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EXPLORING THE MULTIDIMENSIONAL STRUCTURE OF THE NUCLEON

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Recent Review

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The 3-D Structure of the Nucleon

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In this topical collection (17 articles)

Editors: M. Anselmino, M. Guidal, P. Rossi

A few references on GPDs

- Overviews with full bibliography:

  - M. Diehl, Phys. Rep. 388 (2003) 41
  - K. Goeke, M. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401
  - X. Ji, Ann. Rev. Nucl. Part. Sci. 54 (2004) 413
  - A.V. Belitsky, A.V. Radyushkin, Phys. Rept. 418 (2005)
  - S. Boffi, B. Pasquini, Riv. Nuovo Cim. 30 (2007) 387
  - M. Guidal, H. Moutarde, M. Vanderhaeghen, Rept. Prog. Phys. 76 (2013) 066202
  - K. Kumericki, S. Liuti, H. Moutarde, Eur. Phys. J. A52 (2016) 157

Key information from GPDs

- Multidimensional picture of the proton in the 1+2D
- Decomposition of Form Factors as function of x
- Sum rule for Angular Momentum
- Access to Form Factors of Energy Momentum Tensor

⟶ "mechanical" properties of the nucleon

How can we built up
a multidimensional picture
of the nucleon ?

Charges

$$\frac{1}{2P^+}\langle p^+,\vec{0}_\perp,\Lambda'|\bar{\psi}(0)\Gamma\psi(0)|p^+,\vec{0}_\perp,\Lambda\rangle$$

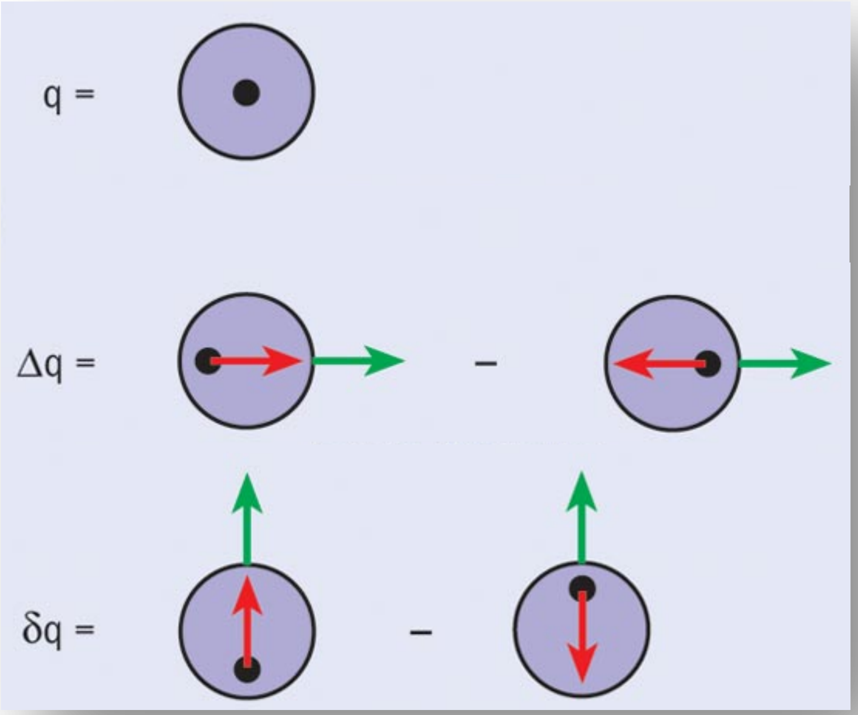
Depend on

Λ, Λ', Γ: nucleon and quark polarizations

Vector: $\Gamma = \gamma^+$
Parton number

Axial: $\Gamma = \gamma^+\gamma_5$
Parton helicity

Tensor: $\Gamma = i\sigma^{+i}\gamma_5$
Parton transversity



Charges

Form Factors (FFs)

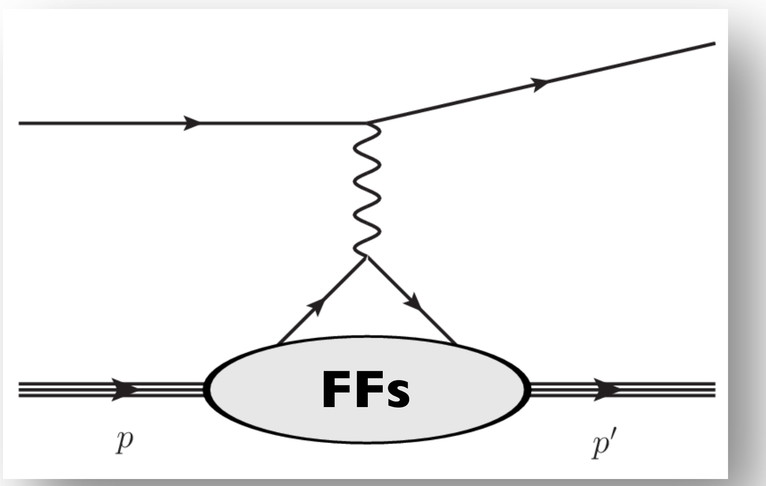
$$\frac{1}{2P^+}\langle p^+,\, \frac{\vec{\Delta}_\perp}{2},\, \Lambda' | \bar{\psi}(0)\,\Gamma\,\psi(0) | p^+,\, -\frac{\vec{\Delta}_\perp}{2},\, \Lambda\rangle$$

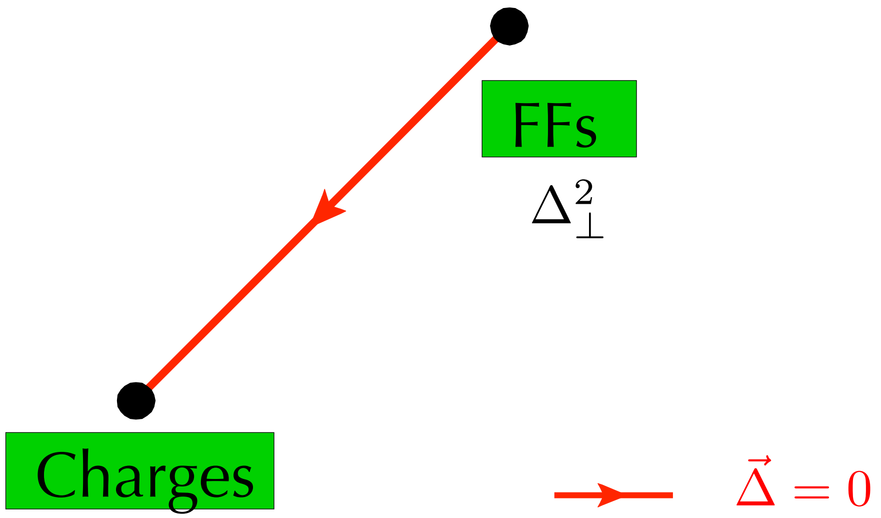
Depend on

$\Lambda,\, \Lambda',\, \Gamma$: nucleon and quark polarizations

Δ : momentum transfer $\vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$: impact parameter

Elastic Scattering

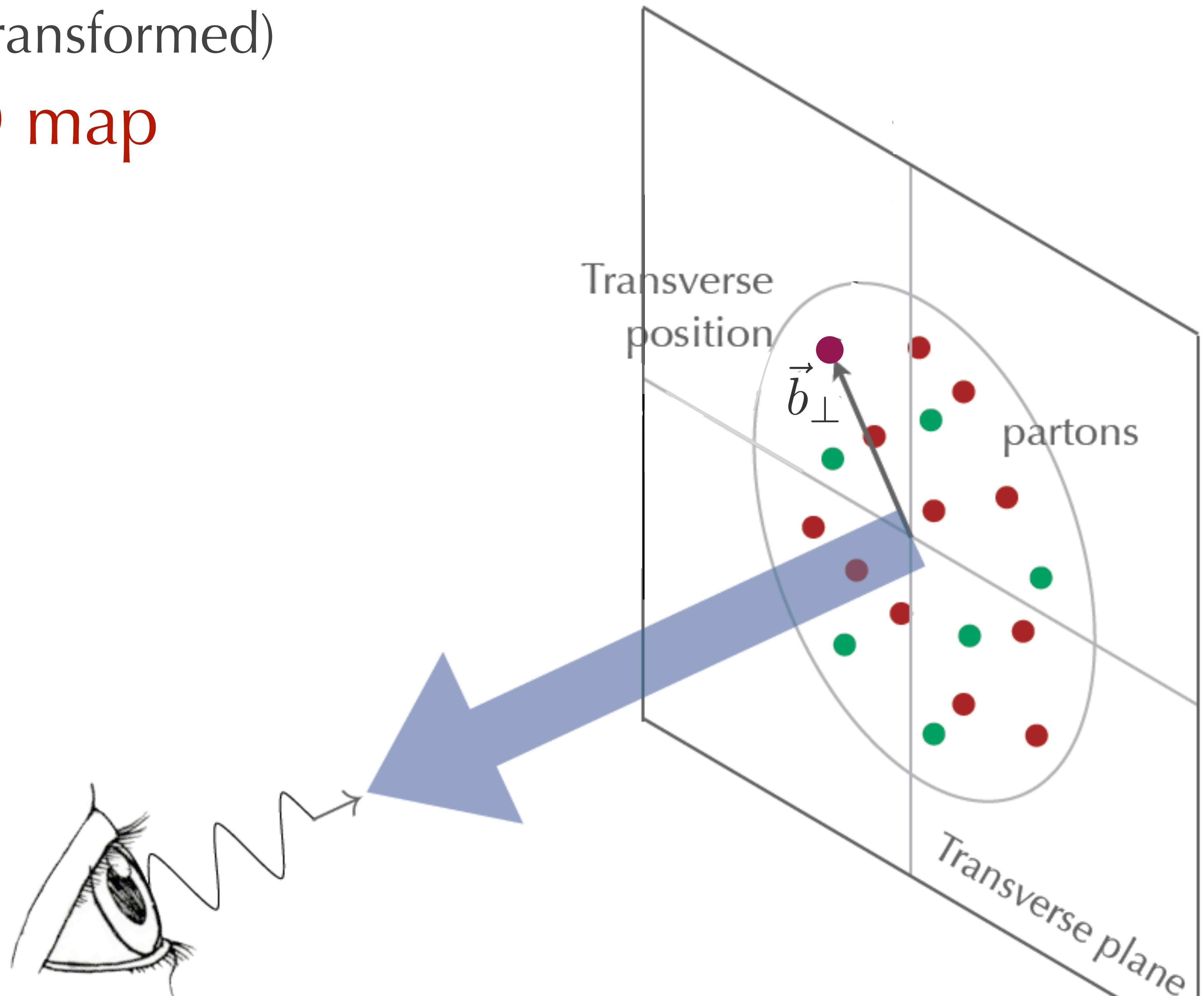




Form Factors:

(Fourier transformed)

0D+2D map



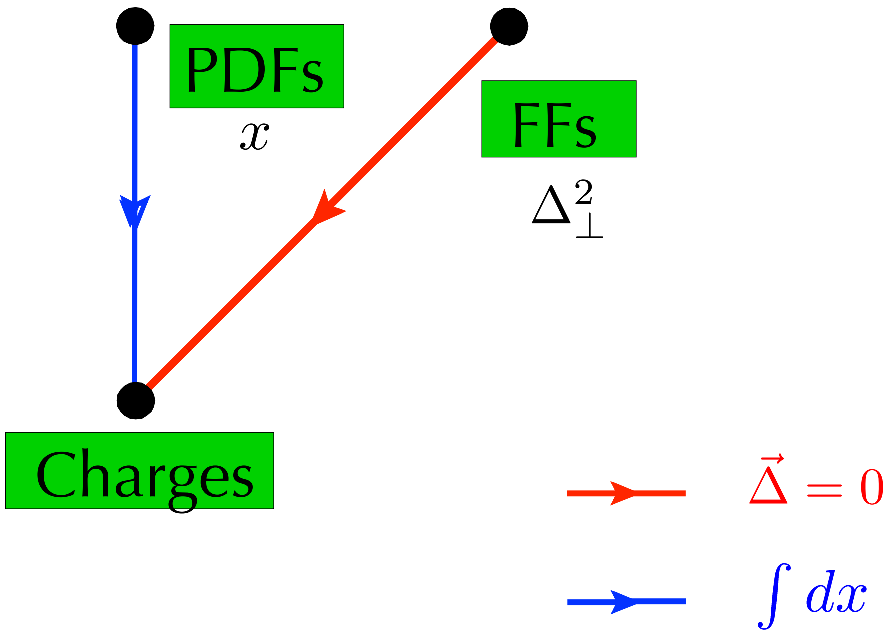
Parton Distribution Functions (PDFs)

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ik^+z^-} \langle p^+,\, \vec{0}_\perp,\, \Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\, \vec{0}_\perp,\, \Lambda\rangle_{z^+=0,z_\perp=0}$$

Depend on

$\Lambda,\, \Lambda',\, \Gamma$: nucleon and quark polarizations

$x = \frac{k^+}{p^+}$: longitudinal momentum fraction



Parton Distribution Functions (PDFs)

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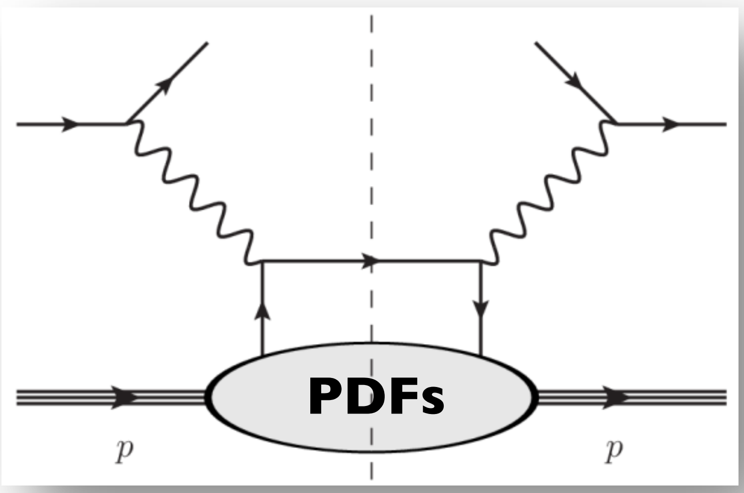
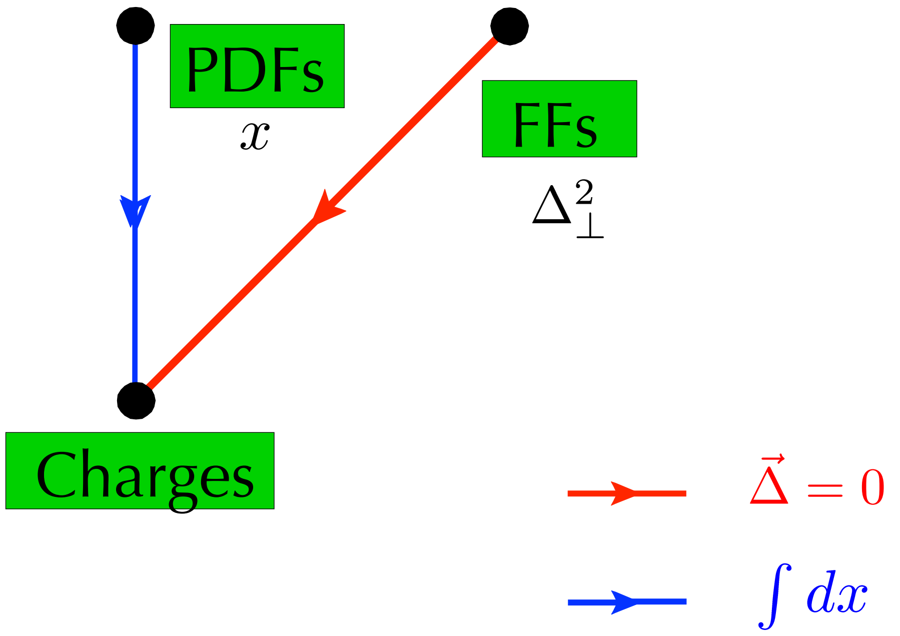
Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

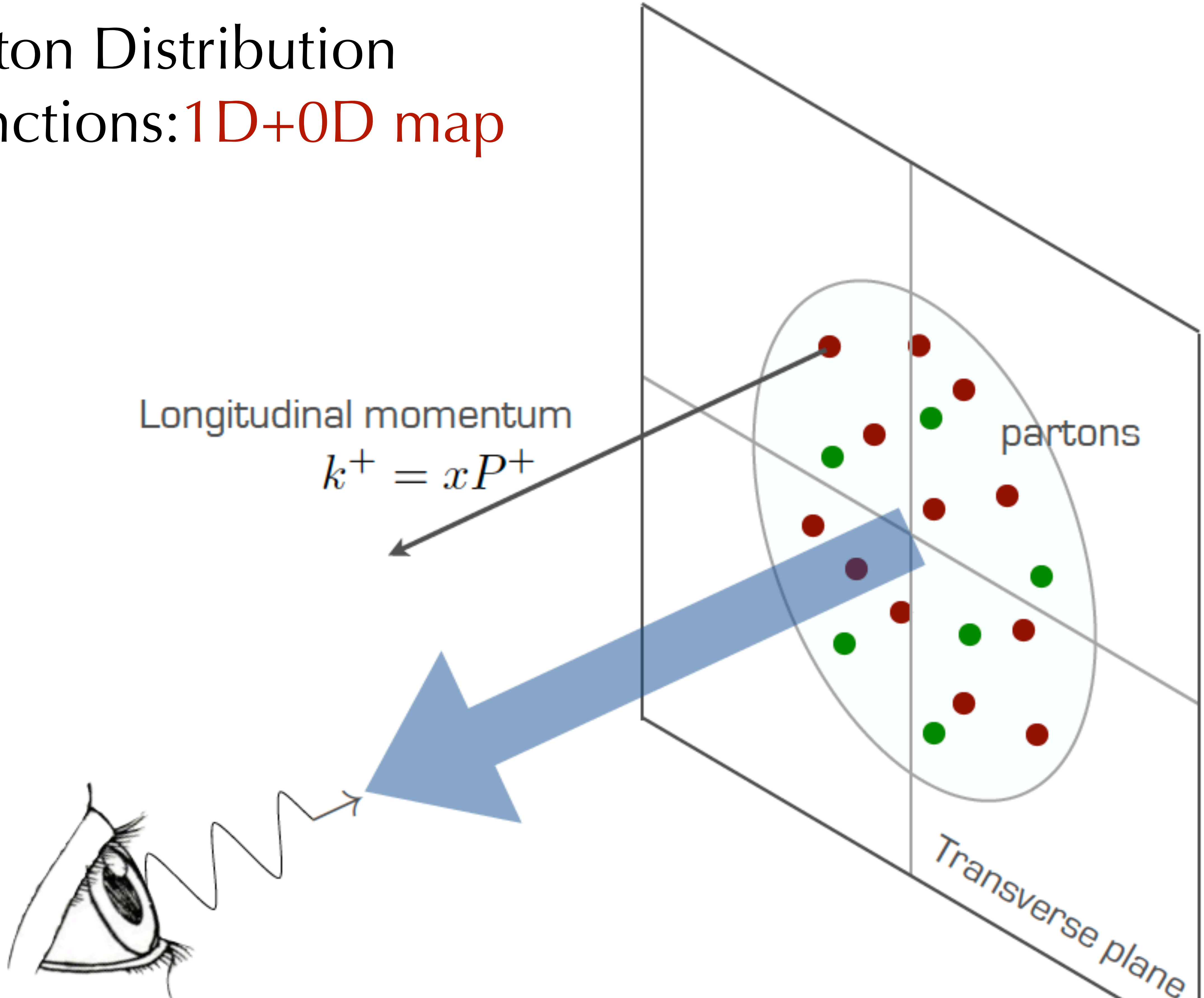
$x=\frac{k^+}{p^+}$: longitudinal momentum fraction

Deep Inelastic Scattering





Parton Distribution Functions:1D+0D map



Generalized Parton Distributions (GPDs)

$$\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}e^{ik^+z^-}\langle p'^+,\,-\frac{\vec{\Delta}_\perp}{2},\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\frac{\vec{\Delta}_\perp}{2},\,\Lambda\rangle_{z^+=0,z_\perp=0}$$

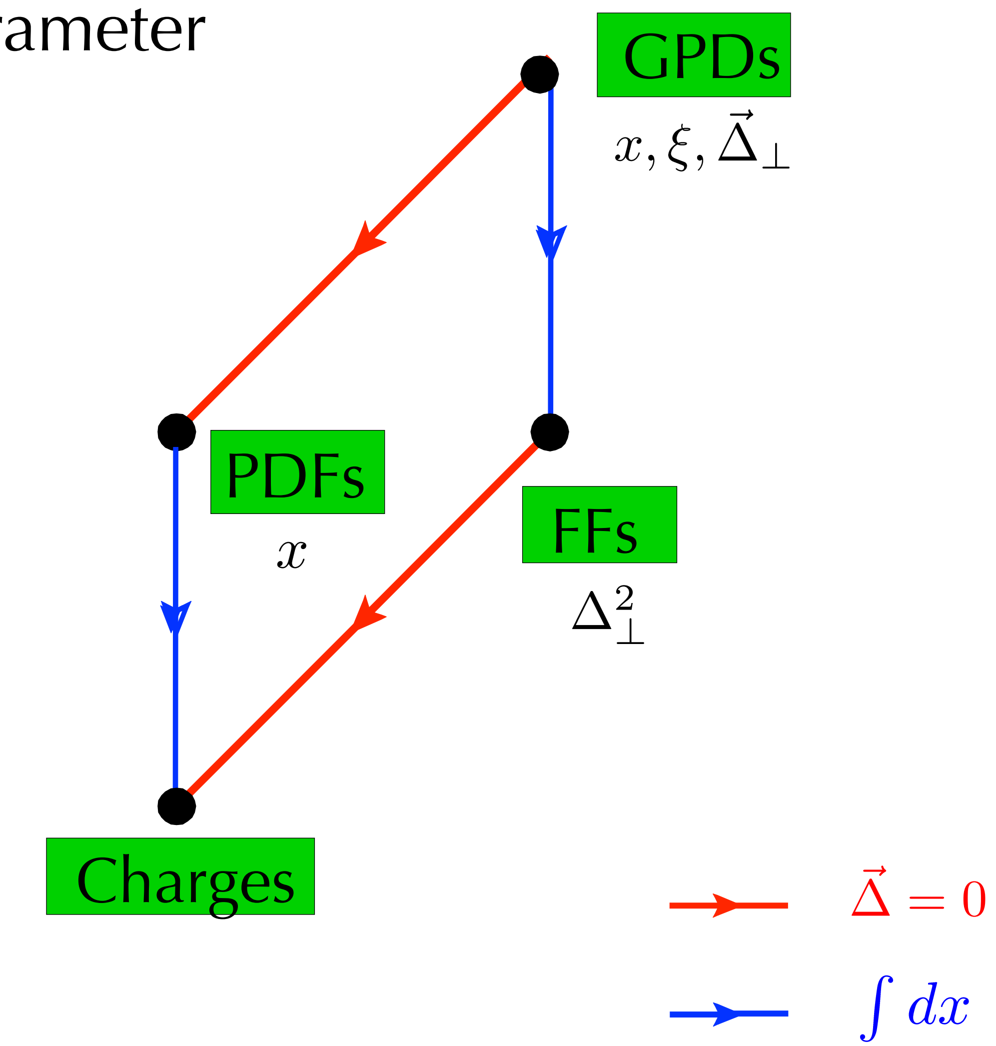
non-diagonal matrix elements

Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

$x=\frac{k^+}{p^+}$: longitudinal momentum fraction

Δ : momentum transfer $\vec{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \vec{b}_\perp$: impact parameter



Generalized Parton Distributions (GPDs)

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik^+z^-} \langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0, z_\perp=0}$$

non-diagonal matrix elements

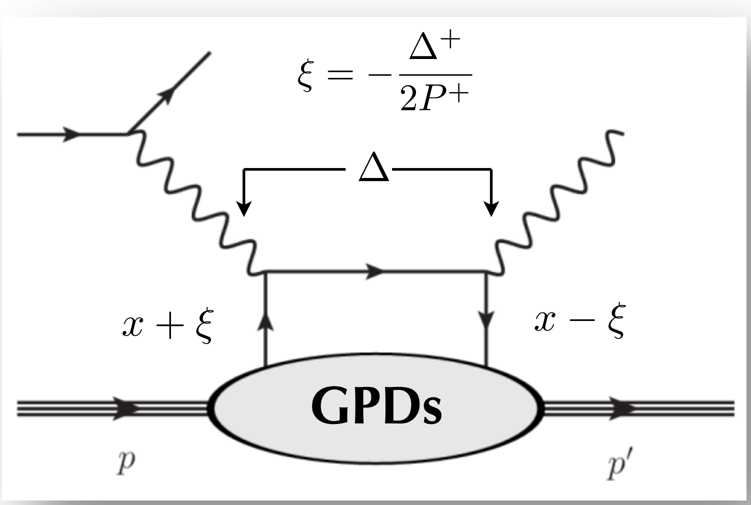
Depend on

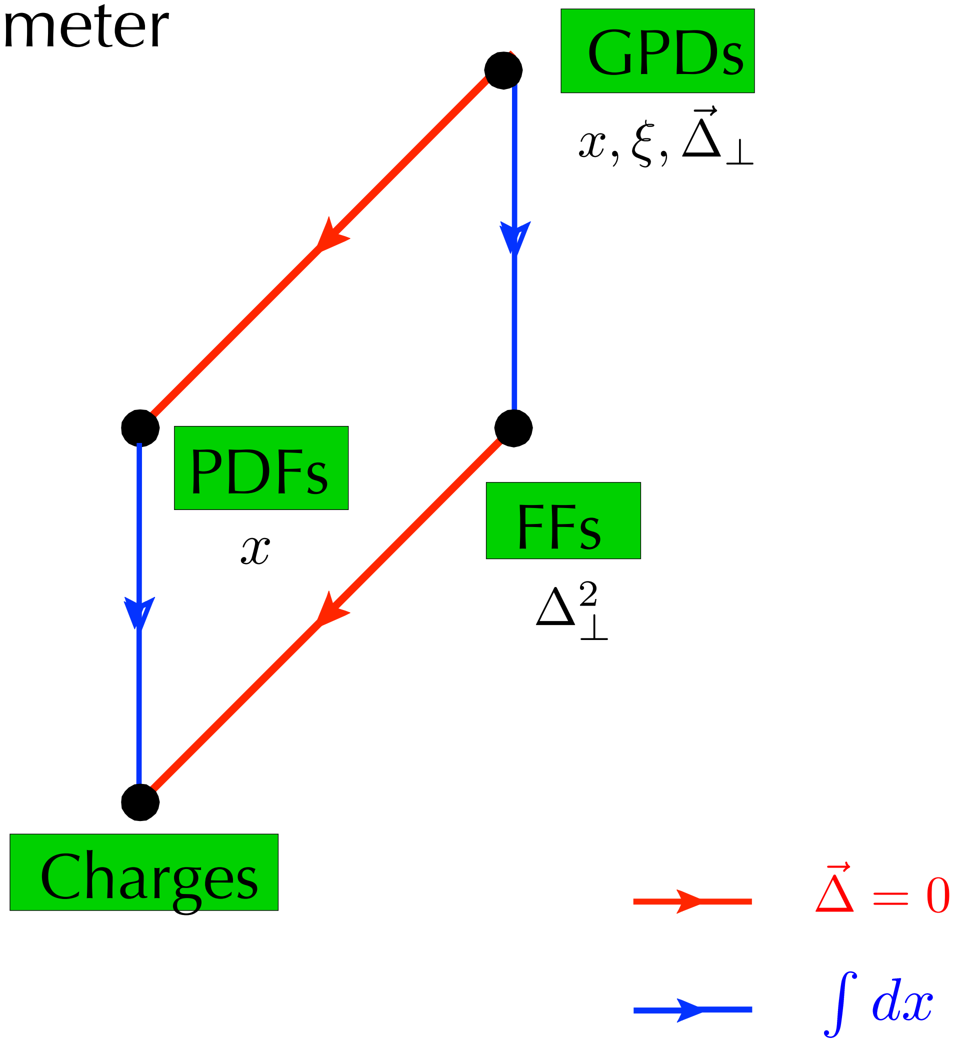
$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

$x = \frac{k^+}{p^+}$: longitudinal momentum fraction

Δ : momentum transfer $\vec{\Delta}_\perp \overset{FT}{\longleftrightarrow} \vec{b}_\perp$: impact parameter

Deeply Virtual Compton Scattering

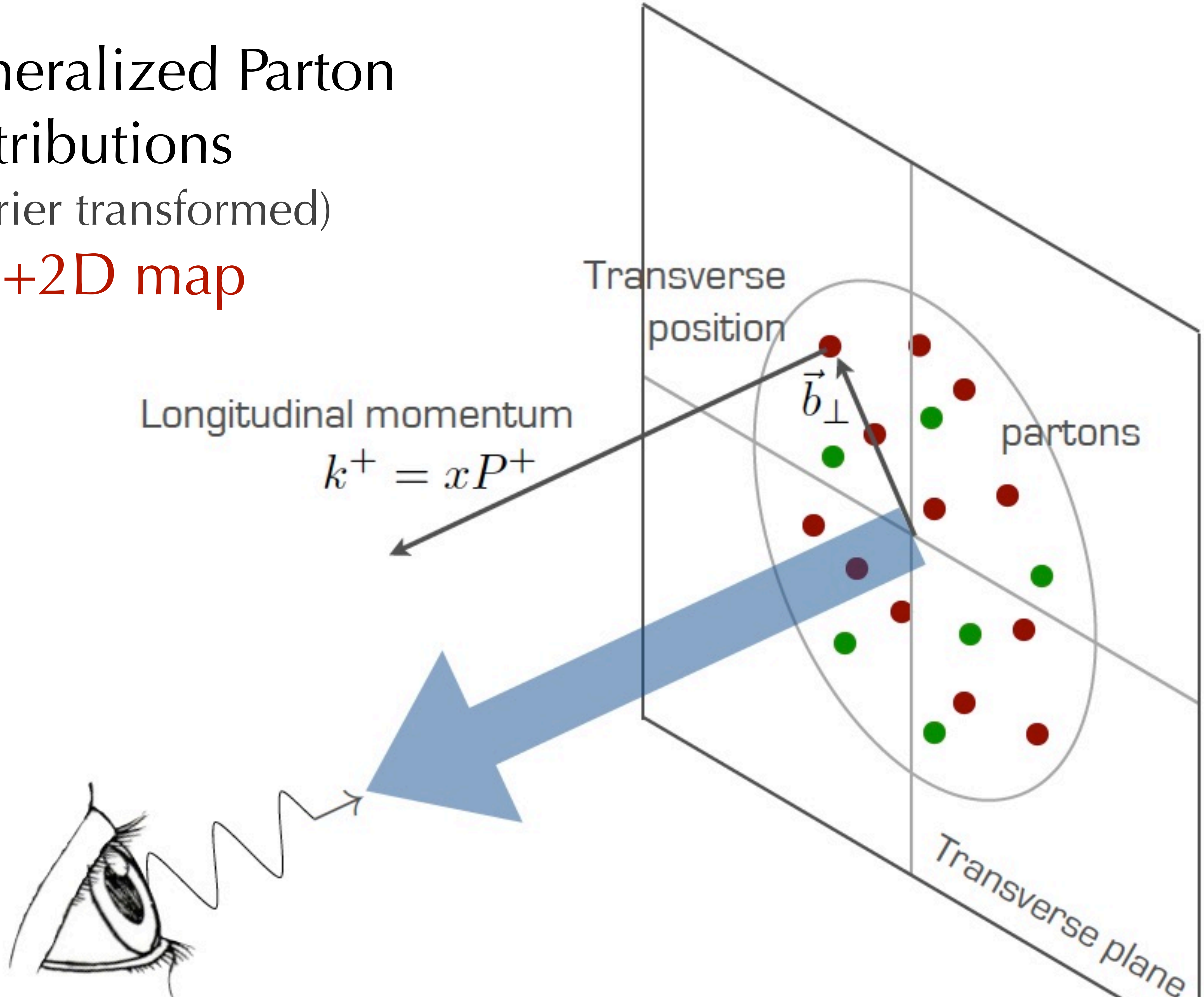




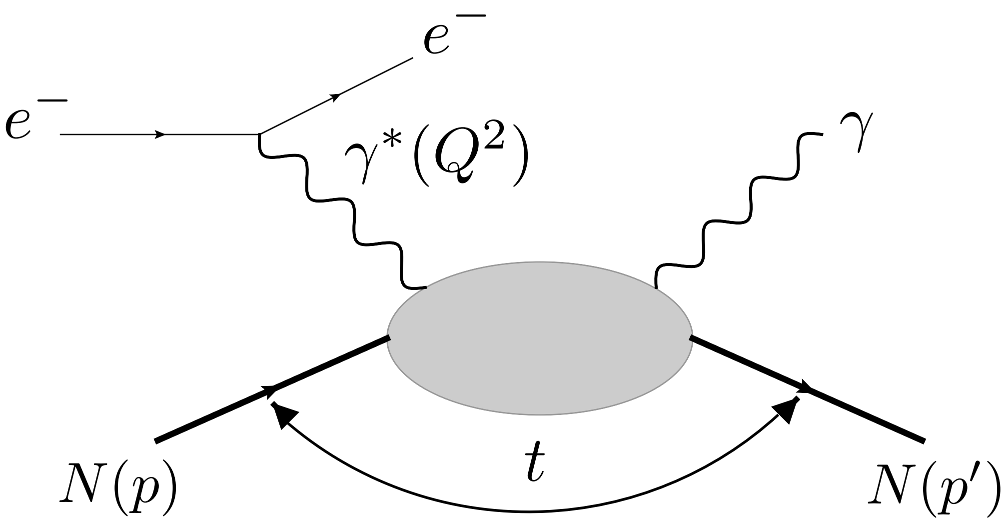
Generalized Parton Distributions

(Fourier transformed)

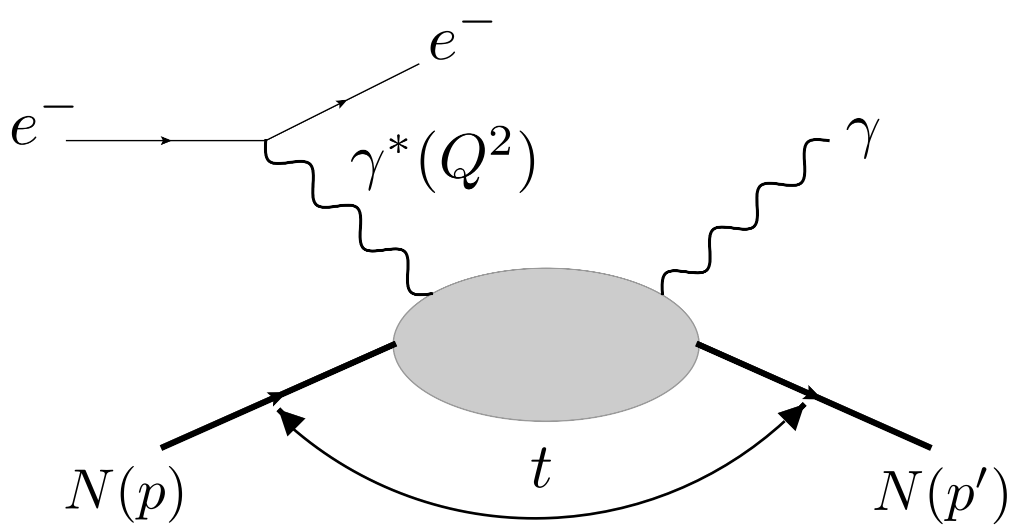
1D+2D map

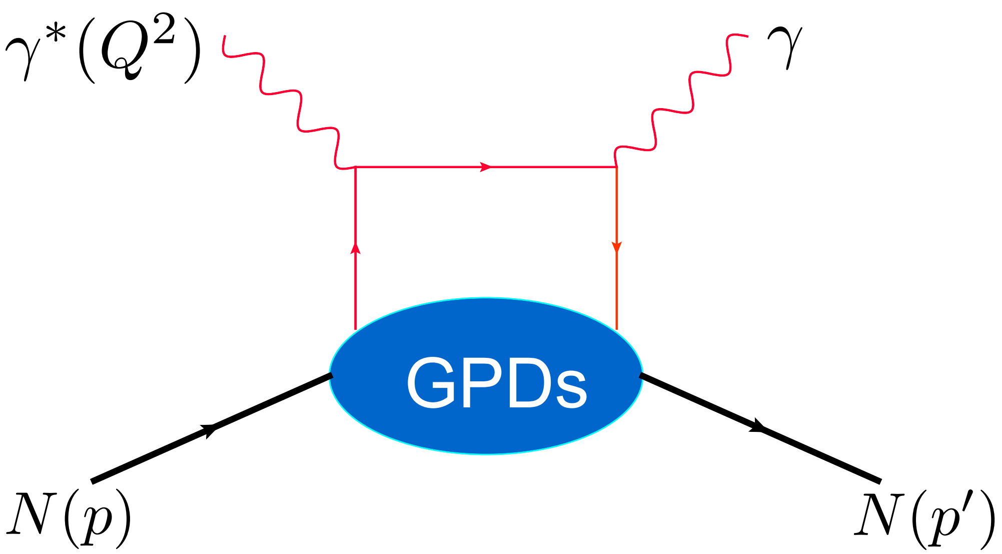


How to access GPDs



How to access GPDs

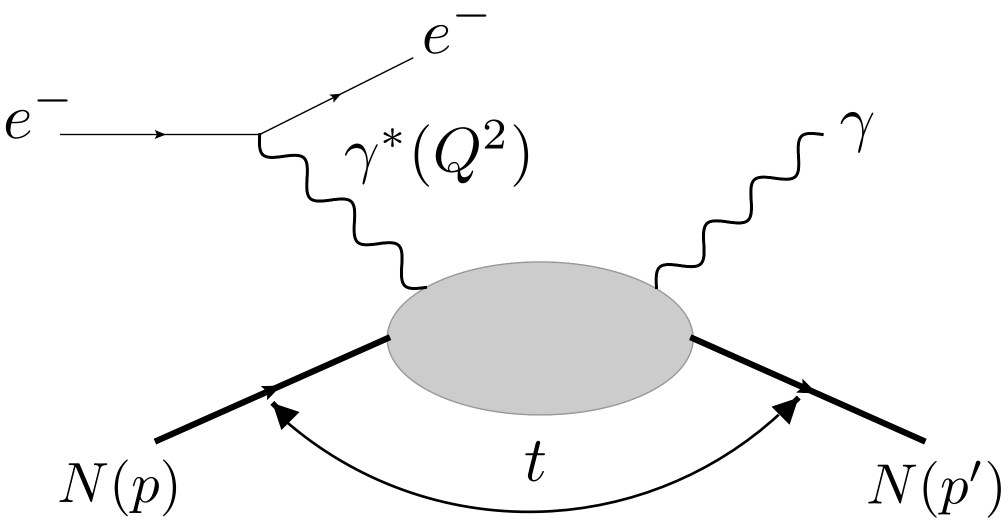


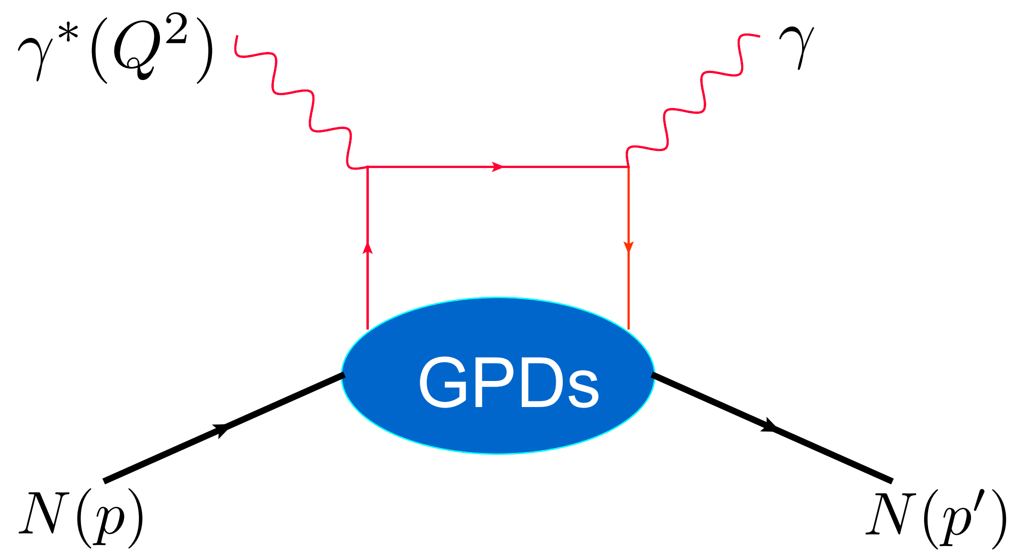


factorization for large Q^2, $|t| << Q^2$, s

$\mathcal{M}$ =[parton Ampl.] $\otimes$ [GPDs]

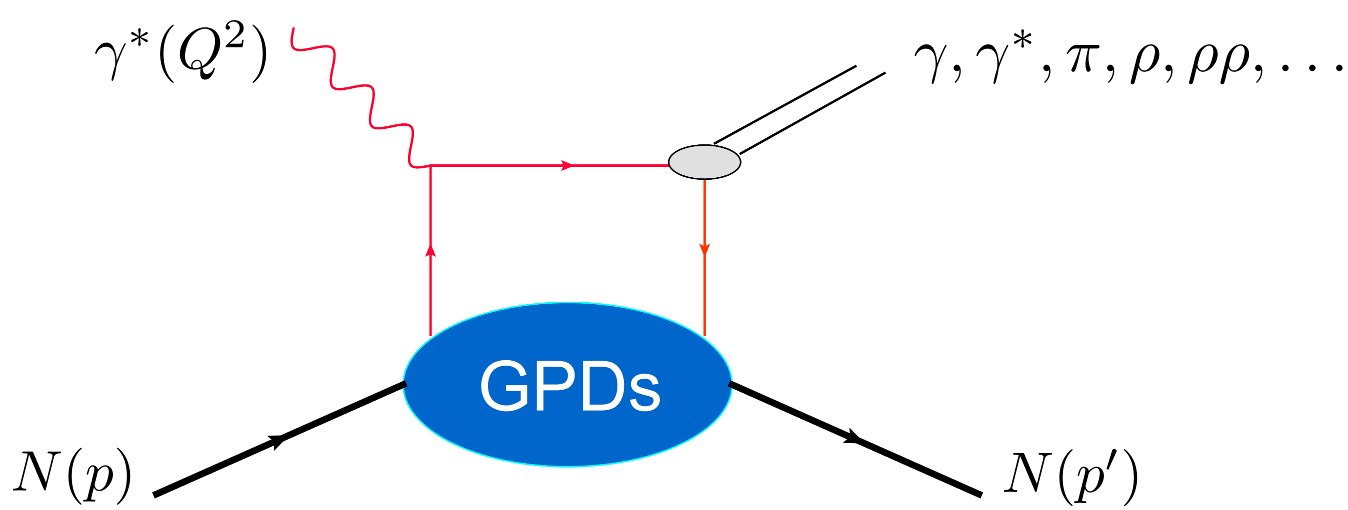
How to access GPDs





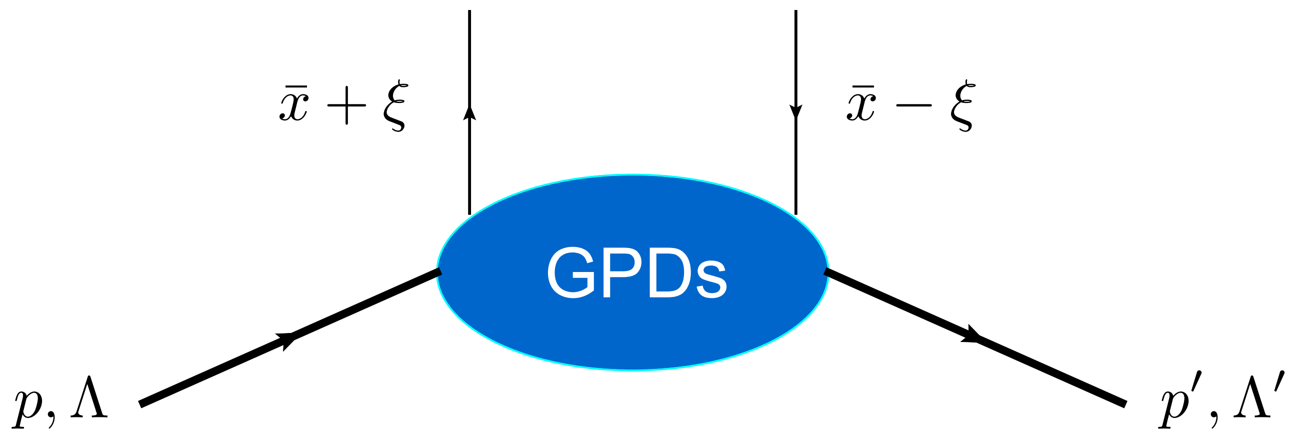
factorization for large Q^2, $|t| << Q^2$, s

$$\mathcal{M} = [\text{parton Ampl.}] \otimes [\text{GPDs}]$$



universality: the same GPDs enter a variety of exclusive reactions

Leading-Twist GPDs



$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-}\bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$$\Gamma = \begin{cases} \gamma^+ & H^q,\ E^q & \text{unpol.} \\ \gamma^+\gamma^5 & \tilde{H}^q,\ \tilde{E}^q & \text{long. pol.} \\ i\sigma^{+i}\gamma^5 & H^q_T,\ E^q_T,\ \tilde{H}^q_T,\ \tilde{E}^q_T & \text{transv. pol.} \end{cases}$$

- $p \neq p' \Rightarrow$ GPDs depend on two momentum fractions $\bar{x}$, ξ, and t

$$\bar{x} = \frac{(k+k')^+}{(p+p')^+} = \frac{\bar{k}^+}{P^+}$$

average fraction of the longitudinal momentum carried by partons

$$\xi = \frac{(p-p')^+}{(p+p')^+} = -\frac{\Delta^+}{2P^+}$$

skewness parameter: fraction of longitudinal momentum transfer

$$t = (p-p')^2 \equiv \Delta^2$$

t-channel momentum transfer squared

Quark polarization

Nucleon pol.

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}_T$	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}_T$	
T_x	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}$
T_y	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

ξ-odd

$$\mathcal{H} = \sqrt{1-\xi^2}\left(H - \frac{\xi^2}{1-\xi^2}\,E\right) \qquad \mathcal{H}_T = \sqrt{1-\xi^2}\left(H_T - \frac{\vec{\Delta}_\perp^2}{2M^2}\,\frac{\tilde{\mathcal{H}}_T}{\sqrt{1-\xi^2}} + \frac{\xi\,\tilde{\mathcal{E}}_T}{\sqrt{1-\xi^2}}\right)$$

$$\mathcal{E} = \frac{E}{\sqrt{1-\xi^2}} \qquad \mathcal{E}_T = \frac{2\tilde{H}_T + E_T - \xi\,\tilde{E}_T}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}} = \sqrt{1-\xi^2}\left(\tilde{H} - \frac{\xi^2}{1-\xi^2}\,\tilde{E}\right) \qquad \tilde{\mathcal{H}}_T = -\frac{\tilde{H}_T}{2\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{E}} = \frac{\xi\tilde{E}}{\sqrt{1-\xi^2}} \qquad \tilde{\mathcal{E}}_T = \frac{\tilde{E}_T - \xi\,E_T}{\sqrt{1-\xi^2}}$$

✦ Helicity structure: nucleon (Λ', Λ) quark (λ', λ)

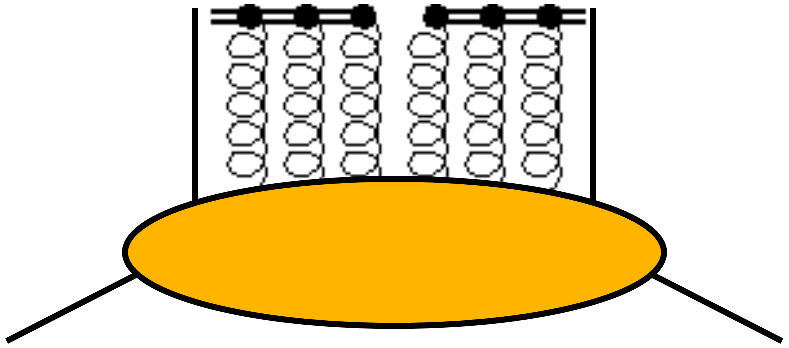
$$U = (++) + (--) \quad T_x = (-+) + (+-) \quad T_y = i[(-+) - (+-)] \quad L = (++) - (--)$$

Need of a gauge link

$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

not invariant under $\psi(z) \to e^{i\alpha(z)}\psi(z)$

$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\mathcal{U}_{[-\frac{z}{2},\frac{z}{2}]}\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$



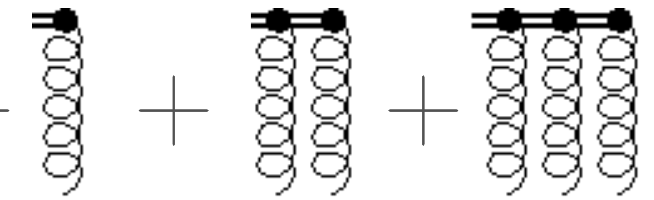
$$\mathcal{U}(-\tfrac{z}{2},\tfrac{z}{2}) \to e^{i\alpha(-\frac{z}{2})}\,\mathcal{U}(-\tfrac{z}{2},\tfrac{z}{2})\,e^{-i\alpha(\frac{z}{2})}$$

$$\mathcal{U}_{[-\frac{z}{2}\ \frac{z}{2}]} = \mathcal{P}\exp\left[-ig\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta^\mu\, A_\mu(\eta)\right]$$

Wilson line definition of GPDs

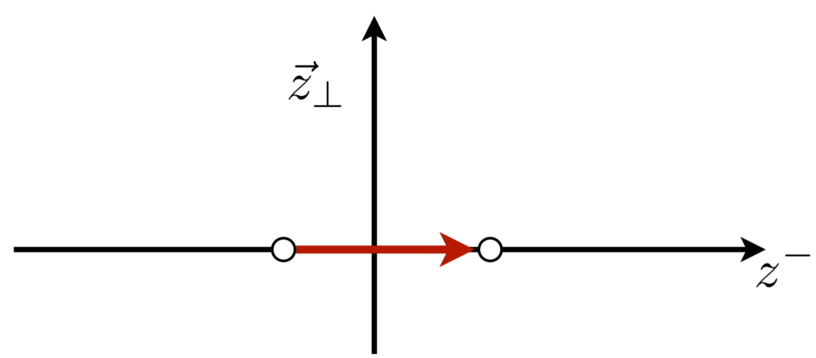
$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{U}_{[-\frac{z}{2},\frac{z}{2}]} \psi\left(\frac{z}{2}\right) |p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right] = (1 + \ldots + \ldots + \ldots + \ldots$$



exchange of more than 2 partons between hard scattering process (H) and soft amplitude (A) is suppressed except for gluons with polarization A^+

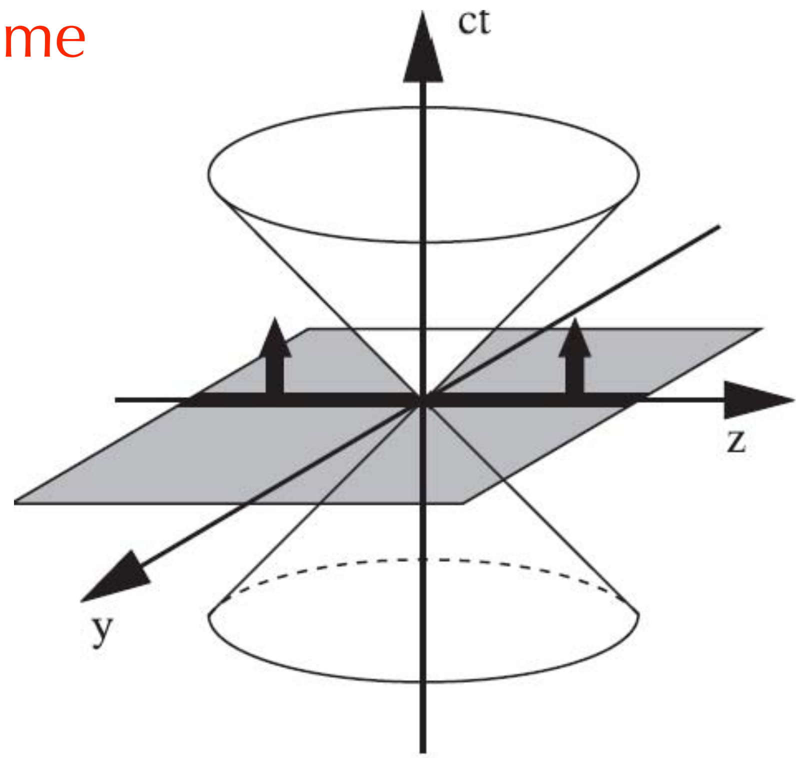
$$\mathcal{U}_{[-\frac{z}{2}\ \frac{z}{2}]} = \mathcal{P}\exp\left[-ig\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta^-\ A^+(\eta)\right]_{z^+=0,z_\perp=0}$$



for convenience, choose light-cone gauge: $A^+ = 0$ in which $U = 1$

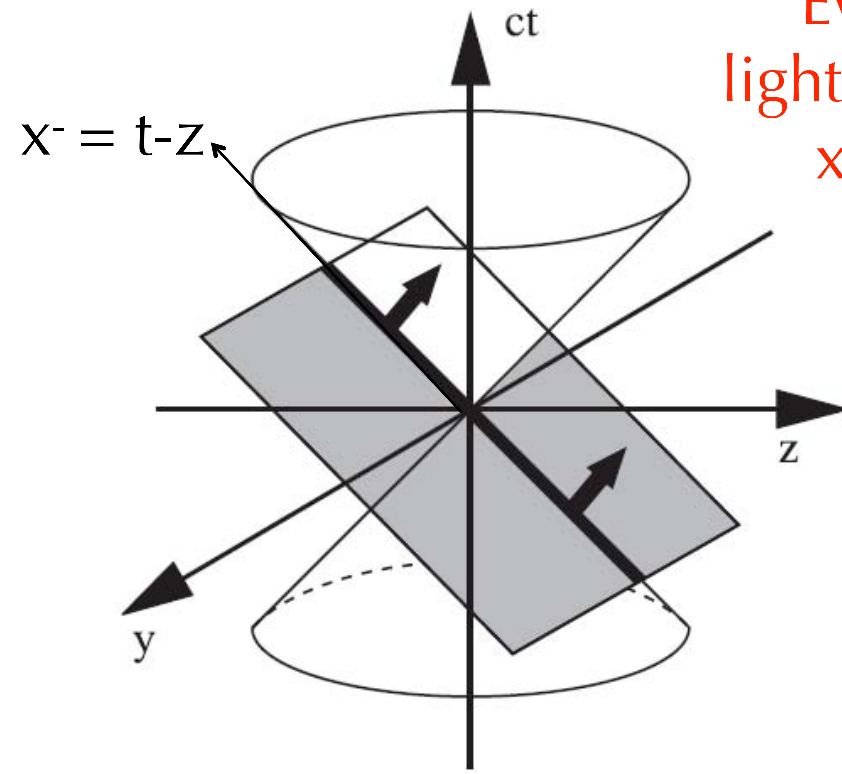
Partonic interpretation

Evolve in ordinary time



Instant Form

Evolve in light-front time $x^+ = t+z$



Light-Front Form

coordinates

Instant Form		Light-Front Form	
x^0	time	$\frac{x^0 + x^3}{\sqrt{2}}$	time
x^1, x^2, x^3	space	$\frac{x^0 - x^3}{\sqrt{2}}, \; x_\perp = (x^1, x^2)$	space

Hamiltonian

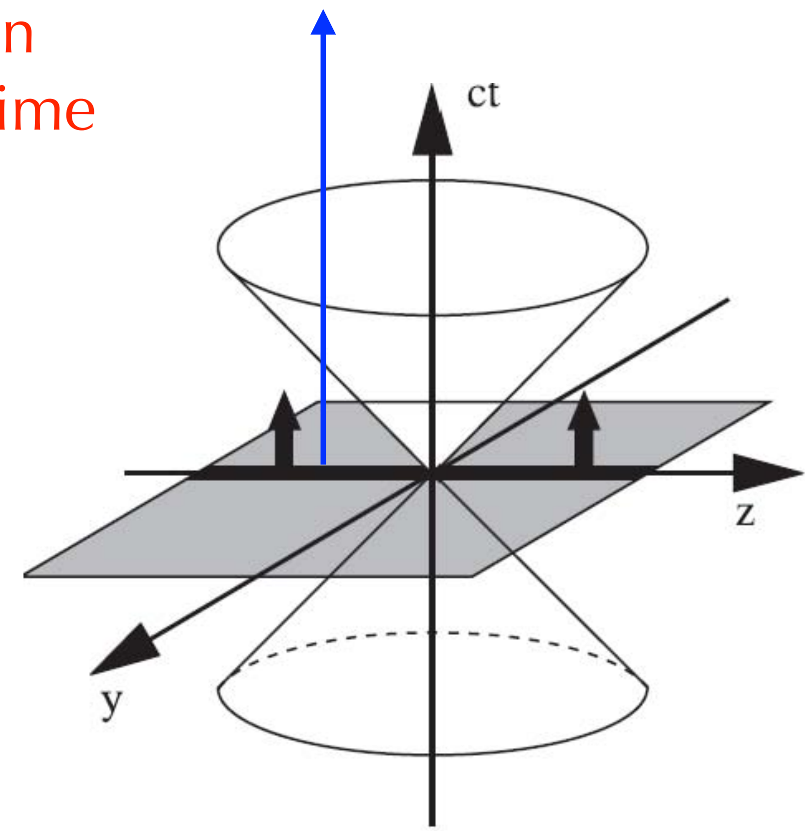
$$H = \sqrt{P^2 + M_0^2} + V \qquad\qquad P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

generators of Poincare′ group interaction independent

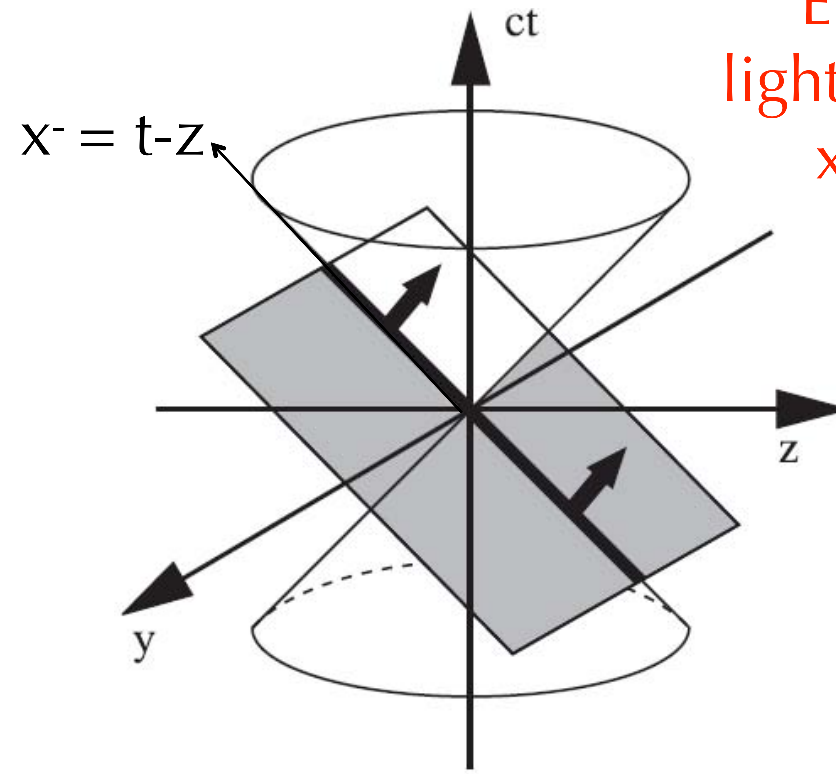
6   7

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in
ordinary time



Instant Form



Evolve in
light-front time
$x^+ = t+z$

Light-Front Form

coordinates

Instant Form		Light-Front Form	
x^0	time	$\frac{x^0+x^3}{\sqrt{2}}$	time
$x^1,\, x^2,\, x^3$	space	$\frac{x^0-x^3}{\sqrt{2}},\; x_\perp = (x^1,\, x^2)$	space

Hamiltonian

$$H = \sqrt{P^2 + M_0^2} + V$$

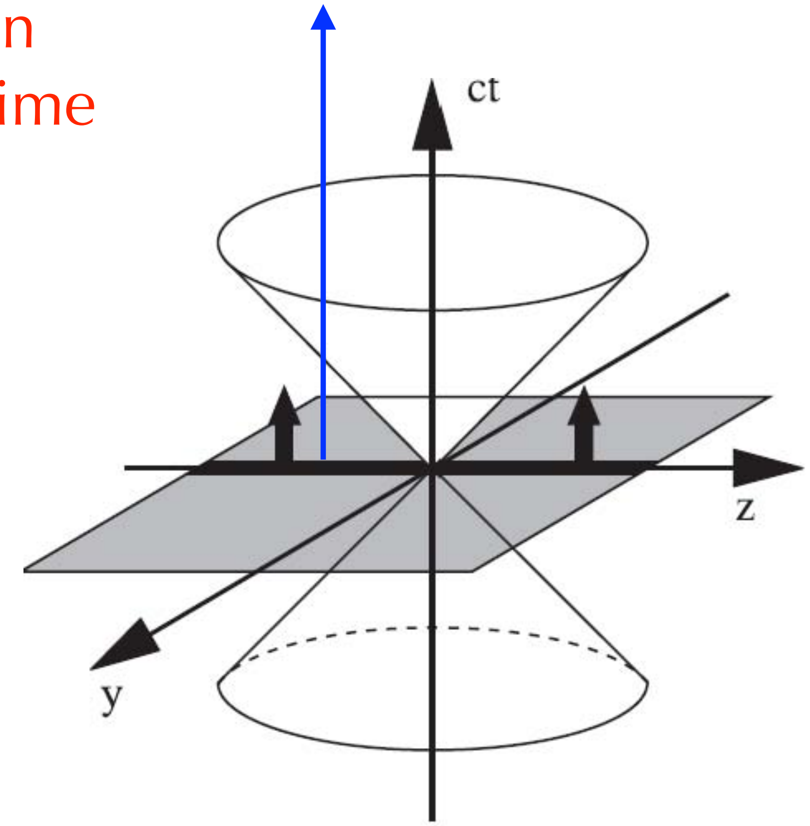
$$P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

generators of Poincare′ group interaction independent

6 7

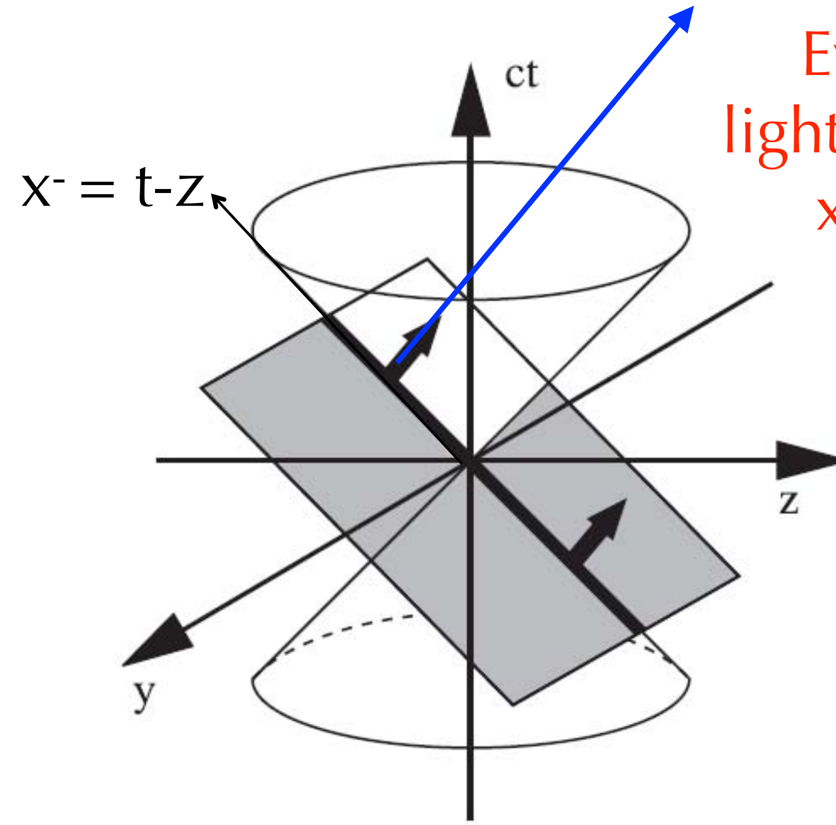
P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time



Instant Form

Evolve in light-front time $x^+ = t+z$



Light-Front Form

Instant Form		Light-Front Form	
coordinates			
x^0	time	$\frac{x^0 + x^3}{\sqrt{2}}$	time
$x^1,\, x^2,\, x^3$	space	$\frac{x^0 - x^3}{\sqrt{2}},\; x_\perp = (x^1,\, x^2)$	space
Hamiltonian			
$H = \sqrt{P^2 + M_0^2} + V$		$P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$	

generators of Poincare′ group interaction independent

6	7

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Good and bad components

- Decompose the four-component fermion field in bad (-) and good (+) components

$$\psi = \psi^+ + \psi^- \qquad \text{with } \psi^+ = P_+\psi \text{ and } \psi^- = P_-\psi$$

- Properties of projector operators: $P_+ = \frac{1}{2}\gamma^-\gamma^+ \quad P_- = \frac{1}{2}\gamma^+\gamma^-$

$$P_+ + P_- = I \qquad (P_+)^2 = P_+ \quad (P_-)^2 = P_- \qquad P_+P_- = P_-P_+ = 0$$

- Projecting the Dirac equation and using the light-cone gauge $A^+ = 0$

$$i\gamma^- \frac{\partial}{\partial x^-}\psi_- = -\vec{\gamma}_\perp \cdot \vec{D}_\perp \psi_+ + m\psi_+ \qquad i\gamma^+ D_+\psi_+ = -\vec{\gamma}_\perp \cdot \vec{D}_\perp \psi_- + m\psi_-$$

constrained field — independent dynamical degree of freedom

R. Jaffe, hep-ph/9602236

Light-Cone Dirac Spinor

$$u(k,\lambda=+1/2) = \frac{1}{\sqrt{2^{3/2}k^+}} \begin{pmatrix} \sqrt{2}k^+ + m \\ k_R \\ \sqrt{2}k^+ - m \\ k_R \end{pmatrix} \qquad P_+ u(k,1/2) = u_+(k,1/2) = \frac{k^+}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

with $k_R = k^x + ik^y$

$$u(k,\lambda=-1/2) = \frac{1}{\sqrt{2^{3/2}k^+}} \begin{pmatrix} -k_L \\ \sqrt{2}k^+ + m \\ k_L \\ -\sqrt{2}k^+ + m \end{pmatrix} \qquad P_+ u(k,-1/2) = u_+(k,-1/2) = \frac{k^+}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

with $k_L = k^x - ik^y$

Partonic interpretation of GPDs

- Unpolarized GPDs

$$\Phi^{[\gamma^+]}(\bar{x},\xi,t) = \langle P',\Lambda'| \int \frac{\mathrm{d}z^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\psi(\tfrac{z}{2})|P,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$\Longrightarrow$ $\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\psi(\tfrac{z}{2}) = \psi_+^\dagger(-\tfrac{z}{2})\psi_+(\tfrac{z}{2})$ good components of the quark fields

$$\Longrightarrow \quad \psi_+(z^-,\mathbf{z}_\perp) = \int \frac{\mathrm{d}k^+\mathrm{d}\mathbf{k}_\perp}{2k^+(2\pi)^3}\theta(k^+) \sum_\mu \left[b_q(w)u_+(w)\exp[-ik^+z^- + i\mathbf{k}_\perp\cdot\mathbf{z}_\perp]\right.$$

$$\left. + d_q^\dagger(w)v_+(w)\exp[ik^+z^- - i\mathbf{k}_\perp\cdot\mathbf{z}_\perp]\right]$$ with $w = (k^+,\mathbf{k}_\perp,\mu)$

$b_q,\, b_q^\dagger$ annihilation and creation operator of quark

$d_q,\, d_q^\dagger$ annihilation and creation operator of antiquark

$\Longrightarrow$ apply momentum conservation

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+ > 0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

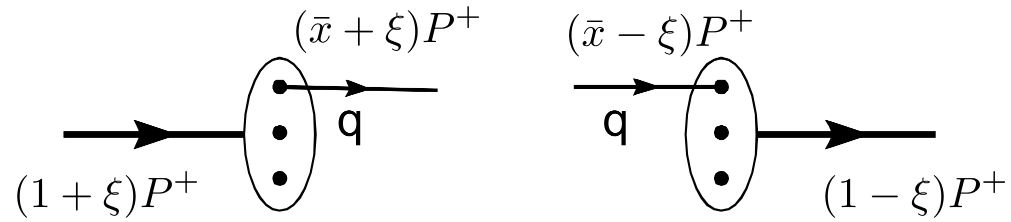
non-diagonal matrix elements of momentum-density matrix

⇩

we loose the probabilistic interpretation of the PDF

we gain information on the quark-momentum correlation

DGLAP region $\xi \leq \bar{x} \leq 1$



$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+ \rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+ > 0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

b, $b^\dagger$ quarks

d, $d^\dagger$ antiquarks

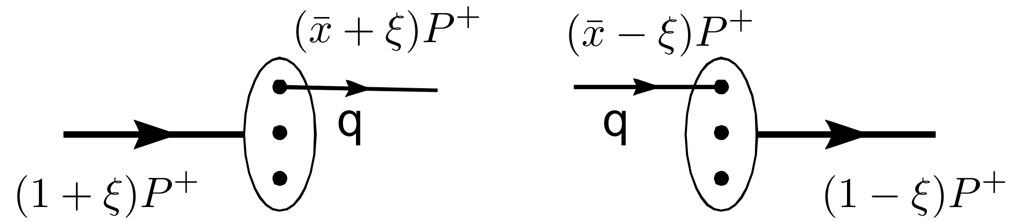
non-diagonal matrix elements of momentum-density matrix

⇓

we loose the probabilistic interpretation of the PDF

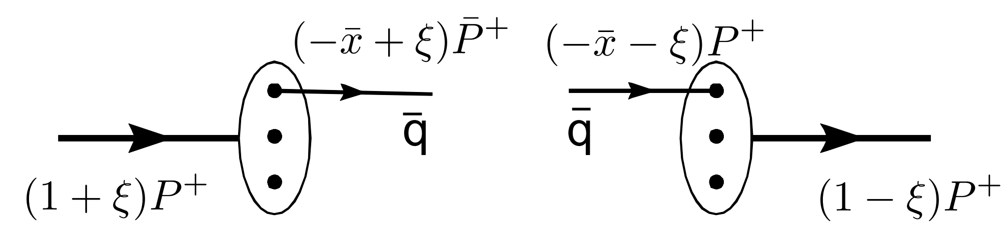
we gain information on the quark-momentum correlation

DGLAP region $\xi \leq \bar{x} \leq 1$



$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

DGLAP region $-1 \leq \bar{x} \leq -\xi$



$$\langle N, (1-\xi)\bar{P}^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition k′+,k+>0 and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

non-diagonal matrix elements of momentum-density matrix

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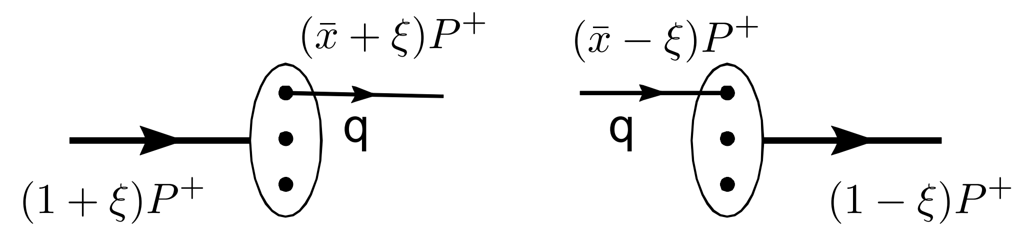
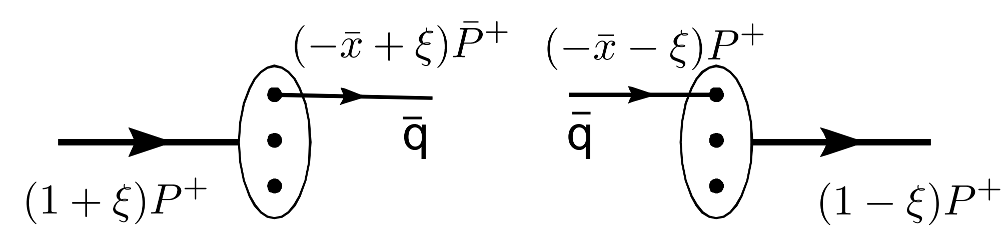


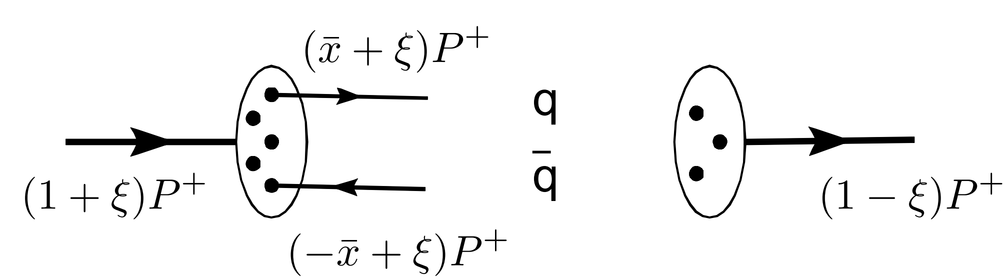
$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

DGLAP region $-1 \leq \bar{x} \leq -\xi$



$$\langle N, (1-\xi)\bar{P}^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

ERBL region $-\xi \leq \bar{x} \leq \xi$



$$\langle N, (1-\xi)\bar{P}^+ | b_{\lambda'}[(\bar{x}+\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+>0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b, b^\dagger$ quarks

$d, d^\dagger$ antiquarks

non-diagonal matrix elements of momentum-density matrix

⇓

we loose the probabilistic interpretation of the PDF

we gain information on the quark-momentum correlation

Spin projection

helicity space

$b^\dagger_\uparrow b_\uparrow + b^\dagger_\downarrow b_\downarrow$

$H^q,\ E^q$

$b^\dagger_\uparrow b_\uparrow - b^\dagger_\downarrow b_\downarrow$

$\tilde{H}^q,\ \tilde{E}^q$

transverse-spin space

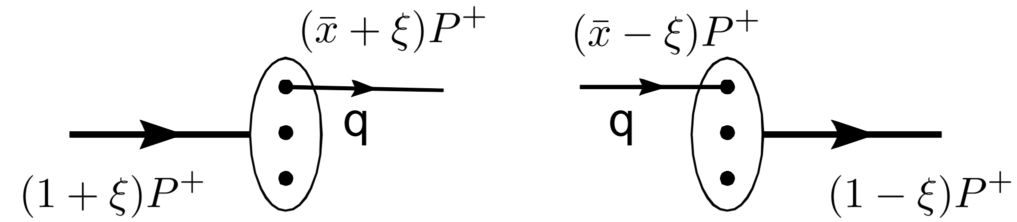
$b^\dagger_\rightarrow b_\rightarrow + b^\dagger_\leftarrow b_\leftarrow$

$H^q_T,\ E^q_T$

$b^\dagger_\rightarrow b_\leftarrow - b^\dagger_\leftarrow b_\rightarrow$

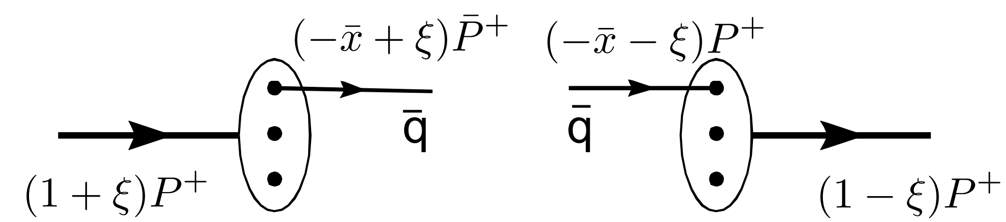
$\tilde{H}^q_T,\ \tilde{E}^q_T$

DGLAP region $\xi \le \bar{x} \le 1$



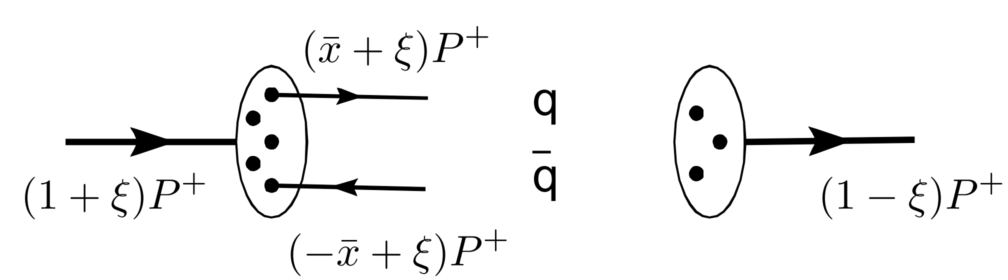
$$\langle N, (1-\xi)\bar{P}^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)\bar{P}^+] b_\lambda[(\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

DGLAP region $-1 \le \bar{x} \le -\xi$



$$\langle N, (1-\xi)\bar{P}^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

ERBL region $-\xi \le \bar{x} \le \xi$

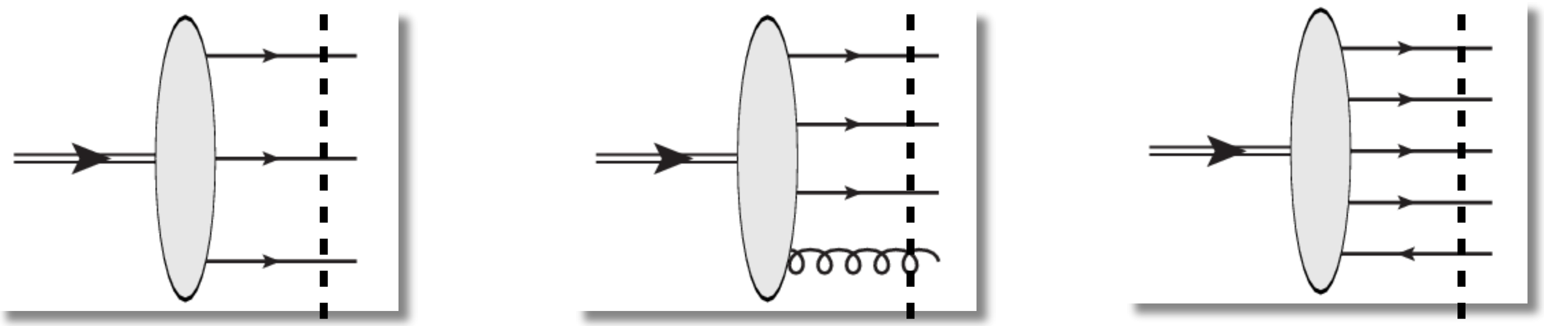


$$\langle N, (1-\xi)\bar{P}^+ | b_{\lambda'}[(\bar{x}+\xi)\bar{P}^+] d_\lambda[(-\bar{x}+\xi)\bar{P}^+] | N, (1+\xi)\bar{p}^+\rangle$$

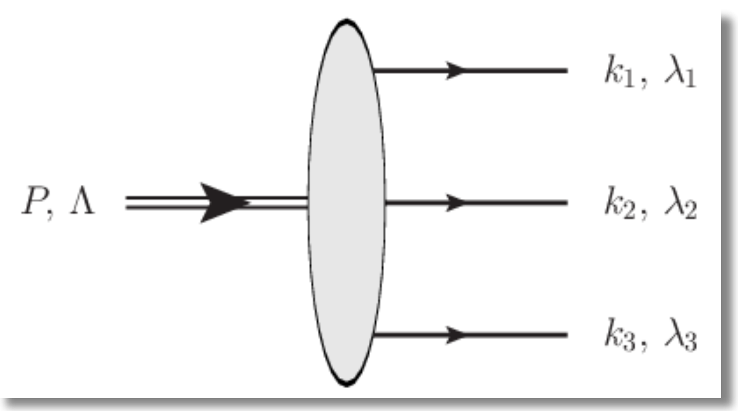
Light-cone Fock expansion

$$|P\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \ldots$$

Fixed light-cone time $x^+=0$



Fock states



Simultaneous eigenstates of

$$P^+ = \sum_i^N p_i^+$$

$$P_\perp = \sum_i^N p_{\perp i}$$

Momentum

$$\lambda_i$$

Light-front helicity

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Proton state

Probability Amplitude for the N,β Fock state

$$|(P^+, \vec{P}_\perp), \Lambda\rangle = \sum_{N,\beta} [dx]_N [d\vec{k}_\perp]_N \Psi^{\Lambda}_{N,\beta}(x_i, \vec{k}_{\perp\, i}) |N, \beta; (x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp\, i}), \lambda_i\rangle$$

Light-front wave functions

Internal variables: $x_i = \frac{p_i^+}{P^+}$ $\sum_{i=1}^{N} x_i = 1$ $\sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz} \Psi^{\Lambda}_{\lambda_1 \ldots \lambda_N} = \lambda_i \Psi^{\Lambda}_{\lambda_1 \ldots \lambda_N}$$

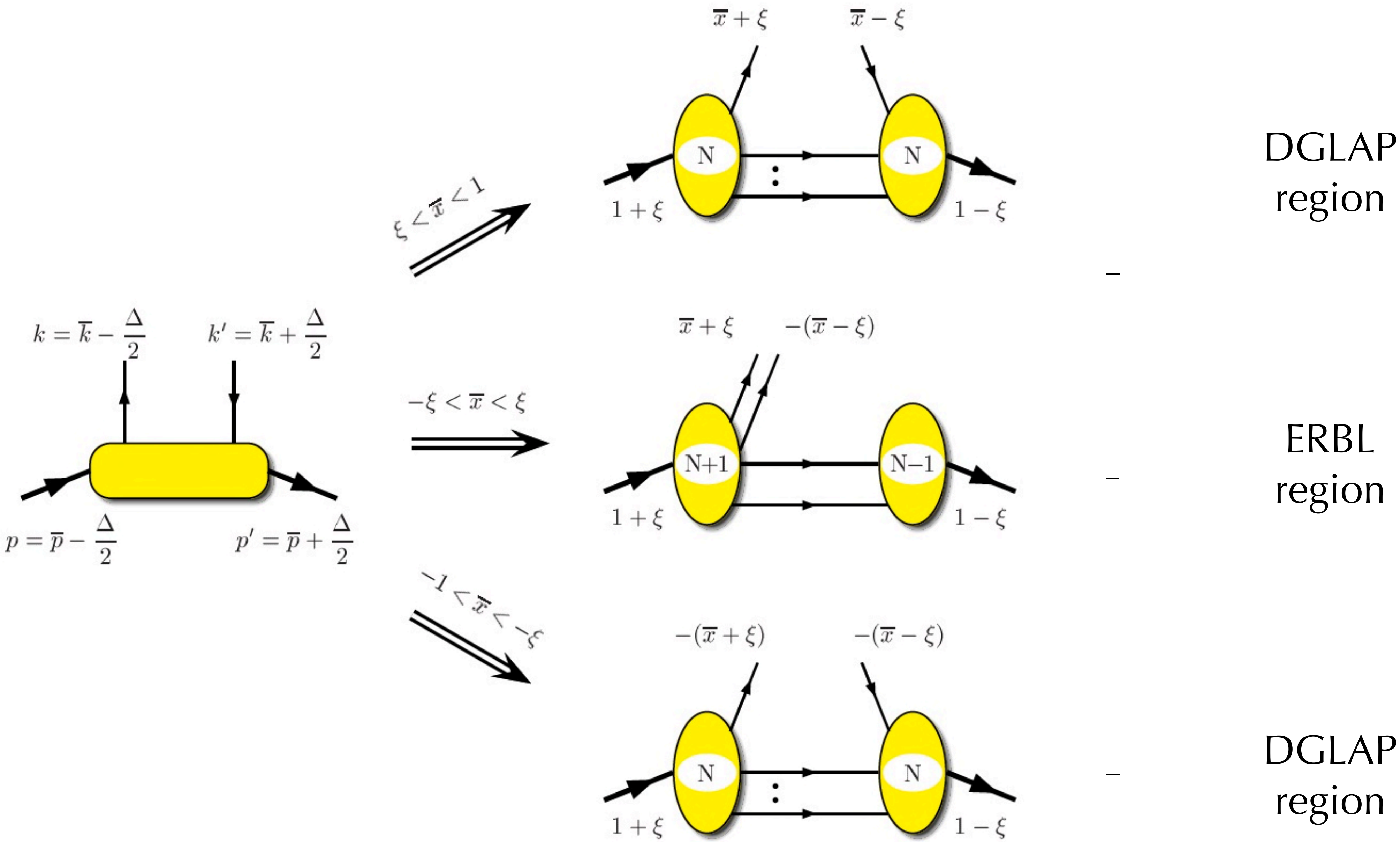
Eigenstates of total OAM

$$\hat{L}_z \Psi^{\Lambda}_{\lambda_1 \ldots \lambda_N} = \ell_z \Psi^{\Lambda}_{\lambda_1 \ldots \lambda_N}$$

$$\Lambda = \sum_{i=1}^{N} \lambda_i + \ell_z$$

$A^+ = 0$ **gauge**

Light-Front Wave Function Overlap Representation



GPDs $\sim \sum_N \int [d^3k]_N \, \Psi^*_N(k'_N)\Psi_N(k_N)\delta(\dots)$ interference of probability amplitudes

PDFs $\sim \sum_N \int [d^3k]_N \, |\Psi_N(k_N)|^2\delta(\dots)$ probability density

Diehl, Feldmann, Jakob, Kroll, NPB596, 2001
Diehl, Hwang, Brodsky, NPB596, 2001
Boffi, Pasquini, NPB649, 2003

Properties of GPDs

- Forward limit: ordinary parton distributions

$H^q(x, \xi = 0, t = 0) = q(x)$ unpolarized quark distributions

$\widetilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$ long. polarized quark distributions

$H_T^q(x, \xi = 0, t = 0) = h_1(x)$ transv. polarized quark distributions

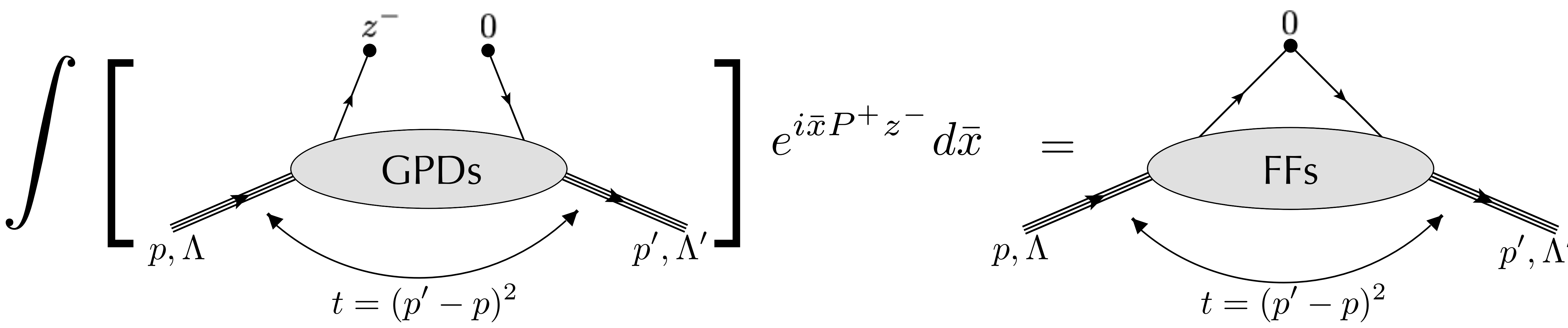
$x > 0$: quarks $x < 0$: antiquarks

analogous relations for gluons, except for transversity distribution

- all the other GPDs do NOT appear in inclusive DIS $\Longrightarrow$ new information

- They all depend on the renormalisation scale ($\mu^2 = Q^2$) with different evolution equations in the DGLAP and ERBL regions

Properties of GPDs



$$\int_{-1}^{1} d\bar{x}\, H^q(\bar{x}, \xi, t) = F_1^q(t)$$ Dirac Form Factor

$$\int_{-1}^{1} d\bar{x}\, E^q(\bar{x}, \xi, t) = F_2^q(t)$$ Pauli Form Factor

$$\int_{-1}^{1} d\bar{x}\, \widetilde{H}^q(\bar{x}, \xi, t) = G_A^q(t)$$ Axial Form Factor

$$\int_{-1}^{1} d\bar{x}\, \widetilde{E}^q(\bar{x}, \xi, t) = G_P^q(t)$$ Pseudoscalar Form Factor

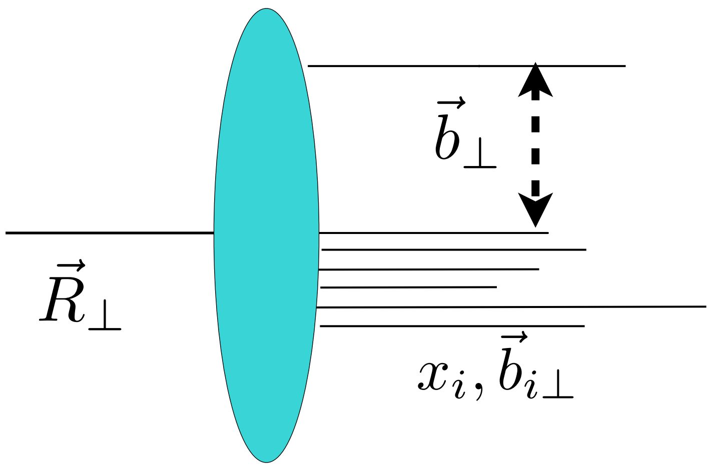
- matrix elements of local operators

  ⟶ can be calculated on the lattice

- renormalisation scale independent

- ξ independence: Lorentz invariance

Impact Parameter Space



- average transverse position of the partons

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

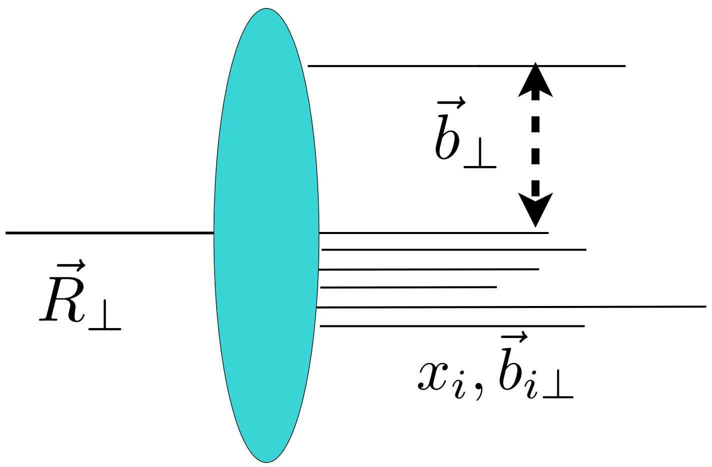
- $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

Isomorphism between Galilei and subgroup of Light-Front operators

Galilei transformation:	Transverse boost:
$m_i \to m_i \qquad \vec{p}_i \to \vec{p}_i - m_i \vec{v}$	$p_i^+ \to p_i^+ \qquad \vec{p}_{\perp i} \to \vec{p}_{\perp i} - p_i^+ \vec{v}$
Center of mass:	Center of plus momentum:
$\vec{r}_* = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$	$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+}$

Impact Parameter Space



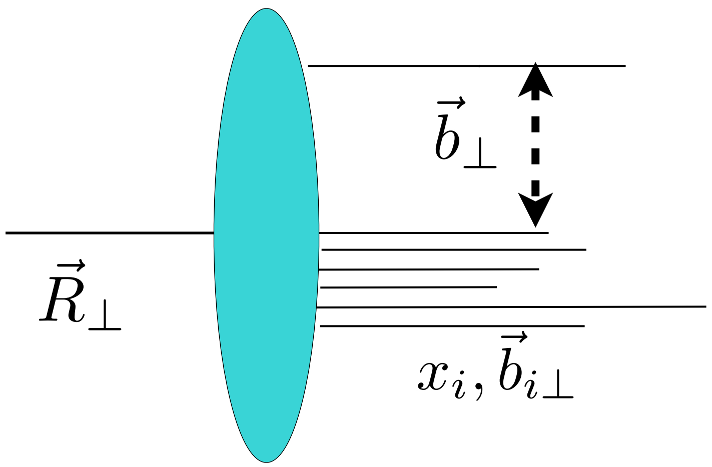
➢ average transverse position of the partons

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

➢ $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

Impact Parameter Space



- average transverse position of the partons

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

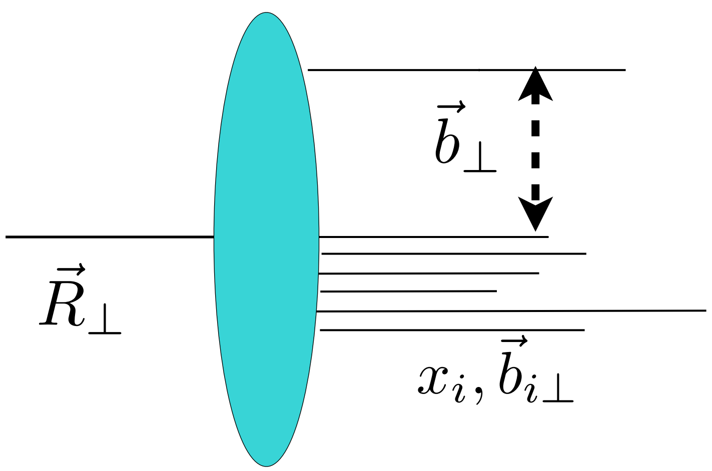
- $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

- Localized wave packet in the transverse plane polarized in the X direction in IMF

$$|p^+, S_x\rangle \equiv \frac{1}{\sqrt{2}}\left(|p^+, \vec{R}_\perp = \vec{0}_\perp, \uparrow\rangle + |p^+, \vec{R}_\perp = \vec{0}_\perp, \downarrow\rangle\right)$$

Impact Parameter Space



➤ average transverse position of the partons

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

➤ $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

- Localized wave packet in the transverse plane polarized in the X direction in IMF

$$|p^+, S_x\rangle \equiv \frac{1}{\sqrt{2}}\Big(|p^+, \vec{R}_\perp = \vec{0}_\perp, \uparrow\rangle + |p^+, \vec{R}_\perp = \vec{0}_\perp, \downarrow\rangle\Big)$$

- Impact parameter dependent GPD for the $\perp$ pol. state

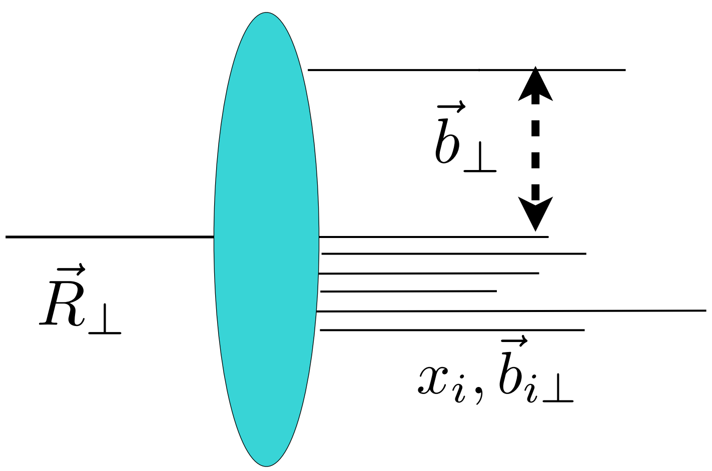
$\Longrightarrow$ quark density in proton state $\perp$ pol.

$$q_x(x, \vec{b}_\perp) = \langle p^+, S_x| \int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}(-\frac{x^-}{2}, \vec{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \vec{b}_\perp)|p^+, S_x\rangle$$

$$q_x(x, \vec{b}_\perp) = H^q(x, \vec{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y} E^q(x, \vec{b}_\perp)$$

$$H^q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x, \vec{\Delta}_\perp) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \qquad E^q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} E^q(x, \vec{\Delta}_\perp) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}$$

Impact Parameter Space



- average transverse position of the partons

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

- $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

- Localized wave packet in the transverse plane polarized in the X direction in IMF

$$|p^+, S_x\rangle \equiv \frac{1}{\sqrt{2}}\left(|p^+, \vec{R}_\perp = \vec{0}_\perp, \uparrow\rangle + |p^+, \vec{R}_\perp = \vec{0}_\perp, \downarrow\rangle\right)$$

- Impact parameter dependent GPD for the ⊥ pol. state

⟹ quark density in proton state ⊥ pol.

$$q_x(x, \vec{b}_\perp) = \langle p^+, S_x| \int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}(-\frac{x^-}{2}, \vec{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \vec{b}_\perp)|p^+, S_x\rangle$$

$$q_x(x, \vec{b}_\perp) = H^q(x, \vec{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y} E^q(x, \vec{b}_\perp)$$

$$H^q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x, \vec{\Delta}_\perp) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \qquad E^q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} E^q(x, \vec{\Delta}_\perp) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}$$

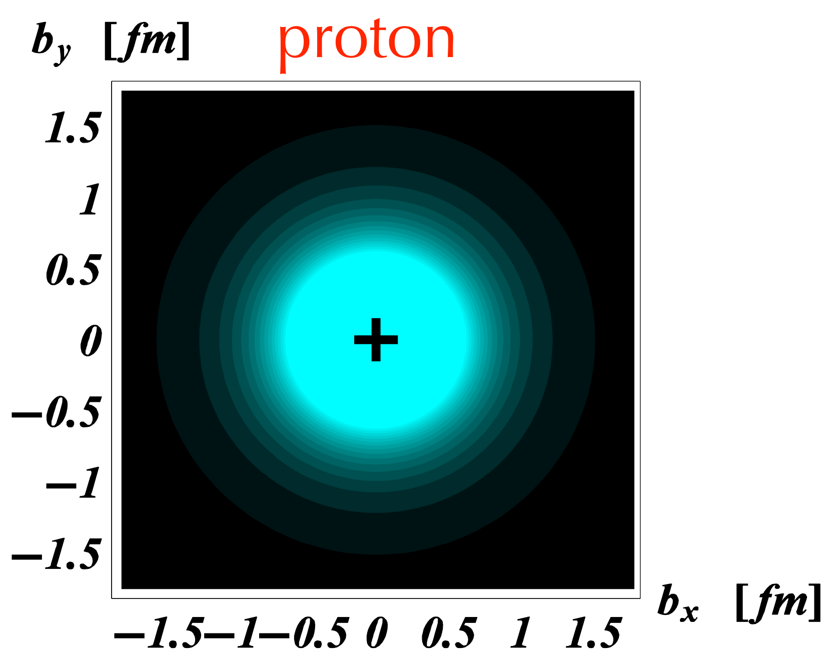
❖ Homework: derive the relation between GPDs and IPDs

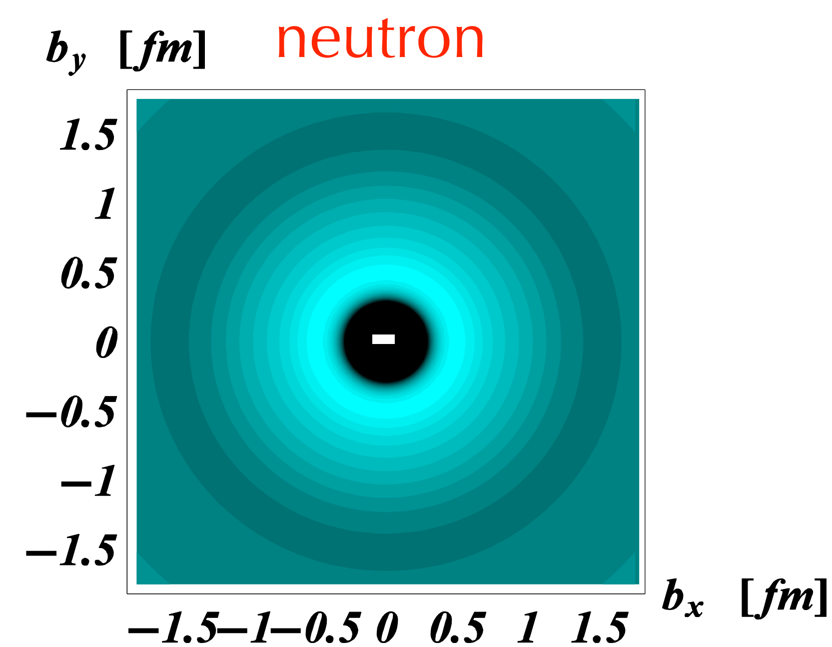
Charge density of partons in the transverse plane

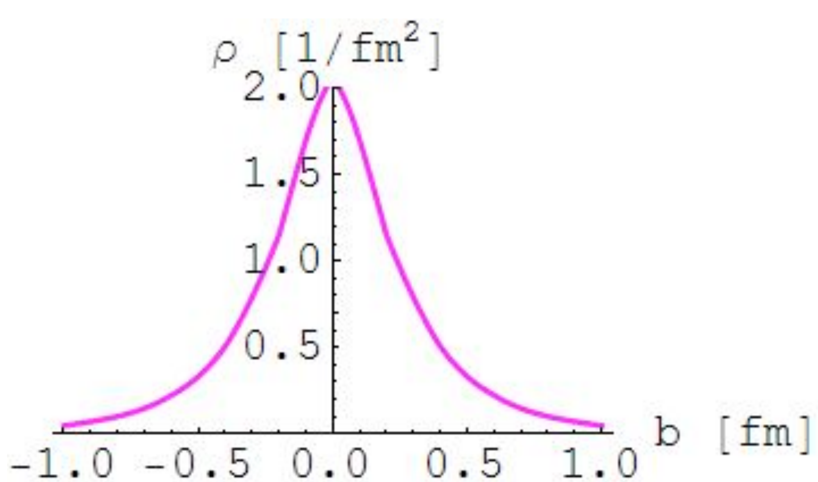
Number density of quark with longitudinal momentum x and transverse position $b_\perp$

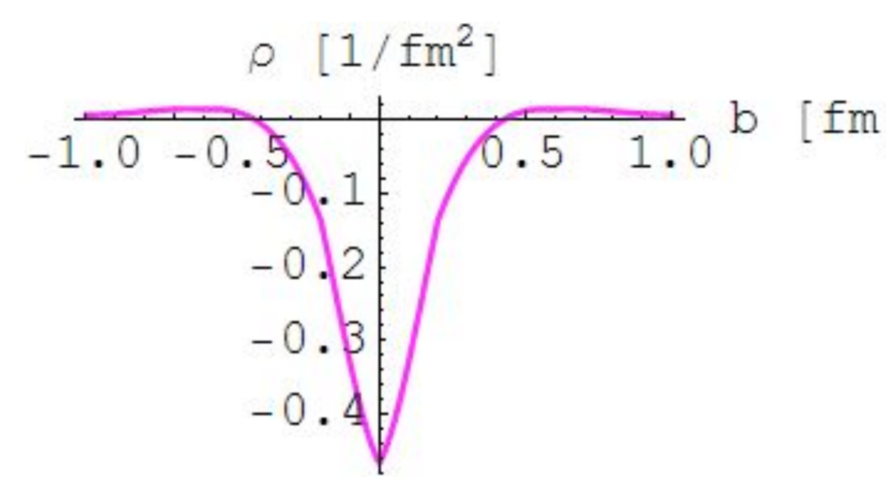
$$\rho^q(b_\perp) = e_q \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} \int dx H^q(x,0,\Delta_\perp^2) = \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} F_1^q(\Delta_\perp^2)$$

Infinite-Momentum-Frame Parton charge density in the transverse plane









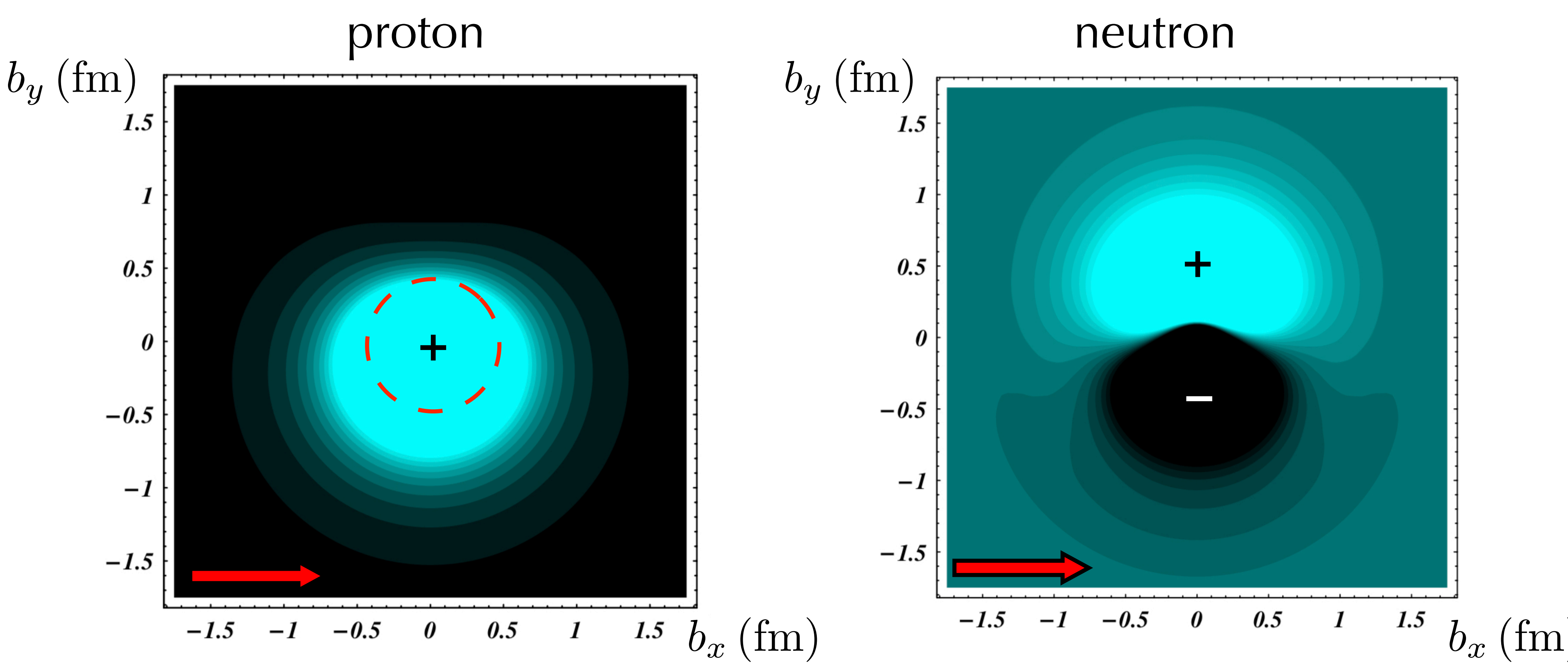
Miller (2007); Burkardt (2007)

Electromagnetic Form Factors

Transversely polarized proton

$$\rho_T(\vec{b}_\perp) = \rho(\vec{b}_\perp) + \sin(\phi_b - \phi_s) \int \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(Qb_\perp) F_2(Q^2)$$

monopole — dipole



nucleon polarized in the x direction

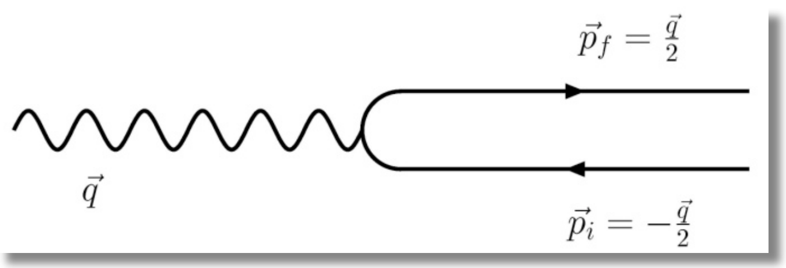
C. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. 100 (2008) 032004

Textbook interpretation

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Breit frame



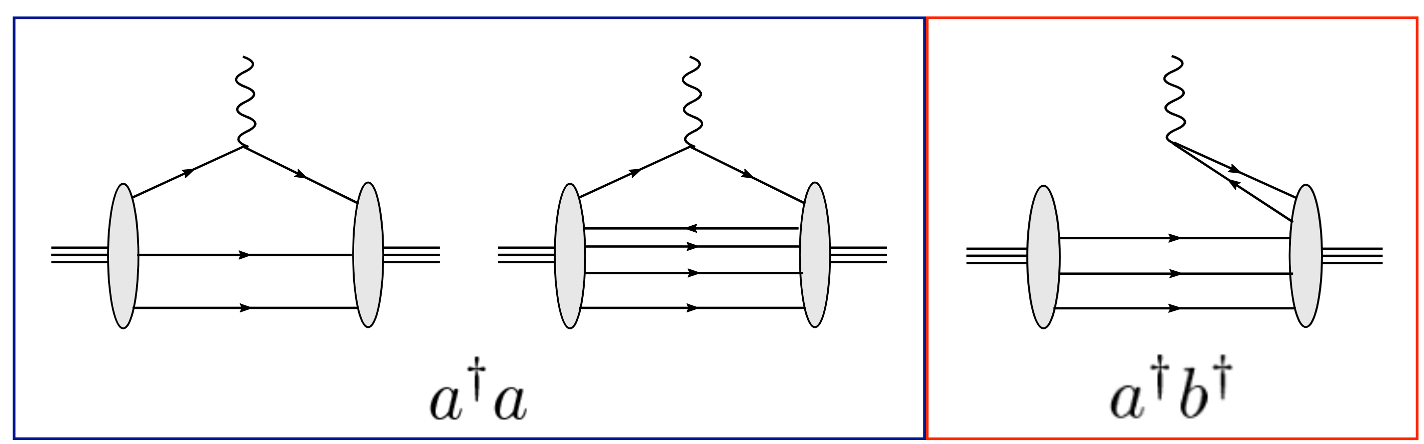
Spatial charge density

$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}} G_E(Q^2)$$

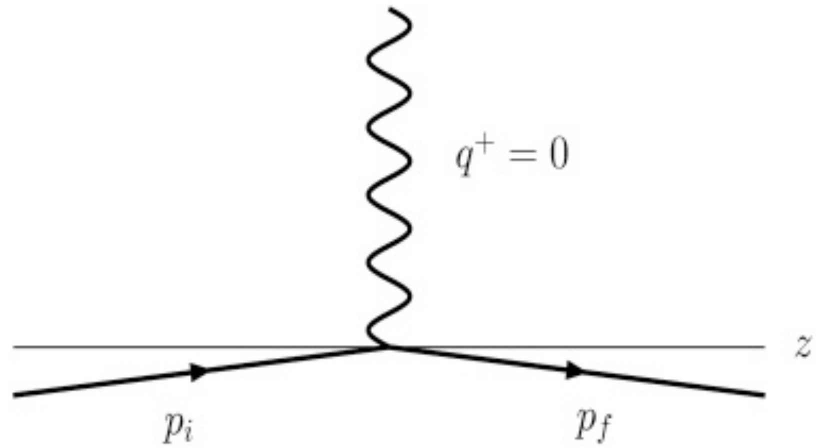
[Ernst, Sachs, Wali (1960)]
[Sachs (1962)]

No probabilistic/charge interpretation

Creation/annihilation of pairs



Drell-Yan-West frame



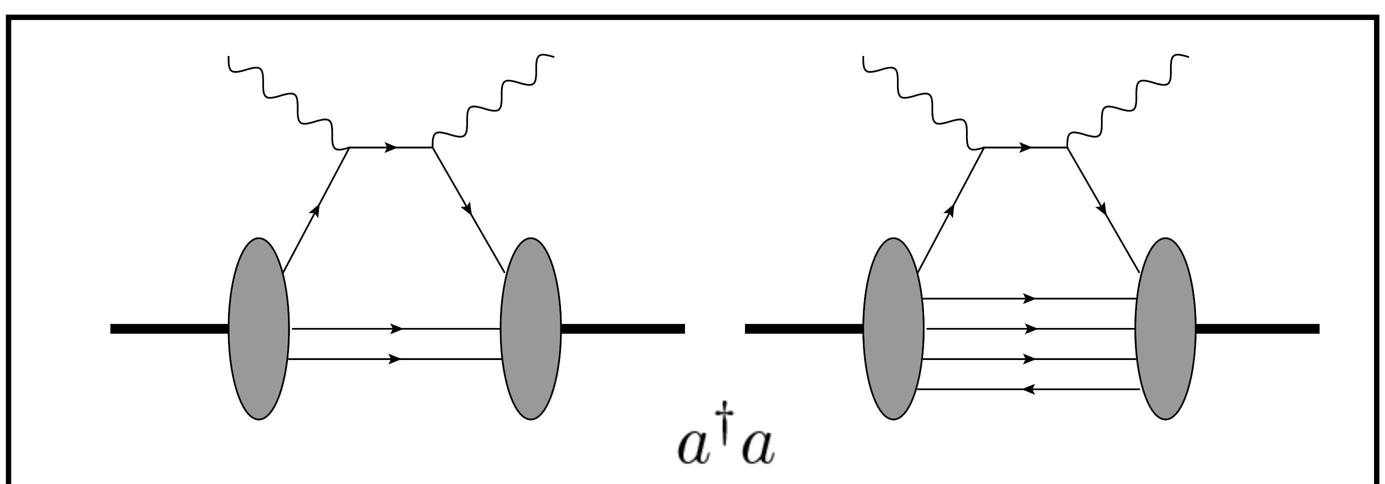
✓ $q^+ = 0$ ⟶ no sensitivity to longitudinal Lorentz contraction

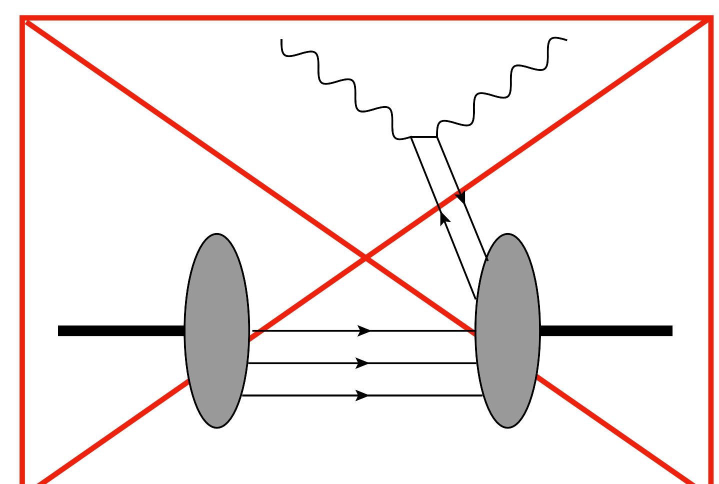
✓ $\vec{q}_\perp \neq 0$: Transverse boosts ⟶ no transverse Lorentz contraction

✓ Particle number is conserved in Drell-Yan frame $\Delta^+ = 0$

$$\rho(\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} \frac{e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}}{2P^+} J^+(\vec{\Delta}_\perp)$$

probabilistic/charge interpretation





Operator	Forward matrix element	Non-forward matrix element	Position-space interpretation
$e_q \bar{\psi}_q(0)\gamma^+ \psi_q(0)$	Q	$F_1(t)$	$\rho(b_\perp^2)$
$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \bar{\psi}_q(-\frac{z}{2})\gamma^+ \psi_q(\frac{z}{2})$	$q(x)$	$H_q(x,0,t)$	$q(x, b_\perp^2)$

$\rho(b_\perp^2)$ 2-dim distribution of charge in the transverse plane

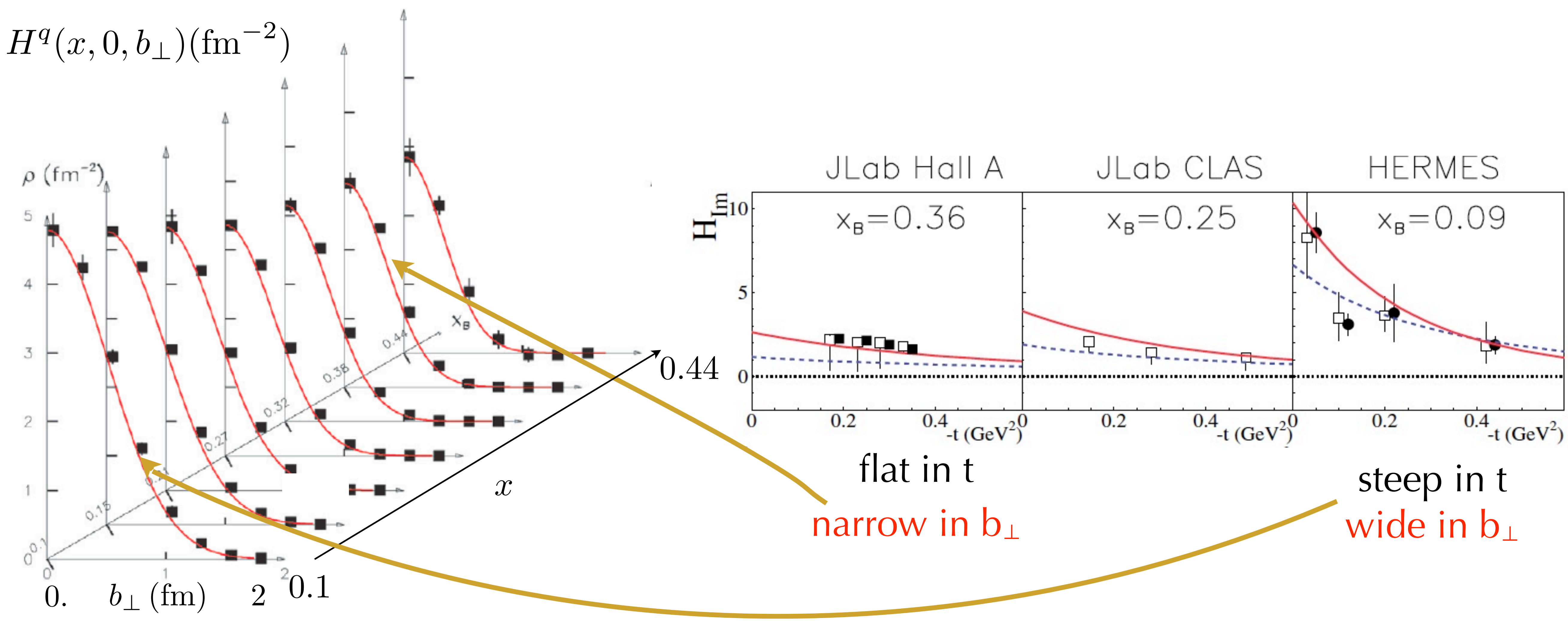
$q(x, b_\perp^2)$ 2-dim. “distribution of the PDF” in the transverse plane

M. Burkardt, Int. J. Mod. Phys. A18 (2003) 173

The unpolarized GPD H

$$H(x,0,\vec{b}_\perp) = \int \mathrm{d}^2\Delta_\perp \, H(x,0,t)\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \qquad (t = -\vec{\Delta}_\perp^2)$$

extrapolation from data

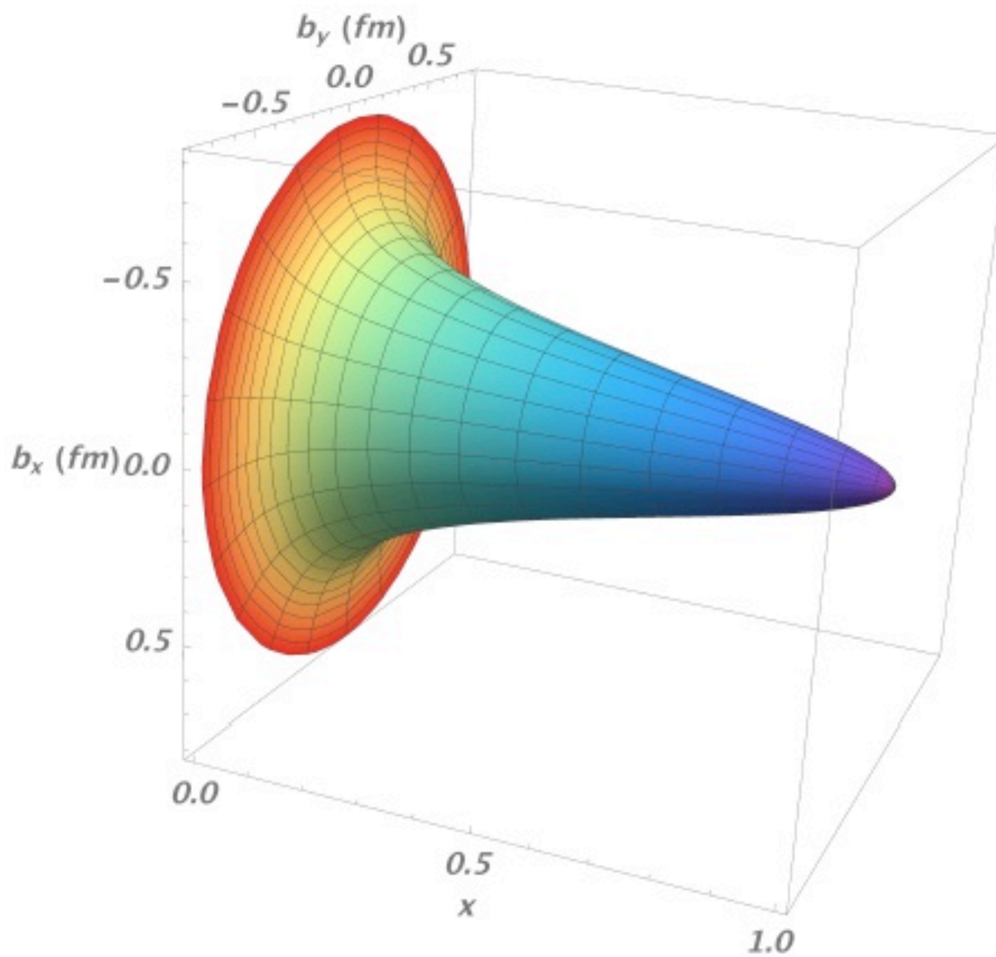


The unpolarized GPD H

$$H(x,0,\vec{b}_\perp) = \int \mathrm{d}^2\Delta_\perp \, H(x,0,t)\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \qquad (t = -\vec{\Delta}_\perp^2)$$

extrapolation from data

$$\langle \vec{b}_\perp^2(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp\, \vec{b}_\perp^2\, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp\, H(x,0,b_\perp)}$$



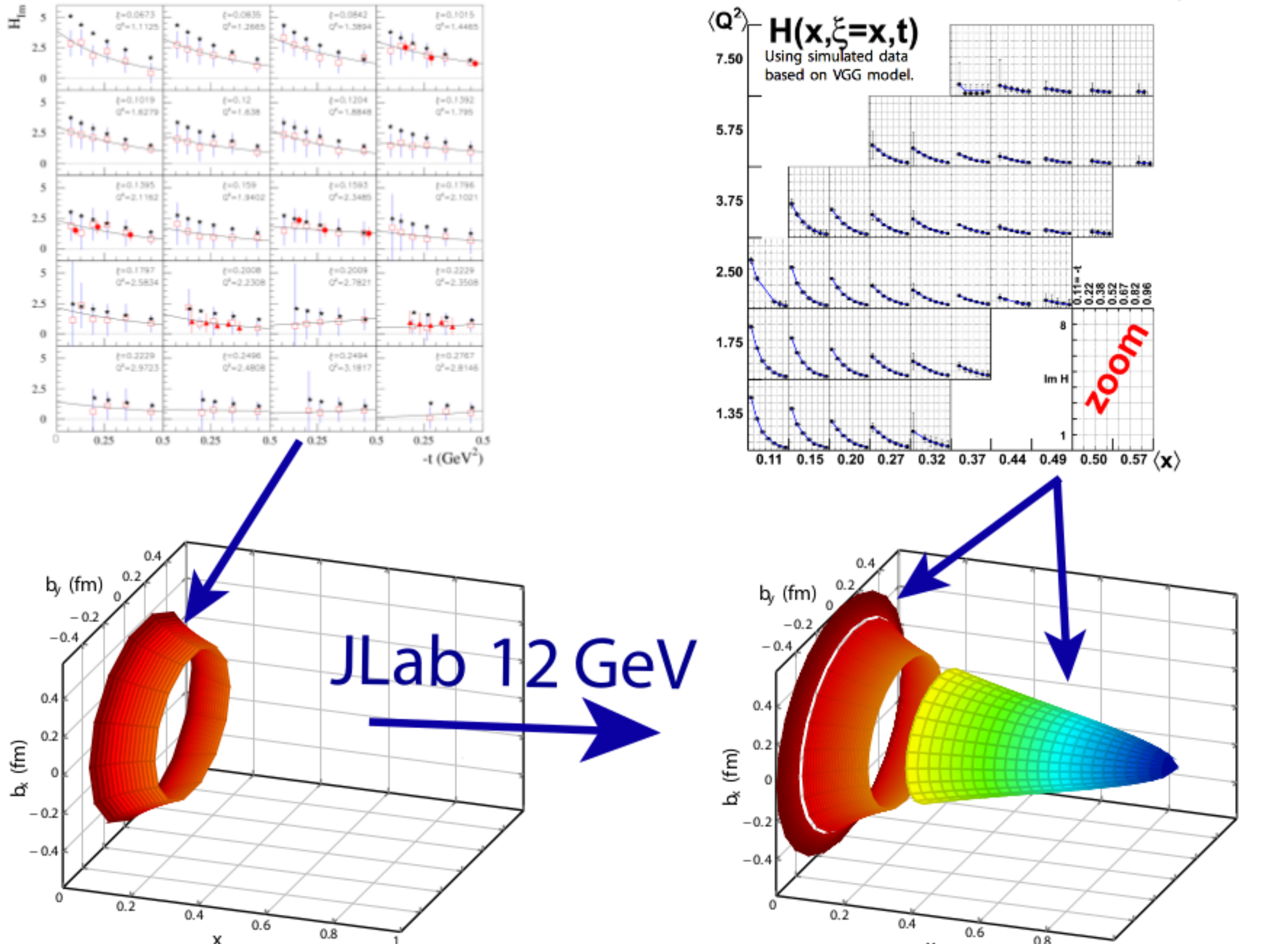
As x ⟶ 1, the active parton carries all the momentum and represents the centre of momentum

Dupré et al., PRD95(2017)011501

The unpolarized GPD H

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)

CLAS12 projections E12-06-119 with DVCS A_{UL} and A_{LU}



Courtesy of R. Dupré, M. Vanderhaeghen and M. Guidal

Energy-momentum tensor and GPDS

The Energy-Momentum Tensor

Energy Density   Momentum Density

$$T^{\mu\nu} = \begin{matrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{matrix}$$

shear forces

pressure

Energy Flux   Momentum Flux

$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle$$

- Where does the spin of the proton come from?
- What are the mechanical properties (pressure, shear forces) inside the proton ?
- What is the origin of the proton mass?

Canonical Energy Momentum Tensor

Emmy Noether (1882-1935)

If a system has a continuous symmetry property,
then there are corresponding quantities whose values are conserved in time

Translation invariance ⟶ Conservation of the canonical EMT $T_C^{\mu\nu}(x)$

Lorentz invariance ⟶ Conservation of the generalized Angular Momentum (AM) density $J_C^{\mu\alpha\beta}(x)$

$$J_C^{\mu\alpha\beta}(x) = L_C^{\mu\alpha\beta} + S_C^{\mu\alpha\beta} \qquad L_C^{\mu\alpha\beta}(x) = x^\alpha T_C^{\mu\beta}(x) - x^\beta T_C^{\mu\alpha}(x)$$

Space components: $J_C^i(x) = \frac{1}{2}\epsilon^{ijk} J_C^{0jk}(x)$

$$\vec{J}_C = \vec{L}_C + \vec{S}_C$$

$\vec{L}_C$ ↓ Orbital AM

$\vec{S}_C$ ↓ Spin

$T_C^{\mu\nu}$ is in general neither gauge-invariant nor symmetric

Belinfante improved EMT

$$T^{\mu\nu}_{\text{Bel}}(x) = T^{\mu\nu}_C(x) + \partial_\lambda G^{\lambda\mu\nu}(x)$$

Belinfante generalized AM

$$J^{\mu\alpha\beta}_{\text{Bel}}(x) = J^{\mu\alpha\beta}_C(x) + \partial_\lambda[x^\alpha G^{\lambda\mu\beta}(x) - x^\beta G^{\lambda\mu\alpha}(x)]$$

with the super-potential

$$G^{\lambda\mu\nu}(x) = \frac{1}{2}[S^{\lambda\mu\nu}_C(x) - S^{\mu\nu\lambda}_C(x) - S^{\nu\mu\lambda}_C(x)] = -G^{\mu\lambda\nu}(x)$$

⬇

$$J^{\mu\alpha\beta}_{\text{Bel}}(x) = x^\alpha T^{\mu\beta}_{\text{Bel}}(x) - x^\beta T^{\mu\alpha}_{\text{Bel}}(x)$$

Belinfante, Rosenfeld (1940)

Canonical ⟷ Belinfante

Canonical ⟷ Belinfante

in general not symmetric

$$T_C^{[\mu\nu]}(x) = -\partial_\alpha S^{\alpha\mu\nu}(x) \neq 0$$

$$[\mu\nu] = \mu\nu - \nu\mu$$

symmetric

$$T_{\mathrm{Bel}}^{[\mu\nu]}(x) = 0$$

Canonical ⟷ Belinfante

Canonical

in general not symmetric

$$T_C^{[\mu\nu]}(x) = -\partial_\alpha S^{\alpha\mu\nu}(x) \neq 0$$

$$[\mu\nu] = \mu\nu - \nu\mu$$

clear distinction between OAM and spin at the density level

$$J_C^{\mu\alpha\beta}(x) = L_C^{\mu\alpha\beta}(x) + S_C^{\mu\alpha\beta}(x)$$

$$L_C^{\mu\alpha\beta}(x) = x^\alpha T_C^{\mu\beta}(x) - x^\beta T_C^{\mu\alpha}(x)$$

Belinfante

symmetric

$$T_{\text{Bel}}^{[\mu\nu]}(x) = 0$$

purely OAM density

$$J_{\text{Bel}}^{\mu\alpha\beta}(x) = x^\alpha T_{\text{Bel}}^{\mu\beta}(x) - x^\beta T_{\text{Bel}}^{\mu\alpha}(x)$$

Canonical ⟷ Belinfante

in general not symmetric

$$T_C^{[\mu\nu]}(x) = -\partial_\alpha S^{\alpha\mu\nu}(x) \neq 0$$

$$[\mu\nu] = \mu\nu - \nu\mu$$

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$$T_{\text{Bel}}^{[\mu\nu]}(x) = 0$$

clear distinction between OAM and spin
at the density level

$$J_C^{\mu\alpha\beta}(x) = L_C^{\mu\alpha\beta}(x) + S_C^{\mu\alpha\beta}(x)$$

$$L_C^{\mu\alpha\beta}(x) = x^\alpha T_C^{\mu\beta}(x) - x^\beta T_C^{\mu\alpha}(x)$$

purely OAM density

$$J_{\text{Bel}}^{\mu\alpha\beta}(x) = x^\alpha T_{\text{Bel}}^{\mu\beta}(x) - x^\beta T_{\text{Bel}}^{\mu\alpha}(x)$$

The total charge does not change:

$$\int T_C^{0\nu}\, d^3x = \int T_{\text{Bel}}^{0\nu}\, d^3x$$

$$\int J_C^{0\alpha\beta}\, d^3x = \int J_{\text{Bel}}^{0\alpha\beta}\, d^3x$$

Kinetic EMT in QCD

Ji, 1997

$$T^{\mu\nu}_{\text{kin}}(x) = T^{\mu\nu}_{\text{kin},q}(x) + T^{\mu\nu}_{\text{kin},g}$$

Quark contribution: $T^{\mu\nu}_{\text{kin},q}(x) = \frac{1}{2}\bar{\psi}(x)\gamma^\mu i\overleftrightarrow{D}^\nu\psi(x)$ $\qquad (D^\mu = \partial^\mu + igA^\mu)$

$$\frac{1}{2}T^{\{\mu\nu\}}_{\text{kin},q}(x) = T^{\mu\nu}_{\text{Bel},q}(x) \qquad \frac{1}{2}T^{[\mu\nu]}_{\text{kin},q}(x) = -\partial_\lambda S^{\lambda\mu\nu}_q(x)$$

$$S^{\lambda\mu\nu}_q(x) = \frac{1}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{\psi}(x)\gamma_\alpha\gamma_5\psi(x)$$

Gluon contribution: $T^{\mu\nu}_{\text{kin},g} = -2\,\text{Tr}[F^{\mu\lambda}(x)F^\nu_{\ \lambda}(x)] + \frac{1}{2}g^{\mu\nu}\,\text{Tr}[F^{\rho\sigma}(x)F_{\rho\sigma}(x)]$

$$T^{\mu\nu}_{\text{kin},g}(x) = T^{\mu\nu}_{\text{Bel},g}(x)$$

Kinetic generalized AM

$$J^{\mu\alpha\beta}_{\text{kin},q}(x) = L^{\mu\alpha\beta}_{\text{kin},q}(x) + S^{\mu\alpha\beta}_q(x) \qquad J^{\mu\alpha\beta}_{\text{Bel},q}(x) = J^{\mu\alpha\beta}_{\text{kin},q}(x) + \frac{1}{2}\partial_\lambda[x^\alpha S^{\lambda\mu\beta}_q(x) - x^\beta S^{\lambda\mu\alpha}_q(x)]$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x J^{0jk} \qquad \int \vec{J}_{\text{Bel},q}\,d^3x = \int \vec{J}_{\text{kin},q}\,d^3x$$

equal total charge