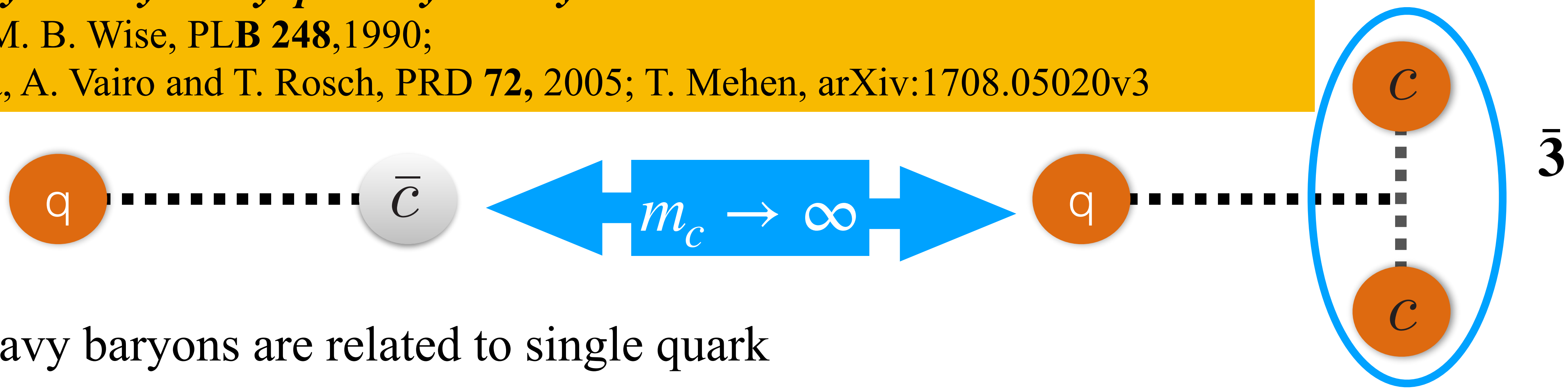
Lecture 6. Born-Oppenheimer approximation for double charm baryons and tetraquarks

- ***Summary***

1. The overall panorama ✓
2. Constituent Quark Model and masses of conventional mesons and baryons ✓
3. Light and Heavy Tetraquarks. First comparison with hadron molecules ✓
4. Tetraquarks and the EightFold Way. Di-J/Ψ resonances. ✓
5. $X(3872)$ and its missing partners ✓
6. Born-Oppenheimer approximation for double charm baryons and tetraquarks ✓
7. Multiquark states in N colours, in the $N \to \infty$ limit
8. Tetraquarks vs. molecules: the Weinberg criterium for $X(3872)$ and the double charm $\mathcal{T}_{cc}^{+}(3875)$

The new sensation: doubly heavy baryons

Single heavy-doubly heavy quark symmetry
M. Savage, M. B. Wise, PL**B 248**,1990;
N. Brambilla, A. Vairo and T. Rosch, PRD **72,** 2005; T. Mehen, arXiv:1708.05020v3



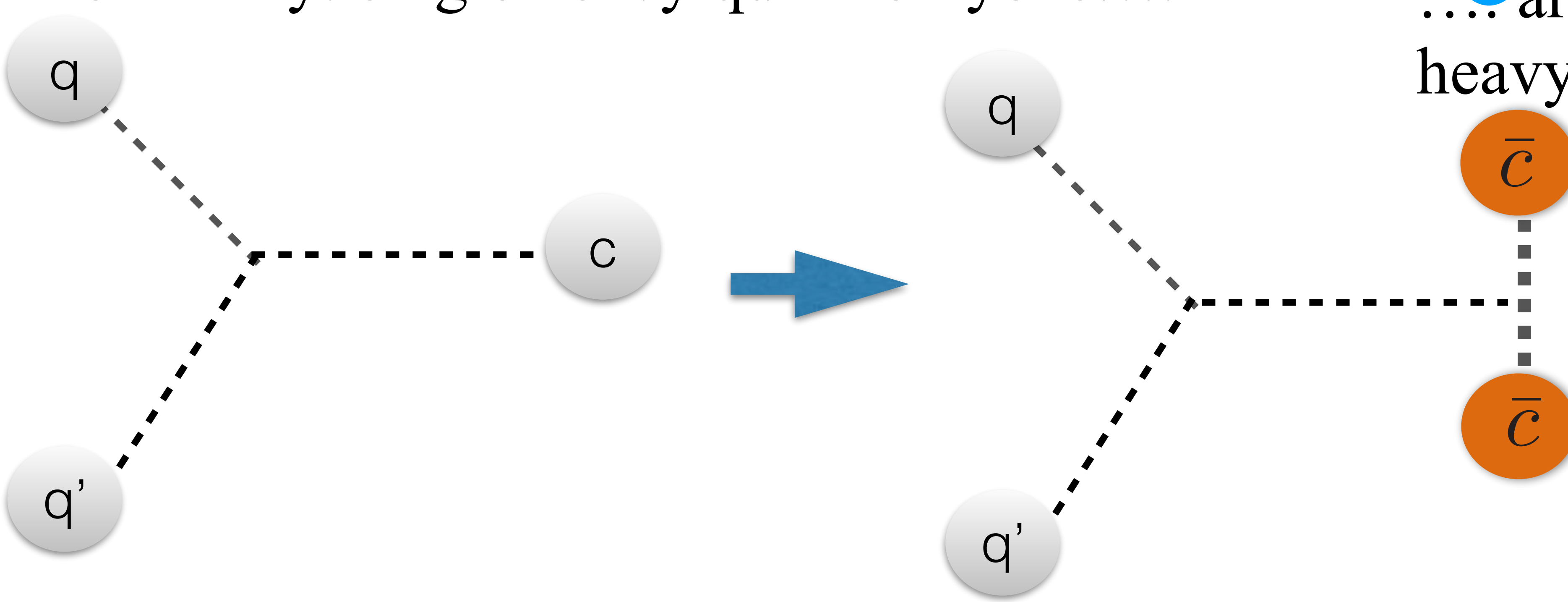
- Doubly heavy baryons are related to single quark heavy mesons
- QCD forces are mainly spin independent, so there is an approximate symmetry relating masses of DH baryons to SH mesons: e.g.

??

$$M(\Xi^*_{cc}) - M(\Xi_{cc}) = \frac{3}{4}[M(D^*) - M(D)]$$

similarly: single heavy quark baryons….

…. are related to doubly heavy tetraquark



Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Phys. Rev. D88, 054029 (2013)
M. Karliner and J. L. Rosner, arXiv:1707.07666 [hep-ph].
E. J. Eichten and C. Quigg, arXiv:1707.09575 [hep-ph].

Born-Oppenheimer Approximation for Quarkonium Hybrids

pioneered by Juge, Kuti, Morningstar 1999

- heavy quark mass $\gg \Lambda_{QCD}$
- Q and $\bar{Q}$ move nonrelativisticly
- gluons respond almost instantaneously to the motion of the Q and $\bar{Q}$

S. Fleck and J. M. Richard, Prog. Theor. Phys. **82** (1989) 760.
E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. **D 90** (2014) 014044.
N. Brambilla, G. Krein, J. Tarrs Castell and A. Vairo, Phys. Rev. **D 97**, 016016 (2018);

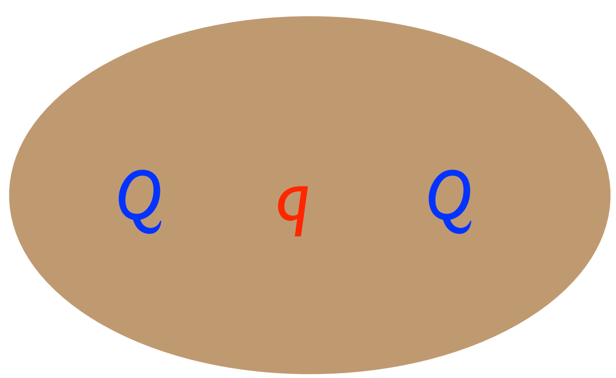
E. Braaten, 10th Workshop on CHARM Physics 2021, Mexico City

Doubly Heavy Hadrons

Born-Oppenheimer approximation

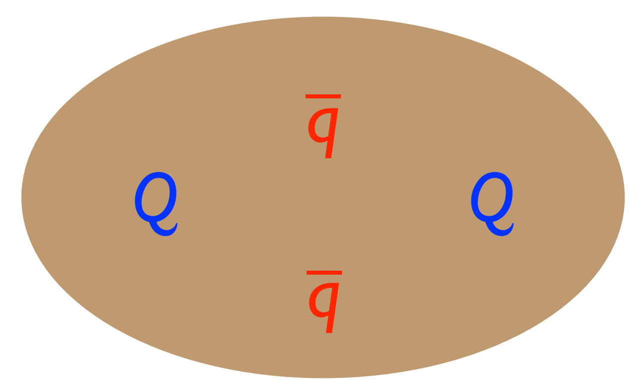
Light QCD fields in the presence of static Q Q sources
can have light-quark flavor q or $\bar{q}$ $\bar{q}$!

doubly heavy baryon



Maiani, Polosa & Riquer
arXiv:1903.10253
Soto & Tarrus Castella
arXiv:2005.00551

doubly heavy tetraquark



Bicudo & Wagner
arXiv:1209.6724
Maiani, Polosa & Riquer
arXiv:1903.10253

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3.The Born-Oppenheimer approximation, in brief

S. Weinberg, *Lectures on Quantum Mechanics,* Cambridge University Press (2015)

- A system with (say, two) heavy and (say, two) light particles

$$H = H_{\text{heavy}} + H_{\text{light}} = \frac{1}{2M}\sum_{\text{heavy}} P_i^2 + V(\mathbf{x}_A, \mathbf{x}_B) + \frac{1}{2m}\sum_{\text{light}} p_i^2 + V_I(\mathbf{x}_A, \mathbf{x}_B, \mathbf{x}_1, \mathbf{x}_2)$$

- First, find the ***ground state of the light particles***, for fixed coordinates of the heavy particles, solving the eigenvalue equation:

$$H_{ligth} f_0 = \mathscr{E} f_0; \;\; f_0 = f_0(\mathbf{x}_A, \mathbf{x}_B, \mathbf{x}_1, \mathbf{x}_2); \;\; \mathscr{E} = \mathscr{E}(\mathbf{x}_A, \mathbf{x}_B)$$

- Then look for solutions of the complete Schroedinger equation, for wave functions of the form:

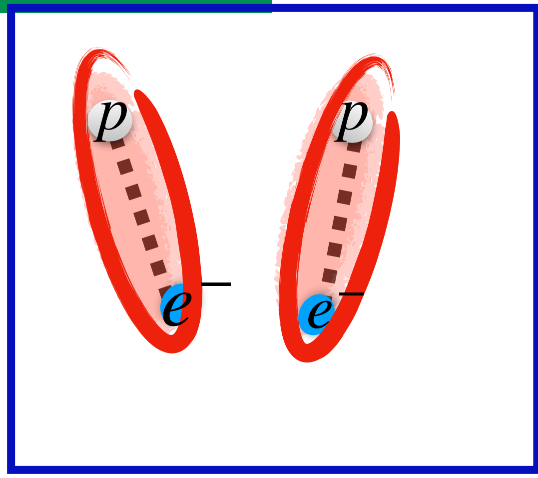
$$\Psi = \psi(\mathbf{x}_A, \mathbf{x}_B)\, f_0(\mathbf{x}_A, \mathbf{x}_B, \mathbf{x}_1, \mathbf{x}_2)$$

- Applying H to Ψ one encounters terms of the kind: -iP Ψ= $\frac{\partial}{\partial x_A}\Psi = \frac{\partial \psi}{\partial x_A} f_0 + \psi \frac{\partial f_0}{\partial x_A}$
- The ***Born-Oppenheimer approximation*** consists in ***neglecting systematically the second with respect to the first term***.
- The error vanishes for m/M ⟶ 0 (see Weinberg's book).
- The upshot is the BO equation:

$$\left(\sum_{\text{heavy}} \frac{P_i^2}{2M} + V_{BO}(\mathbf{x}_A, \mathbf{x}_B)\right)\psi = E\psi$$

$$V_{\text{BO}}(\mathbf{x}_A, \mathbf{x}_B) = V(\mathbf{x}_A, \mathbf{x}_B) + \mathcal{E}(\mathbf{x}_A, \mathbf{x}_B)$$

Orbitals (borrowed from molecular physics)



- To start, we associate each light to one heavy particle, solving Schroedinger for the mutual interaction, if attractive, and neglecting the others

- this wave function is called an "orbital" and we choose f_0 as the product of all orbitals:

$$f_0 = f(x_u - x_c) f(x_{\bar{u}} - x_{\bar{c}})$$

- if there are identical light particles, we have to (anti) symmetrize.

- The interactions left-over from the orbitals, e.g. the interactions between light particles, are taken to first order and $\mathcal{E} =< f_0 | H_{\rm light} | f_0 >$

- The approximation works very well for the hydrogen ion, H_2^+ (one orbital), and the hydrogen molecule, H_2 (two orbitals eP)

Warming-Up with H_2^+

- Light particle hamiltonian: one orbital: e-P(x_A) or e-P(x_B) .

$$H_{\rm light} = \frac{p^2}{2m} - \alpha \left(\frac{1}{|\mathbf{x} - \mathbf{x}_A|} + \frac{1}{|\mathbf{x} - \mathbf{x}_B|} \right) \qquad f(x) = \text{H wave function;}\ \ \mathcal{E}_0 = \frac{1}{2}\alpha^2 m_e$$

$$f_0 = \frac{f(x_e - x_A) + f(x_e - x_B)}{\sqrt{2(1+S)}},\ S = \int dx\ f(x - x_A) * f(x - x_B)$$

$$V_{BO} = -\alpha \frac{1}{|\mathbf{x}_A - \mathbf{x}_B|} + < f_0 | H_{\rm light} | f_0 >$$

Colour gymnastic: couplings…

- Hidden Charm
  - we take $c\bar{c}$ at ***distance R, in color 8***: Tetraquark= $|(\bar{c}c)_{\mathbf{8}}(\bar{q}q')_{\mathbf{8}}>_{\mathbf{1}}$
- correspondingly, we compute the colour couplings

$$\alpha_s \lambda_{c\bar{c}} = \alpha_s[\frac{1}{2}(C_2(\mathbf{8}) - 2C_2(\mathbf{3}))] = \alpha_s \frac{1}{2}(3 - 2\frac{4}{3})] = +\,\alpha_s \frac{1}{6} = \lambda_{q\bar{q}'} \text{ (repulsive)}$$

- $c\bar{c}$ repel each other as $q\bar{q}'$ do, like protons and electrons in H_2 molecule.
- Other arrangements:

?? !!

- Tetraquark= $|(\bar{c}c)_{\mathbf{8}}(\bar{q}q')_{\mathbf{8}}>_{\mathbf{1}} \;=\sqrt{\frac{2}{3}}(\bar{c}q')_{\mathbf{1}}(\bar{q}c)_{\mathbf{1}} - \sqrt{\frac{1}{3}}(\bar{c}q')_{\mathbf{8}}(\bar{q}c)_{\mathbf{8}}$ so that:

$$\lambda_{\bar{q}c} = \frac{2}{3}\lambda_1 + \frac{1}{3}\lambda_8 = \frac{2}{3}(-\frac{4}{3}) + \frac{1}{3}(\frac{1}{6}) = -\,\frac{5}{6} \text{ (attractive)}$$

- similarly: $\lambda_{cq} = \lambda_{\bar{c}\bar{q}} = -\,\frac{7}{6}$ (attractive)

- It is natural to chose the diquark-antidiquark orbitals $[cq]$ and $[\bar{c}\bar{q}']$ and treat the interactions $c - \bar{q}'$, $\bar{c} - q$, $q - \bar{q}'$ as perturbations
  - $V_{BO} = +\,\frac{1}{6}\alpha_s \frac{1}{|x_A - x_B|} + \Delta V(x_A, x_B) + V_{\rm conf}$
  - $\Delta V = <\Psi|V(c\bar{q}) + V(\bar{q}c) + V(q\bar{q})|\Psi>$, $|\Psi>$ product of orbitals wave functions
  - $V_{\rm conf}$, see later

…and strings

- The prototype of non relativistic heavy quark interaction is the Cornell potential, introduced in connection with charmonium spectrum
- A heavy colour triplet pair, $Q\bar{Q}$, in an overall colour singlet state

$$V(r) = -\frac{4}{3}\frac{\alpha_S}{r} + kr + V_0 = V_C(r) + V_{\rm conf}(r) + V_0$$

- V_0 is an unknown constant, to be determined from the mass spectrum, e.g. from the mass of the ground state;
- First term corresponds to one-gluon exchange approximation. It is generalised to any pair of coloured particles in a colour representation R by the combination of C_2 Casimir coefficients
- The second term arises from quark confinement and dominates at large separations;
- In the simplest picture, it is due to the condensation of Coulomb lines of force into a string that joins the quark and the antiquark. In this picture, it is natural to assume that the string tension, k, scales with the Coulomb coefficient: $k_{q_1q_2} \propto |\lambda_{q_1q_2}|$
- For colour charges combined in an overall colour singlet, the assumption leads to $k \propto |C_2(\mathbf{q})|$ (called Casimir scaling).

see e.g. G. S. Bali, Phys. Rept. **343**} (2001) 1. [hep-ph/0001312].

In conclusion, we take k from chamonium spectrum and:

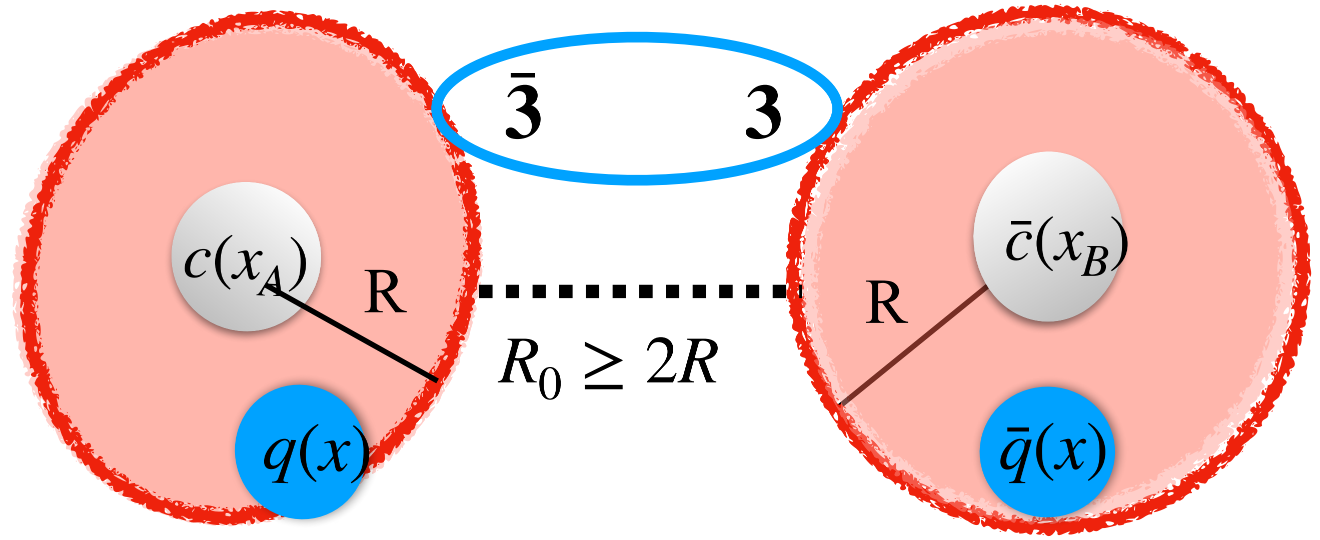
$$V(r) = -\frac{4}{3}\frac{\alpha_S}{r} + \frac{3|\lambda_{q_1q_2}|}{4} kr + V_0 = V_C(r) + V_{\rm conf}(r) + V_0$$

Organizing the QCD calculation

L. Maiani, A. D.Polosa and V. Riquer, PR D**100** (2019) 074002, arXiv:1908.03244 = ***MPR***

- Orbitals with non-vanishing triality are confined and we add to the BO potential the appropriate linearly rising potential, V_{conf}
- Triality zero orbitals, e.g. $(c\bar{q})_{\mathbf{8}}$, are NOT confined because color can be neutralised by extra gluons, and the BO potential vanishes for large separation of the heavy constituent, see our PRD paper.
- Cornell potential contains the additive constant V_0

$$V_{\rm conf}(r) = k \times (r - R_0) \times \theta(r - R_0)$$

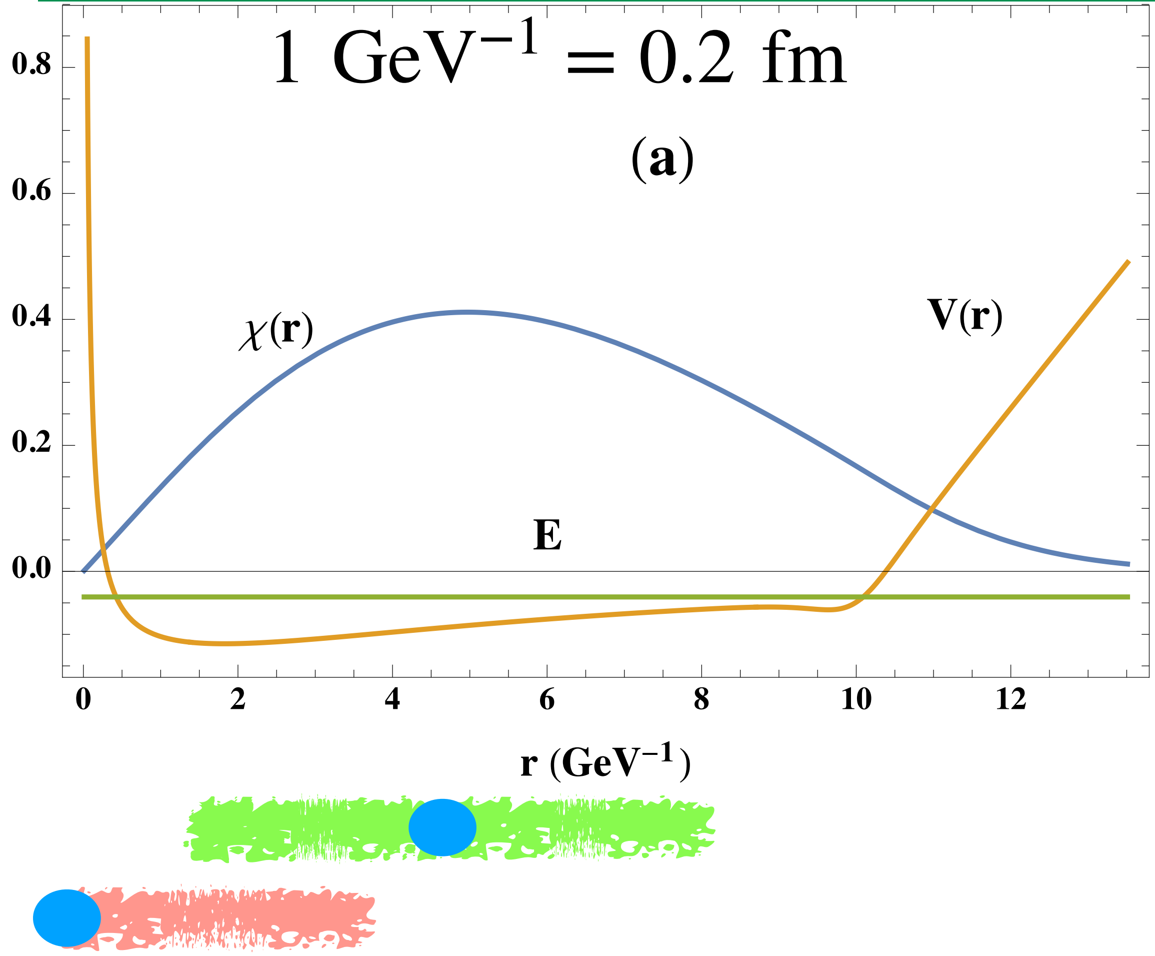


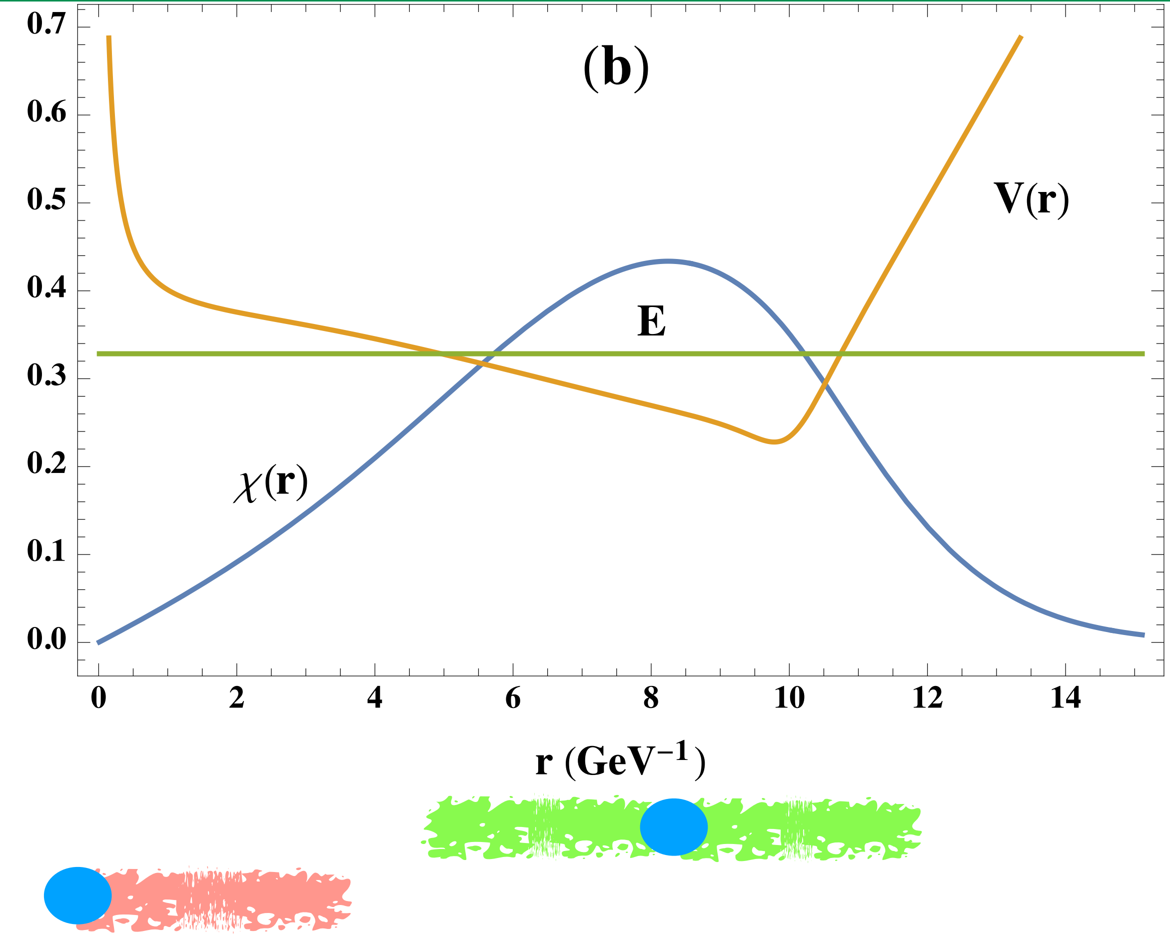
$$V_{BO} = +\frac{1}{6}\alpha_s \frac{1}{|x_A - x_B|} + \Delta V(x_A, x_B) + V_{\rm conf}$$

- in charmonium physics V_0 is determined from one physical mass of the spectrum, e.g. the ground state. We are able to determine V_0 :

- $Q\bar{Q}q\bar{q}$ ***case:*** V_0 is fixed by the ground state mass
- however the wave function gives an independent information about the internal structure.
- ***baryon case***: from a boundary condition from heavy quark-heavy diquark symmetry;
- $QQ\bar{q}\bar{q}'$ ***tetraquarks***: V_0 is fixed by the condition that the potential gives rise at infinite separation to a 2 meson state.

Results for $(c\bar{c})_8\ (q\bar{q}')_8$



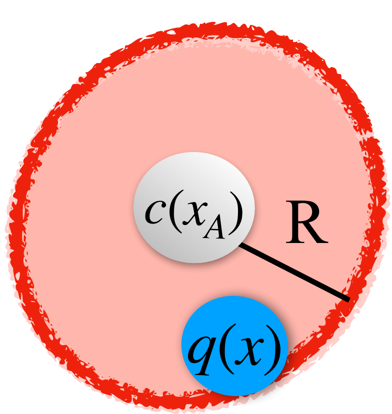


$\Delta V(q\bar{q}')$ ***from one-gluon exchange.***

- Diquark and antidiquark orbitals overlap
- similar to hadrocharmonium

$\Delta V(q\bar{q}') \sim 2\times$ ***one-gluon exchange.***

- Diquark and antidiquark well separated (Lect. 5)
- large potential barrier to $c\bar{c}$ annihilation
- depends crucially upon $(\bar{q}\bar{q})_8$ repulsion



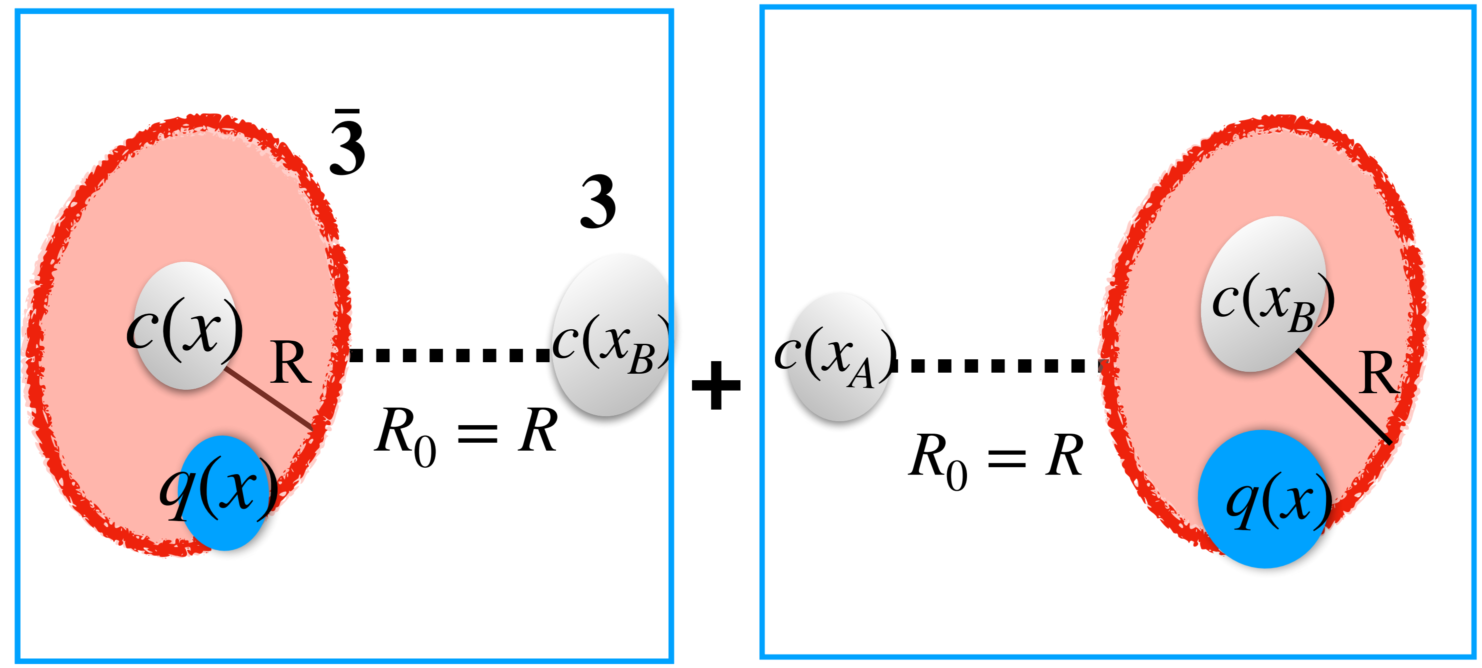
$$R \simeq (0.27\ \mathrm{GeV})^{-1} = 3.7\ GeV^{-1} = 0.74\ \mathrm{fm}$$

Doubly Heavy baryon

We treat it like the H_2^+ ion (PPe^-)

$$H_{\rm light} = \frac{p^2}{2m} + -\frac{2}{3}\alpha_S\left(\frac{1}{|\mathbf{x}-\mathbf{x}_A|}+\frac{1}{|\mathbf{x}-\mathbf{x}_B|}\right)$$

orbital (A) orbital (B)



f(x-x$_A$) f(x-x$_B$)]

- orbital(A)=f(x-x$_A$)=f$_A$(x)
- ground state: $f_0 = \dfrac{f(x-x_A)+f(x-x_B)}{\sqrt{2+2S}}$
- non vanishing ovelap:

$$\boxed{S = \int f(x-x_A)^* f(x-x_B) \neq 0}$$

- energy of the orbital:

$E_0 = V_0 + E_{0,orb} + 2M_c + M_q$

- Energy to first order

$$\mathcal{E}(\mathbf{x}_A-\mathbf{x}_B) = \langle f_0|H_{\rm light}|f_0\rangle = E_0 + \Delta E(r_{AB} =$$

$$= E_0 \; - \frac{2}{3}\alpha_S \frac{1}{1+S}\left[I_1(r_{AB}) + I_2(r_{AB})\right]$$

$$I_1(r_{AB}) = \int d^3\xi \, |f_A(\xi)|^2 \, \frac{1}{|\xi-\mathbf{x}_B|}$$

$$I_2(r_{AB}) = \int d^3\xi \, f_A^*(\xi) f_B(\xi) \, \frac{1}{|\xi-\mathbf{x}_B|}$$

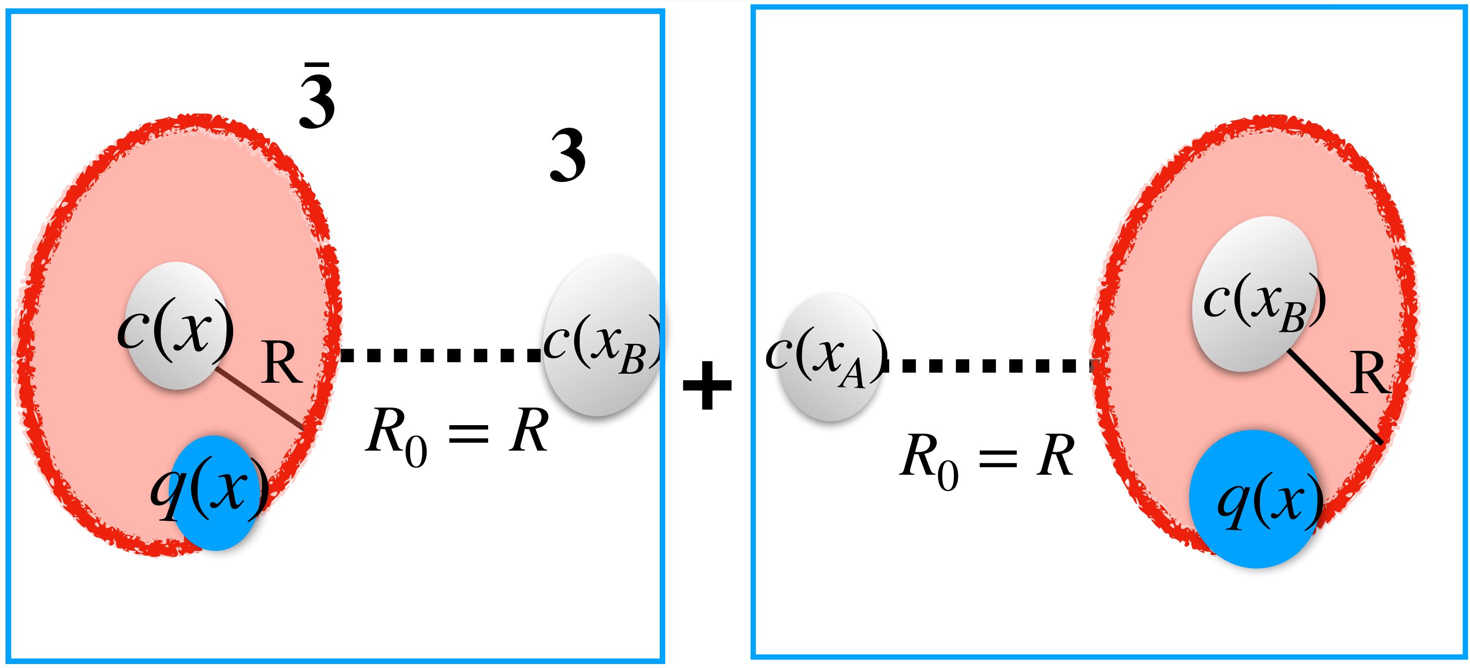
Doubly Heavy baryon (cont'd)

$$\mathcal{E}(\mathbf{x}_A-\mathbf{x}_B) = <f_0|H_{light}|f_0> = E_0 + \Delta E(r_{AB}) =$$

$$= E_0 - \frac{2}{3}\alpha_s \frac{1}{1+S}[I_1(r_{AB}) + I_2(r_{AB})]$$

$$E_0 = V_0 + E_{0,\mathrm{orb}} + 2M_c + M_q$$

- let $r_{AB} \to 0$
- $E_0 + \Delta E_0 - M_c =$
- $= V_0 + E_{0,\mathrm{orb}} + M_c + M_q + \Delta E_0(0)$



- The result must be equal to the mass of $(c\bar{q})$ meson without spin correction, i.e. = M_c+M_q

this is the ***single heavy quark-double heavy quark*** symmetry

- Result: $\mathcal{E}(\mathbf{x}_A-\mathbf{x}_B) = E_0 + \Delta E(r_{AB} = 2M_c + M_q - \Delta E(0) + \Delta E(r_{AB})$

$$V_{\mathrm{BO}}(\mathbf{x}_A,\mathbf{x}_B) = V(\mathbf{x}_A,\mathbf{x}_B) + \mathcal{E}(\mathbf{x}_A,\mathbf{x}_B) + \text{linear confining potential} =$$

$$= 2M_c + M_q - \frac{2}{3}\alpha_S \frac{1}{r_{AB}} + \Delta E(r_{AB}) - \Delta E(0) + k \times (r - R_0) \times \theta(r - R_0)$$

The eigenvalue of the Schroedinge equation of mass M_c and potential inscribed in red is the correction to the naive constituent quark mass formula for Ξcc (spin corrections to be added). Only unknown R_0~1-2 fm ≥ orbital radius

Doubly heavy baryons (QQ'q): Summary

$M(\Xi_{cc})_{\mathrm{LHCb}} = 3621.2 \pm 0.7$ MeV

Contribution	Value (MeV)
$2m_c^b + m_q^b$	3783.9
cc binding	-129.0
$a_{cc}/(m_c^b)^2$	14.2
$-4a/m_q^b m_c^b$	-42.4
Total	3627 ± 12

$}=3655$

$1/2\ \kappa_{cc}$

$-2\kappa_{qc}$

Contribution	Value (MeV)
$2M_c^{(mes)}) + M_q^{(mes)}$	3642
$-\Delta E_0(0) + E$	$+24$
$1/2\ \kappa_{cc}$	$+14$
$-2\ \kappa_{qc}$	-30
Total	$3650^{+17}_{-5} \pm\ 30?$

$}=3666$

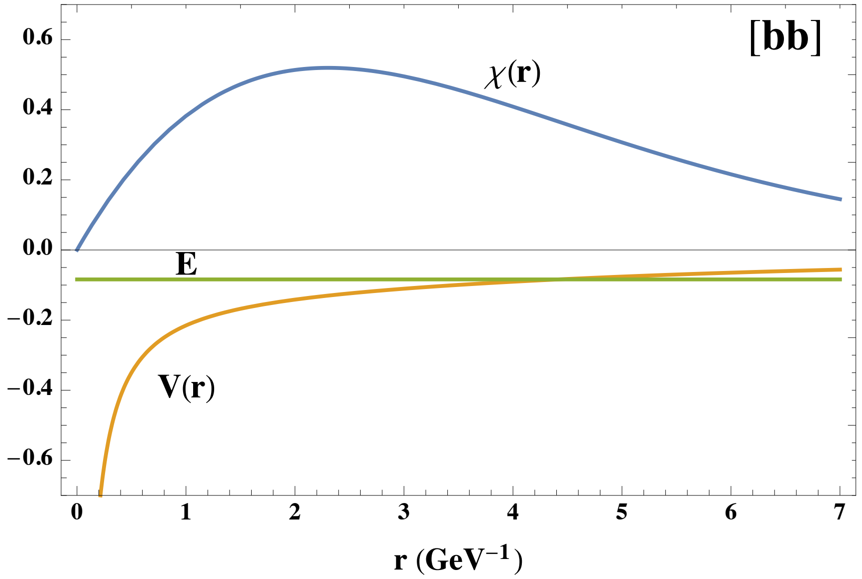
$-$	$\|\Delta E(0)\|$	E	$M[\Xi_{QQ}]$	K&R	Lattice QCD
Ξ_{cc}	$+65$	-41	3650	3628 ± 12	3634(20)
Ω_{cc}	$+75$	-50	3763	3692 ± 16	3712(11)(12)
Ξ_{cb}	$+50$	-41	6957	6920 ± 13	6945(22)(14)
Ξ'_{cb}	$+50$	-41	6989	6935 ± 12	6966(23)(14)
Ξ_{bb}	$+44$	-56	10309	10162 ± 12	—

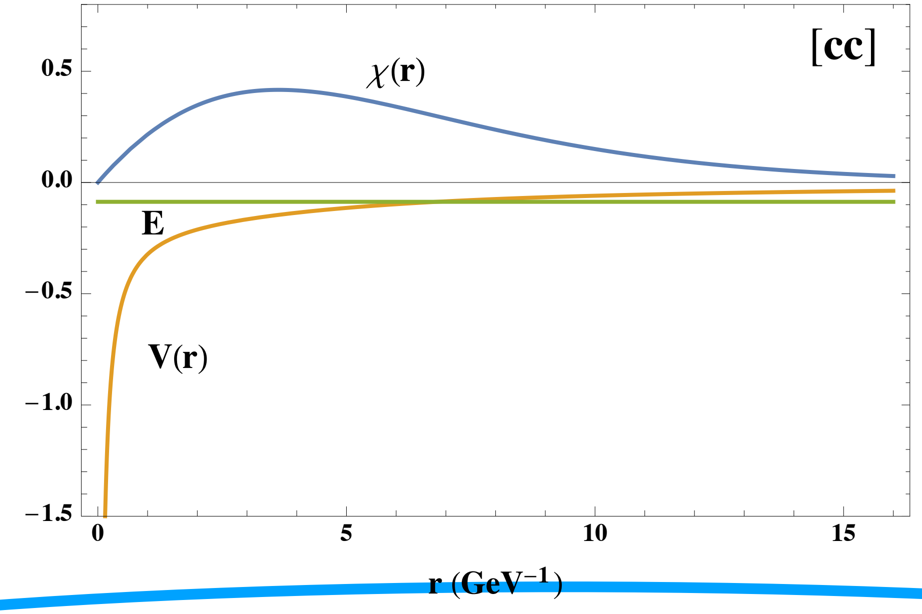
Our work

M. Karliner and J. L. Rosner, Phys. Rev. D 90 (2014), 094007

N. Mathur and M. Padmanath, Phys. Rev. D 99 (2019)

Results for $[(QQ)_{\bar{3}}(\bar{u}\bar{d})_3]$, $Q = b, c$





- k_{bb}=1/4 k, Casimir scaling (or k)
- Orbitals have 0 triality, V_0 determined from boundary conditions

$$M(T_{bb}) = 2(M_b + M_q) + E + \frac{1}{2}\kappa_{bb} - \frac{3}{2}\kappa_{qq}; \quad M_{PS}(b\bar{q}) = M_b + M_q - \frac{3}{2}\kappa_{b\bar{q}}$$

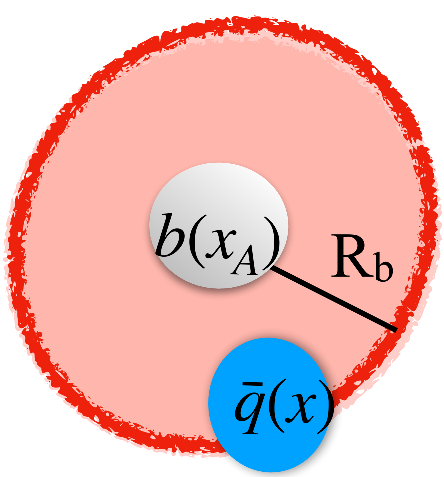
$$Q_{bb} = M(T_{bb}) - 2M_{PS}(b\bar{q}) = E + \frac{1}{2}\kappa_{bb} - \frac{3}{2}\kappa_{qq} + 3\ \kappa_{b\bar{q}}$$

- κ : from meson spectrum

$$Q_{cc} = +7(-10) \quad Q_{bb} = -138(-156)$$

MPR
Masses in MeV

$R_b \simeq (0.27\ \text{GeV})^{-1} =$
$= 3.0\ GeV^{-1} = 0.6$ fm



However:

- Taking κ_{qq} and κ_{bb} from meson spectrum is not justified:
- h.f. couplings depend crucially from the overlap probability of the quark pair, which, in tetraquarks, cannot be assumed to be equal to the overlap probabilities of the same pair in mesons and baryons.
- In our Born-Oppenheimer scheme, we may estimate deviations from this hypothesis and obtain an improved estimate of tetraquark masses.

The revised estimate

- Light antiquarks are each bound to a heavy quark in orbitals with wave functions $\psi(\xi)$ and $\phi(\eta)$; $\xi = \mathbf{x}_A,\ \eta = \mathbf{x}_B$
- the average distance of the light quarks is a function of the heavquarksdistance, r_{AB}

$$D(r_{AB}) = \int d^3\xi d^3\eta\, |\psi(\xi)|^2\, |\phi(\eta)|^2\, |\xi - \eta|$$

- The average distance in the tetraquark is then given by $D_{qq} = \int dr_{AB}\ \chi^2(r_{AB}) D(r_{AB})$
- which leads to: $\kappa_{qq}^{QQ} = \kappa_{qq} \left(\dfrac{R_{qq}^B}{D_{qq}^{QQ}}\right)^3$ $\quad \kappa_{qq}^{cc} = +4.8\ (+4.7)\ \kappa_{qq}^{bb} = +1.2\ (+1.9)$
- analogous reduction for κ_{bb}

$QQ'\bar{u}\bar{d}$	BO revised	[3]	[4]	[6]	Lattice QCD
$cc\bar{u}\bar{d}$	+133 (+117)	$+140$ ~ 0	$+102$ $+83$	$+39$ -108	-23 ± 11 [7] $+8 \pm 23$ [8]
$bb\bar{u}\bar{d}$	-1.4 (-11)	-170	-121	-75	-143 ± 34 [7] $-143(1)(3)$ [9] $-82 \pm 24 \pm 10$ [10]

preliminary !

[3] M. Karliner and J. L. Rosner, arXiv:1707.07666 [hep-ph]; [4] E. J. Eichten and C. Quigg, arXiv:1707.09575 [hep-ph]. [7] Junnarkar et al. arXiv:1810.12285, [8,9,10] A. Francis arXiv:1810.10550, arXiv:1607.05214; 014002 (2019)

Born-Oppenheimer:

- Estimate of T_{cc} mass much closer to the observed mass of the double charm meson:

$$M(T_{cc}^{BO}) = 3863(3847) \leftrightarrow \text{LHCb} : T_{cc}^{+}(3875)$$

- Stable Double Beauty tetraquarks: possible but only marginally