Lecture 4. Tetraquarks and the EightFold Way

• Summary

- 1. The overall panorama \checkmark
- 2. Constituent Quark Model and masses of conventional mesons and baryons \checkmark
- 3. Light and Heavy Tetraquarks. First comparison with hadron molecules \checkmark
- 4. Tetraquarks and the EightFold Way. Di-J/ Ψ resonances \checkmark
- 5. X(3872) and its missing partners
- 6. Born-Oppenheimer approximation for double charm baryons and tetraquarks
- 7. Multiquark states in N colours, in the $N \rightarrow \infty$ limit
- 8. Tetraquarks vs. molecules: the Weinberg criterium for X(3872) and the double charm $\mathcal{T}_{cc}^+(3875)$

1. A few facts about QCD and quark masses

$$\mathscr{L}_{QCD} = \frac{1}{4} Tr[G_{\mu\nu}G^{\mu\nu}] + \sum_{u,d,s} (i\bar{q}_i Dq_i + m_i\bar{q}_i q_i) + \text{ same for heavy quarks}$$

• The light quark mass term can be rewritten as:

$$\mathscr{L}_{m} = \bar{m} \sum_{u,d,s} \bar{q}_{i} q_{i} + m_{3} (\bar{u}u - \bar{d}d) + m_{8} (2\bar{s}s - \bar{u}u - \bar{d}d) = \mathscr{L}_{\bar{m}} + \mathscr{L}_{3} + \mathscr{L}_{8};$$

$$m_{3} = \frac{m_{u} - m_{d}}{2}; m_{8} = \frac{2m_{s} - m_{u} - m_{d}}{6}$$

- $\mathscr{L}_{3,8}$ are color singlets, transform under $SU(3)_f$ like the Gell-Mann's matrices λ_3 , λ_8 and break Isospin and Unitary symmetry, respectively
- In total, we can write

$$\mathscr{L}_{QCD} = \mathscr{L}_{QCD}^0 + \mathscr{L}_3 + \mathscr{L}_8$$

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- \mathscr{L}_{QCD}^0 is invariant under the light flavours symmetry (the EightFold Way) and creates *complete multiplets* of $SU(3)_f$, with the heavy quarks c, b behaving as $SU(3)_f$ singlets
- $\mathscr{L}_3 + \mathscr{L}_8$ are color singlets and can be treated as perturbations that split the degenerate $SU(3)_f$ multiplets created by \mathscr{L}_{QCD}^0 , like e.g. an external, weak magnetic field that splits atomic levels
- quark mass differences are universal, i.e equal for mesons and baryons. Neglecting Isospin breaking, we found in Lect.2:

$$(m_s - m_q)_{baryon} \sim (m_s - m_q)_{meson} = 180 \text{ MeV}$$

Individual quark masses (sensitive to the energy stored in the QCD field) have larger variations

Mesons:
$$(m_q \sim 308, m_s \sim 484)$$
, Baryons: $(m_q \sim 362, m_s \sim 540)$

Nuclei are not like that

- Forces exchanged between color singlets, e.g. nuclei, are strongly dependent from the mass of the exchanged particle, e.g. π , ρ , η ...that is they are strongly flavour dependent.
- Bound states, correspondingly are not expected to form multiplets with a regular pattern of mass differences
- Assuming flavour singlet forces between color singlets, as is done sometime, has no fundamental basis ecept for mimicking color forces.

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2. Exotic mesons: the New Wave

• Starting from 2016, new kinds of exotic hadrons have been discovered:

 $J/\Psi \phi$ resonances, $di - J/\Psi$ resonances, open strangeness Exotics: $Z_{cs}(3082)$ and $Z_{cs}(4003)$

• No pion exchange forces could bind them as hadron molecules made by color singlet mesons: molecular models have to stand on the existence of "phenomenological forces" with undetermined parameters

• The New Exotics arise very naturally as $([cq]^{\bar{3}}[\bar{c}\bar{q}']^3)_1$ bound in color singlet

• the compact tetraquark model makes a firm prediction: hidden charm tetraquarks must form *complete of multiplets flavor SU(3)*, with mass differences determined by the quark mass difference $m_s - m_u$.

• with $Z_{cs}(3082)$ and $Z_{cs}(4003)$ we can almost fill two tetraquark nonets with the expected scale of mass differences

Tetraquarks with hidden charm and strangeness, Z_{cs}

• The first resonance with valence quarks $(csc\bar{u}), Z_{cs}(3985)$ has been seen in 2021 by BES III in $e^+e^- \rightarrow K^+$... annihilation: an excess over the known contributions of conventional charmed mesons is observed near the $D_s^{*-}D^0$ and $D_s^{-}D^{*0}$ threshold, in the K^+ recoil mass spectrum.

• A similar resonance, $Z_{cs}(4003)$, has been observed by LHCb in B^+ decay: $B^+ \rightarrow \phi + Z_{cs}(4003) \rightarrow \phi + K^+ + \Psi$

• LHCb may have a third candidate in the $K^+ \Psi$ channel, $Z_{cs}(4220)$.



BES III(2021): $e^+e^- \to K^+ + Z_{cs}^-(3985) \to K^+(D_s^{*-}D^0 + D_s^-D^{*0})$

3. Hidden charm tetraquarks make $SU(3)_f$ nonets



• X(3872) & X(4140) (the lowest $J/\Psi \phi$ resonance) belong to one $(J^{PC} = 1^{++})$ nonet, as indicated by the mass difference:

 $Z_c(4140) - X(3872) = 275 \text{ MeV}$

- $Z_c(3900)$ needs a second ($J^{PC} = 1^{+-}$) nonet
- $Z_{cs}(3985)$ and $Z_{cs}(4003)$ almost completely fill the two nonets...

Two solutions



Solution 1 is favoured. The scale of the mass differences is in line for both

a well identified shopping list

- $X_{s\bar{s}}, M = 4076 \text{ or } 4121, \text{ decays: } \eta_c \phi, D_s^* \bar{D}_s$:
- the I=1 partners of X(3872), decaying into $J/\Psi + \rho^{\pm}$ (see later)
- the I=0 partners of $Z_c(3900)$ and $Z_c(4020)$, possibly decaying into $J/\Psi + f_0(500)(aka \sigma(500))$
- There is a third nonet associated to Z_c(4020), J^{PC} = 1⁺⁻,
 a third Z_{cs} is required, Mass=4150 4170 MeV;
 LHCb sees indeed a Z_{cs}(4220), J^P = 1⁺ or 1⁻, is it too heavy ?????

A bold suggestion

- $Z_{cs}(3982)$ is a bit too low (Z(3900) is heavier than X(3872))
- if there is a mixing between the two $J^{PC} = 1^{+-}$ nonets:

levels repel eah other, one Z_{cs} would go down, the other would go up



L. Maiani. Exotic Hadrons. 4

4. Charge conjugation for SU(3) nonets

• A charge conjugation quantum number can be given to each self conjugate SU(3) multiplet according to

 $\mathscr{C}T\mathscr{C} = \eta_T \tilde{T}, \ \tilde{T} = \text{transpose matrix}, \ \eta_T = \pm 1$

- η is the sign taken by neutral members, but it can be attributed to all members of the multiplet.
- $\eta = -1$ is given to the electromagnetic current
- for the pseudoscalar mesons: $\mathscr{CPC} = + \tilde{P}$:

$$P = \begin{vmatrix} \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^{+} & K^{+} \\ \pi^{-} - \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^{0} \\ K^{-} & \bar{K}^{0} & -\frac{2\eta}{\sqrt{3}} \end{vmatrix} \qquad \tilde{P} = \mathscr{C}P\mathscr{C} = \begin{vmatrix} \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^{-} & K^{-} \\ \pi^{+} - \frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \bar{K}^{0} \\ K^{+} & \bar{K}^{0} & -\frac{2\eta}{\sqrt{3}} \end{vmatrix}$$

- Trilinear couplings for nonets A, B, C (η_A , η_B , η_C)
 - there are 2 SU(3)_f invariant couplings: Tr(ABC), Tr(ACB)

•
$$\mathscr{C}$$
: $\operatorname{Tr}(ABC) \to \eta_A \eta_B \eta_C \operatorname{Tr}(CBA) = \eta_A \eta_B \eta_C \operatorname{Tr}(ACB)$

• 2 invariants with definite C:

$$D = \operatorname{Tr}\left(A\{B, C\}\right), \text{ for } \eta_A \eta_B \eta_C = +1,$$

$$F = \operatorname{Tr}\left(A[B, C]\right), \text{ for } \eta_A \eta_B \eta_C = -1$$

- Well known cases: C-invariant allowed decays for Vector (C=-1) and Tensor (C=+1) mesons are
 - Vector $\rightarrow 2$ Pseudoscalars, coupling $Tr(V^{\mu}[P, \partial_{\mu}P])$
 - Tensor $\rightarrow 2$ Pseudoscalars, coupling $Tr(T^{\mu\nu}\{\partial_{\mu}P, \partial_{\nu}P\})$

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BES III: $e^+e^- \to K^+ + Z_{cs}^-(3985)$

- Consider the $SU(3)_f$ invariant coupling in two cases
- a) production from continuum: photon $\rightarrow K^+ + Z_{cs}^-(3985)$

$$C_{Z} = +1: g_{\gamma} = iTr(Q[K, Z_{cs}]) = i(\frac{2}{3} + \frac{1}{3})(K^{+}Z_{cs}^{-} - h \cdot c.)$$
$$C_{Z} = -1: g_{\gamma} = Tr(Q\{K, Z_{cs}\}) = \frac{1}{3}(K^{+}Z_{cs}^{-} + h \cdot c.)$$

• b) production from a $Y(cs\bar{c}\bar{s})$ resonance: photon $\rightarrow Y \rightarrow K^+ + Z_{cs}^-(3985)$

•
$$C_Z = \pm 1 : g_{\gamma} = Tr(Y[K, Z_{cs}]_{\mp}) = K^- Z^+ \pm h . c$$
.

• measuring the energy dependence, BES III can distinguish production from continuum (which favors Solution 2) from resonant production (no preference).

•
$$Z_{cs}(C = -1), X_{cs}(C = +1) \rightarrow K \Psi$$
 decay:
 $\mathscr{H}_{I} = \lambda \mu \psi (\operatorname{Tr}\{Z, M\}) ([\mu] = \operatorname{mass}), = \lambda \mu [Z_{cs}^{-}(\psi K^{+}) + c.c.]$
 $\mathscr{H}_{I} = \lambda i \psi \operatorname{Tr}([\epsilon_{8}[X, M]) \sim \lambda (m_{s} - m_{u}) i[X_{cs}^{-}(\psi K^{+}) - c.c.]$

no real preference

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5. di-J/ Ψ resonances: a game changer



Figure 2: Invariant mass spectrum of J/ψ -pair candidates passing the $p_{\rm T}^{\rm move} > 5.2 \,\text{GeV}/c$ requirement with reconstructed J/ψ masses in the (black) signal and (blue) background regions, respectively.

Meson-Meson molecule: tried by some authors assuming SU(3)_f singlet exchange, i.e. J/Ψ exchange
leads to force range of fractions of 1 fm, in the full domain of color forces

Tetraquark constituent picture of 2 J/ Ψ resonances

C. Becchi, J. Ferretti, A. Giachino, L. Maiani and E. Santopinto, Phys. Lett. **B 811** (2020), 135952 arXiv:2006.14388 [hep-ph].

• [cc] in color $\overline{\mathbf{3}}$

$$[cc]_{S=1}[\bar{c}\bar{c}]_{S=1}$$

- total spin of each diquark, S=1 (color antisymmetry and Fermi statistics)
- S-wave: positive parity

S-wave, fully charm tetraquarks

• C=+1 states:
$$J^{PC} = 0^{++}$$
, 2⁺⁺, decay in 2 J/ Ψ , S-wave

• C= -1 states: $J^{PC} = 1^{+-}$, no decay in 2 J/ Ψ

•mass spectrum computed by M.A.Bedolla, J.Ferretti, C.D.Roberts and E.Santopinto, arXiv:1911.00960 [hep-ph]:

- QCD inspired potential (Coulomb+linear potential), gaussian wave functions in the three Jacobi coordinates, ξ_1 , ξ_2 , ξ_3
- parameters of the gaussians are determined by minimizing the energy
- •Authors include computation of the energy levels of radial and orbital excitations.



	ccēē		mass spectrum of $[ccccc]$ states						
J^{PC}	$N[(S_D, S_{\bar{D}})S, L]J$	E th [MeV]		_	_				
	1[(1,1)0,0]0	5883	→ 6537		0++ S-wave				
0++	2[(1, 1)0, 0]0	6573		→ 7227	1st Radial exci	tation			
0^{++}	1[(1, 1)2, 2]0	6835							
0++	3[(1,1)0,0]0	6948							
0++	2[(1,1)2,2]0	7133							
0++	3[(1,1)2,2]0	7387							
1+-	1[(1,1)1,0]1	6120	The prediction inclu	udes an <i>a n</i>	<i>riori</i> unknown ad	lditive			
1+-	2[(1, 1)1, 0]1	6669							
1+-	1[(1,1)1,2]1	6829	constant (to fix the zero of the energy for confined states)						
1+-	3[(1,1)1,0]1	7016	which is to be determined from one mass of the						
1*-	2[(1,1)1,2]1	7128	which is to be determined iron one mass of the						
1+-	3[(1,1)1,2]1	7382	spectrum.						
1	1[(1,1)0,1]1	6580				11 \ 0			
1	1[(1, 1)2, 1]1	6584	In this paper, the constant was taken (provisionally) from						
1	2[(1, 1)0, 1]1	6940	calculations of meson masses						
1	2[(1,1)2,1]1	6943	valuations of muson massus						
1	3[(1,1)0,1]1	7226	•The unchat: you give the mass of 2^{++} (sour 6000 MaV)						
1	3[(1,1)2,1]1	7229	- The upshot. you give the mass of 2^{+1} (say. 0900 wev)						
0-+	1[(1, 1)1, 1]0	6596	and Bedolla <i>et al.</i> predict the mass differences						
0^{-+}	2[(1, 1)1, 1]0	6953							
0^{-+}	3[(1,1)1,1]0	7236							
1++	1[(1,1)2,2]1	6832>		→ 7486	1++ D -wave				
1++	2[(1, 1)2, 2]1	7130							
1++	3[(1, 1)2, 2]1	7384		- 2					
2++	1[(1,1)2,0]2	6246	───→ 6900 (inpເ	ut)	2++ S-wave				
2**	I[(1, 1)2, 2]2	6827							
2**	1[(1,1)0,2]2	6827							
2**	2[(1,1)2,0]2	6739							
2**	3[(1,1)2,0]2	7071							
2**	2[(1, 1)2, 2]2	7125							
2**	2[(1,1)0,2]2	7126							
2**	3[(1,1)2,2]2	7380							
2**	3[(1,1)0,2]2	7380							
arXiv:1911.00960 [hep-ph]			L. Maiani. Exotic Hadrons. 4			15			

Decays and branching fractions

• Decays take place via $c\bar{c}$ annihilation. The starting point is to bring the $c\bar{c}$ pairs together

$$\mathcal{T}(J^{PC} = 0^{++}) = |(cc)^{1}_{\overline{3}}(\bar{c}\bar{c})^{1}_{3} > 0^{1}_{1} = \frac{1}{2} = -\frac{1}{2} \left(\sqrt{\frac{1}{3}} |(c\bar{c})^{1}_{1}(c\bar{c})^{1}_{1} \rangle^{0}_{1} - \sqrt{\frac{2}{3}} |(c\bar{c})^{1}_{8}(c\bar{c})^{1}_{8} \rangle^{0}_{1} \right) + \frac{1}{2} + \frac{\sqrt{3}}{2} \left(\sqrt{\frac{1}{3}} |(c\bar{c})^{0}_{1}(c\bar{c})^{0}_{1} \rangle^{0}_{1} - \sqrt{\frac{2}{3}} |(c\bar{c})^{0}_{8}(c\bar{c})^{0}_{8} \rangle^{0}_{1} \right)$$



- •Four possible annihilations
 - 1. a color singlet pair of spin 1 (0) annihilates into a J/ Ψ (η_c), the other pair rearranges into the available states (near threshold, J/ Ψ or η_c again);
 - 2. a color octet, spin 1 pair annihilates into a pair of light quark flavours, q=u,d,s and the latter recombine with the spectator pair to produce a pair of lower-lying, open-charm mesons. A similar process from color octet spin 0 pair is higher order in α_S and neglected.
- Rates are computed with the formula (well known in atomic physics):

$$\Gamma = |\Psi_T(0)|^2 \cdot |\mathbf{v}| \cdot \sigma(c\bar{c} \to f)$$

- Branching fractions are independent from $|\Psi_T(0)|^2$
- Total rates: see later.

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Results

$[cc\bar{c}\bar{c}]$	η_c + any	$D_q \bar{D}_q \ (m_q < m_c)$	$D_q^* \bar{D}_q^*$	$J/\Psi + any$	$J/\Psi+\mu^+\mu^-$	4μ	4μ
$J^{PC} = 0^{++}$	0.77	0.019	0.057	$7.5\cdot10^{-4}$	$4.5\cdot 10^{-5}$	$2.7\cdot 10^{-6}$	$7.2 \cdot 10^{-7}$
$J^{PC} = 2^{++}$	0	0	0.333	$4.4\cdot 10^{-3}$	$2.6\cdot 10^{-4}$	$1.6 \cdot 10^{-5}$	$20\cdot 10^{-7}$

 ${\rm TABLE \ I: \ Branching \ fractions \ of \ fully-charmed \ tetraquarks, \ assuming \ S-wave \ decay.}$

• Branching ratios in 4 muons are more favorable in 4 *c* than in 4 *b* tetraquarks (a factor 4-10)

[bbbb]

- Among 4 c, the ratio is more favorable for the 2⁺⁺
- In addition 2^{++} is produced in pp collision with a statistical factor 2J+1=5

$B_{4\mu}(2^{++}): B_{4\mu}(0^{++}) \sim 6: 1; \ \sigma(2^{++}): \sigma(0^{++}) = 5: 1$ A visibility ratio 30:1 !!

•The largest decay fraction is in charm-anticharm mesons, perhaps accompanied by a tail of light mesons from gluons irradiated in the decay process (M. Mangano).

•Weak decays of charm-anticharm mesons give rise to a characteristic e_{μ} signal, a good signature for events containing the fully-charm tetraquark.

J/ Ψ -pair and 4 μ cross sections

• We give the upper bounds: $\sigma_{theo.}(T(J) \to 4\mu) \le \sigma(pp \to 2 J/\Psi)[B(T(J) \to 4 \mu)]^2$

with: $\sigma(pp \rightarrow 2 J/\Psi) \simeq 15.2$ nb (LHCb @ 13 TeV, Aaij : 2016bqq)

LHCb arXiv:2006.16957: with luminosity 9 fb⁻¹ sees 258 evts BF in Cross section $[cc][\overline{cc}]$ Decay Channel (a) 6900 MeV peak, corresponding to upper limit (fb) \mathcal{T} decay $\sigma(pp \rightarrow T + ...)B(T \rightarrow 4\mu) \sim 30$ fb $J=0^{++}$ $\mathcal{T}
ightarrow D^{(*)+}D^{(*)-}
ightarrow c+\mu+\dots$ $6.5 \cdot 10^4$ (65 pb) $4.3 \ 10^{-3}$ $\mathcal{T} \to D^{(*)0} \bar{D}^{(*)0} \to e + \mu + \dots \mid 0.67 \ 10^{-3} \mid$ $1.0 \cdot 10^4 (10 \text{ pb})$ consistent with both upper limits. $2.7 \ 10^{-6}$ $\mathcal{T}
ightarrow 4 \mu$ 40• Assuming $T=T(2^{++})$ and our $J = 2^{++}$ $\mathcal{T} \to D^{*+} \bar{D}^{*-} \to e + \mu + \dots$ $6.3 \ 10^{-3}$ $9.6 \cdot 10^4 (96 \text{ pb})$ Bfs, we derive: $\mathcal{T} \to D^{*0} \bar{D}^{*0} \to e + \mu + \dots$ $0.98 \ 10^{-3}$ | $1.5 \cdot 10^4 (15 \text{ pb})$ $\sigma(pp \to T(2^{++}) + ...) \sim 1.8 \text{ nb}$ $\sigma(pp \rightarrow T(2^{++}) + \dots \rightarrow 2 J/\Psi + \dots) \sim 6.4 \text{ pb}$ $\mathcal{T} \to 4\mu$ $1.6 \ 10^{-5}$ 238

• For $T=T(0^{++})$, we predict:

 $\sigma(pp \to T(0^{++} + \dots)B(T(0^{++} \to 4\mu) \sim 1 \text{ fb})$ $\sigma(pp \to T(0^{++} + \dots) \sim 0.37 \text{ nb})$ $\sigma(pp \to T(0^{++}) + \dots \to 2 J/\Psi + \dots) \sim 0.28 \text{ pb}$

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Total widths and J/ Ψ -pair mass spectrum

• Total widths are proportional to the ratio: $\xi = |\Psi_T(0)|^2 / |\Psi_{J/\Psi}(0)|^2$

• we determine ξ from models, and use the spread of values as an error estimate



 $\xi = 4.6 \pm 1.4$ $\Gamma(0^{++}) = 97 \pm 30 \text{ MeV}; \Gamma(2^{++}) = 64 \pm 20 \text{ MeV}$

Wish List spectrum:

- Take mass differences without error and suppose the 6900 MeV peak with 258 events corresponds to $J=2^{++}$
- 9 events expected for J=0⁺⁺ at ~ 6540 MeV
- can the 0⁺⁺ be the small fluctuation seen at ~ 6620 ??

In conclusion....

• The existence of exotic SU(3) flavour multiplets, with a characteristic scale of symmetry breaking is a distinctive prediction of compact tetraquarks.

• The newly found strange exotics are close in mass, like X(3872) and $Z_c(3900)$, and fit into their nonets: a clear score in favour.

• Decays: $Y(4230) \rightarrow \gamma X(3872)$ and $\pi Z_c(3900)$ are consistent with D* and D1 decays

• $Y \rightarrow \pi Z_c(4020)$???? needs clarification

• Much remains to be done, to produce more precise data and to search for still missing particles, some with well defined mass and decay modes

• it is a *tough order*: more luminosity, better energy definition, detectors with exceptional qualities... a lot of work...

• a much closer exchange between theory and experiment is needed.

This exchange took place in the sixties and seventies and it led to the quark picture of mesons and baryons

The 4c tetraquarks: a game changer

- The observation of the 6900 peak opens several exciting possibilities and it may be a real *game changer*.
- 6900 seems to fit well with a 2^{++} , [$cc\bar{c}\bar{c}$] resonance, for production rate and total width
- a direct spin-parity determination is crucial;
- more states are expected:
- a 0⁺⁺ peak is predicted with mass~6600 MeV, similar width, large decay rate in η_c pairs and a 4 muon rate

$$\sigma(pp \rightarrow T + ... \rightarrow 2 J/\Psi + ...)B(J/\Psi \rightarrow 2\mu)^2 \sim 1 \text{ fb}$$

- other peaks at masses above 6900 MeV, corresponding to radial and orbital excitations
- Large cross sections are to be expected for peaks in $D\overline{D}$, even allowing for a reduction for the exclusive channel: worth searching for the 0^{++}