Nucleon partonic structure: concepts and measurements

Part 1: renormalisation

M. Diehl

Deutsches Elektronen-Synchroton DESY

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Plan of lectures

0. Brief introduction
1. Renormalisation, running coupling, running masses
   scale dependence of observables
2. $e^+e^- \to$ hadrons
   some basics of applied perturbation theory
3. Factorisation and parton densities
   using perturbation theory in ep and pp collisions
4. PDFs (parton distribution functions)
   Event generators
5. Exclusive processes and GPDs (generalised parton distributions)
6. TMDs (transverse momentum dependent distributions)
7. Double parton scattering and double parton distributions

Some references (only a small selection)

- more on short-distance factorisation
  J Collins, hep-ph/9907513 and hep-ph/0107252
  J Collins, Foundations of Perturbative QCD, CUP 2011
- short overview of GPDs and TMDs
  MD, arXiv:1512.01328
- full bibliography for GPDs e.g. in reviews
  S Boffi and B Pasquini, arXiv:0711.2625
  A Belitsky and A Radyushkin, hep-ph/0504030
  MD, hep-ph/0307382; K Goeke et al., hep-ph/0106012
- more on TMDs
  S Mert Aybat and T Rogers, arXiv:1101.5057
  T Rogers, arXiv:1509.04766
- multiparton interactions
  Multiple Parton Interactions at LHC, eds. P Bartalini and J Gaunt, 2018
  `https://doi.org/10.1142/10646` (individual chapters on arXiv)

Quantum chromodynamics (QCD)

- theory of interactions between quarks and gluons
- very different from weak and electromagnetic interactions because coupling α_s is large at small momentum scales
  - quarks and gluons are confined inside bound states: hadrons (proton, neutron, pion, ...)
  - perturbative expansion in α_s only at high momentum scales
- symmetries
  - gauge invariance: group SU(3) $\leftrightarrow$ colour charge
    electromagnetism: U(1) $\leftrightarrow$ electric charge
  - Lorentz invariance and discrete symmetries:
    P (parity = space inversion) T (time reversal)
    C (charge conjugation)
  - chiral symmetry for zero masses of u, d and s
- embedded in Standard Model: quarks couple to γ, W, Z and H

Why care about QCD?

- without quantitative understanding of QCD
  would have very few physics results from LHC, Belle, ...
- α_s and quark masses are fundamental parameters of nature
  need e.g.
  - m_t for precision fits in electroweak sector $\rightarrow$ Higgs physics
  - α_s to discuss possible unification of forces
- QCD is the one strongly interacting quantum field theory
  that we can study in experiment
  many interesting phenomena:
  - structure of proton
  - confinement
  - breaking of chiral symmetry
  - convergence of perturbative series

The QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\tfrac{1}{2}\,\mathrm{tr}\,F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,\ldots} \overline{q}\,\big[iD^{\mu}\gamma_{\mu} - m_q\big]q$$

use implicit vector/matrix notation in colour space

- quark field q_i with $i = 1,2,3$
- covariant derivative $D^{\mu}_{ij} = \partial^{\mu}\delta_{ij} + igA^{\mu}_{ij}$
- gluon potential $A^{\mu}_{ij} = A^{\mu a}\,t^{a}_{ij}$ with $a = 1,\ldots,8$
  where $t^{a}_{ij} = \frac{1}{2}\lambda^{a}_{ij}$ are the generators of SU(3)
- gluon field strength

$$F^{\mu\nu} = F^{\mu\nu a}t^{a} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} + ig[A^{\mu},A^{\nu}]$$

$$\Rightarrow\ F^{\mu\nu a} = \partial^{\mu}A^{\nu a} - \partial^{\nu}A^{\mu a} - g\,f^{abc}A^{\mu b}A^{\nu c}$$ (Exercise)

where f^{abc} = structure constants of SU(3): $[t^{a},t^{b}] = if^{abc}\,t^{c}$

Exercise: write out $\mathcal{L}_{\mathrm{QCD}}$ in terms of the fields $q_i, A^{\mu a}, F^{\mu\nu a}$

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$$F^{\mu\nu} = F^{\mu\nu a} t^{a} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} + ig[A^{\mu}, A^{\nu}] = [D^{\mu}, D^{\nu}]/(ig)$$

gauge transformation:

$$q(x) \to U(x)\, q(x) \qquad \text{with } U(x) \in \text{SU(3)}$$

$$D^{\mu}(x) \to U(x)\, D^{\mu}(x) U^{\dagger}(x) \qquad F^{\mu\nu}(x) \to U(x)\, F^{\mu\nu}(x) U^{\dagger}(x)$$

Exercise: show that $\mathcal{L}_{\text{QCD}}$ is gauge invariant

Basics of perturbation theory

- split Lagrangian into free and interacting parts:

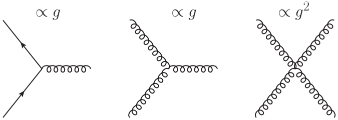
$$\mathcal{L}_{\mathsf{QCD}} = \mathcal{L}_{\mathsf{free}} + \mathcal{L}_{\mathsf{int}}$$

  - $\mathcal{L}_{\mathsf{int}}$: interaction terms $\propto g$ or g^2
  - expand process amplitudes, cross sections, etc. in g
  - Feynman graphs visualise individual terms in expansion

- from $\mathcal{L}_{\mathsf{free}}$: free quark and gluon propagators

  - in position space: propagation from x^μ to y^μ
  - in momentum space: propagation with four-momentum k^μ

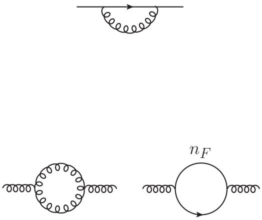
- from $\mathcal{L}_{\mathsf{int}}$: elementary vertices



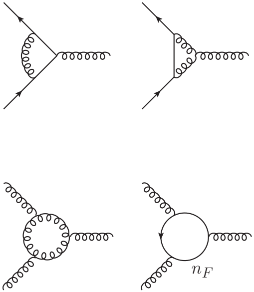
Loop corrections

- in loop corrections find ultraviolet (UV) divergences
- only appear in corrections to

propagators

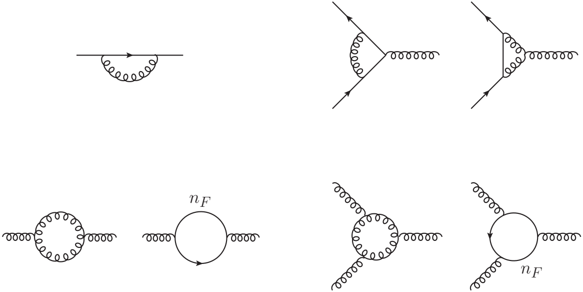


elementary vertices



Exercise: Draw the remaining one-loop graphs for all propagators and elementary vertices

- origin of UV divergences: region of ∞ ly large loop momenta
  $\leftrightarrow$ quantum fluctuations at ∞ ly small space-time distances
- idea: encapsulate UV effects in (a few) parameters
  when describe physics at a given scale μ $\rightsquigarrow$ renormalisation



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  when describe physics at a given scale μ $\rightsquigarrow$ renormalisation
- technically:
  1. regulate: artificial change of theory making div. terms finite
     - physically intuitive: momentum cutoff
     - in practice: dimensional regularisation
  2. renormalise: absorb UV effects into
     - coupling constant $\alpha_s(\mu)$
     - quark masses $m_q(\mu)$
     - quark and gluon fields (wave function renormalisation)
  3. remove regulator: quantities are finite when expressed in terms of renormalised parameters and fields
- renormalisation scheme: choice of which terms to absorb
  "∞" is as good as "$\infty + \log(4\pi)$"

Dimensional regularisation in a nutshell

- choice of regulator ≈ choice between evils
- dim. reg.: little (any?) physics intuition, but keeps intact essential symmetries (gauge and Lorentz invariance)
- idea: integrals for Feynman graphs become UV finite in lower space-time dimension, e.g.

$$\int \frac{d^D k}{(2\pi)^D} \, \frac{1}{k^2 - m^2} \, \frac{1}{(k-p)^2 - m^2}$$

log. div. for $D = 4$
converg. for $D = 3, 2, 1$

- procedure:
  1. formulate theory in D dimensions (with D small enough)
  2. analytically continue results from integer to complex D
     original divergences appear as poles in $1/\epsilon$ $(D = 4 - 2\epsilon)$
  3. renormalise
  4. take $\epsilon \to 0$

Dimensional regularisation in a nutshell: how poles arise

$$\int \frac{d^D k}{(2\pi)^D} \, \frac{1}{k^2 - m^2} \, \frac{1}{(k-p)^2 - m^2} \quad \xrightarrow{\text{UV}} \quad \int_{k > k_0} \frac{d^D k}{k^4}$$

$$\to \int_{k_0}^{\infty} \frac{dk \; k^{D-1}}{k^4} = -\frac{1}{D-4} \, k_0^{D-4} \qquad (D < 4)$$

$$= \frac{1}{2\epsilon} \, k_0^{-2\epsilon} \xrightarrow{\times \mu^{2\epsilon}} \frac{1}{2\epsilon} \left(\frac{\mu}{k_0} \right)^{2\epsilon}$$

$$= \frac{1}{2\epsilon} \exp\left(2\epsilon \ln \mu/k_0\right) = \frac{1}{2\epsilon} + \ln \frac{\mu}{k_0} + O(\epsilon)$$

$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$ in D dimensions

Dimensional regularisation in a nutshell

- enter: a mass scale μ
  - coupling in $4 - 2\epsilon$ dimensions is $\mu^\epsilon g$ with g dimensionless
    Exercise: see how this happens in detail
  - any other regularisation introduces a mass parameter as well

$\rightsquigarrow$ renormalised quantities depend on μ

Renormalisation group equations (RGE)

- scale dependence of renormalised quantities described by differential equations:

$$\frac{d}{d\log\mu^2}\,\alpha_s(\mu) = \beta\big(\alpha_s(\mu)\big)$$

$$\frac{d}{d\log\mu^2}\,m_q(\mu) = m_q(\mu)\,\gamma_m\big(\alpha_s(\mu)\big)$$

- β, $\gamma_m =$ perturbatively calculable functions
  in region where $\alpha_s(\mu)$ is small enough

$$\beta = -b_0\,\alpha_s^2\,\big[1 + b_1\alpha_s + b_2\,\alpha_s^2 + b_3\,\alpha_s^3 + \ldots\big]$$

$$\gamma_m = -c_0\,\alpha_s\,\big[1 + c_1\alpha_s + c_2\,\alpha_s^2 + c_3\,\alpha_s^3 + \ldots\big]$$

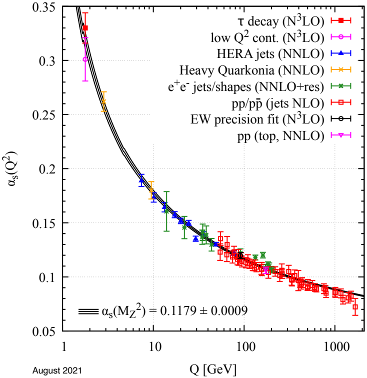
coefficients known including b_4, c_4 (b_4 since 2016)

$$b_0 = \tfrac{1}{4\pi}\big(11 - \tfrac{2}{3}n_F\big) \qquad\qquad c_0 = \tfrac{1}{\pi}$$

The running of α_s

- $\beta_{\text{QCD}} < 0$
  $\Rightarrow \alpha_s(\mu)$ decreases with μ

Nobel prize 2004 for
Gross, Politzer and Wilczek



plot: Review of Particle Properties 2021

- asymptotic freedom at large μ
- perturbative expansion becomes invalid at low μ
  quarks and gluons are strongly bound inside hadrons: confinement
  momenta below $1\,\text{GeV}$ $\leftrightarrow$ distances above $0.2\,\text{fm}$

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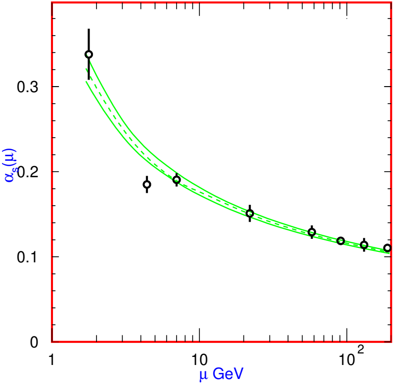

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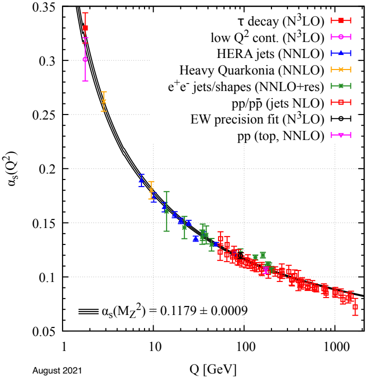
The running of α_s

- truncating $\beta = -b_0 \alpha_s^2 (1 + b_1 \alpha_s)$ get

$$\alpha_s(\mu) = \frac{1}{b_0 L} - \frac{b_1 \log L}{(b_0 L)^2} + \mathcal{O}\Big(\frac{1}{L^3}\Big)$$

with $L = \log \dfrac{\mu^2}{\Lambda^2_{\text{QCD}}}$

Exercise: check this equation



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- dimensional transmutation:
  mass scale Λ_{QCD} not in Lagrangian, reflects quantum effects

Solving the RGE for α_s

$$\frac{d}{d\log\mu^2}\,\alpha_s(\mu) = \beta\big(\alpha_s(\mu)\big) \quad \div \quad \mu(\alpha_s) \quad : \quad \frac{d\ln\mu^2(\alpha_s)}{d\alpha_s} = \frac{1}{\beta(\alpha_s)}$$

$$\Rightarrow \ln\frac{\mu^2}{Q^2} = \int_{\alpha_s(Q)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \overset{LO}{=} \frac{1}{b_0}\left[\frac{1}{\alpha_s(\mu)} - \frac{1}{\alpha_s(Q)}\right]$$

if $\ln\frac{\mu^2}{Q^2} \ll \frac{1}{\alpha_s}$

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1-\alpha_s(\mu)\,b_0\ln\mu^2/Q^2} = \alpha_s(\mu^2)\left[1 + \alpha_s(\mu)\,b_0\ln\frac{\mu^2}{Q^2} + O(\alpha^2)\right]$$

Solving the RGE for α_s

def $\alpha_s(\Lambda) = \infty \implies \alpha_s(\mu) = \dfrac{1}{b_0 \ln \mu^2/\Lambda^2}$

beyond LO : $\beta = -b_0 \alpha_s^2 (1 + b_1 \alpha_s + \dots)$

$$\ln \frac{\mu^2}{\Lambda^2} = \int_{\infty}^{\alpha_s(\mu^2)} \frac{d\alpha}{\beta(\alpha)} + \Delta$$

$$= \frac{1}{b_0} \left\{ \frac{1}{\alpha_s} + b_1 \ln \alpha_s - b_1 \underbrace{\ln(1 + b_1 \alpha_s)}_{O(\alpha_s)} \right\} + \Delta$$

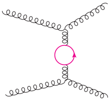
$$\implies 1/\alpha_s(\mu) = b_0 \ln \frac{\mu^2}{\Lambda^2} - b_1 \ln \alpha_s(\mu) - b_0 \Delta + O(\alpha_s)$$

Solving the RGE for α_s

Scale dependence of observables

- observables computed in perturbation theory depend on renormalisation scale μ
  - implicitly through $\alpha_s(\mu)$
  - explicitly through terms $\propto \log(\mu^2/Q^2)$ where $Q =$ typical scale of process

    e.g. $Q = p_T$ for production of particles with high p_T
    $Q = M_H$ for decay Higgs $\to$ hadrons
    $Q =$ c.m. energy for $e^+e^- \to$ hadrons



- μ dependence of observables must cancel at accuracy of the computation

Scale dependence of observables

- see how this works (consider problem without running masses)

$$C(Q) = \alpha_s^n(Q)\Big[C_0 + \alpha_s(Q)\,C_1 + \mathcal{O}\big(\alpha_s^2\big)\Big]$$

$$\alpha_s^n(Q) = \alpha_s^n(\mu)\Big[1 + n\,\alpha_s(\mu)\,b_0 \ln\frac{\mu^2}{Q^2} + \dots\Big]$$

$$\Rightarrow C(Q) = \alpha_s^n(\mu)\Big[C_0 + \alpha_s(\mu)C_1 + \alpha_s(\mu)C_0 \times n\,b_0 \ln\frac{\mu^2}{Q^2} + \mathcal{O}(\alpha_s^2)\Big]$$

Scale dependence of observables

- for generic observable C have expansion

$$C(Q) = \alpha_s^n(\mu)\Big[C_0 + \alpha_s(\mu)\Big\{C_1 + n b_0 C_0 \log\frac{\mu^2}{Q^2}\Big\} + \mathcal{O}\big(\alpha_s^2\big)\Big]$$

- Exercise: check that this satisfies

$$\frac{d}{d\log\mu^2}\, C = \mathcal{O}\big(\alpha_s^{n+2}\big)$$

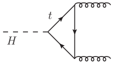
$\Rightarrow$ residual scale dependence when truncate perturbative series

- at higher orders:

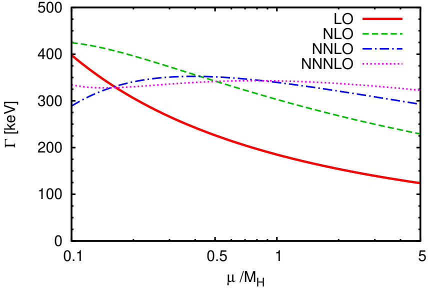
  $\alpha_s^{n+k}(\mu)$ comes with up to k powers of $\log(\mu^2/Q^2)$

  - choose $\mu \sim Q$ so that $\alpha_s \log(\mu/Q) \ll 1$
    otherwise higher-order terms spoil series expansion

Example



- inclusive hadronic decay of Higgs boson via top quark loop (i.e. without direct coupling to b quark)
- in perturbation theory: $H \to 2g$, $H \to 3g$, ... known to N^{3}LO Baikov, Chetyrkin 2006



- scale dependence decreases at higher orders
- scale variation by factor 2 up- and downwards often taken as estimate of higher-order corrections
- choice $\mu < M_H$ more appropriate

Quark masses

- recall: α_s and m_q depend on renormalisation scheme
  - standard in QCD: $\overline{\text{MS}}$ scheme $\rightsquigarrow$ running $\alpha_s(\mu)$ and $m_q(\mu)$
  - for heavy quarks c, b, t can also use pole mass

    def. by condition: quark propagator has pole at $p^2 = m_{\text{pole}}^2$

    possible in perturbation theory, but in nature quarks confined

    scheme transformation:

$$m_{\text{pole}} = m(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\frac{4}{3} - \log\frac{m^2(\mu)}{\mu^2}\right) + \mathcal{O}(\alpha_s^2)\right]$$

- $\overline{\text{MS}}$ masses from Review of Particle Properties 2021

$$m_u = 2.16^{+0.49}_{-0.26}\ \text{MeV} \qquad m_d = 4.67^{+0.48}_{-0.17}\ \text{MeV} \qquad m_s = 93^{+11}_{-5}\ \text{MeV}$$

at $\mu = 2\,\text{GeV}$

$$\overline{m}_c = 1.27 \pm 0.02\ \text{GeV} \qquad \overline{m}_b = 4.18^{+0.03}_{-0.02}\ \text{GeV} \qquad \overline{m}_t = 162.5^{+2.1}_{-1.5}\ \text{GeV}$$

with $m_q(\mu = \overline{m}_q) = \overline{m}_q$

Quark masses

- solution of the RGE

$$\frac{d}{d\log\mu^2}\, m_q(\mu) = m_q(\mu)\,\gamma_m\big(\alpha_s(\mu)\big)$$

- switch variables from μ to α_s $\rightsquigarrow$

$$\frac{d}{d\alpha_s}\,\log m_q(\alpha_s) = \frac{\gamma_m(\alpha_s)}{\beta(\alpha_s)}$$

$$\Rightarrow \quad m_q(\mu) = m_q(\mu_0)\, \exp\left[\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d\alpha\, \frac{\gamma_m(\alpha)}{\beta(\alpha)}\right]$$

Exercise:
evaluate the integral with $\gamma_m(\alpha_s)$ and $\beta(\alpha_s)$ approximated at NLO

- important other RGEs have the same form $\rightsquigarrow$ same form of solution

Summary of part 1: renormalisation

- beyond all technicalities reflects physical idea:
  eliminate details of physics at scales $\gg$ scale Q of problem
- running of α_s $\rightsquigarrow$ characteristic features of QCD:
  - asymptotic freedom at high scales $\rightsquigarrow$ use perturbation theory
  - strong interactions at low scales $\rightsquigarrow$ need other methods
  - introduces mass scale Λ_{QCD} into theory
- dependence of observable on μ governed by RGE
  reflects (and estimates) particular higher-order corrections

  ... but not all

Exercises

1. QCD Lagrangian
   - Deduce the expression of $F^{\mu\nu a}$ from the representation of $F^{\mu\nu} = F^{\mu\nu a}\, t^a$ given on slide 6.
   - Write out $\mathcal{L}_{\text{QCD}}$ with explicit colour indices in terms of the fields $q_i, A^{\mu a}, F^{\mu\nu a}$.
     useful relation: $\mathrm{tr}\; t^a t^b = \delta^{ab}/2$
   - Verify the identity $F^{\mu\nu} = [D^\mu, D^\nu]/(ig)$.
     note: $[\partial^\mu, f]g = \partial^\mu f g - f \partial^\mu g = (\partial^\mu f) g$ for any f and g.
   - Show that the QCD Lagrangian is invariant under local SU(3) gauge transformations.
2. Draw the one-loop graphs for all propagators and elementary vertices that are not given on slide 9.

Exercises

3. Dimensional regularisation

    - Determine the mass dimension of fields q and A^μ for QCD in $D = 4 - 2\epsilon$ space-time dimensions.
    - Deduce the mass dimension of the coupling, taking into account all places where it appears in $\mathcal{L}_{\text{QCD}}$.

Use that the action $\int d^D x \, \mathcal{L}_{\text{QCD}}$ is dimensionless (in natural units, where $\hbar = 1$).

Exercises

4. Scale dependence

    - Prove the expression for $\alpha_s(\mu)$ given on slide 18.
      Start from the expression
      $1/\alpha_s(\mu) = b_0 L - b_1 \log \alpha_s(\mu) - b_0 \Delta + \mathcal{O}(\alpha_s)$
      derived in the lecture.
    - Verify that the scale dependence of C given on slide 24 is of order α_s^{n+2}.
    - Evaluate the explicit expression for the scale dependence of $m_q(\mu)$ on slide 27 to NLO, i.e. keeping the leading and the first subleading term in the exponent.

Notes

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