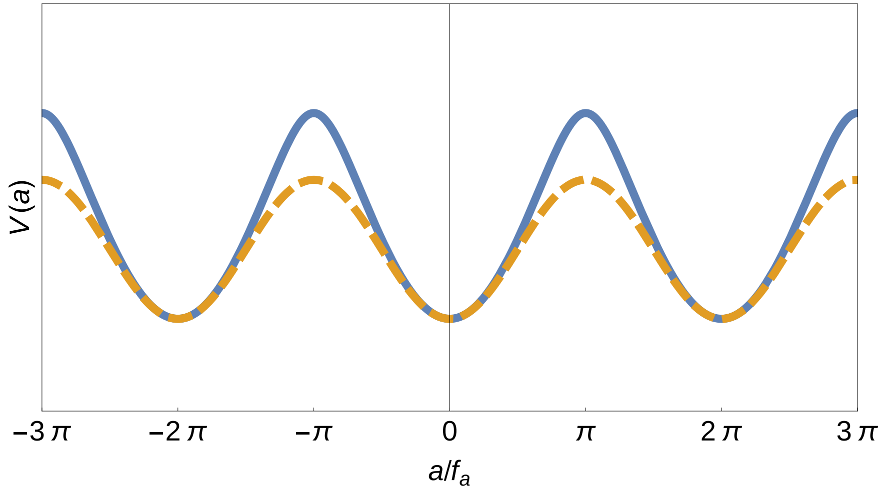
$$\mathcal{L} \supset \frac{f_a^2}{2}\partial_\mu\theta(x)\,\partial^\mu\theta(x) + \theta(x)\,\frac{\alpha_s}{8\pi}G^b_{\mu\nu}(x)\tilde{G}^{b,\mu\nu}(x)$$

- At energies below $\Lambda_{\rm QCD}$,

$$\mathcal{L} \supset \frac{f_a^2}{2}\partial_\mu\theta(x)\partial^\mu\theta(x) - m_\pi^2 f_\pi^2\frac{\sqrt{1+z^2+2z\cos\theta}}{1+z}$$

$$z \equiv m_u/m_d \approx 1/2$$

  - Field dependence of effective potential coincides with theta-dependence of vacuum energy in QCD



The Axion

Peccei-Quinn mechanism [Peccei,Quinn 77]

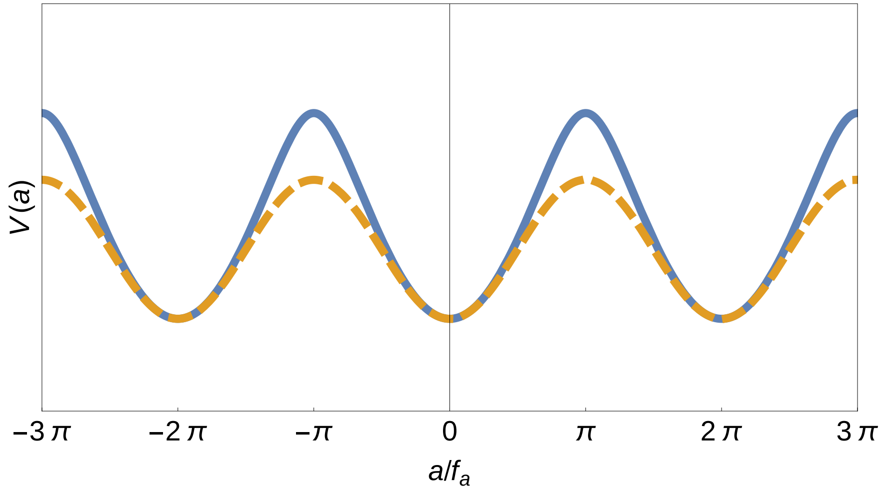
- Dynamics of $\theta(x) \equiv a(x)/f_a$, at energy scales below f_a, but above $\Lambda_{\rm QCD}$, described by

$$\mathcal{L} \supset \frac{f_a^2}{2} \partial_\mu \theta(x)\, \partial^\mu \theta(x) + \theta(x)\, \frac{\alpha_s}{8\pi} G^b_{\mu\nu}(x) \tilde{G}^{b,\mu\nu}(x)$$

- At energies below $\Lambda_{\rm QCD}$,

$$\mathcal{L} \supset \frac{f_a^2}{2} \partial_\mu \theta(x) \partial^\mu \theta(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1 + z^2 + 2z\cos\theta}}{1+z} \qquad z \equiv m_u/m_d \approx 1/2$$

  - Field dependence of effective potential coincides with theta-dependence of vacuum energy in QCD



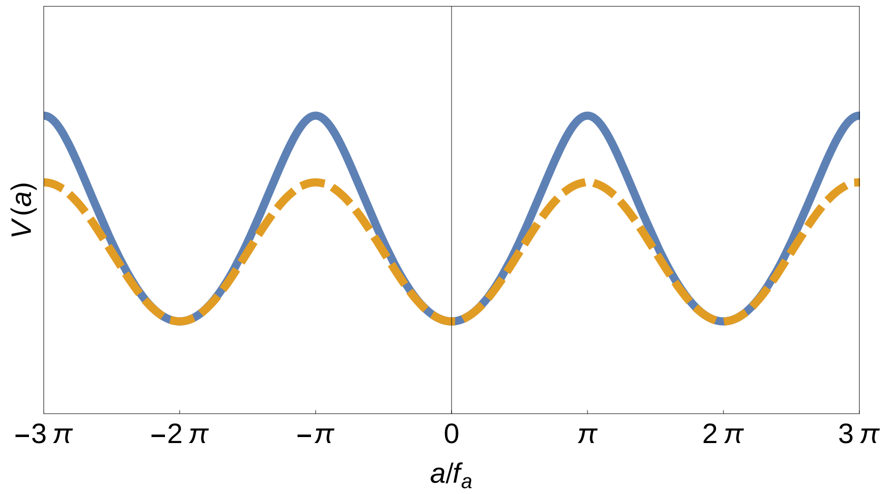
- There is no strong CP violation: $\langle\theta\rangle_0 = 0$

The Axion

PQ mechanism predicts pseudo Nambu-Goldstone boson

[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$

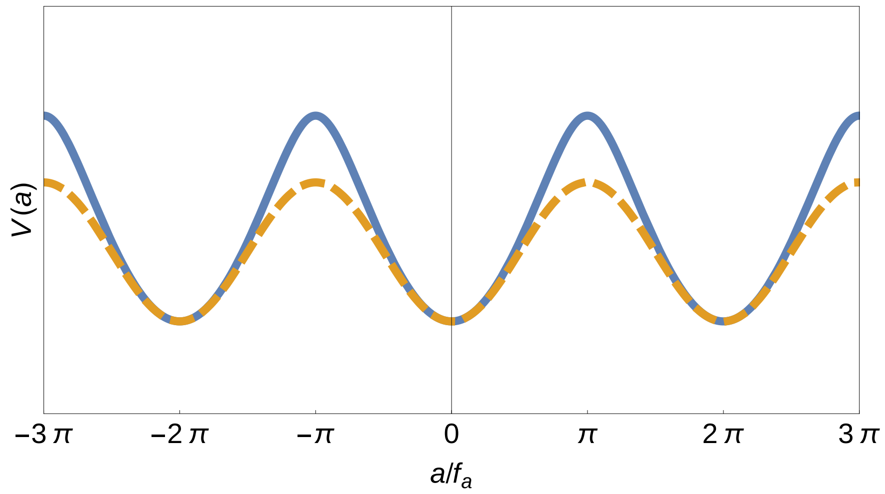


The Axion

PQ mechanism predicts pseudo Nambu-Goldstone boson

[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$



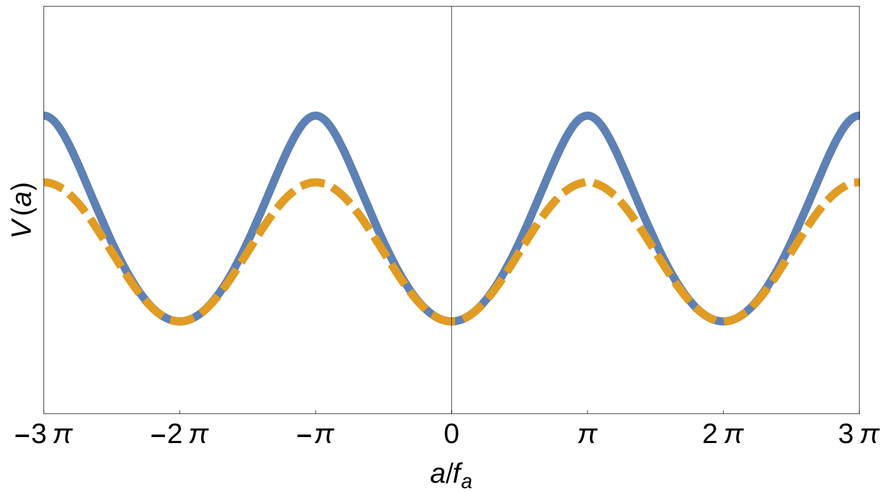
- There is a particle excitation of the field $a(x)$, which was dubbed `axion´

The Axion

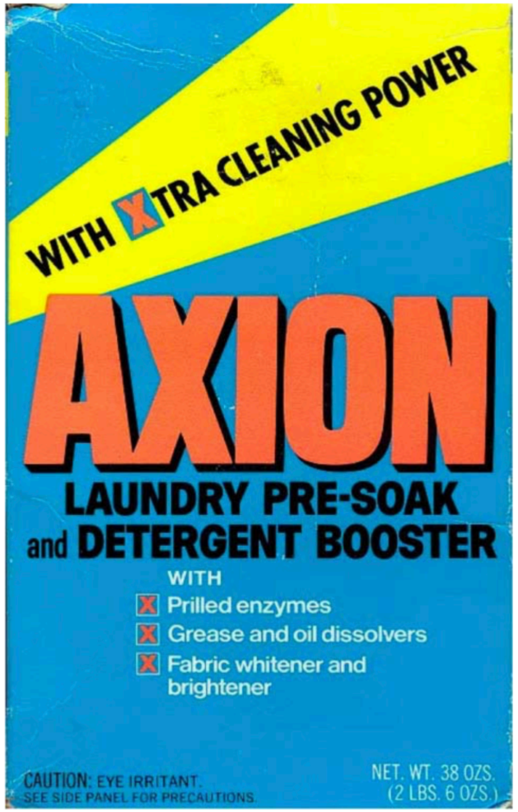
PQ mechanism predicts pseudo Nambu-Goldstone boson

[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$



- There is a particle excitation of the field $a(x)$, which was dubbed `axion´
  - Cleans up a problem associated with an axial current

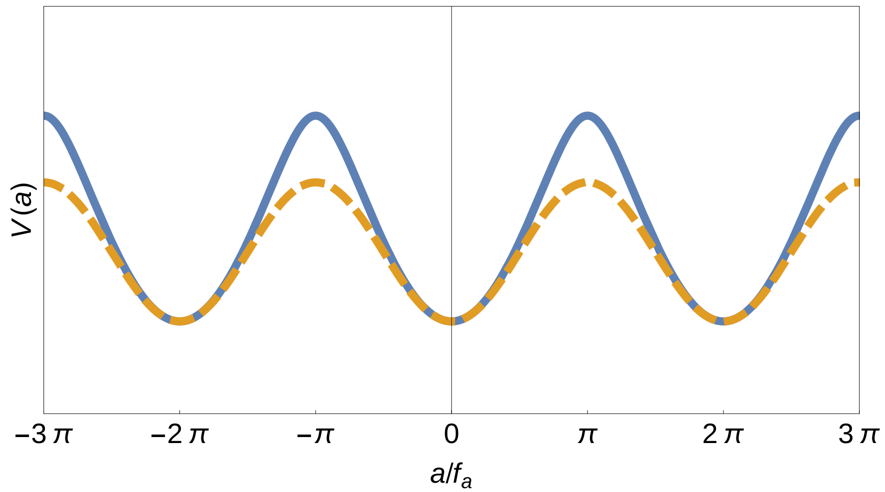


The Axion

PQ mechanism predicts pseudo Nambu-Goldstone boson

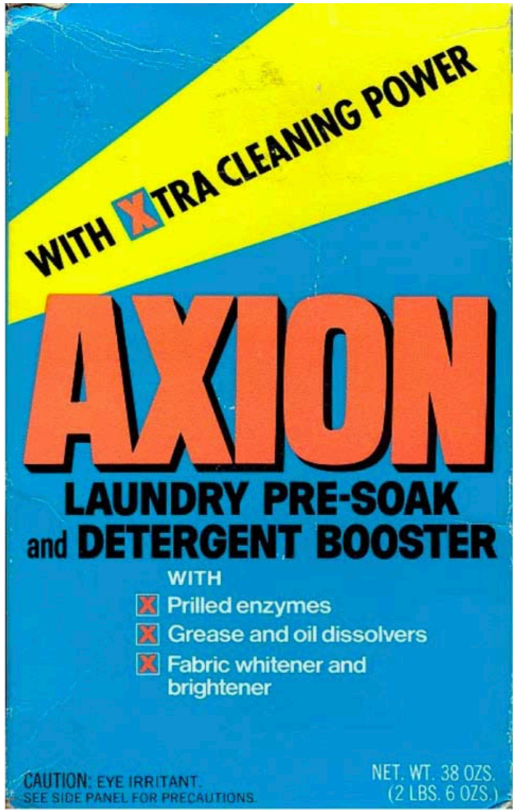
[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$



- There is a particle excitation of the field $a(x)$, which was dubbed \`axion´
  - Cleans up a problem associated with an axial current
- The axion mass determined by the second derivative of the effective potential:

$$m_a \simeq \frac{\sqrt{z}}{1+z}\frac{m_\pi f_\pi}{f_a}$$

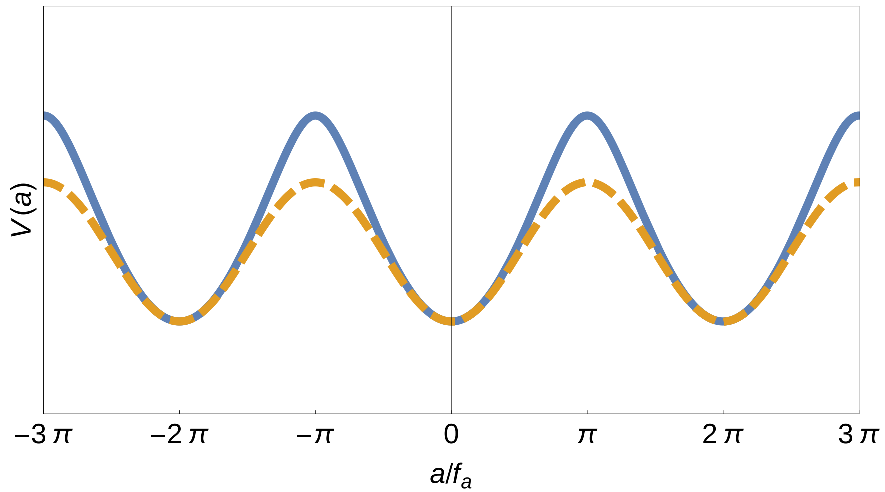


The Axion

PQ mechanism predicts pseudo Nambu-Goldstone boson

[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$

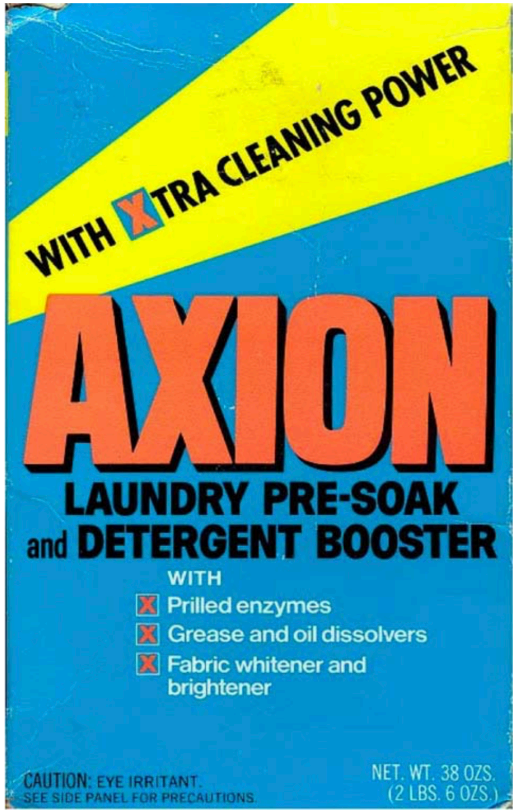


- There is a particle excitation of the field $a(x)$, which was dubbed `axion´
  - Cleans up a problem associated with an axial current
- The axion mass determined by the second derivative of the effective potential:

$$m_a \simeq \frac{\sqrt{z}}{1+z}\frac{m_\pi f_\pi}{f_a}$$

  - Parametrically suppressed by decay constant: axion very light if decay constant large:

$$m_a \approx 6\text{ meV}\left(\frac{10^9\text{ GeV}}{f_a}\right)$$

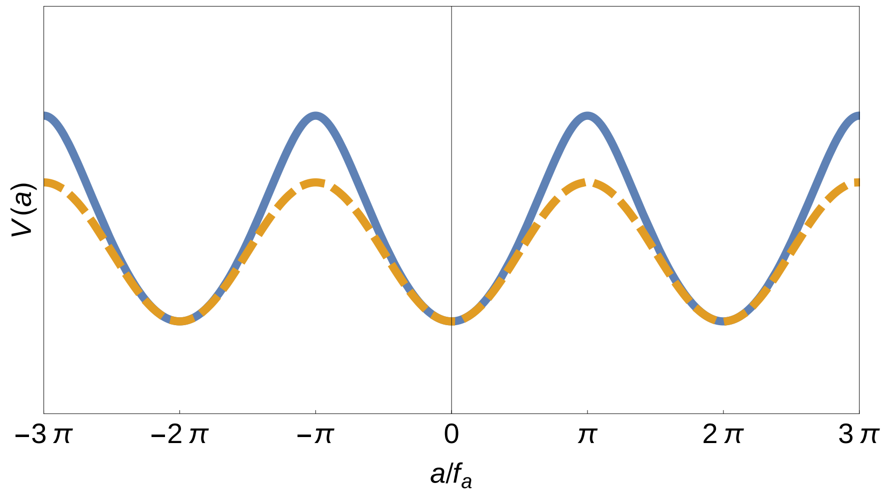


The Axion

PQ mechanism predicts pseudo Nambu-Goldstone boson

[Weinberg 78; Wilczek 78]

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - m_\pi^2 f_\pi^2 \frac{\sqrt{1+z^2+2z\cos\left(\frac{a(x)}{f_a}\right)}}{1+z}$$



- There is a particle excitation of the field $a(x)$, which was dubbed \`axion´
  - Cleans up a problem associated with an axial current
- The axion mass determined by the second derivative of the effective potential:

$$m_a \simeq \frac{\sqrt{z}}{1+z}\frac{m_\pi f_\pi}{f_a}$$

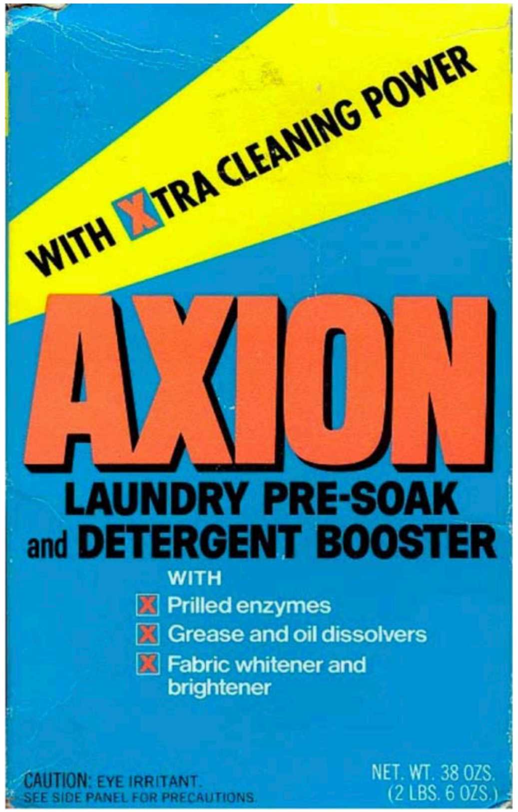
  - Parametrically suppressed by decay constant: axion very light if decay constant large:

$$m_a \approx 6 \text{ meV} \left(\frac{10^9 \text{ GeV}}{f_a}\right)$$

  - Precise calculation, by including $\mathcal{O}(\alpha)$QED corrections and NNLO corrections in chiral perturbation theory:

[Gorghetto et al. 18]

$$m_a = 5.691(51)\left(\frac{10^9\,\text{GeV}}{f_a}\right)\text{meV}$$



The Axion

Coupling to the nucleon EDM

- Axion has a model-independent coupling to the EDM of the nucleon:

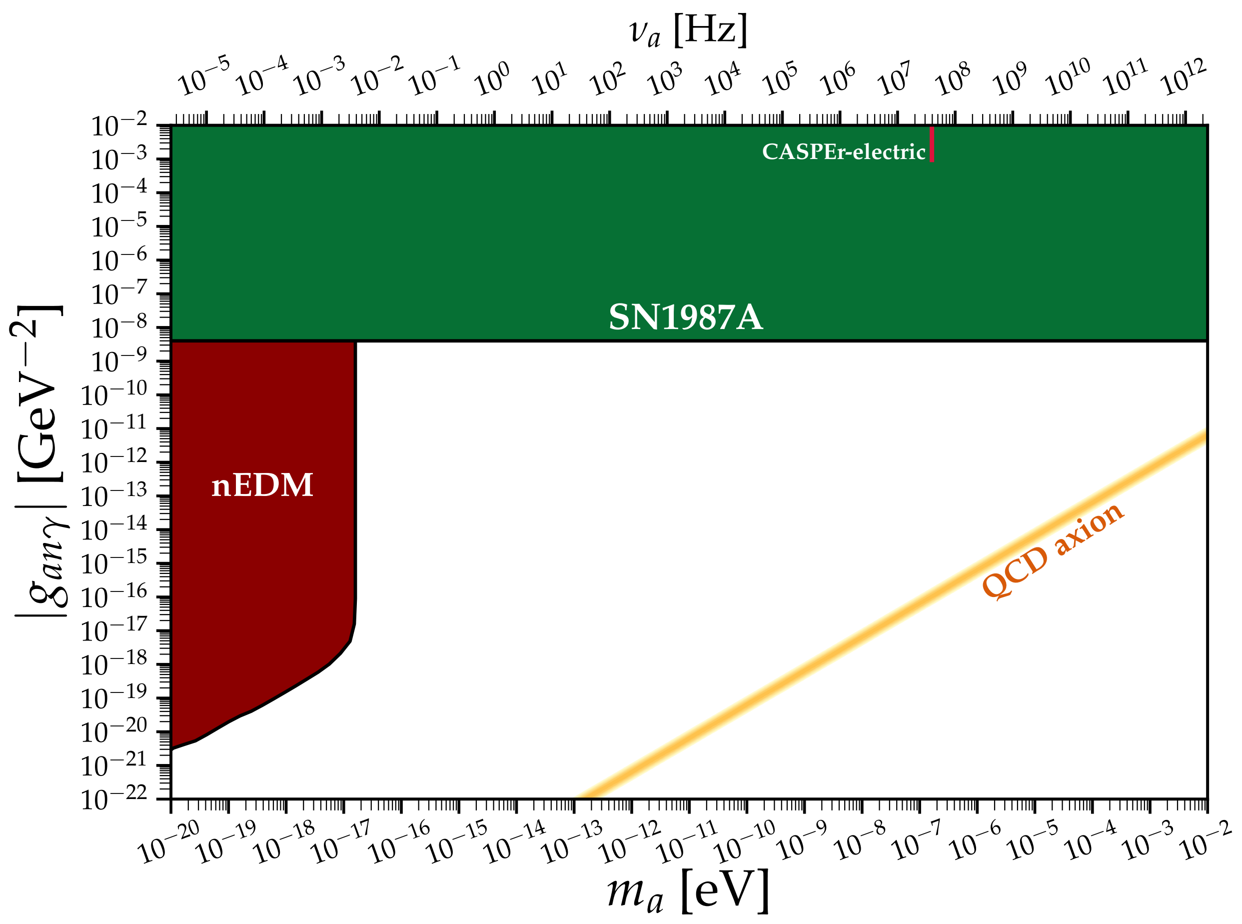
$$\mathcal{L}_{aN\gamma} = -\frac{i}{2} g_{aN\gamma}\, a\, \overline{\Psi}_N \sigma_{\mu\nu}\gamma_5 \Psi_N F^{\mu\nu}$$

$$g_{an\gamma} = -g_{ap\gamma} = e\,\frac{C_{\rm EDM}}{f_a}$$

$$C_{\rm nEDM} = 2.4(1.0)\times 10^{-16}\,{\rm cm}$$

[Pospelov,Ritz 00]

- This is phenomenologically important for experi-ments searching for axion dark matter via osci-llating nucleon electric dipole moments such as nEDM and CASPEr-electric



[AR,Rosenberg,Rybka in: 2021 Update of Review of Particle Physics]

[adapted from https://github.com/cajohare/AxionLimits]

The Axion

Coupling to photons

- From mixing with the pion, the axion inherits at energy scales below the QCD scale, a model-independent contribution to its coupling with two photons:

$$\mathcal{L}_{a\gamma\gamma} \supset \frac{1}{4}\, g_{a\gamma}^{(\mathrm{mi})}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$g_{a\gamma}^{(\mathrm{mi})} = \frac{\alpha}{2\pi f_a}\, C_{a\gamma}^{(\mathrm{mi})}$$

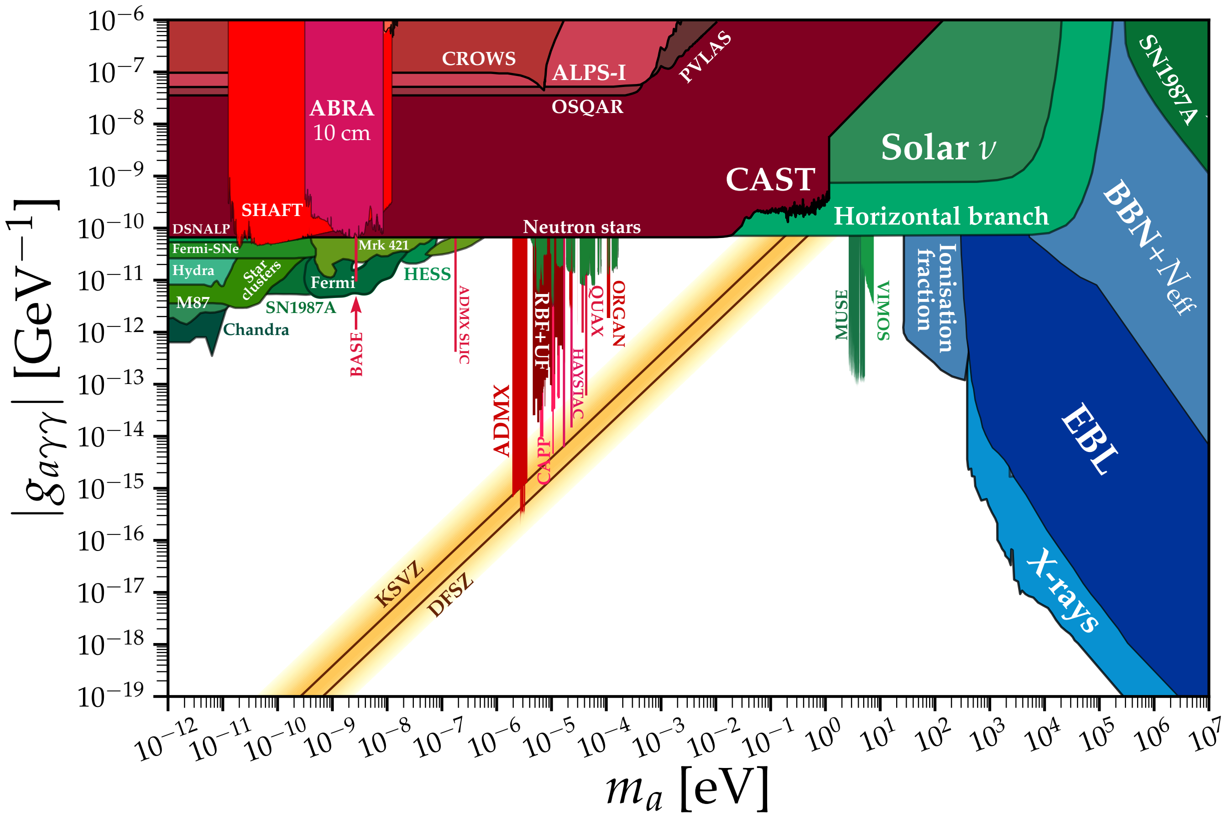
  - LO chiral perturbation theory:

$$C_{\mathrm{a}\gamma}^{(\mathrm{mi})} \simeq -\frac{2}{3}\frac{4+z}{1+z} \approx -2$$ [Kaplan 85;Srednicki `85]

  - Precise NLO determination

$$C_{a\gamma}^{(\mathrm{mi})} = -1.92(4)$$ [Grilli di Cortona et al. `16]

- Phenomenologically important for experiments searching for axions via coupling to photons



[AR,Rosenberg,Rybka in: 2021 Update of Review of Particle Physics]

[adapted from https://github.com/cajohare/AxionLimits]