Lecture 2. The constituent Quark Model

- ***Summary***

1. The overall panorama ✓
2. Constituent Quark Model and masses of conventional mesons and baryons ✓
3. Light and Heavy Tetraquarks. First comparison with hadron molecules
4. Tetraquarks and the EightFold Way
5. $X(3872)$ and the missing partners
6. Born-Oppenheimer approximation for double charm baryons and tetraquarks
7. Multiquark states in N colours, in the $N \to \infty$ limit
8. Tetraquarks vs. molecules: the Weinberg criterium for $X(3872)$ and the double charm $\mathcal{T}_{cc}^{+}(3875)$
9.

The Fundamental Particle Interactions

Three interactions are operative at particle level, distinguished by strength and selection rules

1. Strong interactions, O(1): act on hadrons, conserve
   - Parity, Charge Conjugation, Time reversal,
   - I (isospin): symmetry group $SU(2)$,
     $$-I \leq I_3 \leq +I,\ n(I) = 2I + 1$$
   - S (strangeness), B (baryon number)
   - typical lifetime ~10^{-23}sec, ($\Gamma \sim 100$ MeV)

lowest mass hadrons

$$\begin{bmatrix} p \\ n \end{bmatrix}_{S=0};\ \begin{bmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{bmatrix}_{S=0}$$

$$\Lambda^0_{S=-1};\ \begin{bmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{bmatrix}_{S=-1};\ \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}_{S=+1};\ \begin{bmatrix} \bar{K}^0 \\ K^- \end{bmatrix}_{S=-1}$$

2. Electromagnetic interactions, $\mathcal{O}(1/137)$: act on hadrons and charged leptons (e, μ) ,
   - conserve P (parity), C (charge conjugation) , T (time reversal)
   - Q (electric charge) and S;
   - L (lepton numbers) and B
   - typical lifetimes $\sim 10^{-18}$ sec

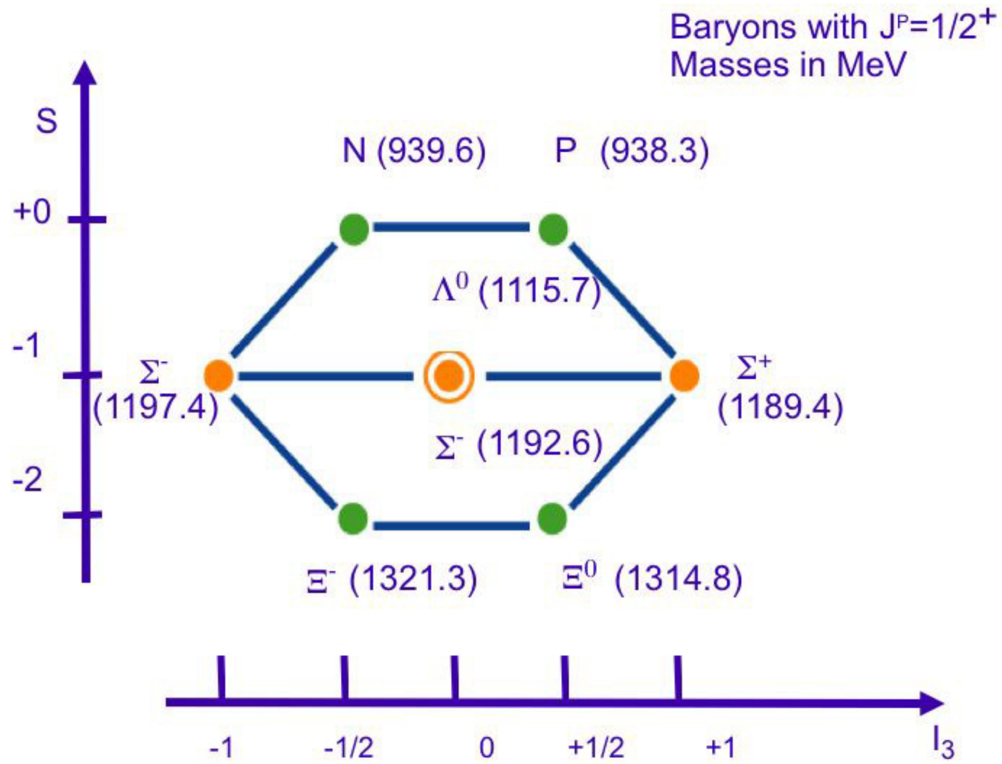
Gell-Mann, Nishijima formula:

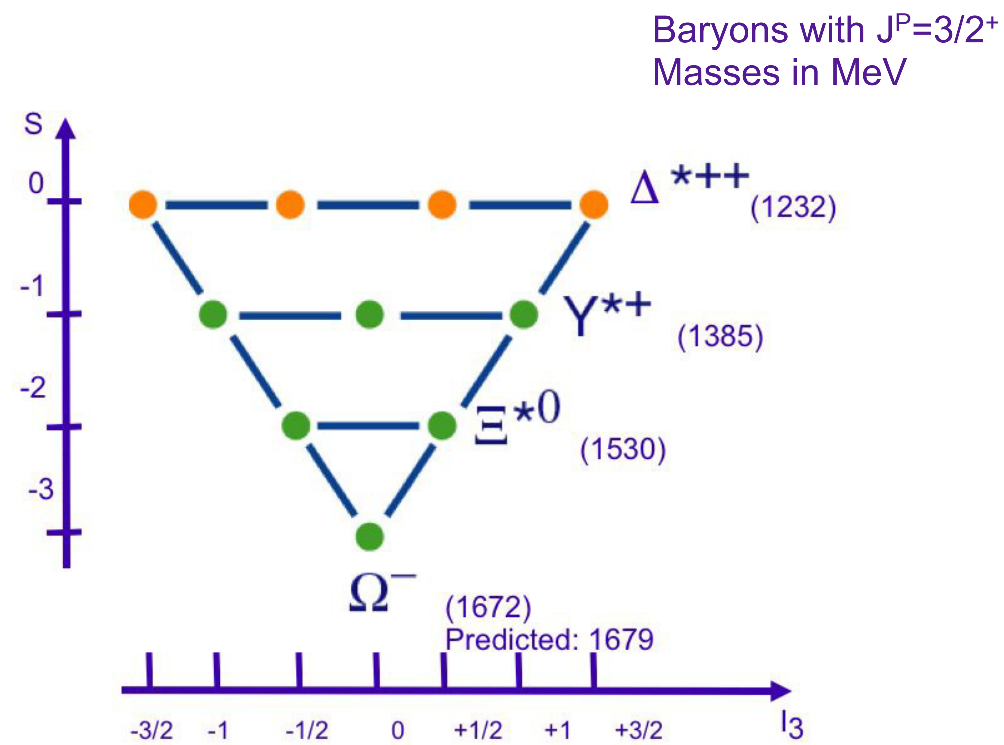
$$Q = I_3 + \frac{S+B}{2} = I_3 + \frac{Y}{2}$$

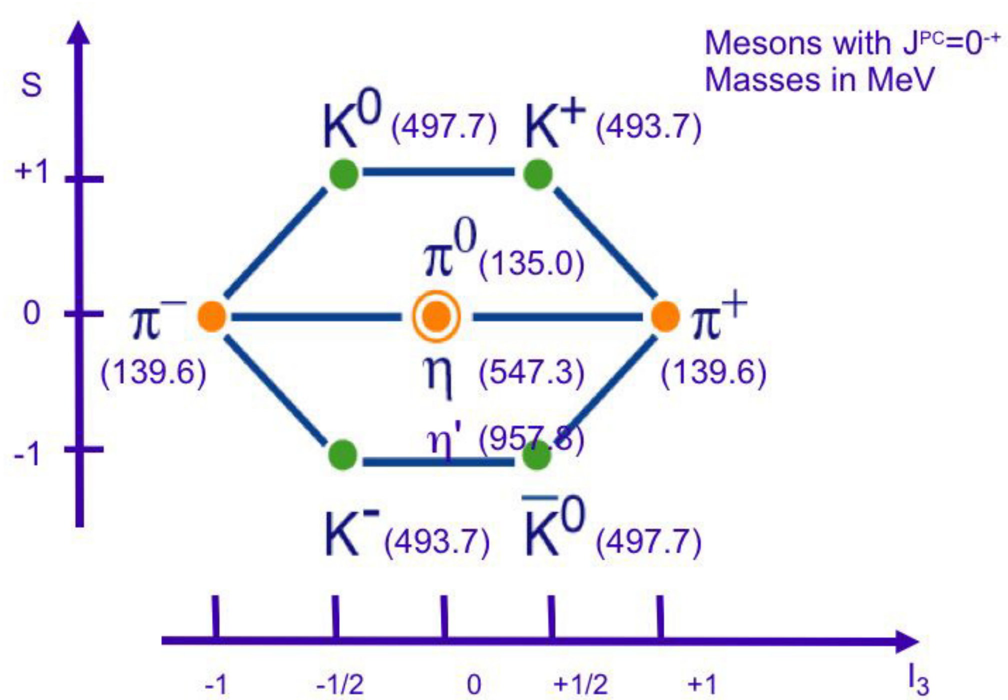
Y= hypercharge

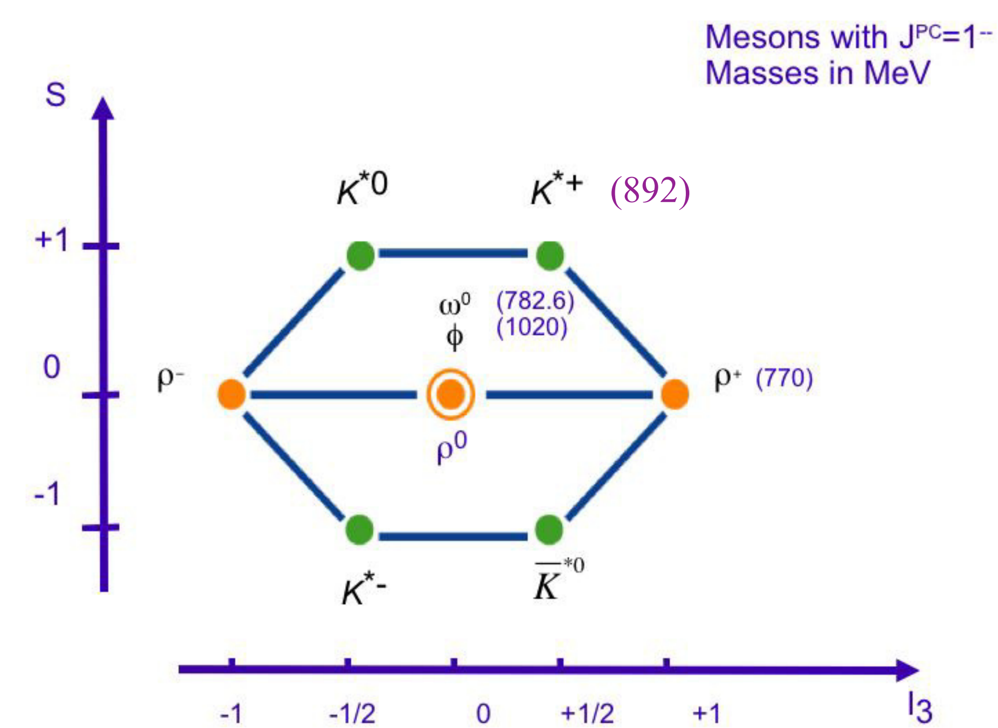
3. Weak Interactions (Fermi, 1932), $\mathcal{O}(10^{-5})$: act on all particles, including ν's,
   - violate: P, CP and T; CPT conserved
   - conserve B, violate: S, L_e and L_μ
   - typical lifetimes $\sim 10^{-12}$ sec, or longer.

Our main characters: the low energy Baryons and Mesons (hadrons)









The constituent way, first attempts

- Fermi&Yang (1949): only F=(p, n) are elementary,

$$\pi - mesons = F\bar{F}$$

- Sakata (1956): one new constituent to account for strange particles: S= (p, n, Λ),

$$mesons = S\bar{S};\ baryons = SS\bar{S}$$

- a clear prediction: there must exist baryons with strangeness S=+1. ***Unfortunately it is a wrong prediction, no such particle has been seen until today*** !

- basic symmetry of Sakata model: SU(2)= isotopic spin symmetry ⇒ SU(3), unitary transformation of the Sakata triplet

Eightfold Way (Gell-Mann, Ne'eman, 1962)

- Symmetry: SU(3)
- Mesons in octet, as in Sakata model
- Baryons in octet and decuplet, forget Sakata!
- assuming SU(3) broken by octet interaction, Gell-Mann and Okubo derived mass-formulae for octet and decuplet

  - octet baryons, a formula very well obeyed:

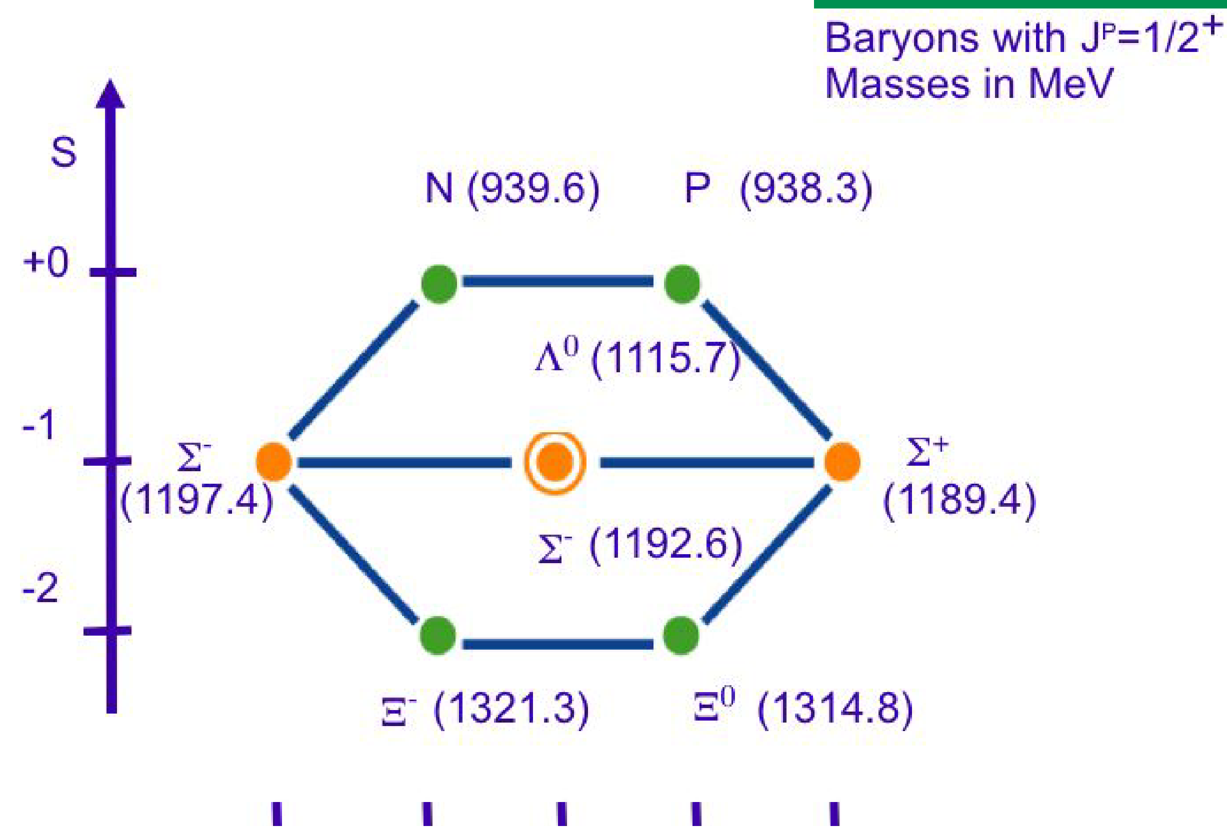
$$\frac{N+\Xi}{2}\ (1128\ \mathrm{MeV}) = \frac{3\Lambda+\Sigma}{4}\ (1136\ \mathrm{MeV})$$

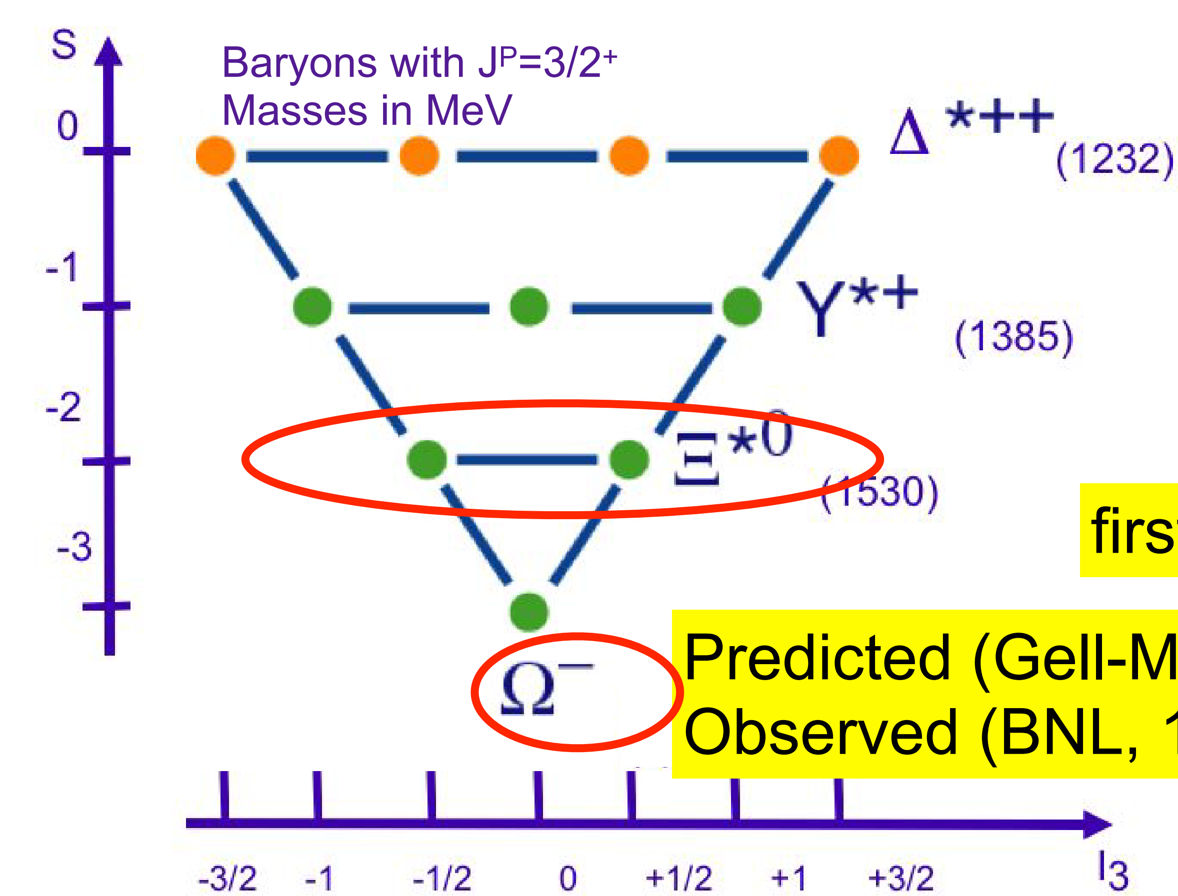
  - decuplet masses equally spaced: from Δ and Σ^* masses one could predict Ξ^* and Ω masses
  - the discovery of two Ξ^* particles was presented at the Ginevra Conference, 1962, and Gell-Mann observed there that their mass checked with equal spacing
  - Ω discovered in 1964 with the expected mass

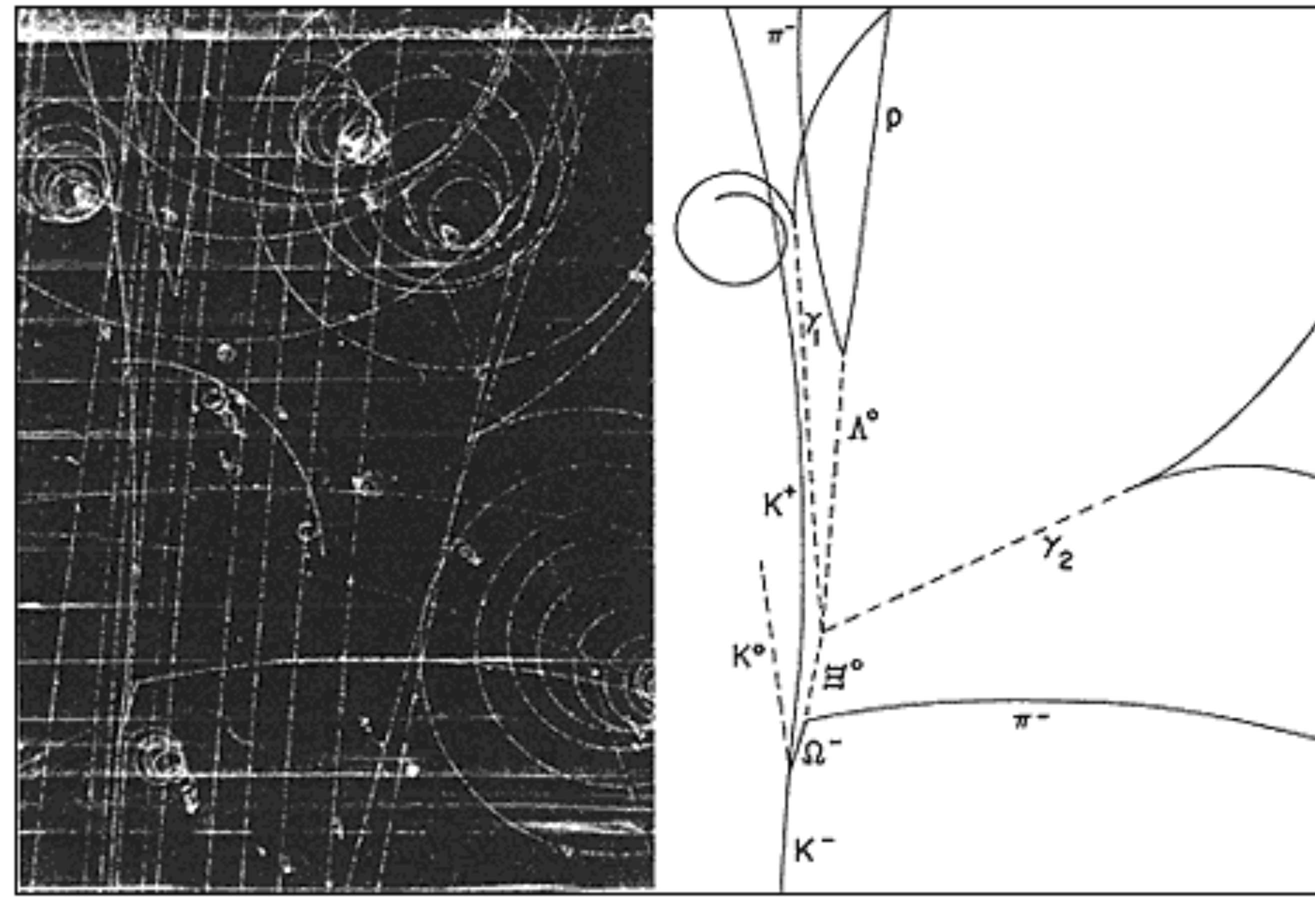
- first mass and quantum number predictions in particle physics !

- SU(3) symmetry was established.

The Ω-







The bubble chamber picture of the first Omega-minus (N. Samios and coworkers)

first confirmation

Predicted (Gell-Mann, 1962) M=1679
Observed (BNL, 1964) M= 1672

Quarks !

- SU(3) representations and symmetry breaking can be studied by pure group theory
- but quarks are much simpler to handle!
- Quarks are the basic SU(3) triplet, first fundamental representation $(\mathbf{3})$
- antiquarks $\bar{q}$: antitriplet, second fundamental representation $(\bar{\mathbf{3}})$

$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix} = \mathbf{3}$$

Quantum numbers :

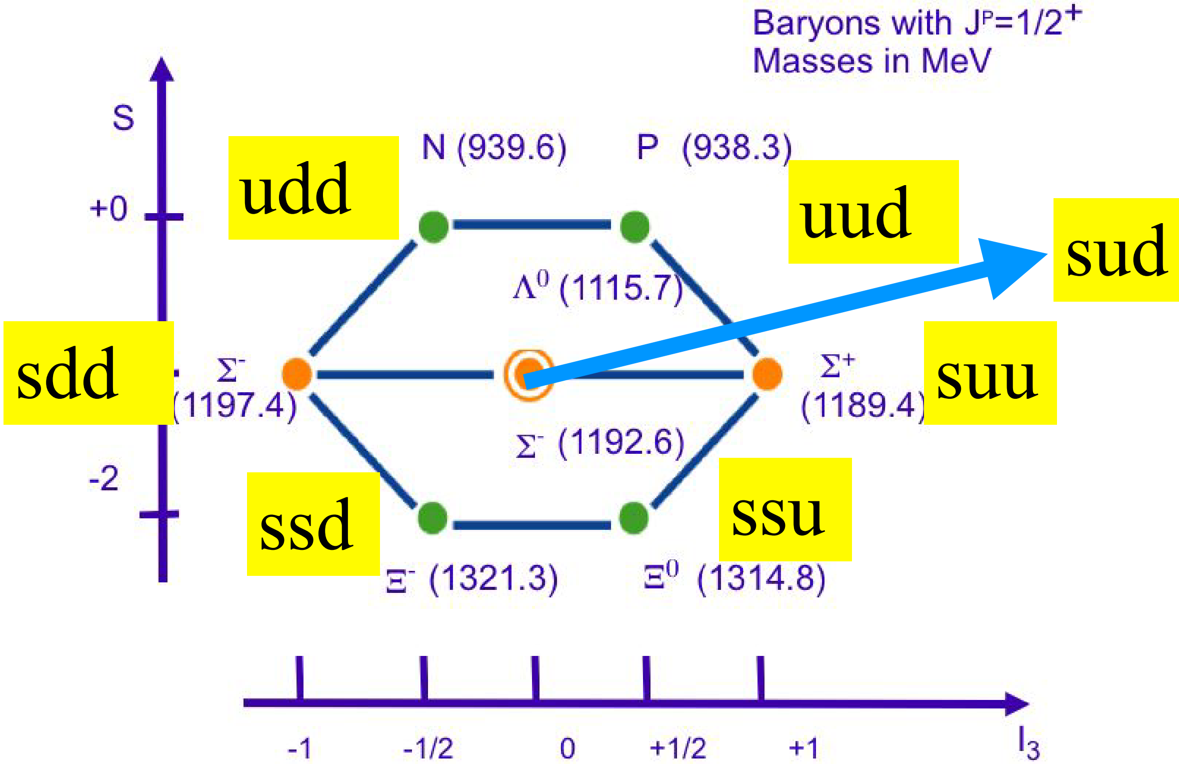
I_3	Y	(S)	Q
$\frac{1}{2}$	$\frac{1}{3}$	(0)	$+\frac{2}{3}$
$-\frac{1}{2}$	$\frac{1}{3}$	(0)	$-\frac{1}{3}$
0	$-\frac{2}{3}$	(-1)	$-\frac{1}{3}$

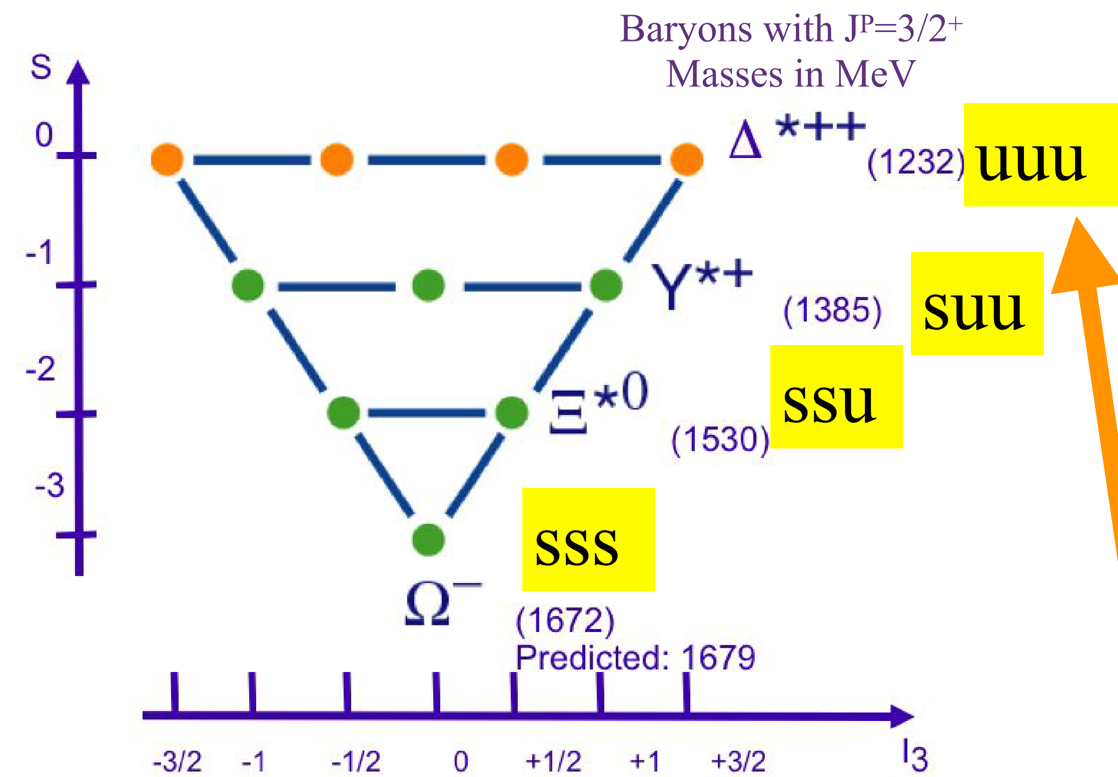
- if spin 1/2, we should be able to construct ***all hadrons*** with ***quark and/or antiquark*** bound states (forget Fermi statistics for a while, we'll come back!)
- how do we make mesons and baryons?

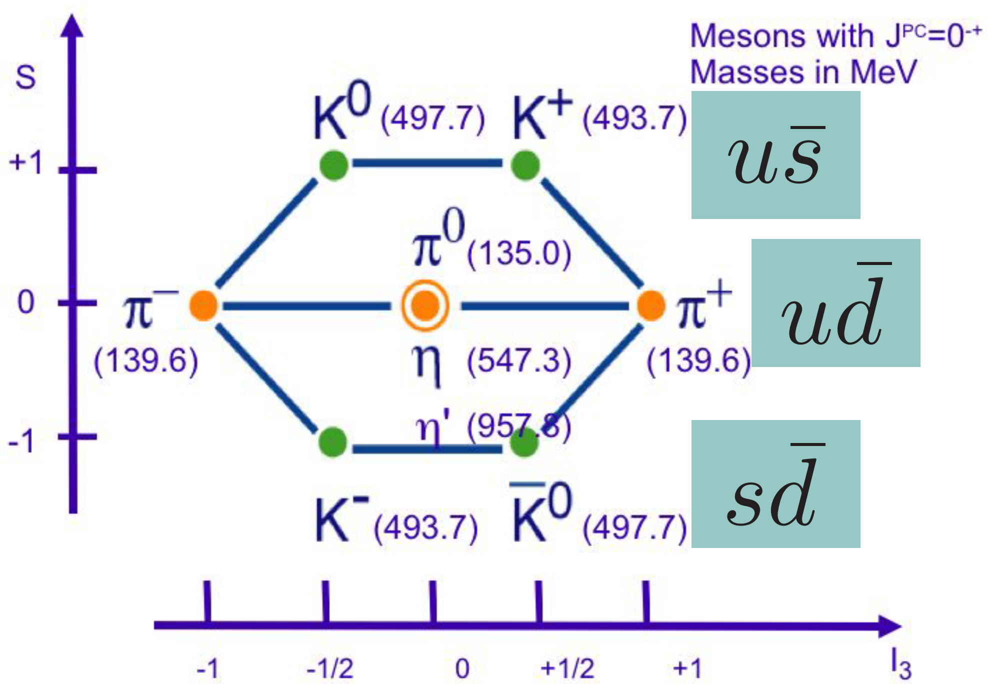
Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc.

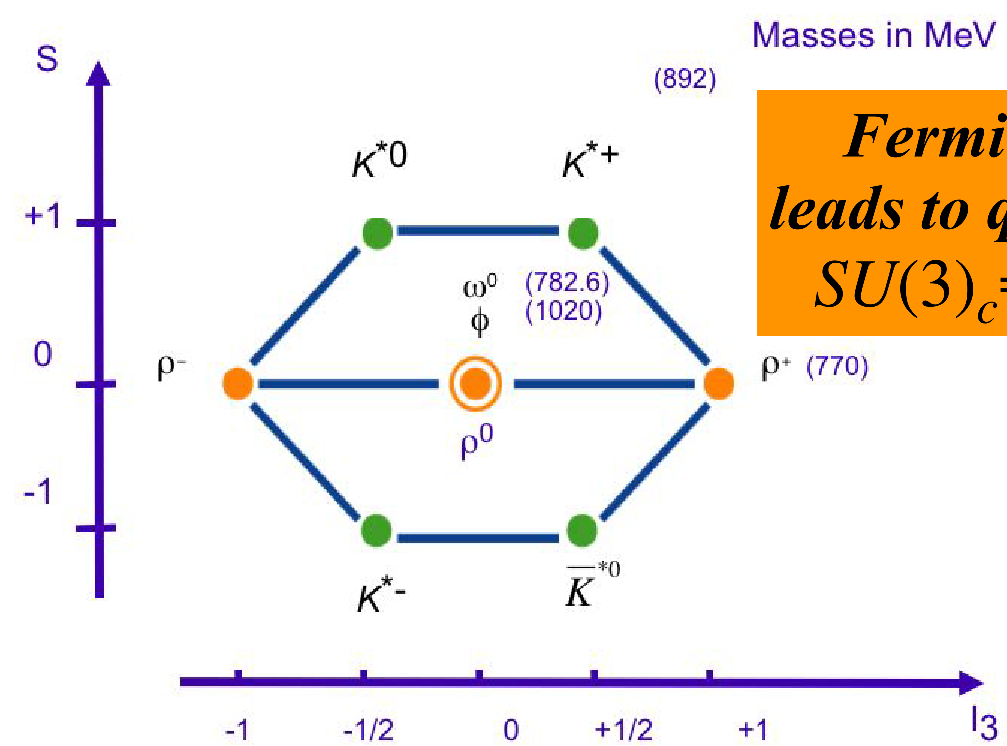
M. Gell-Mann, A Schematic Model of Baryons and Mesons, PL **8**, 214, 1964

Quark composition of the lowest lying Baryons and Mesons





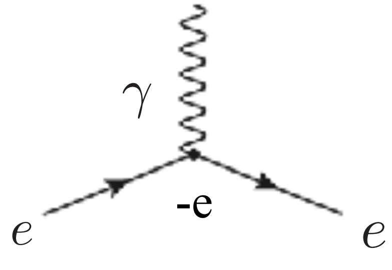




Fermi Statistics leads to quark colour $SU(3)_c$=QCD !!!!

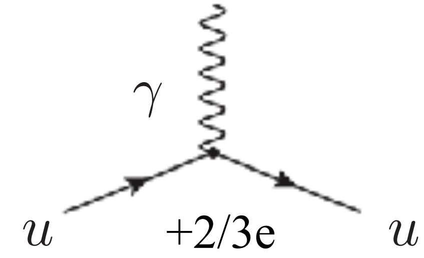
ELETTROMAGNETIC INTERACTIONS

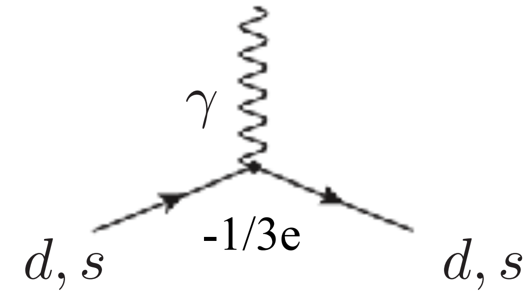
Charged Leptons



$(\mu \to \mu, \; \tau \to \tau)$

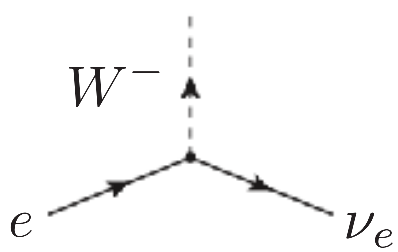
QUARKS





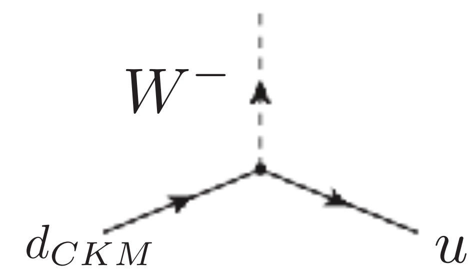
WEAK INTERACTIONS

Leptons-charged currents



$(\mu \to \nu_\mu, \; \tau \to \nu_\tau)$

Quark-charged currents

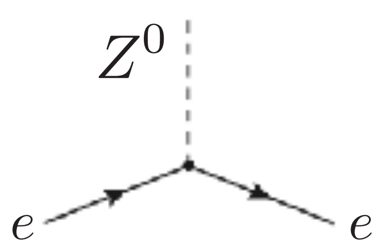


$(s_{CKM} \to c, \; b_{CKM} \to t)$

$$d_C = \cos\theta d + \sin\theta s,$$
$$\theta = \text{Cabibbo angle}$$

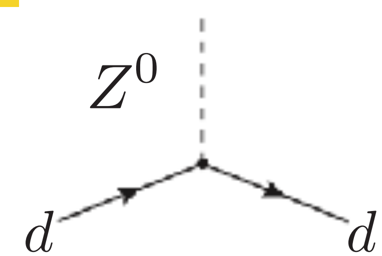
$$\begin{bmatrix} d_{CKM} \\ s_{CKM} \\ b_{CKM} \end{bmatrix} = U_{CKM} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

Leptons-neutral currents



$(\mu \to \mu, \; \tau \to \tau)$

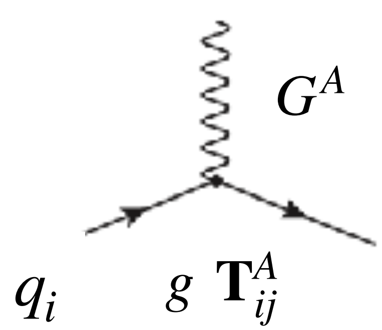
Quark-neutral currents



$(s \to s, \; b \to b)$

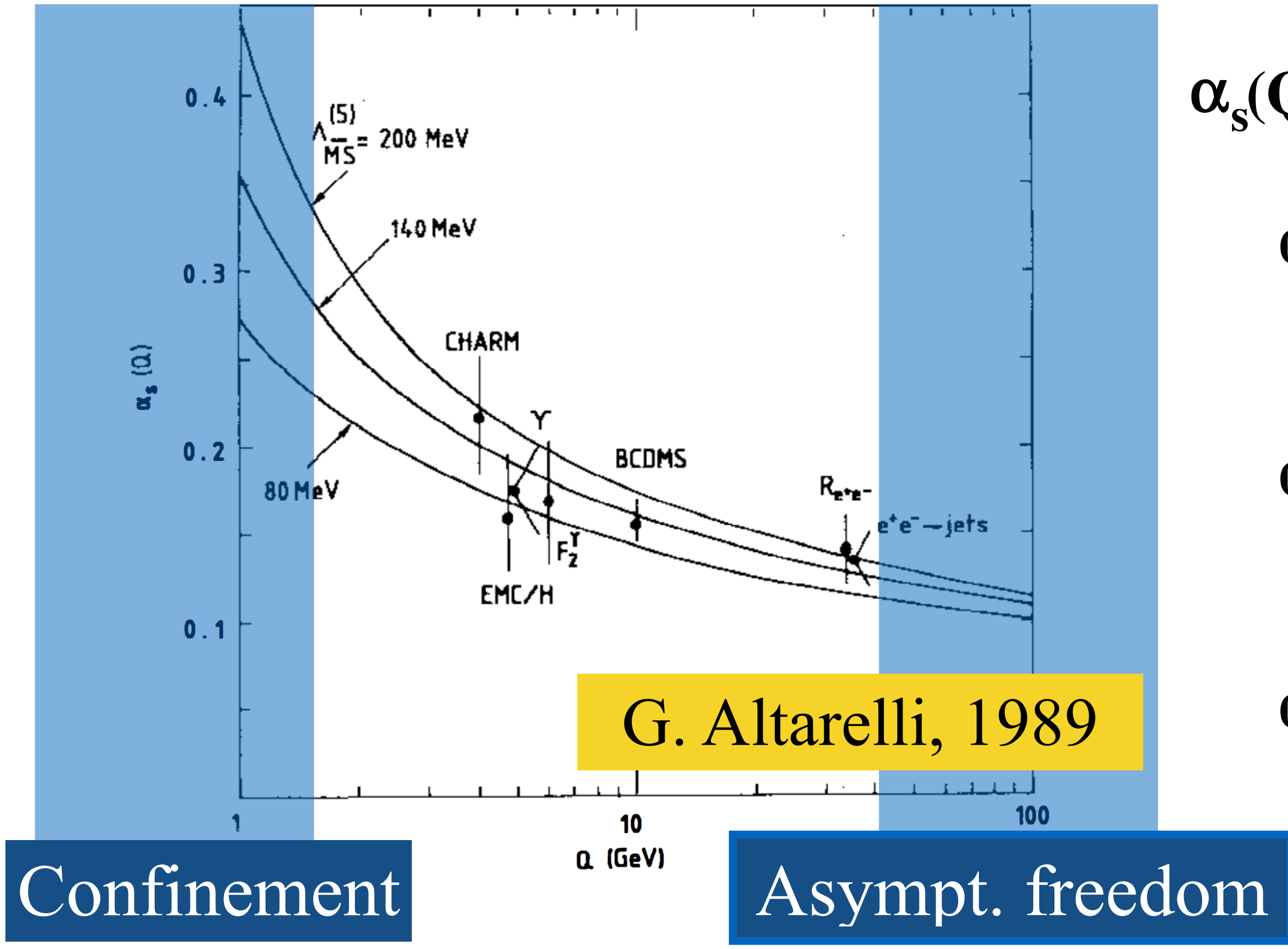
with d_C only, d →s : strangeness changing neutral currents would be allowed (GIM)

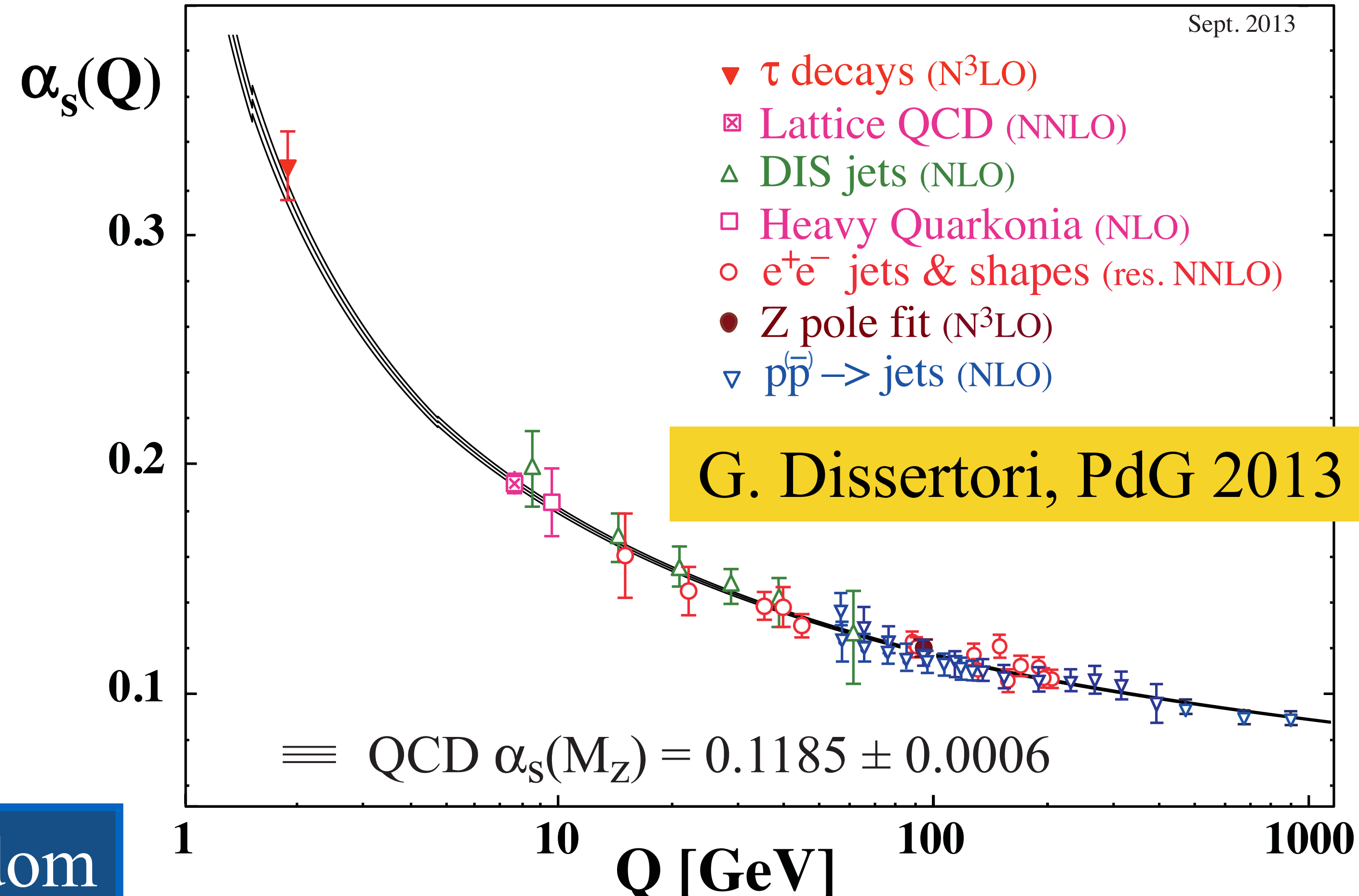
STRONG INTERACTIONS= COLOUR SU(3)



$(A = 1, \; \ldots, \; 8)$

QCD is the answer to (almost) any question





- QCD is asymptotically free
- quarks carry **color symmetry**, $\mathbf{SU(3)_{col}}$, and are confined inside **color singlet hadrons**, e.g. $\Delta^{(++)} = \epsilon^{\alpha\beta\gamma} u^{\uparrow}_{\alpha} u^{\uparrow}_{\beta} u^{\uparrow}_{\gamma}$ Fermi statistics is obeyed

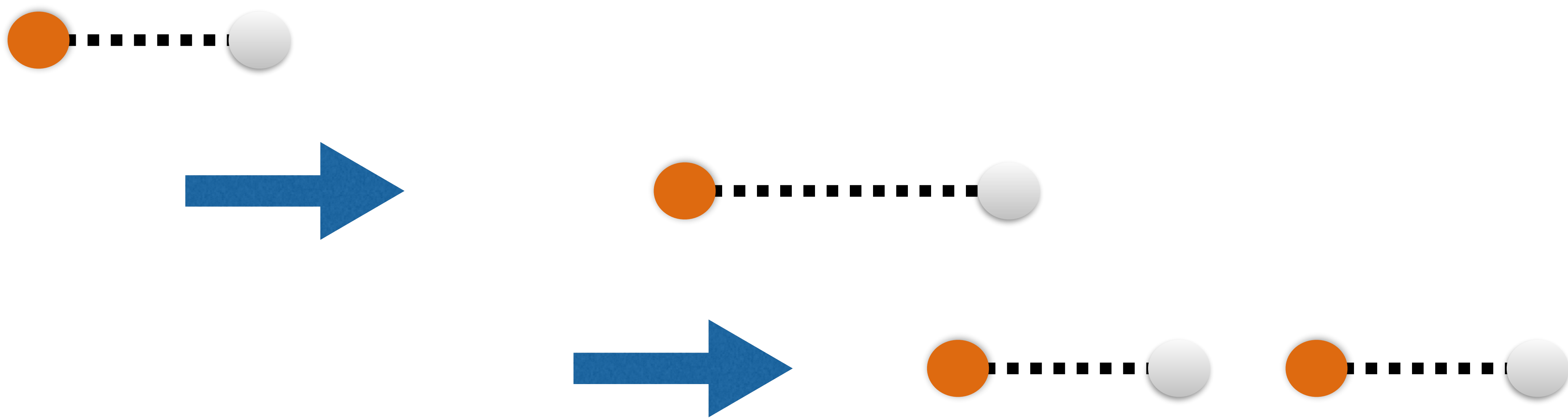
Constituent Quarks

- only one way to make a color singlet with three quarks: no particle proliferation
- proton at rest: only 3 quarks, dressed by strong QCD interactions
- increasing q^2, quarks radiate gluons (the Altarelli-Parisi picture of scaling violations)
- at large q^2, we see quarks and neutral gluons as almost free partons.

QCD Partons

QCD strings and confinement

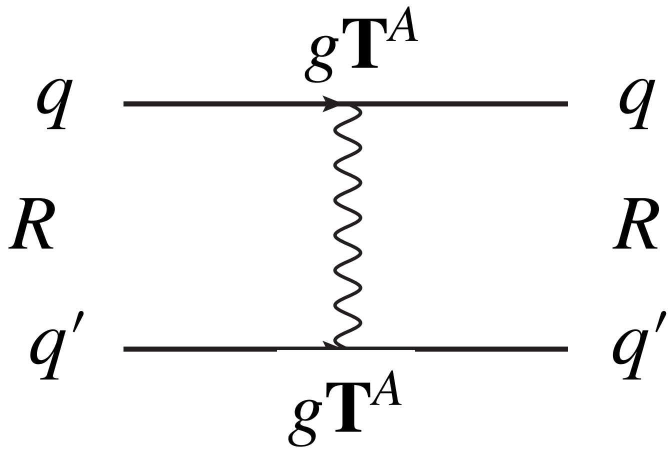
energy on the string is proportional to distance…



quarks do not come out

2. Forces between colored objects:one gluon exchange

(Han&Nambu)



$$\frac{g^2}{k^2} \cdot < R|\mathbf{T}^A \cdot \mathbf{T}^A|R> = \frac{g^2}{k^2}\frac{1}{2}[C_2(R) - 2C_2(q)]$$

$$(\text{note}: C_2(q) = C_2(\bar{q}))$$

$$C_2(R) = \mathbf{T}_R \cdot \mathbf{T}_R$$

R	$C_2(R)$
1	0
8	3
$\mathbf{3}, \bar{\mathbf{3}}$	4/3
6	10/3

- Interaction proportional to $\frac{1}{2}[C_2(R) - C_2(\mathbf{3}) - C_2(\bar{\mathbf{3}})]$:
- quark-antiquark
  - singlet: attractive (-4/3)
  - octet: repulsive (+1/6)
- quark-quark
  - three bar: attractive (-2/3)
  - six: repulsive (+1/3)

- ***quark-antiquark pairs*** bind in color singlet mesons;
- ***diquarks*** bind:

to another quark, to make a color singlet baryon
to an antidiquark, to make a color singlet tetraquark.

Constituent quark model

Old days: Sakharov&Zeldovich
In QCD: De Rujula, Georgi and Glashow, PRL 38 (1977) 317
Revisited & applied to tetraquarks: L.M., A. Polosa, V. Riquer, F, Piccinini, PRD 71 (2005) 014028
An introduction in: H. Georgi, *Lie Algebras in Particle Physics*, Westwiew (1999).

- color string forces produce an overall spin-independent potential that confines quarks inside a definite volume (bag), with some wave functions $\psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$
- residual quark-quark or quark-antiquark interactions are color-magnetic, spin-spin, forces of the form

$$H_{ij} = \frac{g^2}{m_i m_j}(\mathbf{T}_1 \cdot \mathbf{T}_2)(\mathbf{s}_1 \cdot \mathbf{s}_2)\ \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_2)$$

T and **s** are color charges and spin, g the color coupling, the form is derived from the non relativistic limit of QCD

- if i,j are in a color representation R , the formula simplifies to (CF is a color factor similar to the one introduced for color interactions)

$$\mathbf{H}_{ij} = 2\kappa_{ij}\ (\mathbf{s}_i \cdot \mathbf{s}_j) \quad \kappa_{ij} = CF(R) \times \frac{g^2}{m_i m_j} |\psi(0)|^2$$

$$\mathbf{H}_{ij} = 2\kappa_{ij}\,(\mathbf{s}_i \cdot \mathbf{s}_j) \quad \kappa_{ij} = CF(R) \times \frac{g^2}{m_i m_j}\,|\psi(0)|^2$$

- the Hamiltonian can be developed to first order in the small mass differences, like m_s-$m_{u,d}$,
- there is also a first order contribution from the spin-spin interaction, which is very crucial for baryons ($\Lambda - \Sigma^0$ mass difference)
- one usually assumes that the wave function overlap is the same in all mesons and all baryons, but this is dubious in the case of hadrons with a very heavy quark, c or b.
- Works well for mesons and baryons (too well?)
- Few parameters: m_u, m_d, m_s, m_c, m_s, κ_{ij}
- same values of masses, within ±30 MeV, reproduce masses of different hadrons
- κ_{ij} scale approx. like $1/m_i m_j$ and not far from scaling with color factors

Meson masses (c=0), to warm up

- π = $(q\bar{q})$ in S wave (q=u,d), J=0; ρ, ω $(q\bar{q})$ in S wave, J=1,

- since $$2(\mathbf{s}_{q_1}\cdot\mathbf{s}_{q_2}) = \mathbf{J}^2 - \mathbf{s}_{q_1}^2 - \mathbf{s}_{q_2}^2 = J(J+1) - 2s(s+1) = \left\{ \begin{array}{c} +\frac{1}{2} \\ \\ -\frac{3}{2} \end{array} \right\}$$

we find
$$m_\pi = 2m_q - \frac{3}{2}\kappa_{qq};\ m_\rho = m_\omega = 2m_q + \frac{1}{2}\kappa_{qq}$$

- we treat similarly the $(q\bar{s})$ mesons (K and K*) and the $(s\bar{s})$ vector meson φ (η is mixed with η' and is more complicated). We find
$$m_K = m_q + m_s - \frac{3}{2}\kappa_{sq};\ m_{K^\star} = m_q + m_s + \frac{1}{2}\kappa_{sq}$$
$$m_\phi = 2m_s + \frac{1}{2}\kappa_{ss}$$

- we have 5 parameters (2 masses and 3 kappas) and 6 masses, we get only the equality of ρ and ω masses

- but, to first order in the mass difference: $\kappa_{sq}-\kappa_{qq}=\delta\kappa$, and $\kappa_{ss}-\kappa_{qq}=2\delta\kappa$, we have one parameter less and another relation. In conclusion (masses in MeV):
$$m_\rho\ (775) = m_\omega\ (783)$$
$$m_\phi - m_{K^\star}\ (128) = m_{K^\star} - m_\rho (117)$$

Baryons of the octet (N, Λ, Σ, Ξ)

- P, uud: J=1/2 and $s_{u1}+ s_{u2} =1$ (Fermi statistics, since u_1 and u_2 are antisymmetric in color !)

$$H_N = 3m_q + 2\kappa_{qq}[\mathbf{s}_d \cdot (\mathbf{s}_{u_1} + \mathbf{s}_{u_2}) + \mathbf{s}_{u_1} \cdot \mathbf{s}_{u_2}]$$

$$m_N = 3m_q + \kappa_{qq}\left[(\frac{3}{4} - 2 - \frac{3}{4}) + (2 - \frac{3}{2})\right] = 3m_q - \frac{3}{2}\kappa_{qq}$$

- no problem in obtaining the mass formulae for the other particles (see Appendix)

- 4 masses (N, Λ, Σ, Ξ), 4 parameters: m_q-3/2κ_{qq}, m_s-m_q, κ_{sq}-κ_{qq}, κ_{ss}-κ_{qq}

- if we use the relation $\kappa_{sq}-\kappa_{qq}=\delta\kappa$, and $\kappa_{ss}-\kappa_{qq}=2\delta\kappa$, we have one relation: the Gell-Mann, Okubo relation

$$\frac{N+\Xi}{2}\ (1128\ \text{MeV})\ =\ \frac{3\Lambda+\Sigma}{4}\ (1136\ \text{MeV})$$

Baryons of the decuplet

- Δ= uuu:

$$m_\Delta = 3m_q + \frac{3}{2}\kappa_{qq}$$

- this give us the right combination to separate m_q from κ_{qq} and we can compute all decuplet masses in term of known parameters
- we find equally spaced masses with spacing

$$M = m_s - m_q + \kappa_{sq} - \kappa_{qq} = 139$$

experimentally

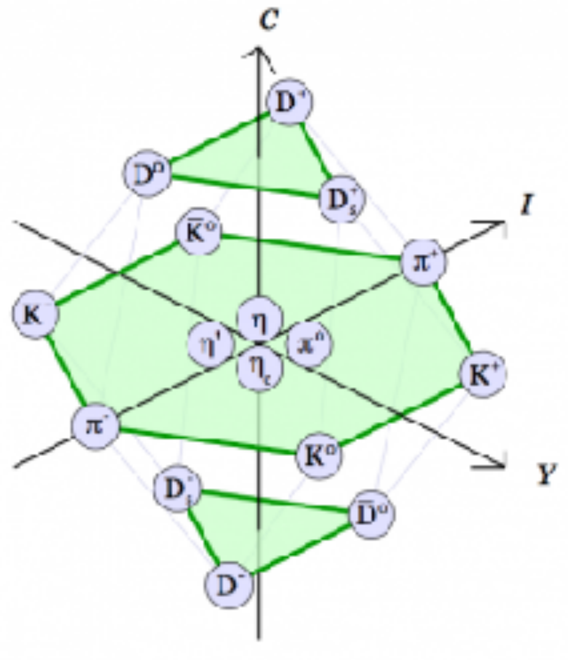
$$m(\Sigma^*) - m(\Delta) = 153$$
$$m(\Xi^*) - m(\Sigma^*) = 145$$
$$m(\Omega) - m(\Xi^*) = 142$$

- the addition to Gell-Mann Okubo is the prediction of the first mass difference (139=153), of similar quality as the other
- octet and decuplet: 8 masses, 4 parameters, 4 relations, satisfied within 20 MeV.

3. Spectroscopy of uncharmed and charmed Mesons and Baryons

- Particle states are now displayed in a 3 dimensional space: I_3, Y and Charm (c) or Beauty (b)
- ***Mesons.*** Quark-antiquark states and fall in a 15+1 dimensional multiplet, both lowest states (spin 0-) and first resonances (spin1-)
- particles made by a pair of the same quark flavor are neutral and fall in the center of the multiplets
- B***aryons.*** Are 3 quark states, classified in two different 20-dimensional multiplets
- c and b Baryons are being observed in several different experiments

MESONS



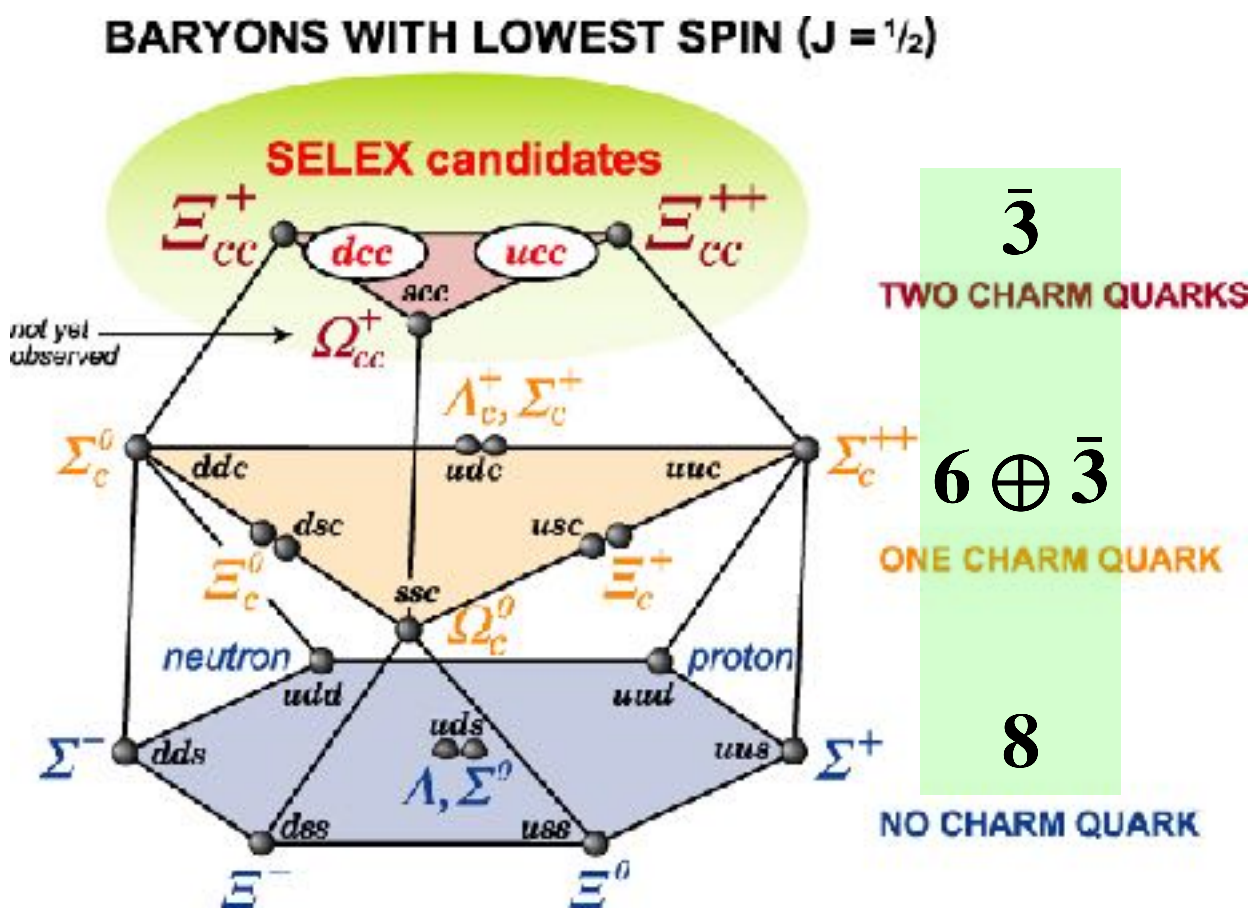
$\bar{3}$

8

3

+ similar multiplet of vector mesons

c- BARYONS



$\bar{3}$

$6 \oplus \bar{3}$

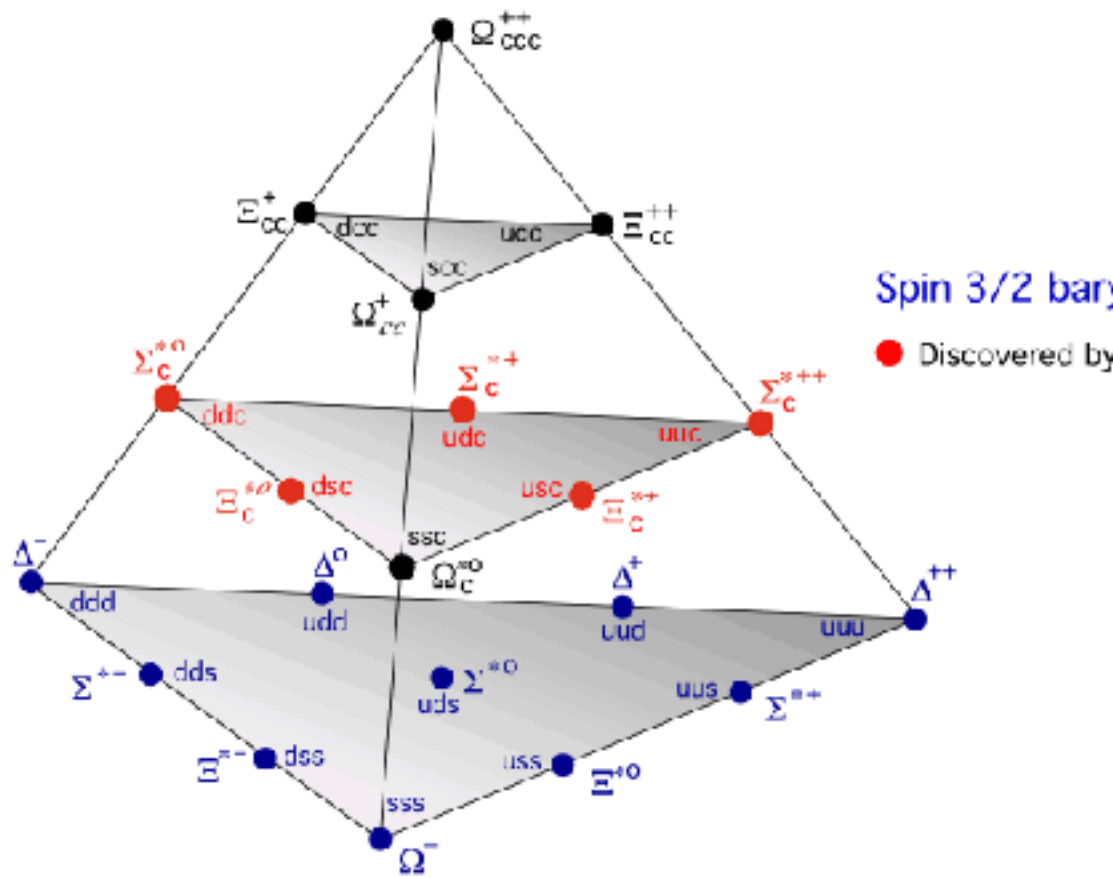
8

1

3

6

8



B and D mesons

- D, D*, D_s, D_s*: three new couplings (m_c and 2 kappas), one relation

$$\frac{3m_{D_s^\star}+m_{D_s}-(3m_{D^\star}+m_D)}{4}=101=$$
$$=\frac{3m_{K^\star}+m_K-(3m_\rho+m_\pi)}{4}=177$$

$= m_s - m_q$

- B, B*, B_s, B_s*: same

$$\frac{3m_{B_s^\star}+m_{B_s}-(3m_{B^\star}+m_B)}{4}=90=$$
$$=\frac{3m_{K^\star}+m_K-(3m_\rho+m_\pi)}{4}=177$$

$= m_s - m_q$

- A puzzling number:
  - B_c has also been observed, with m_{Bc}=6274
  - this is only one GeV heavier than B (m_B=5279), while m_c- m_q~1.4 GeV ?????

These "discrepancies" can be attributed to the different QCD interactions of heavy quarks inside the hadrons, see:
M. Karliner and J. L. Rosner, Phys. Rev. **D 90**, 094007 (2014) [arXiv:1408.5877 [hep-ph]].

Data from PdG live

MESONS	q	s	c	b
mass(MeV)	308	484	1664	5005

BARYONS	q	s	c	b
mass(MeV)	362	540	1710	5044

$m_s - m_q \sim 180$

MESONS	$(q\bar{q})_{\mathbf{1}}$	$(q\bar{s})_{\mathbf{1}}$	$(s\bar{s})_{\mathbf{1}}$	$(c\bar{q})_{\mathbf{1}}$	$(c\bar{s})_{\mathbf{1}}$	$(b\bar{q})_{\mathbf{1}}$	$(b\bar{s})_{\mathbf{1}}$
$(\kappa_{ij})_1$(MeV)	318	200	103	69	72	23	24
$(\kappa_{ij})_1\ m_i m_j/\Lambda^3_{QCD}$	1.9	1.9	1.5	2.3	3.7	0.7	1.2

$(\kappa_{q\bar{q}})_1 >> (\kappa_{cq})_{\bar{3}}$

BARYONS	$(qq)_{\bar{\mathbf{3}}}$	$(qs)_{\bar{\mathbf{3}}}$	$(ss)_{\bar{\mathbf{3}}}$	$(cq)_{\bar{\mathbf{3}}}$	$(cs)_{\bar{\mathbf{3}}}$	$(bq)_{\bar{\mathbf{3}}}$	$(bs)_{\bar{\mathbf{3}}}$
$(\kappa_{ij})_{\bar{3}}$ (MeV)	98	59	23	15	50	2.5	38
$(\kappa_{ij})_{\bar{3}}\ m_i m_j/\Lambda^3_{QCD}$	0.82	0.74	0.43	0.57	2.7	0.29	6.6

−	$\frac{(q\bar{q})_1}{(qq)_{\bar{3}}}$	$\frac{(q\bar{s})_1}{(qs)_{\bar{3}}}$	$\frac{(s\bar{s})_1}{(ss)_{\bar{3}}}$	$\frac{(c\bar{q})_1}{(cq)_{\bar{3}}}$	$\frac{(c\bar{s})_1}{(cs)_{\bar{3}}}$	$\frac{(b\bar{q})_1}{(bq)_{\bar{3}}}$	$\frac{(b\bar{s})_1}{(bs)_{\bar{3}}}$
$\frac{\kappa_{MES}}{\kappa_{BAR}}$	3.2	3.4	4.5	4.7	1.6	9.2	0.6