Interactions with Hyperons

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Outline

- Hyperons and where to find them
- YN and YY interactions
- Theoretical approaches to YN and YY
- YN (and YY) in meson-exchange models
- YN (and YY) in $\chi$EFT
- Hypernuclei
- Bibliography
A hyperon is a baryon containing one or more strange quarks.

<table>
<thead>
<tr>
<th>Hyperon</th>
<th>Quarks</th>
<th>I(J^P)</th>
<th>Mass (MeV)</th>
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</thead>
<tbody>
<tr>
<td>Λ</td>
<td>uds</td>
<td>0(1/2^+)</td>
<td>1115</td>
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<tr>
<td>Σ⁺</td>
<td>uus</td>
<td>1(1/2^+)</td>
<td>1189</td>
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<tr>
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<tr>
<td>Σ⁻</td>
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<tr>
<td>Ξ⁻</td>
<td>dd̅s</td>
<td>1/2(1/2^+)</td>
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</tr>
<tr>
<td>Ω⁻</td>
<td>ss̅s</td>
<td>0(3/2^+)</td>
<td>1672</td>
</tr>
</tbody>
</table>

credit: I. Vidana
On Earth: Hypernuclei

A hyperon is a baryon containing one or more strange quarks.

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</tr>
<tr>
<td>Ω⁻</td>
<td>sss</td>
<td>0(5/2^-)</td>
<td>1672</td>
</tr>
</tbody>
</table>

The study of hypernucleus allows for:
- new spectroscopy
- information on strong and weak interactions between hyperons and nucleons

Laboratories:
BNL, CERN, KEK, JLab, DAPNE, GSI, FAIR

Reactions:
- $\gamma$-ray data
- $(K^-, \pi^-)$
- $(K_{stop}, \pi^-)$
- $(K_{stop}, \pi^0)$
- $(e, e'K^-)$
- $(\pi^+, K^+)$
- $(\pi^-, K^+)$

Physics aspects:
- Hypernuclear structure
- $\Lambda N$ strong force
- $\Lambda N \rightarrow NN$ weak force
Laboratories:
BNL, CERN, KEK, JLab, DAφNE, GSI, FAIR

Reactions:

- Emulsion data
- γ-ray data
- $(K^-, \pi^-)$
- $(K^-_{\text{stop}}, \pi^-)$
- $(K^-_{\text{stop}}, \pi^0)$
- $(e, e'K^+)$
- $(\pi^+, K^+)$
- $(\pi^-, K^+)$

Physics that can be addressed:
- YN and YY interactions
- YN→NN weak decay
- Hypernuclear structure

credit: Axel Perez-Obiol
In Neutron Stars

YN and YY interactions
YN and YY interactions

- Study strangeness in nuclear physics
- Provide input for hypernuclear physics and astrophysics

Scarce YN scattering data due to the short life of hyperons and the low-density beam fluxes

\( \Lambda N \) and \( \Sigma N \): < 50 data points
\( \Xi N \) very few events

\( NN: > 5000 \) data
for \( E_{\text{lab}} < 350 \) MeV

Data from hypernuclei:

- more than 40 \( \Lambda \)-hypernuclei
  \( \Lambda N \) attractive
- few \( \Lambda \Lambda \)-hypernuclei
  \( \Lambda \Lambda \) weak attraction
- few \( \Xi \)-hypernuclei
  \( \Xi N \) attractive
- evidence of 1 \( \Sigma \)-hypernuclei?
  \( \Sigma N \) repulsive

New data on femtoscopy!
Theoretical approaches to YN and YY

- **Meson exchange models (Juelich/Nijmegen models)**
  To build YN and YY from a NN meson-exchange model imposing SU(3)$_{\text{flavor}}$ symmetry
  - **Juelich**: Holzenkamp, Holinde, Speth '89; Haidenbauer and Meißner '05
  - **Nijmegen**: Maesen, Rijken, de Swart '89; Rijken, Nagels and Yamamoto '10

- **Chiral effective field theory approach (Juelich-Bonn-Munich group)**
  To build YN and YY from a chiral effective Lagrangian similarly to NN interaction
  - **Juelich-Bonn-Munich**: Polinder, Haidenbauer and Meißner '06; Haidenbauer, Petschauer, Kaiser, Meißner, Nogga and Weise '13
    Kohno '10; Kohno '18

- **Quark model potentials**
  To build YN and YY within constituent quark models
  - Fujiwara, Suzuki, Nakamoto '07

- **$V_{\text{low } k}$ approach**
  To calculate a “universal” effective low-momentum potential for YN and YY using RG techniques
  - Schaefer, Wagner, Wambach, Kuo and Brown '06

- **Lattice calculations (HALQCD/NPLQCD)**
  To solve YN and YY interactions on the lattice
  - **HALQCD**: Ishii, Aoki, Hatsuda '07; Aoki, Hatsuda and Ishii '10; Aoki et al '12
  - **NPLQCD**: Beane, Orginos and Savage ‘11; Beane et al ‘12
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  - **V\textsubscript{low \textit{k}} approach**
    Garcilazo, Fernandez-Carames and Valcarce ’07 ’10
    Schaefer, Wagner, Wambach, Kuo and Brown ’06

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YN (and YY) in meson-exchange models

Let’s start with NN! Yukawa’s idea

1930’s: NN finite-range nature well established

Yukawa (1935): to construct a force of finite range in analogy to QED

**QED**

A field of particles with zero mass (photons) is assumed and fulfills a field equation (in static approximation)

\[-\Delta A^0(\hat{r}) = e\delta(\hat{r})\]

\[A^0(\hat{r}) = \frac{e}{4\pi} \frac{1}{r} \hat{r}\]

**COULOMB POTENTIAL**

**MESON THEORY**

non-zero mass (mesons) and the solution is

\[(-\Delta + \mu^2)\phi(\hat{r}) = g\delta(\hat{r})\]

\[\phi(\hat{r}) = \frac{g}{4\pi} e^{-\frac{\mu r}{r}} \hat{r}\]

**YUKAWA POTENTIAL**

finite range!!
\[ QED \]
\[ \mathcal{L} = \bar{\psi} (i \gamma \cdot \partial - m) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - e \bar{\psi} X \psi \]

\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

\[ \rightarrow \text{INSERT INTO EULER-LAGRANGE EQ.} \]

\[ \text{Euler-Lagrange} \quad \frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \right) = 0 \]

\[ \frac{\partial}{\partial A_\mu} (-e \bar{\psi} \gamma_\mu \psi) = 0 \]

\[ \rightarrow \text{OBTAIN FIELD EQ. FOR PHOTON FIELD} \]

\[ e A_\mu = e \bar{\psi} \gamma_\mu \psi \]

\[ \rightarrow \text{V=0 [Coulomb GAUGE]} \]

\[ e A_\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) = e \bar{\psi} \gamma_\nu \psi \approx e \psi S(r) \]

\[ \rightarrow \text{STATIC} \quad -\Delta A_\nu (\vec{r}) = e \frac{\delta S(r)}{\delta A_\nu} \quad \text{LAPLACE} \]

\[ \Rightarrow \quad A_\nu (\vec{r}) = \frac{e}{4\pi} \frac{1}{r} \]

[**Meson Theory**]

\[ \mathcal{L} = \bar{\psi} (i \gamma \cdot \partial - m) \psi + \frac{1}{2} (\partial_\mu \psi)(\partial^\mu \psi - m^2 \psi^2) + g \bar{\psi} \gamma_5 \psi \]

\[ \rightarrow \text{EULER-LAGRANGE TO OBTAIN EQ. FOR MESON FIELD} \]

\[ (\partial_\mu \psi)(\partial^\mu \psi - m^2 \psi^2) = g \bar{\psi} \gamma_5 \psi \quad \text{KLEIN-GORDON EQ.} \]

\[ \rightarrow \text{STATIC} \quad (-\Delta + m^2) \psi (\vec{r}) = g \delta (\vec{r}) \]

\[ \Rightarrow \quad \psi (\vec{r}) = \frac{g}{4\pi} \frac{e \mu \gamma_5}{c} \quad \text{FINITE RANGE!} \]
The One Boson Exchange model

Idea: to consider the exchange of bosons among nucleons within quantum field theory in terms of perturbation theory using Feynman diagrams.

At lowest order

Amplitude: $F_\alpha(p', p) = \frac{\bar{u}_1' \Gamma_1 u_1\ P_\alpha\ \bar{u}_2' \Gamma_2 u_2}{q^2 - m^2_\alpha}$

with Dirac spinor $u(p, s) = \sqrt{\frac{E + M}{2M}} \left( \begin{array}{c} \chi_s \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + M} \chi_s \end{array} \right) \approx \left( \begin{array}{c} \chi_s \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + M} \chi_s \end{array} \right) \approx \left( \begin{array}{c} \chi_s \\ 0 \end{array} \right)$

where $E = \sqrt{\vec{p}^2 + M^2}$ and $\chi_s$ is a two-component Pauli spinor.
LIGHT UNFLAVORED MESONS

\( S = C = B = 0 \)

For \( l = 1 \) (\( \pi, b, \rho, a \)): \( \bar{u}d, (\bar{u}u-d\bar{d})/\sqrt{2}, d\bar{u} \);
for \( l = 0 \) (\( \eta, \eta', h, h', \omega, \phi, f, f' \)): \( c_1(\bar{u}u+d\bar{d})+c_2(\bar{s}s) \)

\[ \begin{align*}
\pi^0 & \quad J^G(J^P) = 1^- (0^-) \\
\text{Mass} & \quad m = 139.57018 \pm 0.00035 \text{ MeV} \quad (S = 1.2) \\
\text{Mean life} & \quad \tau = (2.6033 \pm 0.0005) \times 10^{-8} \text{ s} \quad (S = 1.2) \\
\end{align*} \]

\[ \begin{align*}
\pi^- & \quad J^G(J^P) = 1^- (0^-) \\
\text{Mass} & \quad m = 134.9766 \pm 0.0006 \text{ MeV} \quad (S = 1.1) \\
\text{Mean life} & \quad \tau = (8.4 \pm 0.6) \times 10^{-17} \text{ s} \quad (S = 3.0) \\
\end{align*} \]

\[ \begin{align*}
\eta & \quad J^G(J^P) = 0^+ (0^-) \\
\text{Mass} & \quad m = 547.75 \pm 0.12 \text{ MeV} \quad (S = 2.6) \\
\text{Full width} & \quad \Gamma = 1.29 \pm 0.07 \text{ keV} \quad |s| \\
\end{align*} \]

\[ \begin{align*}
\omega(782) & \quad J^G(J^P) = 0^- (1^-) \\
\text{Mass} & \quad m = 782.59 \pm 0.11 \text{ MeV} \quad (S = 1.7) \\
\text{Full width} & \quad \Gamma = 8.49 \pm 0.08 \text{ MeV} \\
\end{align*} \]

\[ \begin{align*}
\omega(782) \text{ DECAY MODES} & \quad \text{Fraction (}\Gamma/\Gamma\text{)} \\
\pi^+\pi^-\pi^0 & \quad (89.1 \pm 0.7) \% \quad S = 1.1 \\
\end{align*} \]
LIGHT UNFLAVORED MESONS
(S = C = B = 0)

For \( l = 1 \) (\( \pi, b, \rho, a \)): \( \bar{u}d, (u\bar{u} - d\bar{d})/\sqrt{2}, du \);
for \( l = 0 \) (\( \eta, \eta', h, h', \omega, \phi, f, f' \)): \( c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s}) \)

**Exchanged bosons**

- PSEUDOSCALAR
- SCALAR
- VECTOR
Lagrangian density
main constraint: Lorentz scalar and Hermitean

$$\mathcal{L}_I = g [\bar{\psi} \tilde{\Gamma} \psi] \phi$$

Vertex ($\Gamma$): “i” times the Lagrangian stripped off the fields

Potential: “i” times the amplitude
Example: One-Pion Exchange for NN

\[ \mathcal{L}_{\pi NN} = -g_{\pi NN} \bar{\psi} i \gamma_5 \vec{r} \psi \, \phi^{(\pi)} \]

Vertex:

\[ \Gamma = g_{\pi NN} \gamma_5 \vec{r} \]

\[ \bar{u}(p'_1) \Gamma_1 u(p_1) = -g_{\pi NN} \frac{\vec{\sigma}_1 \cdot \vec{q}}{2M} \tau_1 \]

\[ \bar{u}(p'_2) \Gamma_1 u(p_2) = g_{\pi NN} \frac{\vec{\sigma}_2 \cdot \vec{q}}{2M} \tau_2 \]

Potential:

\[ (P_{\pi} = i, \quad q^2 \approx -\vec{q}^2) \]

\[ V_\pi = i F_\pi = -\frac{g_{\pi NN}^2}{(2M)^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_{\pi}^2} \tau_1 \cdot \tau_2 \]
\[ V_n = i \Gamma_n = i \bar{u}(p_1, s_1) \Gamma_1 u(p_1, s_1) \cdot \bar{u}(p_2, s_2) \Gamma_2 u(p_2, s_2) \]

\[ \rightarrow \text{full vue} = -i \bar{u}(p_1, s_1) \Gamma_1 u(p_1, s_1) \]

\[ = \bar{g}_{\text{nn}} \bar{u}(p_1, s_1) \partial_5 u(p_1, s_1) \]

\[ = \bar{g}_{\text{nn}} u^+(p_1, s_1) \partial_5 \partial_5 u(p_1, s_1) \]

\[ = \bar{g}_{\text{nn}} \left( \chi_{s_1}^+ \frac{\gamma_4 p_1^i}{2\eta} \chi_{s_1}^+ \right) \partial_5 \partial_5 \left( \frac{\chi_{s_1}}{\sqrt{2m}} \chi_{s_1} \right) \]

\[ \text{full vue} = \bar{u}(p_1, s_1) \Gamma_2 u(p_1, s_1) \]

\[ = \bar{g}_{\text{nn}} \bar{u}(p_1, s_1) \partial_5 \left( \frac{p_1^2 - p_2^2}{2\eta} \right) \partial_5 \]

\[ \text{Potential} \]

\[ = \frac{\bar{g}_{\text{nn}}}{2\eta} \left( \frac{\chi_{s_1}}{\sqrt{2m}} \chi_{s_1} \right) \]

\[ \text{statuc} \]

\[ \text{Potential} \]

\[ = -\frac{\bar{g}_{\text{nn}}}{2\eta} \left( \frac{\chi_{s_1}}{\sqrt{2m}} \chi_{s_1} \right) \]
Using the operator identity

\[ (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) = \frac{\vec{q}^2}{3} [\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\hat{q})] \]

\[ S_{12}(\hat{q}) \equiv 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \]  
(tensor operator)

the one-pion exchange potential (OPEP) can be written

\[ V_\pi = \frac{g^2_{\pi NN}}{3(2M)^2} \frac{\vec{q}^2}{\vec{q}^2 + m^2_\pi} \left[ -\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}(\hat{q}) \right] \vec{\tau}_1 \cdot \vec{\tau}_2 \]

Also OPEP from pseudo-vector or gradient coupling to the nucleon (suggested by chiral symmetry)

\[ \mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \psi \cdot \partial^\mu \phi^{(\pi)} \]

\[ \Gamma_{\pi NN} = (i)^2 \frac{f_{\pi NN}}{m_\pi} \gamma^\mu \gamma_5 \vec{\tau} q_\mu \]
(incoming pion)
Other meson exchanges:

\[ \mathcal{L}_{\sigma NN} = -g_{\sigma NN} \bar{\psi} \psi \phi^{(\sigma)} \]

\[ \mathcal{L}_{\omega NN}^{(\text{vector})} = -g_{\omega} \bar{\psi} \gamma^\mu \psi \phi^{(\omega)}_\mu \]

\[ \mathcal{L}_{\rho NN}^{(\text{tensor})} = -\frac{f_\rho}{4M} \bar{\psi} \sigma^{\mu\nu} \vec{\tau} \psi \cdot (\partial_\mu \phi_\nu^{(\rho)} - \partial_\nu \phi^{(\rho)}_\mu) \]
### Summary

**\( \pi(138) \)**

\[
V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2} \frac{q^2}{q^2 + m_\pi^2} \left[ -\bar{\sigma}_1 \cdot \bar{\sigma}_2 - S_{12}(\hat{q}) \right] \vec{r}_1 \cdot \vec{r}_2
\]

Long-ranged tensor force

**\( \sigma(600) \)**

\[
V_\sigma \approx \frac{g_\sigma^2}{q^2 + m_\sigma^2} \left[ -1 - \frac{\vec{L} \cdot \vec{S}}{2M^2} \right]
\]

Intermediate-ranged, attractive central force plus LS force

**\( \omega(782) \)**

\[
V_\omega \approx \frac{g_\omega^2}{q^2 + m_\omega^2} \left[ +1 + \frac{3 \vec{L} \cdot \vec{S}}{2M^2} \right]
\]

Short-ranged, repulsive central force plus strong LS force

**\( \rho(770) \)**

\[
V_\rho = \frac{f_\rho^2}{12M^2} \frac{q^2}{q^2 + m_\rho^2} \left[ -2\bar{\sigma}_1 \cdot \bar{\sigma}_2 + S_{12}(\hat{q}) \right] \vec{r}_1 \cdot \vec{r}_2
\]

Short-ranged tensor force, opposite to pion
We can describe NN!!

\[ V(r) = V_c(r) + V_T(r) S_{12} + V_{LS}(r) \hat{L}\hat{S} \]

- Short
- Intermediate
- Long range

\[ V_c(r) \]

\[ V_T(r) \]

\[ V_{LS}(r) \]

\[ S_{12} \]

\[ \hat{L}\hat{S} \]

\[ \rho \]

\[ \omega \]

\[ \sigma \]

\[ \pi \]
One Boson Exchange Potential

\[ V_{\text{OBEP}} = \sum_{\alpha=\pi, \sigma, \rho, \omega, \eta, a_0, \ldots} V_\alpha \]

\( \eta(548) \) is a pseudo-scalar meson with \( I = 0 \), therefore, \( V_\eta \) is given by the same expression as \( V_\pi \), except that \( V_\eta \) carries no \( (\vec{\tau}_1 \cdot \vec{\tau}_2) \) factor.

\( a_0(980) \) is a scalar meson with \( I = 1 \), therefore, \( V_{a_0} \) is given by the same expression as \( V_\sigma \), except that \( V_{a_0} \) carries a \( (\vec{\tau}_1 \cdot \vec{\tau}_2) \) factor.

Note: Include FORM FACTORS to implement the substructure of hadrons
Theory (OBEP) vs Experiment

Lippman-Schwinger Equation:

\[ S_{if} = \delta_{if} - i(2\pi)^4 \delta^4(P_i - P_f)T_{if} \]

S-matrix (collision operator)

final state after collision \[ |f \rangle = S |i \rangle \]

initial state

scattering amplitude
Deuteron and Low-Energy Scattering Parameters as Predicted by the Relativistic OBEP Defined in Table 4.1 (Theory) and from Experiment (Experiment)

<table>
<thead>
<tr>
<th></th>
<th>Theory</th>
<th>Experiments(^a)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deuteron</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binding energy-(e_d) (MeV)</td>
<td>2.2246</td>
<td>2.224575 (9)</td>
<td>LA 82</td>
</tr>
<tr>
<td>(D)-state probability (P_D) (%)</td>
<td>4.99</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Quadrupole moment (Q_d) (fm(^2))</td>
<td>0.278(^b)</td>
<td>0.2860 (15)</td>
<td>RV 75, BC 79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2859 (3)</td>
<td>ER 83, BC 79</td>
</tr>
<tr>
<td>Magnetic moment (\mu_d) ((\mu_N))</td>
<td>0.8514(^b)</td>
<td>0.857406 (1)</td>
<td>Lin 65</td>
</tr>
<tr>
<td>Asymptotic S-state (A_S) (fm(^{-1/2}))</td>
<td>0.8860</td>
<td>0.8846 (8)</td>
<td>ER 83</td>
</tr>
<tr>
<td>Asymptotic (D/S)-state (D/S)</td>
<td>0.0264</td>
<td>0.0271 (8)</td>
<td>GKT 82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0272 (4)</td>
<td>Bor+ 82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0256 (4)</td>
<td>RK 86</td>
</tr>
<tr>
<td>Root-mean-square radius (r_d) (fm)</td>
<td>1.9688</td>
<td>1.9635 (45)</td>
<td>Bér+ 73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.9560 (68)</td>
<td>SSW 81, KMS 84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.953 (3)</td>
<td>Kla+ 86</td>
</tr>
<tr>
<td><strong>Neutron-proton low-energy scattering</strong> (scattering length (a), effective range (r))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^1)S(<em>0): (a</em>{np}) (fm)</td>
<td>-23.75</td>
<td>-23.748 (10)</td>
<td>Dum+ 83</td>
</tr>
<tr>
<td>(r_{np}) (fm)</td>
<td>2.71</td>
<td>2.75 (5)</td>
<td>Dum+ 83</td>
</tr>
<tr>
<td>(^3)S(_1): (a) (fm)</td>
<td>5.424</td>
<td>5.419 (7)</td>
<td>Hou 71, Dil 75, KMS 84</td>
</tr>
<tr>
<td>(r_t = \rho(0,0)) (fm)</td>
<td>1.761</td>
<td>1.754 (8)</td>
<td>Hou 71, Dil 75, KMS 84</td>
</tr>
</tbody>
</table>

\(^a\) The figures in parentheses after the values give the one-standard-deviation uncertainties in the last digits.

\(^b\) The meson exchange current contributions to the moments are not included in the theoretical values.
### YN (and YY) meson-exchange models

Built from a NN meson-exchange model imposing SU(3)\text{\textsubscript{flavor}} symmetry

<table>
<thead>
<tr>
<th>NIJMEGEN</th>
<th>JUELICH</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Nagels, Rijken, de Swart, Timmermans, Maessen..)</td>
<td>(Holzenkamp, Reube, Holinde, Speth, Haidenbauer, Meissner, Melnitchouck..)</td>
</tr>
<tr>
<td>✓ Based on Nijmegen NN potential</td>
<td>✓ Based on Bonn NN potential</td>
</tr>
<tr>
<td>✓ Momentum and Configuration Space</td>
<td>✓ Momentum Space, Full Energy Dependence &amp; Non-localities</td>
</tr>
<tr>
<td>✓ Exchange of pseudoscalar, vector and scalar nonets</td>
<td>✓ Exchange of single mesons and higher order processes</td>
</tr>
<tr>
<td>✓ SU(3) symmetry to relate YN to NN vertices</td>
<td>✓ SU(6) symmetry to relate YN to NN vertices</td>
</tr>
<tr>
<td>✓ Gaussian form factors</td>
<td>✓ Dipolar form factors</td>
</tr>
</tbody>
</table>
ΛN and ΣN scattering

\[ T = V + V \frac{1}{E_0 - H_0 + i\eta} T \]

New results from femtoscopy for Σ⁰p

\[ C(k^*) = \mathcal{N} \times \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)} \]

\[ k^* = \frac{1}{2} \times |\mathbf{p}_1^* - \mathbf{p}_2^*| \]

Acharya et al. '19
**QCD**: the interaction at short distances between quarks is weak (asymptotic freedom). However, the interaction is strong at long distances giving rise to confinement in colorless objects.

**Effective Theories** are developed to treat the non-perturbative regime of QCD. The main premise is that the dynamics of low energies does not depend on the details of the dynamics at high energies.

Effective Theory is systematic approach to a certain dynamics (known or not) that governs a physical process in a certain regime of energies. It is not a model, since its systematic character means that, in principle, one can make predictions with arbitrary precision.

However, in order to be possible, some small parameter must govern the systematic approach (expansion). In most physical processes, that parameter it is constructed through the quotient of two of the physical scales present in that process, that are clearly separated.
To describe the interaction between hadrons at low energy, we only need mesons and baryons, as relevant degrees of freedom.

The physics that appears in the fundamental Lagrangian density (QCD) is mimicked in the effective Lagrangian density through a set of operators and associated constants.

If we could solve QCD exactly, we could find the value of these constants comparing the effective theory with the complete theory. But since finding the exact solution is not possible, we use experimental data to determine these constants.

The power of effective theory lies in the fact that:
→ it contains the symmetries of the fundamental theory
→ it provides a power counting that allows us to make consistent calculations order by order
→ it allows us, a priori, to estimate the corrections introduced at each order
→ it is systematic, so independent of models

ALGORITHM
- identify the “soft” and “hard” scales and the appropriate degrees of freedom
- Identify the relevant symmetries of low-energy QCD and investigate whether they are broken or not
- Write the most general possible Lagrangian that contains all the terms consistent with the symmetries of our problem (and with the symmetry breaking)
- Design an organizational scheme to distinguish the more relevant contributions from the non-relevant: the low-momentum expansion

credit: A. Parreno
QCD Lagrangian and Chiral Symmetry

\[ \mathcal{L}_{\text{QCD}} = \sum_{f = u, d, s} \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} F_{\mu \nu}^a F_a^{\mu \nu} \]

\[ D_{\mu} \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} = \partial_{\mu} \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} + ig \sum_{a=1}^{8} \frac{\lambda_a}{2} A_{\mu}^{a} \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} \]

\[ F^{\mu \nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} - g[A^{\mu}, A^{\nu}] \]

assuming \( m_u, m_d \approx 0 \),

\[ \mathcal{L}_{\text{QCD}}^0 = \sum_{l = u, d} \bar{q}_l \not{D} q_l - \frac{1}{4} F_{\mu \nu}^a F_a^{\mu \nu} \]

**Chirality** is the generalization of helicity for particles without mass or highly energetic (ultrarelativistic)

Introducing

\[ P_L = \frac{1 - \gamma_5}{2} \quad \text{"Left - handed"} \]

\[ P_R = \frac{1 + \gamma_5}{2} \quad \text{"Right - handed"} \]

Introducing with

\[ P_R + P_L = 1 \]

\[ P_R^2 = P_R, P_L^2 = P_L, P_R P_L = P_L P_R = 0 \]

Completeness

Idempotent

Orthogonal

Eigenstates of chirality exist for massless particles

\[ q_L = \frac{1 - \gamma_5}{2} q, \quad q_R = \frac{1 + \gamma_5}{2} q \]

\[ (P_L q_L = q_L, P_R q_R = q_R) \]
We can rewrite the (massless) QCD lagrangian

\[ \mathcal{L}_{QCD} = \sum_{l=u,d} \left( \bar{q}_{R,l} i D_{\mu} q_{R,l} + \bar{q}_{L,l} i D_{\mu} q_{L,l} \right) - \frac{1}{4} F_{\mu\nu}^a F_{\alpha\beta}^a \]

**Chiral symmetry:**

the Lagrangian is invariant under a global phase transformation

(interaction between quarks and gluons is independent of flavour and retains helicity)

According to **Noether’s Theorem**

*(to every continuous symmetry of a physical system belongs to a conserved quantity and viceversa)*

there are 3 left-handed and 3 right-handed conserved currents

or 3 vector currents and 3 axial-vectors currents

**SU(2)_L x SU(2)_R symmetry**

(extensible to SU(3)_L x SU(3)_R)

Su(2)_V x Su(2)_A

credit: A. Parreno
The QCD Lagrangian is invariant under SU(2)_V x SU(2)_A for massless quarks, thus, chiral invariant. However, if the mass of quarks is not negligible, then the mass term breaks chiral symmetry explicitly

\[ - \sum_{l=u,d} \bar{q}_l m_l q_l = -\bar{q} M q = - (\bar{q}_R M q_L + \bar{q}_L M q_R) \quad \text{with} \quad M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \]

Moreover, the axial-vector symmetry is spontaneously broken (a spontaneously broken symmetry is a symmetry of the Hamiltonian that is not realized in the ground state)

*Experimental indication:* the hadronic spectrum (fundamental state of QCD) does not contain parity doublets (axial-vector current related to the parity of the state)

Consequence of the spontaneously broken axial-vector symmetry: existence of (pseudo-)Goldstone bosons

*Goldstone theorem:* when a continuous symmetry is spontaneously broken, new scalar particles without mass (or very light, if the symmetry is not exact) appear within the spectrum of possible excitations

\[ \pi^+, \pi^0, \pi^- \]

credit: A. Parreno
Effective Theory: Chiral Perturbation Theory

ALGORITHM

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- “soft” and “hard” scales \( (Q, \Lambda_X) \) and degrees of freedom (pions, nucleons..)

\[
\frac{Q}{\Lambda_X} = \frac{m_\pi}{m_\rho}
\]

- identify relevant symmetry: \textit{chiral symmetry}

- write the most general Lagrangian compatible with the chiral symmetry of QCD

\[
\mathcal{L}_{\text{eff}} = \sum_{i=1}^{\infty} \mathcal{L}_{\pi\pi}^{(2i)} + \sum_{i=1}^{\infty} \mathcal{L}_{\pi N}^{(i)} + \ldots
\]

- while designing an organizational scheme

Chiral Effective Theory for BB Interaction

Baryon-Baryon interaction in SU(3) $\chi$EFT a la Weinberg (1990);

- **power counting** allowing for a systematic improvement by going to higher order
- derivation of **two- and three-baryon forces** in a consistent way

Degrees of freedom: **octet of baryons** ($N, \Lambda, \Sigma, \Xi$) & **pseudoscalar mesons** ($\pi, K, \eta$)

Diagrams: **pseudoscalar-meson exchanges and contact terms**

credit: Haidenbauer

\[ \nu = 2 - B + 2L + \sum v_i \Delta_i, \]
\[ \Delta_i = d_i + \frac{1}{2} b_i - 2, \]

B: number of incoming (outgoing) baryons
L: number of Goldstone boson loops
$\nu$: number of vertices
$d_i$: derivatives
$b_i$: number of internal baryons at vertex

with $a$ and $b$ the Dirac indices of the particles
and $\Gamma_i$ the five elements of Clifford algebra ($1$, $\gamma^\mu$, $\sigma^{\mu\nu}$, $\gamma^\mu\gamma^5$ and $\gamma^5$)
The LO contact potential is

$$V_{L0}^{BB} = C_S^{BB} + C_T^{BB} \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2$$

where the coupling constants for the central and spin-spin parts are linear combinations of the six independent low-energy coefficients

Polinder, Haidenbauer and Meissner, Nucl. Phys. A779 (2006) 244-266
LO: One-Pseudoscalar Meson Exchange

The one-pseudoscalar meson exchange is given by

$$\mathcal{L} = \left\langle i \bar{B} \gamma^\mu D_\mu B - M_0 \bar{B}B + \frac{D}{2} \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} + \frac{F}{2} \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \right\rangle$$

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B],$$

$$\Gamma_\mu = \frac{1}{2} \left[ u^\dagger \partial_\mu u + u \partial_\mu u^\dagger \right],$$

$$P = \left( \begin{array}{cccc} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^-}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\ \frac{\pi^-}{\sqrt{2}} & \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ -K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{array} \right)$$

$$\frac{1}{2} u_\mu = \frac{i}{2} \left( u^\dagger \partial_\mu u - u \partial_\mu u^\dagger \right) = \frac{i}{2} \left\{ u^\dagger, \partial_\mu u \right\} = \frac{i}{2} u^\dagger \partial_\mu U u^\dagger$$

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Acharya et al. ‘19
$\Xi N$ scattering

Cross sections are small

J. Haidenbauer and

New results from femtoscopy

Scarce experimental information.
Hypernuclei

$\Lambda$ hypernuclei

- **Strangeness exchange**: $n(K^-, \pi^-)\Lambda, p(K^-, \pi^+)\Sigma^+$
  - CERN, BNL, KEK
  - FINUDA@DAPHNE

- **Associated production**: $n(\pi^+, K^+)\Lambda$
  - BNL, KEK

- **Electroproduction**: $p(\gamma, K^+)\Lambda, p(e, e'K^+)\Lambda$
  - Jlab, MAMI-C

![Diagram]

Double $\Lambda$ hypernuclei

- **PANDA@FAIR**
  - Hyperon-antihyperon production at threshold; rescattering
  - $K^- + ^{12}C \rightarrow \pi^- + ^{\Lambda}C$

- **Also $\Xi$ hypernuclei** @ BNL, KEK
  - $^{12}C(K^-, K^+)^{12}\bar{B}$, $K^- + p \rightarrow \Xi^- + K^+$

credit: A. Sanchez-Lorente

credit: A. Parreno
Physics that can be addressed:
- YN and YY interactions
- YN→NN weak decay
- Hypernuclear structure
Binding energy of different hypernuclei as function of the mass number

Binding energy saturates at about ~30 MeV for large nuclei

Single-particle model reproduces the data quite well  Gal et al ‘16

Conflicting measurements by STAR and ALICE of the hypertriton lifetime triggered the revived experimental and theoretical interest

Expected $\tau(^3\Lambda H) = \tau(\Lambda)$

$\iff$ observed: $\tau(^3\Lambda H) < \tau(\Lambda)$?
Bibliography

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A. Parreno, Lecture on “Introduccion a las teorias efectivas. Lagrangianos quirales”, Master’s Degree in Nuclear Physics, https://master.us.es/fisicanuclear/


Other references mentioned in the lecture!