Lecture 8. Tetraquarks vs. molecules: the Weinberg criterium for X(3872) and the double charm $\mathcal{T}^{+}_{cc}(3875)$

- ***Summary***

1. The overall panorama ✔
2. Constituent Quark Model and masses of conventional mesons and baryons ✔
3. Light and Heavy Tetraquarks. First comparison with hadron molecules ✔
4. Tetraquarks and the EightFold Way. Di-J/Ψ resonances ✔
5. *X*(3872) and its missing partners ✔
6. Born-Oppenheimer approximation for double charm baryons and tetraquarks ✔
7. Multiquark states in N colours, in the $N \to \infty$ limit ✔
8. Tetraquarks vs. molecules: the Weinberg criterium for *X*(3872) and the double charm $\mathcal{T}^{+}_{cc}(3875)$ ✔

The doubly charmed Tetraquark, T_{cc}^{+}

- The existence of doubly charmed tetraquarks, $[QQ\bar{q}\bar{q}]$, was considered in 2013 by Esposito et al.

  Esposito *et al*, PRD **88**(2013) 054029)

- Starting from the mass of the doubly charmed baryon, Karliner and Rosner estimated of the mass of the lowest lying, I=0 state at $M(T_{cc}^{+}) = 3882 \pm 12$ MeV, 7 MeV above the $D^0 D^{*+}$ threshold.

  M. Karliner and J. L. Rosner, PRL **119**(2017) 202001.

- A similar value was obtained by Eichten and Quigg

  E. J. Eichten and C. Quigg, PRL **119** (2017) 202002

- A value close to the $D^0 D^* \gamma$ threshold is obtained in the Born -Oppenheimer Approximation, using constituent quark masses derived from the meson spectrum

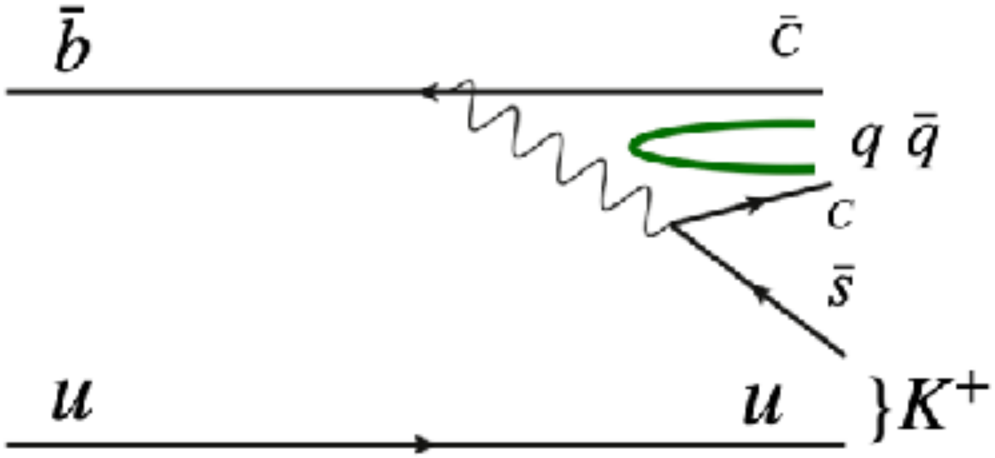
  L. Maiani et al., PRD **100** (2019) 074002

- The value $M(T_{cc}^{+}) - M(D^0 D^{+}) = -23 \pm 11$ MeV is obtained in lattice QCD calculation

  P. Junnarkar *et al*, PRD **99**(2019) 034507

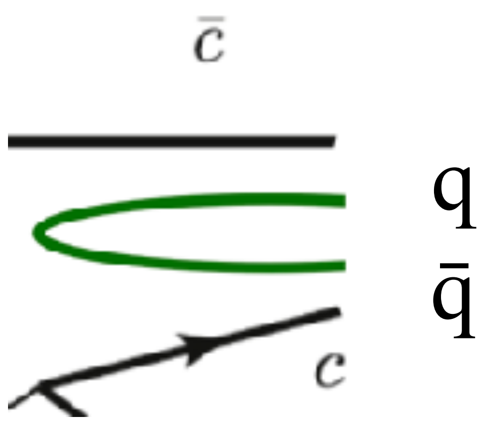
- The closeness to the $D^0 D^{*+}$ threshold has nonetheless invited speculations about a molecular, $D^0 D^{*+}$, nature of T_{cc}^{+}.

2. Molecule or compact? Back to the fundamentals

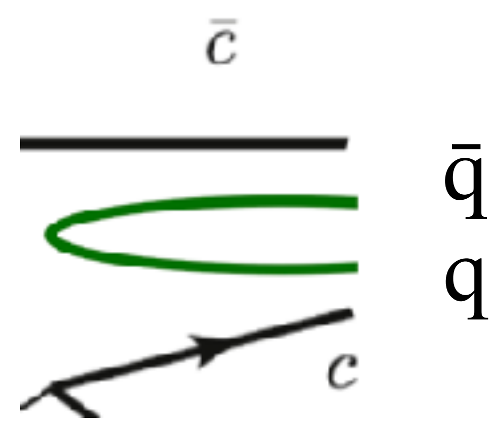


$\rightarrow D^*\bar{D} + \bar{D}^*D$

- The interesting part is the upper left corner, which we can specify in two ways



or



or both?

$[\bar{C}q]^1[C\bar{q}]^1 \rightarrow D^*\bar{D} + \bar{D}^*D \qquad [Cq]^{\bar{3}}\ [\bar{C}\bar{q}]_3 \rightarrow D^*\bar{D} + \bar{D}^*D$

- for a neutron-proton pair, the question posed by Weinberg was;
  is the deuteron a bound state or is there an "elementary" dibaryon ?

Molecule or compact? the QCD framework

- We know for sure that QCD produces hidden charm, confined hadron states: charmonia, $D^*\bar{D} + \bar{D}^*D$. ??? Do confined tetraquarks exist??
- Suppose we switch off the interactions between confined hadrons. The space of possible hidden charm states is made by two components
  - discrete energy states: charmonia and possibly tetraquarks: $|C><C| + |T><T|$
  - continuum charmed meson pairs: $\int d\alpha |D^*\bar{D}(\alpha)><D^*\bar{D}(\alpha)|$

In this limit: $|<X|X>|^2 = 1 = Z + \int d\alpha \, |<X|D^*\bar{D}(\alpha)>|^2$, where,

$Z = |<X|C>|^2 + |<X|T>|^2$

There are ***two regimes*** Z=0: corresponds to a pure molecular state: X results from $D^* - \bar{D}$ interactions only (like the deuteron)

- $Z \neq 0$: some compact, discrete state ***must*** exist
- unlike charmonium states, X decays violate isospin: $\Gamma(\Psi\rho) \sim \Gamma(\Psi\omega)$

so that:

- $Z \neq 0$ ➜ ***Tetraquark with X quantum numbers most likely exists***.

How can we know?

- The key is the $D^*\bar{D}$ scattering amplitude, *f*, that near threshold (k=center of mass momentum~0) can be written as $1/f = k\cot\delta(k) - ik = -\kappa_0 + \frac{1}{2}r_0k^2 - ik + \dots$

- With Weinberg, we find

$$\kappa_0^{-1} = 2\frac{1-Z}{2-Z}\kappa^{-1} + O(1/m_\pi);\;\; r_0 = -\frac{Z}{1-Z}\kappa^{-1} + O(1/m_\pi)$$

$\kappa^{-1} = \sqrt{2\mu B}$, $B = M(D^*) + M(D) - M(X)$ (the "binding energy")

- It turns out that ***the parameters*** κ_0, r_0 ***can be determined from the X(3872) (or T^+_{cc}) line-shape***

R. Aaij *et al*. (LHCb), PRD **102**, 092005 (2020

- in the molecular case (Z=0) one has $r_0 = O(1/m_\pi)$

- In the molecular case and for attractive potentials, one can show that the unspecified part of $O(1/m_\pi)$ is positive: $r_0 > 0$

A little theorem (Landau-Smorodinski)

- Consider the Schroedinger's equation for the radial wave function of the molecular constituents

$$u_k''(r) + \left[k^2 - U(r)\right] u_k(r) = 0$$

with $U(r) = 2\mu V(r)$, $V(r) < 0$ is the potential, assumed to be attractive everywhere.

- We consider the wave function for two values of the momentum: $u_{k_{1,2}} \equiv u_{1,2}$

With simple manipulations we find the identity

$$u_2 u_1' - u_2' u_1 \Big|_0^R = (k_2^2 - k_1^2) \int_0^R dr\, u_2 u_1 \quad \text{(A)}$$

R >> a_0, the range of the potential ($\simeq 1/m_\pi$).

- Consider now the free equation, $\psi_k''(r) + k^2 \psi_k(r) = 0$, from which we also obtain

$$\psi_2 \psi_1' - \psi_2' \psi_1 \Big|_0^R = (k_2^2 - k_1^2) \int_0^R dr\, \psi_2 \psi_1 \quad \text{(B)}$$

- Normalizing to unity at r=0, the general expression for ψ_k is

$$\psi_k(r) = \frac{\sin(kr + \delta(k))}{\sin \delta(k)}, \text{ and: } \psi_k'(0) = k \cot \delta(k).$$

- The radial wave function u_k vanishes at r=0, and we normalize so that it tends exactly to the corresponding ψ_k for large enough radii.
- Now, subtract (A) from (B) and let $R \to \infty$ (the integral now is convergent) to find

$$k_2 \cot \delta(k_2) - k_1 \cot \delta(k_1) = (k_2^2 - k_1^2) \int_0^\infty dr\, (\psi_2 \psi_1 - u_2 u_1)$$

$$k_2 \cot\delta(k_2) - k_1 \cot\delta(k_1) = (k_2^2 - k_1^2)\int_0^\infty dr\,(\psi_2\psi_1 - u_2 u_1) \quad \text{(C)}$$

We compare (C) with the parameters of the scattering amplitude.

First we set $k_1 = 0$. Since $\lim_{k_1\to 0} k_1 \cot\delta(k_1) = -\kappa_0$

$$k_2 \cot\delta(k_2) = -\kappa_0 + k_2^2 \int_0^\infty dr\,\left(\psi_2\psi_0 - u_2 u_0\right)$$

For small momenta: $\quad k_2 \cot\delta(k_2) = -\kappa_0 + \frac{1}{2} r_0 k_2^2 + \dots$ so that

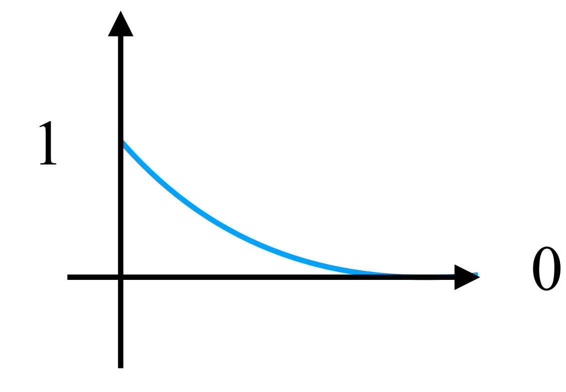
$$\boxed{r_0 = 2\int_0^\infty dr\,(\psi_0^2 - u_0^2)}$$

We know that $u_0(0) = 0,\ \psi_0(0) = 1$. Defining $\Delta(r) = \psi_0(r) - u_0(r)$ we have

$$\Delta(0) = +1,\ \Delta(\infty) = 0$$

The equations of motion imply $\Delta''(r) = -U(r)u_0(r)$. In presence of a single bound state, where u(r) has no nodes, we get

$$\Delta''(r) > 0 \rightarrow \psi_0(r) > u_0(r) \qquad \text{that is}$$



$$\boxed{r_0 > 0}$$

- reassuringly: $r_0(\text{deuteron}) = +1.75\ \text{fm}$,
- conversely a negative value of $r_0 > 0$ implies $Z > 0$

X lineshape: from Breit-Wigner to scattering lengths

- Consider $D^{*0}\bar{D}^0$scattering above threshold. If there is a resonance slight below, the amplitude takes the Breit-Wigner form

$$f = -\frac{\frac{1}{2}g^2_{\rm BW}}{E - m_{\rm BW} + \frac{i}{2}g^2_{\rm BW}k} \quad \text{(A)}$$

- for E= $\frac{k^2}{2\mu}$, the BW has the same form of the scattering amplitude

$$f = \frac{1}{-\kappa_0 + \frac{1}{2}r_0k^2 - ik + \dots} \quad \text{(B)}$$

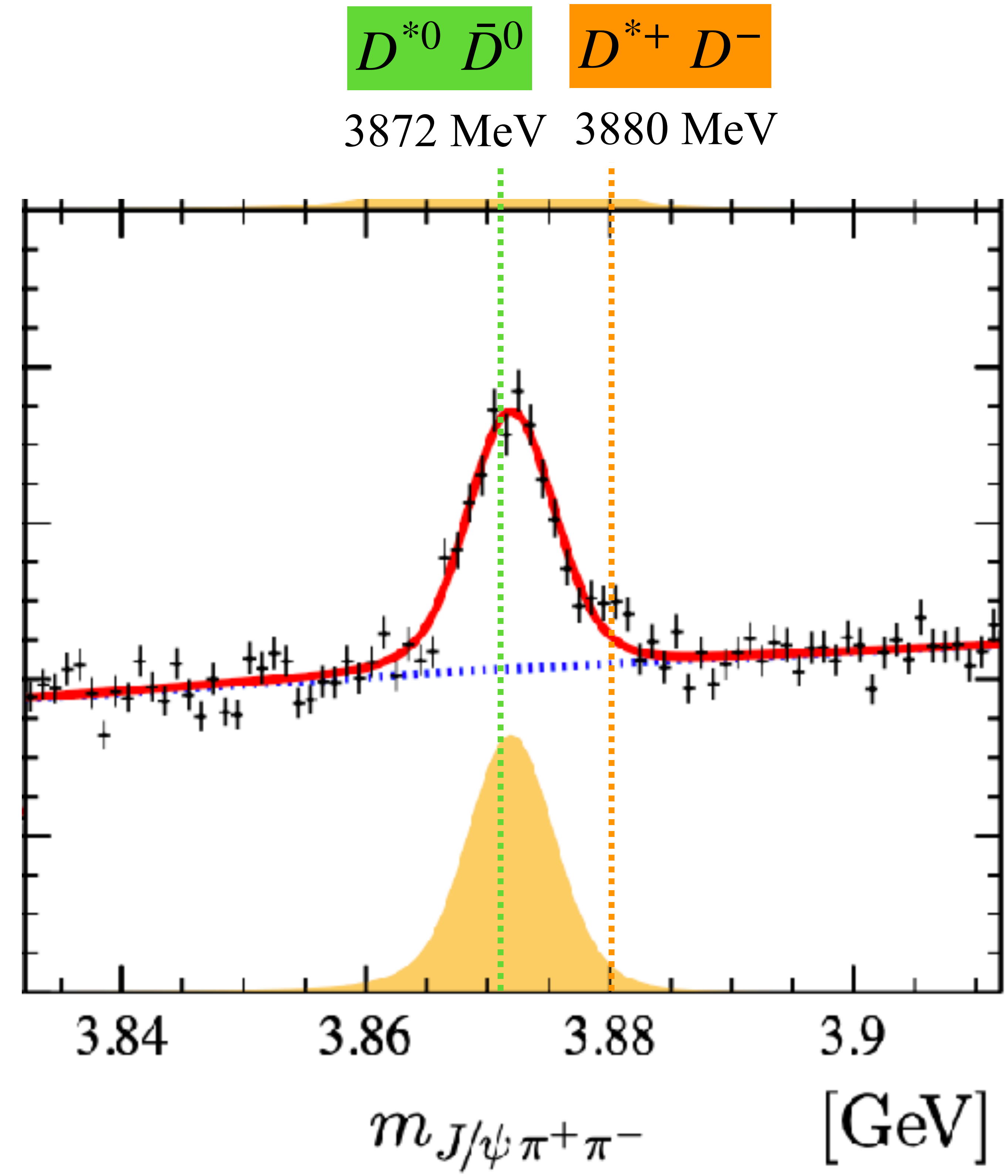
thus, from the parameters of the line-shape (A) we can determine κ_0 and r_0 (B)

- neglecting experimental errors on the parameters, for the X(3872) and the LHCb data, we find:
- $\kappa_0 \simeq 6.92$ MeV; $r_0 = -5.3$ fm, well into the compact tetraquark region.
- using a more recent error analysis, the effective radius is found to be in the range

$$-1.6 \text{ fm} > r_0 > -5.3 \text{ fm}$$

A.Esposito *et al*, arXiv:2108.11413

V. Baru et al., arXiv:2110.07484


$D^{*0}\ \bar{D}^0$
$D^{*+}\ D^-$
3872 MeV
3880 MeV
3.84
3.86
3.88
3.9
$m_{J/\psi\,\pi^+\pi^-}$
[GeV]

Details

- Flattte' function to fit the X(3972) lineshape to determine the parameters m_X^0, g_{LHCb}

$$f\left(X \to J/\psi\pi^+\pi^-\right) = -\frac{N}{E - m_X^0 + \frac{i}{2} g_{\text{LHCb}}\left(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)}\right) + \frac{i}{2}\left(\Gamma_\rho^0(E) + \Gamma_\omega^0(E) + \Gamma_0^0\right)}$$

$$\mu = \frac{D^{*0}\bar{D}^0}{D^{*0} + \bar{D}^0} = 967 \text{ MeV} \qquad \mu^+ = \frac{D^{*+}D^-}{D^{*+} + D^-} = 969 \text{ MeV} \qquad \delta = D^{*+} + D^- - D^{*0} - \bar{D}^0 = 8.3 \text{ MeV}$$

- Parametrization of the denominator

$$Den = E - m_X^0 + \frac{i}{2} g_{\text{LHCb}}\left(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)}\right) \sim \frac{2}{g_{\text{LHCb}}}(T - m_X^0) - \sqrt{2m_+\delta} + T\sqrt{\frac{m_+}{2\delta}} + ik \qquad T = \frac{k^2}{2\mu}$$

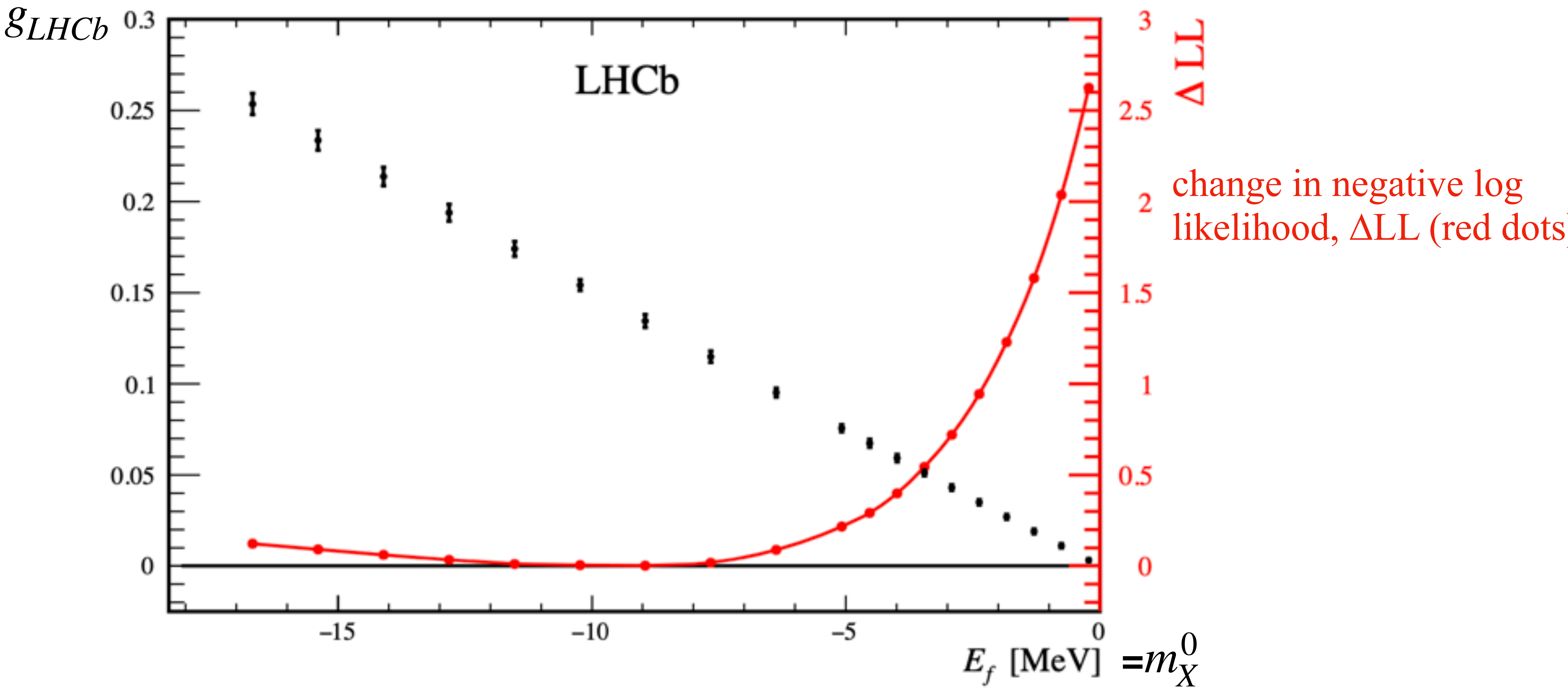
$$\kappa_0 = -\frac{2m_X^0}{g_{\text{LHCb}}} - \sqrt{2\mu_+\delta} \simeq 6.92 \text{ MeV}$$

$$r_0 = -\frac{2}{\mu g_{\text{LHCb}}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34\ fm$$

best fit:
$g_{LHCb} = 0.108$
$m_X^0 = -7.18$ MeV

- Taking into account the error on g_{LHCb}:

$$-1.7 \text{ fm} > r_0 > -5.3 \text{ fm}$$



- Log Likelihood is very insensitive to the value of g_{LHCb}
- $10 > g_{LHCb} > 0.108$
- leads to the range r_0

The value of Z

- From previous Weinberg formulae, we derive

$$Z = \frac{-r_0\kappa}{1 - r_0\kappa}$$

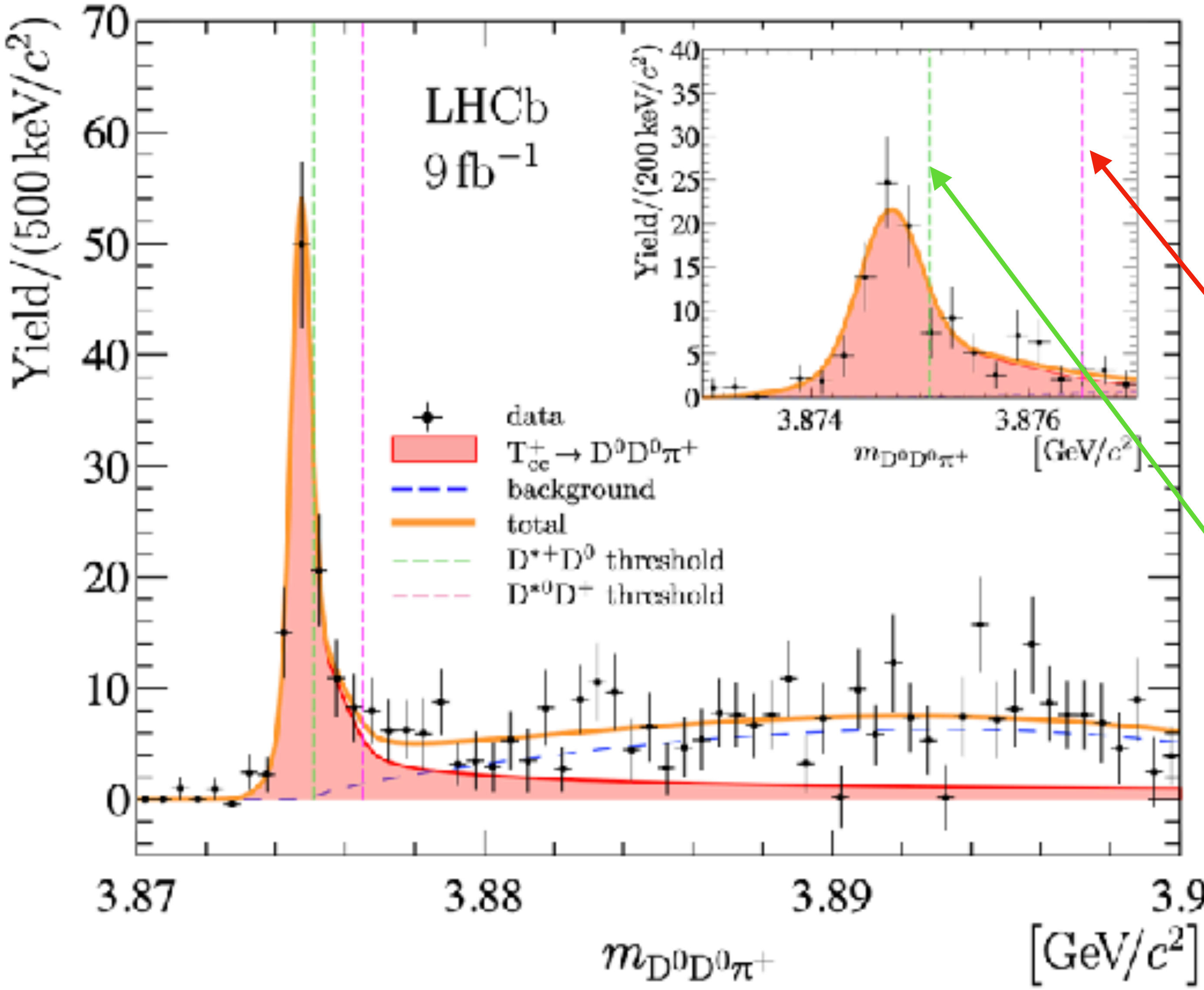
and with $\kappa \simeq \kappa_0$, we find

$$0.14 > Z > 0.052 > 0$$

- Z is often idenified with the admixture of X with the compact (tetraquark) state. In this case one would say that X is "essentially" a molecule
- This interpretation of Z, however, holds in the free theory only. With interaction, the compact state vector may be renormalized and the sternght of Z losses its meaning.
- With Weinberg, we think that what counts is that Z is non vanishing, indicating that ***there are***, in the Hilbert space, states, states different from D D* states: *the true token that the deuteron is composite is an effective range r_0 small and positive rather than large and negative* and *an elementary deuteron would have* 0<Z<1.

Latest: double charm tetraquark

LHCb arXiv:2109.01056v2



Mass : $D^{*0}D^{+} = 3876.5$ MeV

Mass : $D^{*+}D^{0} = 3875.1$ MeV

for $\mathcal{T}_{cc}^{+}$

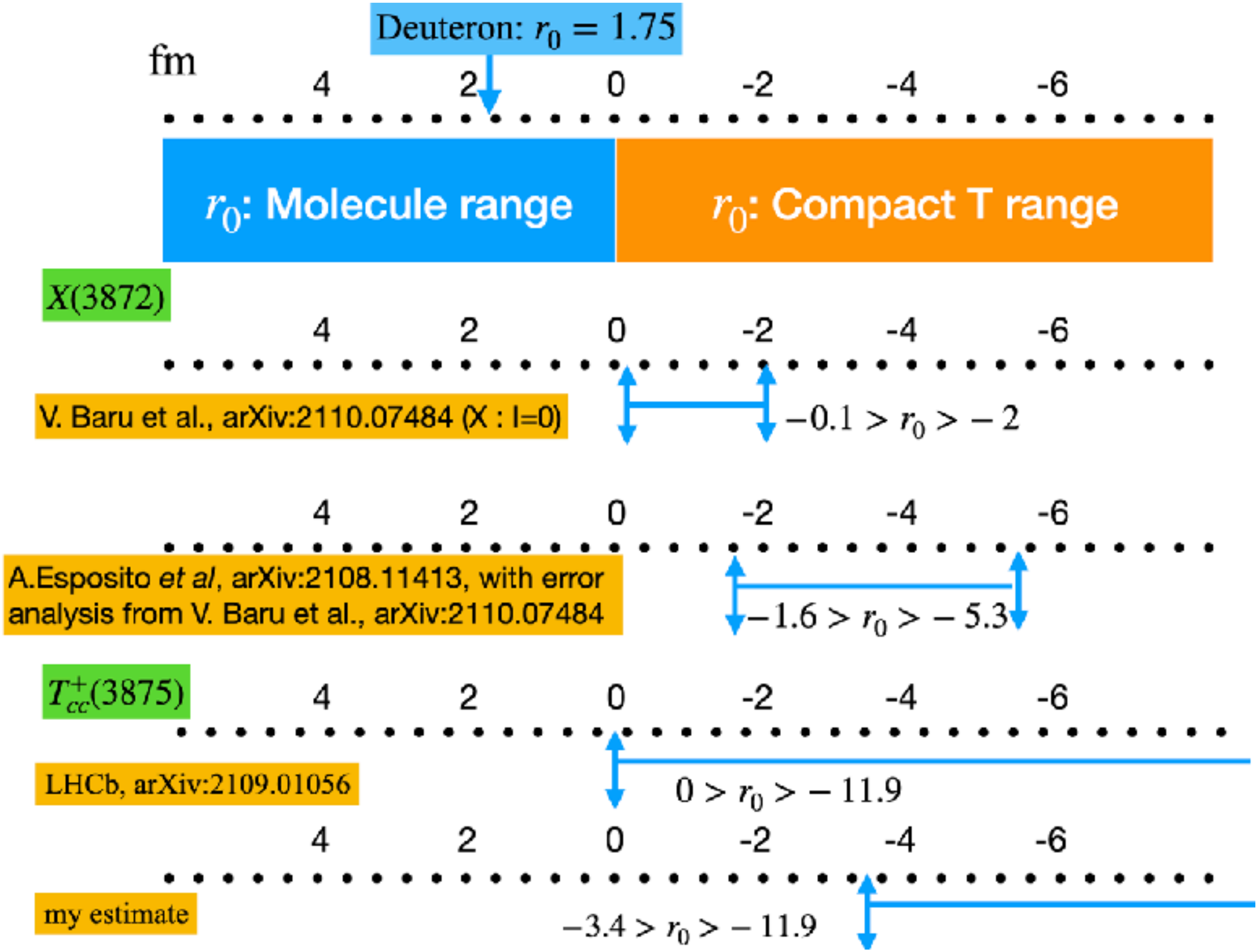
$$\mu = \frac{D^{*+}D^0}{D^{*+} + \bar{D}^0} = 967.5 \text{ MeV} \quad \mu^+ = \frac{D^{*0}D^+}{D^{*0} + D^+} = 968.0 \text{ MeV} \quad \delta = D^{*0} + D^+ - D^{*+} - \bar{D}^0 = 1.7 \text{ MeV}$$

$$(r_0)_{u.l.} = \sqrt{\frac{\mu_+}{2\mu^2\delta}} = -3.4 \text{ fm}$$

lower limit

$(r_0)_{l.l.} = -11.9$ fm LHCb, arXiv:2109.01056

r_0 : My Summary



A new analysis by the Valencia group claims $r_0 \sim +1$ fm for T_{cc}^+.

No consensus yet, but it seems we are on a very promising road.
Stay tuned!!