Nucleon partonic structure: concepts and measurements
Part 6: TMDs

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Measured transverse momentum

- consider
  - Drell-Yan with measured small $q_T$ of $\gamma^*$
  - SIDIS with measured small $q_T$ of hadron
  - $e^+e^- \rightarrow h_1h_2 + X$ with $h_1$, $h_2$ approx. opposite momenta and small relative $q_T$

- $k_T \sim m$ from collinear graphs matters in final state
  - can still neglect parton $k_T$ in hard scattering
  - but do not $\int d^2k$ in parton densities and fragm. fcts.
    $\sim k_T$ dependent/unintegrated PDFs
    also called TMDs (transverse-momentum distributions)

- theoretical framework: TMD factorisation
  - also called $k_T$ factorisation
different from (but related to) $k_T$ factorisation at small $x$
**Measured transverse momentum**

\[ e + p \rightarrow e + h^\pm + X \]

\( h^\pm = \text{charged hadron} \)

H1, hep-ex/9610006

\[ p + p \rightarrow \ell^+\ell^- + X \]

CMS, arXiv:1909.04133
$k_T$ dependent parton densities

$k_T$ integrated:

$$f_1(x) = \int \frac{dz^-}{4\pi} e^{iz^- p^+ x} \langle p, s | \bar{q}(0) \gamma^+ W(0, \infty) W(\infty, z^-) q(z^-) | p, s \rangle \bigg|_{z^+ = 0, z = 0}$$

$k_T$ dependent:

$$\int \frac{dz^-}{4\pi} \frac{d^2z}{(2\pi)^2} e^{iz^- p^+ x} e^{-ikz} \langle p, s | \bar{q}(0) \gamma^+ W(0, \infty) W(\infty, z^-, z) q(z^-, z) | p, s \rangle \bigg|_{z^+ = 0}$$

- fields at different transv. positions
- implications on Wilson lines $\rightarrow$ later

- notation: write $k_T$ in text, omit $T$ in boldface vector $\mathbf{k}$ etc.
\( k_T \) dependent parton densities

\( k_T \) integrated:

\[
f_1(x) = \int \frac{dz^-}{4\pi} e^{iz^-p^+x} \left\langle p, s \left| \bar{q}(0)\gamma^+ W(0, \infty) W(\infty, z^-) q(z^-) \right| p, s \right\rangle \bigg|_{z^+ = 0, z = 0}
\]

\( k_T \) dependent:

\[
\int \frac{dz^-}{4\pi} \frac{d^2 z}{(2\pi)^2} e^{iz^-p^+x} e^{-ikz} \left\langle p, s \left| \bar{q}(0)\gamma^+ W(0, \infty) W(\infty, z^-, z) q(z^-, z) \right| p, s \right\rangle \bigg|_{z^+ = 0}
\]

\[
= f_1(x, k^2) - \frac{\epsilon^{ij} k^i s^j}{m} f_{1T}(x, k^2)
\]

\( \epsilon^{12} = -\epsilon^{21} = 1 \)

\( \epsilon^{11} = \epsilon^{22} = 0 \)

- fields at different transv. positions
  - implications on Wilson lines → later

- correlations between spins and transv. momentum
  - e.g. Sivers function \( f_{1T}^{\perp} \)
A zoo of distributions

- **collinear twist 2 densities:**
  - $f_1$ unpol. quark in unpol. proton
  - $g_1$ correlate $s_L$ of quark with $S_L$ of proton
  - $h_1$ correlate $s_T$ of quark with $S_T$ of proton

- **$k_T$ dependent twist 2 densities:**
  - $f_1, g_1, h_1$ as above
  - $f_{1T}$ correlate $k_T$ of quark with $S_T$ of proton (Sivers)
  - $h_{1T}$ correlate $k_T$ and $s_T$ of quark (Boer-Mulders)
  - $g_{1T}, h_{1T}^T, h_{1L}^T$ three more densities

- analogous for fragmentation functions:
  - $f_1 \leftrightarrow D_1$ unpolarized
  - $h_{1T}^T \leftrightarrow H_{1T}^T$ Collins fragm. fct.
TMD factorisation (SIDIS as example)

- take $Q$ large and $q_T$ small ($\sim m$ for power counting purposes)

- transverse-momentum dep’t distribution and fragmentation fcts.
- only virtual corrections to hard subgraph
  no radiation of high-$p_T$ partons allowed
**TMD factorisation** (SIDIS as example)

- take $Q$ large and $q_T$ small ($\sim m$ for power counting purposes)

- soft gluon exchange does not cancel in sum over hadronic final state at leading-power accuracy gives **soft factor** in factorisation formula

$$S' = \text{universal non-perturbative fct}$$
$$\rightarrow 1 \text{ when integrate over } k_T$$

cancellation between real
TMD factorisation (SIDIS as example)

▷ take $Q$ large and $q_T$ small ($\sim m$ for power counting purposes)

▷ soft gluon exchange does not cancel in sum over hadronic final state at leading-power accuracy gives soft factor in factorisation formula

$S' = \text{universal non-perturbative fct}$

$\rightarrow 1 \text{ when integrate over } k_T$

Cancellation between real and virtual graphs
SIDIS at low $q_T$

- factorisation formula

$$\frac{d\sigma_{\gamma^* p}}{dz \, d\mathbf{q}^2} = (\text{kin. fact.}) \times |H(\mu)|^2 \int d^2 \mathbf{p} \, d^2 \mathbf{k} \, d^2 \mathbf{l} \, \delta^{(2)}(\mathbf{p} - \mathbf{k} + \mathbf{l} + \mathbf{q})$$

$$\times \sum_{i=q,\bar{q}} e_i^2 \, f^i(x, \mathbf{p}, \mu) \, D^i(z, \mathbf{k}, \mu) \, S(\mathbf{l}, \mu)$$

- no $\int d^2 \mathbf{k}$ in parton densities $\leadsto$ no DGLAP type evolution !!

- various azimuthal and spin asymmetries
SIDIS at low $q_T$

- factorisation formula

$$\frac{d\sigma_{\gamma^* p}}{dz \, dq^2} = (\text{kin. fact.}) \times |H(\mu)|^2 \int d^2p \, d^2k \, d^2l \, \delta^{(2)}(p - k + l + q) \times \sum_{i=q,\bar{q}} e_i^2 \, f^i(x, p, \mu) \, D^i(z, k, \mu) \, S(l, \mu)$$

- simplifies if Fourier transform

$$f(p) \rightarrow f(b), \ D(k) \rightarrow D(b), \ S(l) \rightarrow S(b):$$

$$\frac{d\sigma_{\gamma^* p}}{dz \, dq^2} = (\text{kin.fact.}) \times |H(\mu)|^2 \int d^2b \, e^{-ibq} \sum_{i=q,\bar{q}} e_i^2 \, f^i(x, b, \mu) \, D^i(z, b, \mu) \, S(b, \mu)$$

note: $b$ here not the same as $b$ in GPDs (will later call $z$)

- redefine $f$ and $D$ to each absorb factor $\sqrt{S}$
SIDIS at low $q_T$

- TMD distributions also depend on rapidity parameter $\zeta$
- Collins-Soper equation and RGE for $f$ (same for $D$):

$$\frac{d}{d \ln \sqrt{\zeta}} f(x, b, \zeta, \mu) = K(b, \mu) f(x, b, \zeta, \mu)$$

$$\frac{d}{d \ln \mu} f(x, b, \zeta, \mu) = \gamma_F(\zeta, \mu) f(x, b, \zeta, \mu) \quad \text{(no } x \text{ integral as in DGLAP eq.)}$$

- “cusp anomalous dimension”

$$\frac{dK(b, \mu)}{d \ln \mu} = \frac{d\gamma_F(\zeta, \mu)}{d \ln \sqrt{\zeta}} = -\gamma_K(\mu) = -C_F \frac{2\alpha_s(\mu)}{\pi} + \ldots$$

$$C_F = \frac{N_c^2 - 1}{2N_c} \text{ for colour group } SU(N_c)$$
SIDIS at low $q_T$

- TMD distributions also depend on rapidity parameter $\zeta$
- Collins-Soper equation and RGE for $f$ (same for $D$):

\[
\frac{d}{d \ln \sqrt{\zeta}} f(x, b, \zeta, \mu) = K(b, \mu) f(x, b, \zeta, \mu)
\]

\[
\frac{d}{d \ln \mu} f(x, b, \zeta, \mu) = \gamma_F(\zeta, \mu) f(x, b, \zeta, \mu) \quad \text{(no $x$ integral as in DGLAP eq.)}
\]

- solution:

\[
\frac{f(x, b, \zeta, \mu)}{f(x, b, \zeta_0, \mu_0)} = \exp \left\{ - \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_K(\mu') \ln \frac{\sqrt{\zeta}}{\mu'} - \gamma_F(\mu'^2, \mu') \right] + K(b, \mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}
\]

- exp{\ldots} = Sudakov factor
- in exponent have “double logarithms” $\ln^2(\mu/\mu_0)$ for $\zeta \sim \mu^2$
  in cross section set $\sqrt{\zeta} \sim \mu \sim Q$ and $\sqrt{\zeta_0} \sim \mu_0 \sim q_T$
- $K(b, \mu)$ calculable in pert. theory only if $b$ is small
  need to interpolate between small and large $b$
Compare with collinear factorisation for large $q_T$

\[ \frac{d\sigma_{\gamma^* p}}{dz\, dq^2} = (\text{kin. fact.}) \times \int_x^1 \frac{dx}{\hat{x}} \int_z^1 \frac{dz}{\hat{z}} \delta \left( \frac{q^2}{Q^2} - \frac{(1 - \hat{x})(1 - \hat{z})}{\hat{x}\hat{z}} \right) \]
\[ \times \sum_{i,j=q,\bar{q},g} f_i \left( \frac{x}{\hat{x}}, \mu^2 \right) D_j \left( \frac{z}{\hat{z}}, \mu^2 \right) C_{ij} \left( \hat{x}, \hat{z}, \ln \frac{\mu^2}{Q^2} \right) \]

- $C_{ij}$ start at $\mathcal{O}(\alpha_s)$, must emit partons recoiling against $q$
- convolution in momentum fractions
Wilson lines in short-distance factorisation

- exchange of $>2$ partons between $H$ and $B$ power suppressed
- except for $A^+$ gluon exchange
  $\leadsto$ resum to all orders

- $H_\mu(l)$ all components big
- $B_\mu(l) \propto l^\mu$ only plus-component big ($p^\mu$ and $l^\mu$ collinear momenta)
  $\leadsto H_\mu B^\mu \approx H^- B^+$
Wilson lines in short-distance factorisation

- exchange of $>2$ partons between $H$ and $B$ power suppressed
- except for $A^+$ gluon exchange
  $\rightsquigarrow$ resum to all orders

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  $\rightsquigarrow H_\mu B^\mu \approx H^- B^+$

- Ward identities $\rightsquigarrow$ gluons removed from $H$
  in $B$ obtain Wilson line $W(a, b) = \mathbb{P} \exp \left[ ig \int^b_a dz^- A^+(z) \right]_{z^+ = 0, z = 0}$

$$q(x) \propto \int dz^- e^{i \frac{\pi x}{p^+} z^-} \langle p | \bar{q}(0) W(0, z) \gamma^+ q(z) | p \rangle \Big|_{z^+ = 0, z = 0}$$
Wilson lines in TMDs

\[ q(x, k^2) \propto \int dz^- d^2z e^{ixp^+z^-} e^{-ikz} \langle p | \bar{q}(0) W_P(0, z) \gamma^+ q(z) | p \rangle \bigg|_{z^+=0} \]

- space-time structure of process \( \leadsto \) path \( P \) in Wilson line
- **SIDIS**: interactions **after** quark struck by photon
- **DY**: interactions **before** quark annihilates
Wilson lines in TMDs

\[ q(x, k^2) \propto \int d^2 z e^{ixp^+z^-} e^{-i k z} \langle p | \bar{q}(0) \ W_P(0, z) \gamma^+ q(z) | p \rangle \mid_{z^+ = 0} \]

- space-time structure of process \( \rightsquigarrow \) path \( P \) in Wilson line
- SIDIS: interactions after quark struck by photon
- DY: interactions before quark annihilates
- obtain “staple like” paths
  - Feynman gauge: pieces at \( z^- \rightarrow \pm \infty \) not important
  - light-cone gauge \( A^+ = 0 \): straight sections \( \rightarrow 1 \)
    - all effects from \( z^- \rightarrow \pm \infty \)
Wilson lines at lowest order

\[
\begin{align*}
\bar{u}(\hat{k} + q)\left(\mathbf{i}q\gamma^8\right) & = \\
\left(\hat{k} + q - \hat{e}\right)^8 & = \\
\mathbf{k} & \rightarrow \hat{k} = (k^+, 0, \mathbf{0}) \\
\mathbf{e} & \rightarrow \hat{e} \\
\ldots & = \bar{u}(\hat{k} + q) \frac{-i\gamma_8}{k^+} \frac{\hat{e} \cdot \gamma}{(\hat{k} + q - \hat{e})^8} = \bar{u}(\hat{k} + q) \mathbf{i} \frac{i\gamma_8}{k^+ - i\varepsilon} \\
\left(\hat{k} + q\right) - (\hat{k} + q\hat{e}) & = \\
\frac{i}{(\hat{k} + q - \hat{e})^8} & = \frac{i(\hat{k} + q - \hat{e})^8}{(\hat{k} + q - \hat{e})^8 + i\varepsilon} = -\frac{(\hat{k} + q - \hat{e})^8}{2q^-} \mathbf{i} \frac{i}{k^+ - i\varepsilon} \\
2q^-(-e^+) & = 
\end{align*}
\]
Wilson lines at lowest order

\[ \int \text{d}y^- \frac{i}{-y^+ - i\epsilon} \int \frac{\text{d}^2 y_r}{(2\pi)^2} \int \text{d}e^+ \frac{\text{d}^2 x_r}{2\pi} \ e^{i(k^+-k_r^+)} e^{i(k^-+k_r^-)} e^{iS_{T_r}} e^{iS_{T_r}} \]

\[ \frac{1}{e^{+ - i\epsilon}} \int \text{d}y^- \frac{i}{y^+ - i\epsilon} \int \frac{\text{d}^2 y_r}{(2\pi)^2} \int \text{d}e^+ \frac{\text{d}^2 x_r}{2\pi} \ e^{iS_{T_r}} e^{iS_{T_r}} \]

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Wilson lines at lowest order

\[ \frac{i}{(q - \hat{k} + \hat{z})^2 + i\epsilon} = \frac{i}{2q^-} + \frac{i}{2q^+ + i\epsilon} \]
Rapidity divergences revisited

- arise as $\int \frac{d\ell^+}{\ell^+}$ from region $\ell^+ \to 0$ at nonzero $\ell^-$
  - negative rapidity, should not be inside TMD

- need to regulate $\sim$ rapidity “cutoff” parameter $\zeta$

\[
\frac{d}{d \ln \sqrt{\zeta}} = \frac{d}{d(\text{rapidity})}
\]

- $\int d^2k \sim$ divergences cancel between real and virtual graphs
  - not present in usual PDFs (or GPDs)
Wilson line has physical consequences

- **transverse** proton polarization $\rightsquigarrow$ anisotropic $k$ distribution

$$f_{q/p^\uparrow}(x, k) = f_1(x, k^2) + \frac{(S \times k) \cdot p}{m |p|} f_{1T\perp}(x, k^2)$$

- induces anisotropic $p_T$ distribution in SIDIS (Sivers effect) observed experimentally

- time reversal changes sign of $(S \times k) \cdot p$
  $\rightsquigarrow$ Sivers function $= 0$ ??
Wilson line has physical consequences

- **transverse** proton polarization $\Rightarrow$ anisotropic $k$ distribution

$$f_{q/p\uparrow}(x, k) = f_1(x, k^2) + \frac{(S \times k) \cdot p}{m|p|} f_{1T\perp}(x, k^2)$$

- induces anisotropic $p_T$ distribution in SIDIS (Sivers effect) observed experimentally

- time reversal changes sign of $(S \times k) \cdot p$

  $\Rightarrow$ Sivers function $= 0$ ??

- **no**: time reversal interchanges Wilson lines for SIDIS (future pointing) and DY (past pointing)

  $\Rightarrow f_{1T}^{\perp,\text{SIDIS}}(x, k^2) = -f_{1T}^{\perp,\text{DY}}(x, k^2)$

  J. Collins ’02
More complicated processes

▶ examples: \( pp \rightarrow \gamma + \text{jet} + X, \ pp \rightarrow \pi + \text{jet} + X \)

▶ more partons in initial and final state
  \( \leadsto \) more complicated Wilson lines
  \( \leadsto \) more parton densities and fragm. functions

Bomhof, Mulders, Pijlman, Buffing '04-'15

two-loop analysis \( \leadsto \) breakdown of TMD factorisation

Mulders, Rogers '10
Relation between high-$q_T$ and low-$q_T$ descriptions

- for $q_T \gg m$ calc. $k_T$ dependent densities from coll. ones:

\[
f^i_1(x, k^2; \zeta, \mu) = \frac{1}{k^2} \sum_j \int_1^1 \frac{dx'}{x'} K^{ij} \left( \frac{x}{x'}, \ln \frac{k^2}{\zeta} \right) f^j_1(x'; \mu)
\]

$K$ closely related with DGLAP splitting functions $P$
Relation between high-\(q_T\) and low-\(q_T\) descriptions

- for \(q_T \gg m\) calc. \(k_T\) dependent densities from coll. ones:

\[
f_i(x, b^2; \zeta, \mu) = f_i(x; \mu) + \sum_j \int_0^1 dx' \frac{d x'}{x'} \tilde{K}^{ij} \left( \frac{x}{x'}, \ln \frac{\mu^2}{\zeta}, \ln(\mu^2 b^2) \right) f_j(x'; \mu)
\]

\(\tilde{K}\) closely related with DGLAP splitting functions \(P\)
Comparison between high-$q_T$ and low-$q_T$ descriptions

- collinear fact. requires $q_T \gg m$
  $k_T$ fact. requires $q_T \ll Q$
  $\sim$ in region $m \ll q_T \ll Q$ both approaches are valid

- compare $q_T \gg m$ limit of $k_T$ fact. result
  with $q_T \ll Q$ limit of coll. fact. result
  $\sim$ full agreement for unpol. cross section

  Collins, Soper, Sterman '85; Bacchetta et al. '08

- detailed comparison also for various spin asymmetries
  e.g. Sivers asy. in SIDIS or Drell-Yan at low $q_T$ (Sivers fct.) and
  high $q_T$ (Qiu-Sterman fct.)
  Ji, Qiu, Vogelsang, Yuan '06; Koike, Vogelsang, Yuan '07
Correspondence at level of graphs

**high-\(q_T\) calculation**

**low-\(q_T\) calculation with \(q_T \gg m\)**

\[ (k-l)^+ \]

\[ D(z,k_T) \]
Correspondence at level of graphs

high-$q_T$ calculation

low-$q_T$ calculation with $q_T \gg m$
Transverse momentum vs. position

- variables related by 2d Fourier transforms, e.g.
  - quark fields \( \bar{q}(k, z^-) = \int d^2 z \, e^{izk} q(z, z^-) \)
  - proton states \( |p^+, b\rangle = \int d^2 p \, e^{-i bp} |p^+, p\rangle \)

- in bilinear operators

\[
\bar{q}(k) \bar{q}(l) = \int d^2 y \, d^2 z \, e^{-i(yk-zl)} \bar{q}(y) q(z)
\]
\[
y k - z l = \frac{1}{2} (y + z)(k - l) + \frac{1}{2} (y - z)(k + l)
\]

'average' transv. momentum ↔ position difference
transv. momentum transfer ↔ 'average' position

- 'average' transv. mom. and position not Fourier conjugate
Mind the difference

TMDs

\[ \int d^2 z \ e^{-izk} \langle 0 | \bar{q}(-\frac{1}{2}z) \cdots q(\frac{1}{2}z) | 0 \rangle \]

impact parameter distributions

\[ \int d^2 \Delta \ e^{-ib\Delta} \langle -\frac{1}{2} \Delta | \bar{q}(0) \cdots q(0) | \frac{1}{2} \Delta \rangle \]

(longitudinal variables not shown for simplicity)

Fourier conjugates:

average transv. momentum  \[ q(x, k) \]

difference of transv. momenta  \[ H(x, \Delta)_{\xi=0} \]

\[ \leftrightarrow \]

difference of transv. positions

Wilson lines, Sudakov resummation, ...

average transv. position  \[ q(x, b) \]
Mind the difference

TMDs

\[ \int d^2 z \, e^{-i z k} \langle 0 | \bar{q}(\frac{1}{2} z) \ldots q(\frac{1}{2} z) | 0 \rangle \]

impact parameter distributions

\[ \int d^2 \Delta \, e^{-i b \Delta} \langle -\frac{1}{2} \Delta | \bar{q}(0) \ldots q(0) | \frac{1}{2} \Delta \rangle \]

more general:

GTMDs

\[ \int d^2 z \, e^{-i z k} \langle -\frac{1}{2} \Delta | \bar{q}(\frac{1}{2} z) \ldots q(\frac{1}{2} z) | \frac{1}{2} \Delta \rangle \]
Mind the difference

TMDs

\[ \int d^2 z \ e^{-i z \vec{k}} \langle 0 | \bar{q}(-\frac{1}{2} z) \ldots q(\frac{1}{2} z) | 0 \rangle \]

impact parameter distributions

\[ \int d^2 \Delta \ e^{-i \Delta \vec{b}} \langle -\frac{1}{2} \Delta | \bar{q}(0) \ldots q(0) | \frac{1}{2} \Delta \rangle \]

more general:

GTMDs

\[ \int d^2 z \ e^{-i z \vec{k}} \langle -\frac{1}{2} \Delta | \bar{q}(-\frac{1}{2} z) \ldots q(\frac{1}{2} z) | \frac{1}{2} \Delta \rangle \]

Fourier transf. from \( \Delta \) to \( b \)

\( \sim \) Wigner functions

parton momentum and position within limits of uncertainty rel’n
Relations

\[ H(x, z, \Delta) \quad \text{FT} \ z \leftrightarrow k \quad H(x, k, \Delta) \quad \text{FT} \ \Delta \leftrightarrow b \quad W(x, k, b) \]

\[ \Delta = 0 \]

\[ q(x, z) \quad \text{FT} \ z \leftrightarrow k \quad q(x, k) \]

\[ \int d^2k \]

\[ H(x, \Delta) \quad \text{FT} \ \Delta \leftrightarrow b \quad q(x, b) \]

\[ \int d^2b \]

\[ \int d^2k \]

\[ \int d^2b \]

\[ \Delta = 0 \]

\[ \int dx x^{n-1} \]

\[ q(x) \]

\[ F_n(\Delta) \]

\[ \int d^2k \] needs UV regularization

GPDs taken at zero skewness \( \xi = 0 \)

\( k \) dep’t functions have \( \zeta \) dep’ce
More relations

\[ \int d^2k \text{ needs UV regularization} \quad k \text{ dep't functions have } \zeta \text{ dep'ce} \]
naive:

\[ \int d^2k \, q(x, k) = q(x) \]

cannot be true because \( q(x, k) \sim 1/k^2 \) at large \( k \)

correct:

\[ \int d^2k \, q(x, k; \zeta, \mu) = q(x; \mu) + \text{calculable terms of } \mathcal{O}(\alpha_s) \]

\[ k^2 < \mu^2 \]

Fourier trf. from \( k \) to \( z \):

instead of \( \int d^2k \) take \( \int d^2k \, e^{ikz} \) with \( |z| = 1/\mu \)

oscillations suppress region \( |k| \gg 1/|z| \)

\[ q(x, z; \zeta, \mu) = q(x; \mu) + \text{calculable terms of } \mathcal{O}(\alpha_s) \]

see earlier slide
Summary of part 6

- TMD factorisation for measured $p_T \ll$ hard scale
- important differences with collinear factorisation, different evolution
- subtle dynamical effects due to gluons $\sim$ Wilson lines
- valid for restricted class of processes
  for some cases smooth theoretical transition to high $p_T$ regime
- theoretically controlled access to transverse parton momentum
- Wigner functions: unifying framework for describing transverse momentum and position
Notes
## Notes

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