Axions as Dark Matter Candidates

Andreas Ringwald
FNHP 2022
Frontiers in Nuclear and Hadronic Physics
Galileo Galilei Institute School
Online Event
Feb 21, 2022 - Mar 04, 2022

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

DESY.

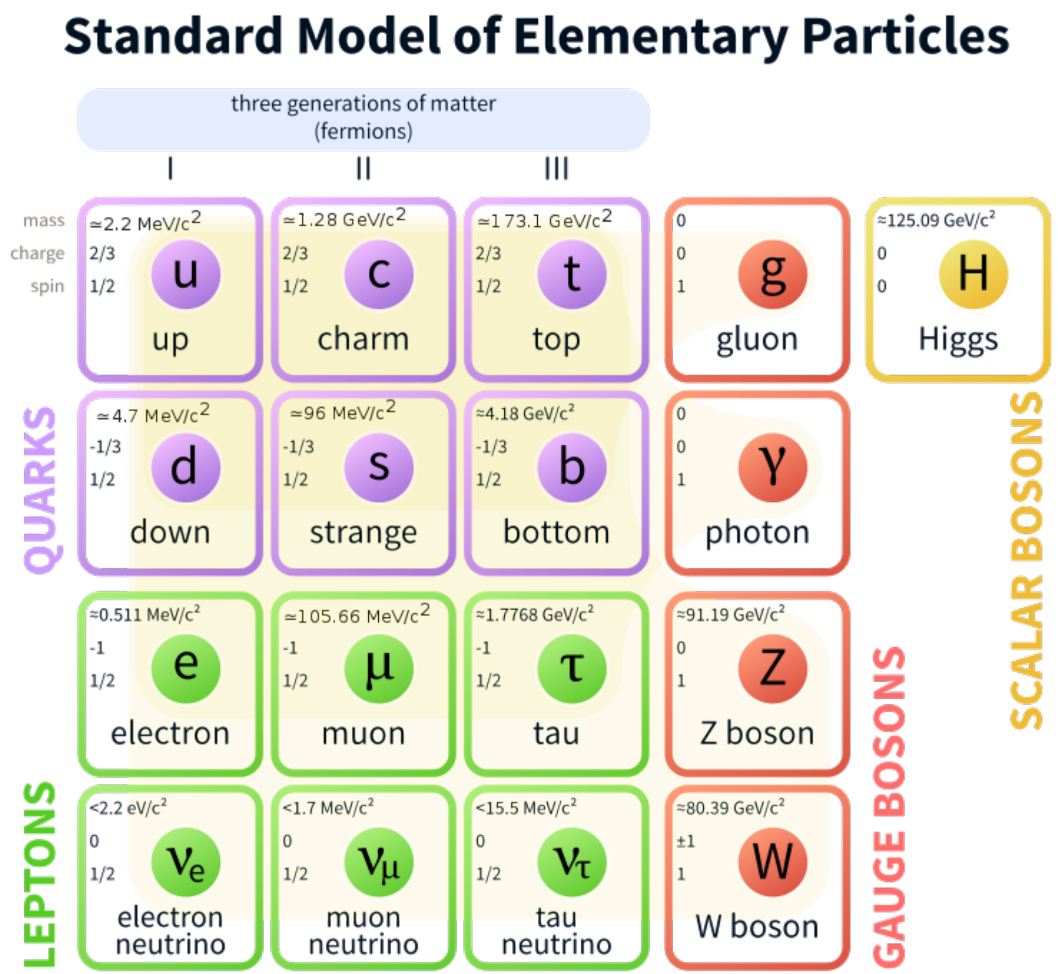
Today’s Plan

- Vacuum Structure in Quantum Chromodynamics
- Strong CP Puzzle
- The Axion
- **Axion Dark Matter**
- Axion Experiments

Axion Dark Matter

Big Success: Standard Model of Particle Physics

- Standard Model of elementary particle physics (SM) describes interactions of all known particles



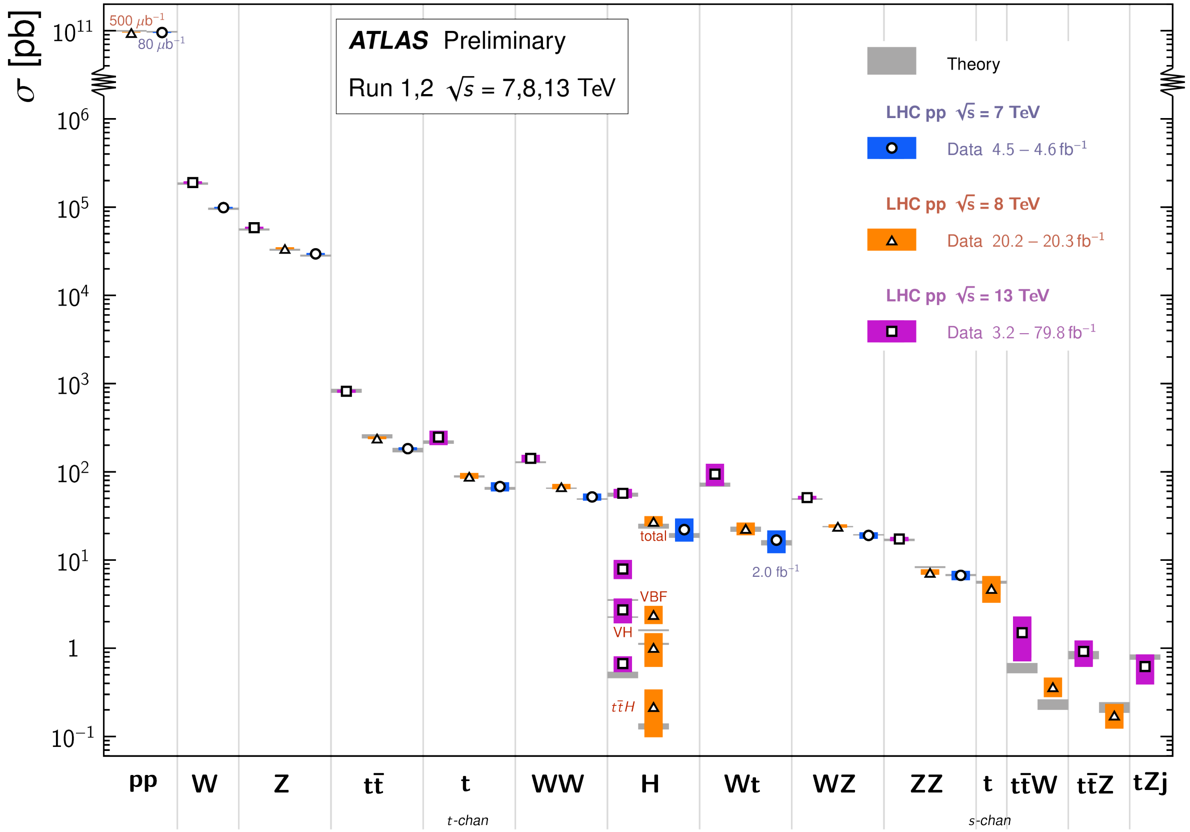
[Wikipedia]

Axion Dark Matter

Big Success: Standard Model of Particle Physics

- Standard Model of elementary particle physics (SM) describes interactions of all known particles with remarkable accuracy

[cern.ch]

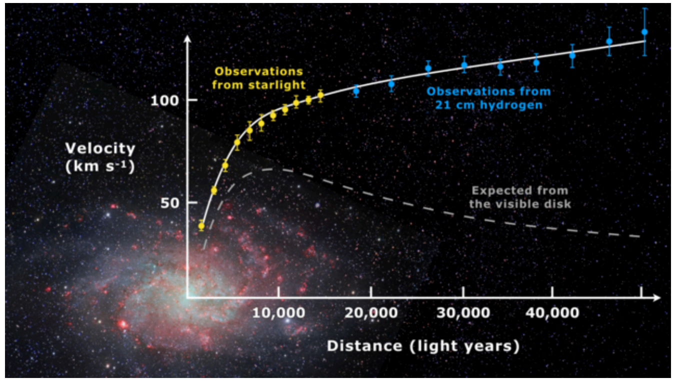


[atlas.web.cern.ch]

Axion Dark Matter

Big Flaw: SM accounts only for few percent of total energy of universe

- Astronomical observations on galactic



Axion Dark Matter

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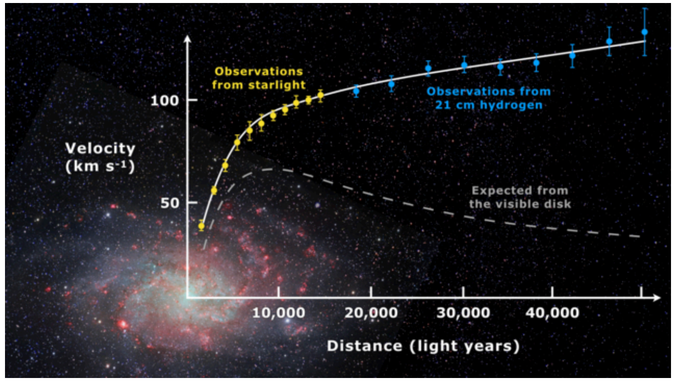
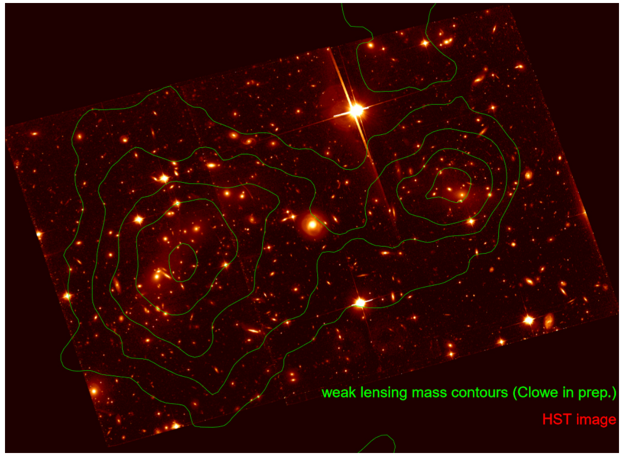


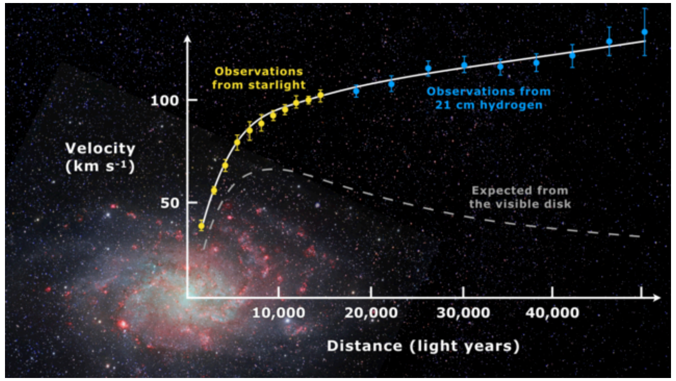


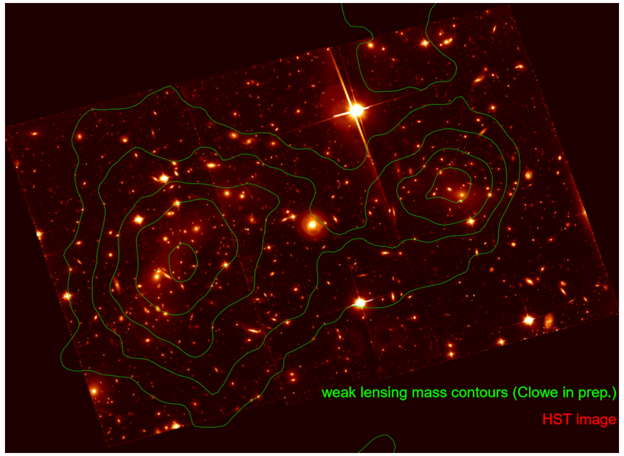
[Wikipedia]

Axion Dark Matter

Big Flaw: SM accounts only for few percent of total energy of Universe

- Astronomical observations on galactic, galaxy cluster and cosmological scales:





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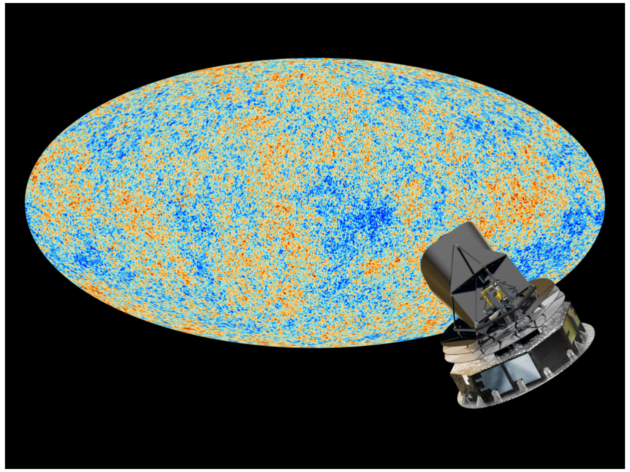
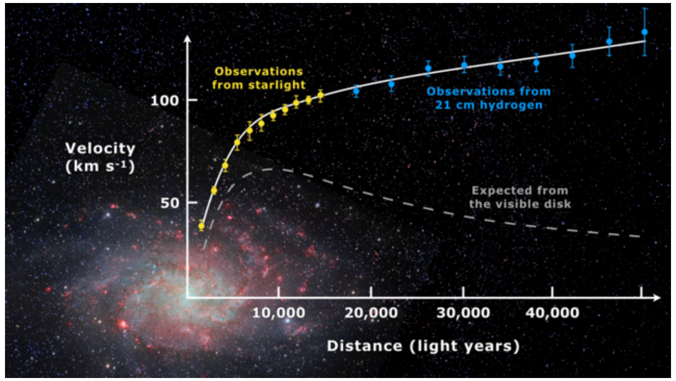


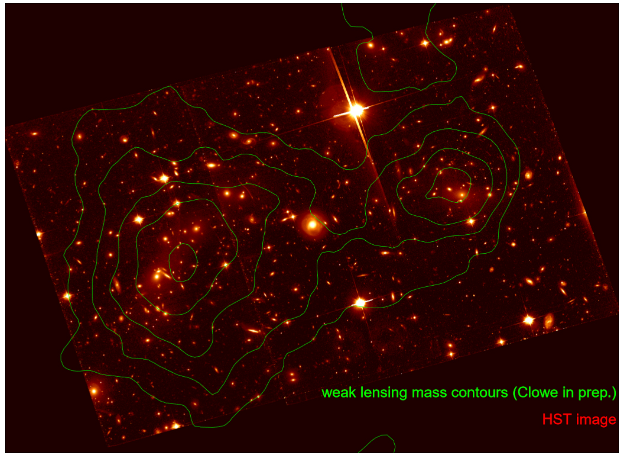
[NASA]

Axion Dark Matter

Big Flaw: SM accounts only for few percent of total energy of Universe

- Astronomical observations on galactic, galaxy cluster and cosmological scales show that about 85 % of the matter in Universe not comprised of ordinary matter





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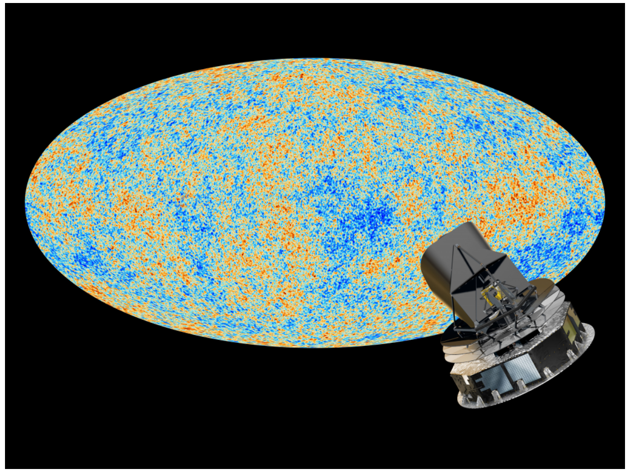

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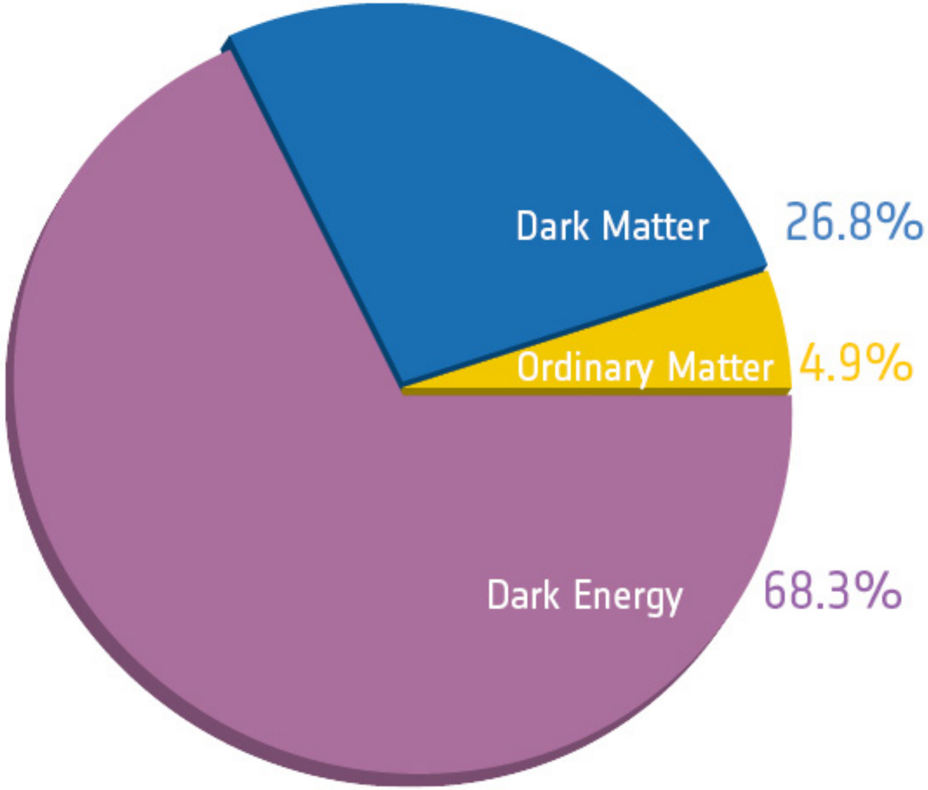
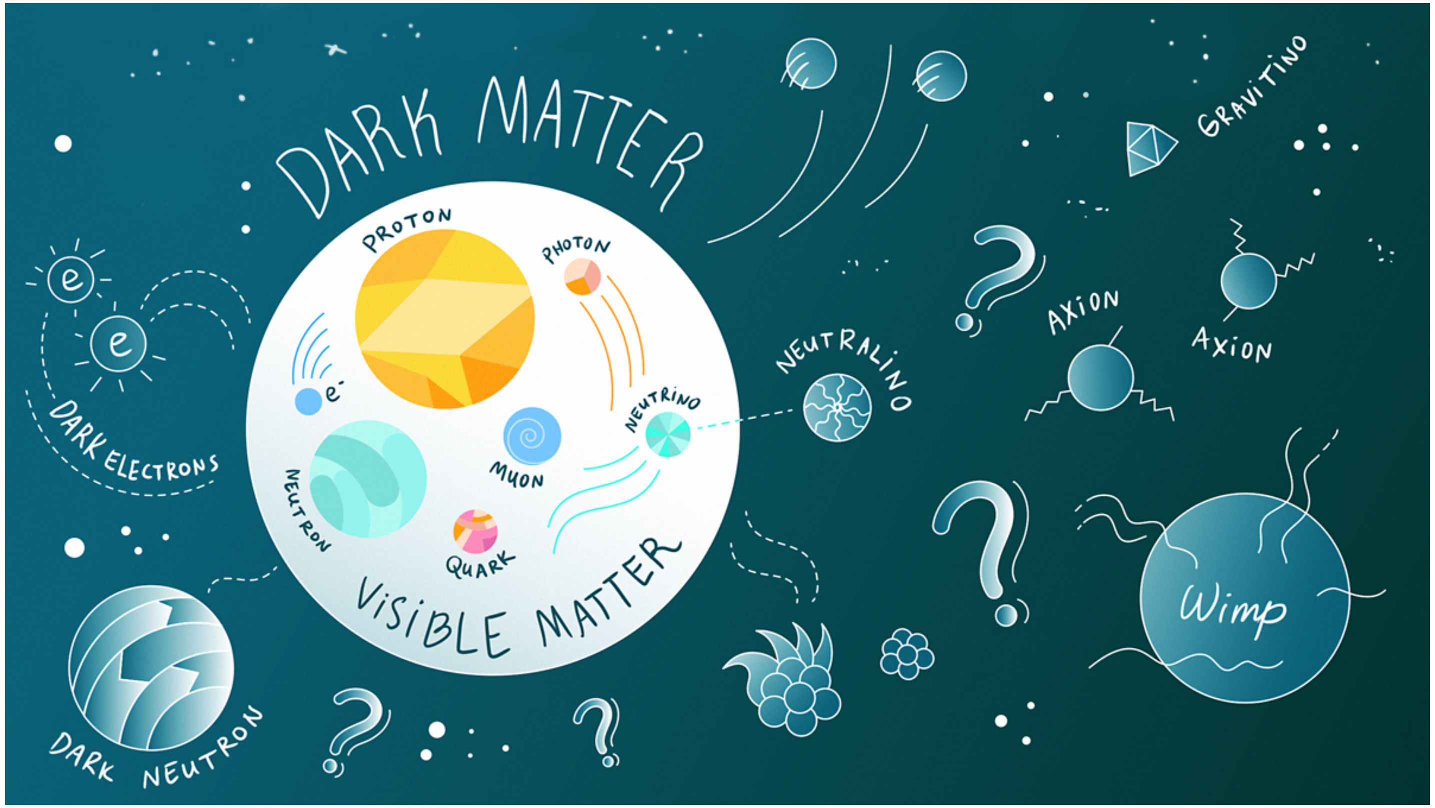


[PLANCK]

Axion Dark Matter

Big Variety: Zoo of Dark Matter candidates

- Theoretical particle physicists have proposed a plenitude of Dark Matter (DM) candidates:

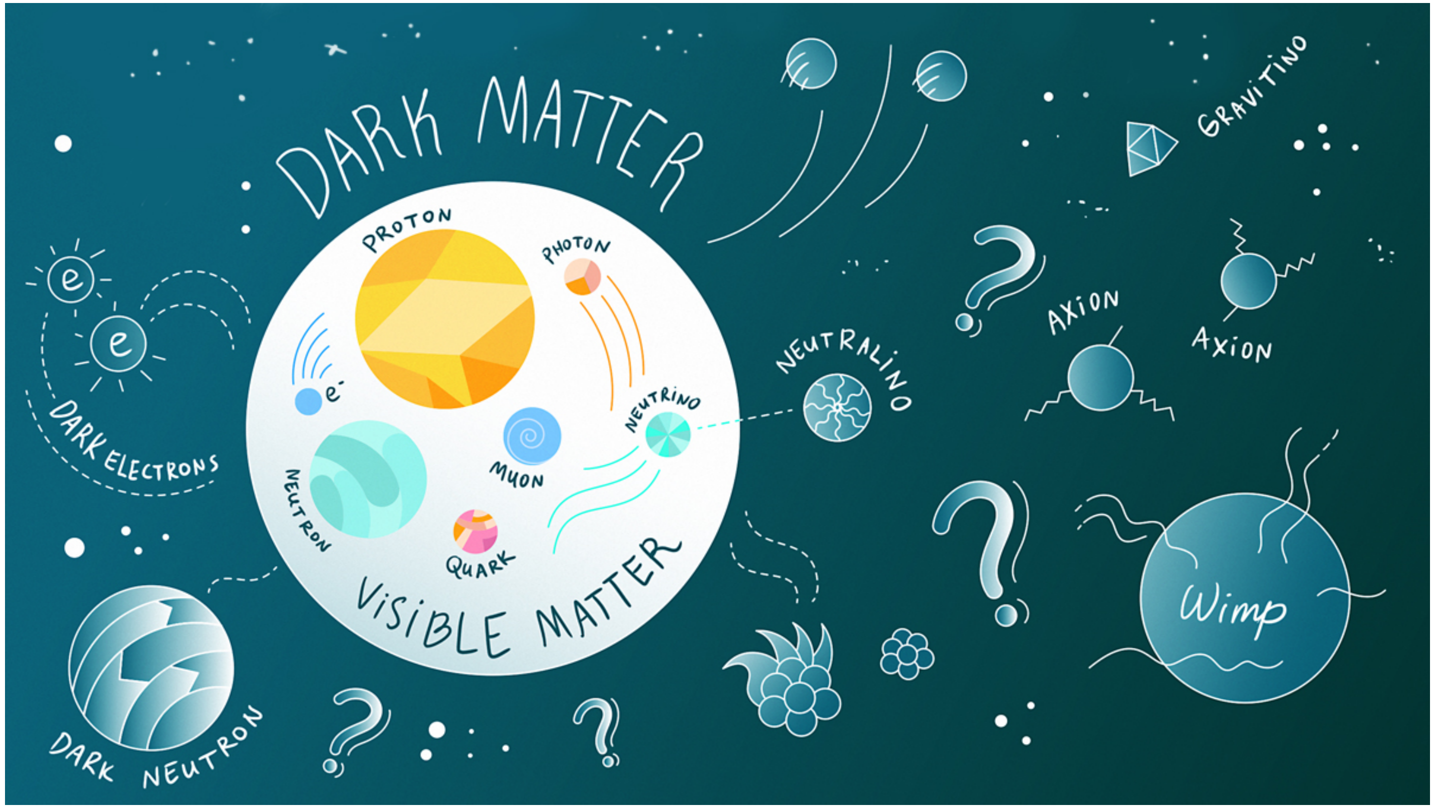


[Symmetry Magazine]

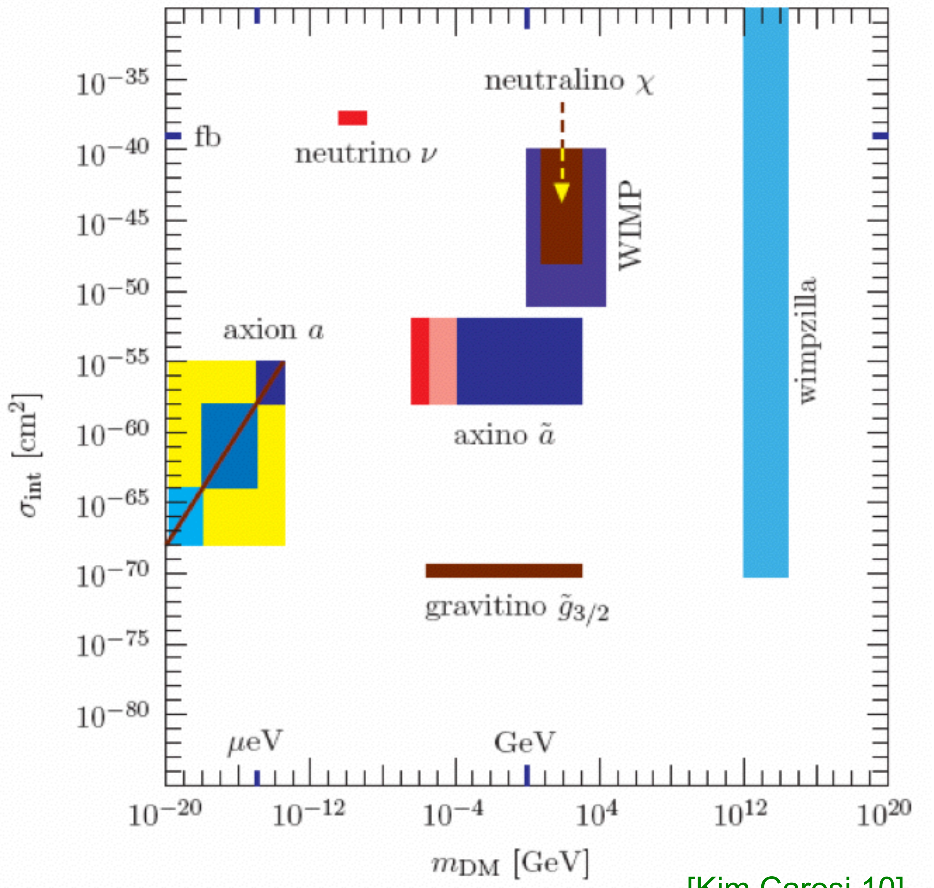
Axion Dark Matter

Big Variety: Zoo of Dark Matter candidates

- Theoretical particle physicists have proposed a plenitude of Dark Matter (DM) candidates, spanning a huge range in masses and interaction strength:



[Symmetry Magazine]

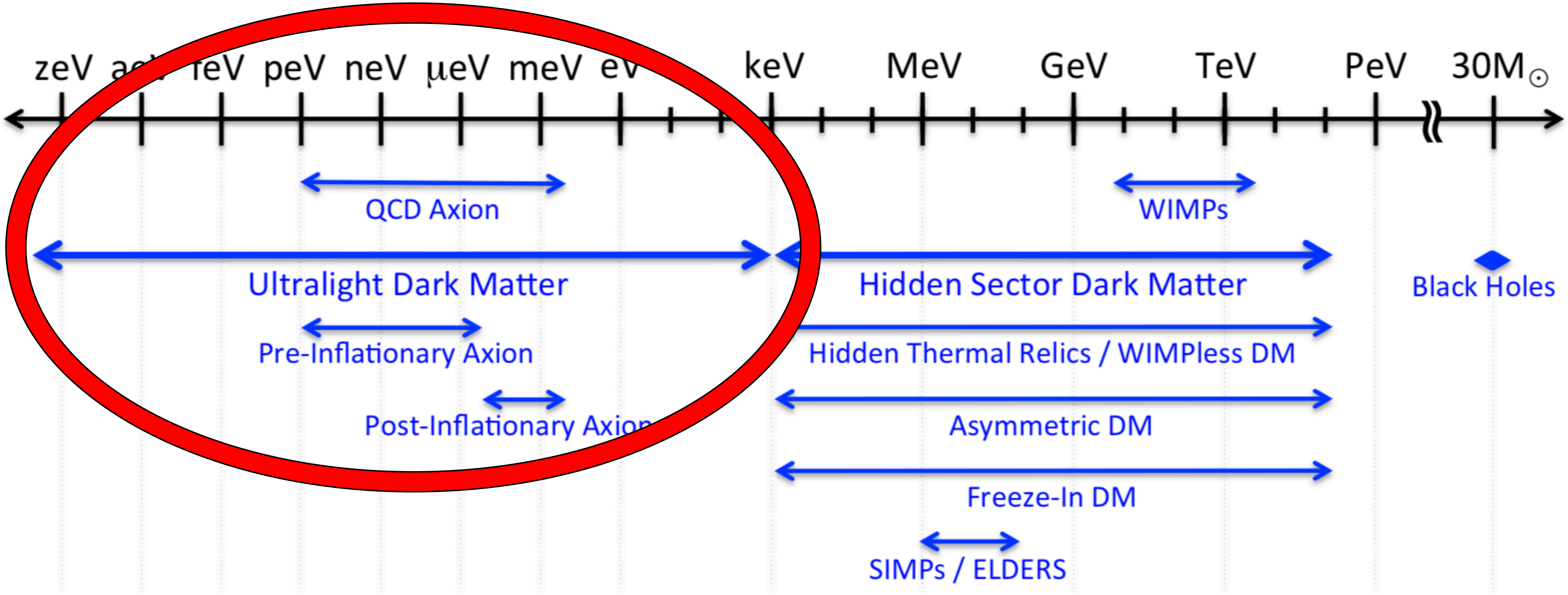


[Kim,Carosi 10]

Axion Dark Matter

Big Variety: Zoo of Dark Matter candidates

- Concentrate here on QCD axions, belonging to the generic class of **Ultralight Dark Matter**



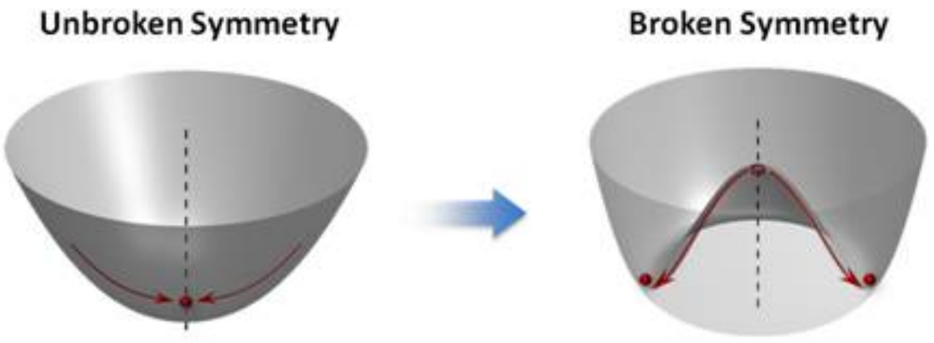
[US Cosmic Visions: New Ideas in Dark Matter 2017]

Axion Dark Matter

Production via vacuum misalignment

[Preskill,Wise,Wilczek 83; Abbott,Sikivie 83; Dine,Fischler 83,....]

- PQ phase transition takes place when $T \lesssim T_c^{\rm PQ} \sim v_{\rm PQ} = N_{\rm DW} f_a$



[Peking University]

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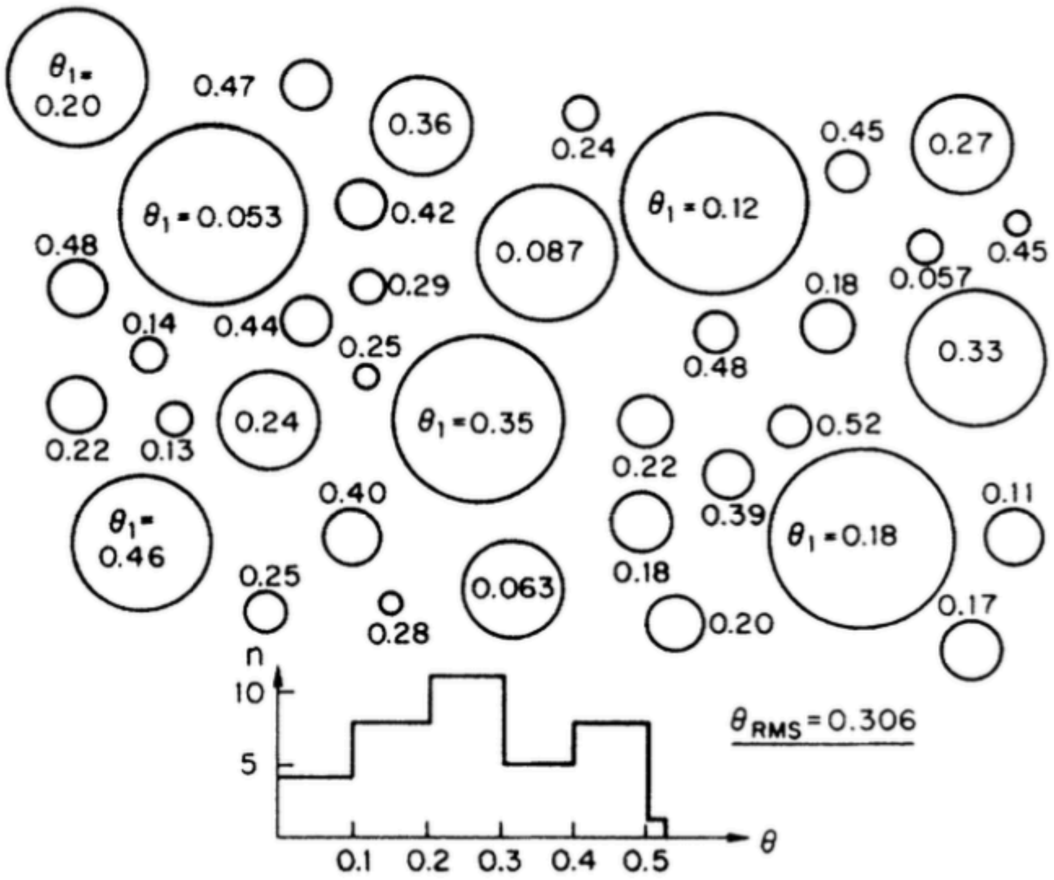

[Turner `86]

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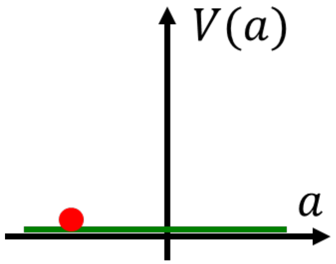
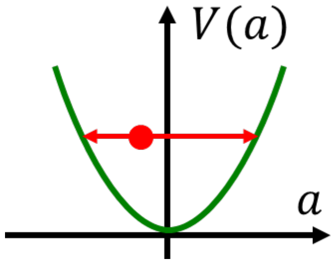
$$\frac{d^2\theta}{dt^2} + 3H(T)\frac{d\theta}{dt} + m_a^2(T)\sin\theta = 0$$





[Raffelt]

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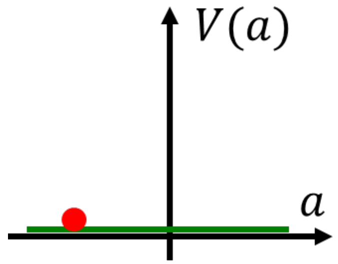

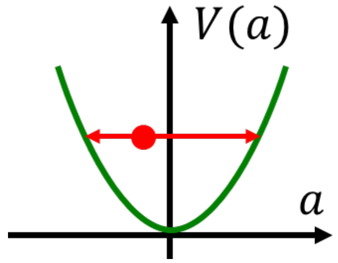

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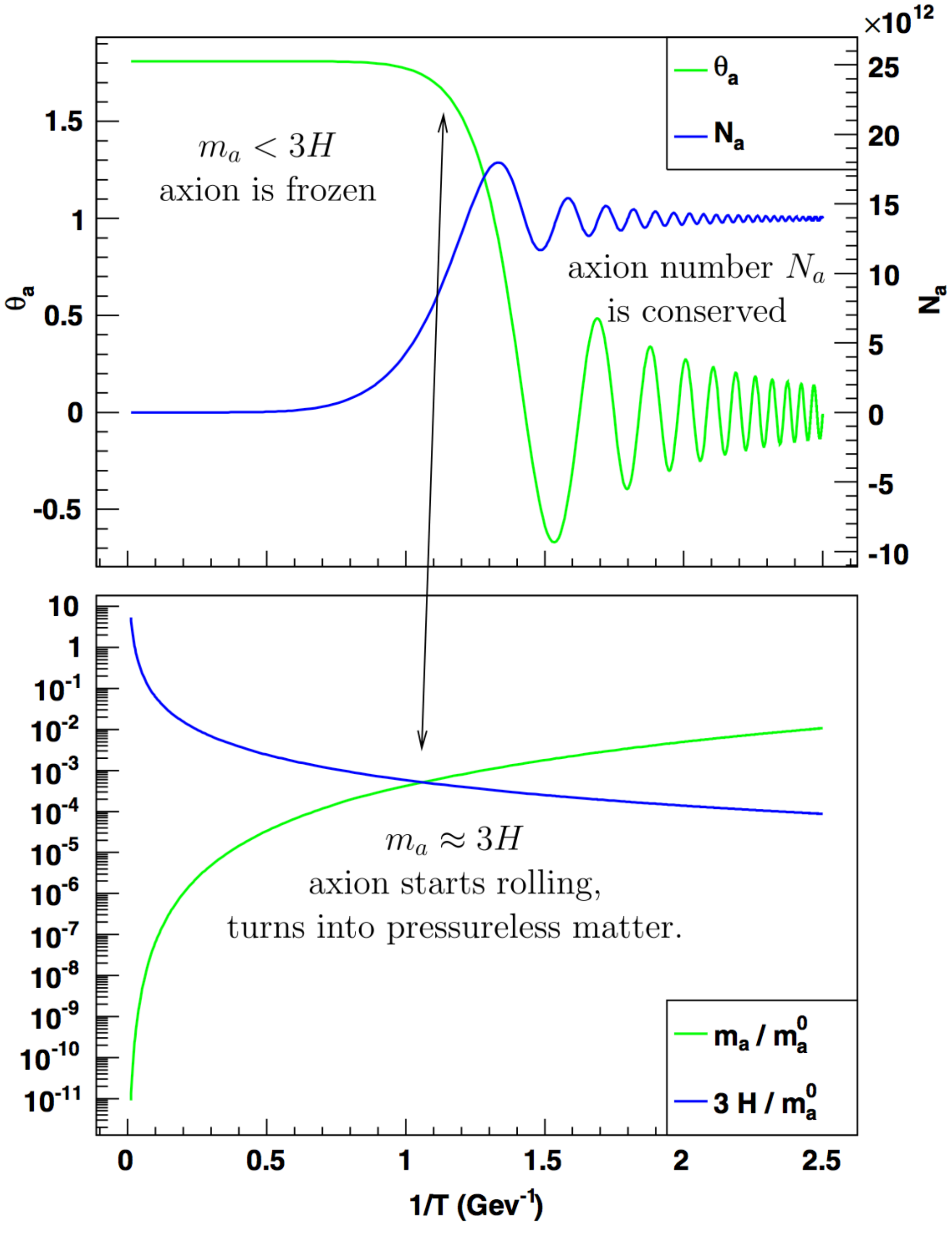

[Wantz,Shellard `09]

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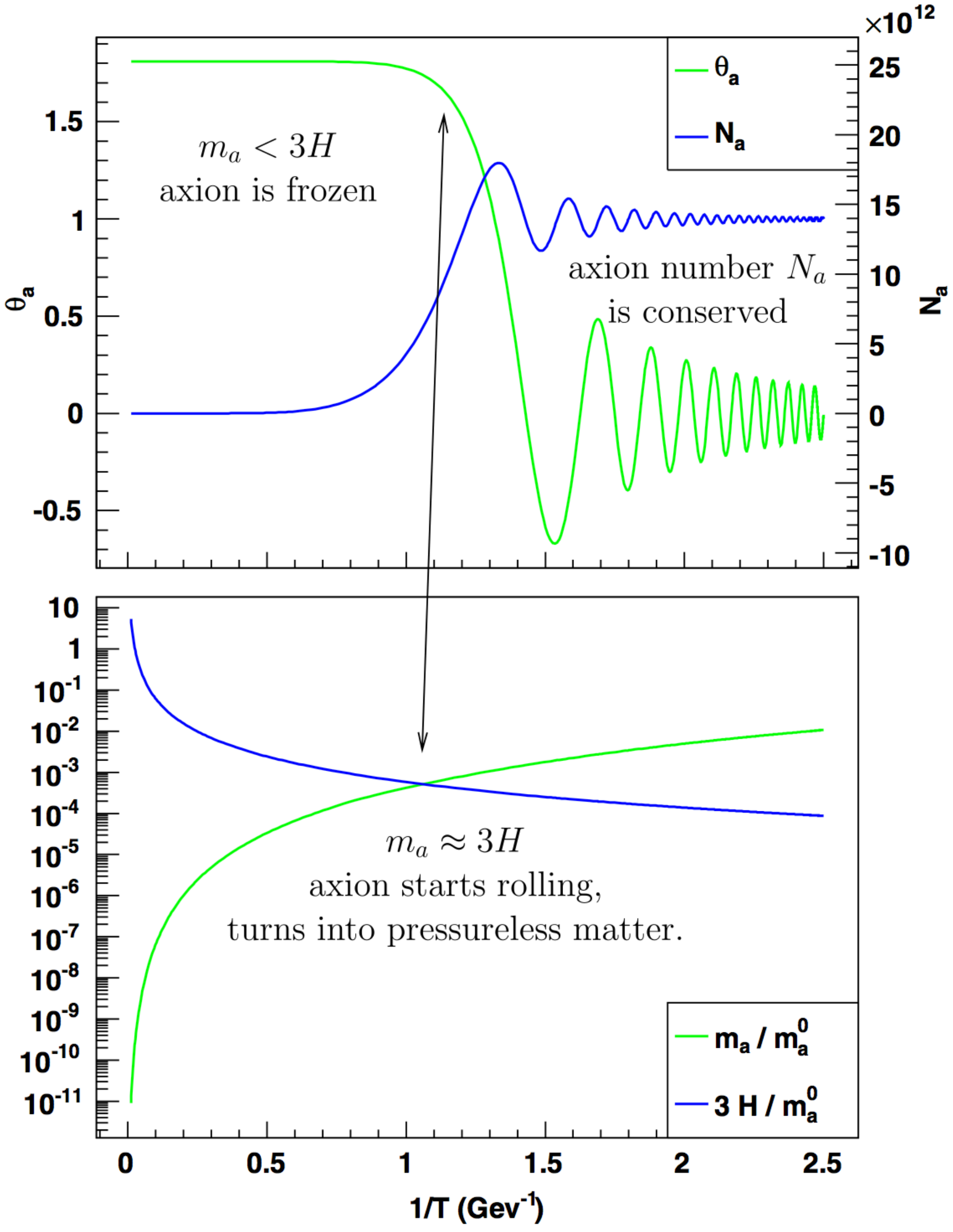

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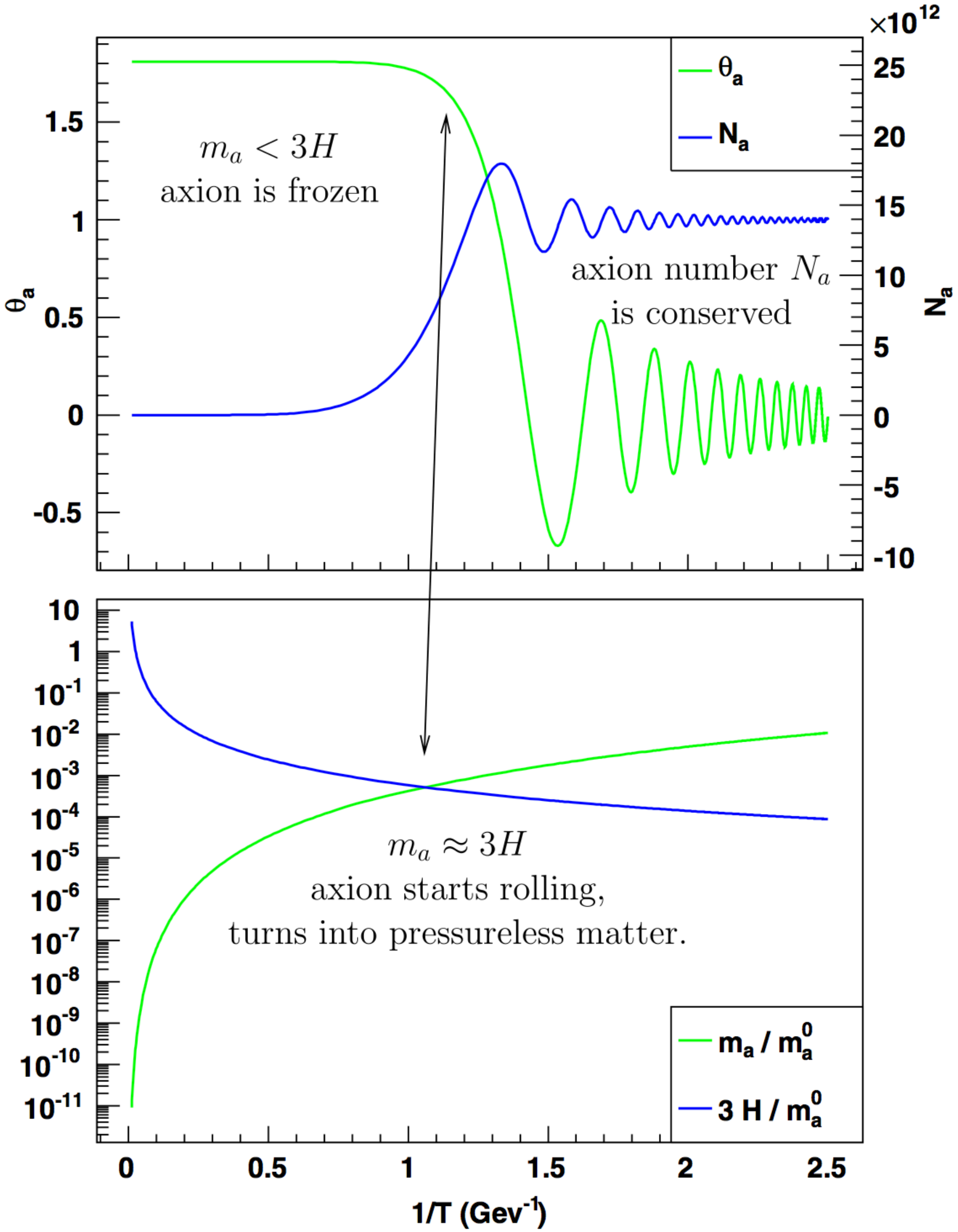
  - Coherent and spatially uniform oscillations of axion field correspond to a coherent state of non-relativistic axion particles



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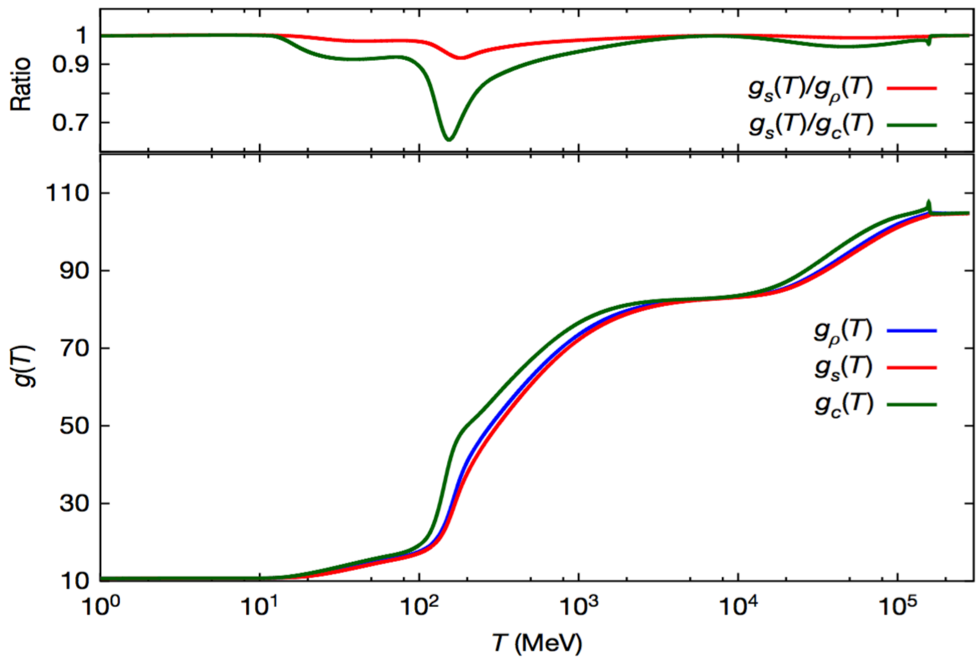

[Borsanyi et al., Nature `16 [1606.0794]]

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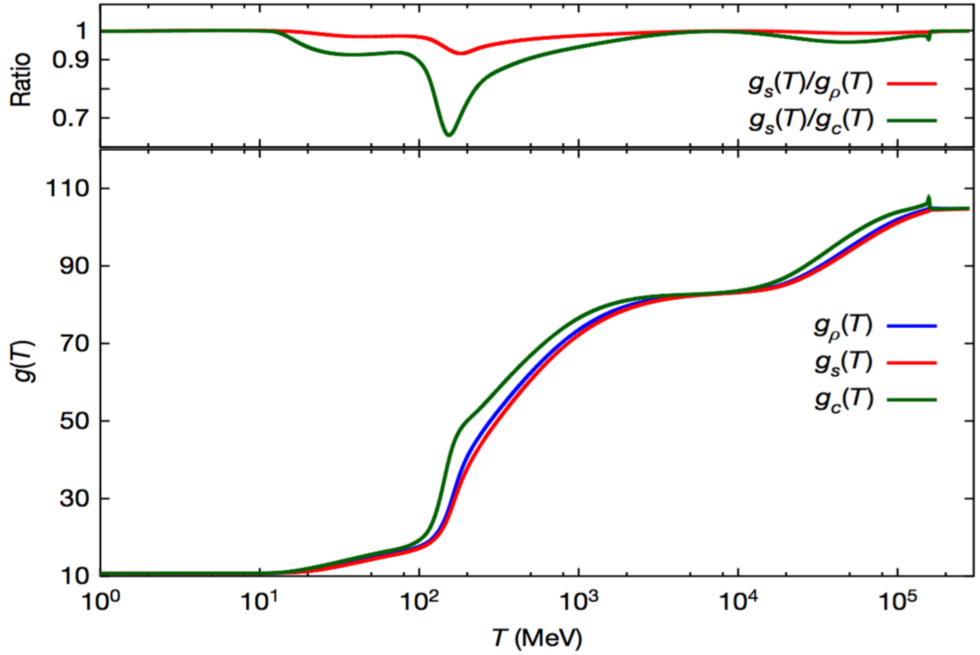

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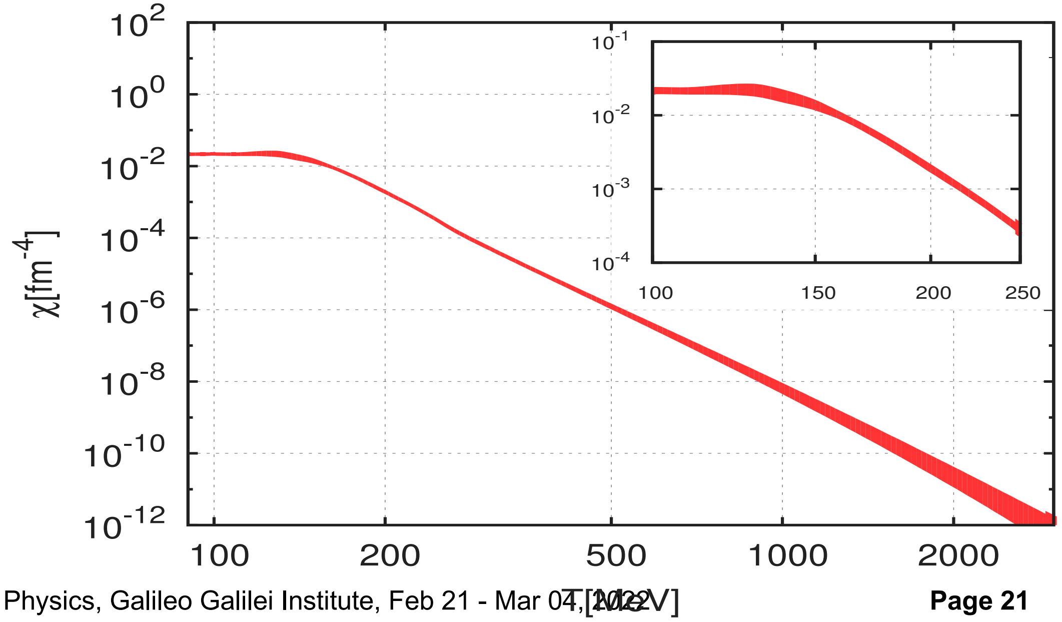
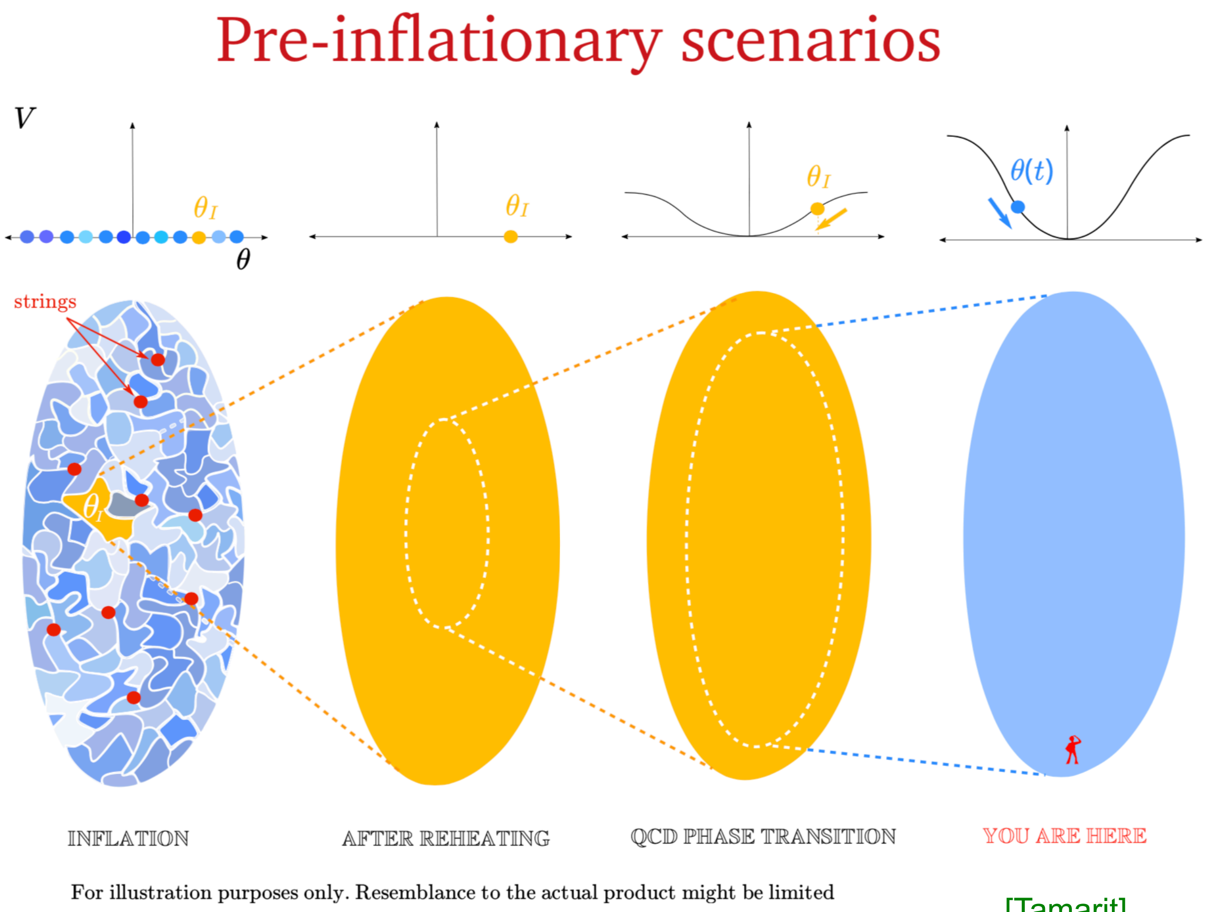


Axion Dark Matter

Pre-inflationary PQ SSB scenario

- If PQ symmetry broken before or during inflation ($f_a > H_{\rm inf}/(2\pi)$) and not restored afterwards



[Tamarit]

Axion Dark Matter

Pre-inflationary PQ SSB scenario

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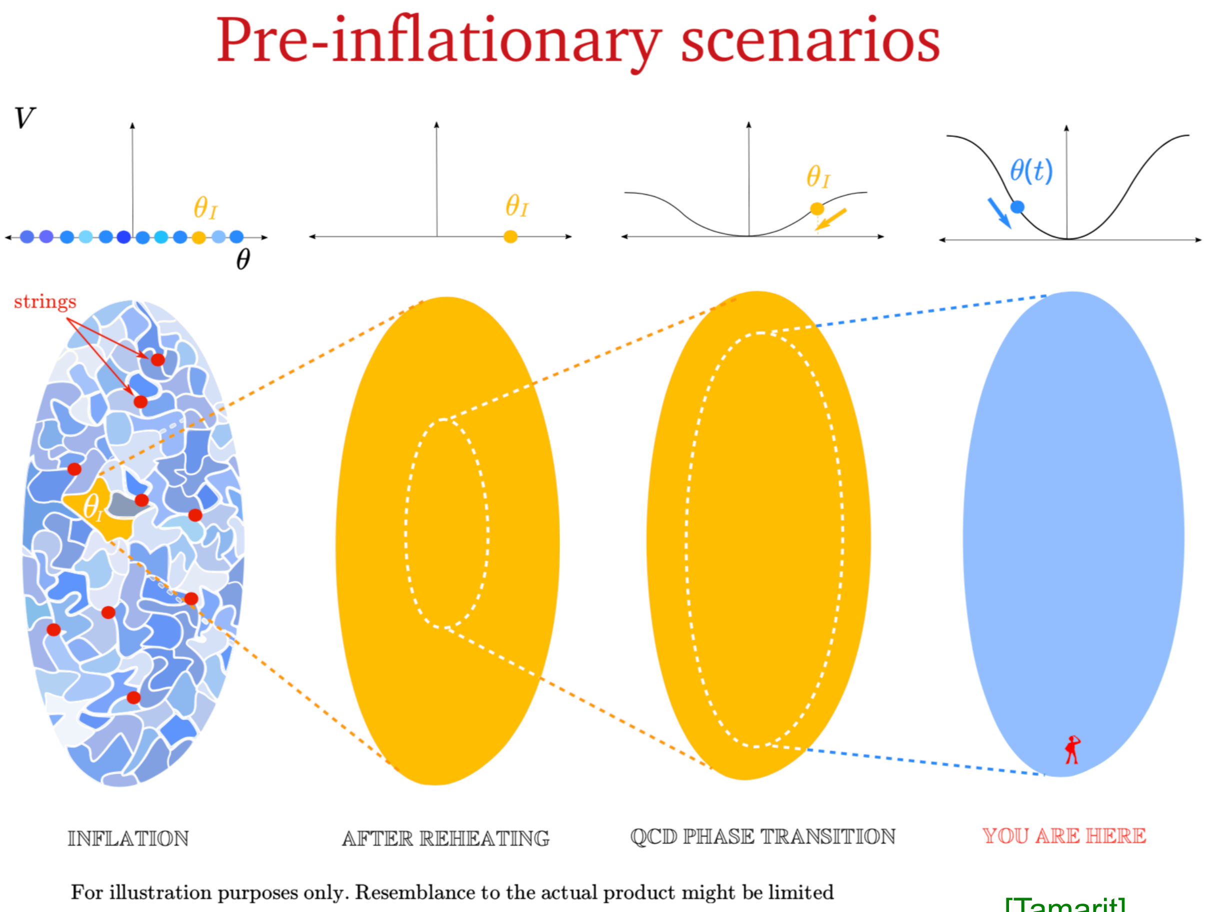

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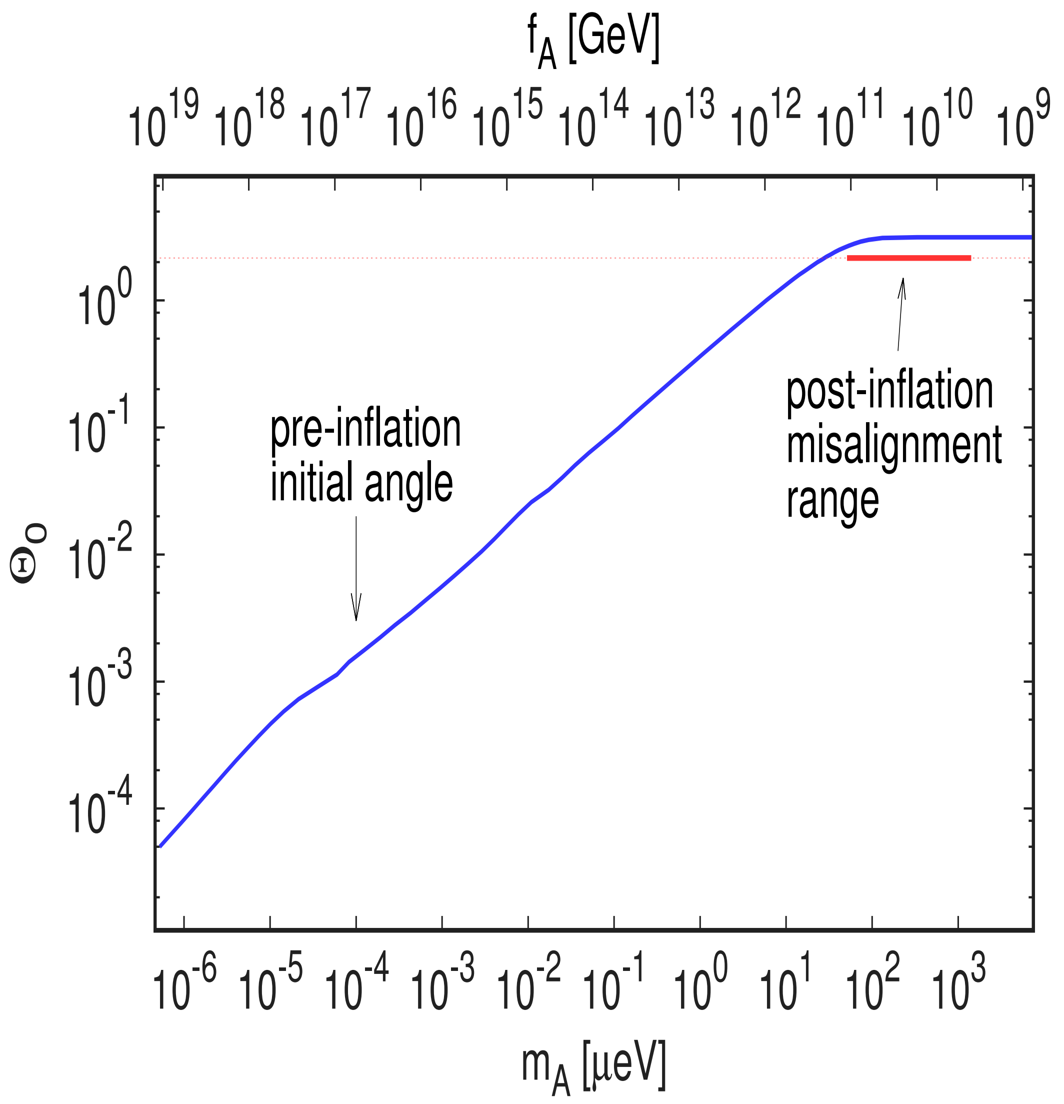
$$\Omega_a^{\rm MIS} h^2 \approx 0.12 \left(\frac{f_a}{9\times 10^{11}\ {\rm GeV}} \right)^{1.165} \theta_{\rm i}^2$$
$$\approx 0.12 \left(\frac{6\ \mu{\rm eV}}{m_a} \right)^{1.165} \theta_{\rm i}^2 ,$$



[Borsanyi et al., Nature `16]

Axion Dark Matter

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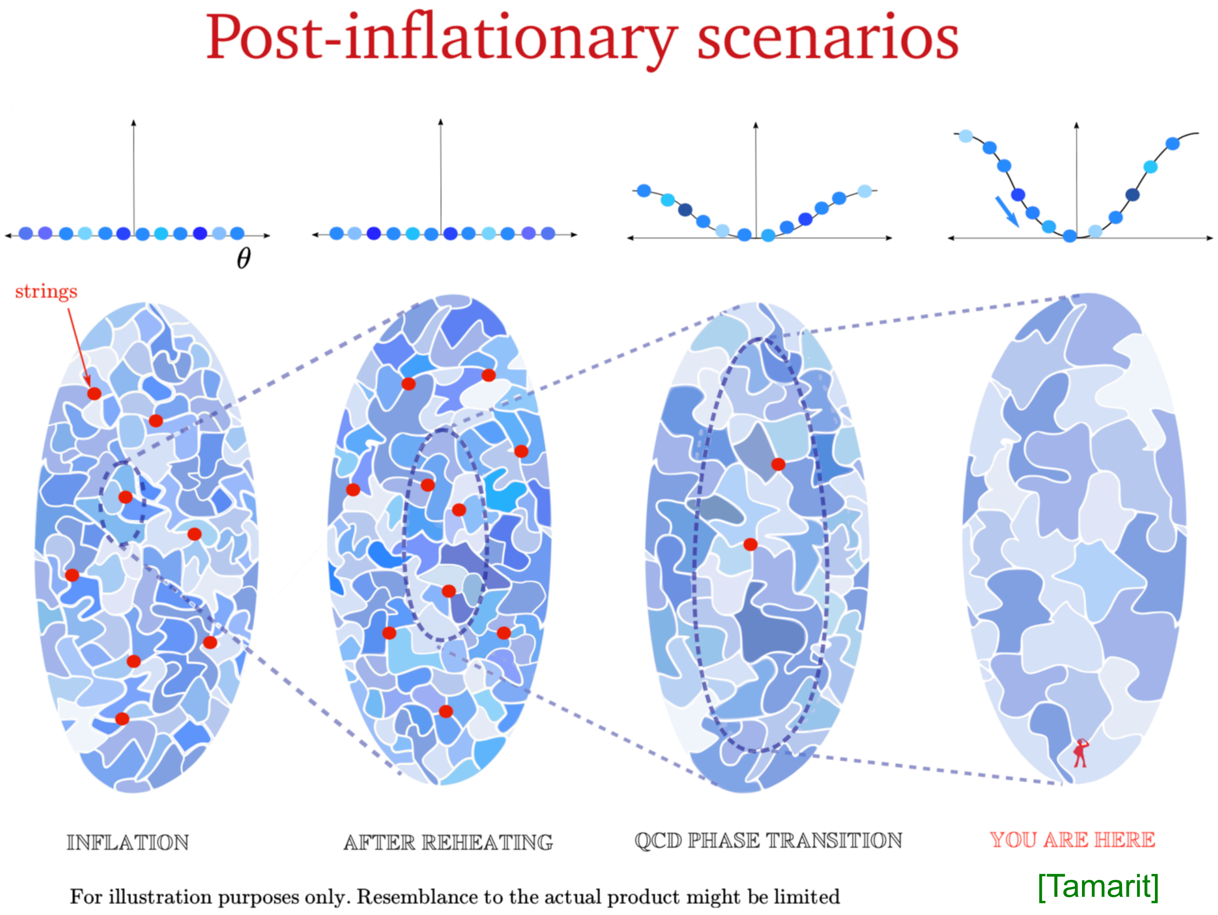
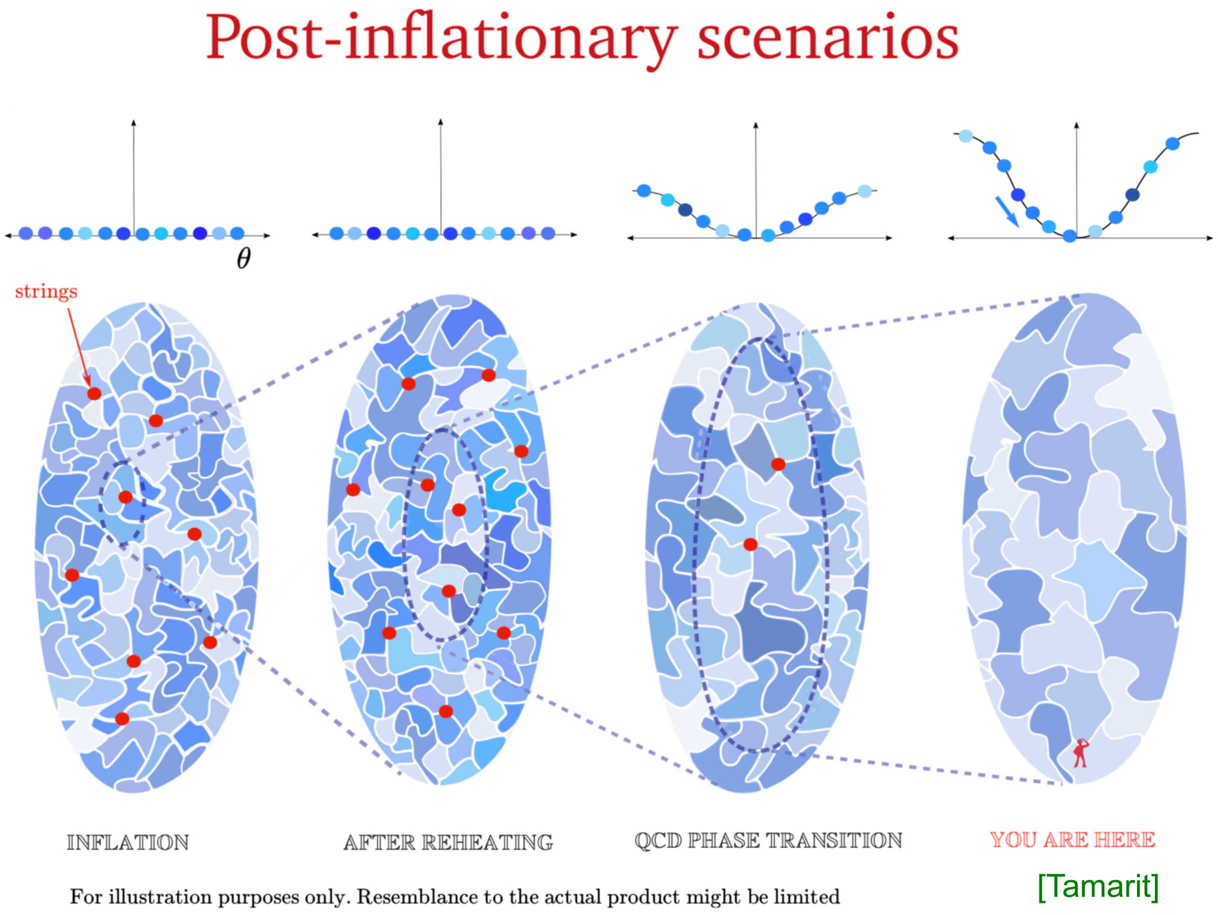


[Tamarit]

Axion Dark Matter

Post-inflationary PQ SSB scenario

- If PQ symmetry broken after inflation
  - Axion field took on different values in different patches of the present universe



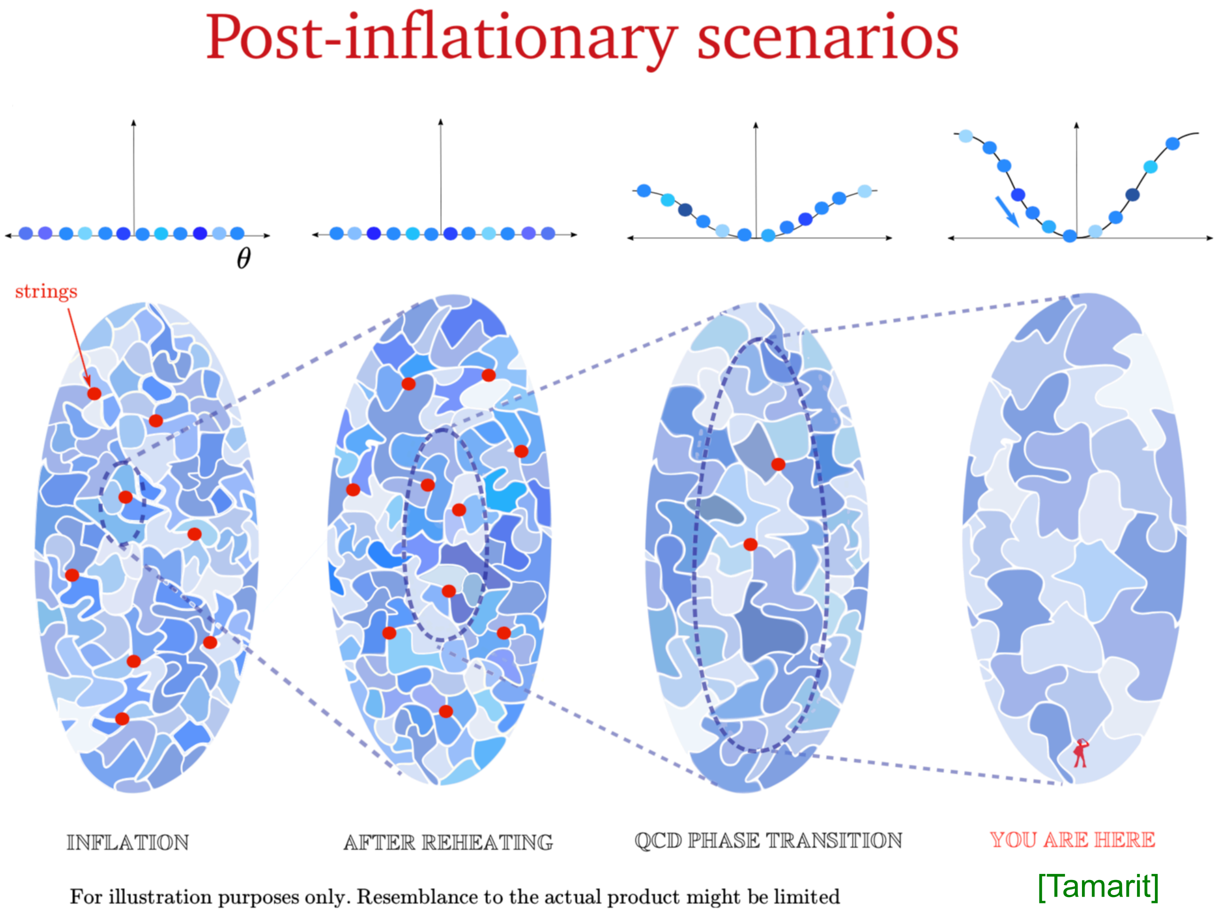
[Tamarit]

Axion Dark Matter

Post-inflationary PQ SSB scenario

- If PQ symmetry broken after inflation
  - Axion field took on different values in different patches of the present universe
  - Need to average over random initial axion field values:

$$\Omega_a^{\rm MIS} h^2 \approx 0.12 \left(\frac{30\ \mu{\rm eV}}{m_a}\right)^{1.165}$$



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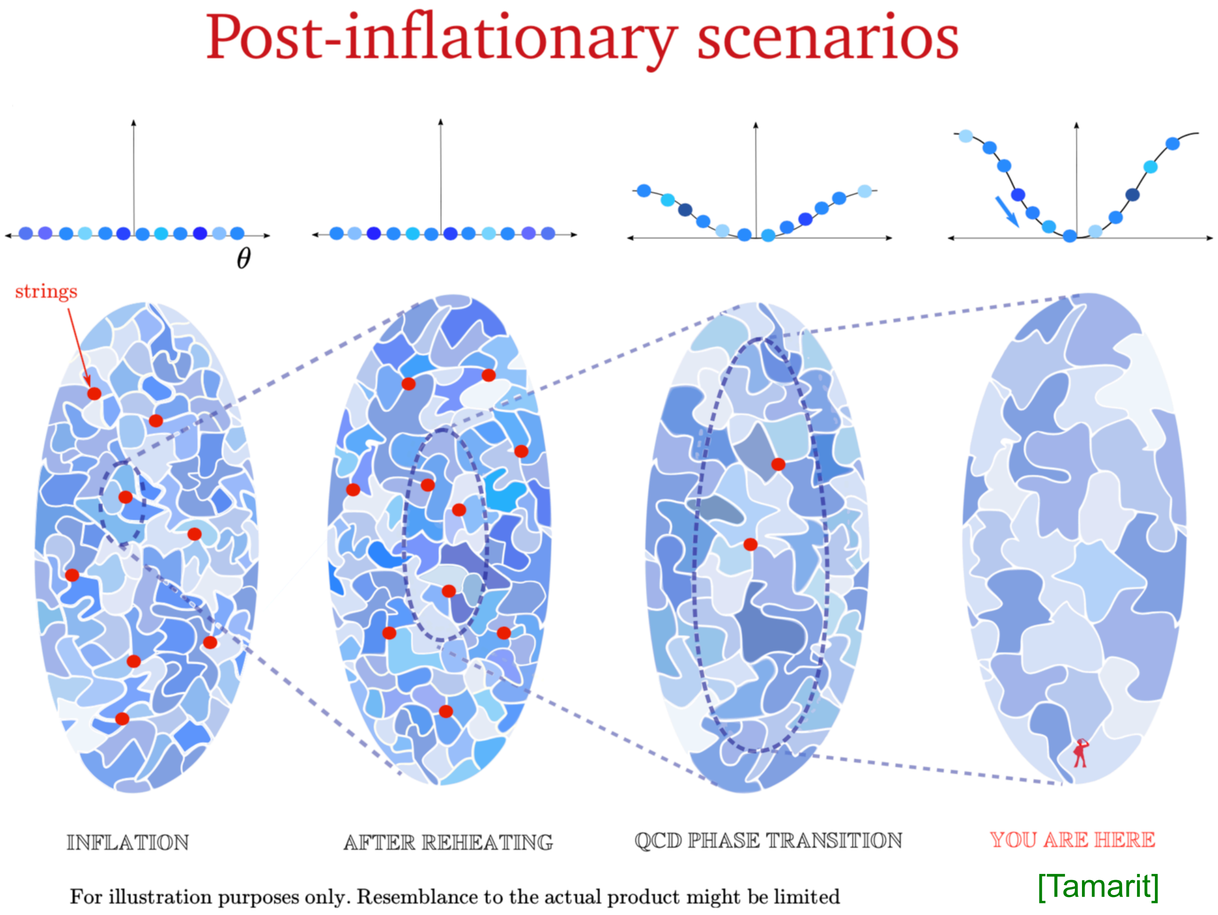
- Observed cold dark matter abundance puts lower bound on axion mass:

$$m_a > 28(2)\ \mu\mathrm{eV}$$

[Borsanyi et al., Nature `16 [1606.0794]]

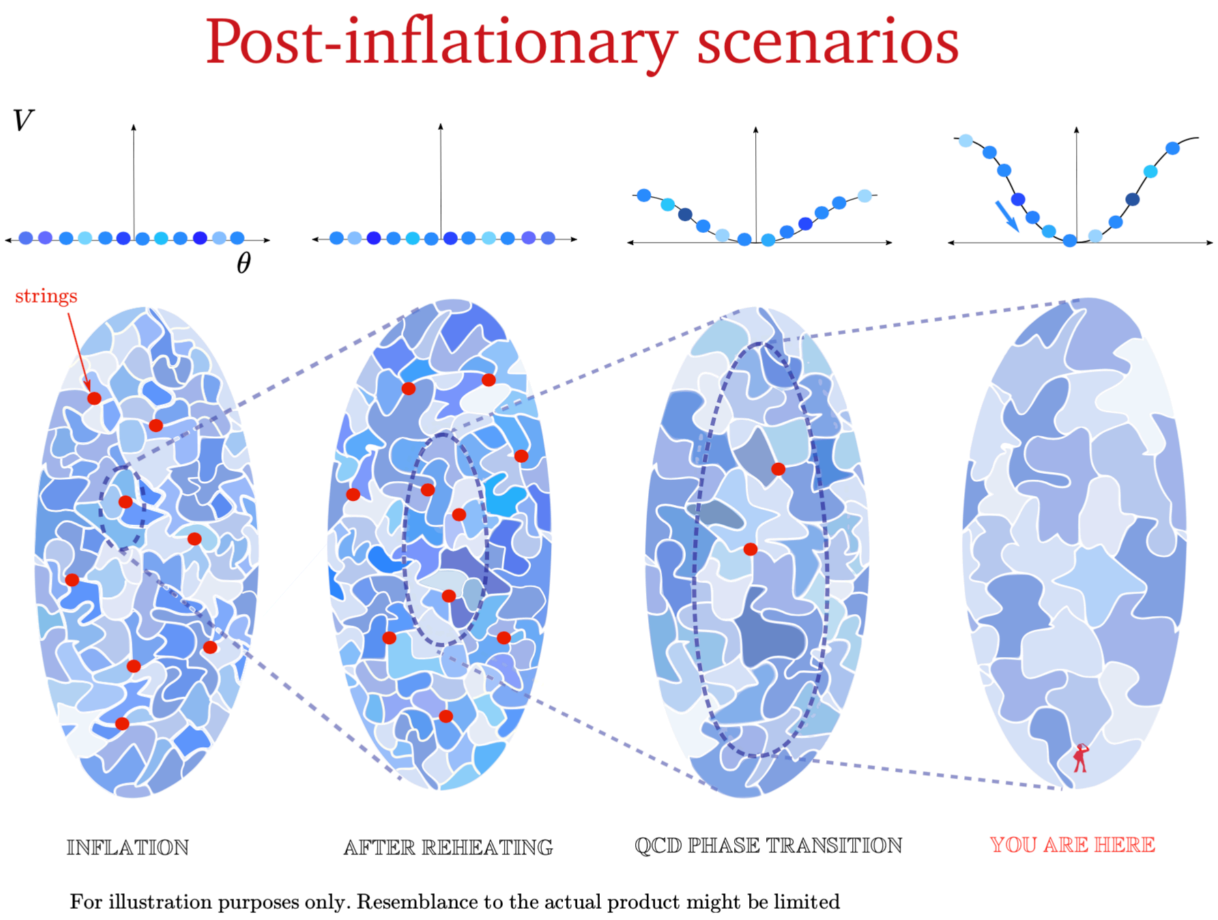


[Tamarit]

Axion Dark Matter

Post-inflationary PQ SSB scenario

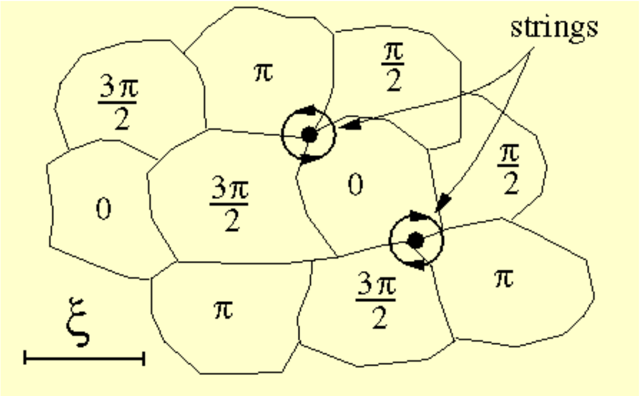
- If PQ symmetry broken after inflation
  - Random initial values in patches of size of causal contact after PQ phase transition

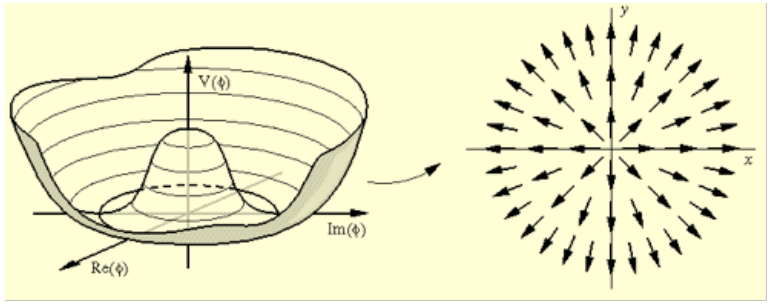


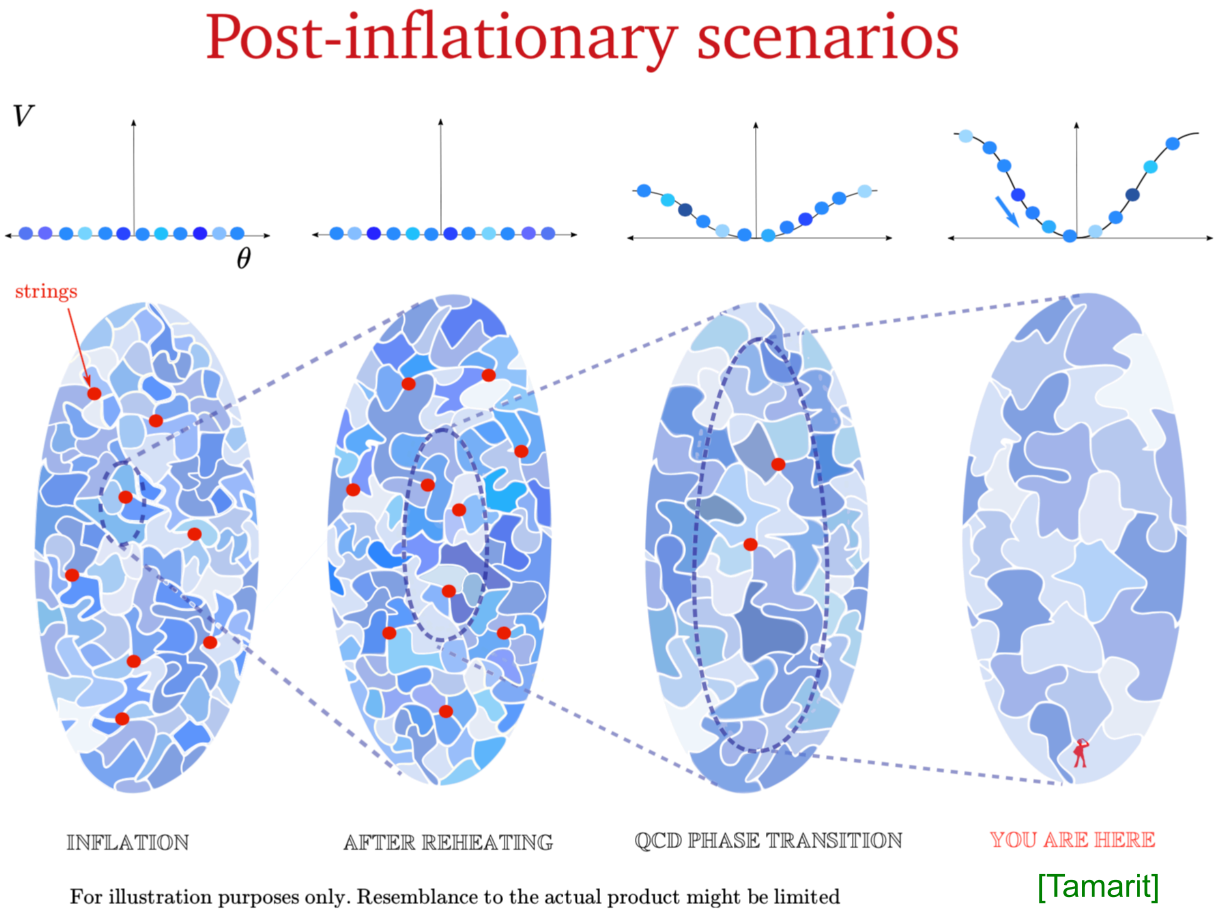
Axion Dark Matter

Post-inflationary PQ SSB scenario

- If PQ symmetry broken after inflation
    - Random initial values in patches of size of causal contact after PQ phase transition
    - Network of cosmic strings generated by Kibble mechanism







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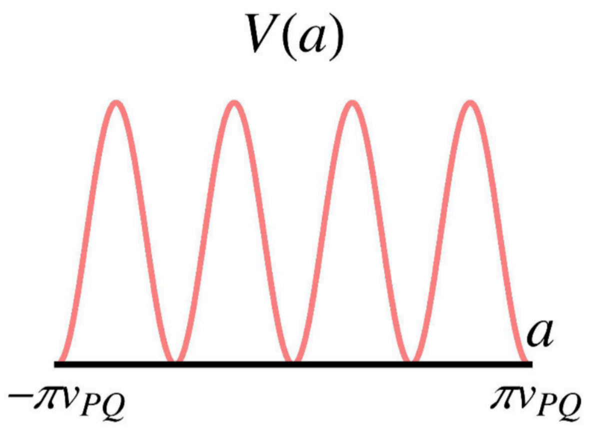
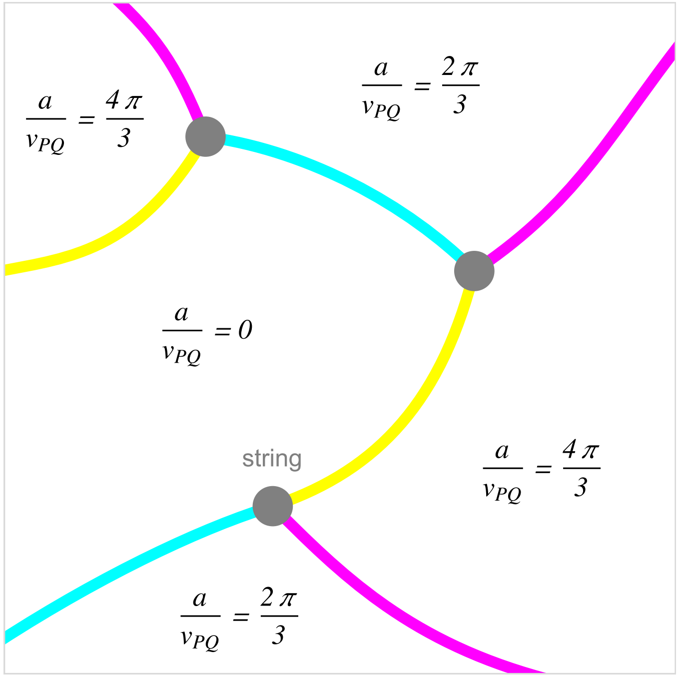


[Saikawa]



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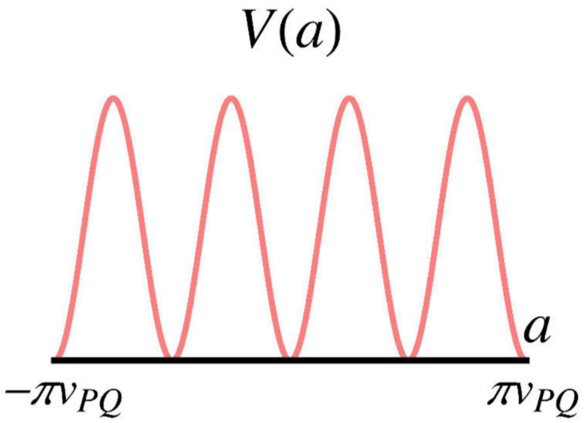
  - Around QCD phase transition, axion potential develops $$V(A,T) = \chi(T)\left[1-\cos\left(N_{\rm DW}\frac{A}{v_{\rm PQ}}\right)\right]$$
    - $N_{\rm DW}$ domain walls attached to a string



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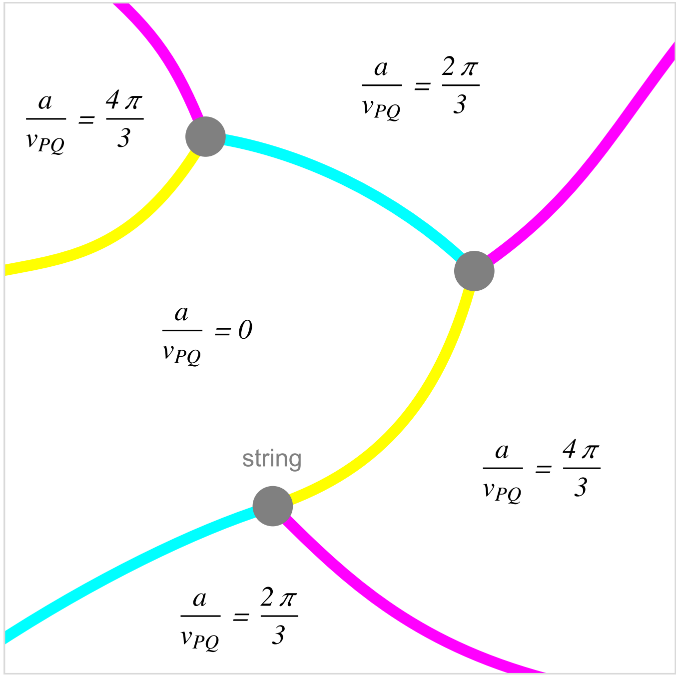

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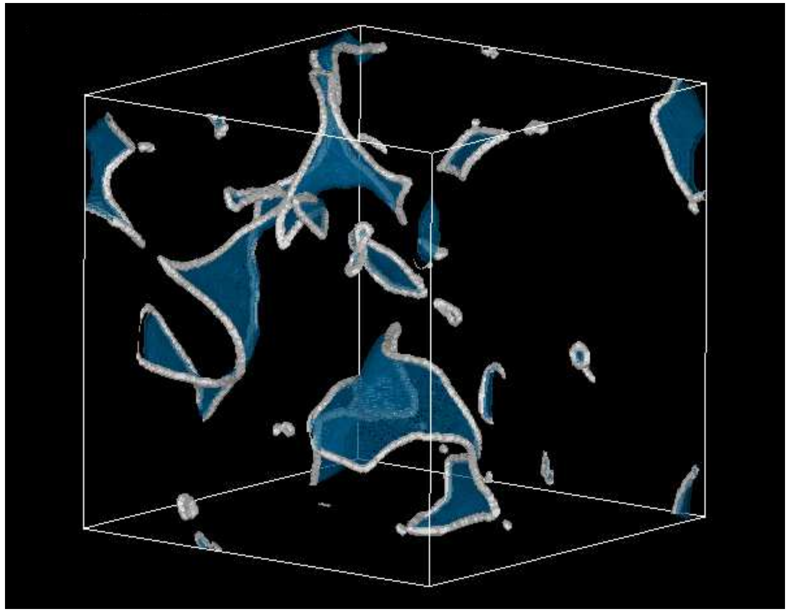
      - $N_{\rm DW}=1$: string wall system decays

$N_{\rm DW}=1$



[Hiramatsu et al.]

Axion Dark Matter

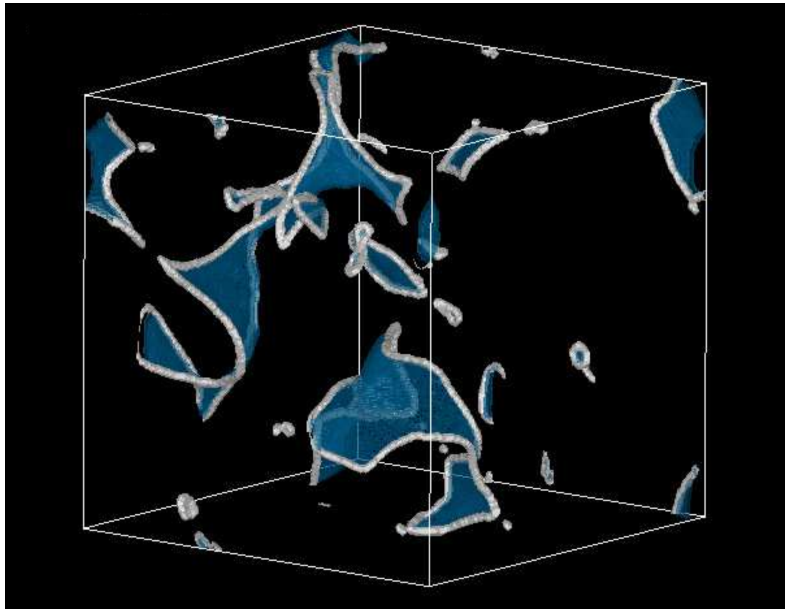
Post-inflationary PQ SSB scenario

- If PQ symmetry broken after inflation
  - Random initial values in patches of size of causal contact after PQ phase transition
  - Network of cosmic strings generated by Kibble mechanism
  - Around QCD phase transition, axion potential develops

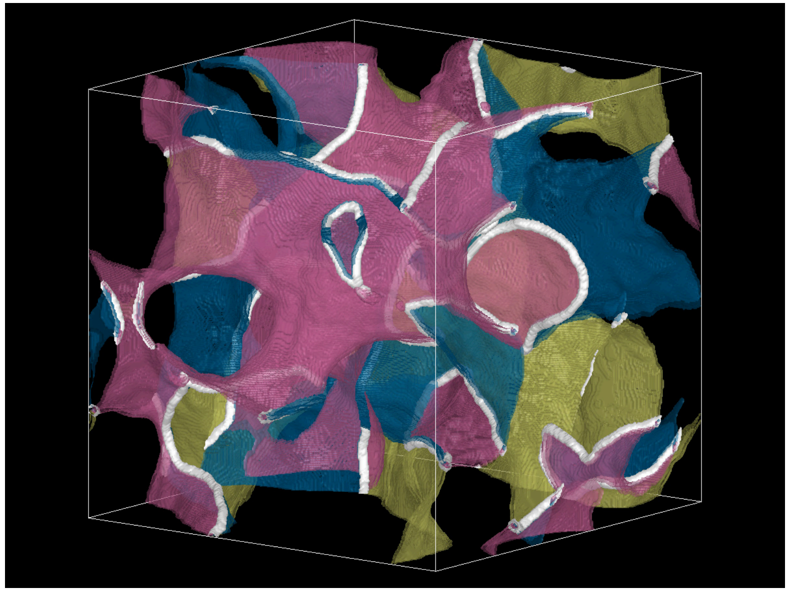
$$V(A,T) = \chi(T)\left[1-\cos\left(N_{\rm DW}\frac{A}{v_{\rm PQ}}\right)\right]$$

    - $N_{\rm DW}$ domain walls attached to a string
      - $N_{\rm DW}=1$: string wall system decays
      - $N_{\rm DW}>1$: string wall system long lived

$N_{\rm DW}=1$



$N_{\rm DW}=3$



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  - Amount of axions produced by collapse of network of strings and domain wall currently under debate
    [Hiramatsu et al. 11,12,13; Kawasaki,Saikawa,Segikuchi 15; AR,Saikawa `16; Klaer,Moore `17; Gorghetto,Hardy,Villadoro `18; Buschmann et al. 19; Hindmarsh 19; Gorghetto,Hardy,Villadoro ’20; Buschmann et al. 21]

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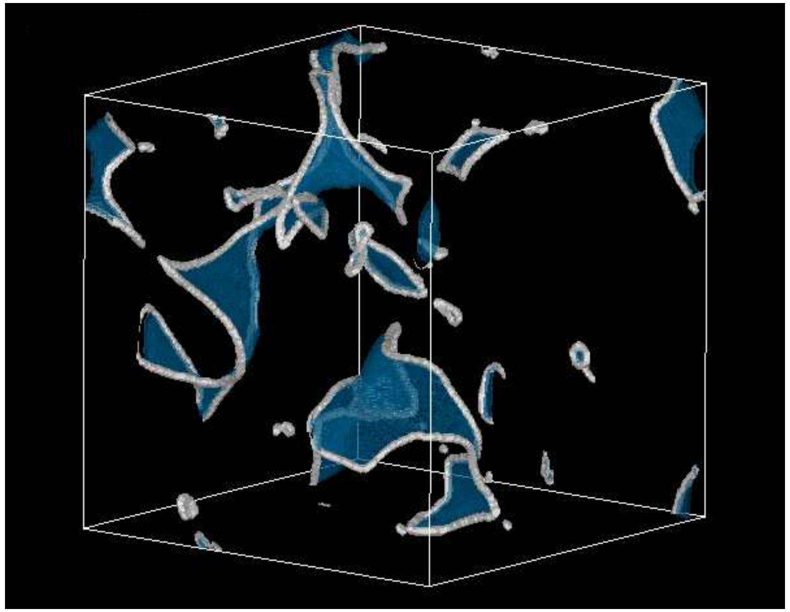

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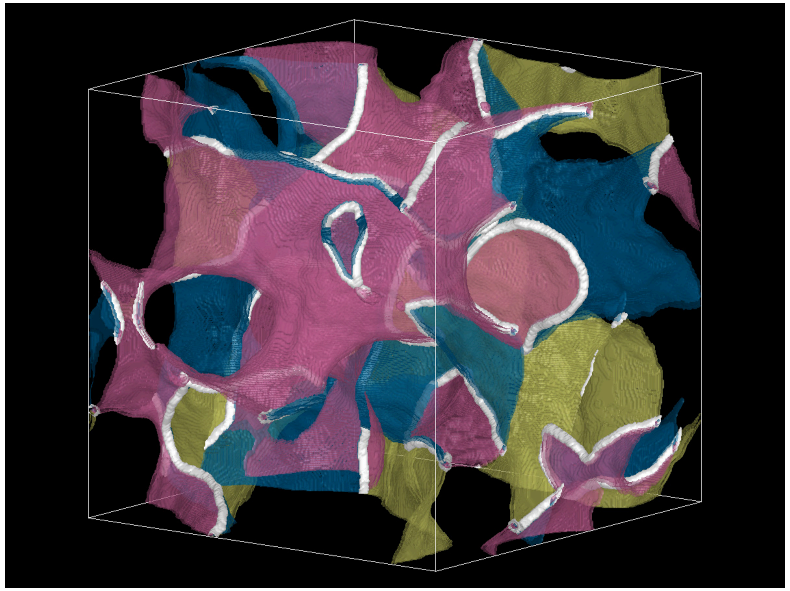

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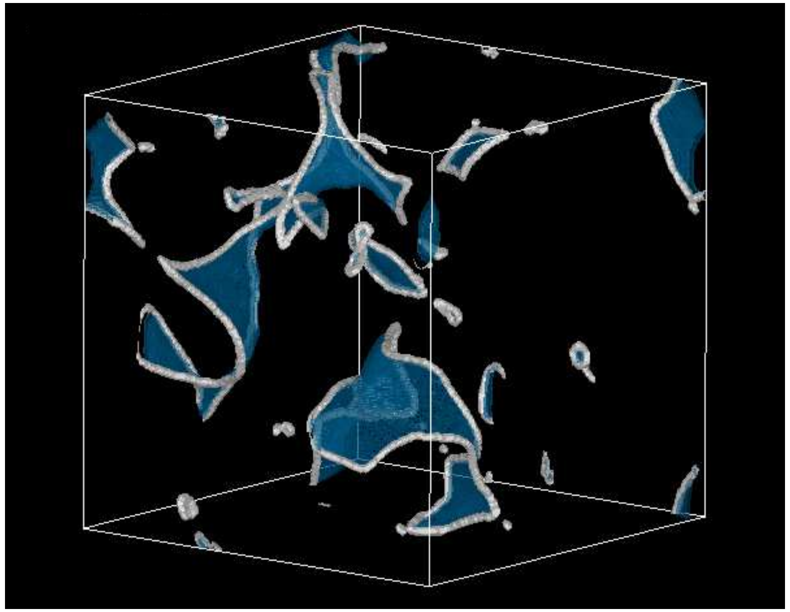
  - Required axion mass to explain 100% of CDM abundance: $N_{\rm DW} = 1: \;\; m_a \approx 26\ \mu{\rm eV} - 0.5\ {\rm meV}$
    $N_{\rm DW} > 1: \;\; m_a \approx (0.58 - 130)\,{\rm meV}$

$N_{\rm DW} = 1$



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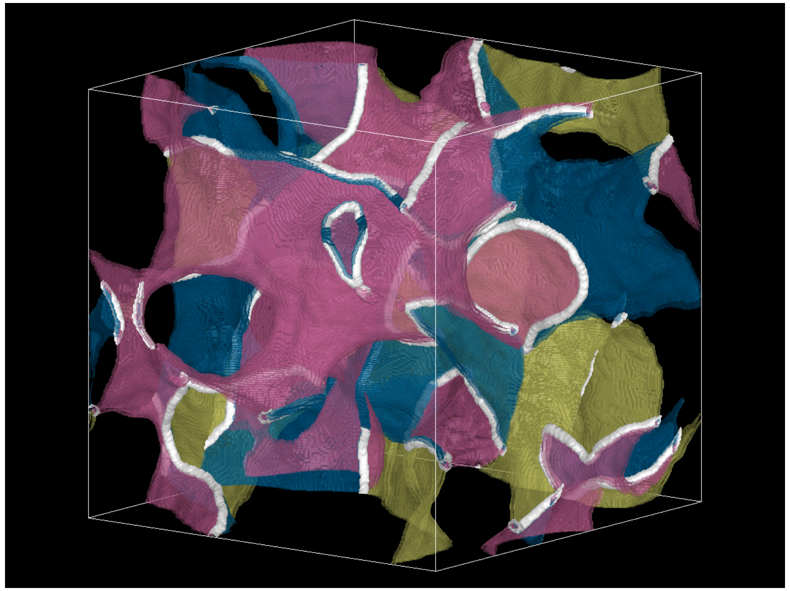
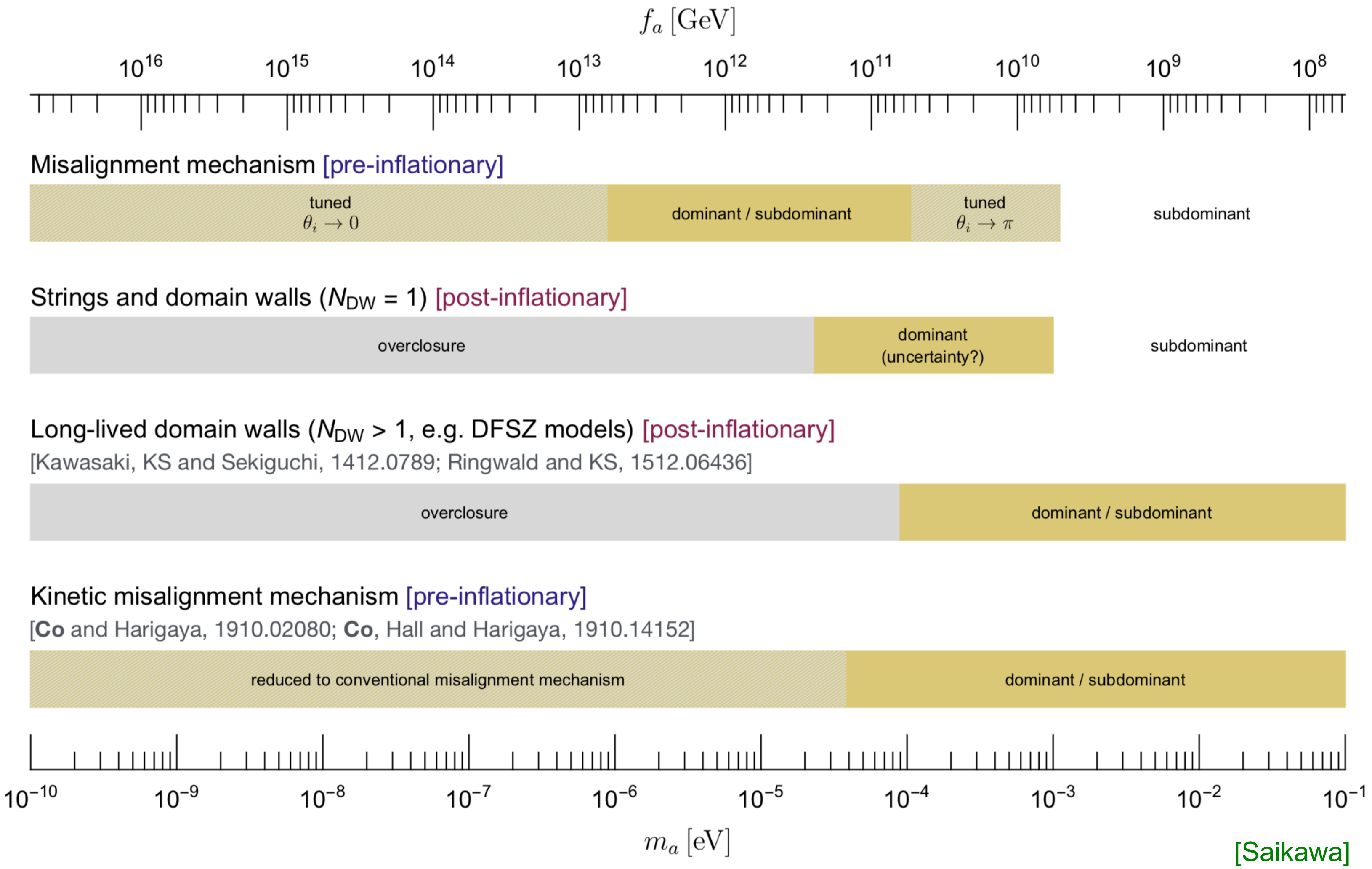


[Hiramatsu et al.]

Axion Dark Matter

Dark Matter axion mass spans a huge range



[Saikawa]

Today’s Plan

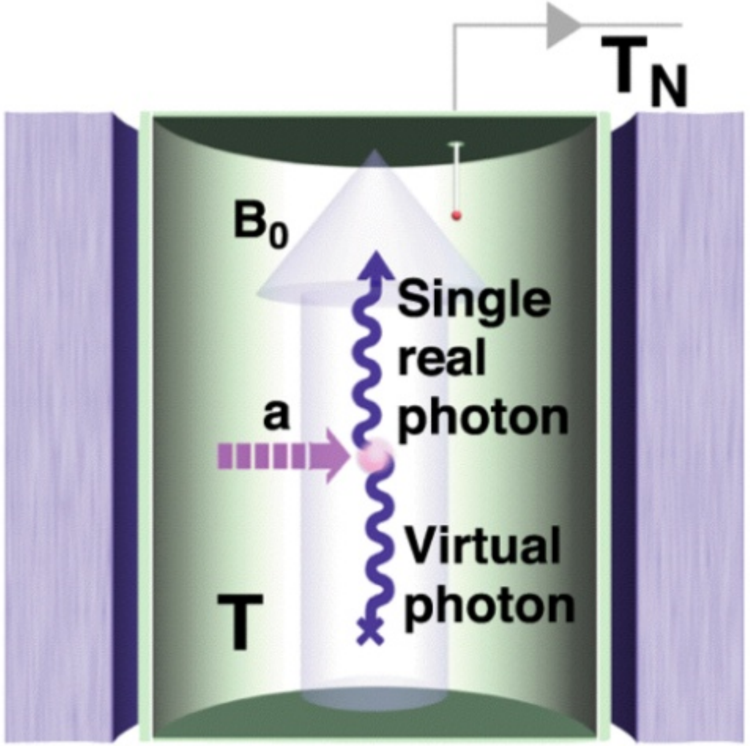
- Vacuum Structure in Quantum Chromodynamics
- Strong CP Puzzle
- The Axion
- Axion Dark Matter
- **Axion Experiments**

Axion Dark Matter Detection

Microwave Cavities

- Axion DM – photon conversion in microwave cavity placed in magnetic field [Sikivie 83]

$$\mathcal{L} \supset -\frac{g_{a\gamma}}{4}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv g_{a\gamma}\, a\, \mathbf{E}\cdot\mathbf{B}$$



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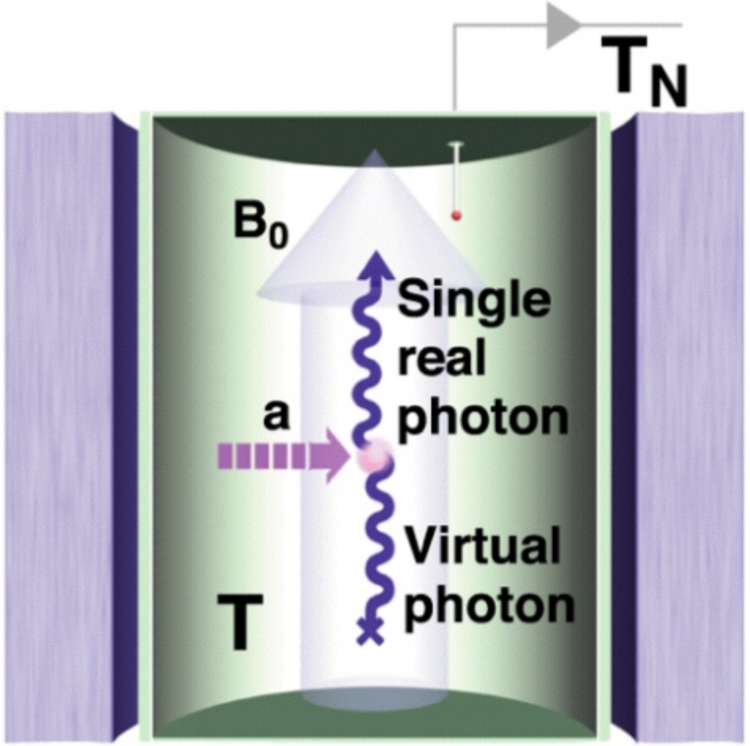

- Best sensitivity: mass = resonance frequency $m_a = 2\pi\nu \sim 4\ \mu\text{eV}\left(\frac{\nu}{\text{GHz}}\right)$

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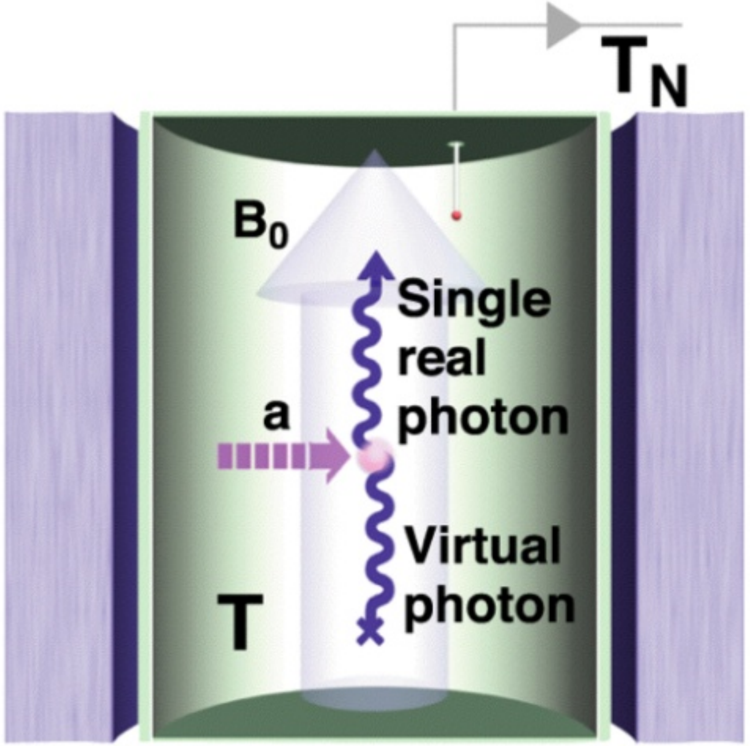
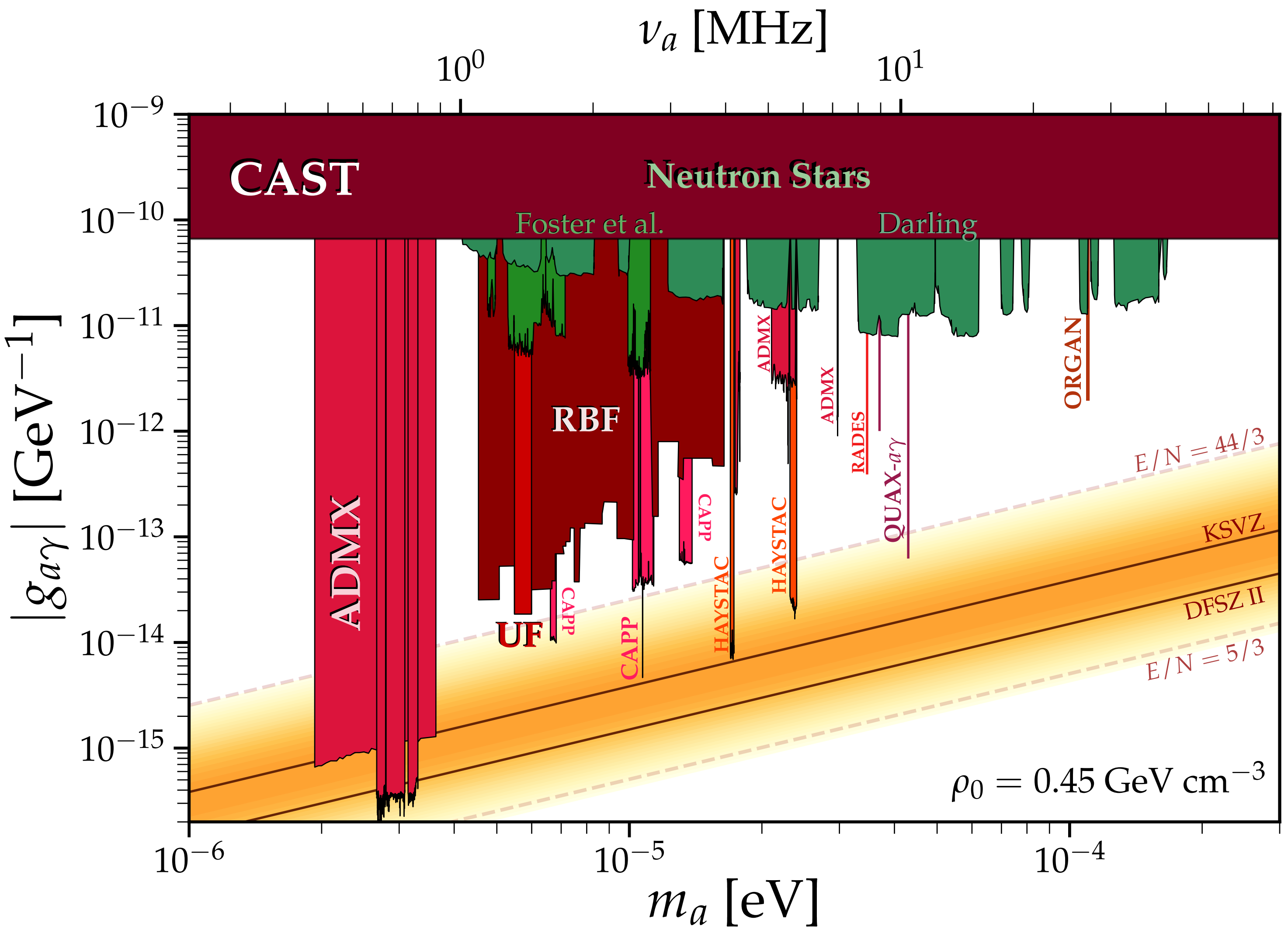


- Best sensitivity: mass = resonance frequency $m_a = 2\pi\nu \sim 4\ \mu\mathrm{eV}\left(\frac{\nu}{\mathrm{GHz}}\right)$
- Power output: $P_{\mathrm{out}} \sim g^2 \mid \mathbf{B}_0 \mid^2 \rho_{\mathrm{DM}} V Q / m_a$

Axion Dark Matter Detection

Microwave Cavities

- Currently running:
  - ADMX
  - CAPP
  - HAYSTAC
  - ORGAN
  - QUAX a gamma
  - RADES



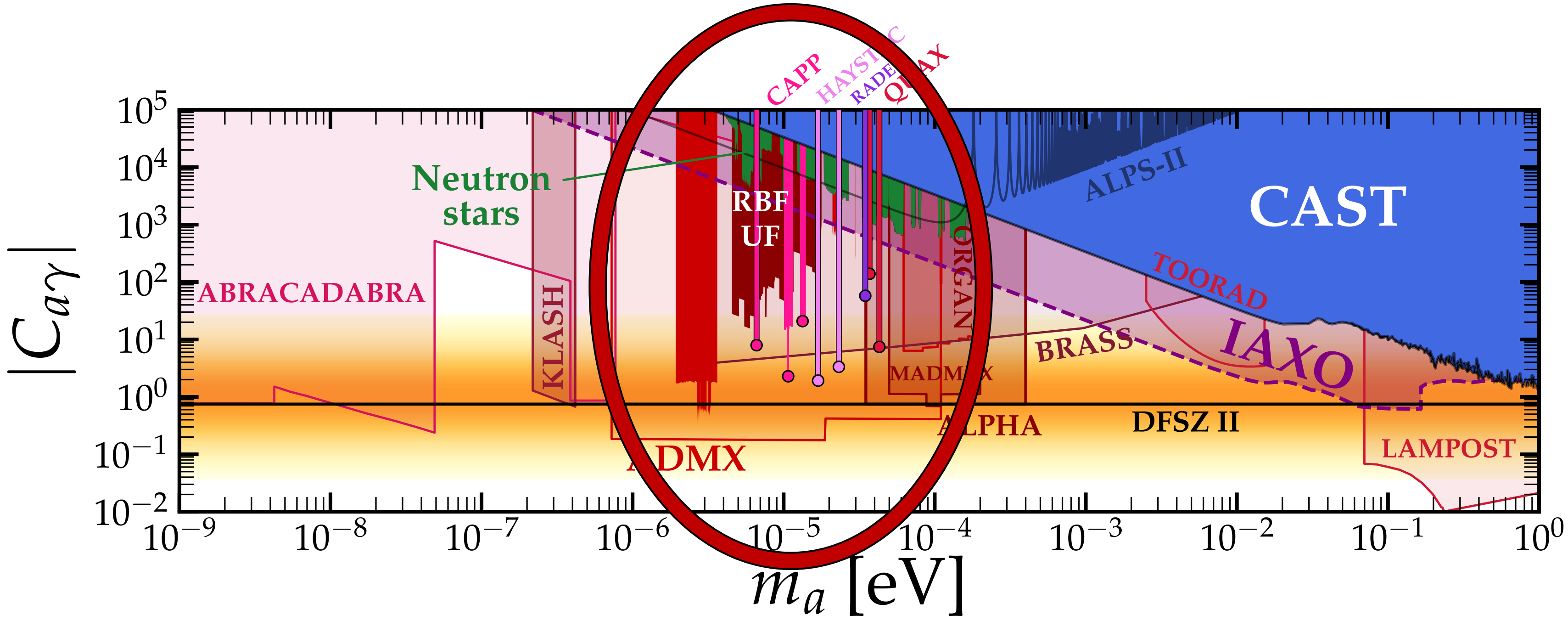
[https://github.com/cajohare/AxionLimits/blob/master/plots/AxionPhoton_RadioFreqCloseup.pdf]

Haloscope Searches

Microwave Cavities

- Currently running:
  - ADMX
  - CAPP
  - HAYSTAC
  - ORGAN
  - QUAX a gamma
  - RADES
- Axion reach deep into axion band for

$$\mu\mathrm{eV} \lesssim m_a \lesssim 100\,\mu\mathrm{eV}$$



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Axion Dark Matter Detection

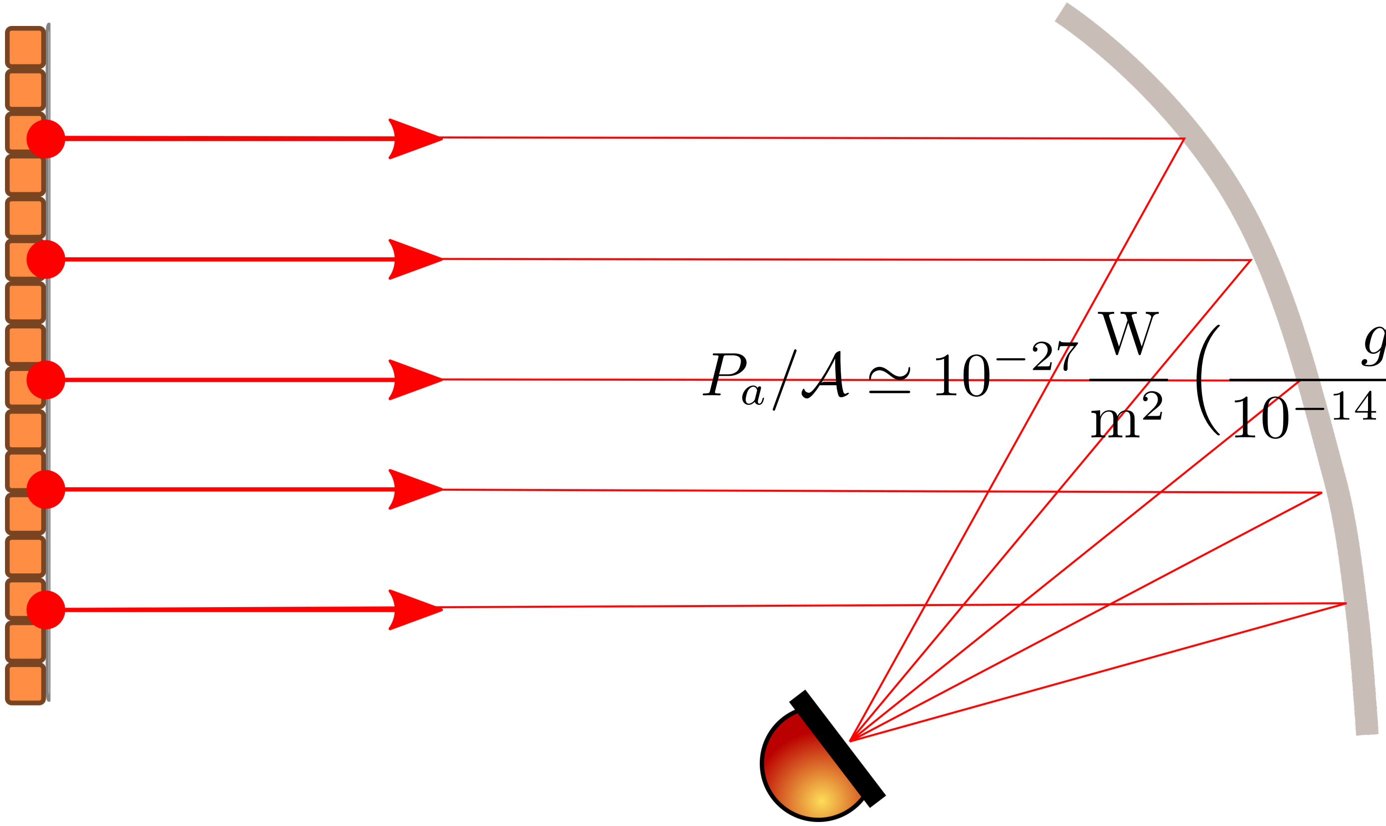
Dish Antennas

- Oscillating axion DM in a background magnetic field carries a small electric field component

Axion Dark Matter Detection

Dish Antennas

- Oscillating axion DM in a background magnetic field carries a small electric field component
- A magnetised mirror in axion/ALP DM background radiates photons [Horns,Jaeckel,Lindner,Lobanov,Redondo,AR 13]

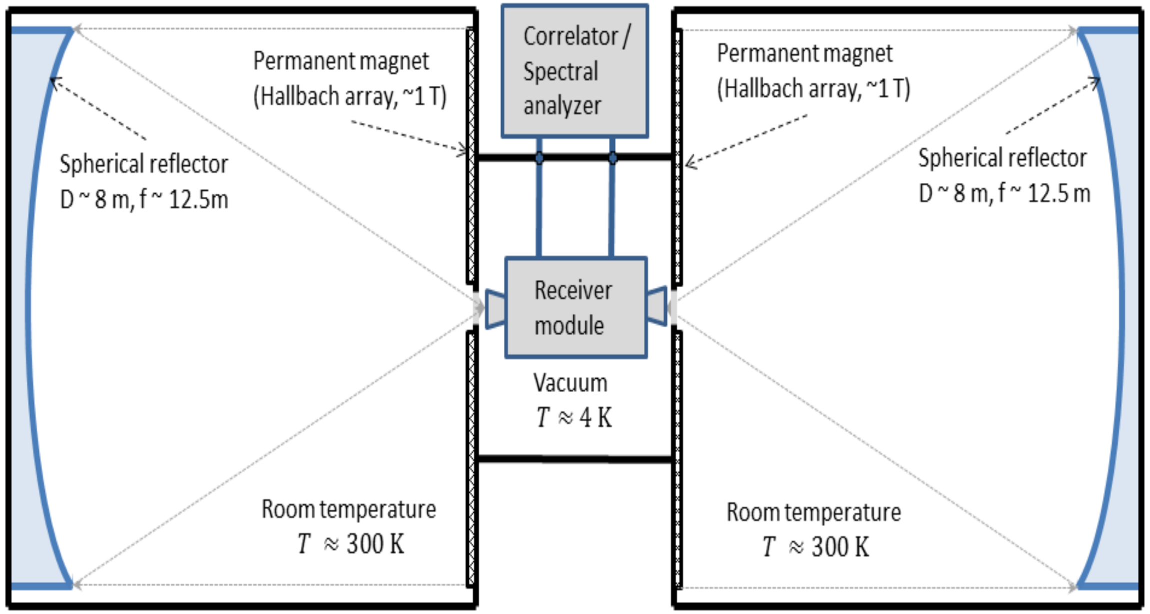


$$P_a/\mathcal{A} \simeq 10^{-27}\,\frac{\mathrm{W}}{\mathrm{m}^2}\left(\frac{g_{a\gamma}}{10^{-14}\,\mathrm{GeV}^{-1}}\right)^2\left(\frac{10^{-4}\,\mathrm{eV}}{m_a}\right)^2\left(\frac{B}{10\,\mathrm{T}}\right)^2$$

Axion Dark Matter Detection

Dish Antennas

- Oscillating axion DM in a background magnetic field carries a small electric field component
- A magnetised mirror in axion/ALP DM background radiates photons [Horns,Jaeckel,Lindner,Lobanov,Redondo,AR 13]
- Axion/ALP DM dish antenna experiment: BRASS (U Hamburg)



[Horns et al. (unpublished)]

- Permanently magnetized surface for axion/ALP photon conversion
- Dish antenna for photon signal concentration
- Broadband acquisition (16 GHz bandwidth, 10^7 channels)

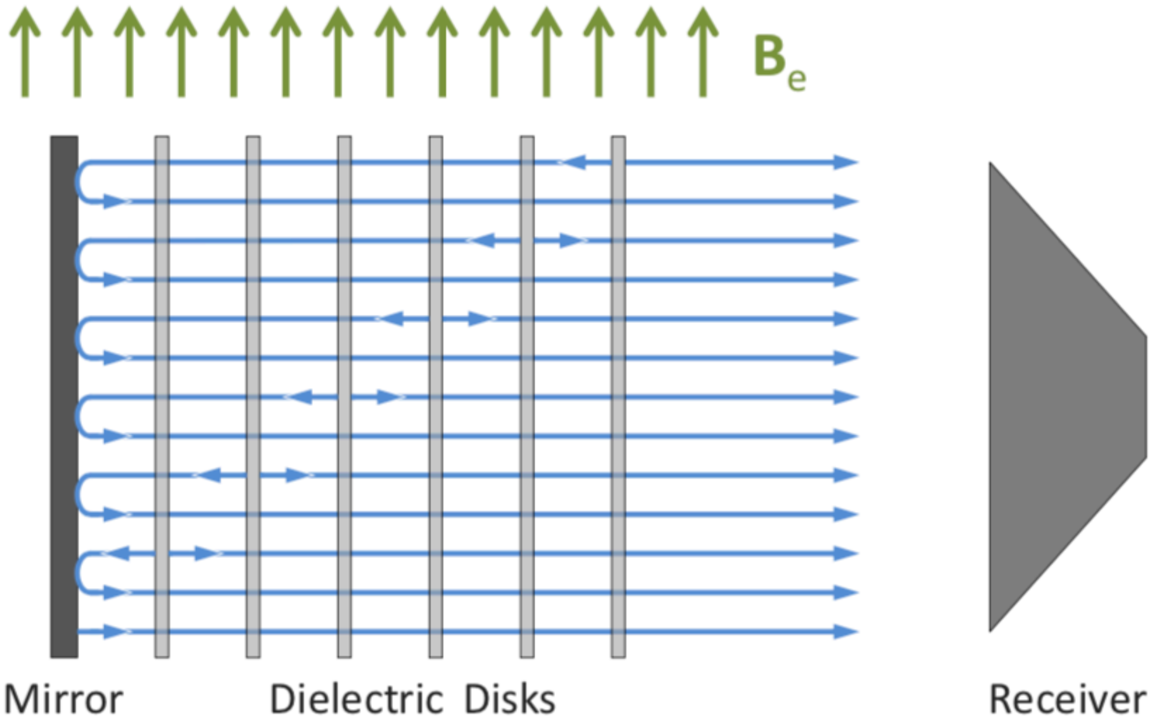
Axion Dark Matter Detection

Dish Antennas

- **Boosted dish antenna:** Open dielectric resonator
  - Add stack of dielectric disks with $\sim \lambda/2$ spacing in front of mirror (all immersed in magnetic field) [Jaeckel,Redondo 13]
  - Constructive interference of photon part of wave function

[Millar,Raffelt,Redondo,Steffen 16]

[Baryakhtar,Huang,Lasenby18]



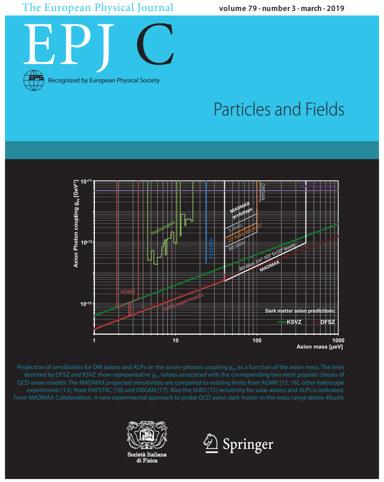
[Caldwell et al. `16]

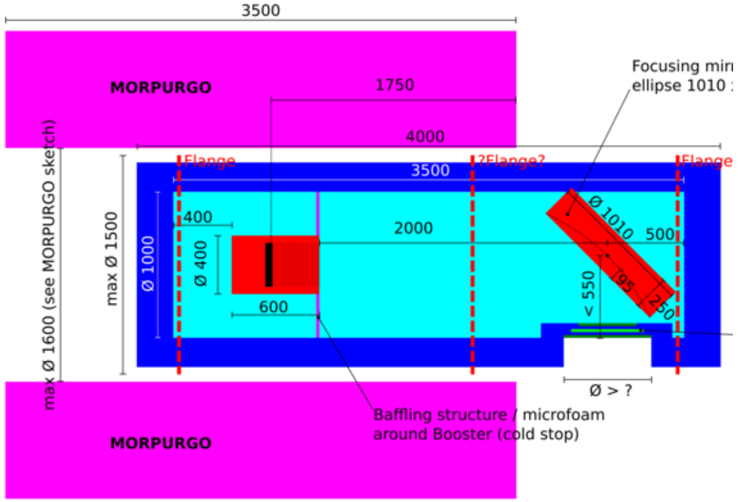
Axion Dark Matter Detection

Dish Antennas

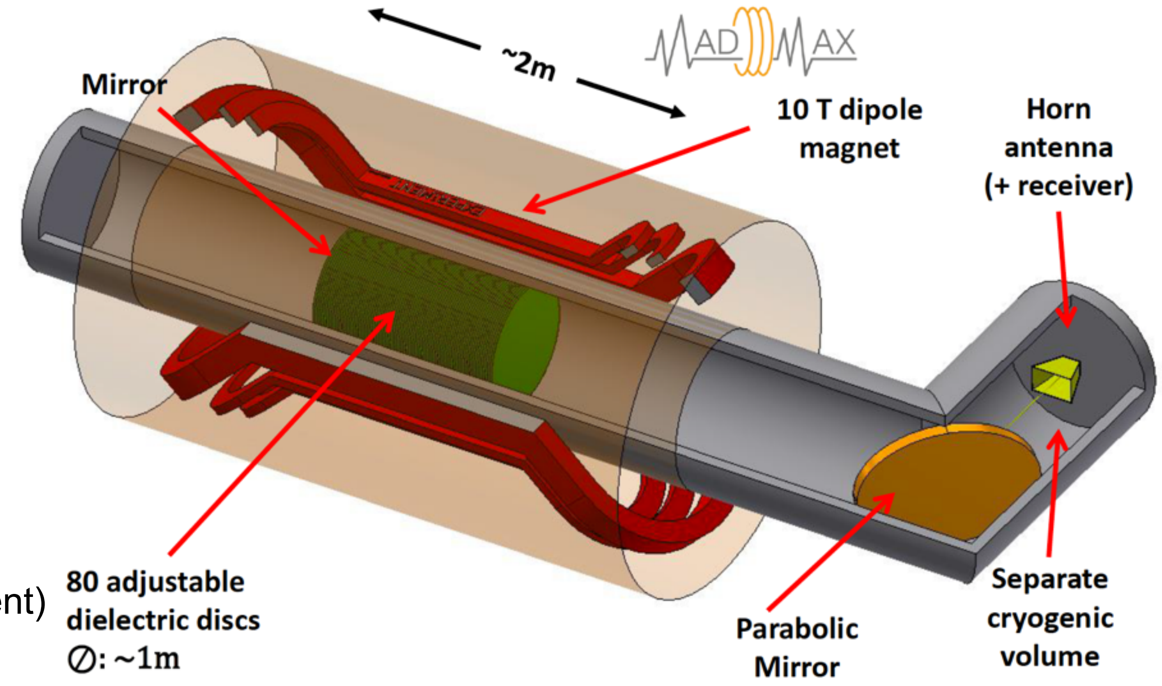
- **Boosted dish antenna:** Proposed MADMAX experiment [Caldwell et al. `16; Bruns et al. 19]

2017 -2019 Design	2020 -2028 Prototype	2028 -2038 Experiment
	@CERN	@DESY





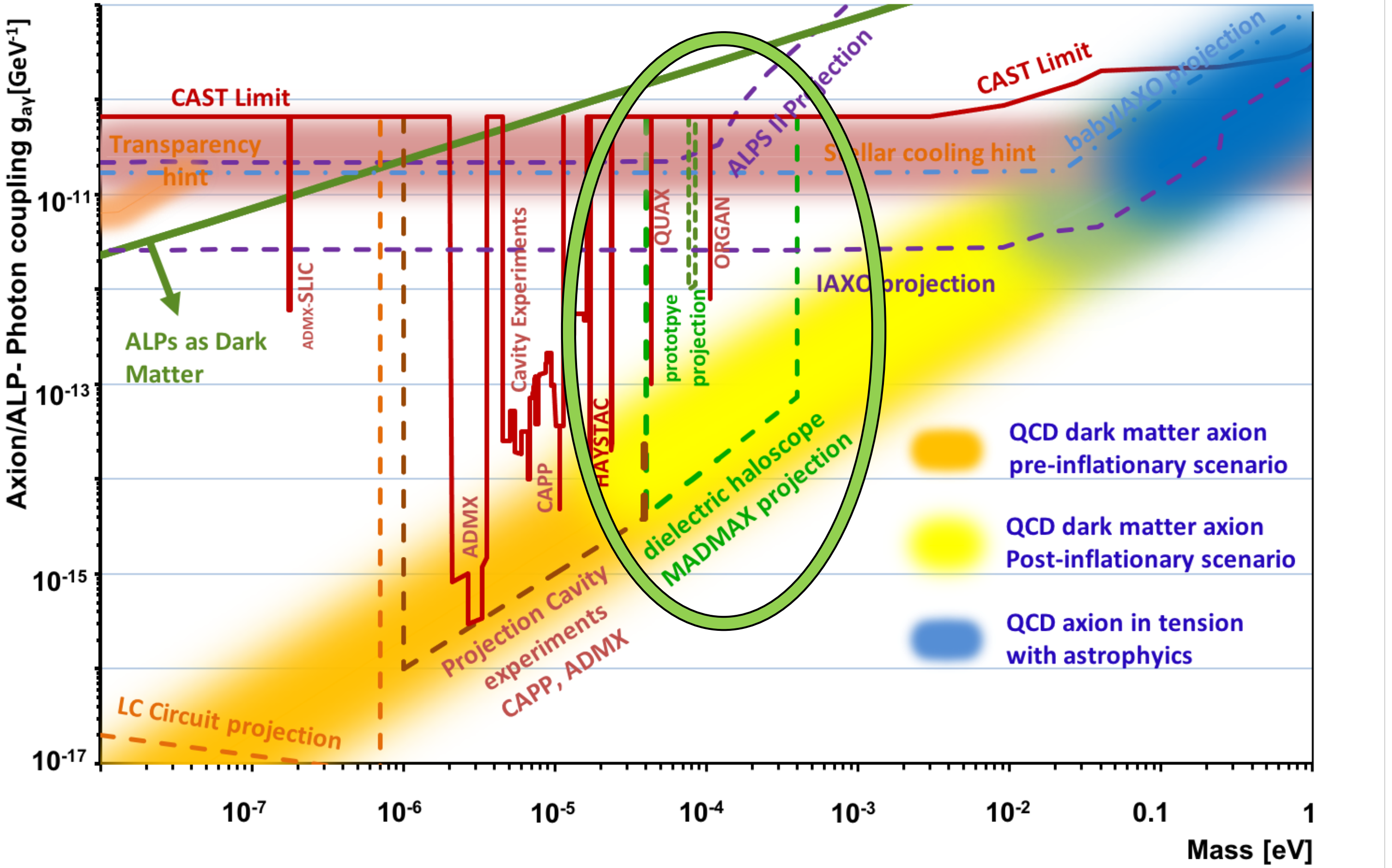
Scaling: Area 1/10 (of final experiment)
\# discs 1/4
B [T] 1/5



Axion Dark Matter Detection

Dish Antennas

- MADMAX projected to probe deep into axion band in the mass range prefered by the post-inflationary PQ symmetry breaking scenario:



[Lindner,Majorovits,AR `21]

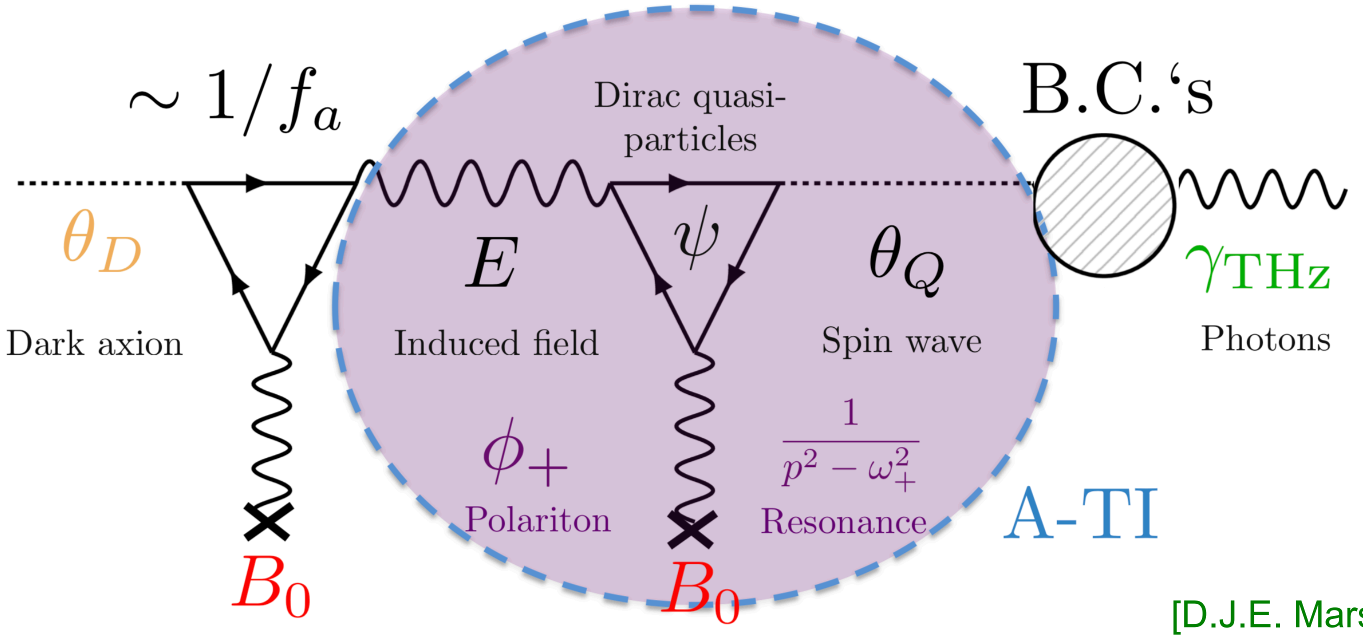
Axion Dark Matter Detection

Dish Antennas

- **Boosted dish antenna:** Replace magnetised dish by antiferromagnetic topological insulator (A-TI)

[Marsh et al. 19; Schütte-Engel et al. `21]

  - Some AF-TIs predicted to feature axionic quasiparticles (AQ) - longitudinal A spin fluctuations coupled to E.B [Li et al. `10]
  - In presence of magnetic field, the induced oscillating electric field associated with the DM axion field, mixes with the AQ, leading to a resonant conversion into photons



[D.J.E. Marsh `21]

Axion Dark Matter Detection

Dish Antennas

- **TOORAD:** Projected sensitivity reaches axion band in meV region

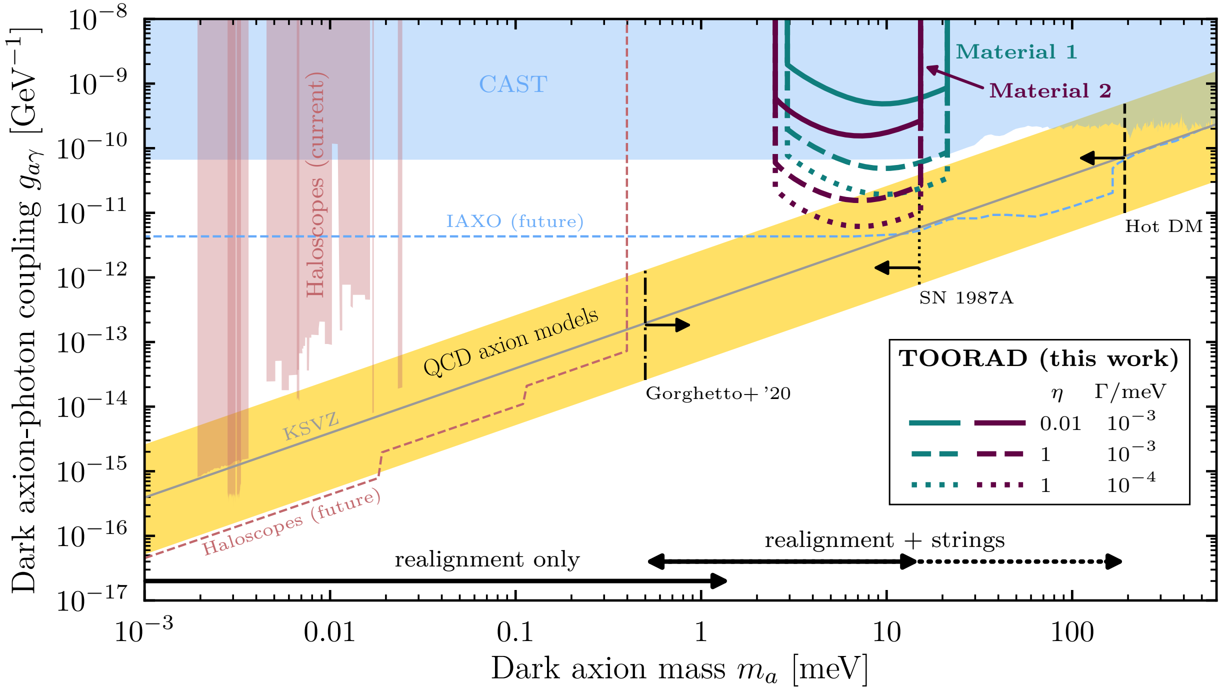


Table 8: Parameter reference values and ranges. Our benchmark material is "Material 2", based on $Mn_2Bi_2Te_5$.

Parameter name & symbol		Range	Benchmark
TMI parameters			
Decay constant	f_Θ	[50, 200] eV	70 eV
AQ mass	m_Θ	$\sim \mathcal{O}$(meV)	1.8 meV
Permittivity	ϵ	[9,49]	25
Magnetic permeability	μ	$\sim \mathcal{O}(1)$	1
Magnon losses	Γ_m	$[10^{-5}, 10^{-3}]$ meV	
Specific conductance	Γ_ρ	$[10^{-5}, 10^{-3}]$ meV	
Area of crystal face	A	$(0.2\,\mathrm{m})^2$	
Thickness	d	d_{opt}, cf. Eq. (4.51)	
Experimental parameters			
External *B*-field	B_e	[1, 10] T	2 T
Detection effciency	η	[0.01, 1]	0.01
Dark count rate	λ_d	$\gtrsim$ 1 mHz	1 mHz

[Schütte-Engel et al. `21]