

Hyperons in Matter



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FRONTIERS IN NUCLEAR AND HADRONIC PHYSICS

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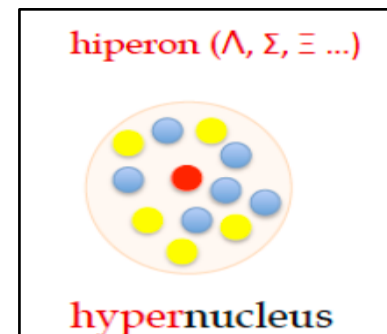
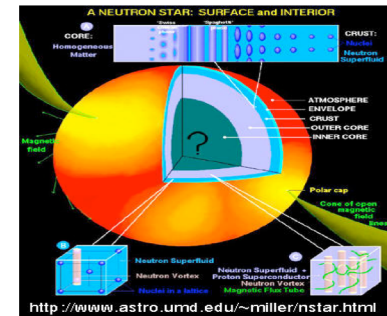
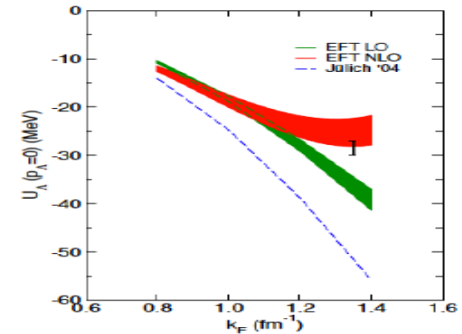
Outline

- Why hyperons in matter?
- Brueckner-Hartree-Fock approach
 - Many-body problem
 - Goldstone Theorem
 - Brueckner-Goldstone Theory:
The Bethe-Goldstone Equation
- Lowest order Brueckner Theory
- Hyperons in matter
- Bibliography

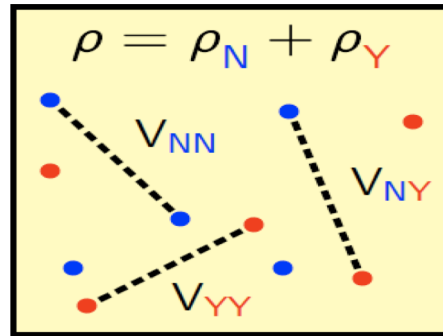
Why hyperons in matter?

To understand..

- hadron-hadron interaction in matter
- optical potential of hadrons in matter
- equation of state for hadronic matter: neutron star matter
- nuclear and hypernuclear structure: hypernuclei
-



Experimental data scattering and hypernuclei

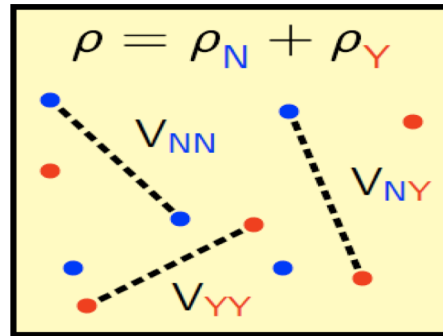


Theoretical models for hyperons in neutron stars

- **YN:** < 50 scattering data points
- **N Λ :** Λ -hypernuclei for $A=3-209$, $U_{\Lambda}(\rho_0) = -30$ MeV
- **N Σ :** Σ^- atoms but one Σ -hypernuclei, $U_{\Sigma}(\rho_0) = 30$ MeV ?
- **N Ξ :** few Ξ hypernucleus $U_{\Xi}(\rho_0) = -24$ MeV ?
- **$\Lambda\Lambda$:** few $\Lambda\Lambda$ hypernuclear events, slightly attractive ?
- **YY:** $Y = \Lambda, \Sigma, \Xi$ unknown!

- **Relativistic mean field models**
Glendenning '85; Knorren, Prakash & Ellis '95; Schaffner & Mishustin '96..
- **Non-relativistic potential model**
Balberg & Gal '97...
- **Quark-meson coupling model**
Pal et al '99..
- **Chiral effective lagrangians**
Hanuske et al. '00...
- **Density dependent hadron field model** Hofmann, Keil & Lenske '01..
- **DBHF/BHF approaches**
Brockmann & Machleidt '90; Baldo, Burgio, Schulze '00; Vidana et al. '00; Jong and Lenske '98..
- **Low-momentum interactions**
Schwenk, Pethick, Hebeler, Friman, LT, Djapo..
- **Quantum Monte Carlo**
Leonardi et al '14..

Experimental data scattering and hypernuclei



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- **ΛΛ**: few ΛΛ hypernuclear events, slightly attractive ?
- **YY**: Y=Λ, Σ, Ξ **unknown!**

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Brueckner-Hartree-Fock approach

Many-body problem

- **NUCLEAR MATTER** is hypothetical system with the same number of protons and neutrons, which fill out the whole space with a uniform density. It is a dilute system: the range of the repulsive “core” ($a \sim 0.4 - 0.5$ fm) is much less than the distance among nucleons ($r_0 \sim 1-2$ fm),
→ $a/r_0 \sim 1/3$

$$\rho = \frac{A}{V} = \frac{A}{\frac{4\pi}{3}r_0^3 A} = 0.17 \text{ fm}^{-3}$$

$$\rho = \frac{A}{V} = 4 \int \frac{d^3k}{(2\pi)^3} \theta(k_F - k) = \frac{2}{3\pi^2} k_F^3 \quad (k_F = 1.36 \text{ fm}^{-1})$$

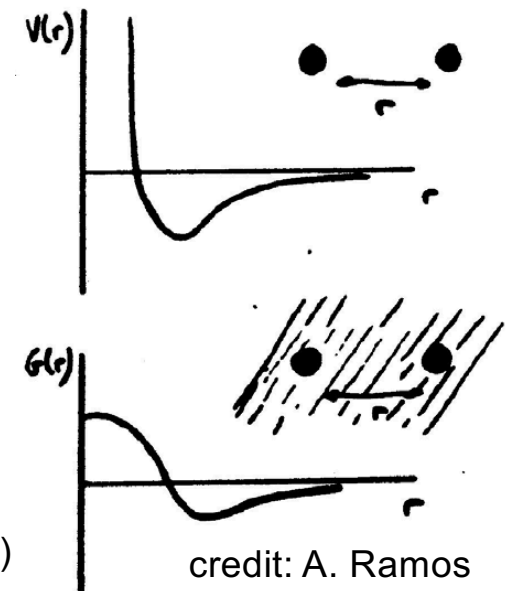
- One of the **main difficulties** in nuclear many-body systems (finite nuclei and nuclear matter) arises from the fact that **baryon-baryon interaction V is repulsive at short distances** and, hence, any expansion in terms of V becomes meaningless. A description in terms of a model of independent particles seems “a priori” not reasonable!

- However, the **shell model** reproduces a great number of nuclear properties under the assumption that **each nucleon** moves in a single orbital and its dynamics **is independent of other surrounding nucleons**.

- The success in describing the properties of nucleons with a model of independent particles even if NN is repulsive at short distances induces to think that the **NN interaction is shielded when nucleons are submerged in nuclear matter**.

- The **Brueckner-Goldstone Theory** provides the scheme to obtain the effective NN interaction starting from the bare potential V by resumming ladder diagrams. The effective interaction is called **G-matrix** (or Brueckner reaction matrix) and it is deduced from the **Bethe-Goldstone Equation**

B. D. Day, Reviews of Modern Physics, Vol. 39, 719 (1967); Vol. 50, 495 (1978)



Goldstone Theorem

The starting point of the **Brueckner-Goldstone Theory** is the expression of the energy of the fundamental state of an interacting many-body system deduced by Goldstone, **Goldstone Theorem**:

$$E = E_0 + \langle \Phi_0 | H_1 \sum_{n=0}^{\infty} \left[\frac{1 - |\Phi_0\rangle\langle\Phi_0|}{E_0 - H_0} H_1 \right]^n | \Phi_0 \rangle_l$$

where H_0 is the free Hamiltonian (or one-body interactions at most)

H_1 is the two-body interaction Hamiltonian

Φ_0 is the fundamental state without correlations

E_0 is the energy of the fundamental state without correlations

$1 - |\Phi_0\rangle\langle\Phi_0|$ is the Pauli operator

and “l” indicates that **only linked diagrams** contribute to the Goldstone expansion, i.e., those diagrams which cannot be separated in two pieces by a vertical cut which would not cross any line



DIAGRAM FOR INTERACTIONS

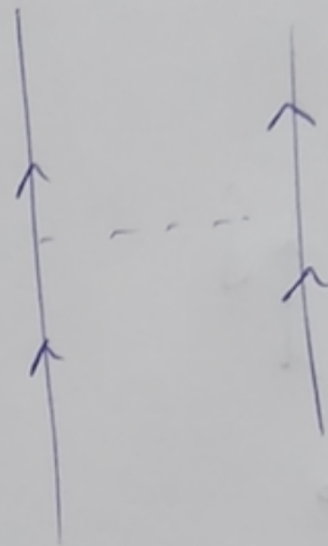


DIAGRAM FOR SELF-ENERGY

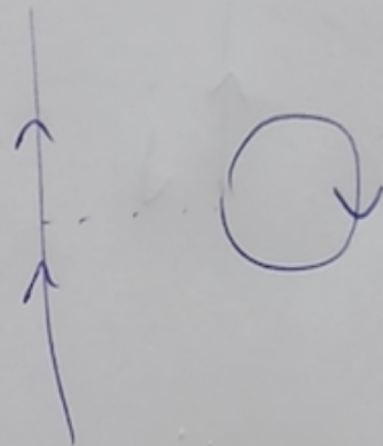
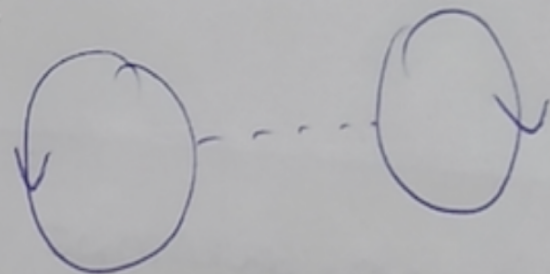


DIAGRAM FOR ENERGY OF THE SYSTEM



Example: Second-order expansion

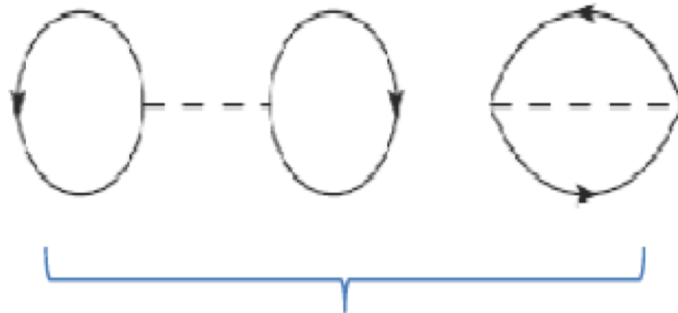
$$E - E_0 = \langle \Phi_0 | \hat{H}_1 | \Phi_0 \rangle + \sum_{m \neq 0} \frac{\langle \Phi_0 | \hat{H}_1 | \Phi_m \rangle \langle \Phi_m | \hat{H}_1 | \Phi_0 \rangle}{E_0 - E_m}$$

due to translational invariance,
single-particle states are plane waves

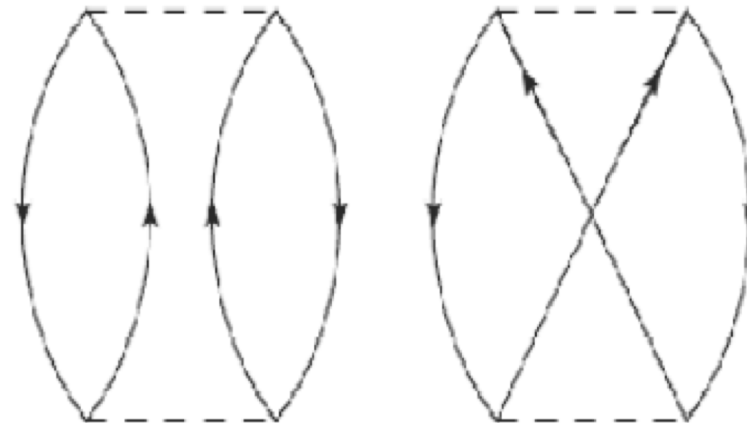
$$|k\rangle = \frac{1}{\sqrt{V}} e^{i\vec{k}\vec{r}}$$

$$\frac{1}{4} \sum_{k,j < k_F} \sum_{a,b > k_F} \frac{\langle kj | \hat{H}_1 | ab - ba \rangle \langle ab - ba | \hat{H}_1 | kj \rangle}{\varepsilon(k) + \varepsilon(j) - \varepsilon(a) - \varepsilon(b)}$$

$$\langle \Phi_0 | \hat{H}_1 | \Phi_0 \rangle = \frac{1}{2} \sum_{k,j < k_F} \langle kj | \hat{H}_1 | kj - jk \rangle$$



first order



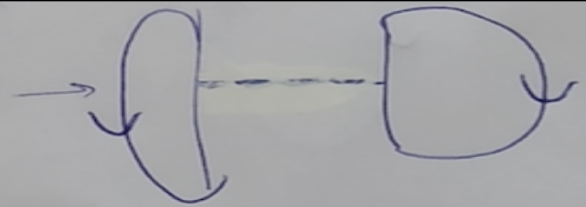
second order

HARTREE
(FIRST ORDER)

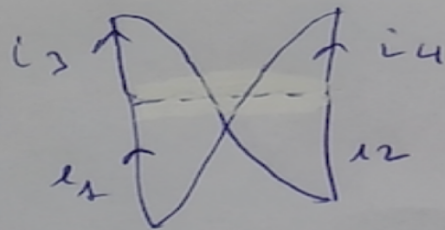


$$i_1 = i_3$$

$$i_2 = i_4$$

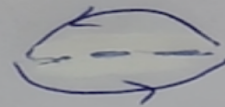


FOCK DIAGRAM
(FIRST ORDER)

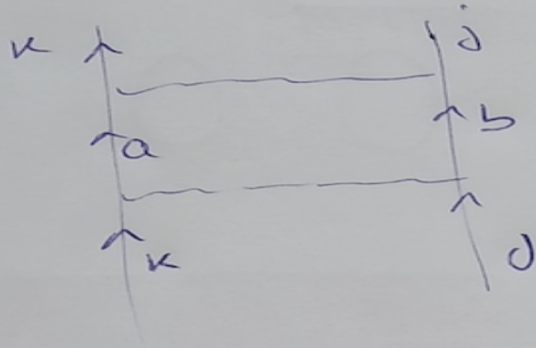


$$i_3 = i_2$$

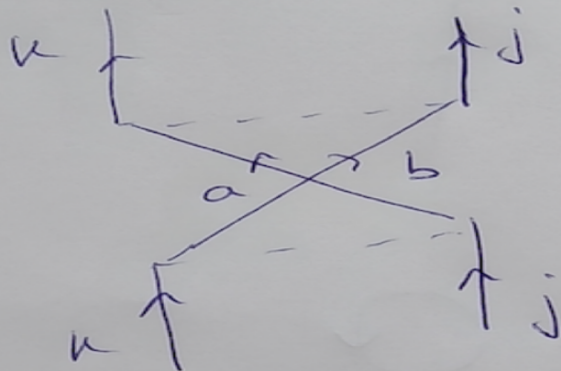
$$i_1 = i_4$$



SECOND ORDER DIAGRAMS

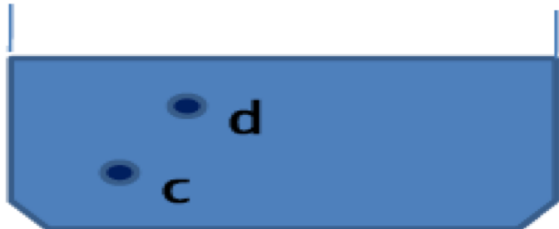
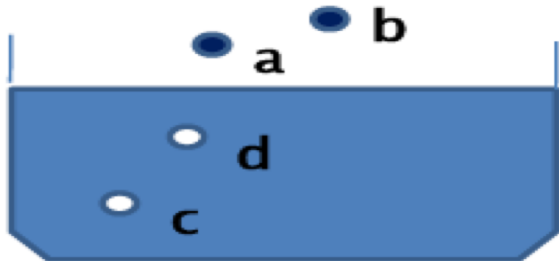
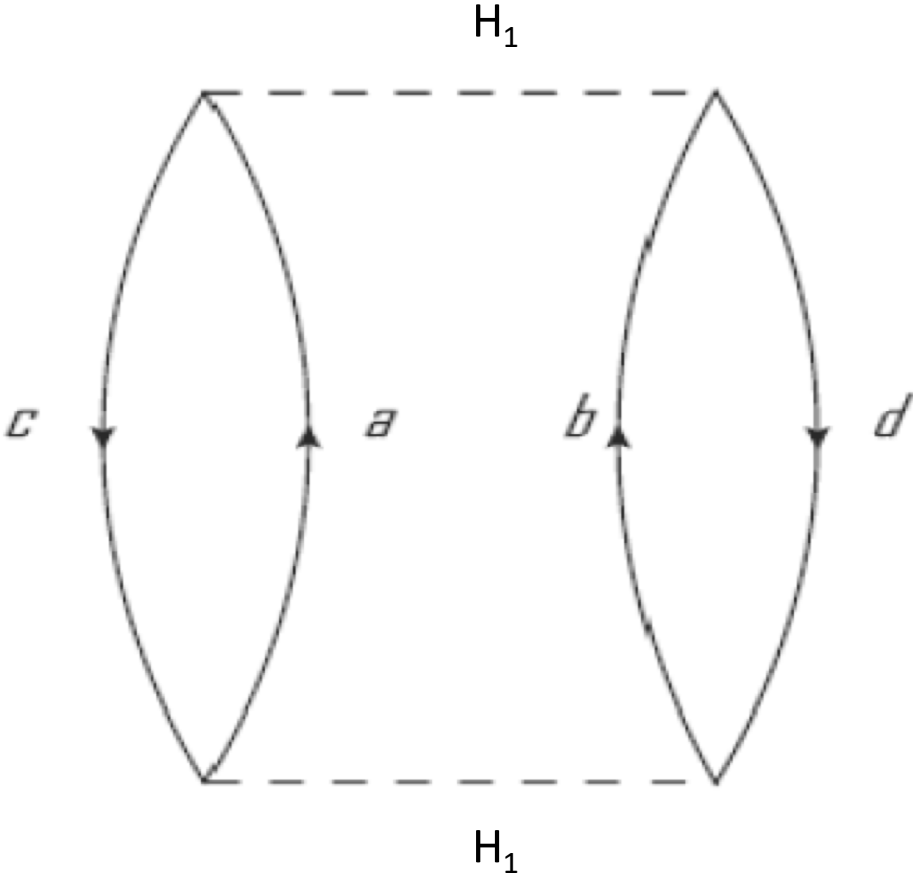


$$\langle k_j | \dots | a \rangle \langle a | \dots | k_d \rangle$$

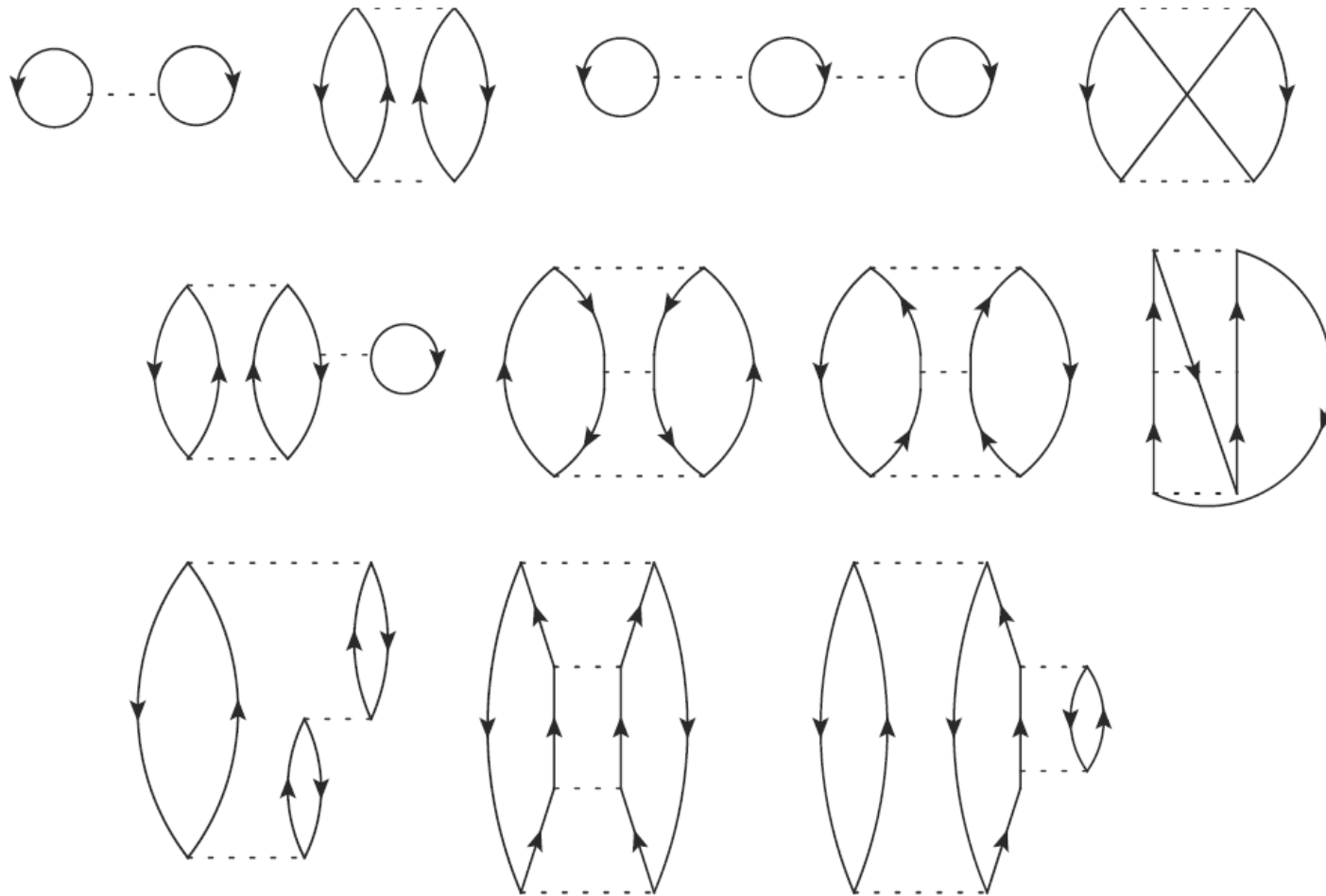


$$\langle k_j | \dots | b \rangle \langle a | \dots | k_d \rangle$$

How to understand those diagrams??



The expansion consists of all possible different linked diagrams:



However, the expansion order by order in V cannot converge because the short-range repulsion in the NN interaction makes all matrix elements very large. Which is the solution???

Brueckner-Goldstone Theory: The Bethe-Goldstone Equation

The solution is given within the **Brueckner-Goldstone Theory** which **resums partially the most relevant terms up to infinity**

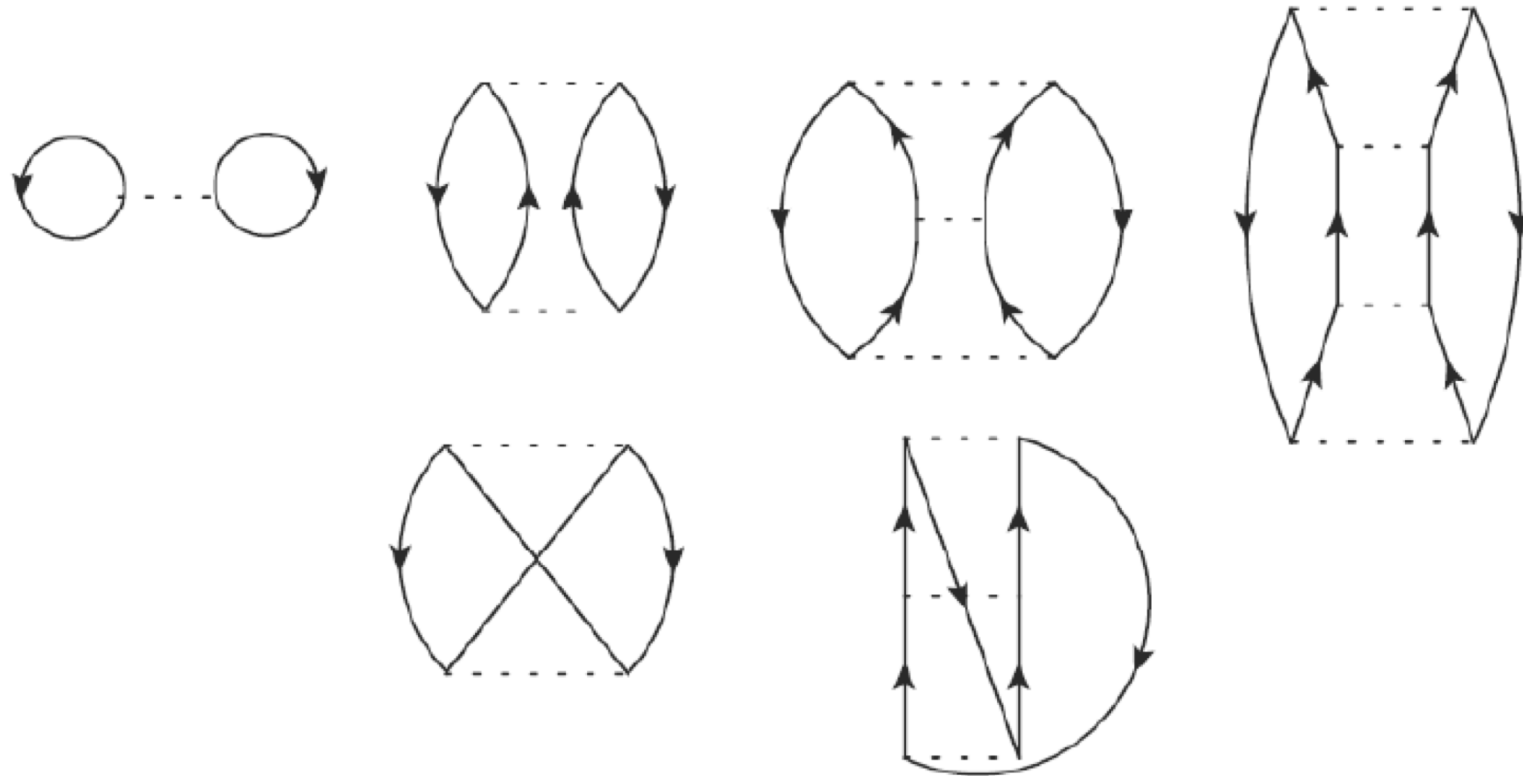
How to select them??

Nuclear matter is a dilute system ($a/r_0 \ll 1$, $a \cdot k_F \ll 1$).

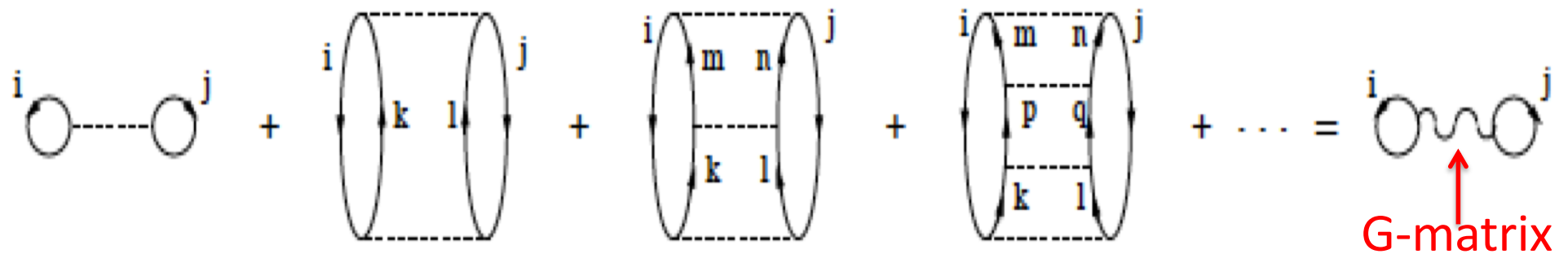
For each diagram, each “hole” line implies an integral over momentum up to k_F while each “particle” line implies integrating above k_F . For low density systems (k_F small), the contribution of “hole” line integrals is smaller than the contribution of “particle” line integrals: **the more “hole” lines there are, the more subdominant the diagram is for $a \cdot k_F \ll 1$**

Diagrams are ordered according to the number of holes and not by number of interactions. The dominant diagrams in the perturbative expansion for the energy (and other observables) are those with less “hole” lines. Those form the **ladder expansion**.

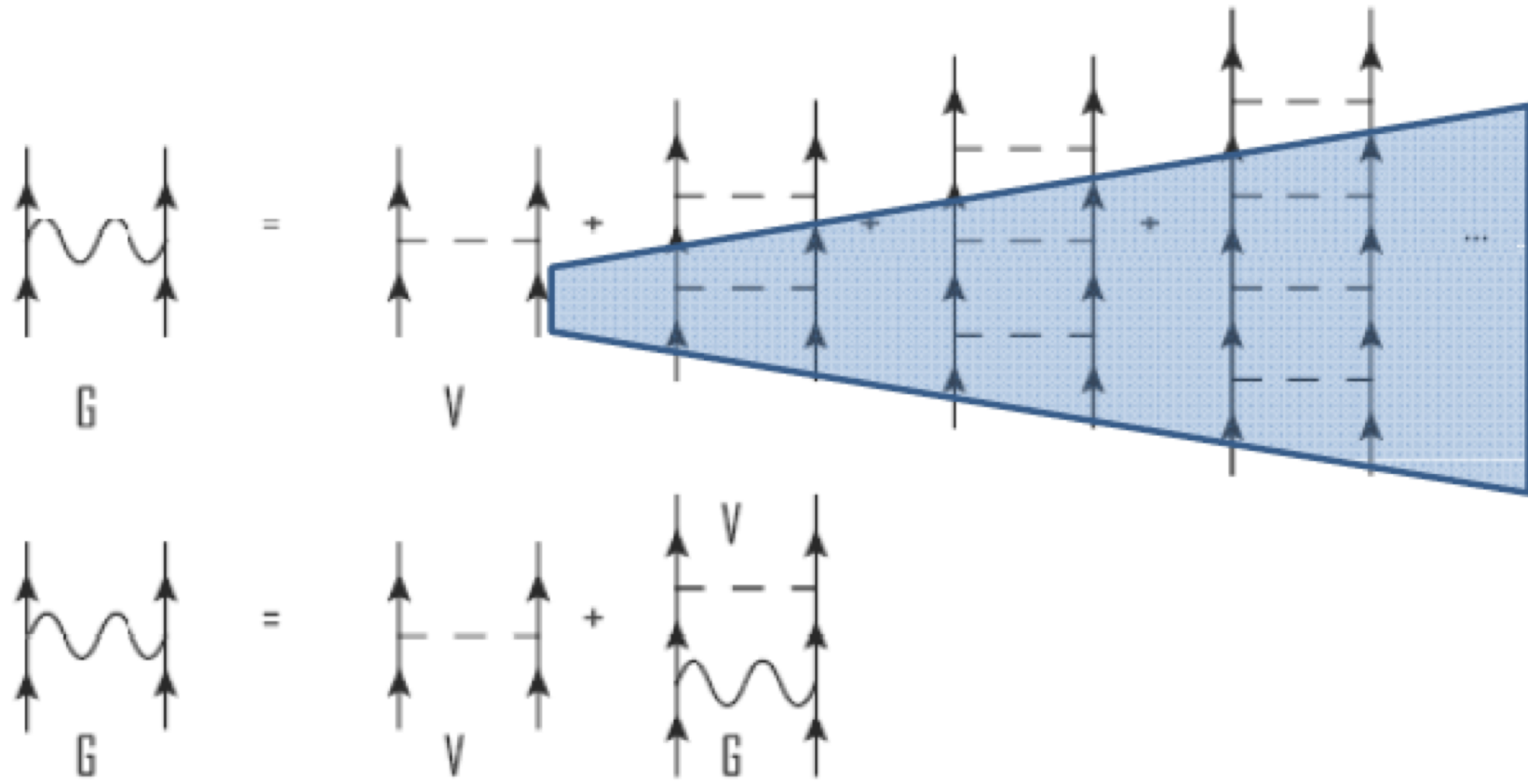
Some **ladder diagrams** with two-hole lines:



which form part of the ladder series for **the effective interaction or G-matrix**



The **G-matrix** (or Brueckner reaction matrix) is obtained by solving the **Bethe-Goldstone Equation**



$$G = V + V \frac{Q_{\text{pauli}}}{E_0 - H_0} G$$

Bethe-Goldstone Equation

Formally one has
$$G = V + V \frac{Q_{\text{pauli}}}{E_0 - H_0} G$$

- Q_{Pauli} : it only allows two particles above k_F in the intermediate states. This operator is known as **Pauli operator**

$$Q_{\text{Pauli}} |p_1 p_2\rangle = \begin{cases} |p_1 p_2\rangle & \text{if } |p_1| \text{ \& } |p_2| > k_F \\ 0 & \text{otherwise} \end{cases}$$

- The **energy denominator**, with E_0 being the energy of the fundamental state without correlations and H_0 is the free Hamiltonian acting on the intermediate state (in this case, 2 particle-2 hole)

$$E_0 - H_0 |p_1 p_2 h_1 h_2\rangle = E_0 - [E_0 + \epsilon_{p_1} + \epsilon_{p_2} - \epsilon_{h_1} - \epsilon_{h_2}] |p_1 p_2 h_1 h_2\rangle = \epsilon_{h_1} + \epsilon_{h_2} - \epsilon_{p_1} - \epsilon_{p_2} |p_1 p_2 h_1 h_2\rangle = \omega - \epsilon_{p_1} - \epsilon_{p_2} |p_1 p_2 h_1 h_2\rangle, \omega: \text{starting energy}$$

Then, the **Bethe-Goldstone Equation** reads

$$G(\omega) = V + V \sum_{p_1, p_2 > k_F} \frac{|p_1 p_2 \rangle \langle p_1 p_2|}{\omega - \epsilon_{p_1} - \epsilon_{p_2}} G(\omega)$$

Brueckner found that the **G-matrix** could be interpreted as describing the **collision of two particles in the presence of a medium**. There is, therefore, a parallelism with the **Lippman-Schwinger equation** for the scattering of two free particles satisfied by the **T-matrix**:

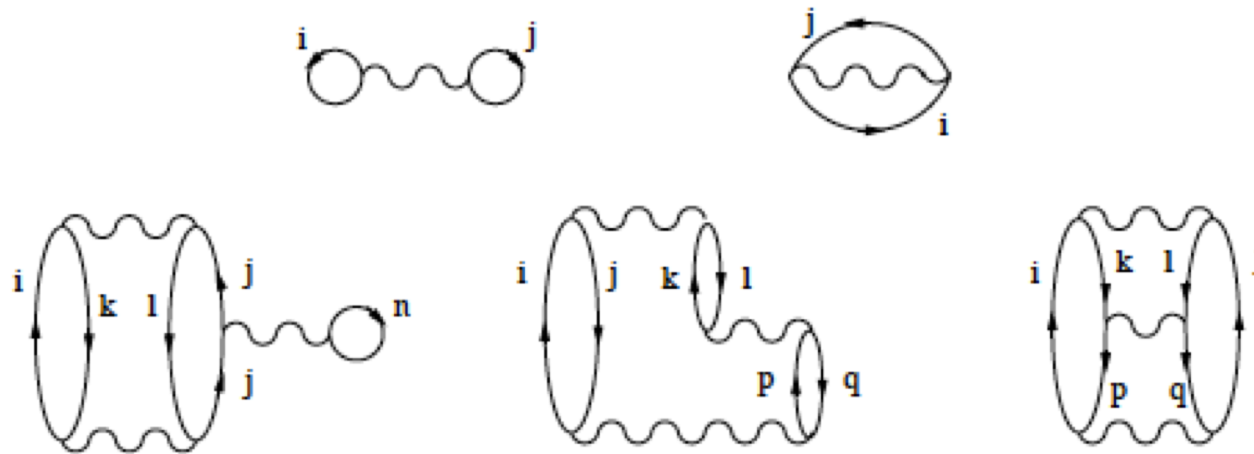
$$T = V + V \frac{1}{E_0 - H_0 + i\eta} T$$

The most important difference lies in the Pauli operator, which indicates that two nucleons interact in the presence of other nucleons and, therefore, there are occupied states not accessible after the interaction.

$$G = V + V \frac{Q_{\text{pauli}}}{E_0 - H_0} G$$

The summation of ladder diagrams has led to a single diagram where the interaction V has been replaced by the reaction matrix G . Now the matrix elements are well-behaved even with short-range repulsion. This suggests to convert each V interaction into a G -matrix line by summing the proper sequences of ladder diagrams avoiding double counting. In this way, one obtains the so-called **Brueckner-Goldstone Expansion**, in which every term is finite and well behaved.

Some of the diagrams appearing in the **Brueckner-Goldstone Expansion**



How to organize this expansion? The solution was given by Brueckner, Goldstone and Gammel with the **Hole-Line Expansion**: the diagrams with h (hole) lines being more important than those with $h+1$ lines (not a theorem!)

Lowest order Brueckner Theory

The leading term of the hole-line expansion for the energy

$$E = E_0 + \text{[diagram 1]} + \text{[diagram 2]}$$

$$E = \sum_{m < k_F} \langle m | T | m \rangle + \frac{1}{2} \sum_{m, j < k_F} \langle mj | G[\varepsilon(m) + \varepsilon(j)] | mj \rangle - \langle jm | G[\varepsilon(m) + \varepsilon(j)] | mj \rangle$$

$$\langle mn | G | ij \rangle = \langle mn | V | ij \rangle + \sum_{p, l > k_F} \langle mn | V | pl \rangle \frac{1}{\varepsilon(i) + \varepsilon(j) - \varepsilon(p) - \varepsilon(l)} \langle pl | G | ij \rangle$$

This approximation is also usually called **Brueckner-Hartree-Fock (BHF)** approximation due to its analogy with Hartree-Fock approximation. The only difference is that **the interaction V has been replaced by the Brueckner reaction matrix or G -matrix**

Self-consistent calculation of E:

1. Starting from a single-particle spectrum $\epsilon(m)$, one solves the **Bethe-Goldstone Equation** for $G(\omega)$ for a range of $\omega [= \epsilon(i) + \epsilon(j)]$

$$\langle mn | G | ij \rangle = \langle mn | V | ij \rangle + \sum_{p,l > k_F} \langle mn | V | pl \rangle \frac{1}{\epsilon(i) + \epsilon(j) - \epsilon(p) - \epsilon(l)} \langle pl | G | ij \rangle$$

1. Calculate **single-particle potential U(m)**

$$U(m) = \sum_{j < k_F} \langle mj - jm | G[\epsilon(m) + \epsilon(j)] | mj \rangle$$

2. Adding the kinetic energy to obtain a **new single-particle spectrum $\epsilon(m)$** and compare it with the starting one, until both coincide

$$\epsilon(m) = \langle m | T | m \rangle + \frac{1}{2} U(m)$$

1. Once self-consistency is reached, calculate **E (energy density) summing over all single-particle energies up to the Fermi level**

$$E = \sum_{m < k_F} \left[\langle m | T | m \rangle + \frac{1}{2} U(m) \right]$$

Summary: Brueckner-Hartree-Fock approach

credit: I. Vidana

- Bethe-Goldstone Equation

$$G(\omega)_{B_1 B_2; B_3 B_4} = V_{B_1 B_2; B_3 B_4} + \sum_{B_5 B_6} V_{B_1 B_2; B_5 B_6} \frac{Q_{B_5 B_6}}{\omega - E_{B_5} - E_{B_6} + i\eta} G(\omega)_{B_5 B_6; B_3 B_4}$$

Pauli blocking

Particle dressing

- Single-particle energy & single-particle potential

$$E_{B_i}(k) = M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + \text{Re}[U_{B_i}(k)]$$

$$U_{B_i}(k) = \sum_{B_j} \sum_{k \leq k_{F B_j}} \langle \bar{k}_i \bar{k}_j | G(\omega = E_{B_i} + E_{B_j}) | \bar{k}_i \bar{k}_j \rangle$$

- Energy density

$$\varepsilon = 2 \sum_{B_i} \int_0^{k_{F B_i}} \frac{d^3 k}{(2\pi)^3} \left[M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + \frac{1}{2} \text{Re}[U_{B_i}^N] + \frac{1}{2} \text{Re}[U_{B_i}^Y] \right]$$

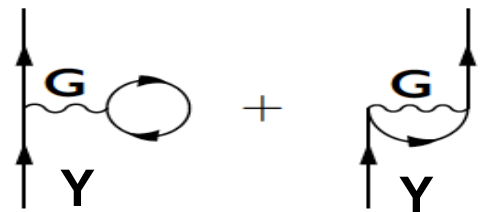
Self-consistency in coupled channels!!!

	S = 0	S = -1	S = -2	S = -3	S = -4
I = 0	(NN → NN)		$\begin{pmatrix} \Lambda\Lambda \rightarrow \Lambda\Lambda & \Lambda\Lambda \rightarrow \Xi N & \Lambda\Lambda \rightarrow \Sigma\Sigma \\ \Xi N \rightarrow \Lambda\Lambda & \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Lambda\Lambda & \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$		(ΞΞ → ΞΞ)
I = 1/2		$\begin{pmatrix} \Lambda N \rightarrow \Lambda N & \Lambda N \rightarrow \Sigma N \\ \Sigma N \rightarrow \Lambda N & \Sigma N \rightarrow \Sigma N \end{pmatrix}$		$\begin{pmatrix} \Lambda\Sigma \rightarrow \Lambda\Sigma & \Lambda\Sigma \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Lambda\Sigma & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$	
I = 1	(NN → NN)		$\begin{pmatrix} \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Lambda\Sigma & \Xi N \rightarrow \Sigma\Sigma \\ \Lambda\Sigma \rightarrow \Xi N & \Lambda\Sigma \rightarrow \Lambda\Sigma & \Lambda\Sigma \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Lambda\Sigma & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$		(ΞΞ → ΞΞ)
I = 3/2		(ΣN → ΣN)		(ΣΞ → ΣΞ)	
I = 2			(ΣΣ → ΣΣ)		

credit: I. Vidana

Hyperons in matter

Λ and Σ in dense matter



$$G = V + V \frac{Q_{\text{pauli}}}{E_0 - H_0} G$$

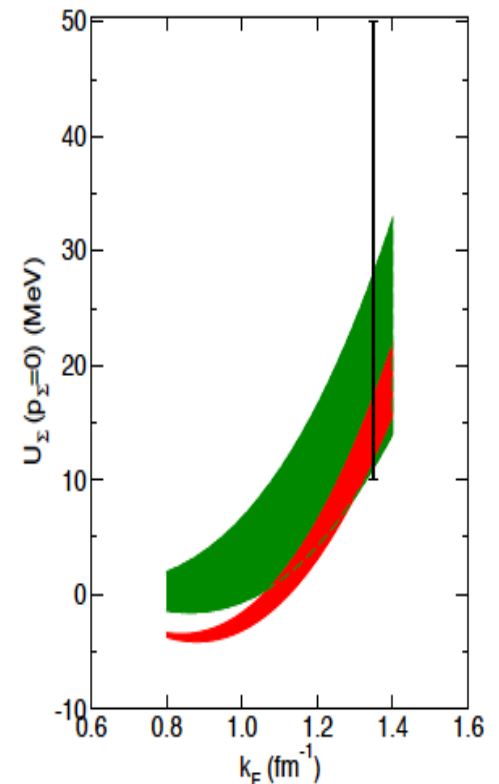
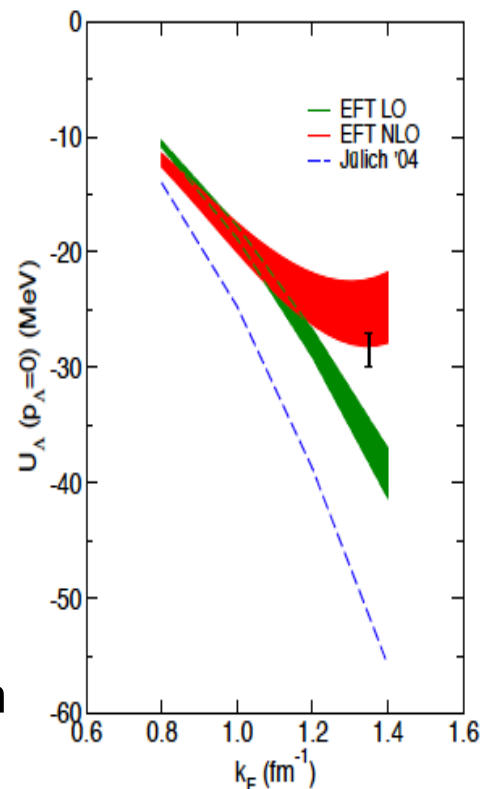
$$k_F = 1.35 \text{ fm}^{-1} \quad (\rho_0 = 0.166 \text{ fm}^{-3})$$

	EFT LO	EFT NLO
Λ [MeV]	550 ... 700	500 ... 650
$U_\Lambda(0)$	-38.0 ... -34.4	-28.2 ... -22.4
$U_\Sigma(0)$	28.0 ... 11.1	17.3 ... 11.9

- Empirical value of Λ binding in nuclear matter $\sim 27\text{-}30$ MeV

- ΣN ($I=3/2$): 3S_1 - 3D_1 decisive for Σ properties in nuclear matter. YN data can be reproduced with attractive and repulsive 3S_1 - 3D_1 interaction. It is chosen to be repulsive in accordance to data on Σ^- atoms and (π, K^+) inclusive spectra for Σ^- formation in heavy nuclei.

Lattice* supports repulsion!



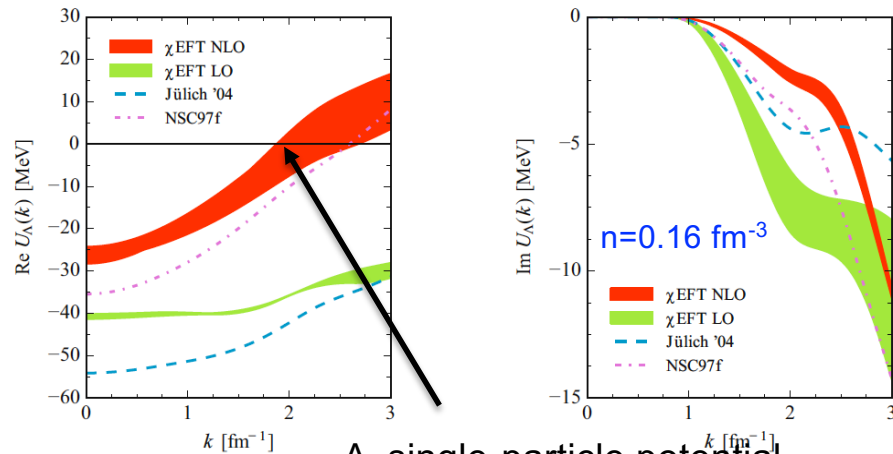
Haidenbauer and Meißner, NPA 936 (2015) 29

* Nemura et al EPJ Web of Conferences 175 (2018) 05030; Hatsuda (HALQCD) SQM2019

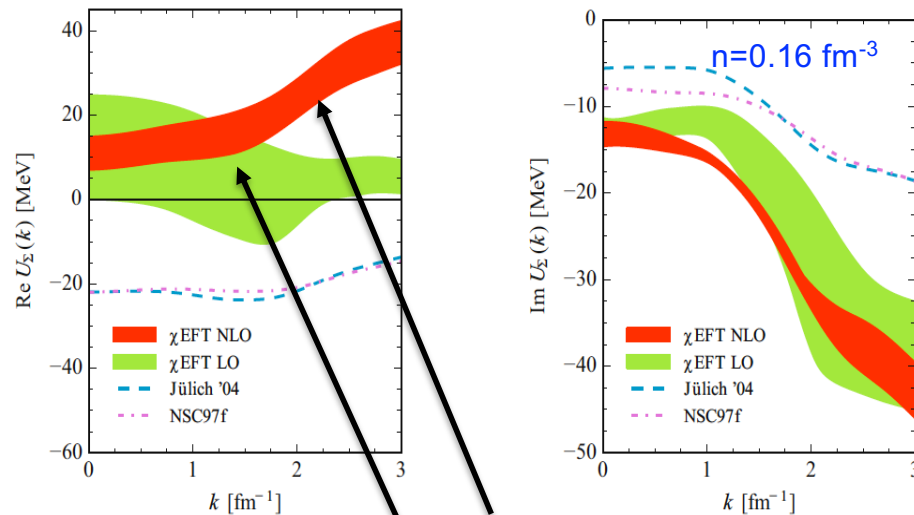
Improving on the calculation by using χ EFT NN interaction and continuous choice in Brueckner-Hartree-Fock approach while investigating isospin-asymmetric matter

S. Petschauer, J. Haidenbauer, N. Kaiser, U.G. Meißner and W. Weise EPJA 52 (2016) 15

symmetric nuclear matter

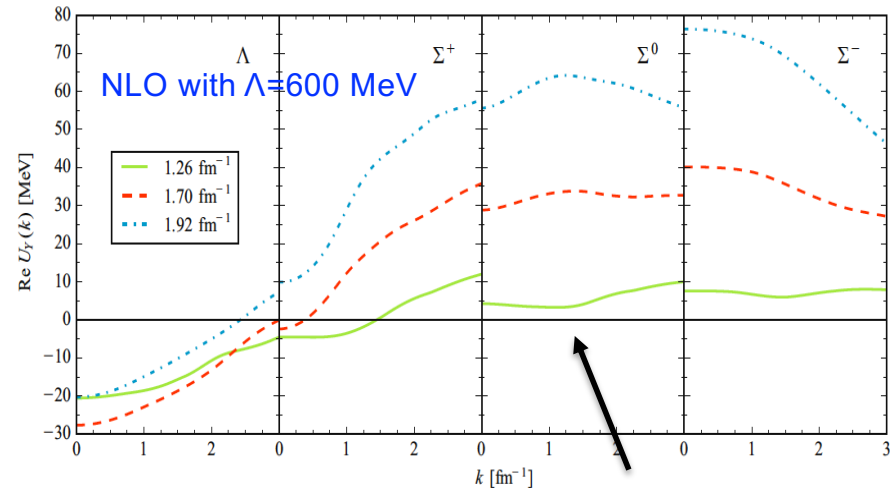


Λ single-particle potential at NLO turns repulsive $k \sim 2 \text{ fm}^{-1}$

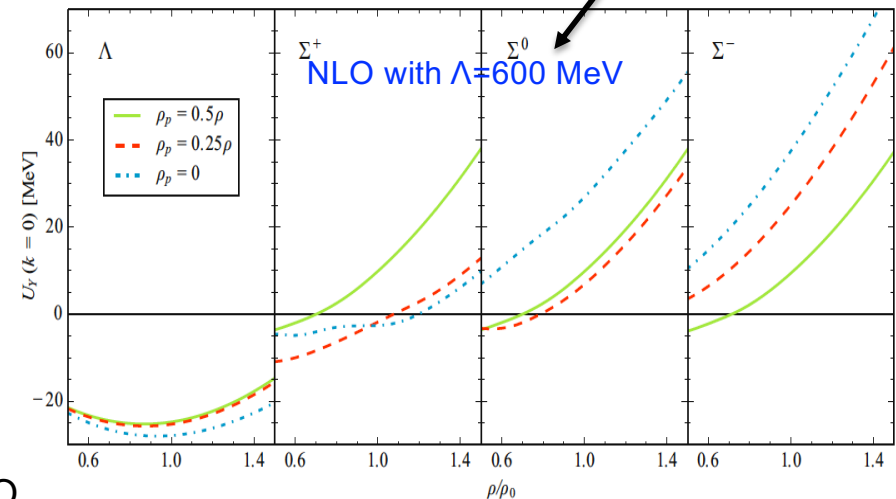


Σ -nuclear potential is moderately repulsive for LO and NLO

neutron matter



strong isospin dependence of the Σ N interaction

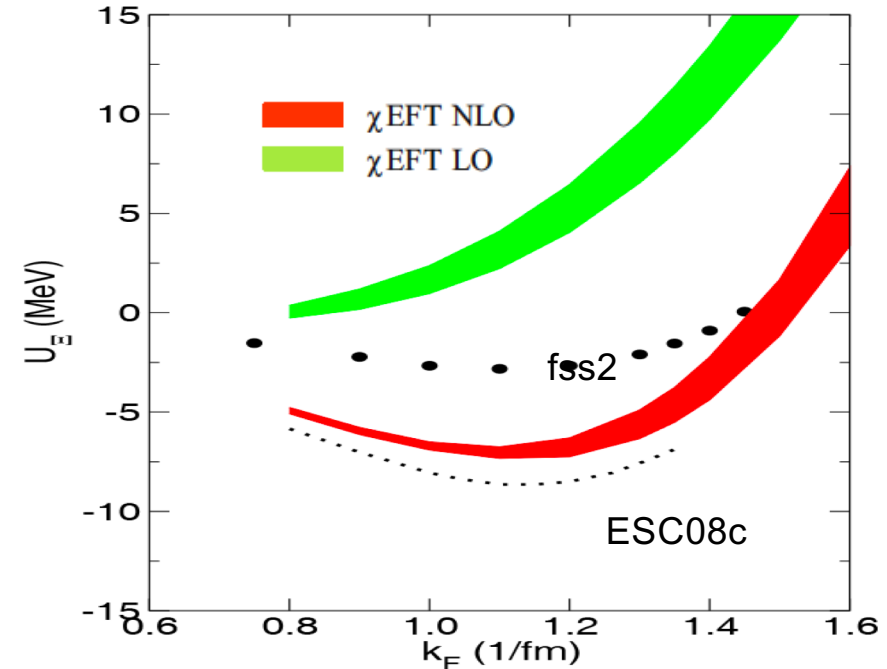
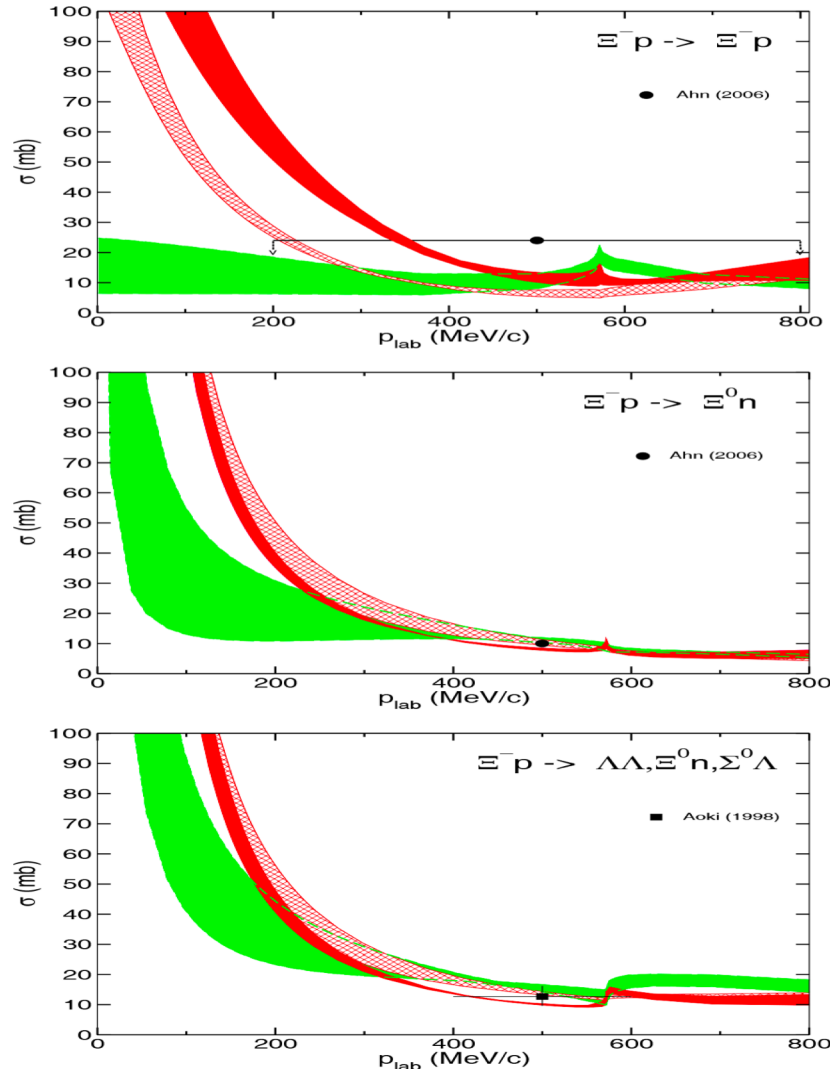


Ξ in dense matter

Using experimental constraints on $\Lambda\Lambda$ scattering length to be mildly attractive, whereas ΞN cross sections are small

J. Haidenbauer and
U.G. Meißner EPJA 55 (2019) 23

Ξ in dense matter



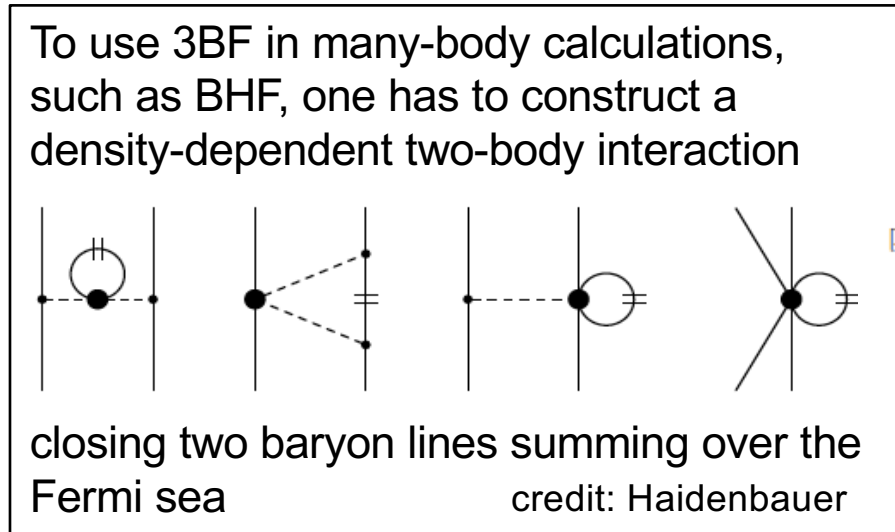
Moderately attractive Ξ -nuclear interaction, with $U_{\Xi}(0) \sim -3$ to -5 MeV.

Smaller than $U_{\Xi}(n_0) \sim -14$ MeV Khaustov et al'00 and in line with other BHF studies with phenomenological ΞN potentials.

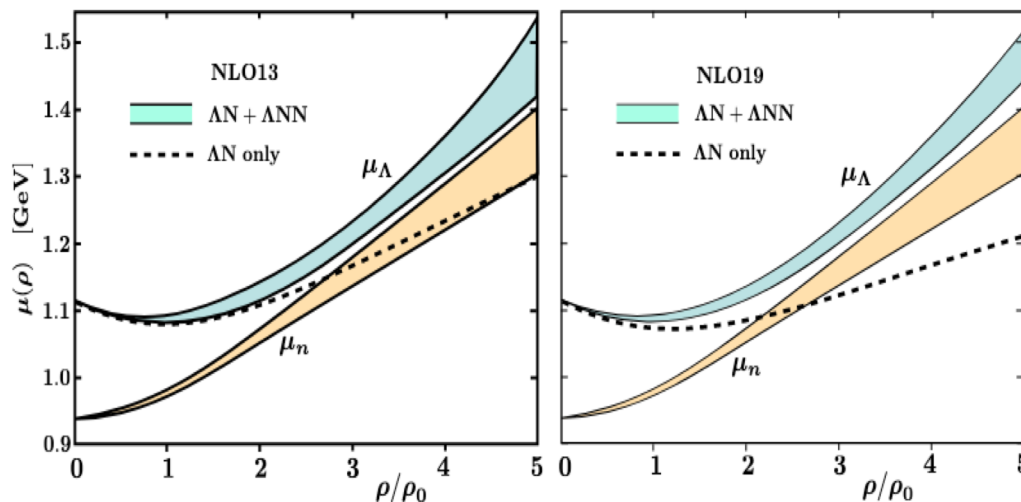
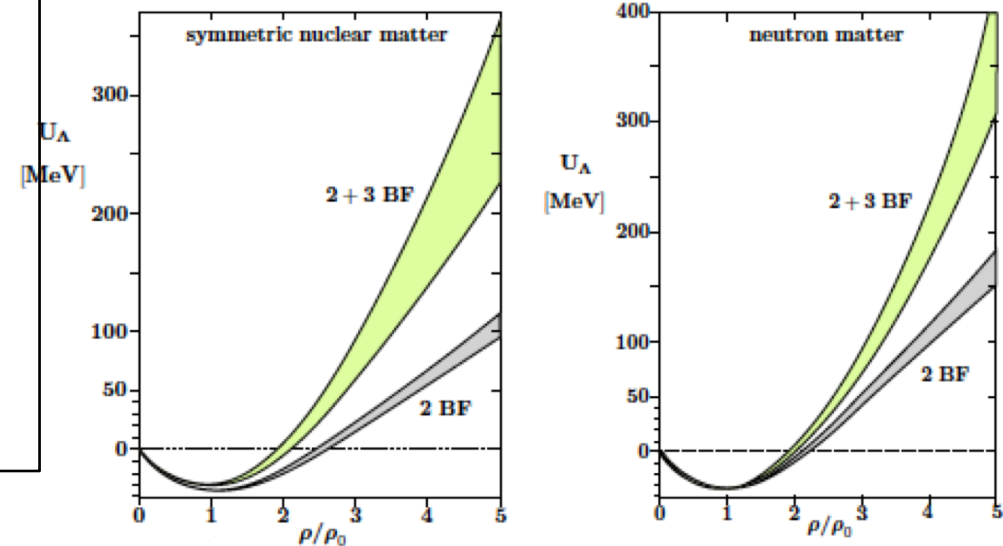
Recent results on $U_{\Xi}(n_0) \sim -24$ MeV Friedman et al '21

Λ in dense matter: including three-body forces

Three-body forces are required to reproduce few-nucleon binding energies, scattering observables and nuclear saturation in non-relativistic many-body approaches



Λ in dense matter



$\mu_\Lambda(\rho) \leq \mu_n(\rho) \Rightarrow$ energetically favorable to replace n by Λ

D. Gerstung et al. EPJA 56 (2020) 175

Λ hyperons will not appear in neutron stars (including 3BF)

Solution of the Hyperon Puzzle?

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