Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi}\nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2) \frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi}\nu^2 \mathcal{P}\int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2)\frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

- t-channel DRs for subtraction function

$$\Delta(t, Q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t} + \frac{1}{\pi}\int_{-\infty}^{-a} \mathrm{d}t' \frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t}$$

$$\downarrow$$

$$-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$$

Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi}\nu^2 \mathcal{P}\int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2)\frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

- t-channel DRs for subtraction function

$$\Delta(t, Q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t} + \frac{1}{\pi}\int_{-\infty}^{-a} \mathrm{d}t' \xcancel{\frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t}}$$

Dispersion Relations for DVCS amplitudes

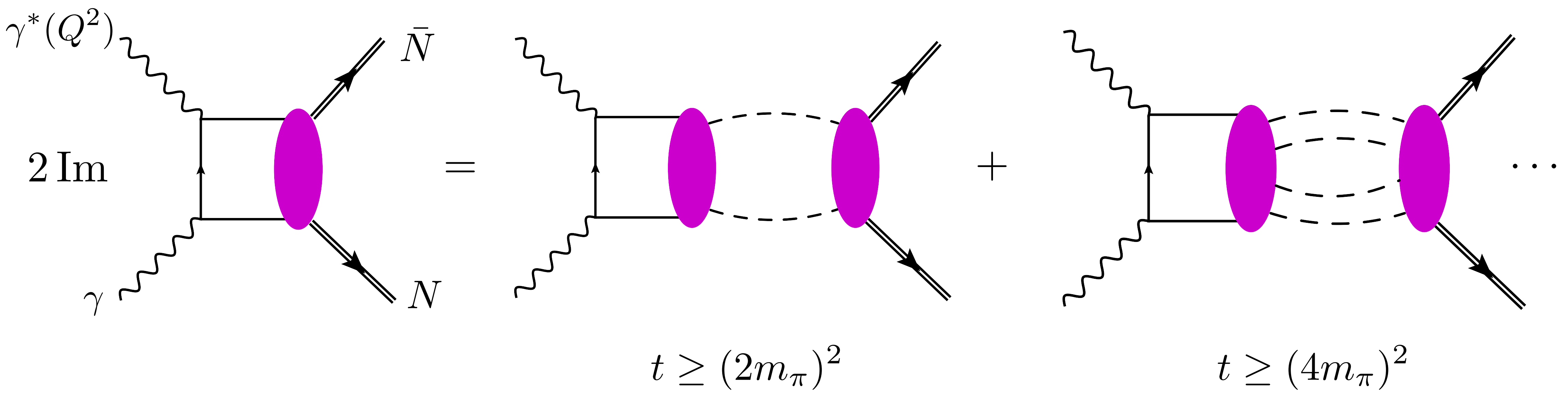
- s-channel subtracted DRs:

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi}\nu^2 \mathcal{P}\int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2)\frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

- t-channel DRs for subtraction function

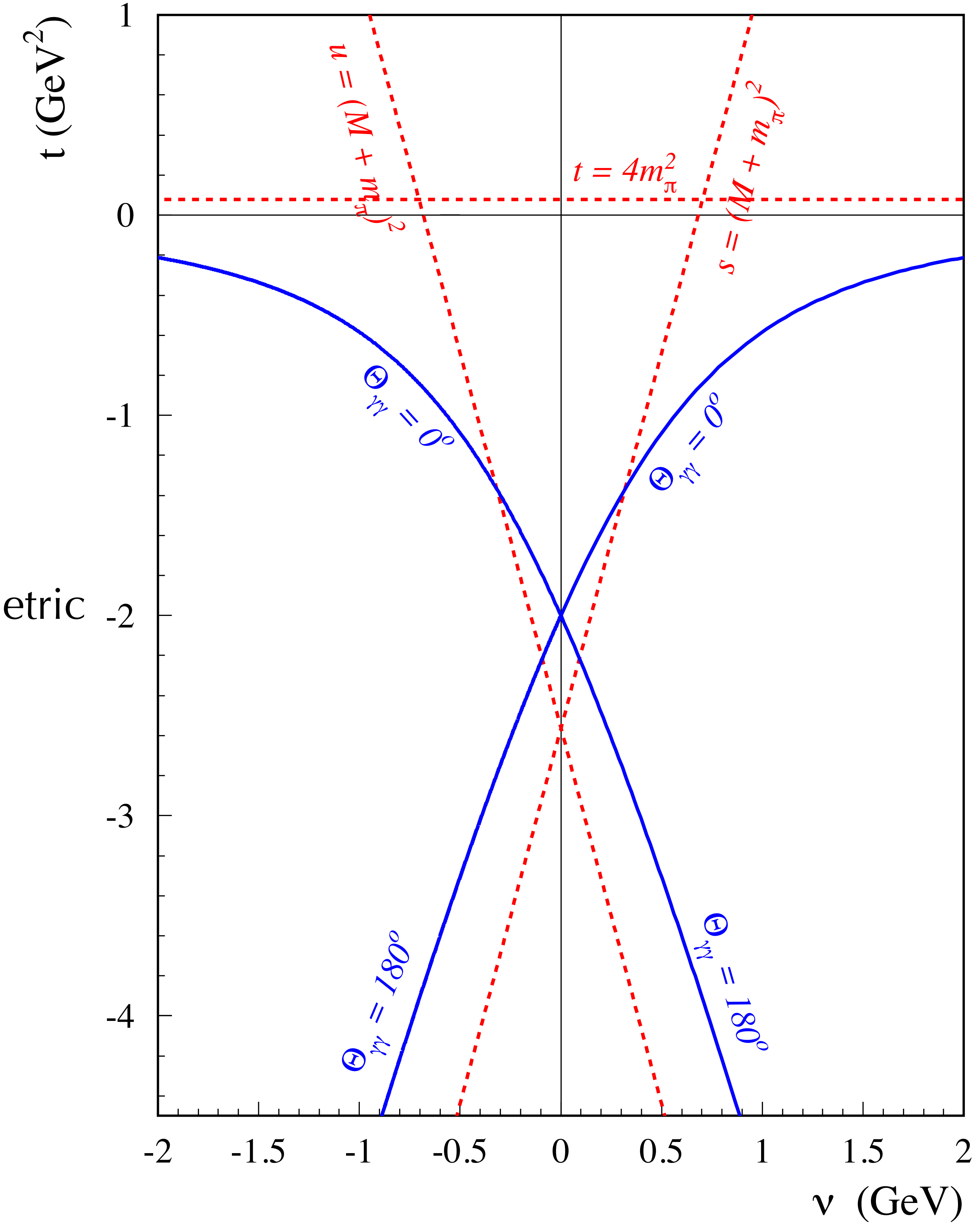
$$\Delta(t, Q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t} + \frac{1}{\pi}\int_{-\infty}^{-a} \mathrm{d}t' \xcancel{\frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t}}$$

Unitarity relation in t-channel



Fixed

$Q^2 = 2\,\mathrm{GeV}^2$

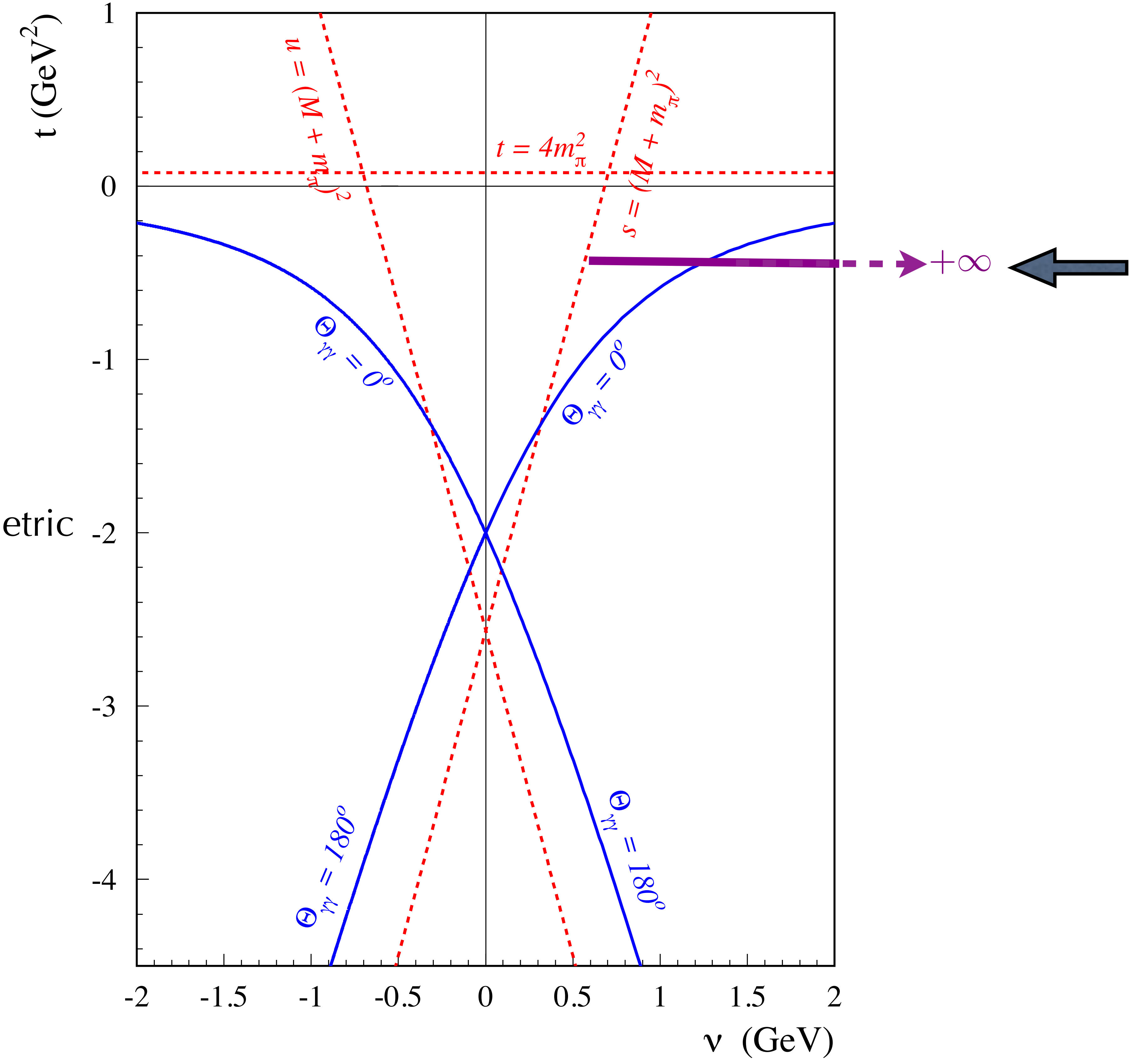


$\nu = \frac{s-u}{4M_N}$

crossing symmetric variable

Fixed

$Q^2 = 2\,\mathrm{GeV}^2$

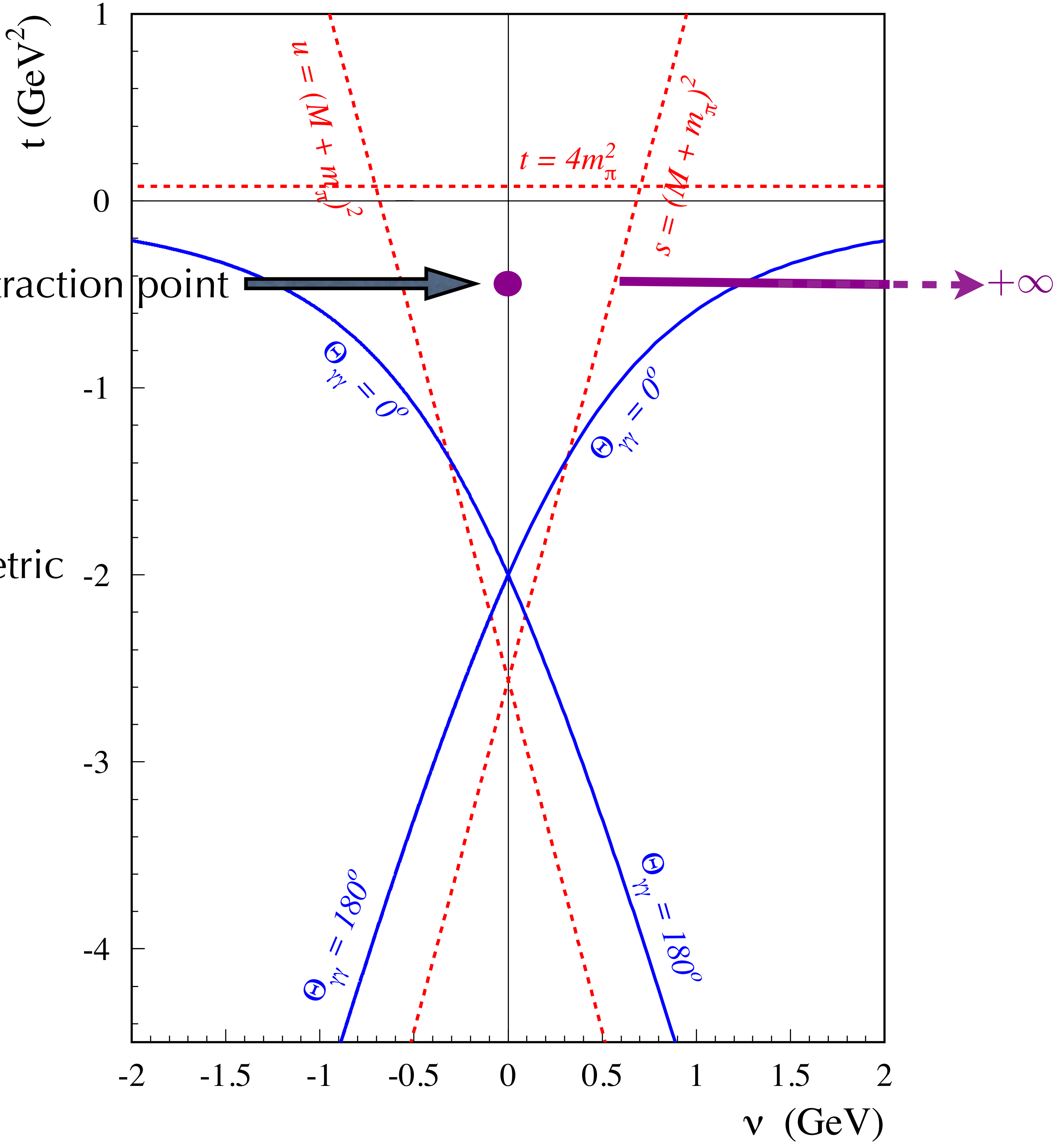


DRs
in the s-channel
at fixed t

$\nu = \frac{s-u}{4M_N}$

crossing symmetric
variable

Fixed
$Q^2 = 2\,\mathrm{GeV}^2$



Subtraction point

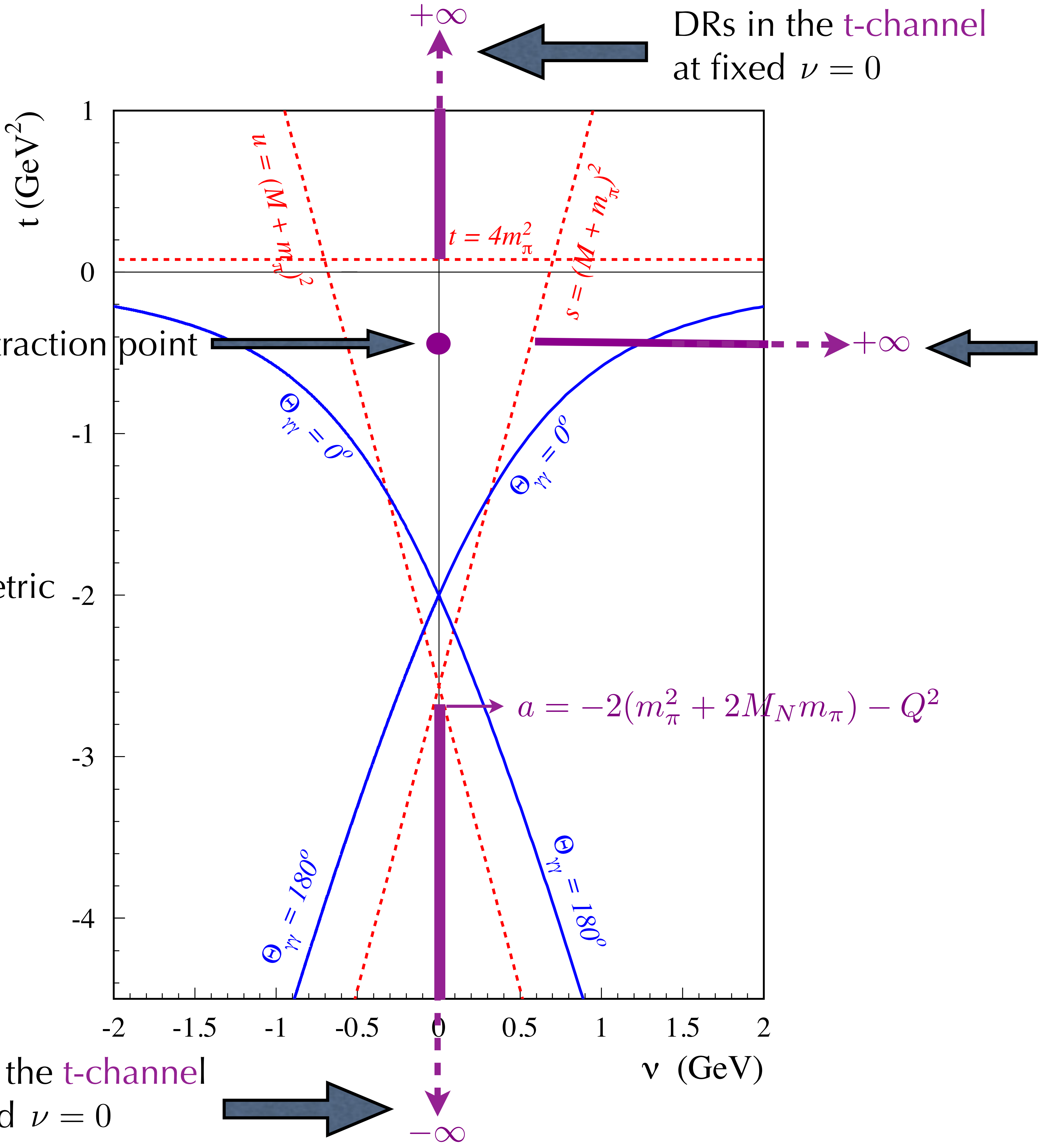
DRs
in the s-channel
at fixed t

$\nu = \frac{s-u}{4M_N}$

crossing symmetric
variable

Fixed

$Q^2 = 2\,\mathrm{GeV}^2$



DRs in the t-channel
at fixed $\nu = 0$

DRs
in the s-channel
at fixed t

Subtraction point

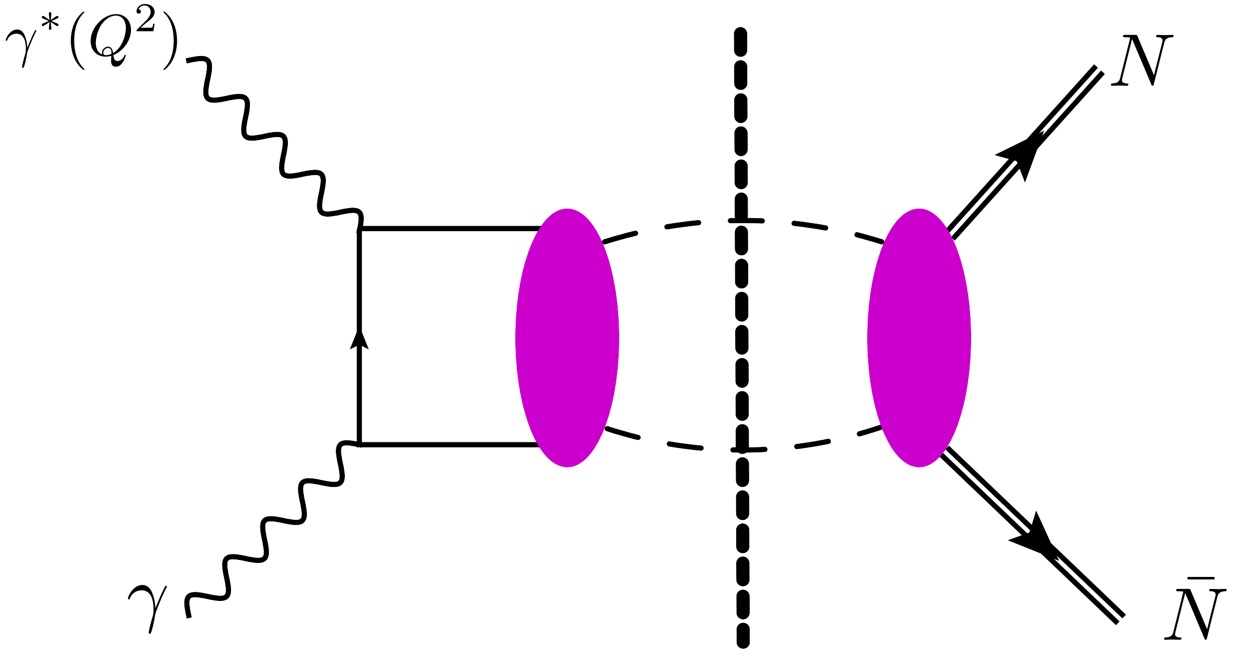
$a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$

$\nu = \frac{s-u}{4M_N}$

crossing symmetric
variable

DRs in the t-channel
at fixed $\nu = 0$

Unitarity Relations in the t-channel



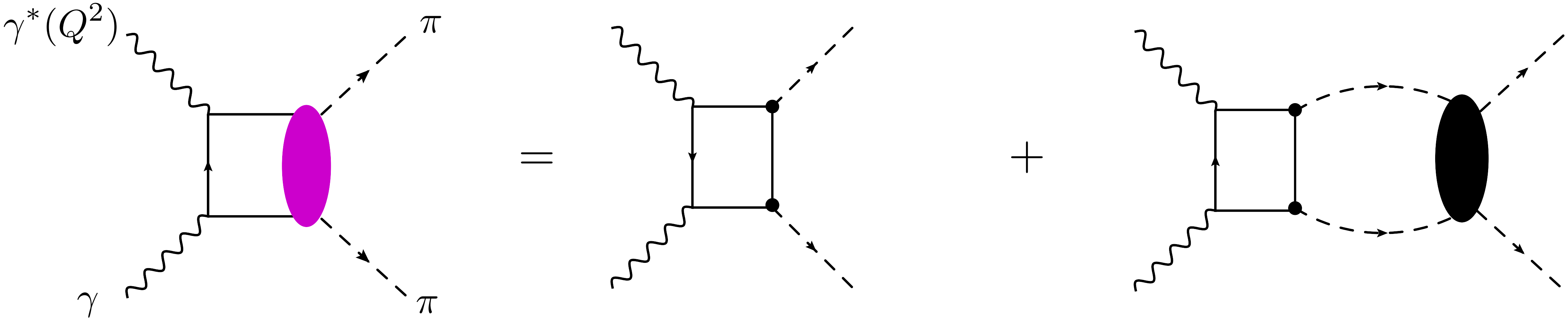
- Charge conjugation

- Partial wave expansion with $\nu = 0 \rightarrow \theta_t = 90^o$

two-pion intermediate state with

$I = 0 \qquad J = 0, 2, \cdots$

Two-Pion Generalized Distribution Amplitude (GDA)



Two-pion intermediate states with $I = 0$ and $J = 0,\ 2$

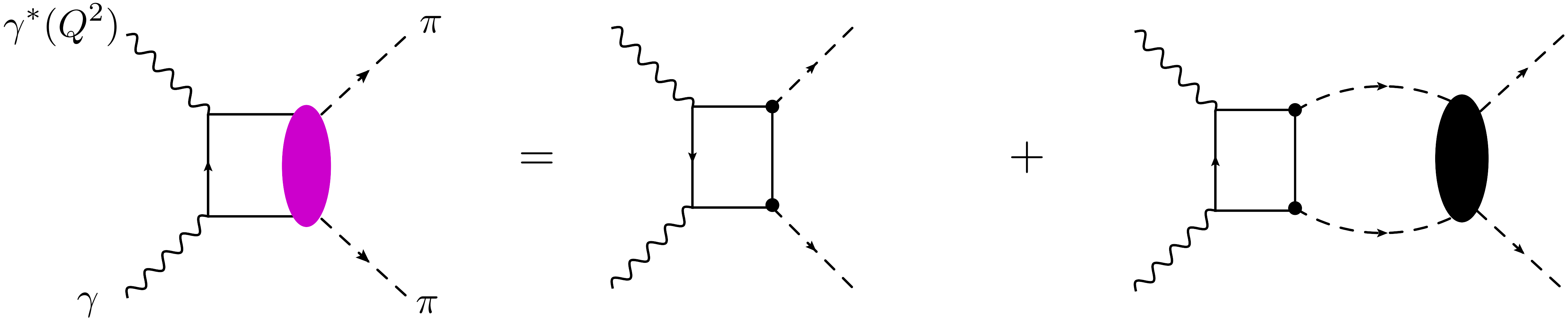
$$\Delta(t, Q^2) = \frac{4}{N_f} \sum_{n\ \mathrm{odd}} d_n(t, Q^2) \longrightarrow \text{only } d_1(t, Q^2)$$

input ⇒ $\pi\pi$ phase shifts

pion PDFs

M. Polyakov, 1999

Two-Pion Generalized Distribution Amplitude (GDA)



Two-pion intermediate states with $I = 0$ and $J = 0,\ 2$

$$\Delta(t, Q^2) = \frac{4}{N_f} \sum_{n\ \mathrm{odd}} d_n(t, Q^2) \longrightarrow \text{only } d_1(t, Q^2)$$

input

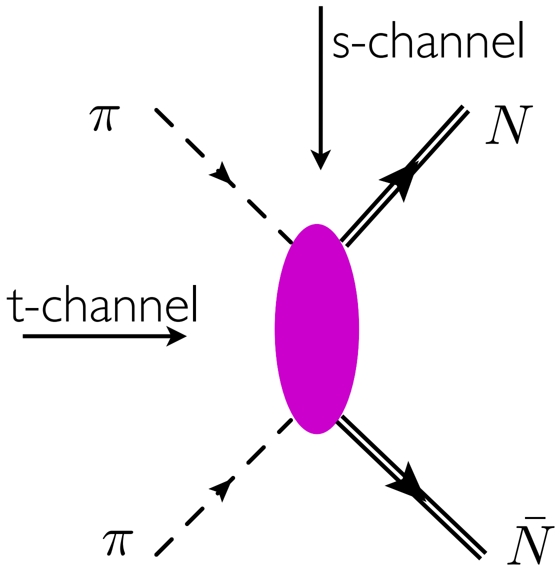
$\pi\pi$ phase shifts

pion PDFs

M. Polyakov, 1999

$\pi\pi \to N\bar{N}$ scattering amplitude

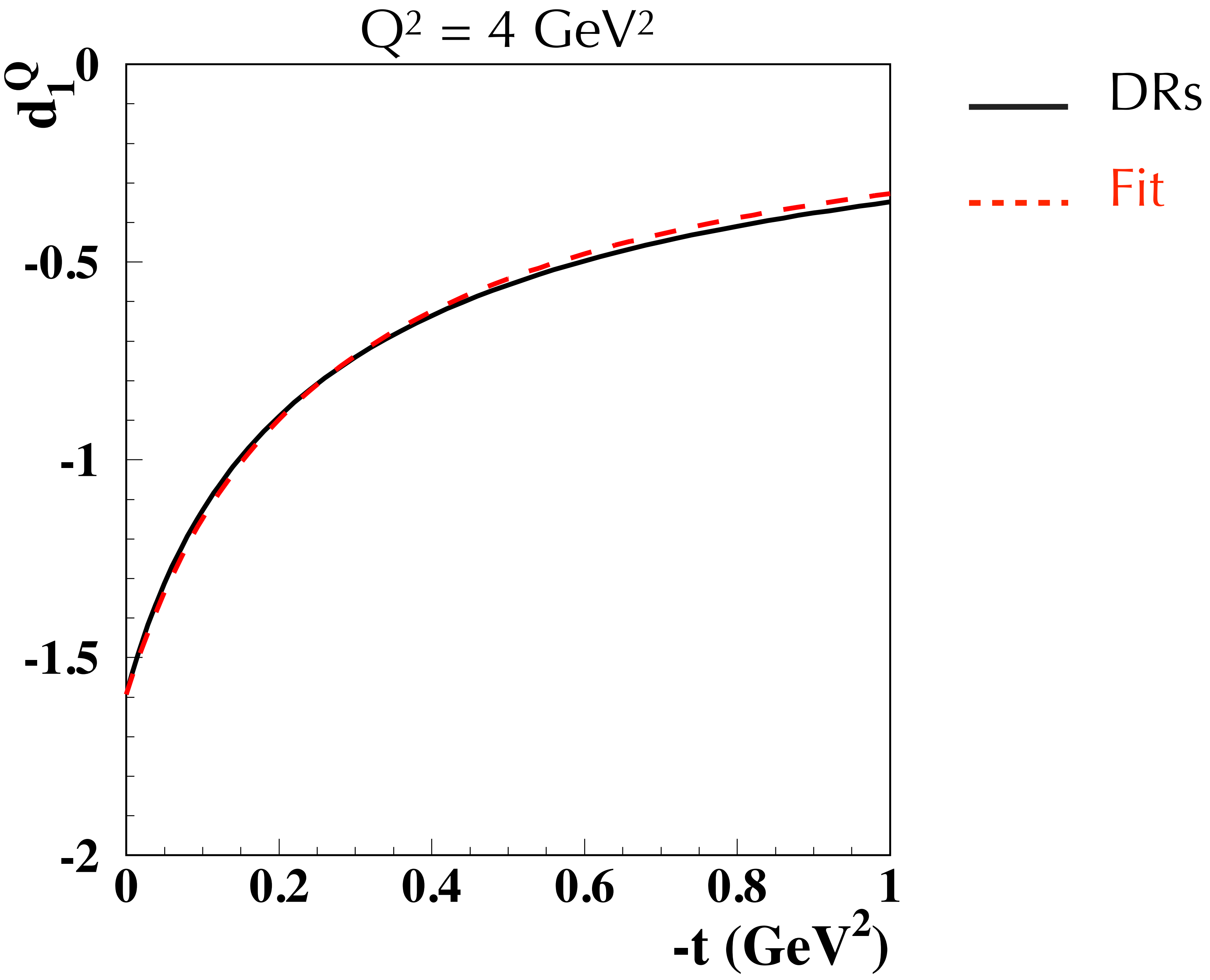
analytical continuation of s-channel helicity amplitudes



G. Hoehler, 1983

D-term form factor: t-dependence

Q= u + d

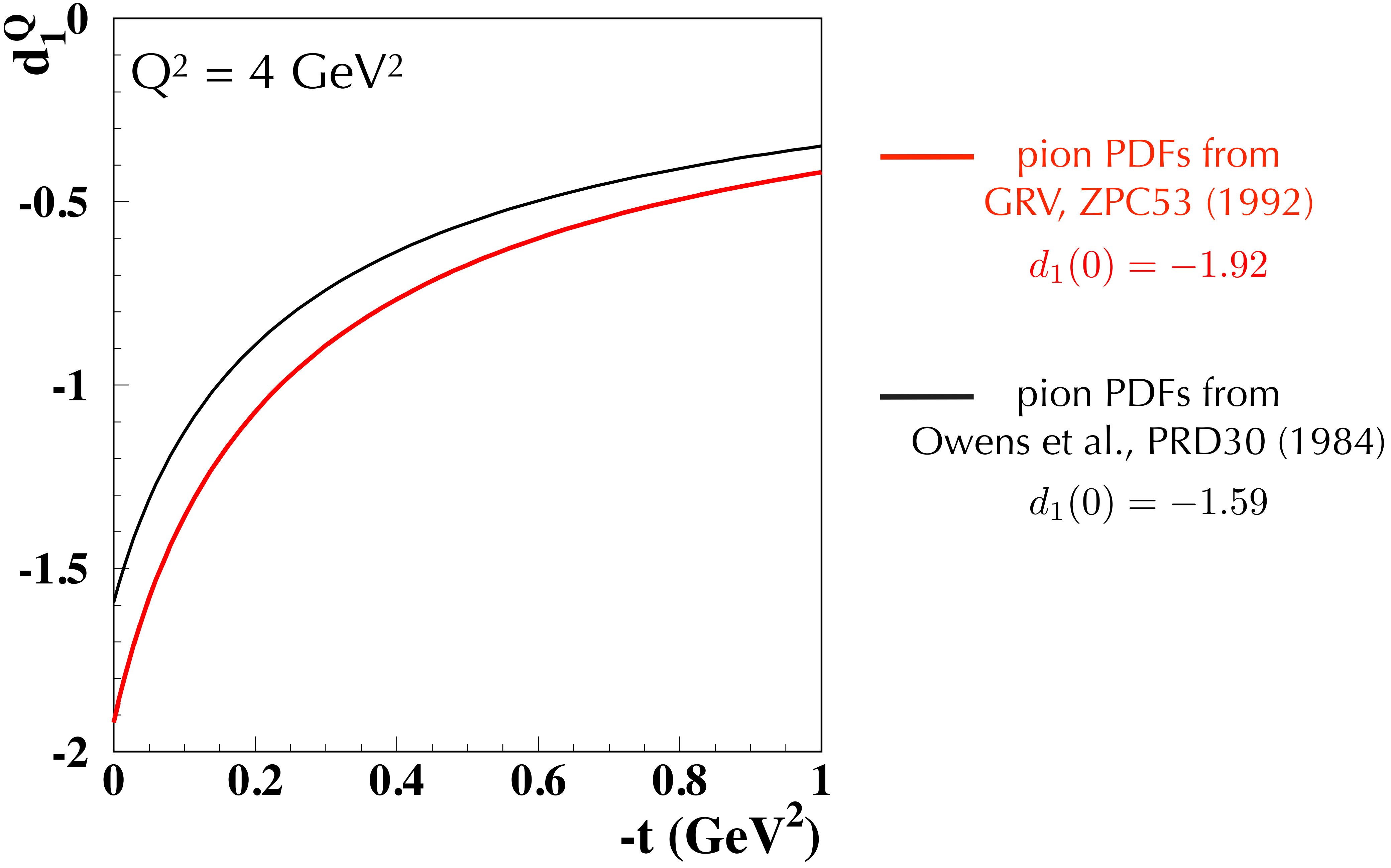


Fit: $F^Q(t) = \dfrac{d_1^Q(0)}{[1 - t/(\alpha M_D^2)]^\alpha}$ with $M_D = 0.487\,\mathrm{GeV}$

$\alpha = 0.841$

D-term form factor: dependence on pion PDFs

Q= u + d



χQSM

$d_1^Q(0) = -2.35$

Schweitzer et al., (2007)

Skyrme model

$d_1^Q(0) = -4.48$

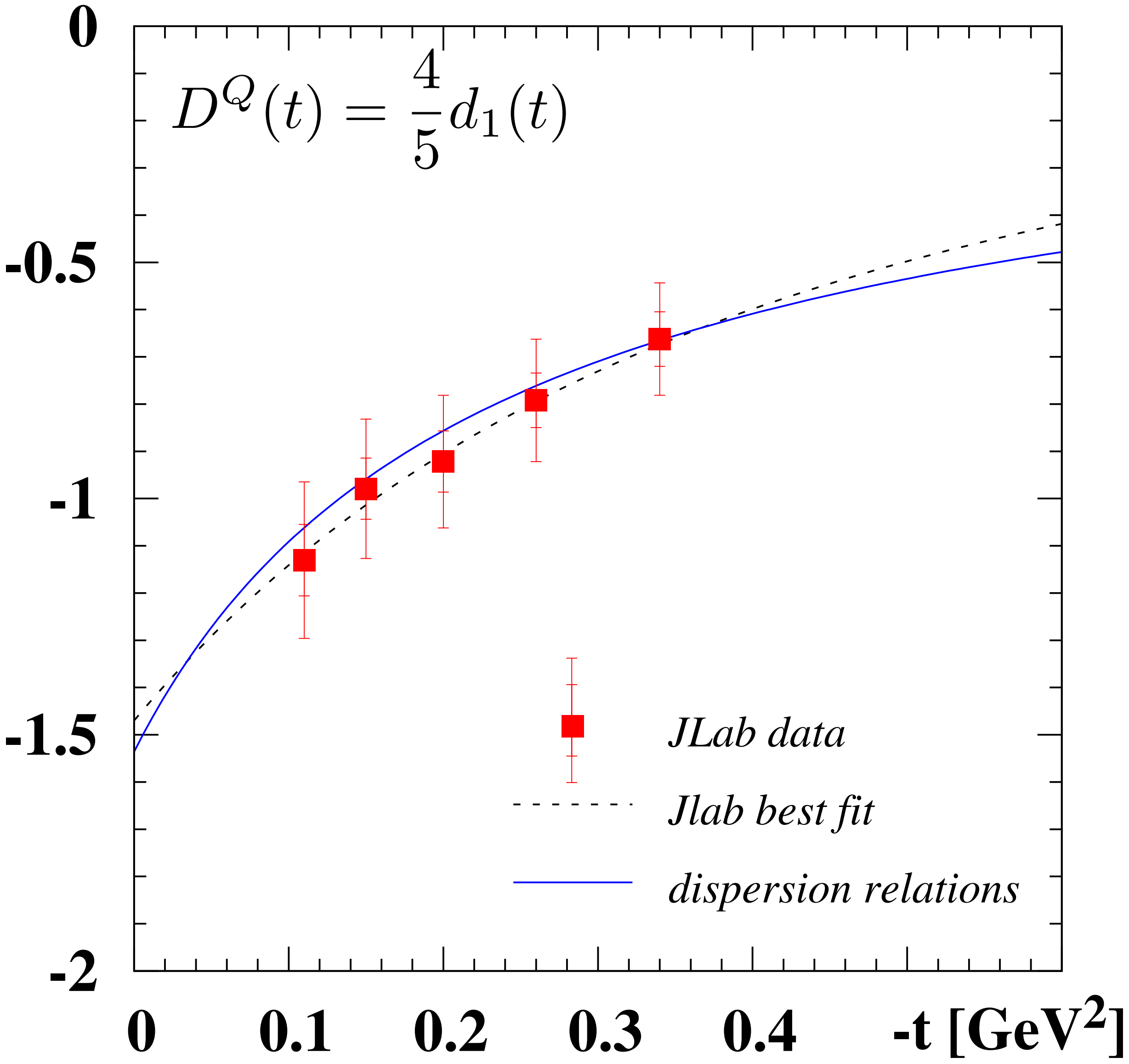
Schweitzer et al., (2007)

Effective LFWFs

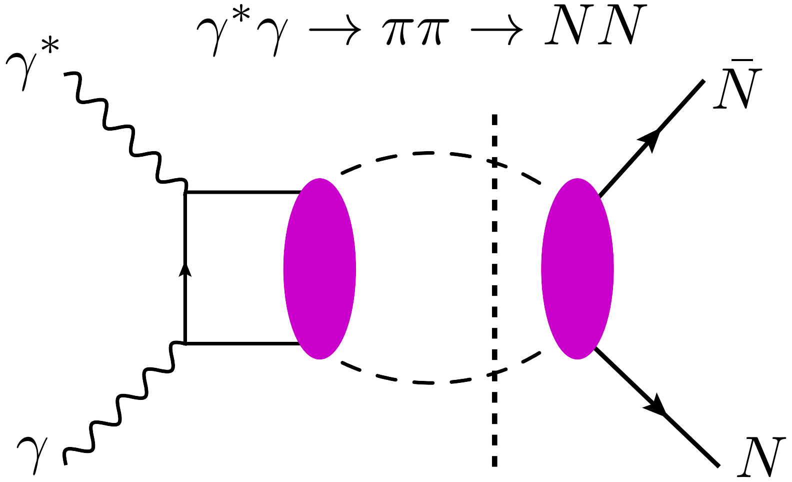
$d_1^Q(0) = -2.01$

Mueller and Hwang, (2014)

D-term form factor and radial pressure distribution



D-term from t-channel dispersion relations



Extraction from data:

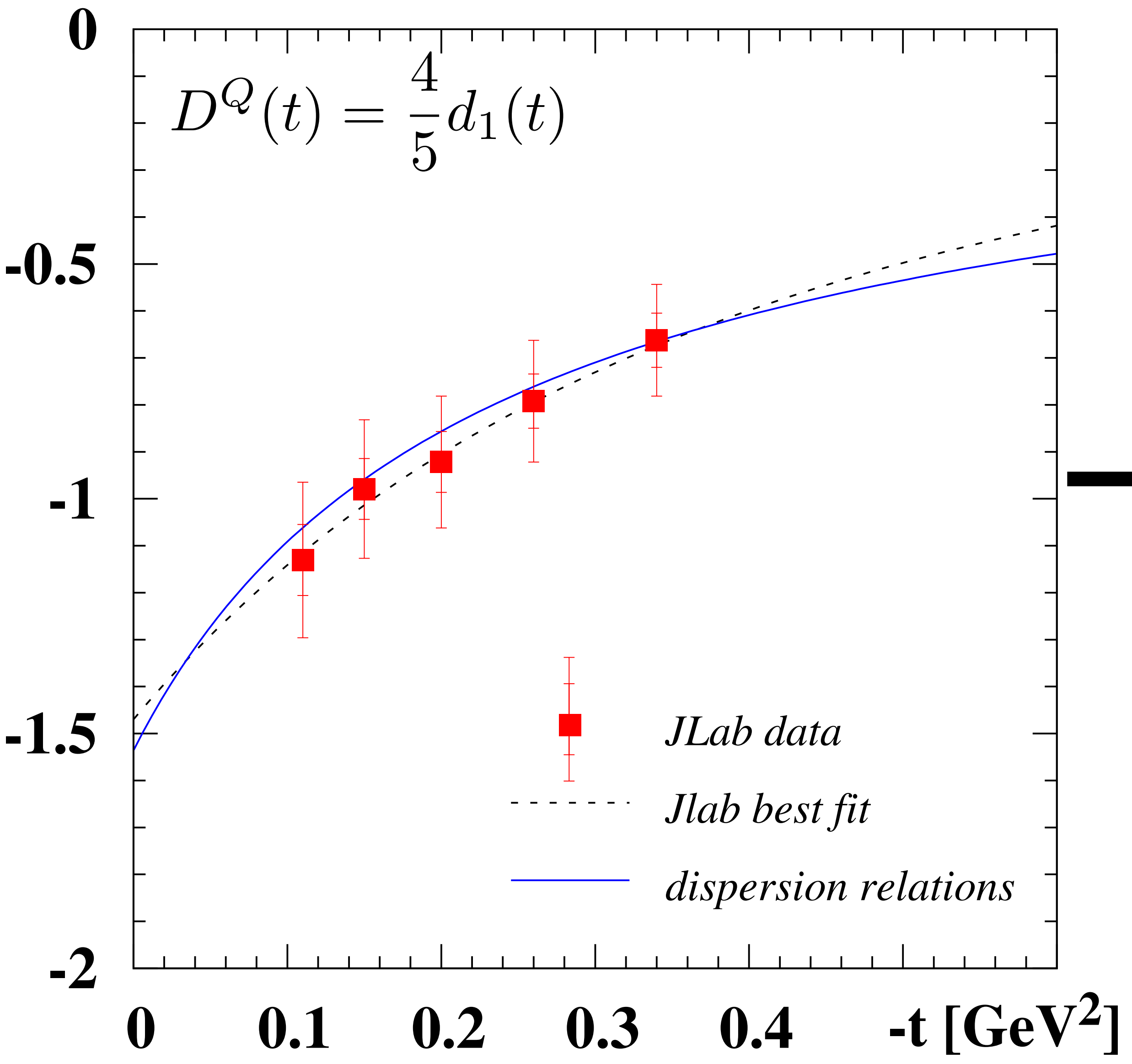
- neglecting gluon contribution
- assuming:

$$\Delta(t, Q^2) = \frac{4}{N_f} \sum_{n \text{ odd}} d_n(t, Q^2) \longrightarrow \text{only } d_1(t, Q^2)$$

BP, Polyakov, Vanderhaeghen, PLB 739 (2014) 133

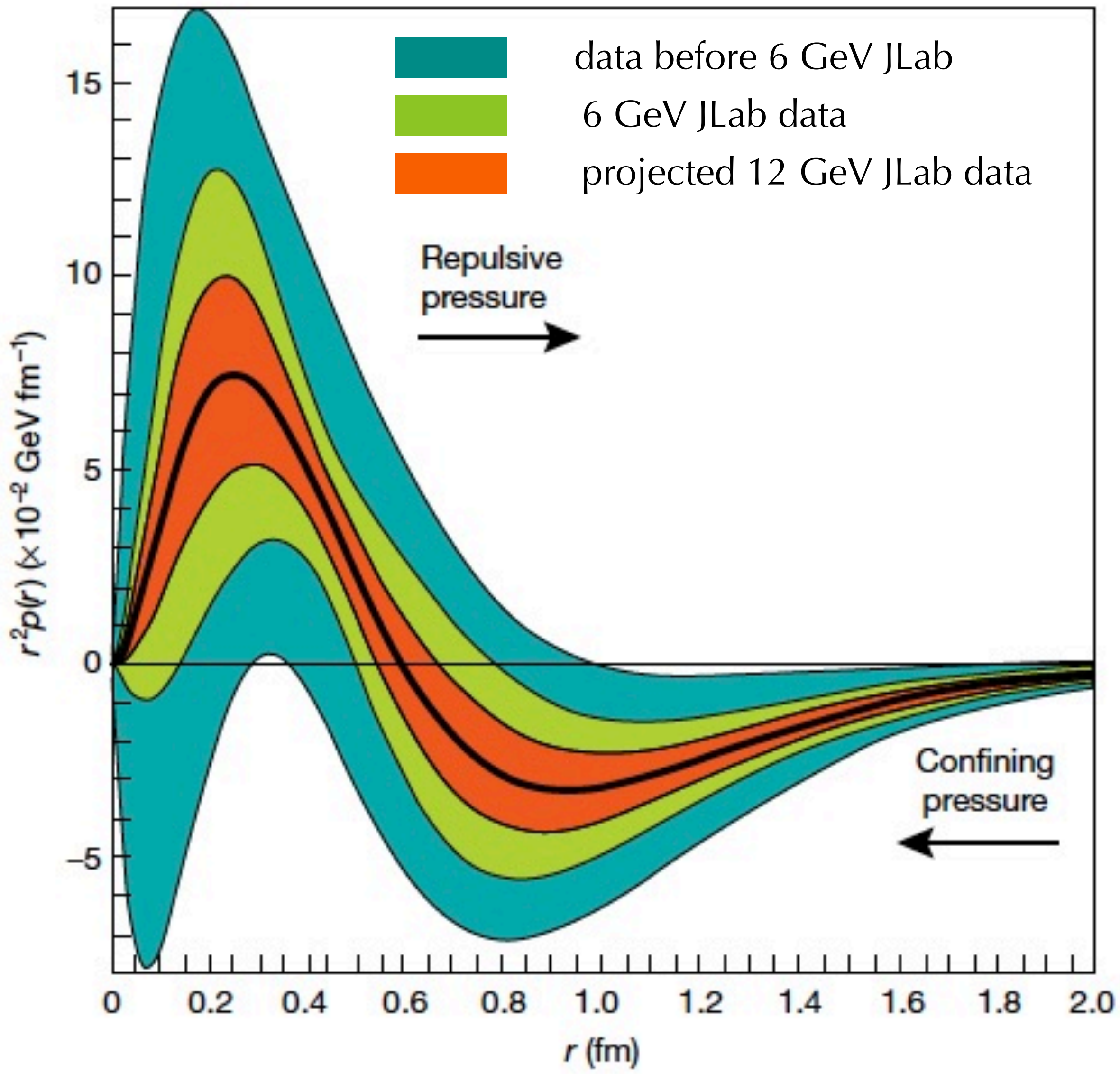
Girod, Elouadrhiri, Burkert,Nature 557 (2018) 7705

D-term form factor and radial pressure distribution



$$\xrightarrow{\text{FT}}$$

$r^2 p(r)$ radial pressure distribution



$$T_{ij}(\vec{r}) = s(\vec{r}) \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij} \right) + p(\vec{r})\delta_{ij}$$

↓ shear forces

↓ pressure

neglecting gluon contribution:

$$r^2 p(r) = \frac{1}{3} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$

$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$

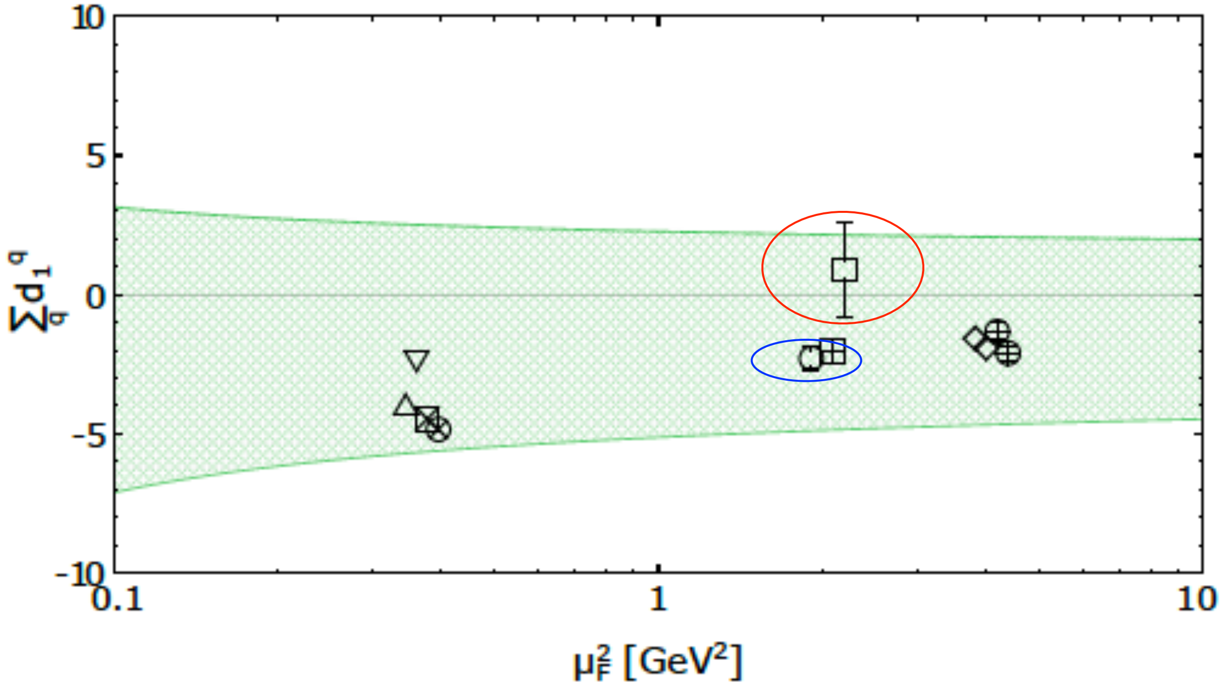
BP, Polyakov, Vanderhaeghen, PLB 739 (2014) 133

Girod, Elouadrhiri, Burkert,Nature 557 (2018) 7705

Necessary to verify model assumptions in the exp extraction with more data coming from JLab, COMPASS and the future EIC, EIcC

Kumericki, Nature 570 (2019) 7759; Dutrieux et al, Eur. Phys. J. C81 (2021) 4

▨ global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param., Girod et al.

CLAS data, with neural networks Kumericki

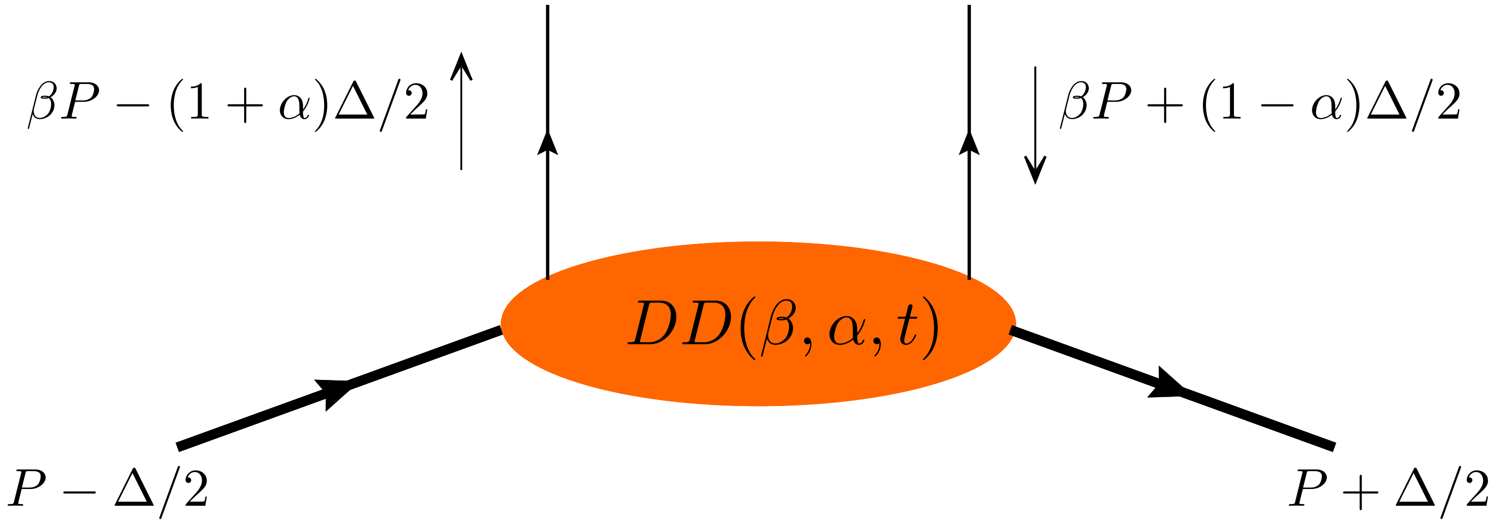
$$\sum_q d_1^q < 0$$

in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
□	0.88 ± 1.69	2.2	2	from experimental data
◇	-1.59	4	2	*t*-channel saturated model
	-1.92	4	2	*t*-channel saturated model
△	-4	0.36	3	χQSM
▽	-2.35	0.36	2	χQSM
⊠	-4.48	0.36	2	Skyrme model
⊞	-2.02	2	3	LFWF model
⊗	-4.85	0.36	2	χQSM
⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)

Parametrizations of GPDs

GPDs from Double Distributions (DDs)



$$\langle p'|\bar{q}(-\frac{1}{2}z)\not{z}\,q(\frac{1}{2}z)|p\rangle|_{z^2=0}$$

$\Delta = 0$

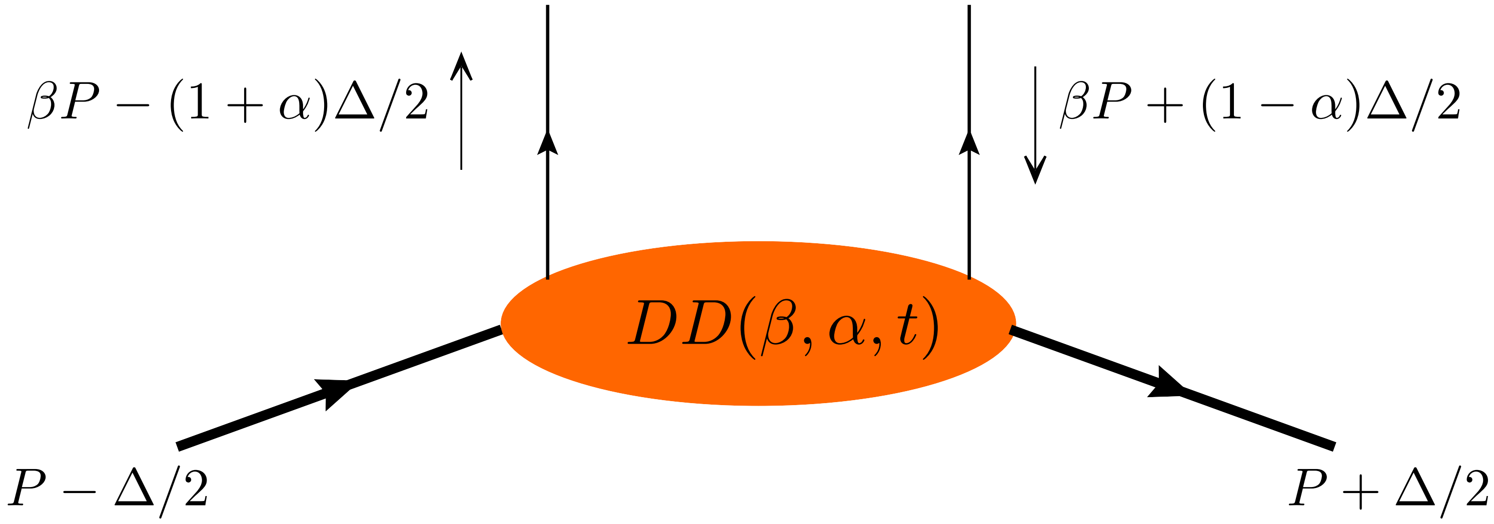
forward matrix element
defining the collinear PDFs

$P = 0$

probability amplitude of finding
$q\bar{q}$ pair in the nucleon

[Mueller et al., 94; Radyushkin 97, Polyakov,Weiss 99]

GPDs from Double Distributions (DDs)

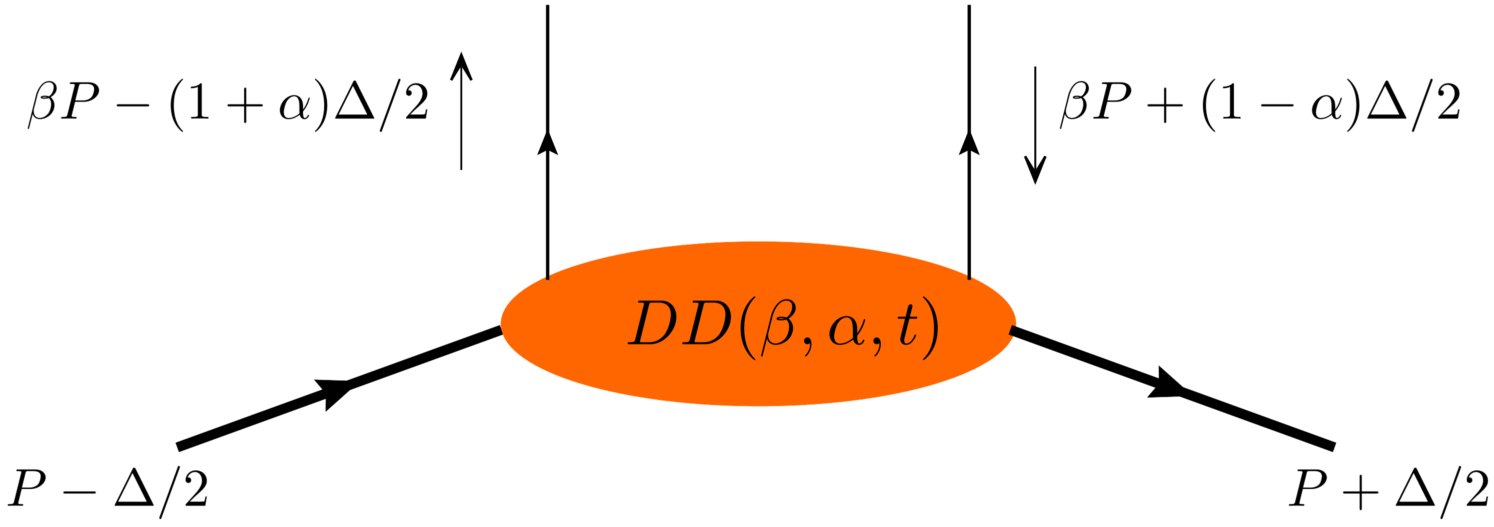


$$\langle p'|\bar{q}(-\frac{1}{2}z)\not{z}q(\frac{1}{2}z)|p\rangle|_{z^2=0}$$

$$\begin{aligned}\langle p'|\bar{q}(-\frac{1}{2}z)\not{z}q(\frac{1}{2}z)|p\rangle|_{z^2=0} &= \bar{u}(p')\not{z}u(p)\int d\beta d\alpha\, e^{-i\beta(Pz)+i\alpha(\Delta z)/2}f^q(\beta,\alpha,t) \\ &+\bar{u}(p')\frac{i\sigma^{\mu\alpha}z_\mu\Delta_\alpha}{2M}u(p)\int d\beta d\alpha\, e^{-i\beta(Pz)+i\alpha(\Delta z)/2}k^q(\beta,\alpha,t) \\ &-\bar{u}(p')\frac{\Delta\cdot z}{2M}u(p)\int d\beta d\alpha\, e^{i\alpha(\Delta z)/2}D^q(\alpha,t)\end{aligned}$$

[Mueller et al., 94; Radyushkin 97, Polyakov,Weiss 99]

GPDs from Double Distributions (DDs)



$$\langle p'|\bar{q}(-\frac{1}{2}z)\not{z}q(\frac{1}{2}z)|p\rangle|_{z^2=0}$$

$$\begin{aligned}\langle p'|\bar{q}(-\frac{1}{2}z)\not{z}q(\frac{1}{2}z)|p\rangle|_{z^2=0} &= \bar{u}(p')\not{z}u(p)\int d\beta d\alpha\, e^{-i\beta(Pz)+i\alpha(\Delta z)/2}f^q(\beta,\alpha,t)\\ &+\bar{u}(p')\frac{i\sigma^{\mu\alpha}z_\mu\Delta_\alpha}{2M}u(p)\int d\beta d\alpha\, e^{-i\beta(Pz)+i\alpha(\Delta z)/2}k^q(\beta,\alpha,t)\\ &-\bar{u}(p')\frac{\Delta\cdot z}{2M}u(p)\int d\beta d\alpha\, e^{i\alpha(\Delta z)/2}D^q(\alpha,t)\end{aligned}$$

⟶ take $z^+ = \vec{z}_\perp = 0$ and compare with GPD correlator

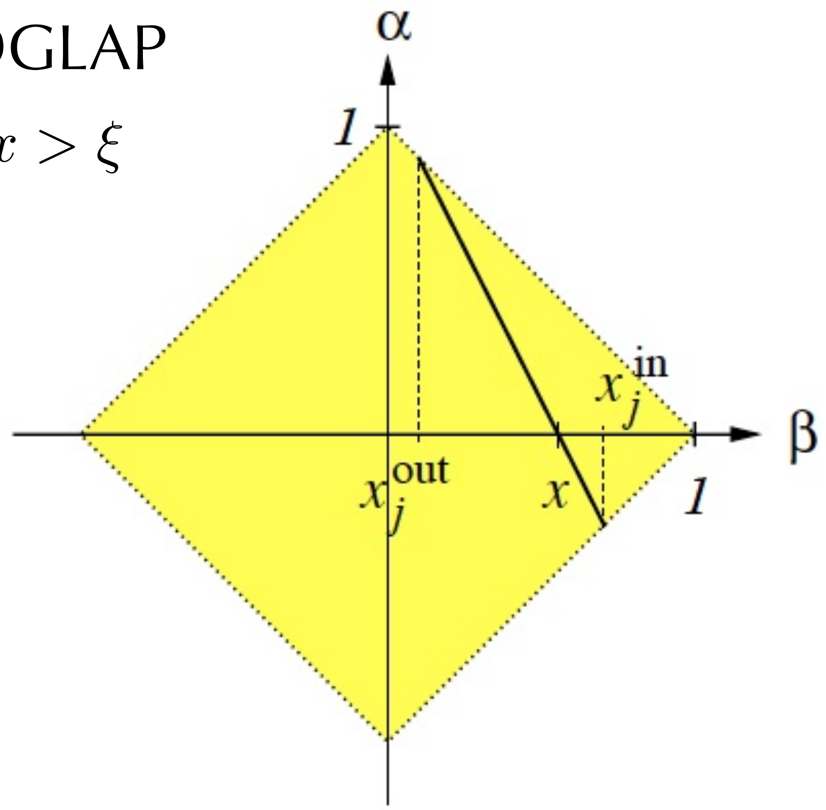
[Mueller et al., 94; Radyushkin 97, Polyakov,Weiss 99]

$$\mathrm{GPD}^q(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \delta(x-\beta-\xi\alpha)\mathrm{DD}(\alpha,\beta,t)$$ + D-term for GPDs H and E

- Support region: $|\beta| + |\alpha| \leq 1$

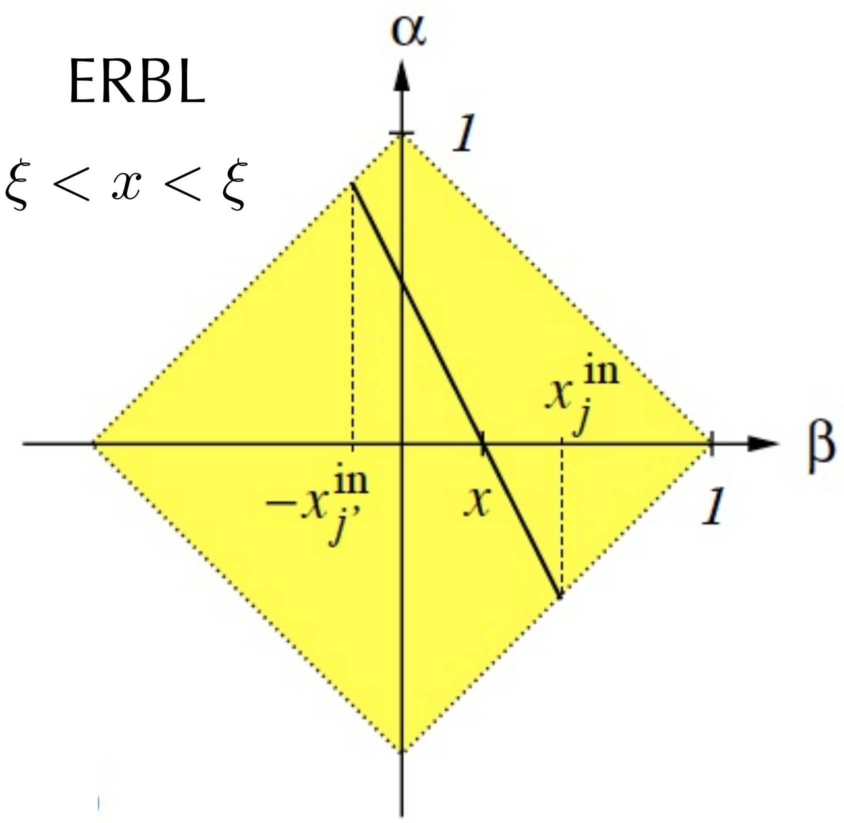
DGLAP

$x > \xi$



ERBL

$-\xi < x < \xi$



Model for GPDs from DDs

$$H(x,\xi,t) = \int \mathrm{d}\beta \int \mathrm{d}\alpha\, \delta(x-\beta-\xi\alpha) f(\beta,\alpha,t) + \theta(\xi - |x|)\frac{1}{N_f} D\left(\frac{x}{\xi},t\right)$$

- Ansatz for DD which smoothly interpolates between a PDF and DA

$f(\beta,\alpha) = h(\beta,\alpha)\, H(\beta,0,t)$ with $\quad h(\beta,\alpha) \propto \dfrac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$

b free parameter which governs the ξ dependence

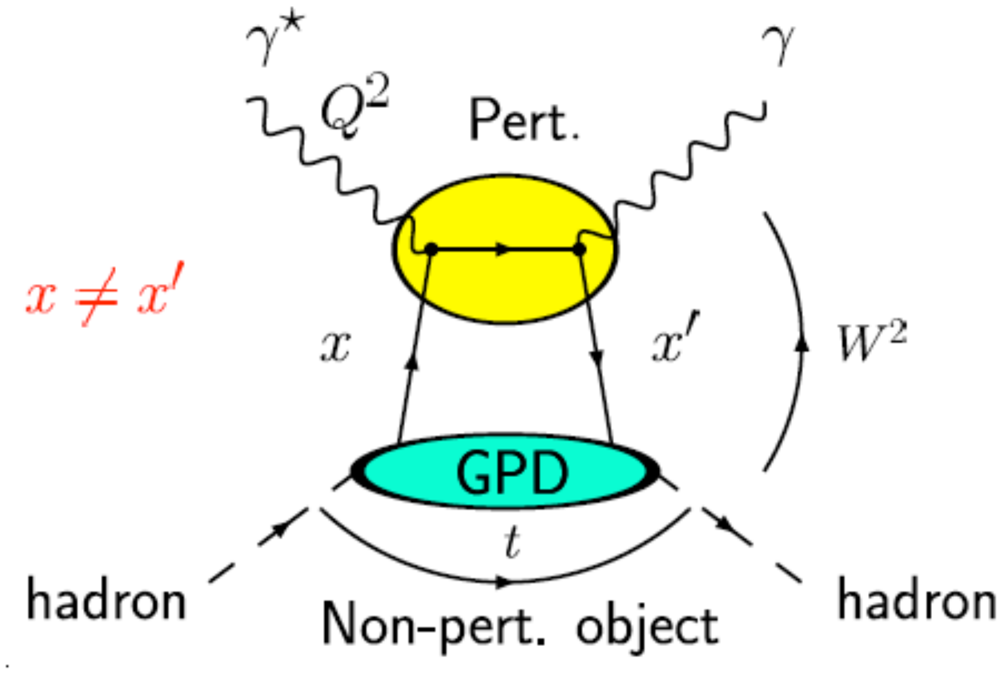
- forward limit: $\int \mathrm{d}\alpha\, f(\beta,\alpha,0) = H(\beta,0,0) = f_1(\beta)$

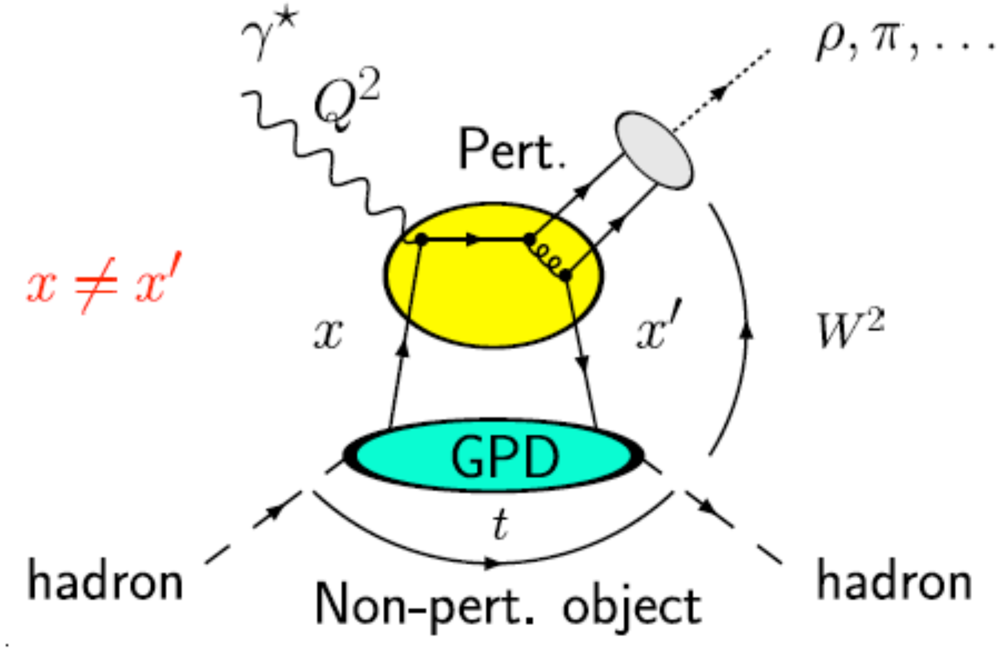
- t-dependence to reproduce behaviour of e.m. form factor

- D-term fitted to data (assuming a dipole functional form with adjustable mass parameter)

[Radyushkin, PRD59, 014030 (1998); PLB 449, 81 (1999)]

How to measure GPDs





- accessible in exclusive reactions
- factorization for large Q^2, $|t| << Q^2$, W^2
- depend on 3 variables: x, ξ, t

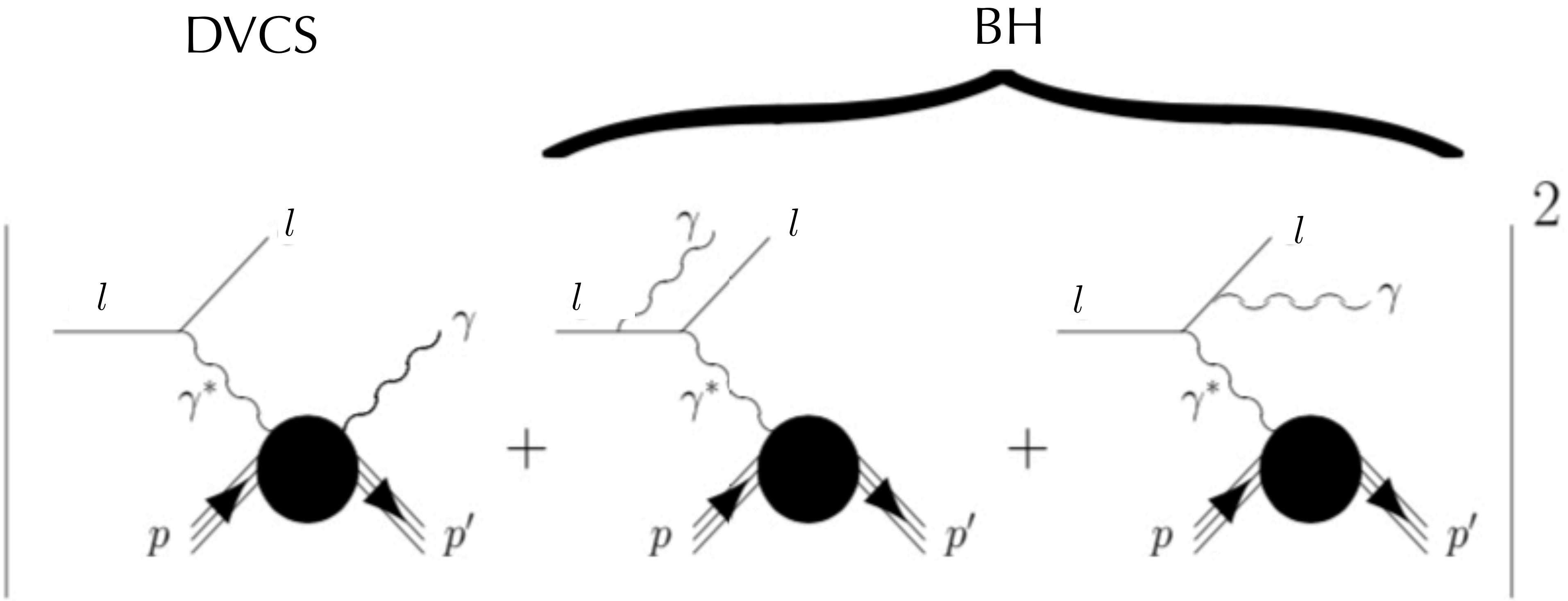
Compton Form Factors

$$\mathrm{Im}\,\mathcal{H}(\xi, t) \overset{\mathrm{LO}}{=} H(\xi, \xi, t)$$

$$\mathrm{Re}\,\mathcal{H}(\xi, t) \overset{\mathrm{LO}}{=} \mathcal{P} \int_{-1}^{1} \mathrm{d}x\ \mathrm{H}(x, \xi, t) \frac{1}{x-\xi}$$

Golden channel: deeply virtual Compton scattering

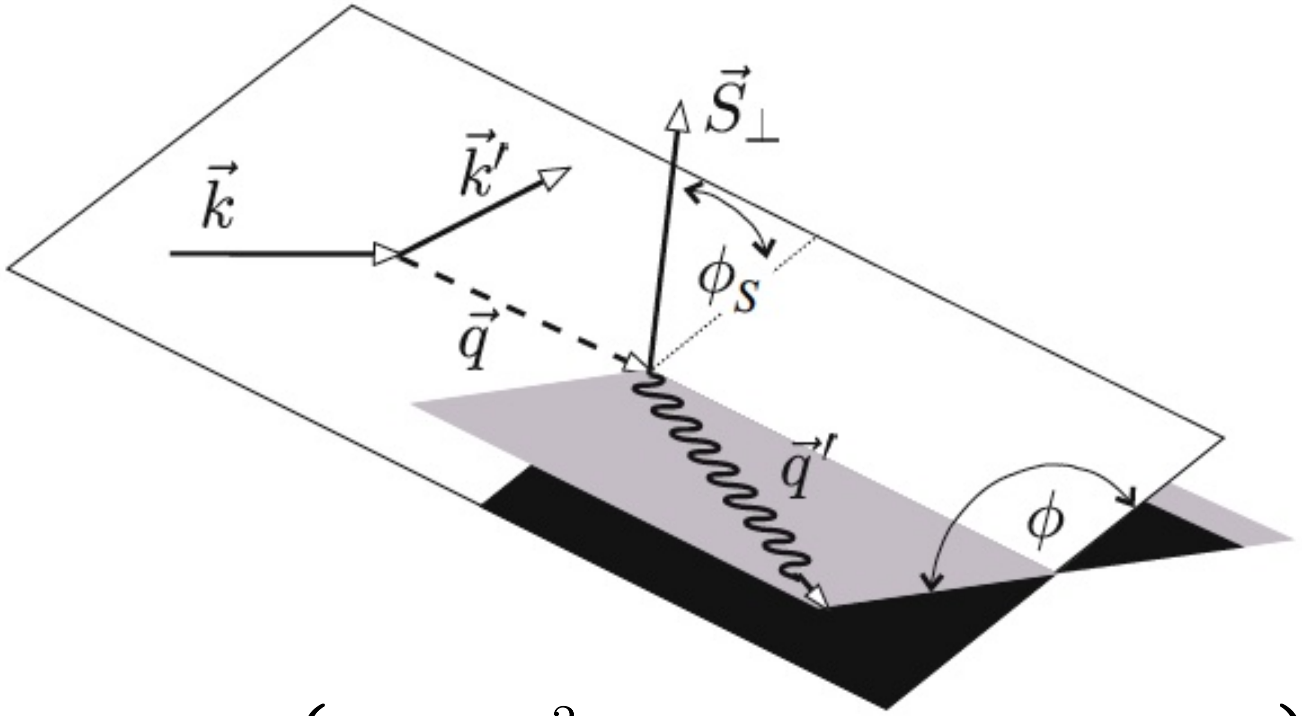
$\sigma(lp \to l\gamma p) \propto$



$$\sigma(lp \to l\gamma p) \propto |\mathcal{T}^{\mathrm{BH}}|^2 + |\mathcal{T}^{\mathrm{DVCS}}|^2 + e_l\,\mathcal{I}$$

- BH: calculable in QED with ~1% knowledge of e.m. at low momentum transfer
- |DVCS|^2: bilinear in GPDs
- $\mathcal{I}(\mathrm{BH} \cdot \mathrm{DVCS})$: linear combination of GPDs

DVCS cross section



Filter out interference term using cross section dependence on

- beam charge
- azimuth
- beam polarization
- target polarization

$$|\mathcal{T}_{\mathrm{BH}}|^2 \propto \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) + s_1^{\mathrm{BH}} \sin\phi \right\}$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 \propto \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right\}$$

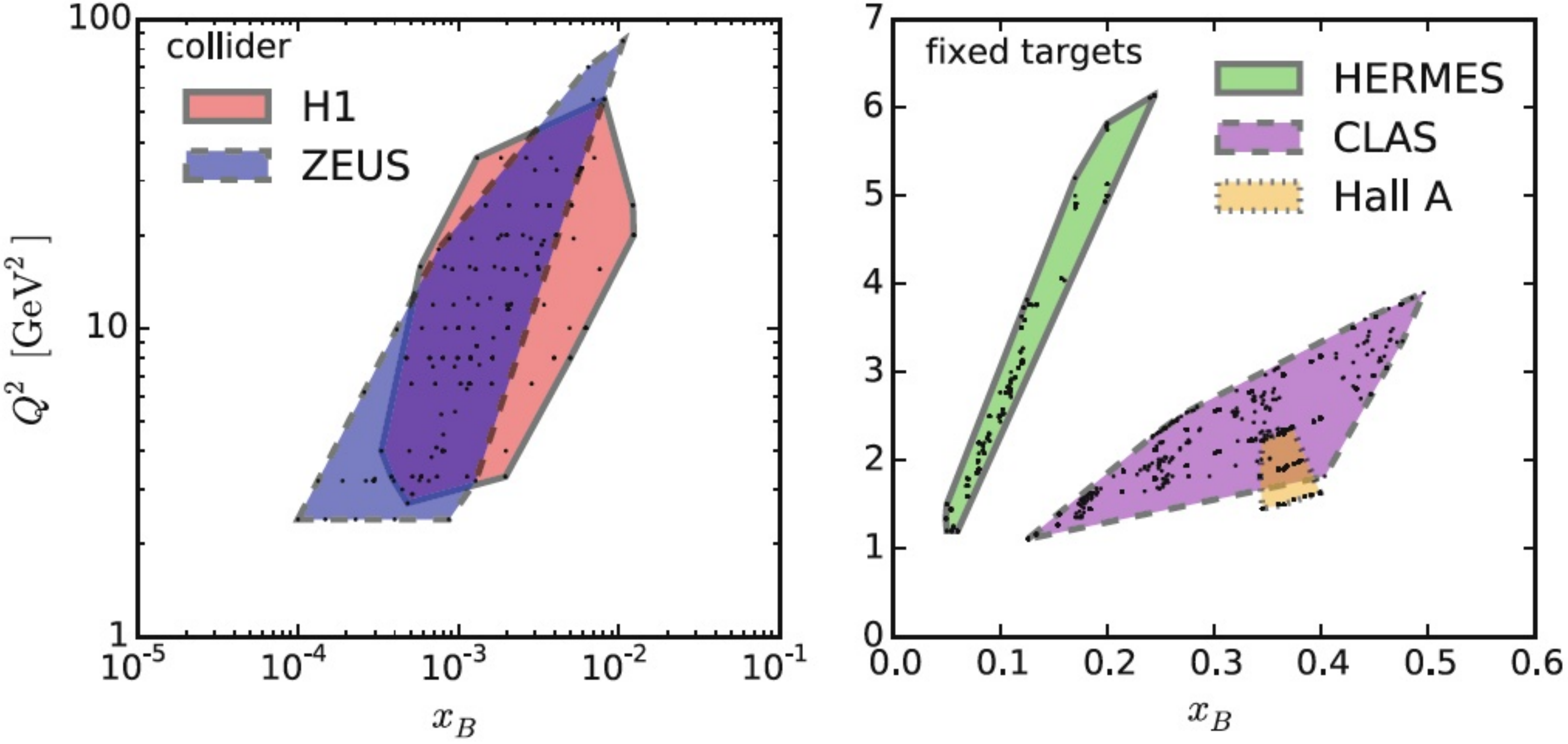
$$\mathcal{I} \propto \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right\}$$

- $c_0^{\mathrm{DVCS},\mathcal{I}}$, $(c,s)_1^{\mathcal{I}}$ dominated by leading-twist GPDs

- Similar decomposition for various polarization states of the target, but different dependence of the coefficients on the CFFs

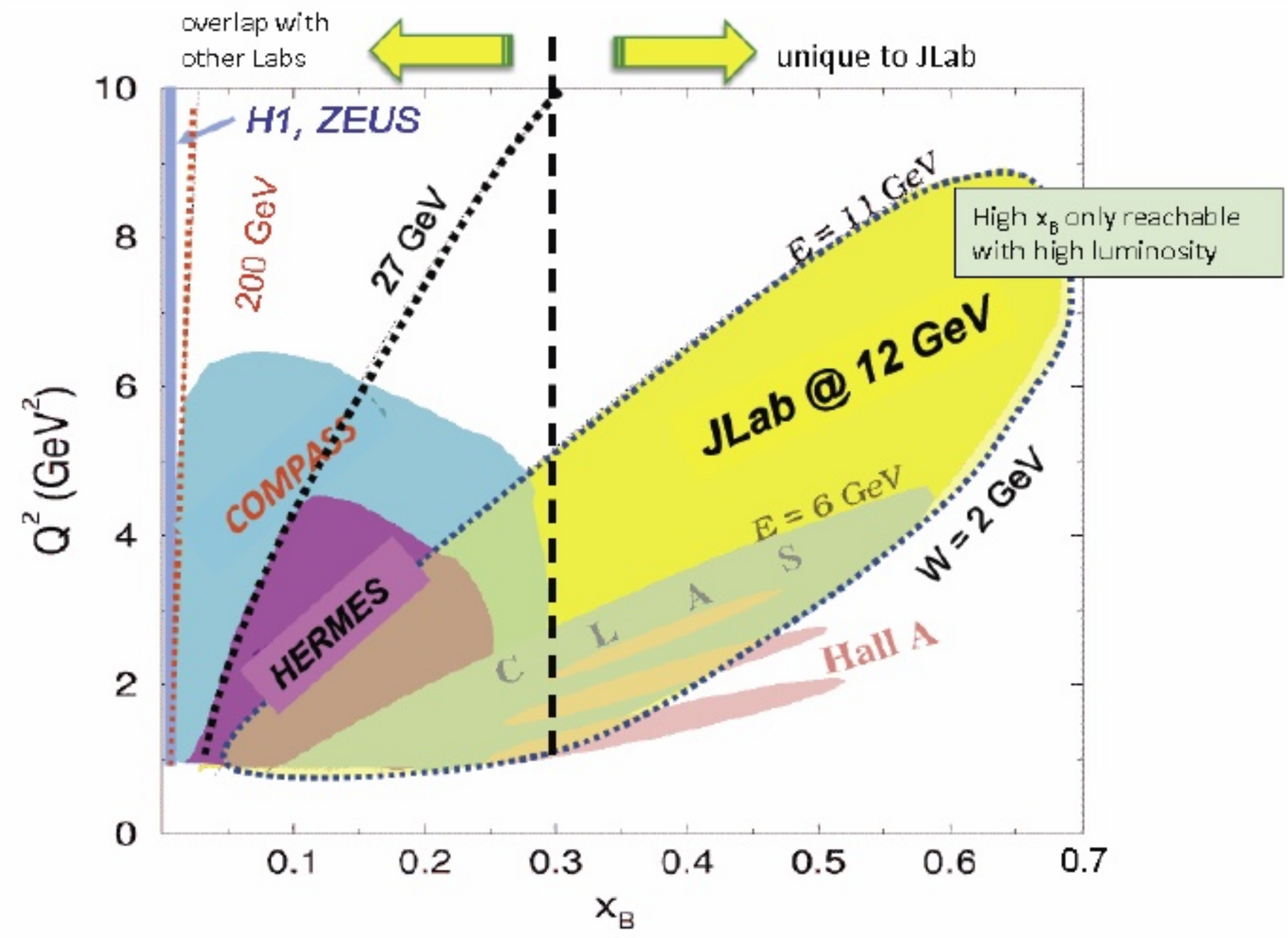
Belitsky, Kirchner, Müller, NPB629, 323 (2001)

World-data kinematic coverage



Kumericki, Liuti, Moutarde, EPJA52, 157 (2016)

JLab 12 kinematics



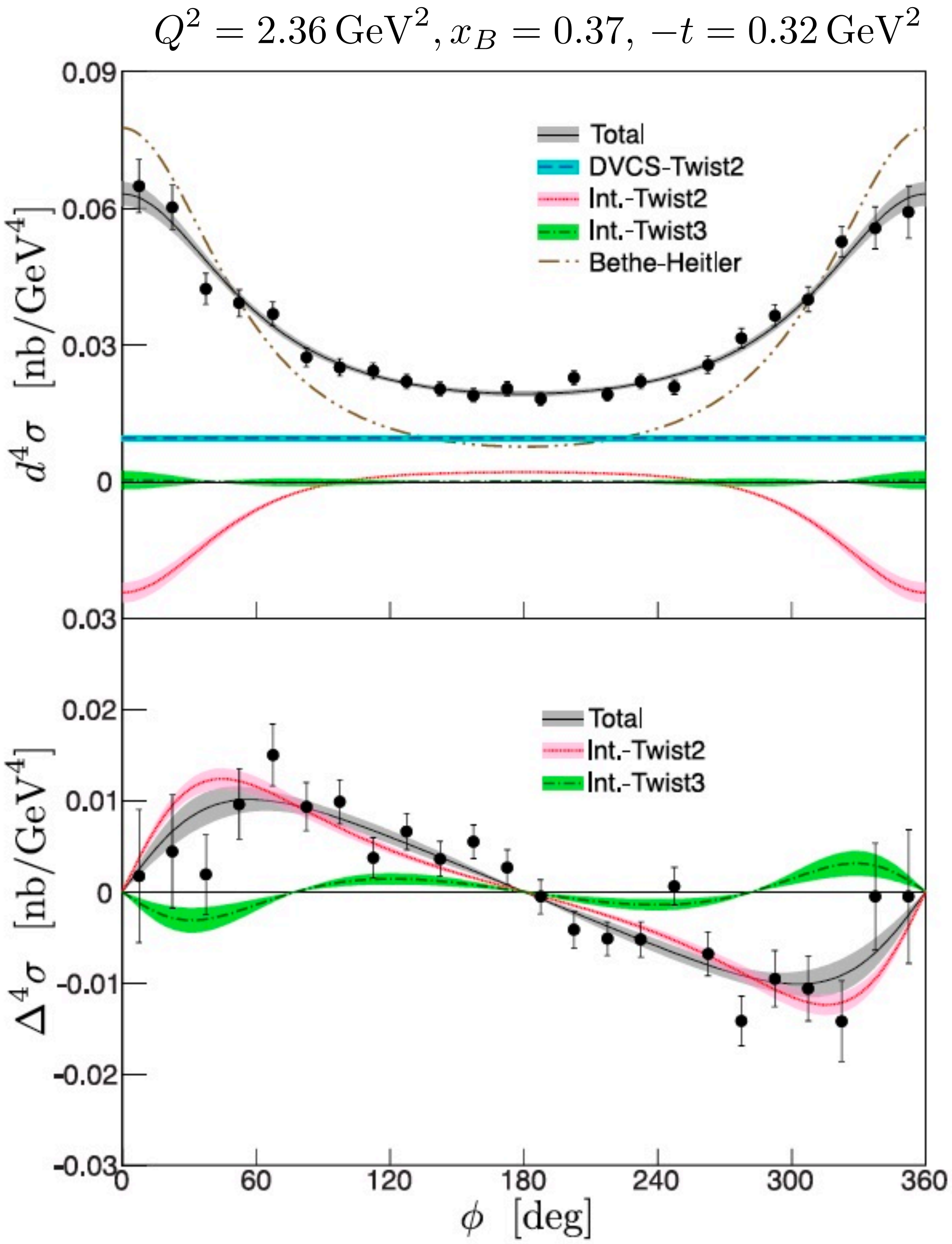
overlap with other Labs

unique to JLab

High x_B only reachable with high luminosity

Arrington et al., arXiv:2112.00060

A sample of typical results



$$d^4\sigma = |\mathcal{T}_{\rm BH}|^2 + \mathcal{T}_{\rm BH}\,\mathrm{Re}(\mathcal{T}_{\rm DVCS}) + |\mathcal{T}_{\rm DVCS}|^2$$

$$\mathrm{Re}(\mathcal{T}_{\rm DVCS}) \sim c_0^{\mathcal{I}} + c_1^{\mathcal{I}}\cos(\phi) + c_2^{\mathcal{I}}\cos(2\phi)$$

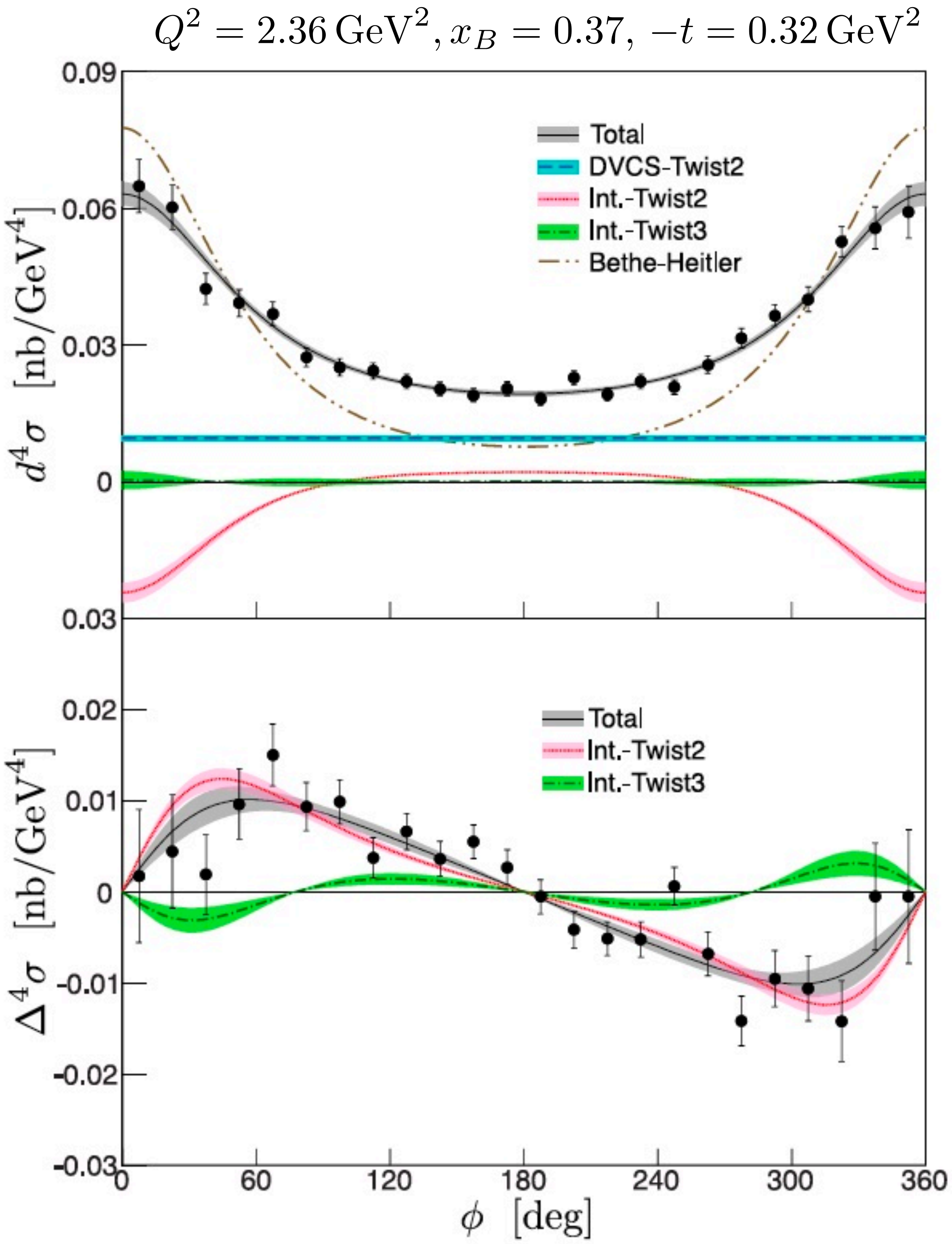
$$|\mathcal{T}_{\rm DVCS}|^2 \sim c_0^{\rm DVCS} + c_1^{\rm DVCS}\cos\phi$$

$$\Delta^4\sigma = \frac{d^4\overrightarrow{\sigma} - d^4\overleftarrow{\sigma}}{2} \sim \mathrm{Im}(\mathcal{T}_{\rm DVCS})$$

$$\mathrm{Im}(\mathcal{T}_{\rm DVCS}) \sim s_1^{\mathcal{I}}\sin\phi + s_2^{\mathcal{I}}\sin(2\phi)$$

Defurne, et al., PRDC 92, 055202 (2015)

A sample of typical results



$$d^4\sigma = |\mathcal{T}_{\mathrm{BH}}|^2 + \mathcal{T}_{\mathrm{BH}}\,\mathrm{Re}(\mathcal{T}_{\mathrm{DVCS}}) + |\mathcal{T}_{\mathrm{DVCS}}|^2$$

$$\mathrm{Re}(\mathcal{T}_{\mathrm{DVCS}}) \sim c_0^{\mathcal{I}} + c_1^{\mathcal{I}}\cos(\phi) + c_2^{\mathcal{I}}\cos(2\phi)$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 \sim c_0^{\mathrm{DVCS}} + \xcancel{c_1^{\mathrm{DVCS}}}\cos\phi$$

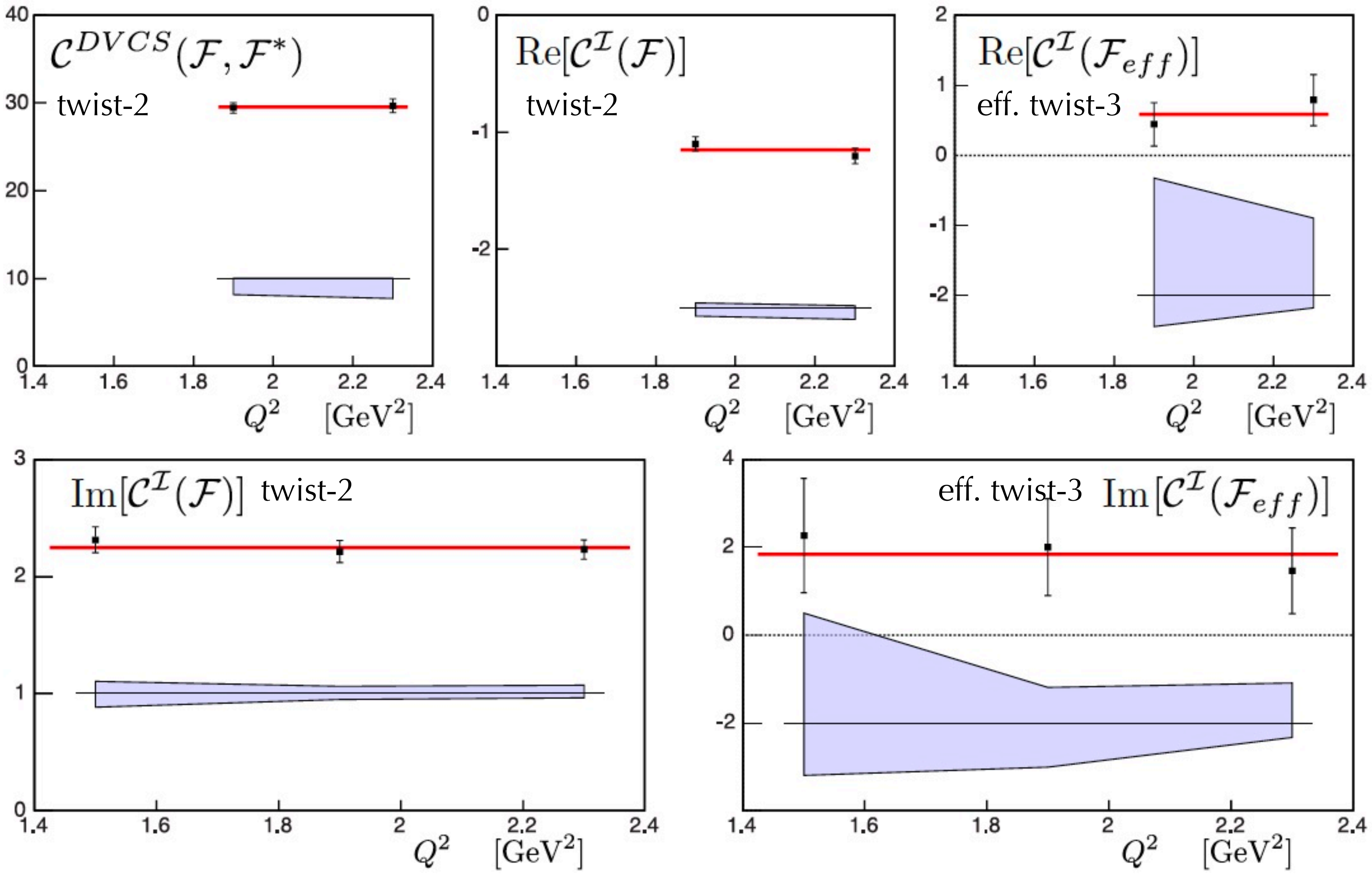
keeping only twist-2 contribution

$$\Delta^4\sigma = \frac{d^4\overrightarrow{\sigma} - d^4\overleftarrow{\sigma}}{2} \sim \mathrm{Im}(\mathcal{T}_{\mathrm{DVCS}})$$

$$\mathrm{Im}(\mathcal{T}_{\mathrm{DVCS}}) \sim s_1^{\mathcal{I}}\sin\phi + s_2^{\mathcal{I}}\sin(2\phi)$$

Defurne, et al., PRDC 92, 055202 (2015)

DVCS cross section: Q^2 dependence



unpolarized cross section

helicity dependent cross section

- Limited range in Q^2
- No Q^2 dependence observed
- Support leading-twist dominance

Defurne, et al., PRDC 92, 055202 (2015)

Extraction of Compton form factors

- Domain space of the unknown functions: 3 dimensions for GPDs (x, ξ, t) vs 1 dimension for PDF (x)

  curse of dimensionality: *"It is easy to find a coin lost on a 100 meter line, but difficult to find it on a football field."*
  Here we could say that we deal with a haystack, 100 m per side!

- Mapping of GPDs will significantly improve with the release of new data of unprecedented accuracy (JLab) and data in a larger kinematic domain (EIC)

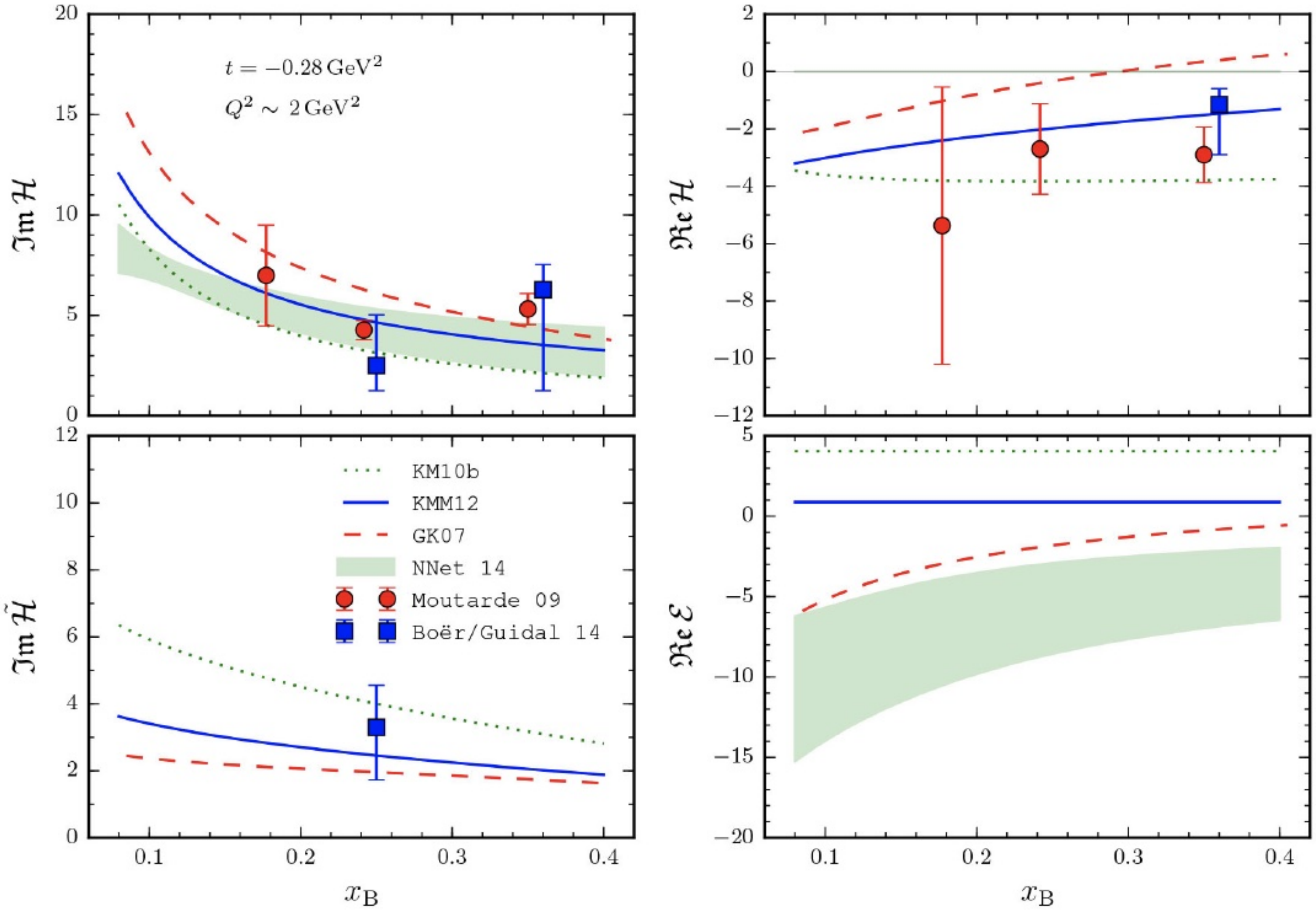
- Fitting strategies:

Global fits: use a parametrization of GPD and consider all kinematic bins at the same time

Local fits: take each kinematic bin independently and fit CCF-value at this point

Artificial neural network: already used for PDF fits. In progress for GPDs.

Comparison of different extraction methods



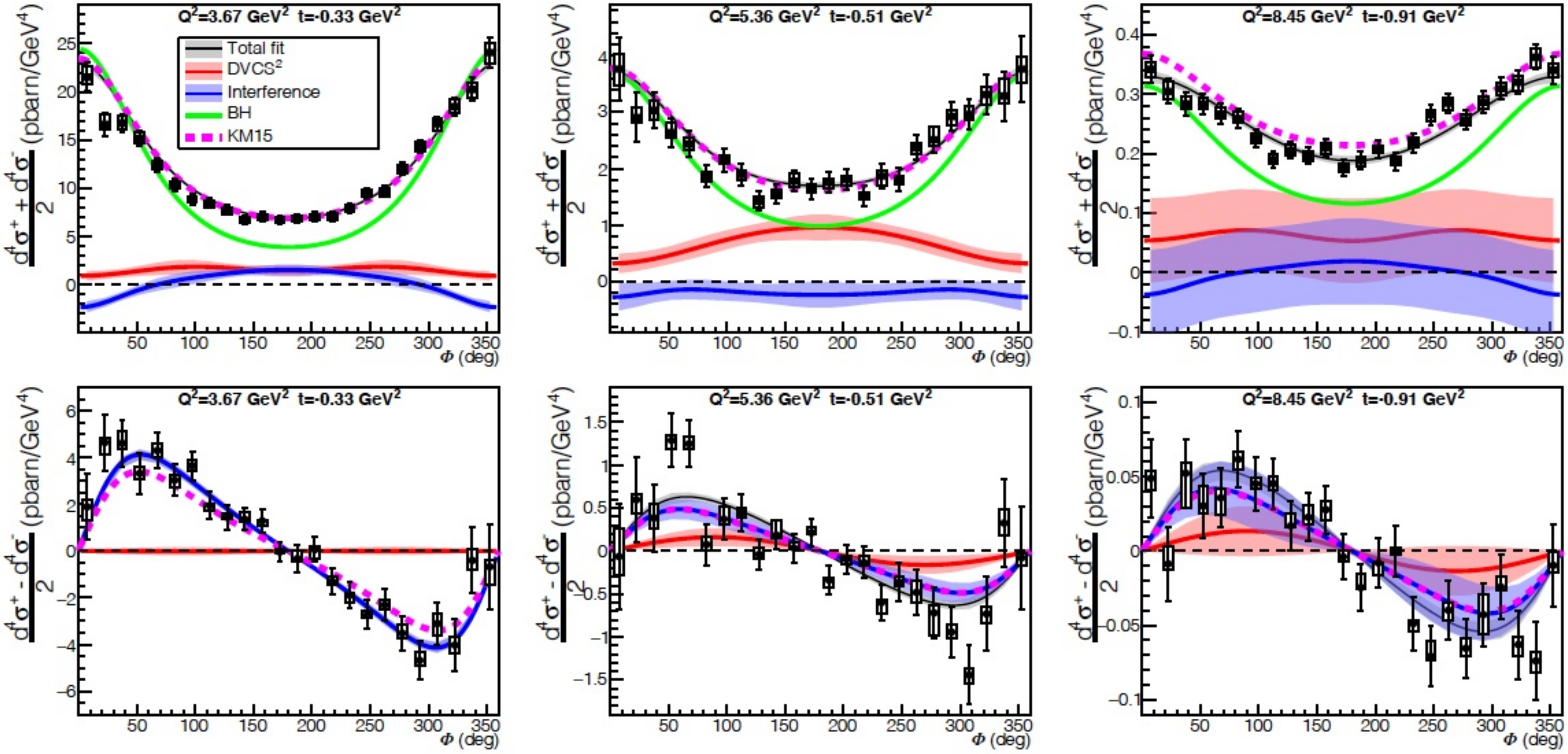
- Continuous curves: global fits based on double distributions or dispersion relations
- Two fit methods are compatible: good consistency check!

K. Kumericki, S. Liuti, H. Moutarde, Eur. Phys. J. A52 (2016) 157

Recent JLab12 results

Exploit energy dependence of cross section to separate $|\mathrm{DVCS}|^2$ and $\mathcal{I}$ contributions

$$|\mathcal{T}^{\mathrm{DVCS}}|^2 \propto 1/y^2 \qquad \mathcal{I} \propto 1/y^3 \quad \text{with} \quad y = E_b/\nu$$

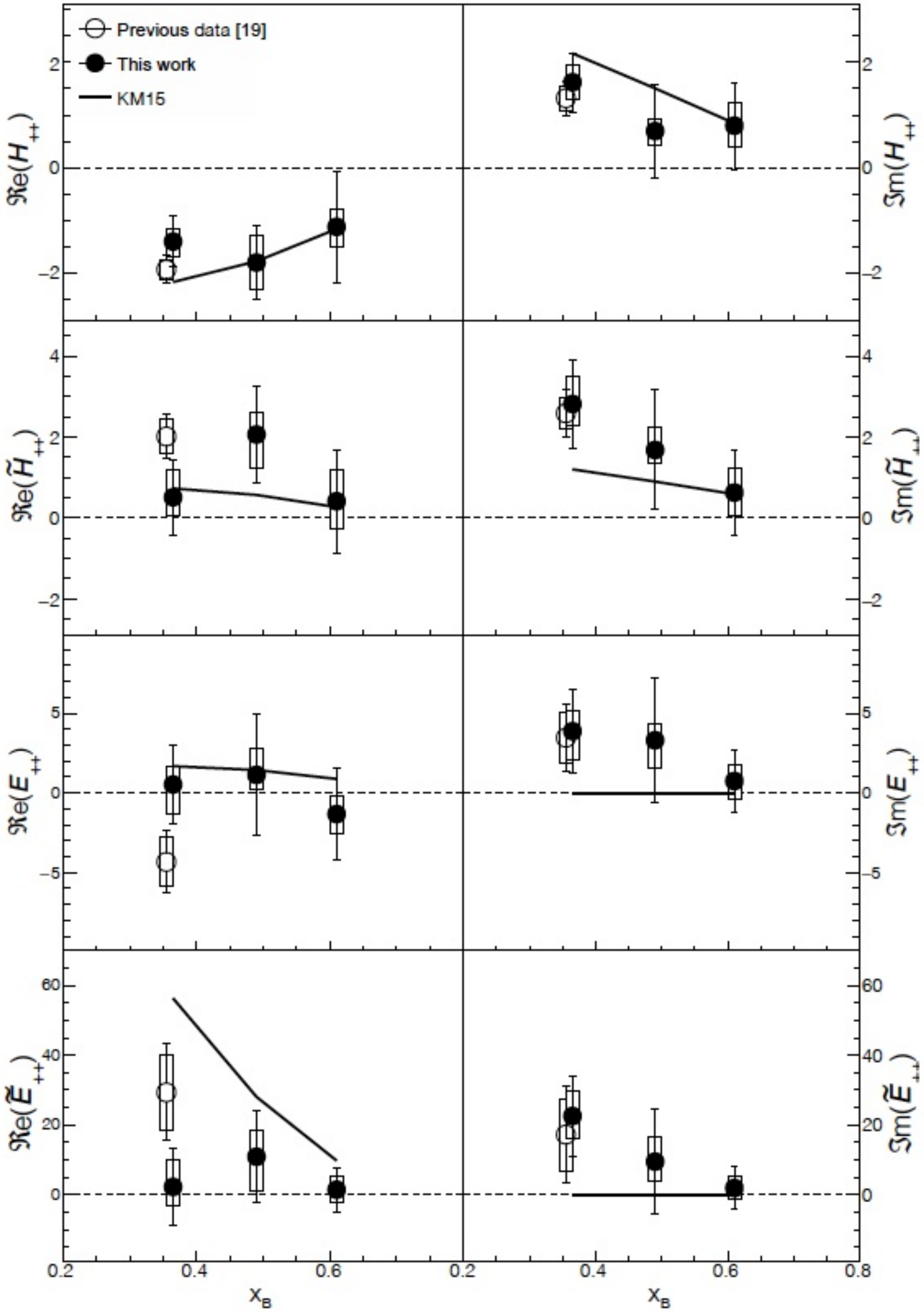


F. Georges et al. (Hall A Coll.), arXiv:2201.03714

Recent JLab12 results

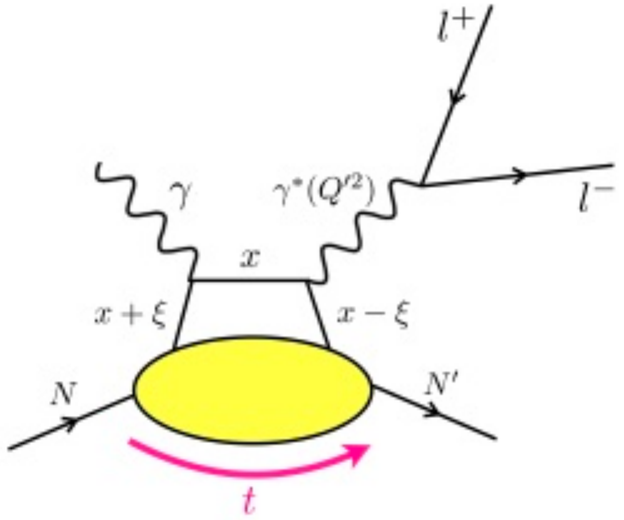
First extraction of all four helicity conserving CFFs!

F. Georges et al. (Hall A Coll.), arXiv:2201.03714



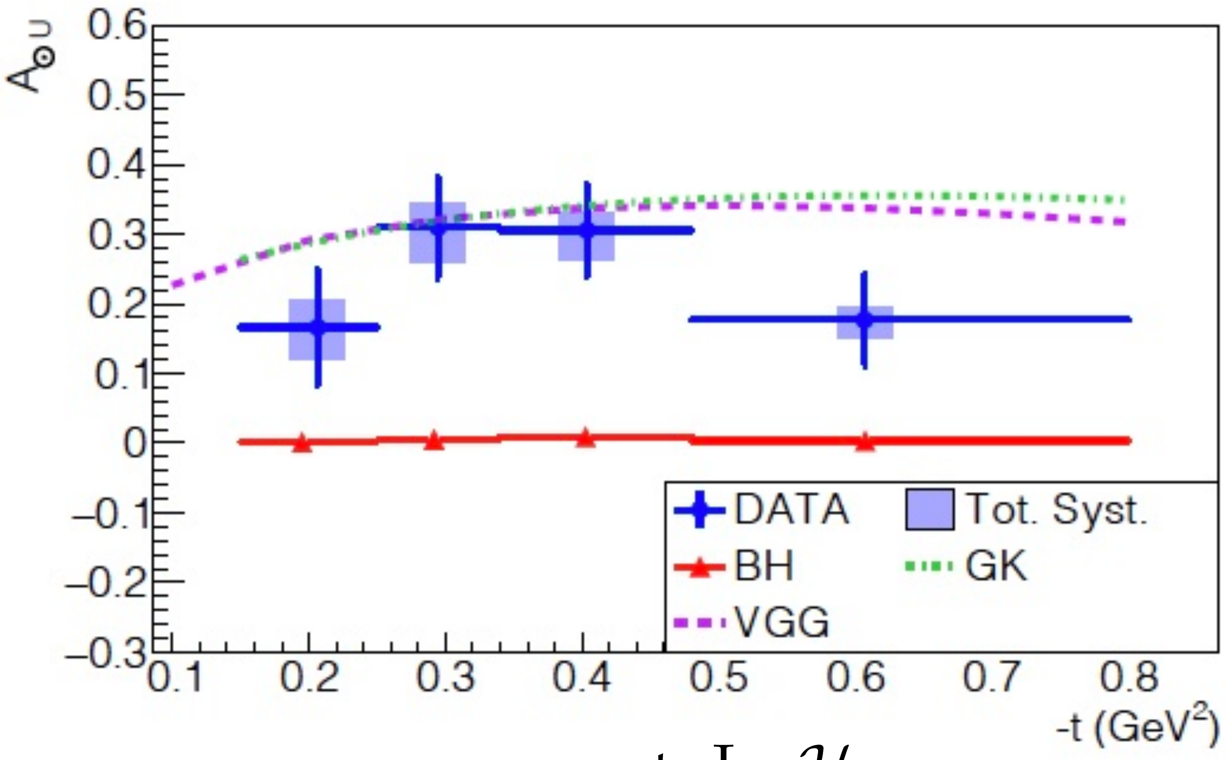
Timelike Compton scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)



photon polarization asymmetry

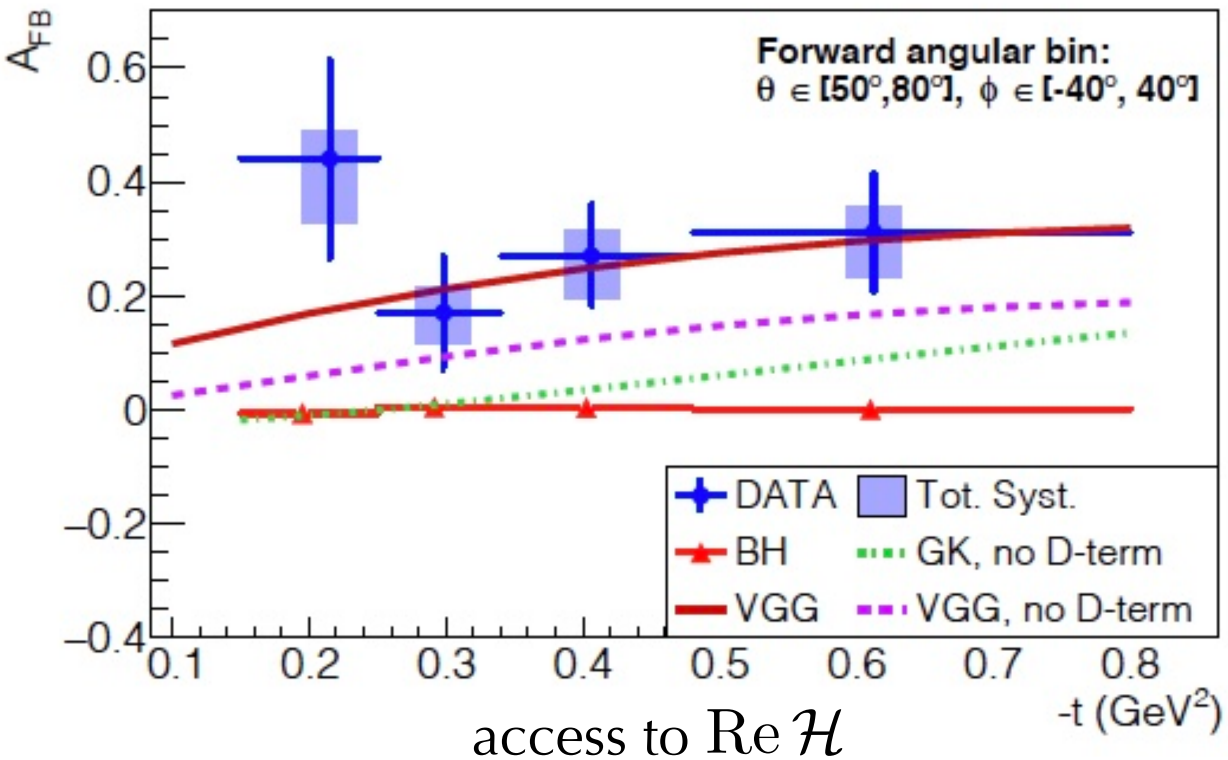
$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$



access to $\mathrm{Im}\,\mathcal{H}$

forward-backward asymmetry

$$A_{FB} = \frac{d\sigma(\theta,\phi) - d\sigma(180^\circ-\theta, 180^\circ+\phi)}{d\sigma(\theta,\phi) + d\sigma(180^\circ-\theta, 180^\circ+\phi)}$$



access to $\mathrm{Re}\,\mathcal{H}$

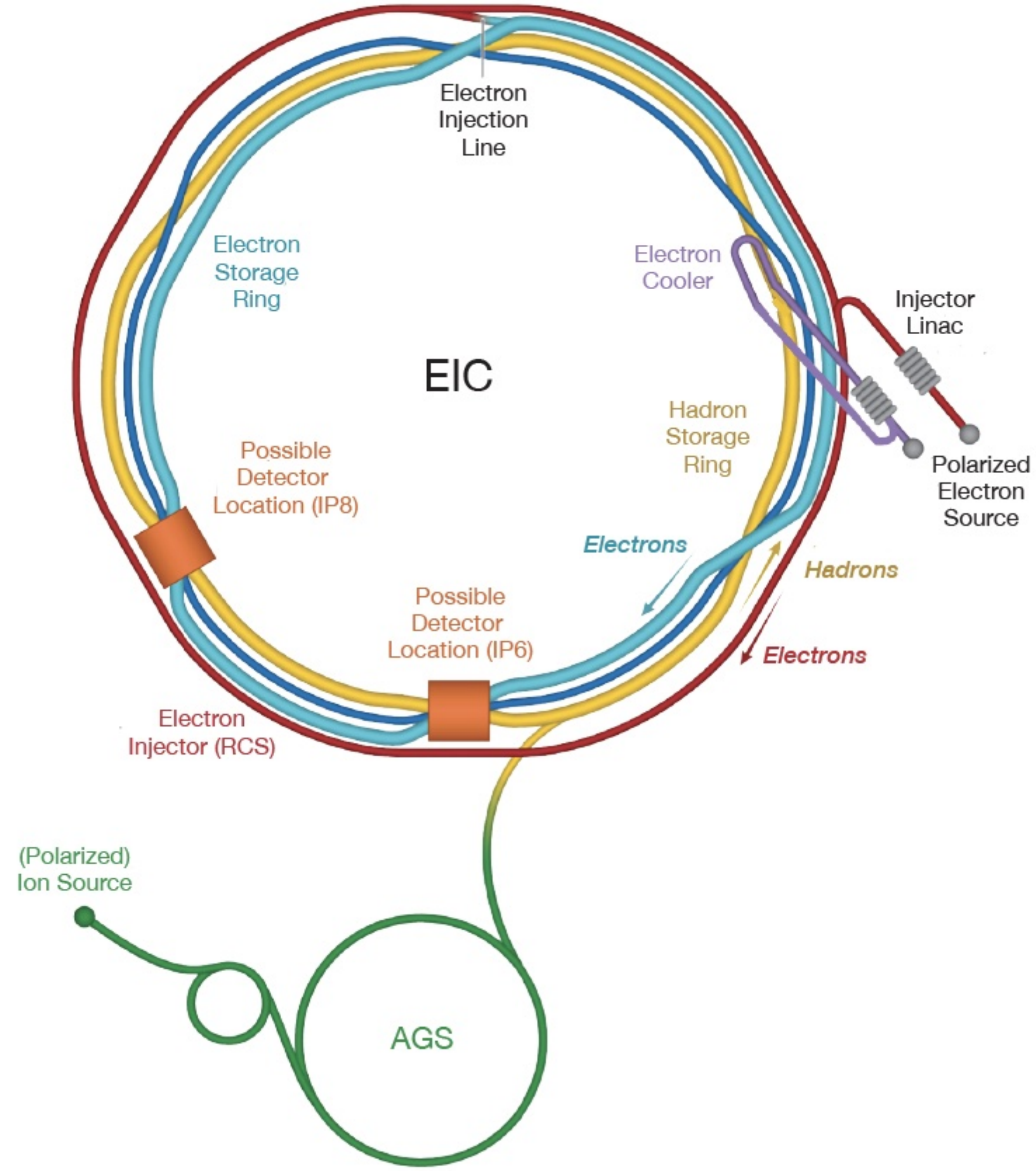
✓ Test of the universality of GPDs

✓ Further data from JLab12 and future EIC

✓ New promising path towards the extraction of $\mathrm{Re}\,\mathcal{H}$ and then the D-term

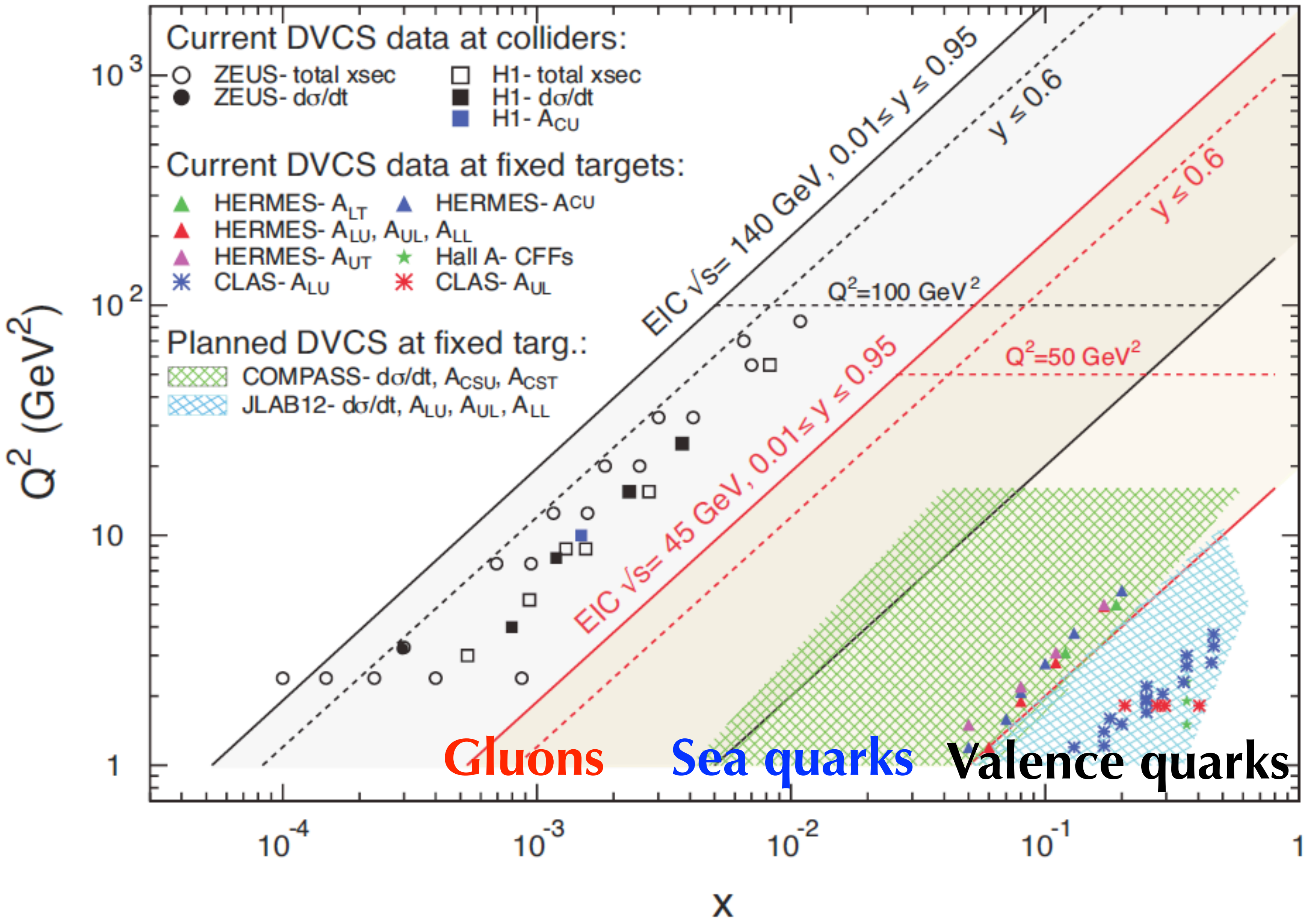
The Electron Ion Collider

is a future electron-proton and electron-ion collider to be constructed in the United States in this decade and foreseen to start operation in 2030



- Large center of mass energy range, $\sqrt{s} = 20 - 140\,\mathrm{GeV}$
- Polarized electron, proton and light nuclear beam, $\geq 70\%$
- Nuclear beams with heavy ions, up to U
- High luminosity (100 X HERA), $10^{33-34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$

Paste, present and future DVCS experiments



DESY

hermes

Jefferson Lab

COMPASS

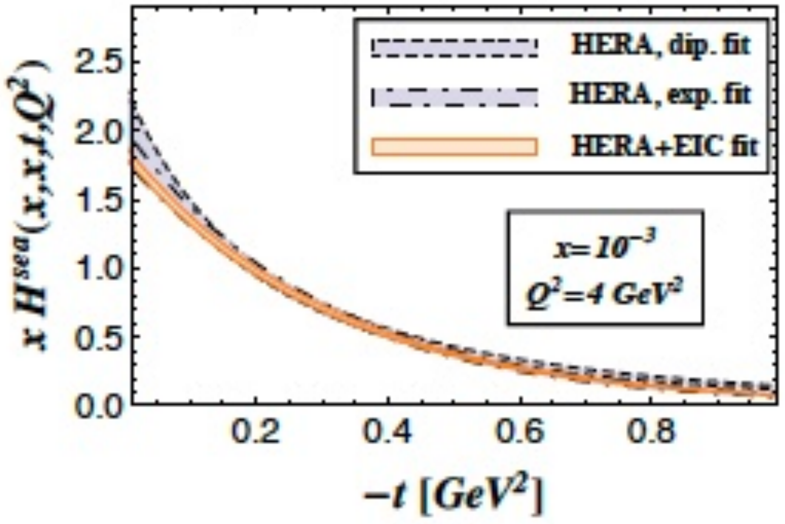
EIC

Accardi et al., The Electron Ion Collider: the next QCD Frontier, Eur. Phys. J., A52, 2016

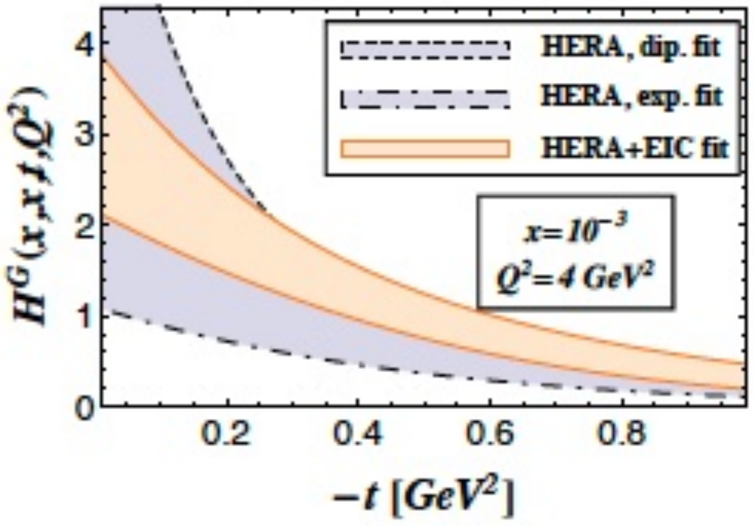
Impact of EIC on GPD measurements

EIC Yellow Report: arXiv: 2103.05419

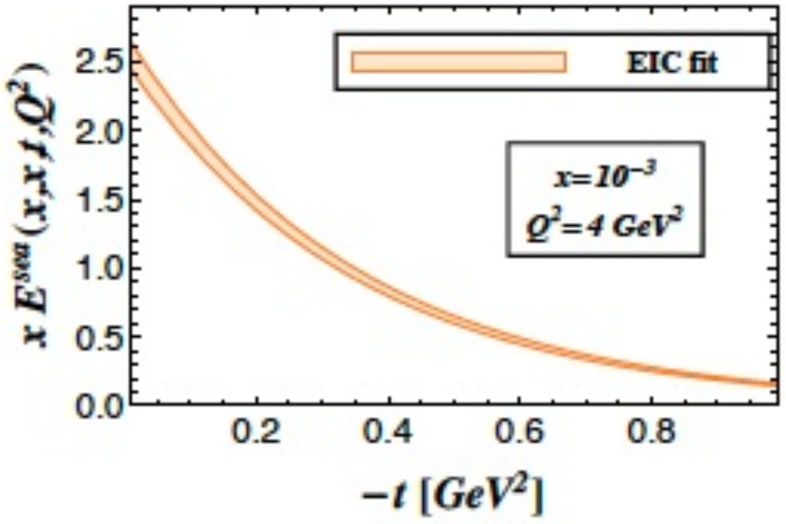
H - sea



H - gluon



E - sea



$x = 10^{-3}, Q^2 = 4\,\mathrm{GeV}^2$