Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

\[
\text{Re } A_2(\nu, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \text{Im } A_2(\nu', t, Q^2) \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)}
\]
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\]

• t-channel DRs for subtraction function

\[
\Delta(t, Q^2) = \frac{1}{\pi} \int_{4m^2_\pi}^{\infty} dt' \frac{\text{Im}_t A_2(0, t', Q^2)}{t' - t} + \frac{1}{\pi} \int_{-\infty}^{-a} dt' \frac{\text{Im}_t A_2(0, t', Q^2)}{t' - t}
\]

\[
-a = -2(m^2_\pi + 2M_N m_\pi) - Q^2
\]
Dispersion Relations for DVCS amplitudes

• s-channel subtracted DRs:

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Dispersion Relations for DVCS amplitudes

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• t-channel DRs for subtraction function

\[ \Delta(t, Q^2) = \frac{1}{\pi} \int_{4m_{\pi}^2}^{\infty} dt' \frac{\text{Im}_t A_2(0, t', Q^2)}{t' - t} + \frac{1}{\pi} \int_{-\infty}^{-\alpha} dt' \frac{\text{Im}_t A_2(0, t', Q^2)}{t' - t} \]

Unitarity relation in t-channel

\[ \gamma^*(Q^2) N = 2 \text{ Im } \gamma N \]

\[ t \geq (2m_{\pi})^2 \]

\[ t \geq (4m_{\pi})^2 \]

[Diagram of dispersion relations]
Fixed

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ \nu = \frac{s-u}{4M_N} \]

crossing symmetric variable

\[ s = (M + m^2) \]

\[ u = (M + m^2) \]

\[ t = 4m^2 \]

\[ t = \mu^2 + 4m^2 \]

\[ s = M + m^2 \]
Fixed

\( Q^2 = 2 \text{ GeV}^2 \)

\[ t = 4m_x^2 \]

\[ s = (M + m_{W})^2 \]

\[ u = (M + m_{W})^2 \]

\[ \Theta_{\gamma\gamma} = 0^\circ \]

\[ \Theta_{\gamma\gamma} = 180^\circ \]

\[ \nu = \frac{s-u}{4M_N} \]

crossing symmetric variable

DRs in the \( s \)-channel at fixed \( t \)
Fixed
$Q^2 = 2 \text{ GeV}^2$

$\nu = \frac{s-u}{4M_N}$

crossing symmetric variable

$\Theta_{\chi} = 0^\circ$

$\Theta_{\chi} = 180^\circ$

Subtraction point

DRs in the s-channel at fixed $t$
Fixed

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ \nu = \frac{s-u}{4M_N} \]

crossing symmetric variable

\[ a = -2(m^2_\pi + 2M_N m_\pi) - Q^2 \]
Unitarity Relations in the t-channel

- Charge conjugation
- Partial wave expansion with $\nu = 0 \rightarrow \theta_t = 90^\circ$

two-pion intermediate state with $I = 0 \quad J = 0, 2, \cdots$
Two-Pion Generalized Distribution Amplitude (GDA)

Two-pion intermediate states with $I = 0$ and $J = 0, 2$

$$\Delta(t, Q^2) = \frac{4}{N_f} \sum_{n \text{ odd}} d_n(t, Q^2) \quad \text{only } d_1(t, Q^2)$$

$\pi\pi$ phase shifts

input $\rightarrow$ pion PDFs

M. Polyakov, 1999
Two-Pion Generalized Distribution Amplitude (GDA)

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input

$\pi\pi$ phase shifts

pion PDFs

$\pi\pi \rightarrow N\bar{N}$ scattering amplitude

analytical continuation of s-channel helicity amplitudes

M. Polyakov, 1999

G. Hoehler, 1983
D-term form factor: $t$-dependence

$Q = u + d$

Fit: $F^Q(t) = \frac{d^Q_1(0)}{[1 - t/(\alpha M_D^2)]^{\alpha}}$ with $M_D = 0.487 \text{ GeV}$

$\alpha = 0.841$
D-term form factor: dependence on pion PDFs

\[ Q^2 = 4 \text{ GeV}^2 \]

\[ d_1^Q(0) = -1.92 \quad \text{GRV, ZPC53 (1992)} \]

\[ d_1^Q(0) = -1.59 \quad \text{Owens et al., PRD30 (1984)} \]

\[ d_1^Q(0) = -2.01 \quad \text{Mueller and Hwang, (2014)} \]

\[ \chi_{QSM} \]

\[ d_1^Q(0) = -2.35 \quad \text{Schweitzer et al., (2007)} \]

\[ \text{Skyrme model} \]

\[ d_1^Q(0) = -4.48 \quad \text{Schweitzer et al., (2007)} \]

\[ \text{Effective LFWFs} \]

\[ d_1^Q(0) = -2.01 \quad \text{Mueller and Hwang, (2014)} \]
D-term form factor and radial pressure distribution

\[ D^Q(t) = \frac{4}{5} d_1(t) \]

D-term from t-channel dispersion relations

\[ \gamma^* \gamma \rightarrow \pi\pi \rightarrow N\bar{N} \]

Extraction from data:
- neglecting gluon contribution
- assuming:

\[ \Delta(t, Q^2) = \frac{4}{N_f} \sum_{n_{\text{odd}}} d_n(t, Q^2) \rightarrow \text{only } d_1(t, Q^2) \]
D-term form factor and radial pressure distribution

\[ D^Q(t) = \frac{4}{5} d_1(t) \]

\[ r^2 p(r) \text{ radial pressure distribution} \]

Neglecting gluon contribution:

\[ r^2 p(r) = \frac{1}{3} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r) \]

\[ \int_0^\infty dr r^2 p(r) = 0 \]

JLab data

FT

JLab best fit

Dispersion relations

Shear forces

Pressure

BP, Polyakov, Vanderhaeghen, PLB 739 (2014) 133

Necessary to verify model assumptions in the experiment with more data coming from JLab, COMPASS and the future EIC, EICcC


**Global fit to DVCS data with artificial neural networks**

\[ \sum d_1^q < 0 \]

in all model calculations for a stable proton

---

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Parametrizations of GPDs
GPDs from Double Distributions (DDs)

\[ \beta P - (1 + \alpha)\Delta/2 \]

\[ \beta P + (1 - \alpha)\Delta/2 \]

\[ DD(\beta, \alpha, t) \]

\[ P - \Delta/2 \]

\[ P + \Delta/2 \]

\[ \langle p'|\bar{q}(-\frac{1}{2}z)\gamma_q\frac{1}{2}z|p\rangle |z^2=0 \]

\[ \Delta = 0 \]

forward matrix element defining the collinear PDFs

\[ P = 0 \]

probability amplitude of finding \( q\bar{q} \) pair in the nucleon

[Mueller et al., 94; Radyushkin 97, Polyakov, Weiss 99]
GPDs from Double Distributions (DDs)

\[ \langle p' | \bar{q} (-\frac{1}{2} z) \not{q} (\frac{1}{2} z) | p \rangle \big|_{z^2=0} = \bar{u}(p') \not{u}(p) \int d\beta d\alpha e^{-i\beta(Pz)+i\alpha(\Delta z)/2} f_q(\beta, \alpha, t) \]

\[ + \bar{u}(p') i\sigma^{\mu\alpha} \frac{z_\mu \Delta_\alpha}{2M} u(p) \int d\beta d\alpha e^{-i\beta(Pz)+i\alpha(\Delta z)/2} k_q(\beta, \alpha, t) \]

\[ - \bar{u}(p') \frac{\Delta \cdot z}{2M} u(p) \int d\beta d\alpha e^{i\alpha(\Delta z)/2} D^q(\alpha, t) \]

[Mueller et al., 94; Radyushkin 97, Polyakov, Weiss 99]
GPDs from Double Distributions (DDs)

\[ \langle p' | \bar{q}(\frac{-1}{2}z) q(\frac{1}{2}z) | p \rangle | z^2 = 0 \]

\[ \langle p' | \bar{q}(\frac{-1}{2}z) q(\frac{1}{2}z) | p \rangle | z^2 = 0 = \bar{u}(p') u(p) \int d\beta d\alpha e^{-i\beta(Pz) + i\alpha(\Delta z)/2} f^q(\beta, \alpha, t) \]

\[ + \bar{u}(p') \frac{i\sigma^{\mu\nu} z_\mu \Delta_\nu}{2M} u(p) \int d\beta d\alpha e^{-i\beta(Pz) + i\alpha(\Delta z)/2} k^q(\beta, \alpha, t) \]

\[ - \bar{u}(p') \frac{\Delta \cdot z}{2M} u(p) \int d\beta d\alpha e^{i\alpha(\Delta z)/2} D^q(\alpha, t) \]

\[ \text{take } z^+ = z_\perp = 0 \text{ and compare with GPD correlator} \]

[Mueller et al., 94; Radyushkin 97, Polyakov, Weiss 99]
\( \text{GPD}^q(x, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi \alpha) \text{DD}(\alpha, \beta, t) + \text{D-term for GPDs H and E} \)

- Support region: \(|\beta| + |\alpha| \leq 1\)
Model for GPDs from DDs

\[ H(x, \xi, t) = \int d\beta \int d\alpha \delta(x - \beta - \xi \alpha) f(\beta, \alpha, t) + \theta(\xi - |x|) \frac{1}{N_f} D \left( \frac{x}{\xi}, t \right) \]

- Ansatz for DD which smoothly interpolates between a PDF and DA

\[ f(\beta, \alpha) = h(\beta, \alpha) H(\beta, 0, t) \quad \text{with} \quad h(\beta, \alpha) \propto \frac{[(1 - |\beta|)^2 - \alpha^2]^b}{(1 - |\beta|)^{2b+1}} \]

\( b \) free parameter which governs the \( \xi \) dependence

- forward limit: \( \int d\alpha f(\beta, \alpha, 0) = H(\beta, 0, 0) = f_1(\beta) \)

- t-dependence to reproduce behaviour of e.m. form factor

- D-term fitted to data (assuming a dipole functional form with adjustable mass parameter)

[Radyushkin, PRD59, 014030 (1998); PLB 449, 81 (1999)]
accessible in exclusive reactions

factorization for large $Q^2$, $|t| \ll Q^2, W^2$

depend on 3 variables: $x, \xi, t$

Compton Form Factors

\[
\text{Im } H(\xi, t) \overset{\text{LO}}{=} H(\xi, \xi, t)
\]

\[
\text{Re } H(\xi, t) \overset{\text{LO}}{=} \mathcal{P} \int_{-1}^{1} dx \ H(x, \xi, t) \frac{1}{x - \xi}
\]
Golden channel: deeply virtual Compton scattering

\[ \sigma(lp \rightarrow l\gamma p) \propto |T_{BH}|^2 + |T_{DVCS}|^2 + e_l I \]

- BH: calculable in QED with \( \sim 1\% \) knowledge of e.m. at low momentum transfer
- \( |DVCS|^2 \): bilinear in GPDs
- \( I(BH \cdot DVCS) \): linear combination of GPDs
Filter out interference term using cross section dependence on

- beam charge
- azimuth
- beam polarization
- target polarization

\[ |T_{BH}|^2 \propto \left\{ c_0^{BH} + \sum_{n=1}^{2} c_n^{BH} \cos(n\phi) + s_1^{BH} \sin \phi \right\} \]

\[ |T_{DVCS}|^2 \propto \left\{ c_0^{DVCS} + \sum_{n=1}^{2} c_n^{DVCS} \cos(n\phi) + s_n^{DVCS} \sin(n\phi) \right\} \]

\[ \mathcal{I} \propto \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right\} \]

- Similar decomposition for various polarization states of the target, but different dependence of the coefficients on the CFFs

Belitsky, Kirchner, Müller, NPB629, 323 (2001)
World-data kinematic coverage

Kumericki, Liuti, Moutarde, EPJA52, 157 (2016)
JLab 12 kinematics

High $x_b$ only reachable with high luminosity

Arrington et al., arXiv:2112.00060
A sample of typical results

\[ d^4 \sigma = |T_{BH}|^2 + T_{BH} \text{Re}(T_{DVCS}) + |T_{DVCS}|^2 \]

\[
\text{Re}(T_{DVCS}) \sim c_0^T + c_1^T \cos(\phi) + c_2^T \cos(2\phi)
\]

\[
|T_{DVCS}|^2 \sim c_0^{DVCS} + c_1^{DVCS} \cos \phi
\]

\[
\Delta^4 \sigma = \frac{d^4 \overline{\sigma} - d^4 \sigma}{2} \sim \text{Im}(T_{DVCS})
\]

\[
\text{Im}(T_{DVCS}) \sim s_1^T \sin \phi + s_2^T \sin(2\phi)
\]
A sample of typical results

\[ Q^2 = 2.36 \text{ GeV}^2, \ x_B = 0.37, \ -t = 0.32 \text{ GeV}^2 \]

\[ d^4 \sigma = |T_{\text{BH}}|^2 + T_{\text{BH}} \text{Re}(T_{\text{DVCS}}) + |T_{\text{DVCS}}|^2 \]

\[
\text{Re}(T_{\text{DVCS}}) \sim c_0^T + c_1^T \cos(\phi) + c_2^T \cos(2\phi)
\]

\[
|T_{\text{DVCS}}|^2 \sim c_0^{\text{DVCS}} + c_1^{\text{DVCS}} \cos \phi
\]

keeping only twist-2 contribution

\[
\Delta^4 \sigma = \frac{d^4 \sigma - d^4 \sigma^*}{2} \sim \text{Im}(T_{\text{DVCS}})
\]

\[
\text{Im}(T_{\text{DVCS}}) \sim s_1^T \sin \phi + s_2^T \sin(2\phi)
\]
DVCS cross section: $Q^2$ dependence

- Limited range in $Q^2$
- No $Q^2$ dependence observed
- Support leading-twist dominance
Extraction of Compton form factors

- Domain space of the unknown functions: 3 dimensions for GPDs $(x, \xi, t)$ vs 1 dimension for PDF $(x)$
- Curse of dimensionality: “It is easy to find a coin lost on a 100 meter line, but difficult to find it on a football field.”
  Here we could say that we deal with a haystack, 100 m per side!

- Mapping of GPDs will significantly improve with the release of new data of unprecedented accuracy (JLab) and data in a larger kinematic domain (EIC)

- Fitting strategies:
  
  **Global fits**: use a parametrization of GPD and consider all kinematic bins at the same time
  
  **Local fits**: take each kinematic bin independently and fit CCF-value at this point
  
  **Artificial neural network**: already used for PDF fits. In progress for GPDs.
Comparison of different extraction methods

- Continuous curves: global fits based on double distributions or dispersion relations
- Two fit methods are compatible: good consistency check!
Exploit energy dependence of cross section to separate $|\mathcal{T}^{DVCS}|^2$ and $\mathcal{I}$ contributions

$$|\mathcal{T}^{DVCS}|^2 \propto 1/y^2 \quad \mathcal{I} \propto 1/y^3 \quad \text{with} \quad y = E_b/\nu$$
Recent JLab12 results
First extraction of all four helicity conserving CFFs!

F. Georges et al. (Hall A Coll.), arXiv:2201.03714
Timelike Compton scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)

- Test of the universality of GPDs
- Further data from JLab12 and future EIC
- New promising path towards the extraction of $\text{Re } \mathcal{H}$ and then the D-term
The Electron Ion Collider
is a future electron-proton and electron-ion collider to be constructed in the United States in this decade and foreseen to start operation in 2030

• Large center of mass energy range, $\sqrt{s} = 20 - 140$ GeV
• Polarized electron, proton and light nuclear beam, $\geq 70\%$
• Nuclear beams with heavy ions, up to U
• High luminosity (100 X HERA), $10^{33-34} \text{ cm}^{-2} \text{ s}^{-1}$
Paste, present and future DVCS experiments

Impact of EIC on GPD measurements


$x = 10^{-3}, Q^2 = 4 \text{ GeV}^2$