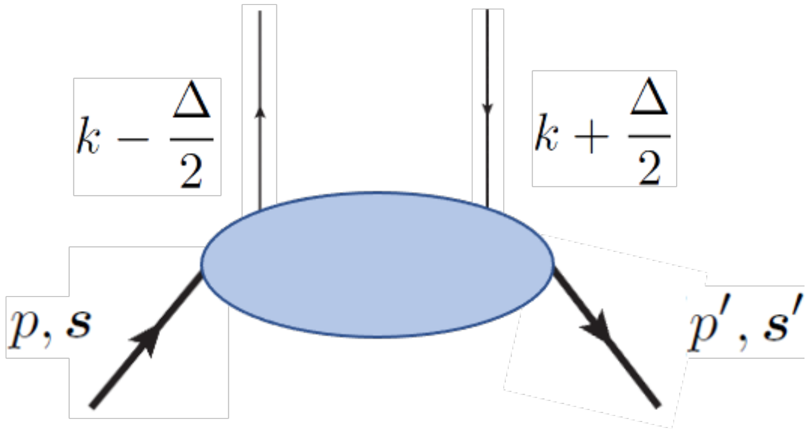
Form Factors of the EMT



$$\Delta = p' - p$$

$$t = \Delta^2$$

$$P = \frac{p + p'}{2}$$

Belinfante-Rosenfeld EMT

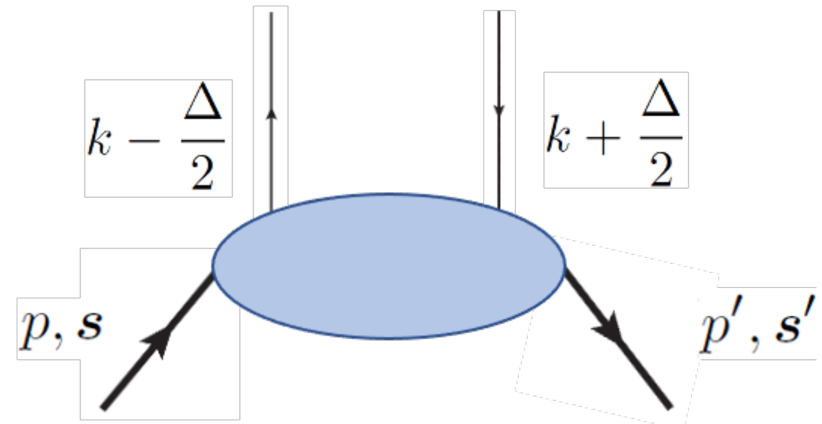
$$\langle p', s' | T^{\mu\nu}_{\mathrm{Bel},a}(0) | p, s \rangle = \bar{u}(p', s')\, \Gamma^{\mu\nu}_{\mathrm{Bel},a}(P, \Delta)\, u(p, s)$$

$$\Gamma^{\mu\nu}_{\mathrm{Bel},a}(P, \Delta) = \frac{P^{\{\mu}\gamma^{\nu\}}}{2} A_a(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\Delta}}{4M} B_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} D_a(t) + M g^{\mu\nu} \bar{C}_a(t)$$

$(a = q, g,\ \{\mu, \nu\} = \mu\nu + \nu\mu)$

Bakker et al. (2004)

Form Factors of the EMT



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Kinetic EMT

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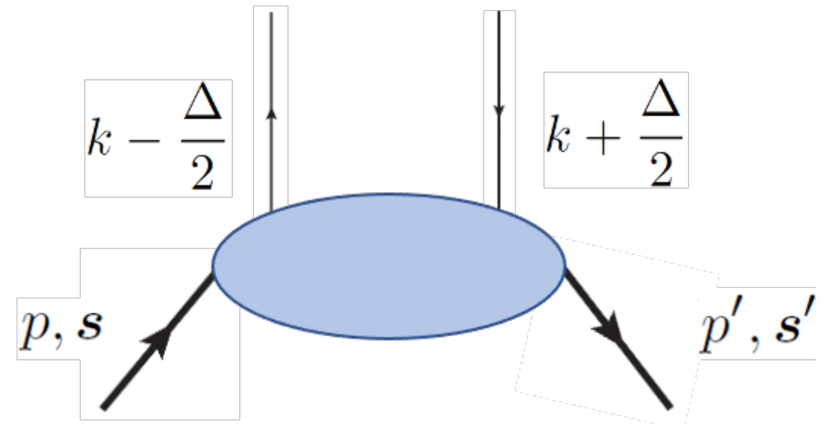
$$\Gamma^{\mu\nu}_{\mathrm{kin},a}(P, \Delta) = \Gamma^{\mu\nu}_{\mathrm{Bel},a}(P, \Delta) + \frac{P^{[\mu}\gamma^{\nu]}}{2} C_a(t)$$

$$C_g(t) = 0$$

Ji (1997)

Form Factors of the EMT



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$$C_g(t) = 0$$

Angular momentum relation

$$J^a_z = \frac{1}{2}(A_a + B_a)$$

$$J^q_z = L^q_{\text{kin},z} + S^q_{\text{kin},z}$$

$$L^q_{\text{kin},z} = \frac{1}{2}(A_q + B_q + C_q)$$

$$S^q_{\text{kin},z} = -\frac{1}{2} C_q$$

Ji (1997)

Form Factors of the quark spin operator

$$S_q^{\lambda\mu\nu}(x) = \frac{1}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{\psi}(x)\gamma_\alpha\gamma_5\psi(x)$$

$$\langle p', S'|S_q^{\lambda\mu\nu}(0)|p, S\rangle = \frac{1}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{u}(p', s')\Big[\gamma_\alpha\gamma_5\, G_A^q(t) + \frac{\Delta_\alpha\gamma_5}{2M}\, G_P^q(t)\Big]u(p, s)$$

$G_A^q(t)$ axial form factor

$G_P^q(t)$ pseudoscalar form factor

QCD equation of motion $\quad \frac{1}{2}T^{[\mu\nu]}_{\mathrm{kin},q}(x) = -\partial_\lambda S_q^{\lambda\mu\nu}(x)$

$$\Downarrow$$

$$C_q(t) = -G_A^q(t)$$

Lorcé, Mantovani, Pasquini, Phys. Lett. B776, 38 (2018)

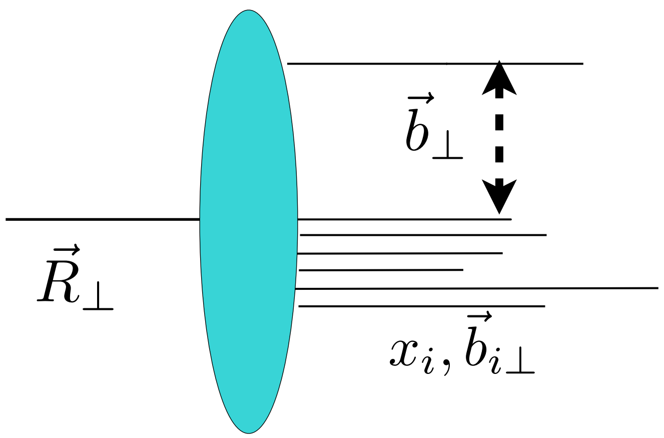
AM Distribution in the impact parameter space

Drell-Yan frame

$$\Delta^+ = 0 \quad \vec{\Delta}_\perp \neq 0$$

impact parameter space

$$\vec{\Delta}_\perp \overset{\text{FT}}{\longleftrightarrow} \vec{b}_\perp$$



Lorcé, Mantovani, Pasquini, Phys. Lett. B776, 38 (2018)

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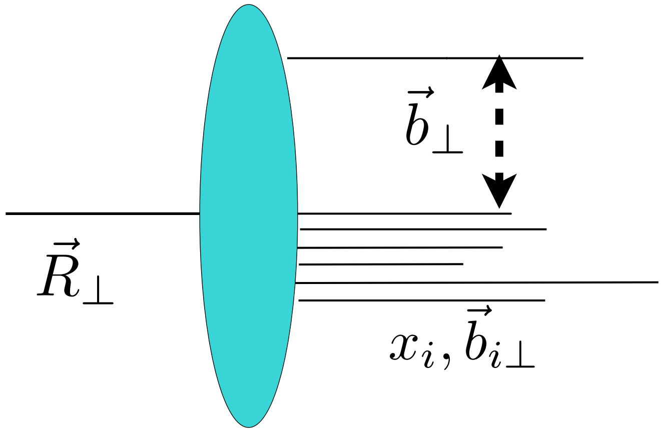

$$J^z_{\text{kin}}(\vec{b}_\perp) = J^z_{\text{naive}}(\vec{b}_\perp) + J^z_{\text{corr}}(\vec{b}_\perp)$$

$$J^z_{\text{naive}}(\vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} J^z(t = -\vec{\Delta}^2_\perp)$$

$$J^z(t) \rightarrow \frac{1}{2}(A(t) + B(t))$$

Lorcé, Mantovani, Pasquini, Phys. Lett. B776, 38 (2018)

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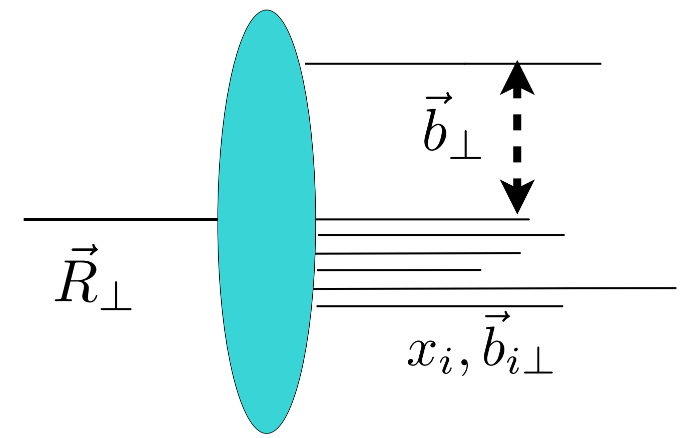
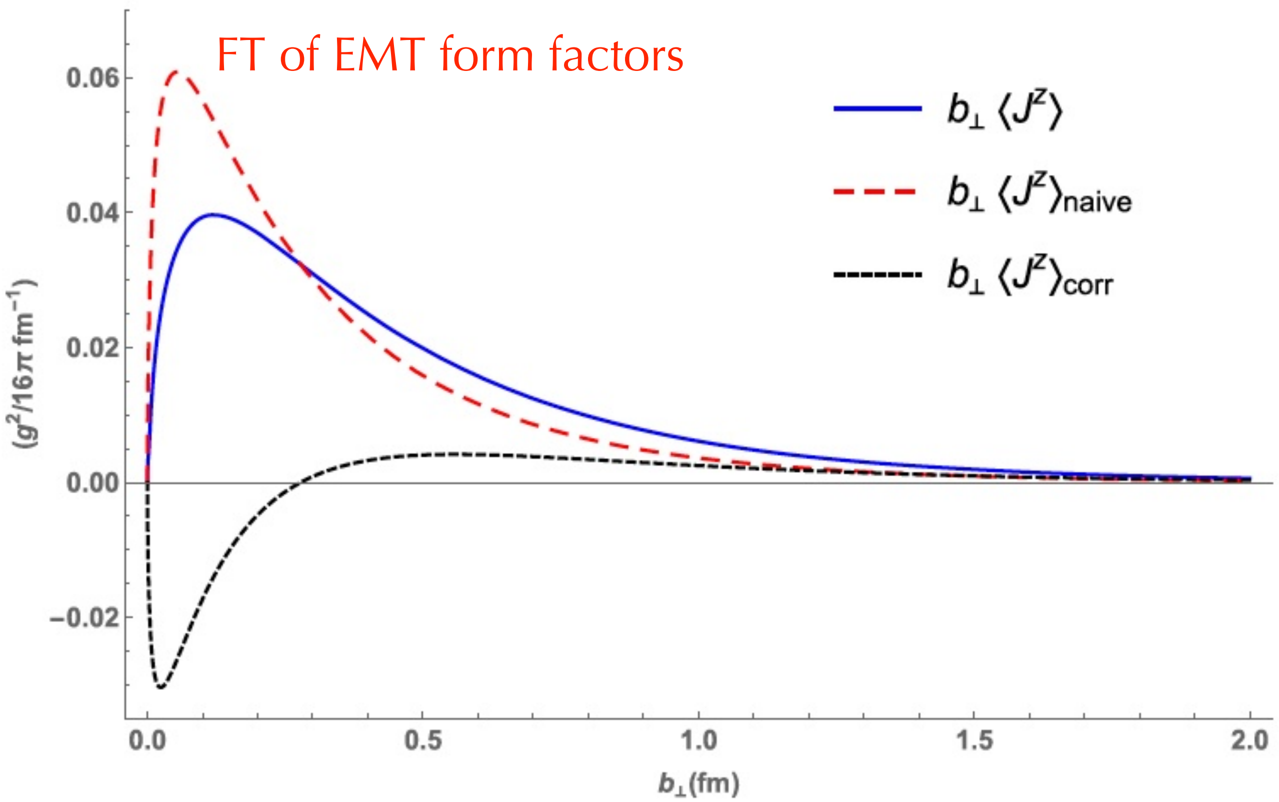


$$J^z_{\text{kin}}(\vec{b}_\perp) = J^z_{\text{naive}}(\vec{b}_\perp) + J^z_{\text{corr}}(\vec{b}_\perp) \qquad J^z_{\text{naive}}(\vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} J^z(t = -\vec{\Delta}^2_\perp)$$

$$J^z \downarrow \ \frac{1}{2}(A(t) + B(t))$$

Scalar diquark model



Lorcé, Mantovani, Pasquini, Phys. Lett. B776, 38 (2018)

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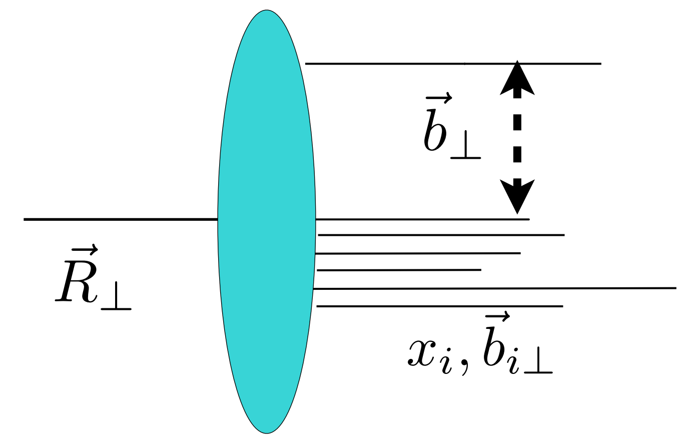

$$J^z_{\text{kin}}(\vec{b}_\perp) = J^z_{\text{naive}}(\vec{b}_\perp) + J^z_{\text{corr}}(\vec{b}_\perp)$$

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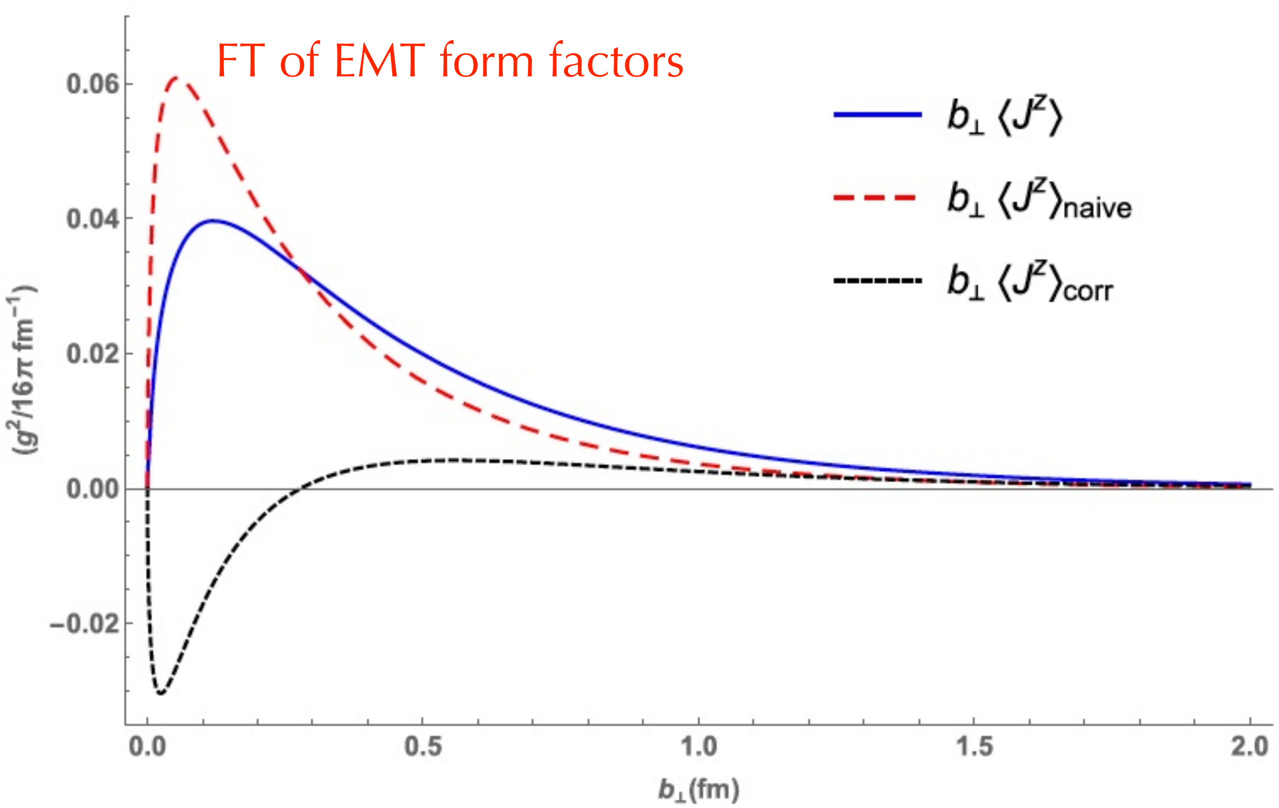
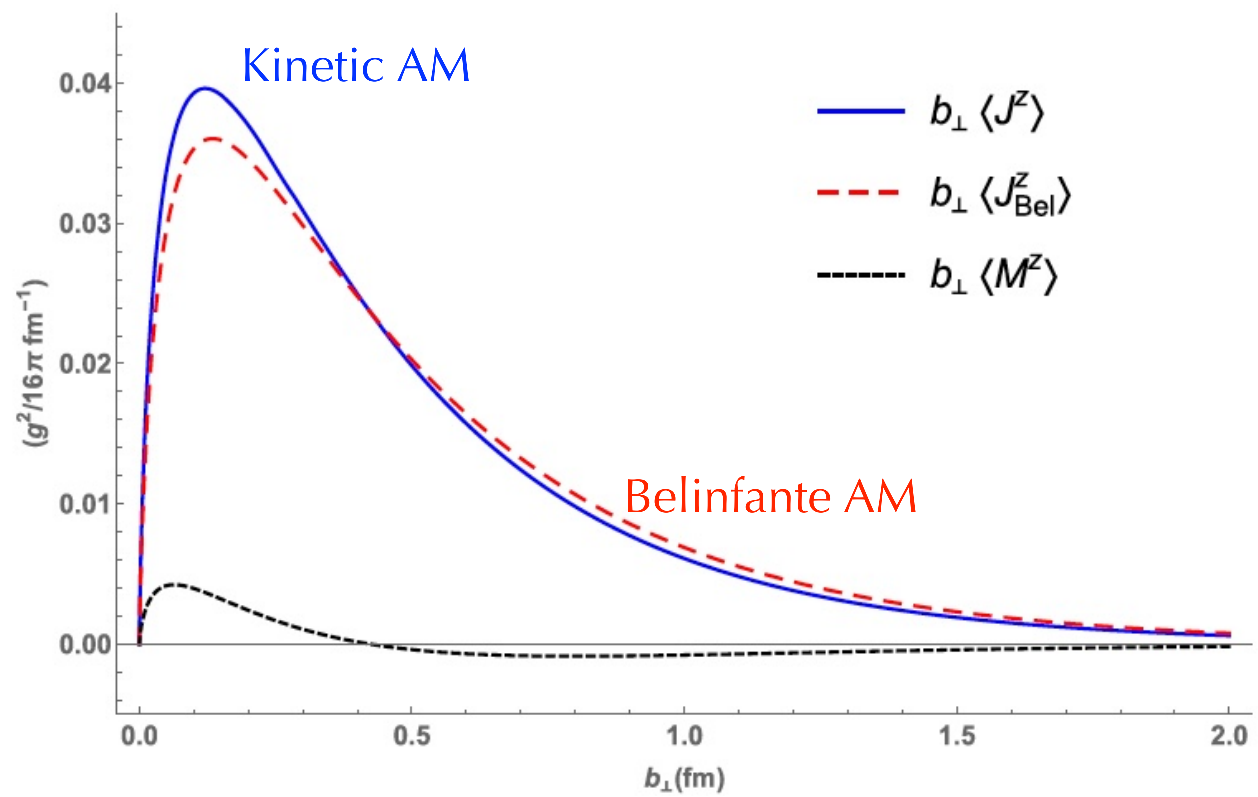
$$J^z(t = -\vec{\Delta}^2_\perp) \rightarrow \frac{1}{2}(A(t) + B(t))$$

$$J^z_{\text{kin}}(\vec{b}_\perp) = J^z_{\text{Bel}}(\vec{b}_\perp) + M^z(\vec{b}_\perp)$$

Scalar diquark model





Lorcé, Mantovani, Pasquini, Phys. Lett. B776, 38 (2018)

Link to generalized parton distributions

- Polinomiality:

$$\int dx\, xH(x,\xi,t) = A(t) + D(t)\,\xi^2 \qquad \int dx\, xE(x,\xi,t) = B(t) - D(t)\,\xi^2$$

- Momentum sum rule $\quad A^q(0) + A^g(0) = 1 = \int dx\, x(H^q(x,\xi,0) + H^g(x,\xi,0))$

- Angular momentum sum rule
  (Ji's relation)

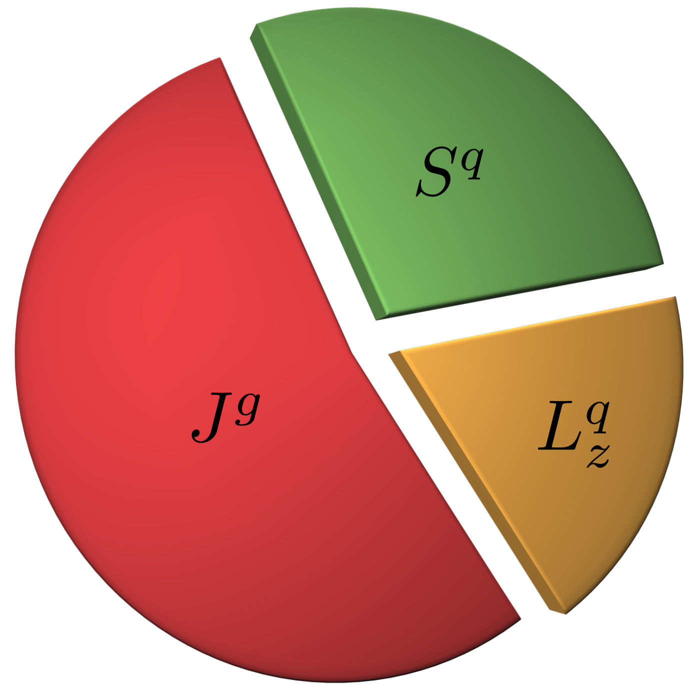
$$J^{q,g} = \frac{1}{2}[A^{q,g}(0) + B^{q,g}(0)] = \frac{1}{2}\int dx\, x\,\left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right)$$

- Gravitomagnetic sum rule

$$J^{\mathrm{TOT}} = J^q + J^g = \frac{1}{2} = \frac{1}{2}[A^q(0) + A^g(0) + B^q(0) + B^g(0)] = \frac{1}{2}[1 + B^q(0) + B^g(0)]$$

$$B^q(0) + B^g(0) = 0 = \int dx\, x(E^q(x,\xi,0) + E^g(x,\xi,0))$$

Link to generalized parton distributions



Ji's sum rule: $J^{q,g}(t=0) = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right)$ $\quad (\xi = -\frac{\Delta^+}{2P^+})$

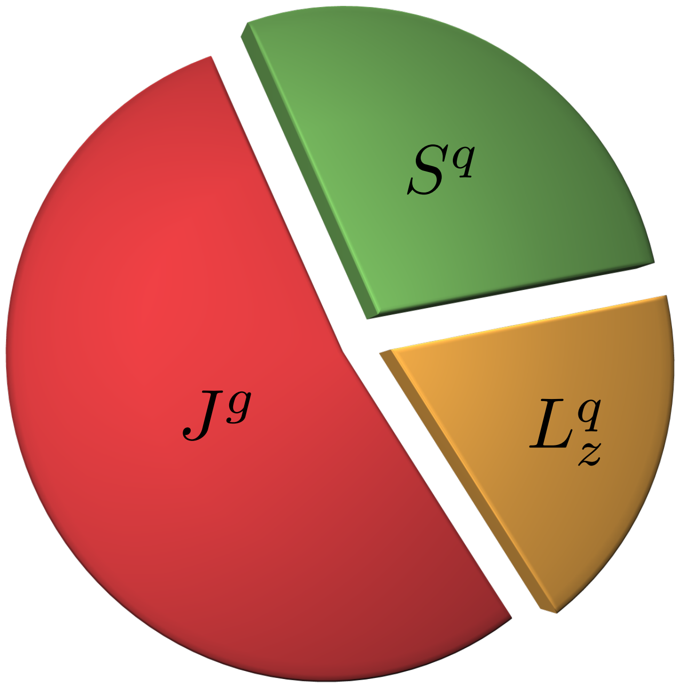
$H^{q,g}$: at $\xi = 0$ unpolarized PDF

$E^{q,g}$: not directly accessible

$$J^q = L^q + S^q \qquad S^q = \frac{1}{2}\int_0^1 dx\, g_1(x)$$

- Requires extrapolation at t=0
- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)
- $J^q(x) \neq \frac{1}{2}x(H^q(x,0,0) + E^q(x,0,0)$ contribution from surface term

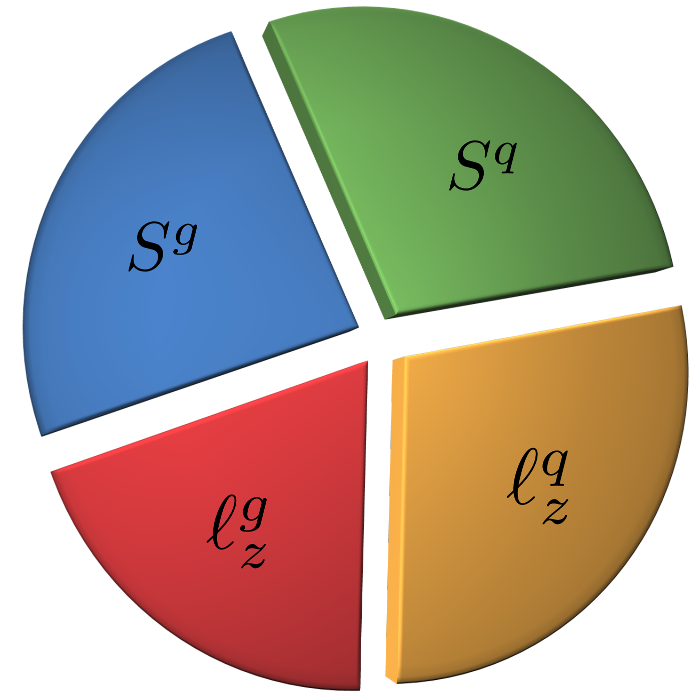
Ji (kinetic EMT) Sum Rule



$$\frac{1}{2} = \underbrace{S^q(\mu) + L_z^q(\mu)}_{J^q} + J^g(\mu)$$

- each term is gauge invariant
- frame independent
- it works also for the transverse AM in the infinite momentum frame
- J^q and J^g can be obtained from moments of GPDs

Jaffe-Manohar (canonical EMT) Sum Rule



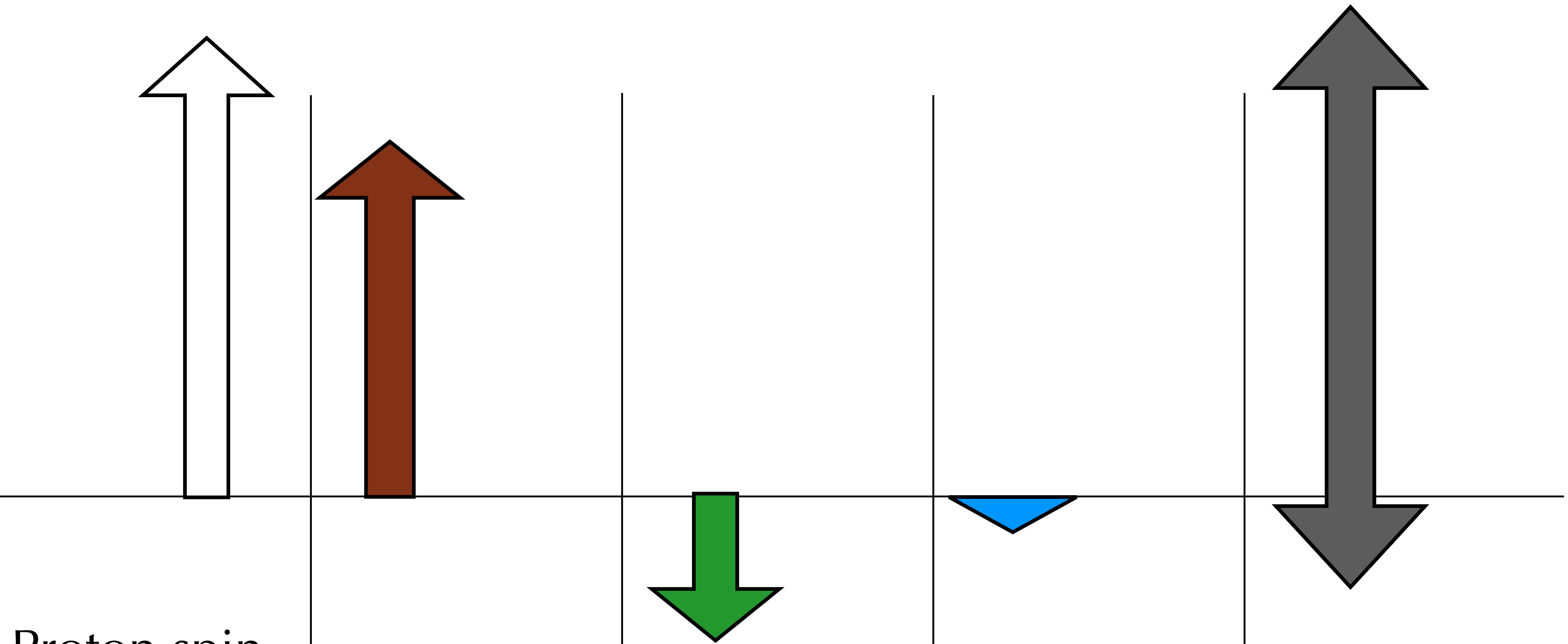
$$\frac{1}{2} = S^q(\mu) + \ell_z^q(\mu) + \ell_z^g(\mu) + S^g(\mu)$$

- ℓ_z^q, ℓ_z^g, S^g are gauge dependent, BUT measurable
- S^q, S^g can be obtained from pol. PDFs
- ℓ_z^q, ℓ_z^g can be obtained from twist-3 GPDs and Wigner distributions
- simple partonic interpretation in the IMF

$$\ell_z^q - l_z^{q,\text{pot}} = L_z^q \longrightarrow \ell_z^g + S^g + l_z^{q,\text{pot}} = J^g$$

Y. Guo, X. Ji, K. Shiells, NPB 96 (2021) 115440; X. Ji, F. Yuan, Y. Zhao, Nature Rev. Phys. 3 (2021) 1

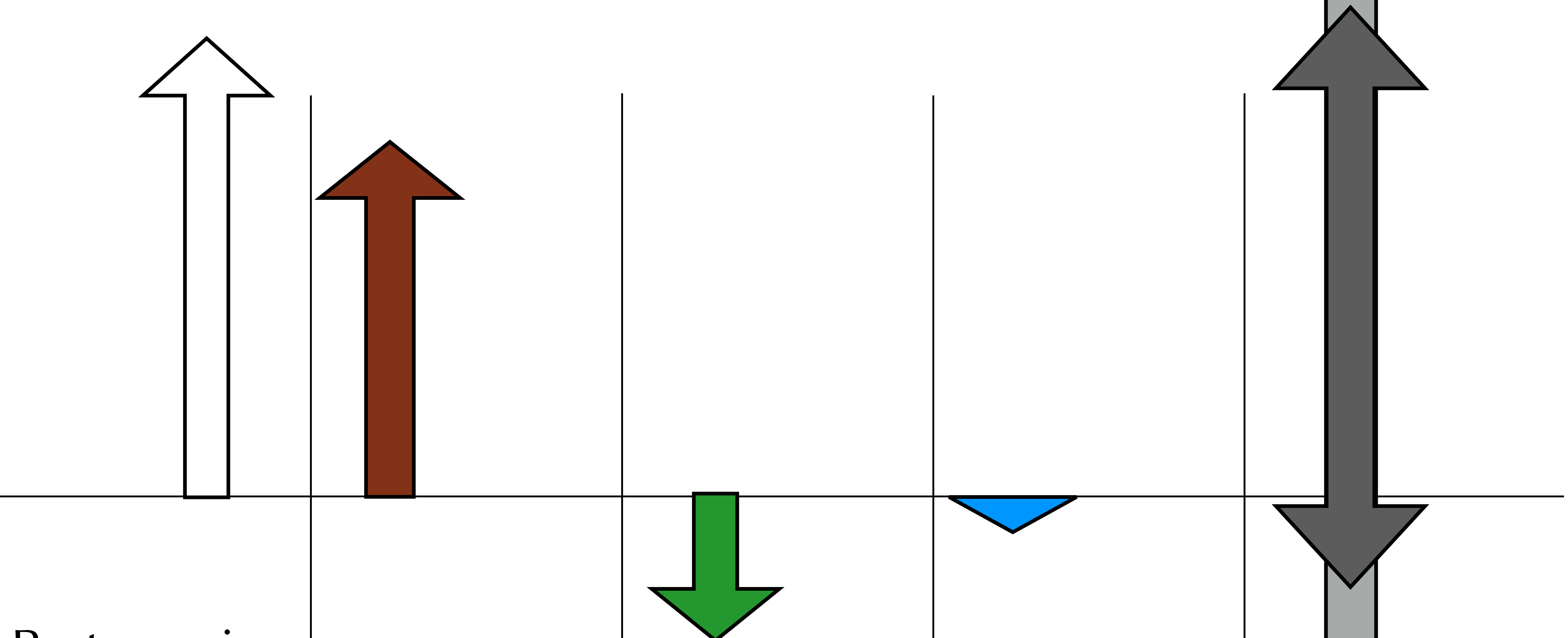
Status of Spin Sum Rule



Proton spin
(Q^2=10 GeV^2)

$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
81 ± 1		-43 ± 1		-11 ± 2		40 ± 60		DSSV14
79 ± 5		-45 ± 5		12 ± 8		3 ± 324		NNPDF

Status of Spin Sum Rule



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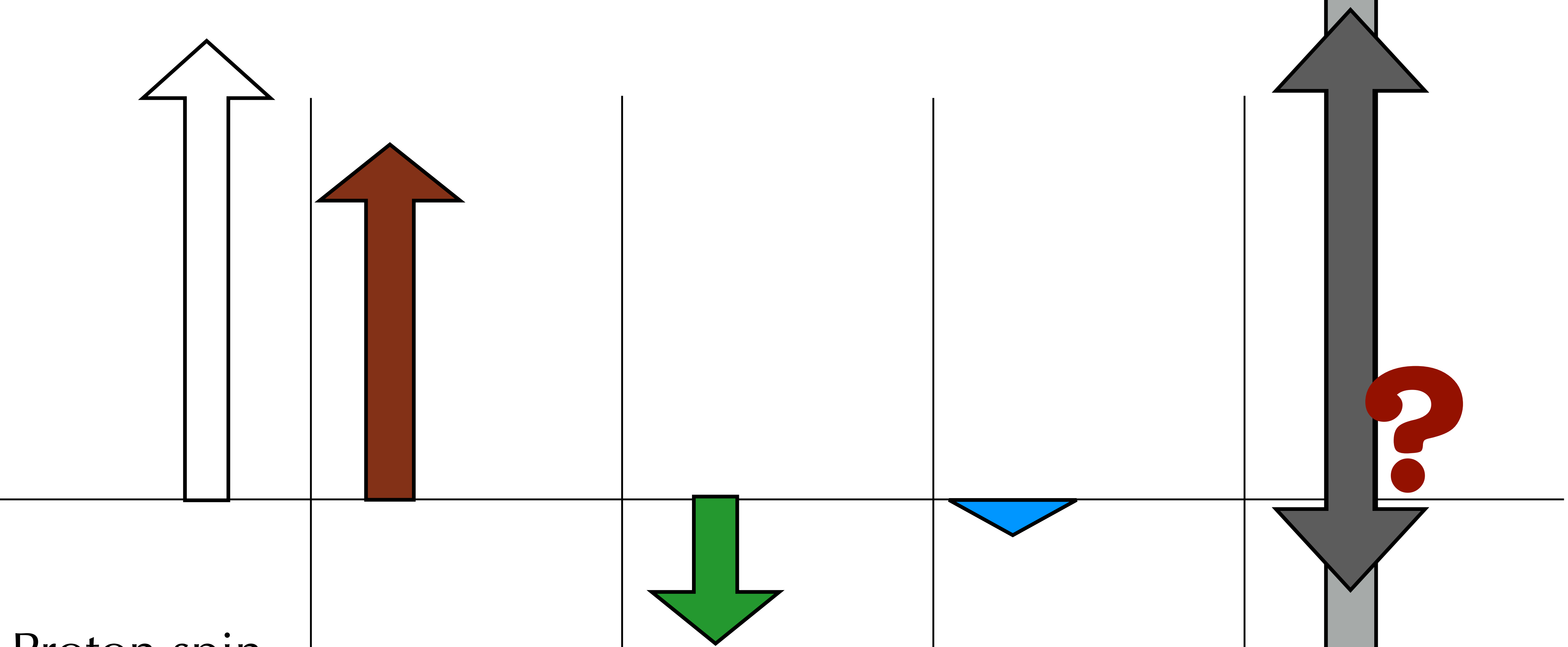

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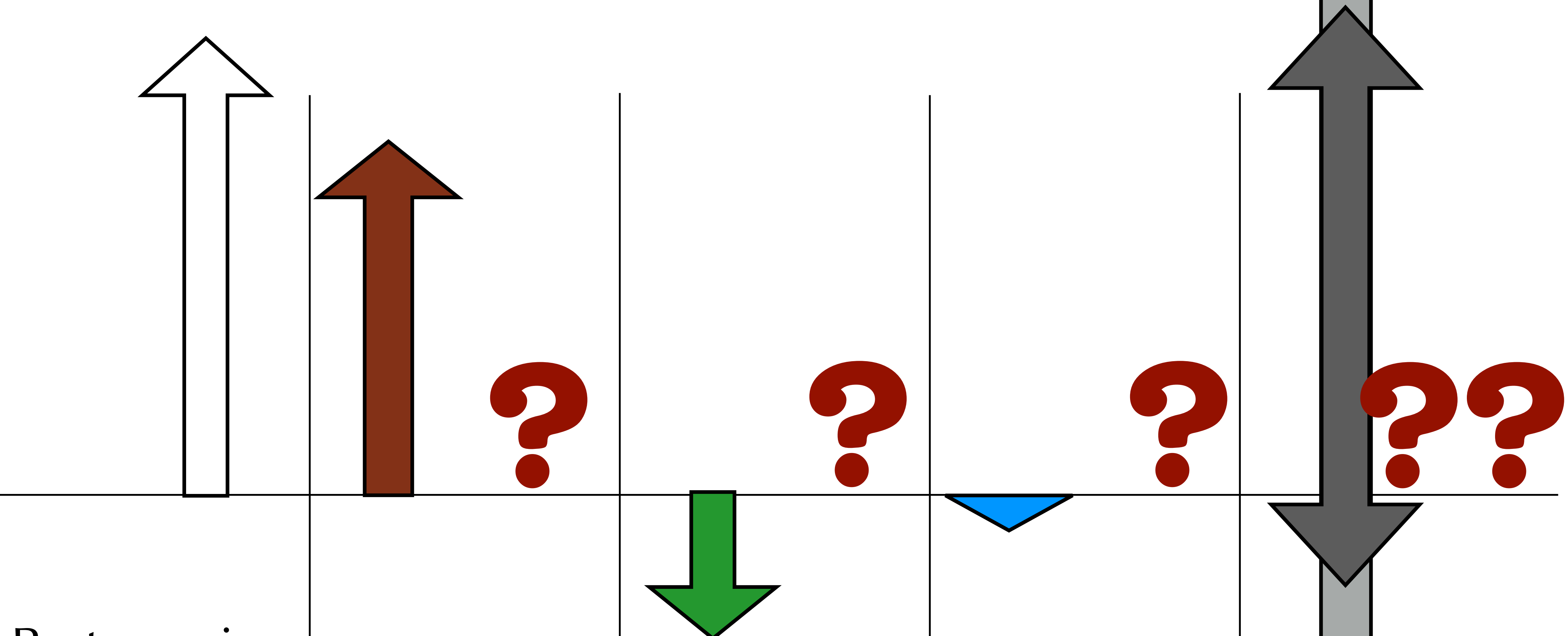

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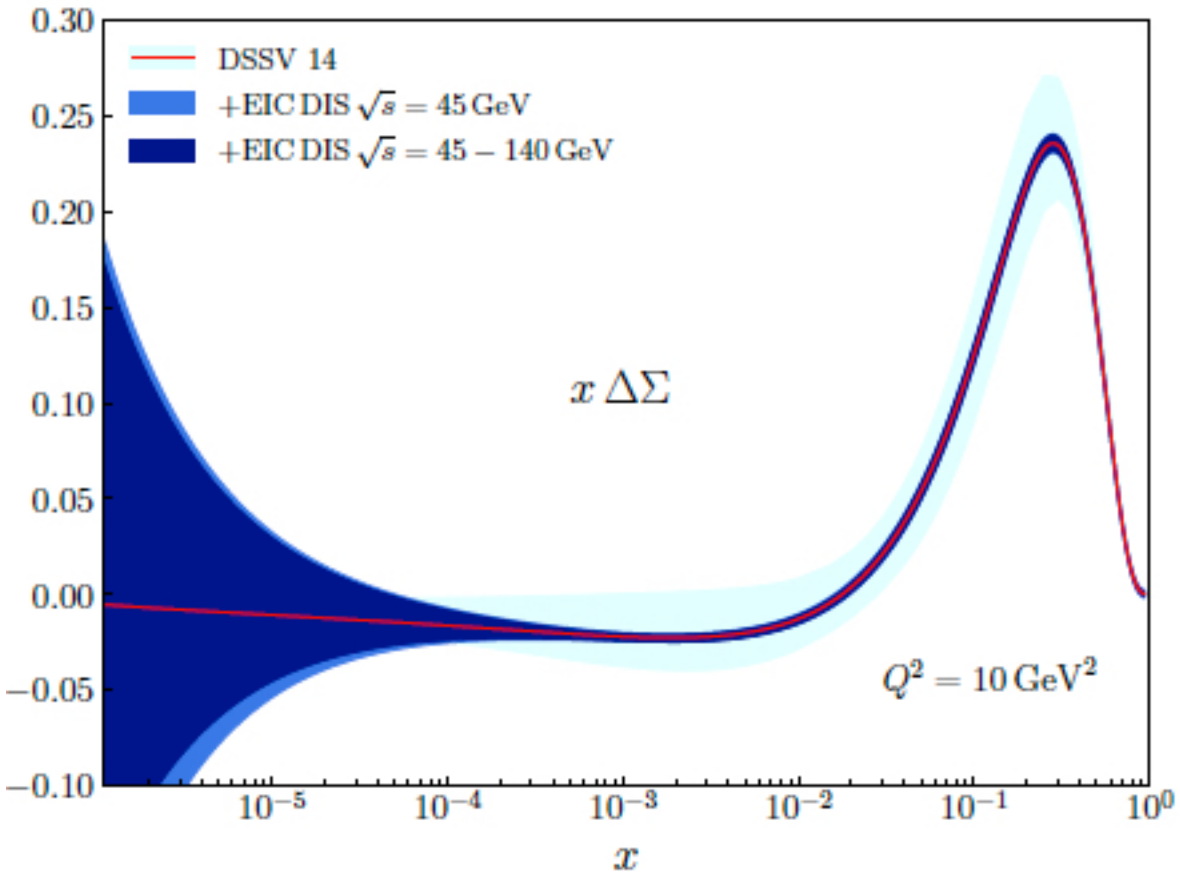
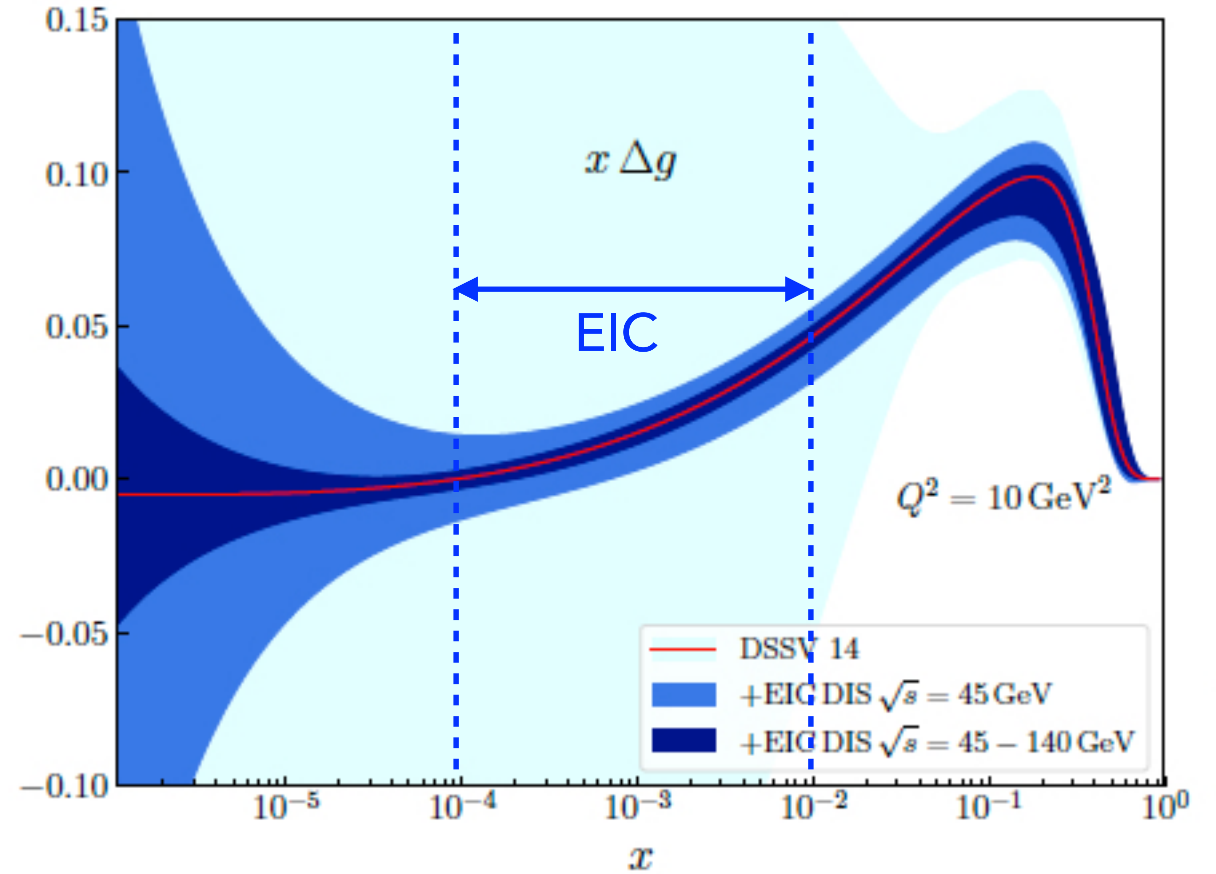
79 ± 5		-45 ± 5		12 ± 8		3 ± 324		NNPDF

Impact of future EIC for quark and gluon spin contributions

Quark Spin



Gluon Spin



EIC Yellow Report: arXiv: 2103.05419

We are constantly improving the knowledge of the contributions to the spin of the nucleon

However the details on the flavor and sea contributions are still sketchy

Orbital Angular momentum of the proton from available GPD measurements

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right) \qquad L^q = J^q - S^q$$

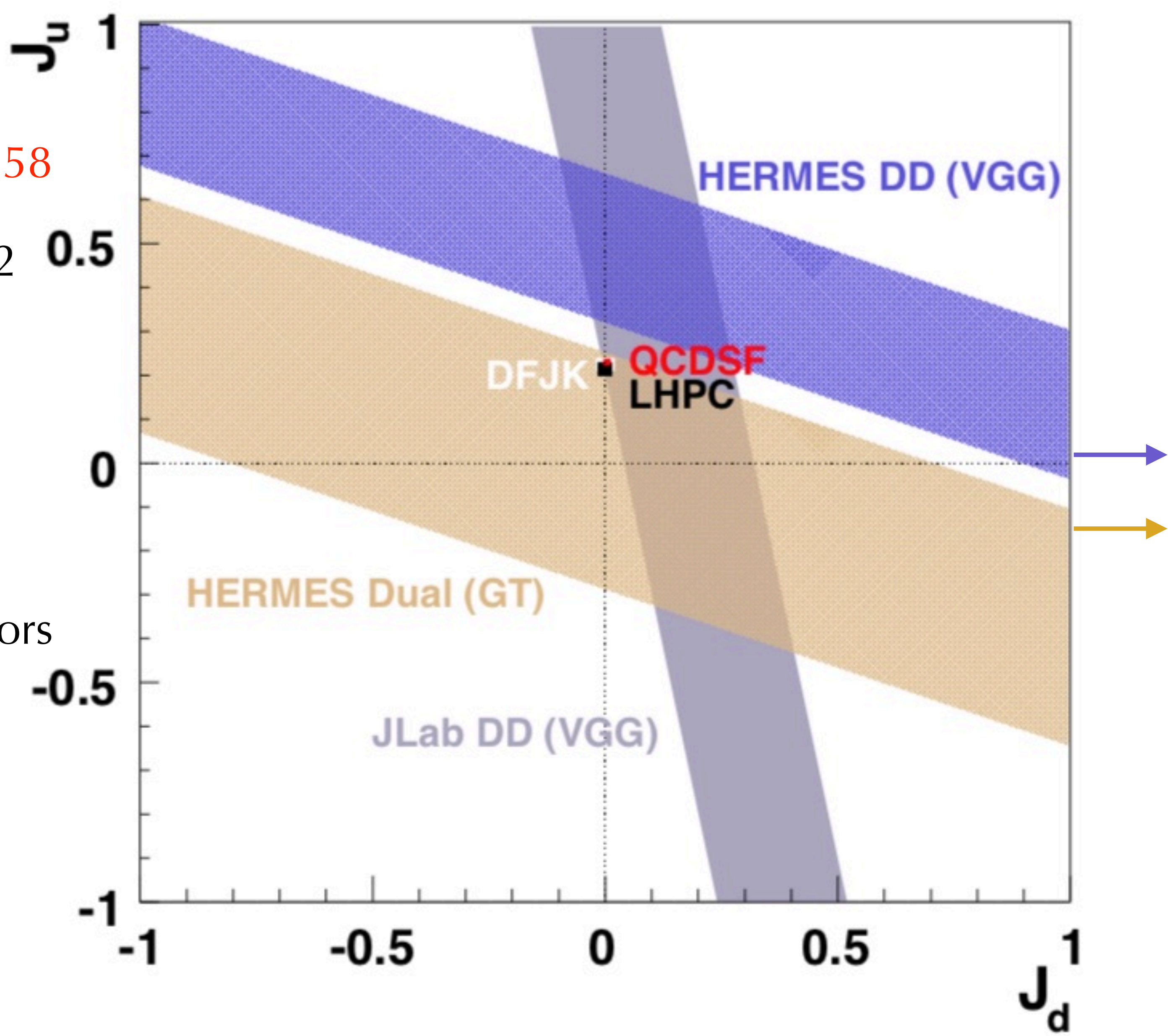
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



extractions from HERMES data using two different models

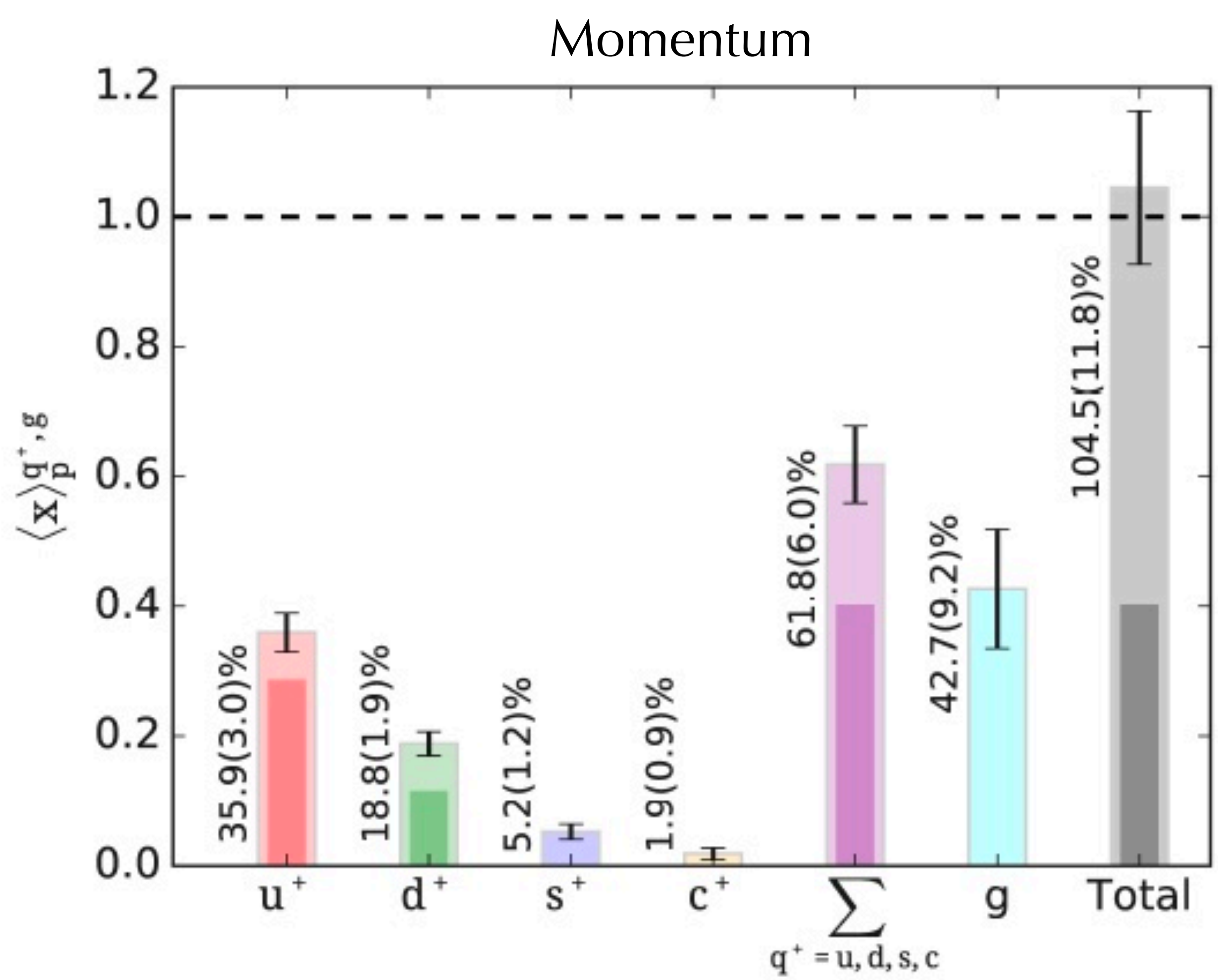
JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

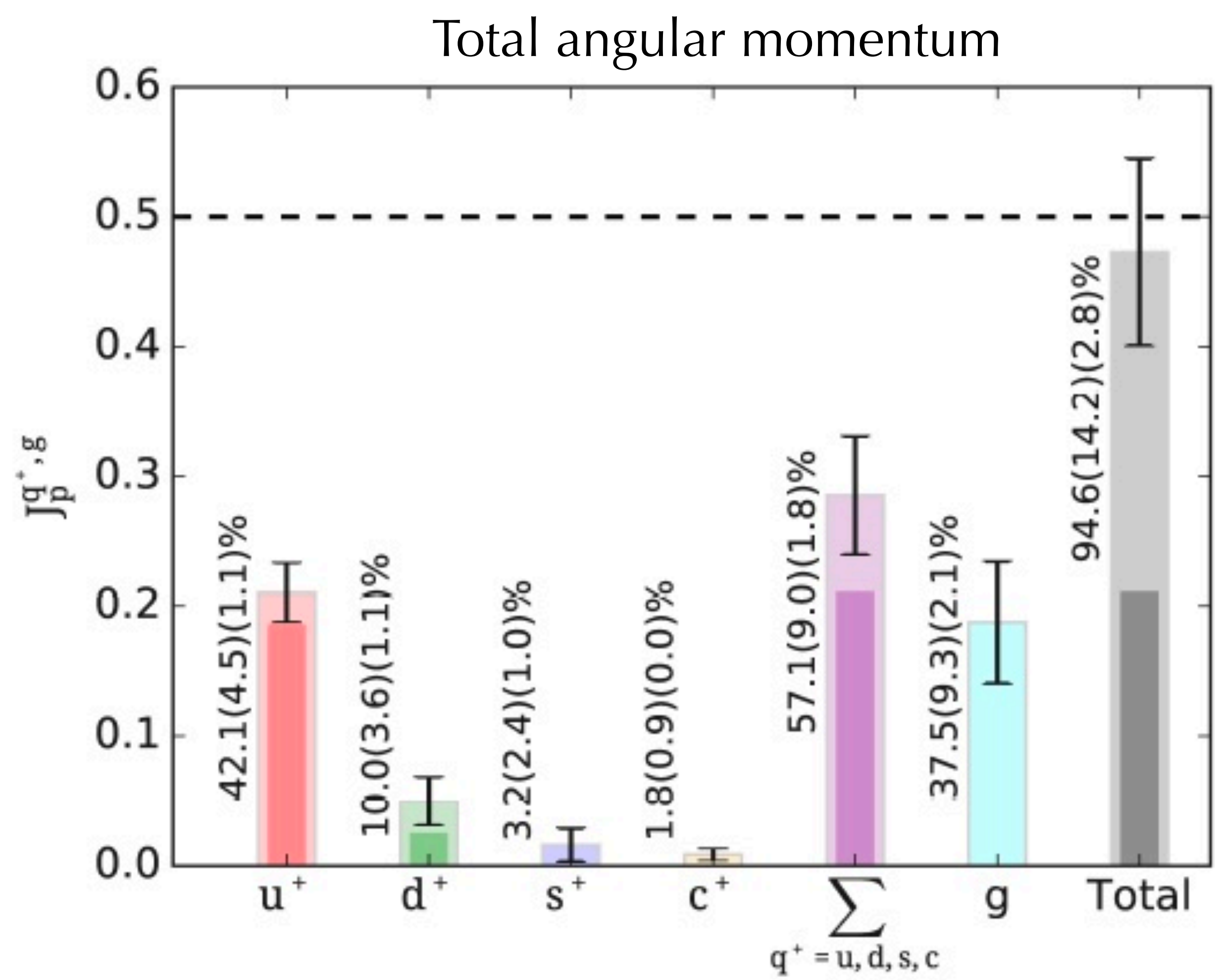
Hermes Coll., JHEP 06 (2008) 066

Improved accuracy with JLab12 and future EIC measurements!

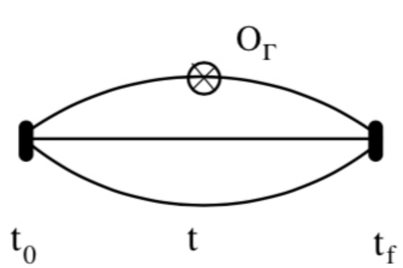
Sum rules from lattice QCD

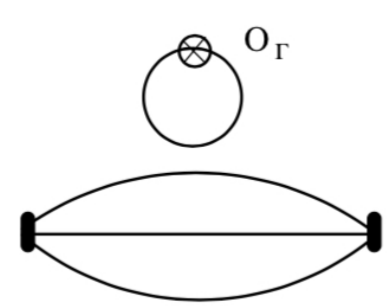
- Results at physical pion mass at the scale of 2 GeV





- dark bars: connected
- light bars: disconnected contributions (quarks & gluons)



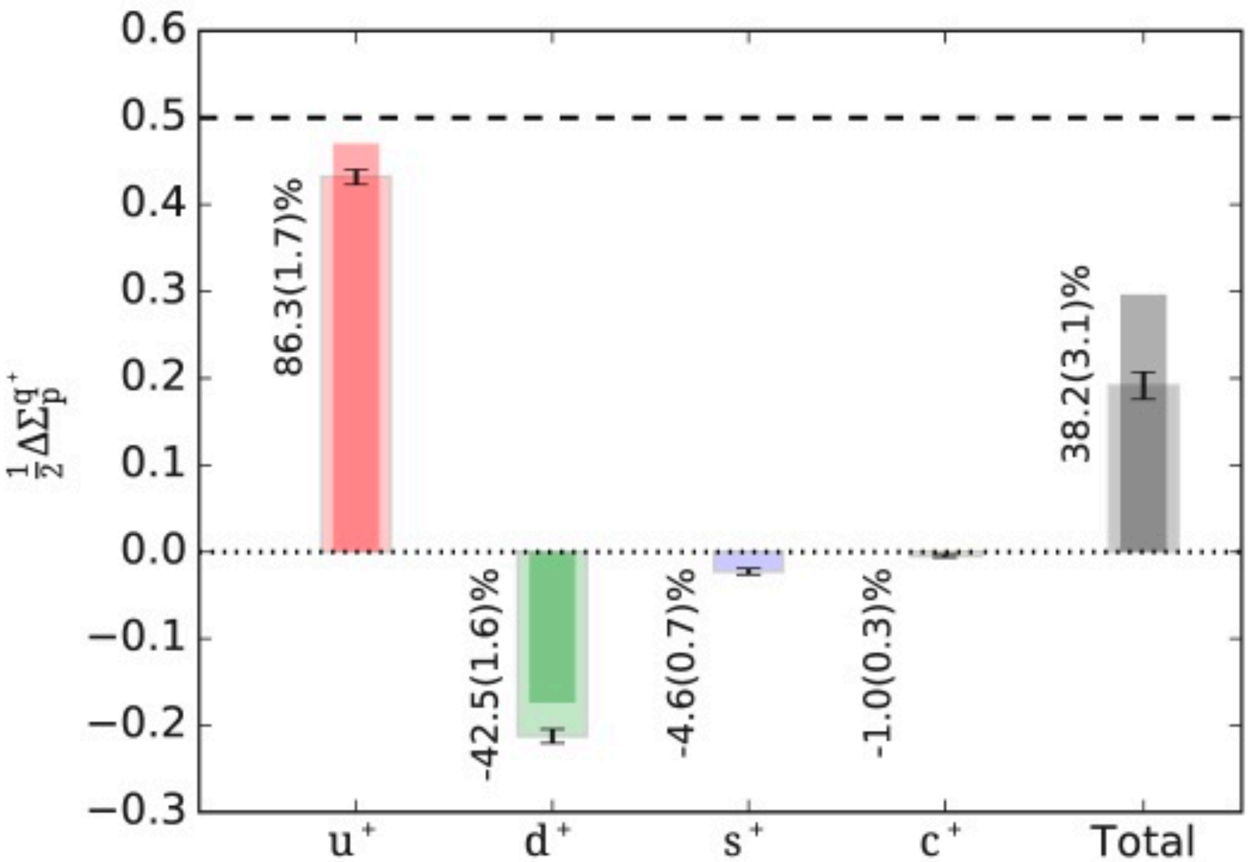


- gravitomagnetic sum rule: $\sum_{q=u,d,s} B^q(0) + B^g(0) = -0.099(91)(28)$

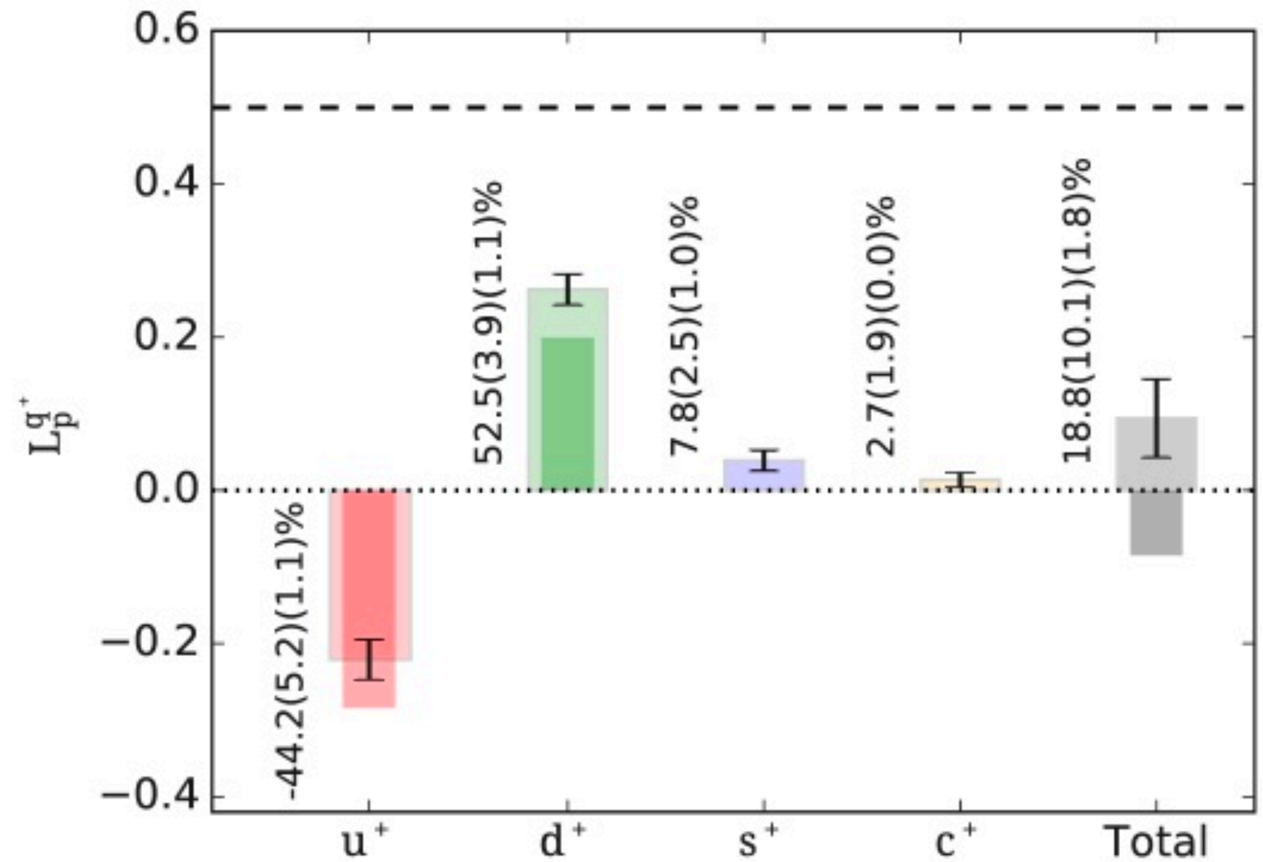
Alexandrou et al. [Extended Twist Mass Coll.], Phys.Rev.D 101 (2020), 094513

Spin and orbital angular momentum from lattice QCD

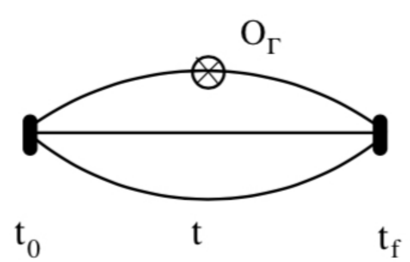
Quark spin



Quark orbital angular momentum $J^q - \frac{1}{2}\Delta\Sigma^q$



- dark bars: connected



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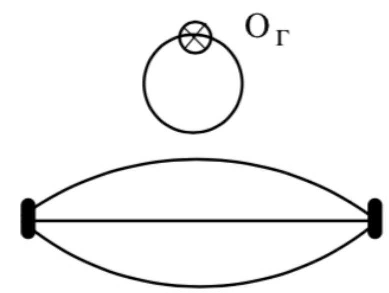
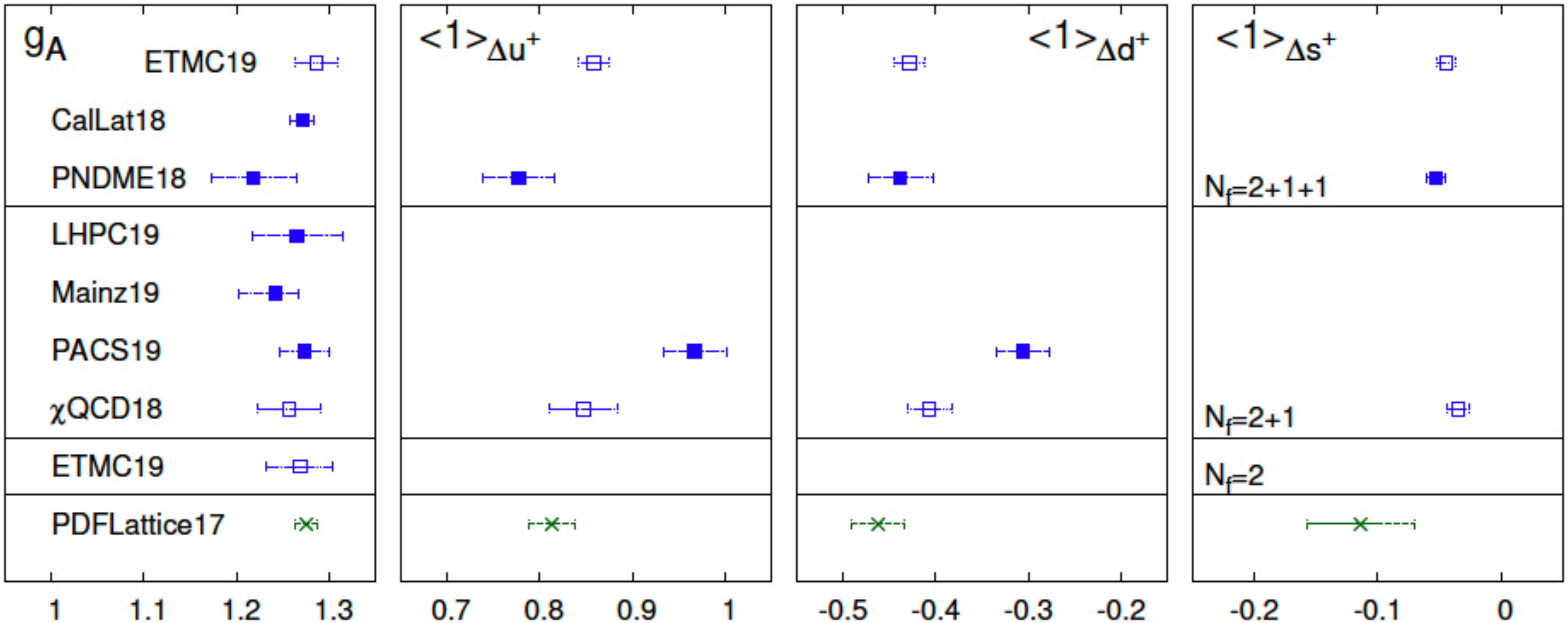


Alexandrou et al., Phys.Rev.D 101 (2020), 094513

Comparison with Lattice calculations

Quark spin $Q^2 = 4\ \mathrm{GeV}^2$



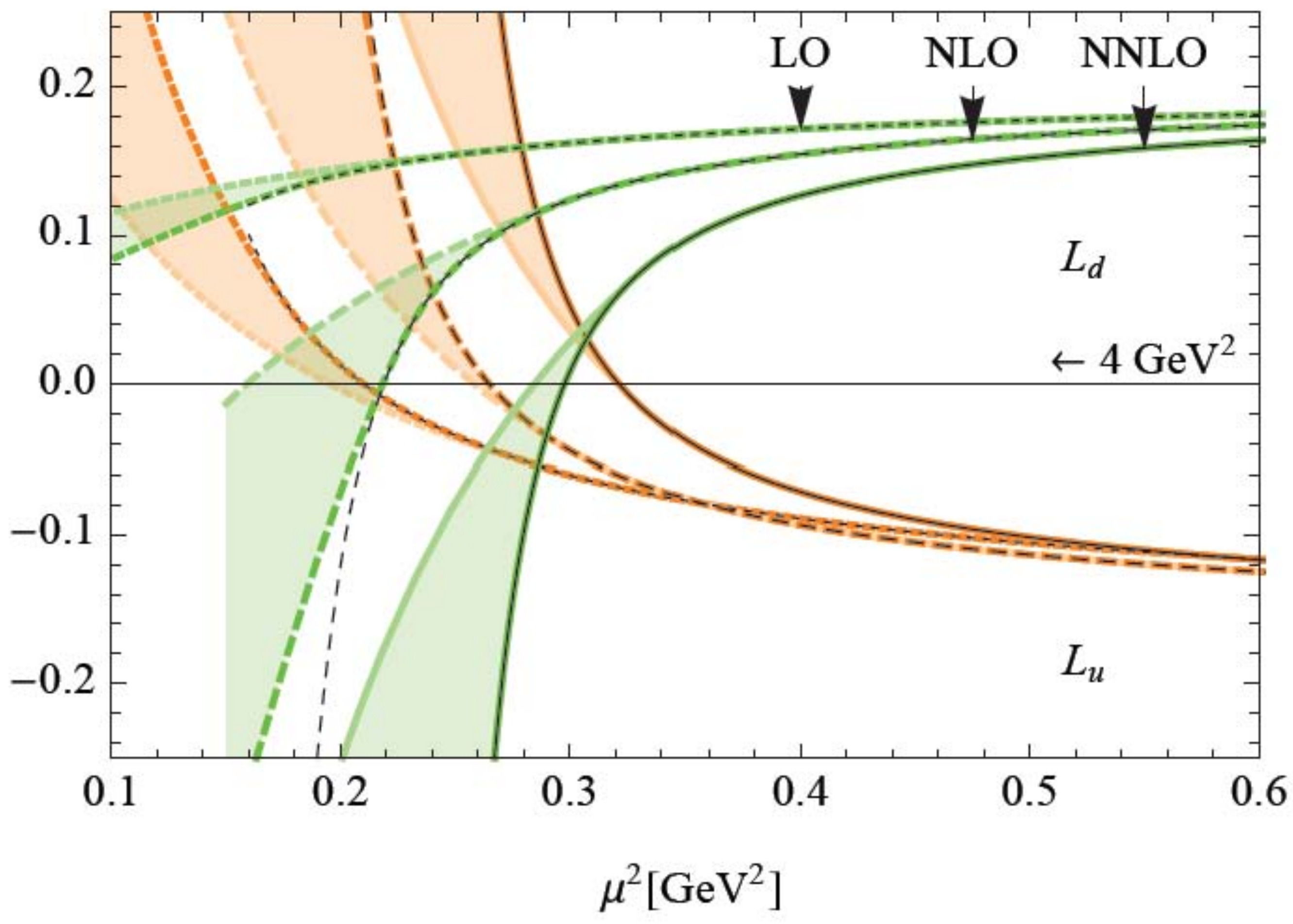
PDFLattice17*: average of phenomenological extractions: JAM15, NNPDFpol1.1, DSSV08 from the community white paper, Prog.Part.Nucl.Phys. 100 (2018) 107*

Overall fair agreement between lattice calculations and phenomenological fits

The uncertainties of the two have comparable size

Lattice QCD results could provide useful inputs to global fits of polarized PDFs

Scale dependence



Altenbuchinger, Hägler, Weise, EPJA47, 140 (2011)

Nucleon Structure Properties

em	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\vert J^\mu_{\text{em}}\vert N\rangle$	$\longrightarrow$	$Q,\ \mu$
weak	$\partial_\mu J^\mu_{\text{weak}} = 0$	$\langle N'\vert J^\mu_{\text{weak}}\vert N\rangle$	$\longrightarrow$	$g_A,\ g_p$
gravity	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\text{grav}}\vert N\rangle$	$\longrightarrow$	M_N, J, D

$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19}\,\text{C}$

$\mu_{\text{prot}} = 2.792847356(23)\mu_N$

$M_{\text{prot}} = 938.272013(23)\,\text{MeV}$

$g_p = 8 - 12$

$g_A = 1.2694(28)$

$J = \frac{1}{2}$

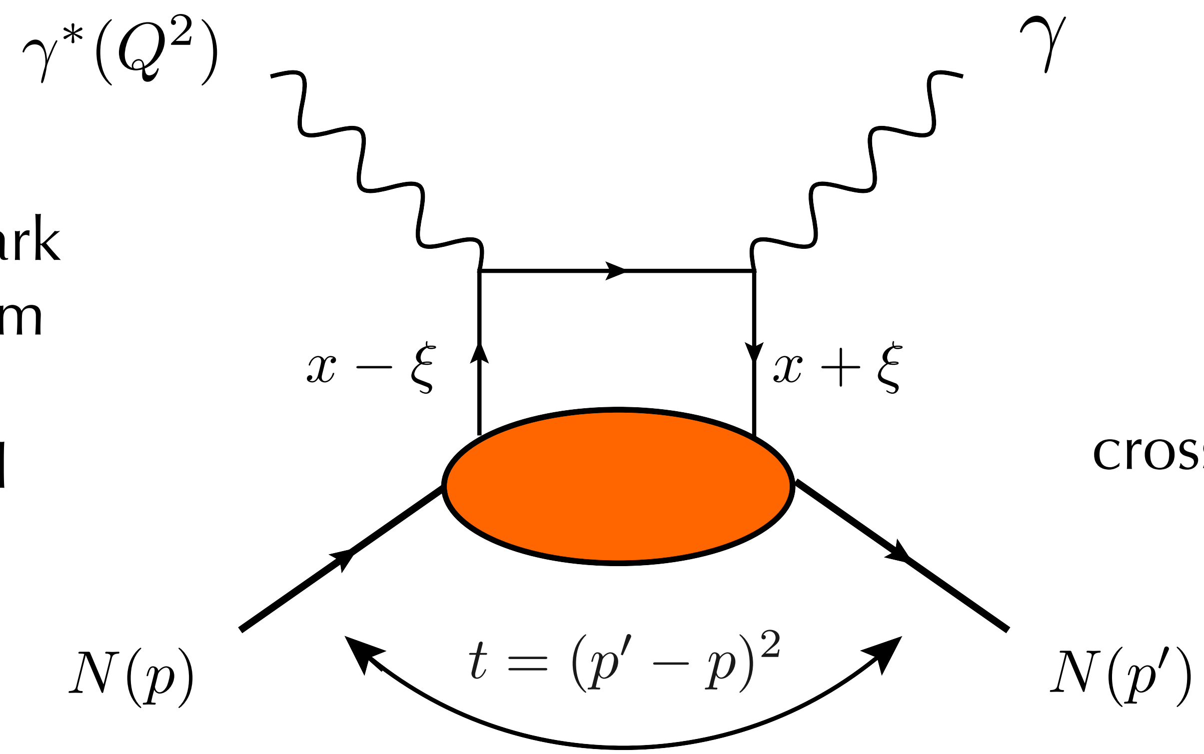
$$D = \frac{4}{5}d_1 = ??$$

can be accessed from GPDs in hard exclusive reactions

M. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A33 (2018) 1830025

Dispersion relation approach

DVCS at leading twist



x : average fraction of quark longitudinal momentum

ξ : fraction of longitudinal momentum transfer

t : nucleon momentum transfer

$t \ll \nu,\ Q^2$

↓

crossing symmetric variable

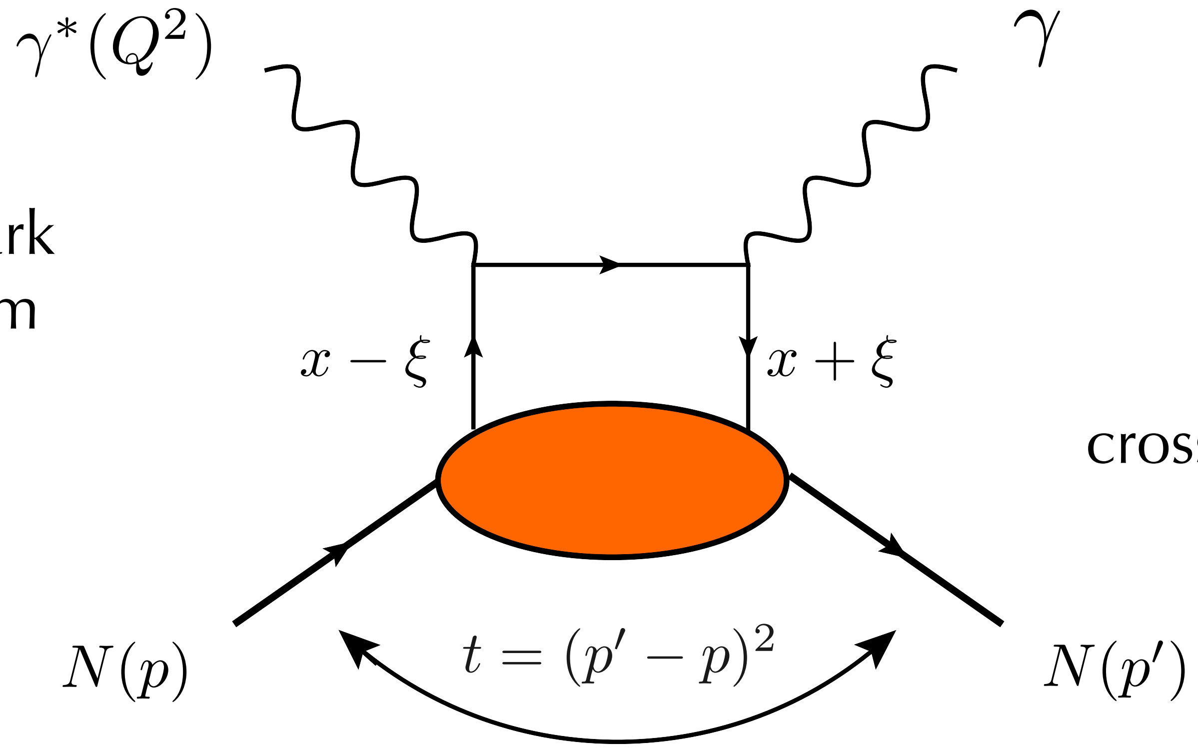
$$\nu = \frac{s - u}{4M_N}$$

DVCS at leading twist

x : average fraction of quark longitudinal momentum

ξ : fraction of longitudinal momentum transfer

t : nucleon momentum transfer



$t \ll \nu,\ Q^2$

crossing symmetric variable

$$\nu = \frac{s-u}{4M_N}$$

DVCS tensor at twist 2: $T^{\mu\nu} = \sum_{i=1}^{4} A_i(\nu,t,Q^2)\, O_i^{\mu\nu}$

unpolarized quark

$$A_1 = \mathcal{H} + \mathcal{E}$$

$$A_2 = -\mathcal{E}$$

long. polarized quark

$$A_3 = \tilde{\mathcal{H}}$$

$$A_4 = \tilde{\mathcal{E}}$$

Compton form factors: $\mathcal{F} = \int_0^1 \mathrm{d}x\, F^+(x,\xi,t,Q^2) \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right]$ $\quad F = \left\{H,\ E,\ \tilde{H},\ \tilde{E}\right\}$

singlet GPDs: $F^+(x,\xi,t) = F(x,\xi,t) - F(-x,\xi,t)$

Dispersion relations at fixed t and Q^2

$A_i(\nu, t, Q^2)$: analytical functions in the complex ν plane, with cuts on the real axis

Drechsel, Pasquini, Vanderhaeghen (2003)

Dispersion relations at fixed t and Q²

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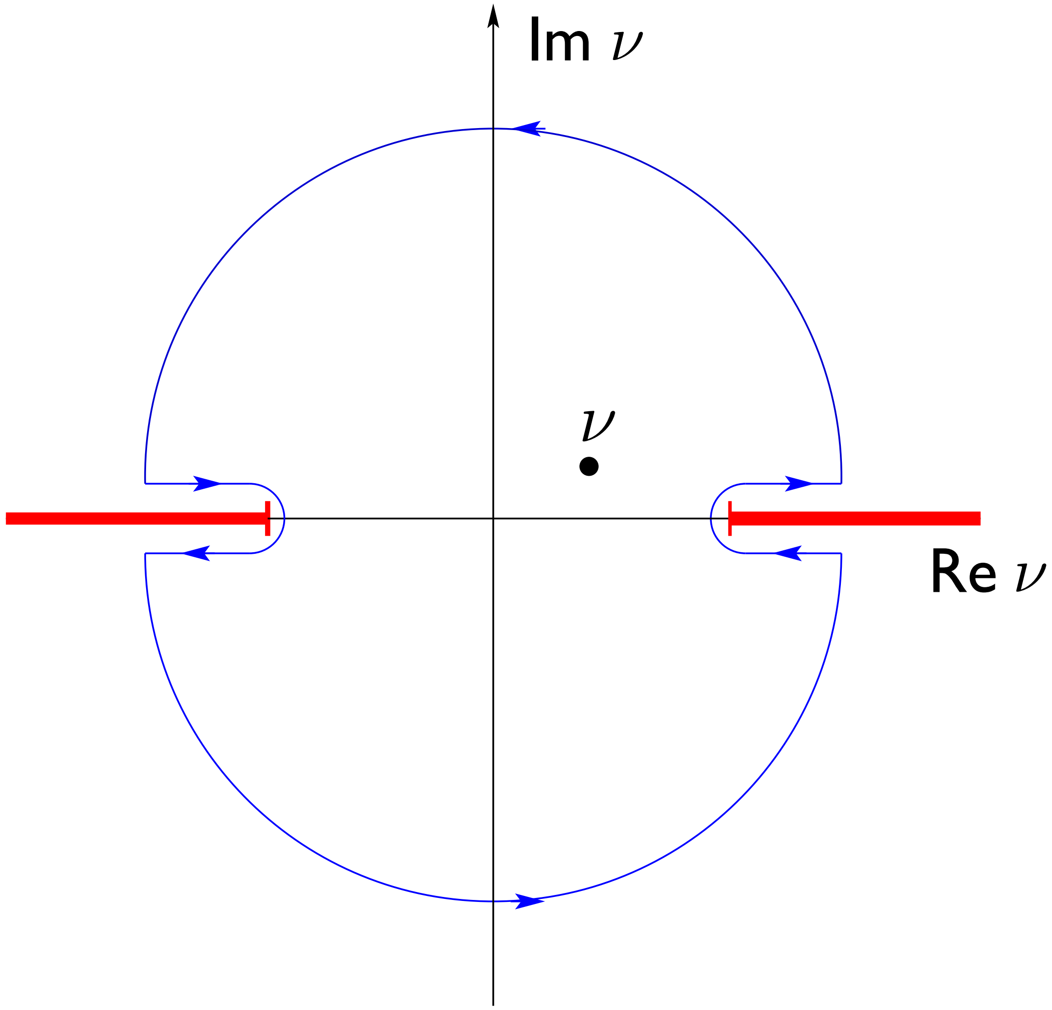

- Cauchy integral formula

$$A_i(\nu, t, Q^2) = \frac{1}{2\pi i} \oint_C d\nu' \frac{A_i(\nu', t, Q^2)}{\nu' - \nu}$$

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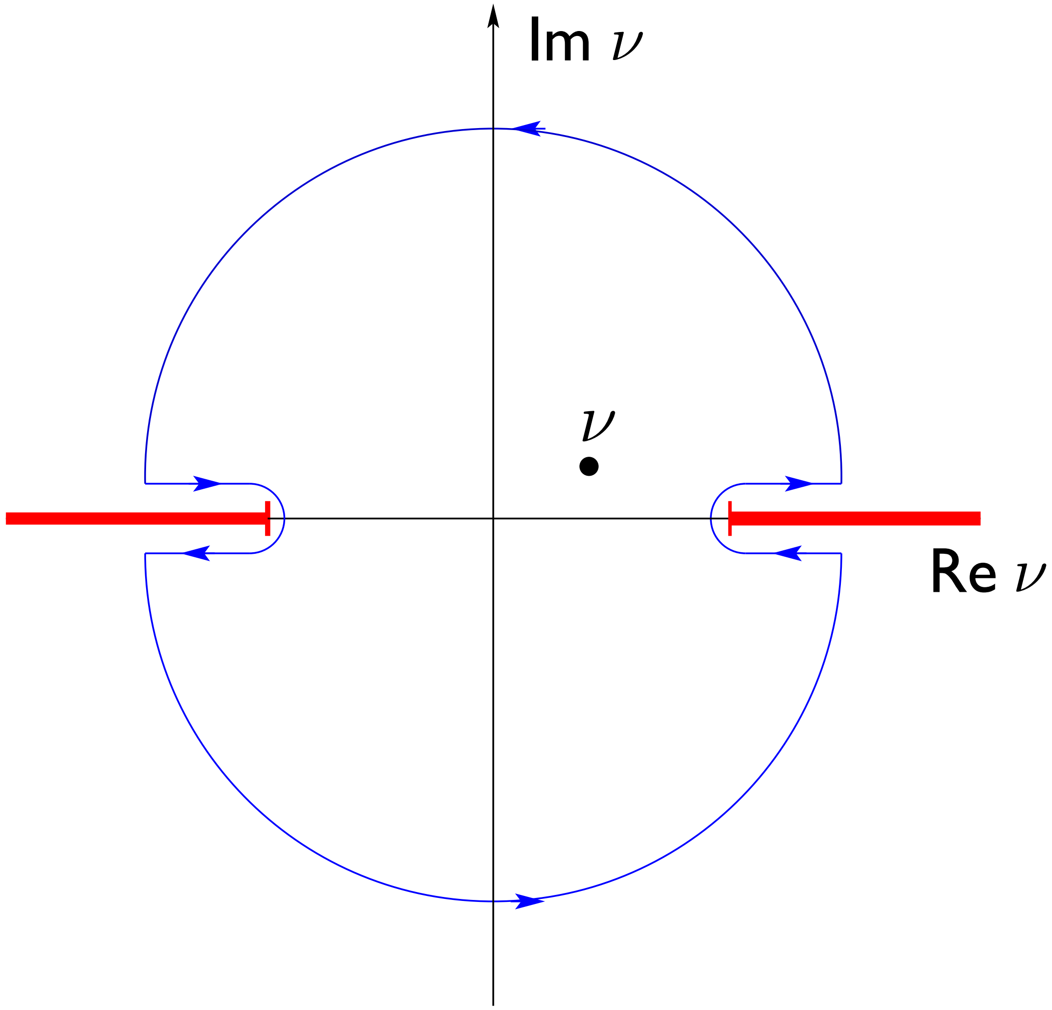

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$$A_i(\nu, t, Q^2) = A_i(-\nu, t, Q^2) \qquad A_i(\nu^*, t, Q^2) = A_i^*(\nu, t, Q^2)$$

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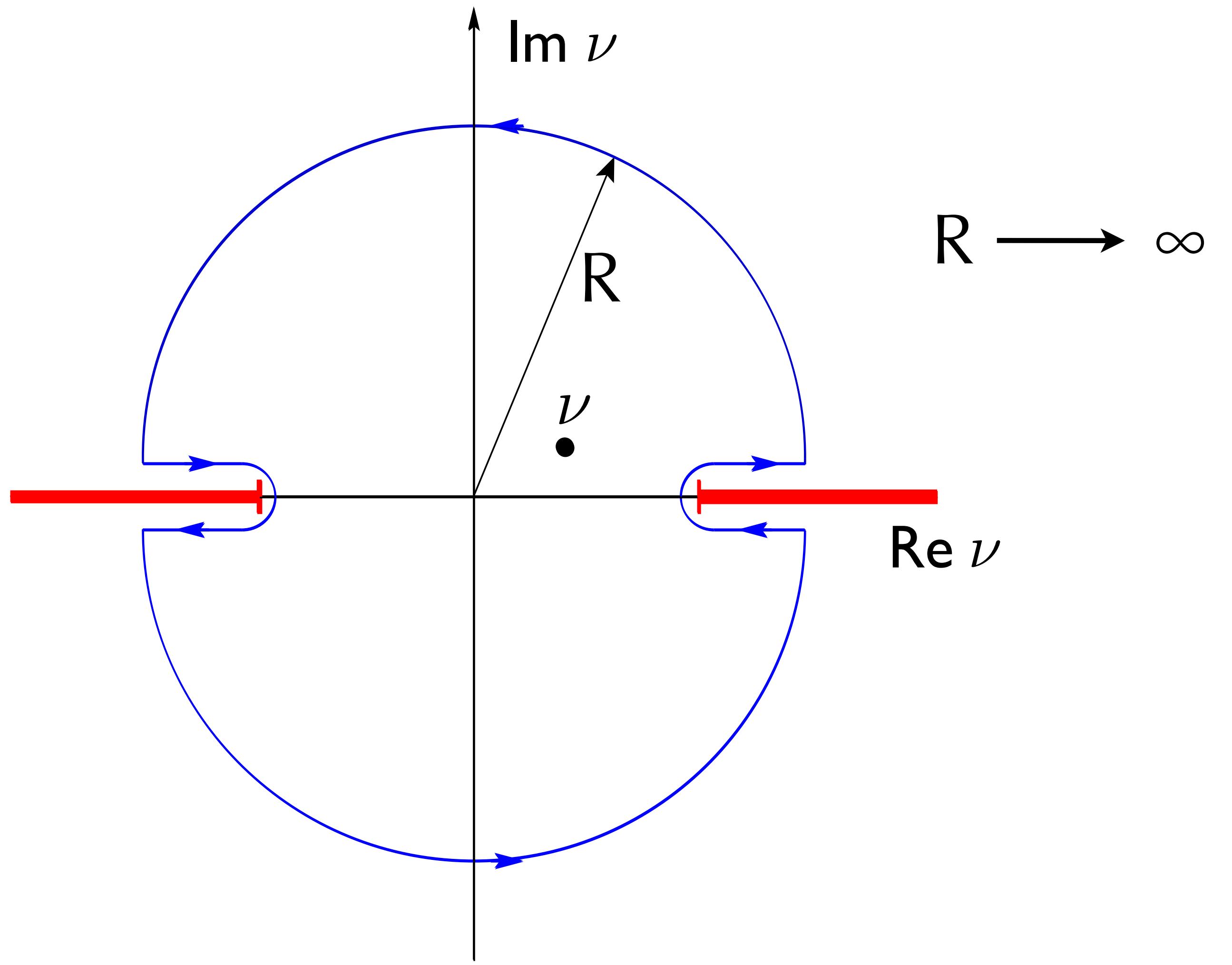

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Unsubtracted Dispersion Relations

$$\mathrm{Re}\, A_i(\nu, t, Q^2) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_i(\nu', t, Q^2) \frac{\nu' \mathrm{d}\nu'}{\nu'^2 - \nu^2} \qquad (i = 1, \ldots, 4)$$

non-convergent integrals

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↓

Subtracted Dispersion Relations

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = A_2(0, t, Q^2) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2) \frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

↓

subtraction at $\nu = 0$

Dispersion relations in terms of GPDs

energy variables $\longrightarrow$ $\nu = \frac{Q^2}{4M_N\xi}$ $\qquad$ $\nu' = \frac{Q^2}{4M_N x}$

once subtracted fixed-t DRs in the variable x

$$\mathrm{Re}\, A_2(\xi, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi}\mathcal{P}\int_0^1 \frac{dx}{x}\frac{\mathrm{Im}\, A_2(x, t, Q^2)}{(\xi^2/x^2 - 1)}$$

link to twist-2 GPDs: $\mathrm{Im}\, A_2(\xi, t, Q^2) = \pi E^+(x, \xi = x, t, Q^2)$

$$\mathrm{Re}\, A_2(\xi, t, Q^2) = \Delta(t, Q^2) - \mathcal{P}\int_0^1 dx E^+(x, x, t, Q^2)\left[\frac{1}{x-\xi} + \frac{1}{x+\xi}\right]$$

Anikin, Teryaev (2007); Kumericki-Passek, Mueller, Passek (2008); Diehl, Ivanov (2007);

Polyakov, Vanderhaeghen (2008); Goldstein, Liuti (2009)

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Subtraction Function

$$\Delta(t, Q^2) = -\mathcal{P}\int_0^1 \mathrm{d}x \left[E^+(x,\xi,t,Q^2) - E^+(x,x,t,Q^2)\right]\left[\frac{1}{x-\xi} + \frac{1}{x+\xi}\right]$$

Make use of:

- ξ-independence $\longrightarrow$ take $\xi = 0$
- Time-Reversal invariance
- Use representation of GPD with double distribution (E_{DD})+ D-term (E_D)
- Vanishing contribution from E_{DD}

$$\frac{E_D(x,0,t)}{x} = -\delta(x)\frac{1}{N_f}\int_{-1}^1 \mathrm{d}z\,\frac{D(z,t)}{z} \qquad \frac{E_D(x,x,t)}{x} = -\delta(x)\frac{1}{N_f}\int_{-1}^1 \mathrm{d}z\,\frac{D(z,t)}{z(1-z)}$$

Anikin, Teryaev (2007); Radyushkin (2012)

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$$\Delta(t,Q^2) = -\mathcal{P}\int_0^1 \mathrm{d}x\,[E^+(x,\xi,t,Q^2) - E^+(x,x,t,Q^2)]\left[\frac{1}{x-\xi} + \frac{1}{x+\xi}\right]$$

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$\Rightarrow$ $\boxed{\Delta(t,Q^2) = -\frac{4}{N_f}D(t,Q^2)}$ with $D(t,Q^2) = \frac{1}{2}\int_{-1}^1 \mathrm{d}z\,\frac{D(z,t,Q^2)}{1-z}$

Polyakov-Weiss D-term

Anikin, Teryaev (2007); Radyushkin (2012)