Transverse Momentum PDFs (TMDs)

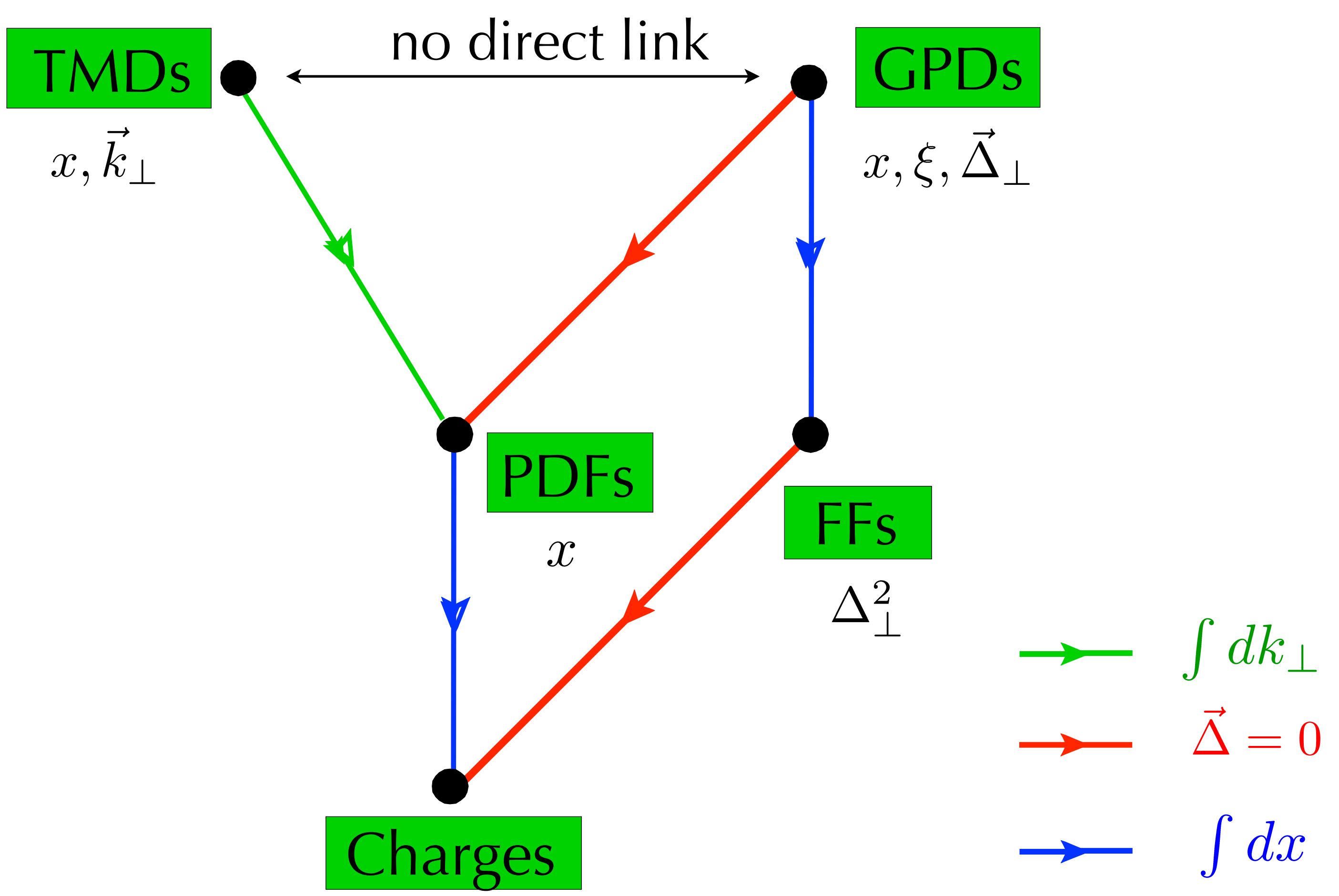
$$\boxed{\frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik\cdot z} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(-\frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\frac{z}{2}) | p^+, \vec{0}_\perp, \Lambda\rangle_{z^+=0}}$$

Depend on

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

$x = \dfrac{k^+}{p^+}$: longitudinal momentum fraction

$k_\perp$: parton transverse momentum



Transverse Momentum PDFs (TMDs)

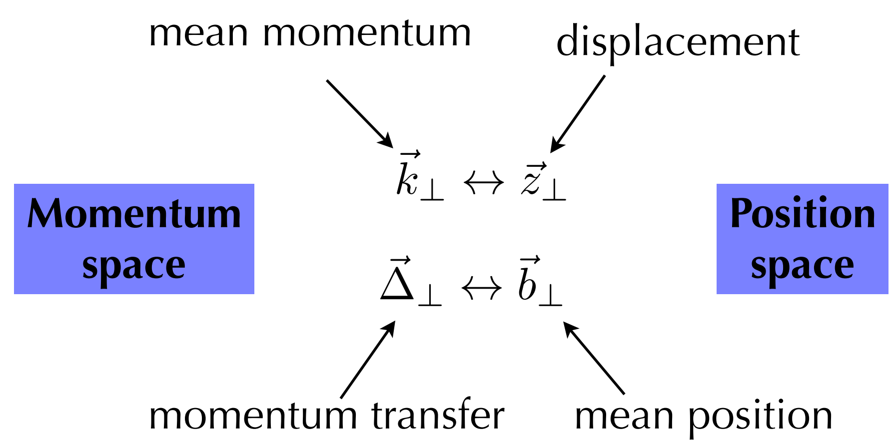
$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}e^{ik\cdot z}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0}}$$

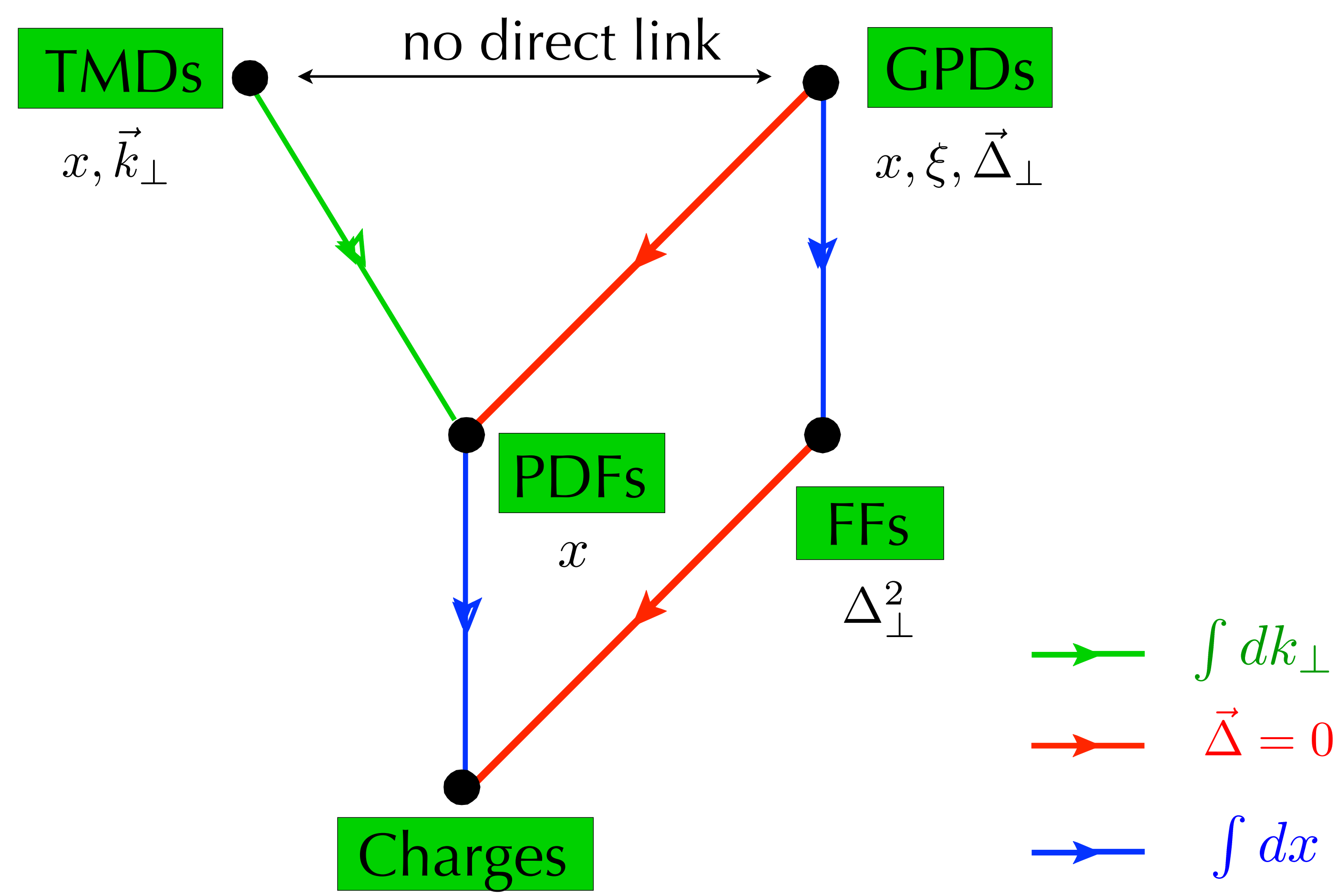
Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

$x=\dfrac{k^+}{p^+}$: longitudinal momentum fraction

$k_\perp$: parton transverse momentum





Transverse Momentum PDFs (TMDs)

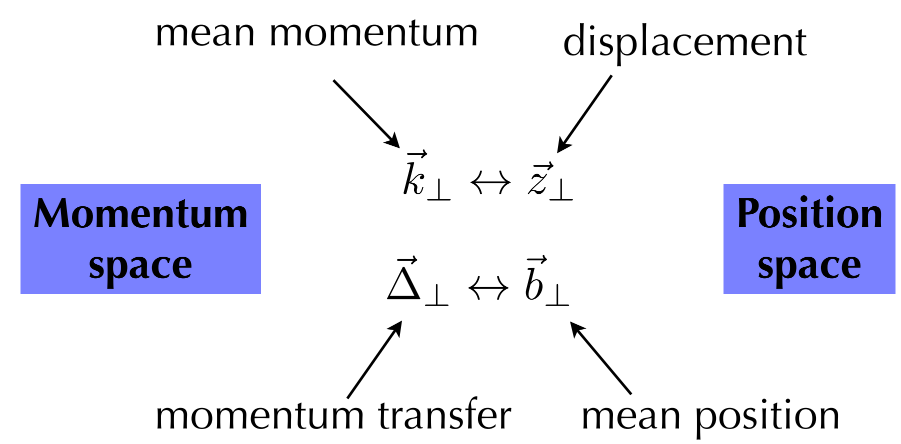
$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}e^{ik\cdot z}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0}}$$

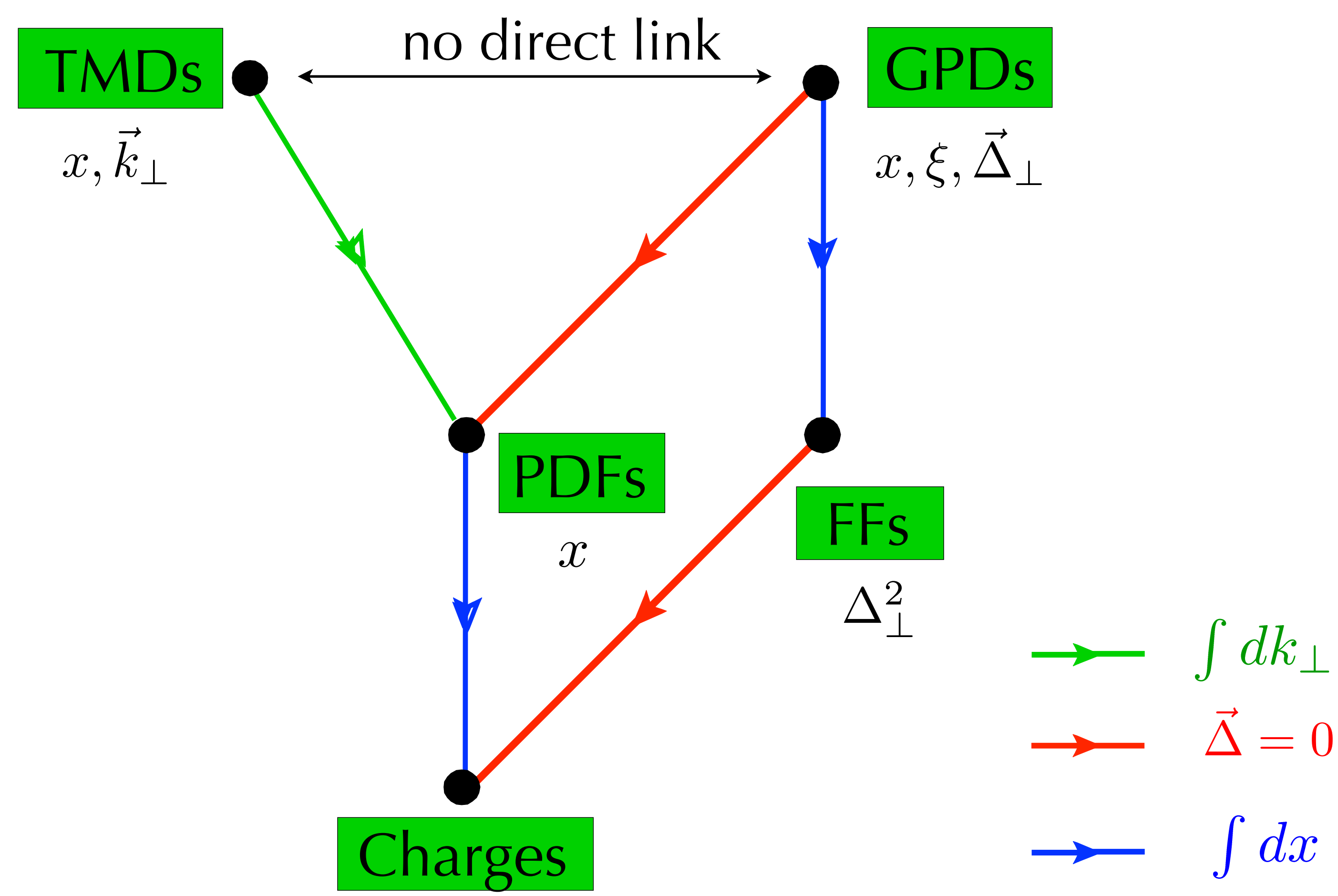
Depend on

$\Lambda,\,\Lambda',\,\Gamma$: nucleon and quark polarizations

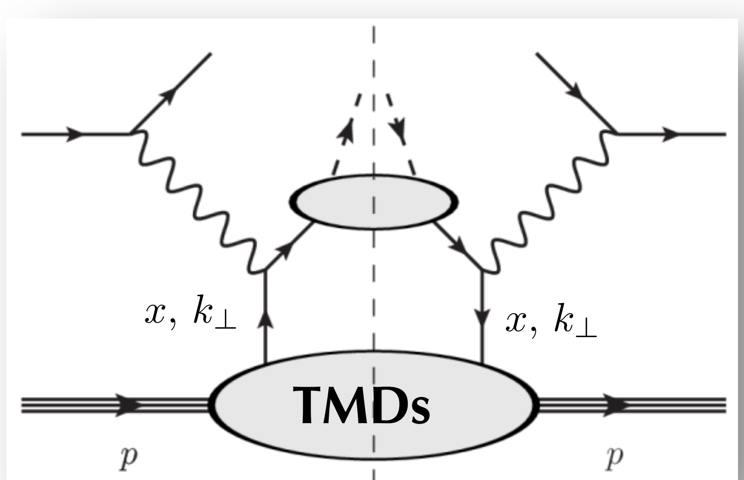
$x=\dfrac{k^+}{p^+}$: longitudinal momentum fraction

$k_\perp$: parton transverse momentum

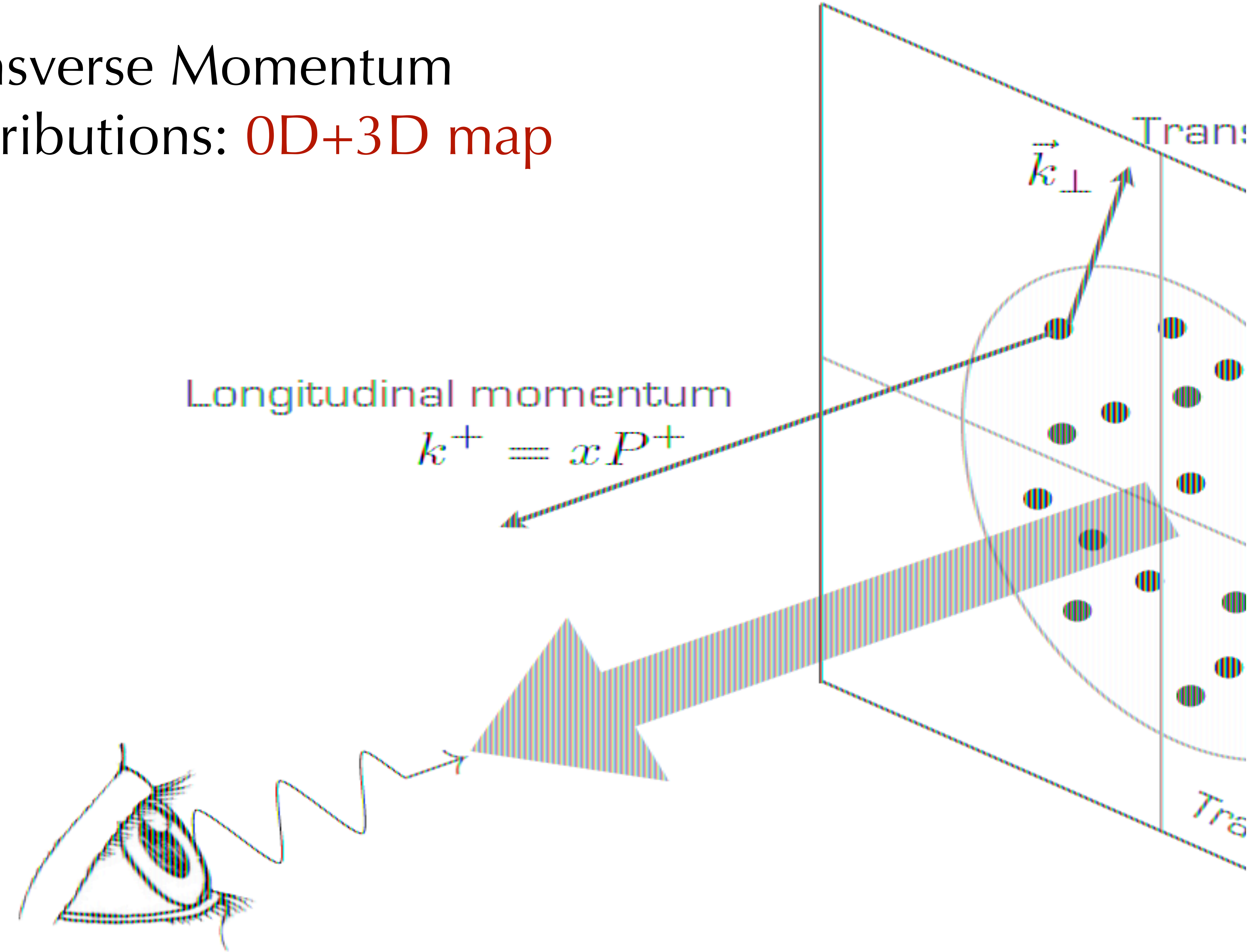




Semi-Inclusive
Deep Inelastic Scattering



Transverse Momentum Distributions: 0D+3D map



Key information from TMDs

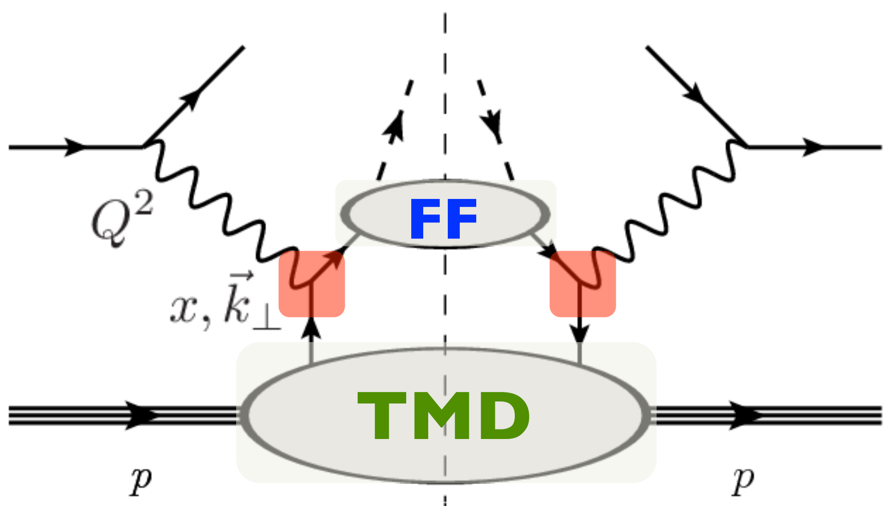
- Complete momentum spectrum of single particle
- Transverse momentum size as function of x (3D map)
- Spin-Spin and Spin-Orbit Correlations of partons
- Information on parton orbital angular momentum
  (no direct model-independent relation)
- Study interesting new non-trivial aspects of pQCD: role of re-scattering of active partons, factorization, universality, evolution,….
- Non-perturbative structure we cannot calculate with QCD

A few references on TMDs

- V. Barone, A. Drago, P. Ratcliffe, Phys. Rept. 359 (2002) 1
- U. D'Alesio, F. Murgia, Prog. Part. Nucl. Phys. 61 (2008) 394
- A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. Mulders, M. Schlegel, JHEP 0702 (2007) 93
- M. Anselmino, et al., Eur. Phys. J. A47 (2011) 35
- C. Aidala, S. Bass, D. Hasch, G. Mallot, Rev. Mod. Phys. 85 (2013) 655
- Collins, *Foundations of Perturbative QCD*, Cambridge U. Press, 2011
- A. Metz, A. Vossen, Prog. Part. Nucl. Phys. 91 (2016) 136

How to measure TMDs

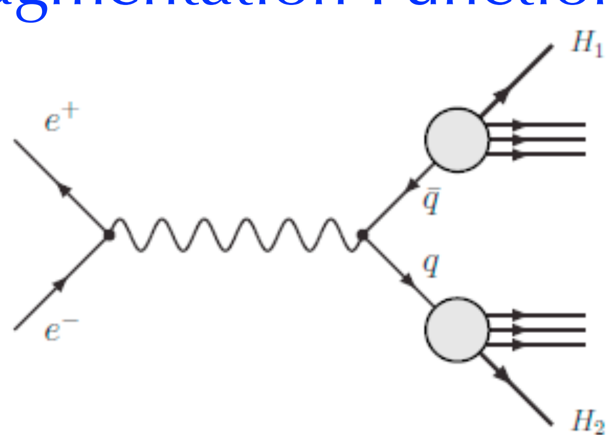
$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$



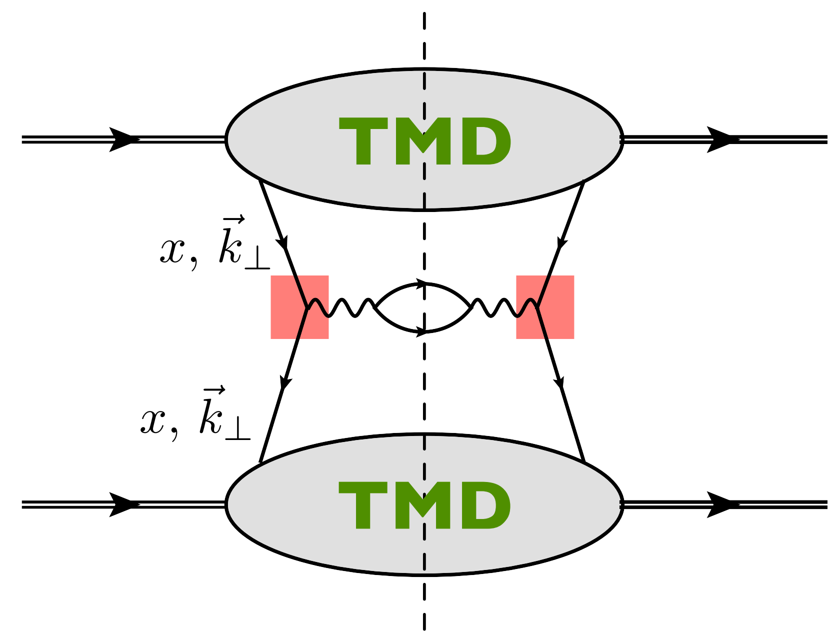
$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Fragmentation Functions

$$e^+e^- \to hh'X$$



$$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$$



$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

✓Factorization

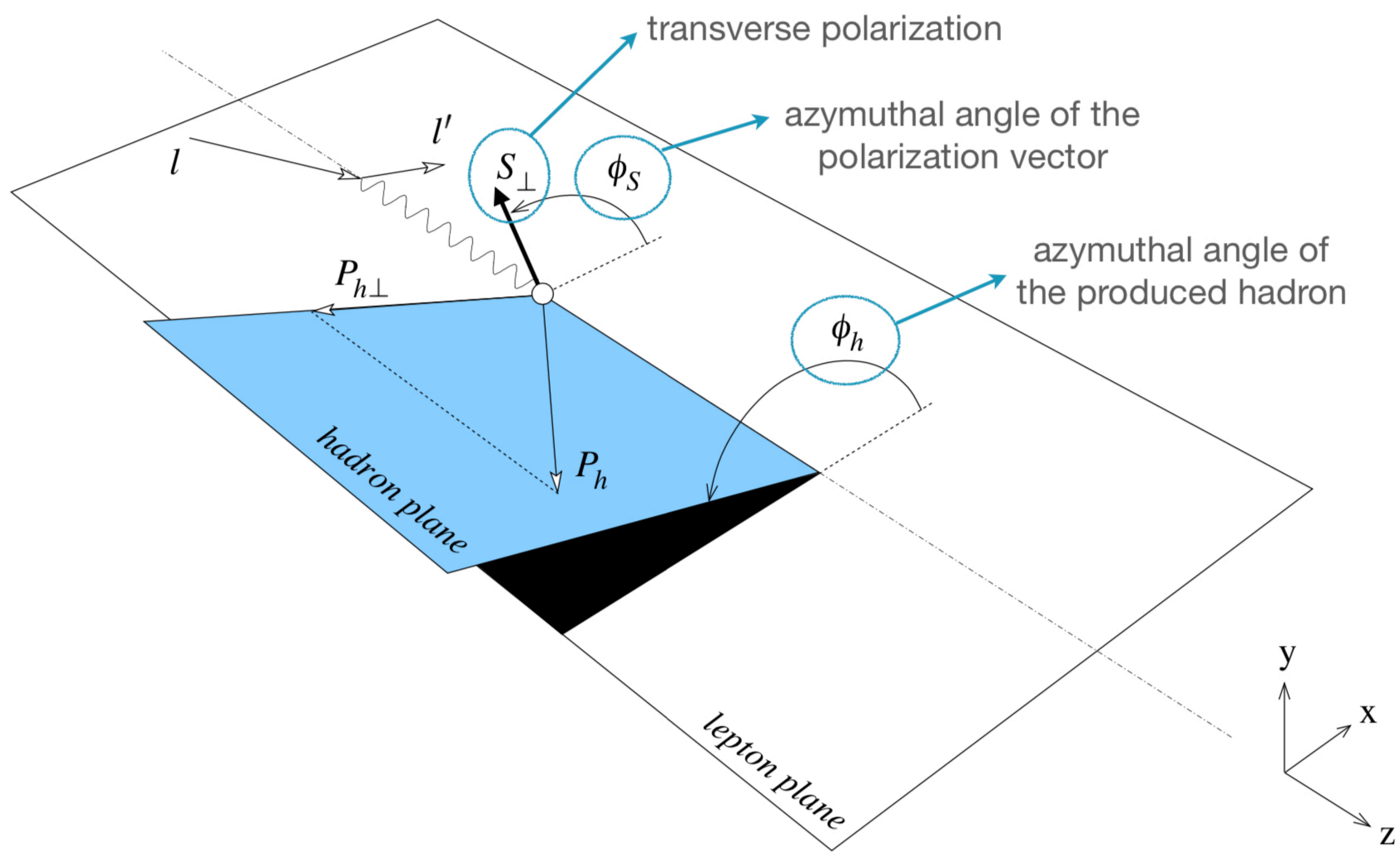
✓Universality

SIDIS

$$\ell(l,\lambda_\ell)+N(P,S)\to\ell(l',\lambda'_\ell)+h(P_h,S_h)+X$$

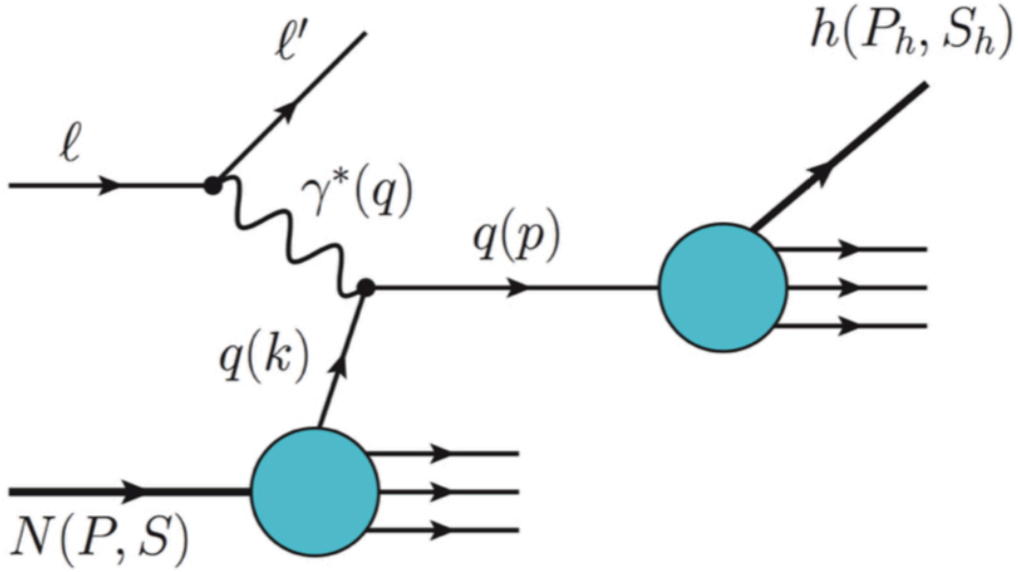
- 6 independent kinematical variables

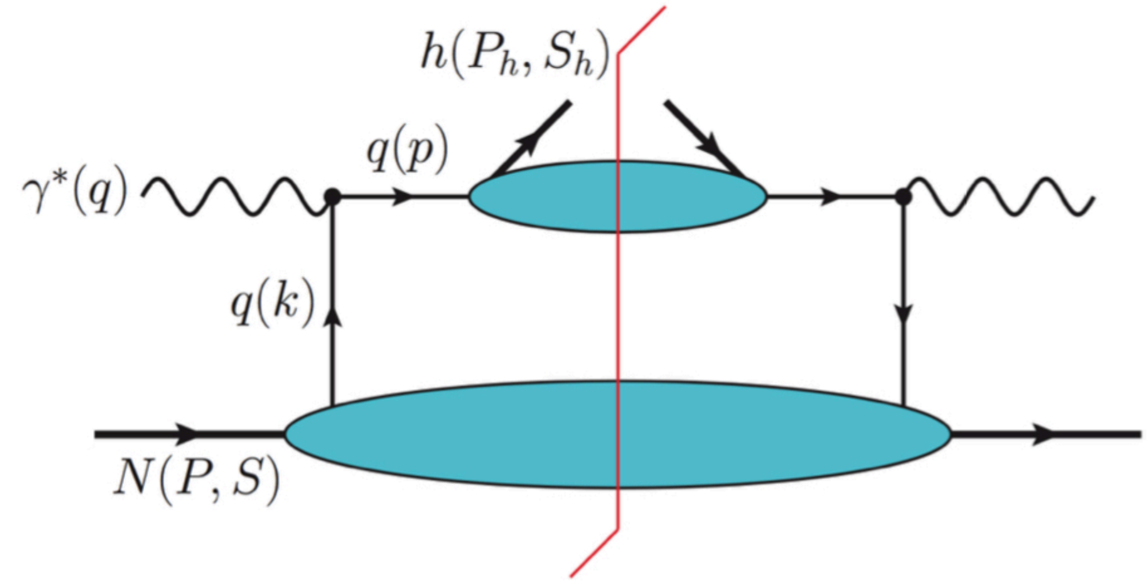
$$x_B=\frac{Q^2}{2P\cdot q} \qquad Q^2 \qquad \phi_S \qquad z_h=\frac{P\cdot P_h}{P\cdot q} \qquad P_{h\perp}=|\vec{P}_{h\perp}| \qquad \phi_h$$



$$d\sigma\sim L^{\mu\nu}W_{\mu\nu}$$

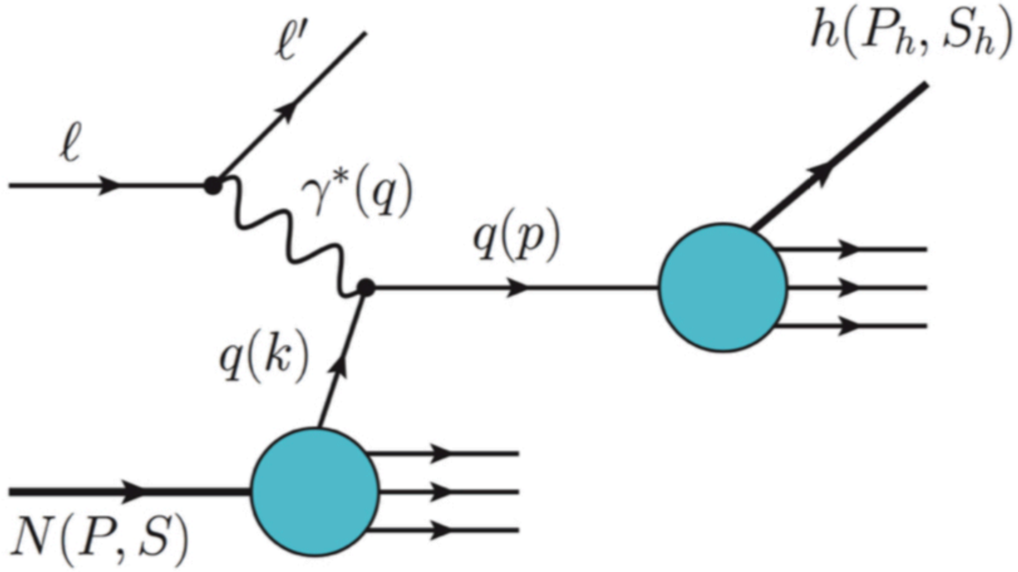
Hadronic tensor at tree level

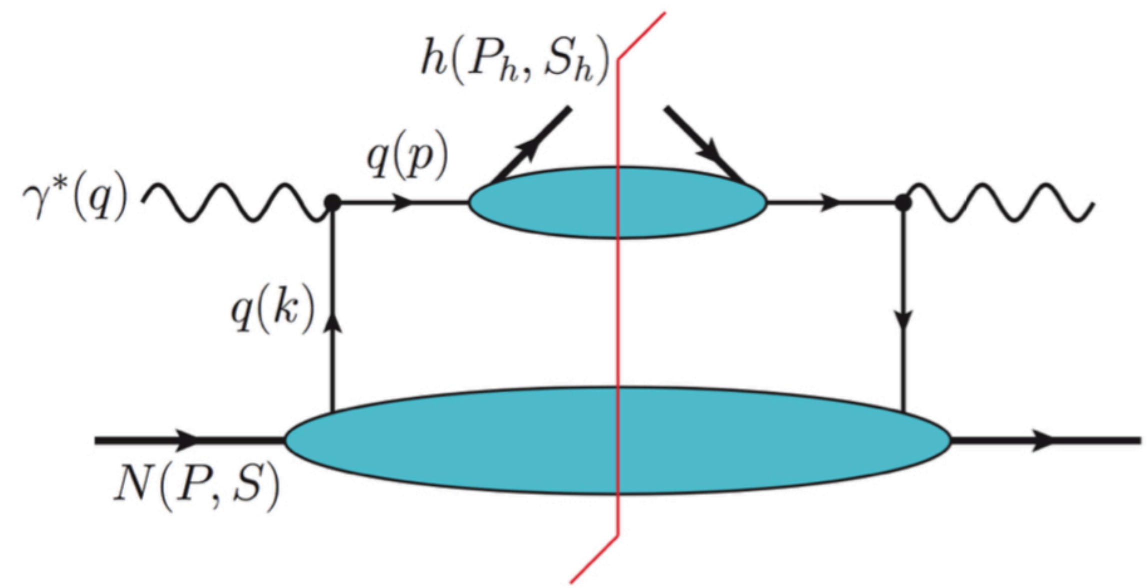




$$W^{\mu\nu} = \frac{1}{2M} \sum_X \int \frac{d^3 \vec{P}_X}{(2\pi)^3 2P_X^0} \delta^{(4)}(q + P - P_X - P_h)$$
$$\times \langle PS|J^\mu(0)|P_X; P_h S_h\rangle \langle P_X; P_h S_h|J^\nu(0)|PS\rangle$$

Hadronic tensor at tree level

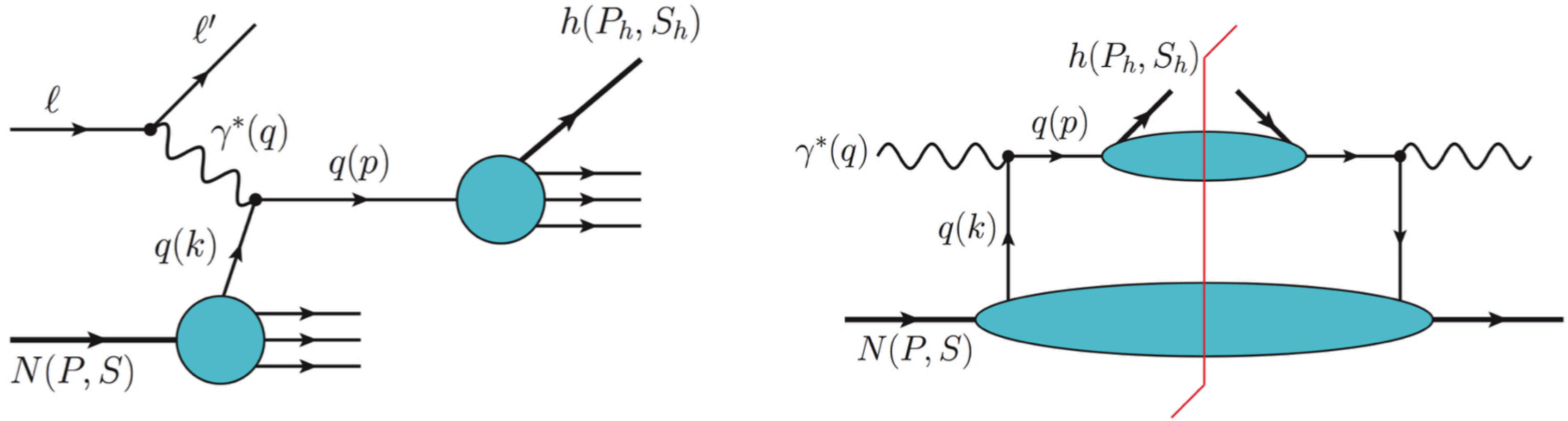




$$W^{\mu\nu} = \frac{1}{2M} \sum_X \int \frac{d^3 \vec{P}_X}{(2\pi)^3 2P_X^0} \delta^{(4)}(q + P - P_X - P_h)$$

$$J^\mu(x) =: \bar{\psi}\gamma^\mu\psi(x) : \longrightarrow \qquad \times \langle PS|J^\mu(0)|P_X; P_h S_h\rangle\langle P_X; P_h S_h|J^\nu(0)|PS\rangle$$

Hadronic tensor at tree level



$$W^{\mu\nu} = \frac{1}{2M} \sum_X \int \frac{d^3 \vec{P}_X}{(2\pi)^3 2P_X^0} \delta^{(4)}(q + P - P_X - P_h)$$

$$J^\mu(x) =: \bar{\psi}\gamma^\mu\psi(x) : \longrightarrow \qquad \times \langle PS|J^\mu(0)|P_X; P_h S_h\rangle\langle P_X; P_h S_h|J^\nu(0)|PS\rangle$$

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k \, d^4p \, \delta^{(4)}(k + q - p) \, \mathrm{Tr}\left[\Phi^q(k, P, S)\, \gamma^\mu \, \Delta^q(p, P_h)\, \gamma^\nu\right]$$

$$\Phi(k, P, S) = \frac{1}{(2\pi)^4} \int d^4z \, e^{ik\cdot z} \, \langle PS|\bar{\psi}(-\frac{z}{2})\, \psi(\frac{z}{2})|PS\rangle$$

$$\Delta(p, P_h) = \frac{1}{(2\pi)^4} \int d^4z \, e^{ip\cdot z} \, \langle 0|\psi(\frac{z}{2}) \sum_X |X; P_h S_h\rangle\langle X; P_h S_h|\bar{\psi}(-\frac{z}{2})|0\rangle$$

Hadronic tensor in SIDIS

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k\, d^4p\, \delta^{(4)}(k+q-p)\, \mathrm{Tr}\left[\Phi^q(k,P,S)\, \gamma^\mu\, \Delta^q(p,P_h)\, \gamma^\nu\right]$$

- consider P^+ and P_h^- large $\longrightarrow$ large $k^+ = xP^+,\; P_h^- = z_h p^-$
- consider frame with $\vec{P}_{h\perp} = 0$ and small $\vec{q}_\perp \neq 0$
- neglect small light-cone components of parton momenta $(k^-,\, p^+)$

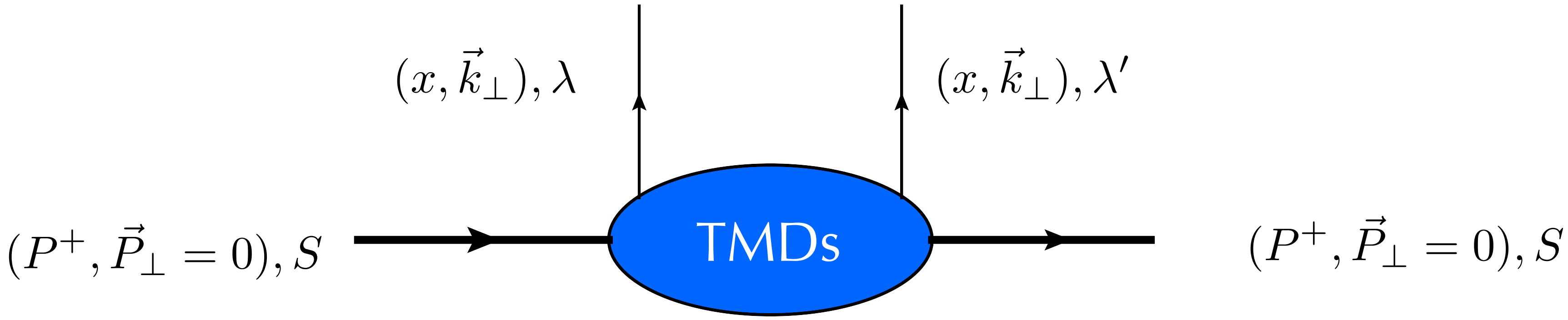
$$\delta^{(4)}(k+q-p) \approx \delta(k^+ + q^+)\delta(q^- - p^-)\delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)$$

$$W^{\mu\nu} \sim \frac{2x_B z_h}{Q^2} \sum_q e_q^2 \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)$$

$$\times \mathrm{Tr}\left[\int dk^-\, \Phi^q(k,P,S)\, \gamma^\mu \int dp^+\, \Delta(p,P_h)\, \gamma^\nu\right]\Bigg|_{\substack{k^+ = x_B P^+ \\ p^- = P_h^-/z_h}}$$

TMD Correlators

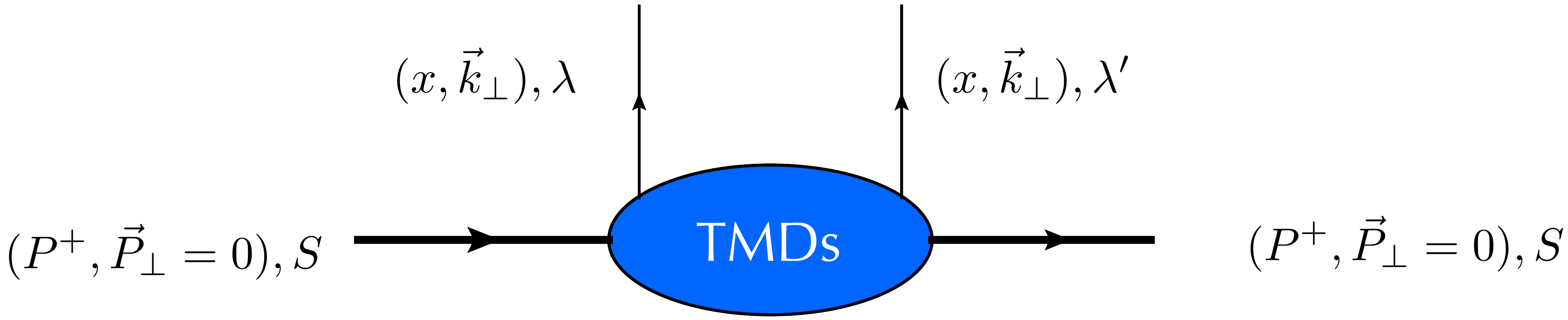
$$\Phi^q(x, \vec{k}_\perp, P, S) = \int dk^- \, \Phi^q(k, P, S) = \int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ik\cdot z} \langle P, S | \bar{\psi}(-\frac{z}{2}) \psi(\frac{z}{2}) | P, S \rangle \Big|_{z^+=0}$$



TMDs parametrize quark-quark correlator

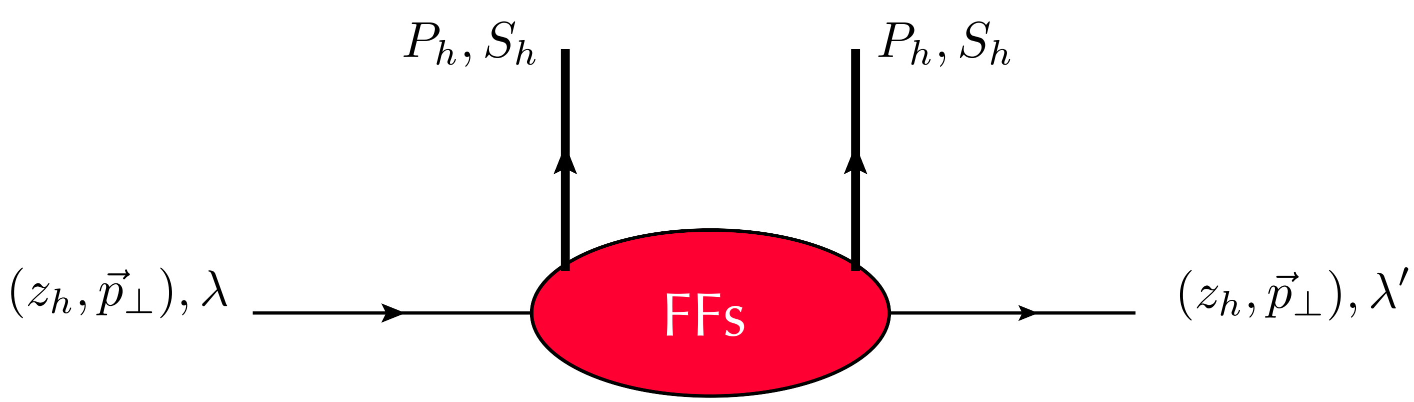
TMD Correlators

$$\Phi^q(x, \vec{k}_\perp, P, S) = \int dk^- \, \Phi^q(k, P, S) = \int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ik\cdot z} \langle P, S|\bar{\psi}(-\frac{z}{2})\psi(\frac{z}{2})|P, S\rangle\Big|_{z^+=0}$$



TMDs parametrize quark-quark correlator

$$\Delta(z_h, \vec{p}_\perp, P_h, S_h) = \int dp^+ \, \Delta(p, P_h, S_h) = \sum_X \int \frac{dz^+ d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ip\cdot z} \langle 0|\psi(\frac{z}{2})|P_h, S_h; X\rangle\langle P_h, S_h; X|\bar{\psi}(-\frac{z}{2})|0\rangle\Big|_{z^-=0}$$



FFs parametrize fragmentation correlator

SIDIS cross section

$$\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,dP_{h\perp}^2} \sim \Big\{ (1-y+\frac{1}{2}y^2)\, F_{UU,T} + (1-y)\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h}$$

$$+\Lambda(1-y)\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} + \lambda_l \Lambda y(1-\frac{1}{2}y)\, F_{LL}$$

$$+|\vec{S}_\perp|(1-y+\frac{1}{2}y^2)\sin(\phi_h-\phi_S)\, F_{UT,T}^{\sin(\phi_h-\phi_S)}$$

$$+|\vec{S}_\perp|(1-y)\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+|\vec{S}_\perp|(1-y)\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\lambda_\ell|\vec{S}_\perp|y(1-\frac{1}{2}y)\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \text{10 additional terms} \Big\}$$

- Structure functions depend on 4 variables: $F_i = F_i(x_B, z_h, P_{h\perp}^2, Q^2)$

$F_{XY,L(T)}^{\text{weight}}$

X beam polarization

Y target polarization

weight angular distribution of produced hadron

$L(T)$ virtual-photon polarization

Bacchetta, et al., JHEP 0702 (2007) 93

SIDIS structure functions at tree level

$$F_{UU,T} = x_B \sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp\, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)\, f_1(x, \vec{k}_\perp^{\,2})\, D_1(z_h, \vec{p}_\perp^{\,2})$$

$$F_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$$

$$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

$$F_{LL} \sim g_1 \otimes D_1$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^\perp \otimes D_1$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^\perp$$

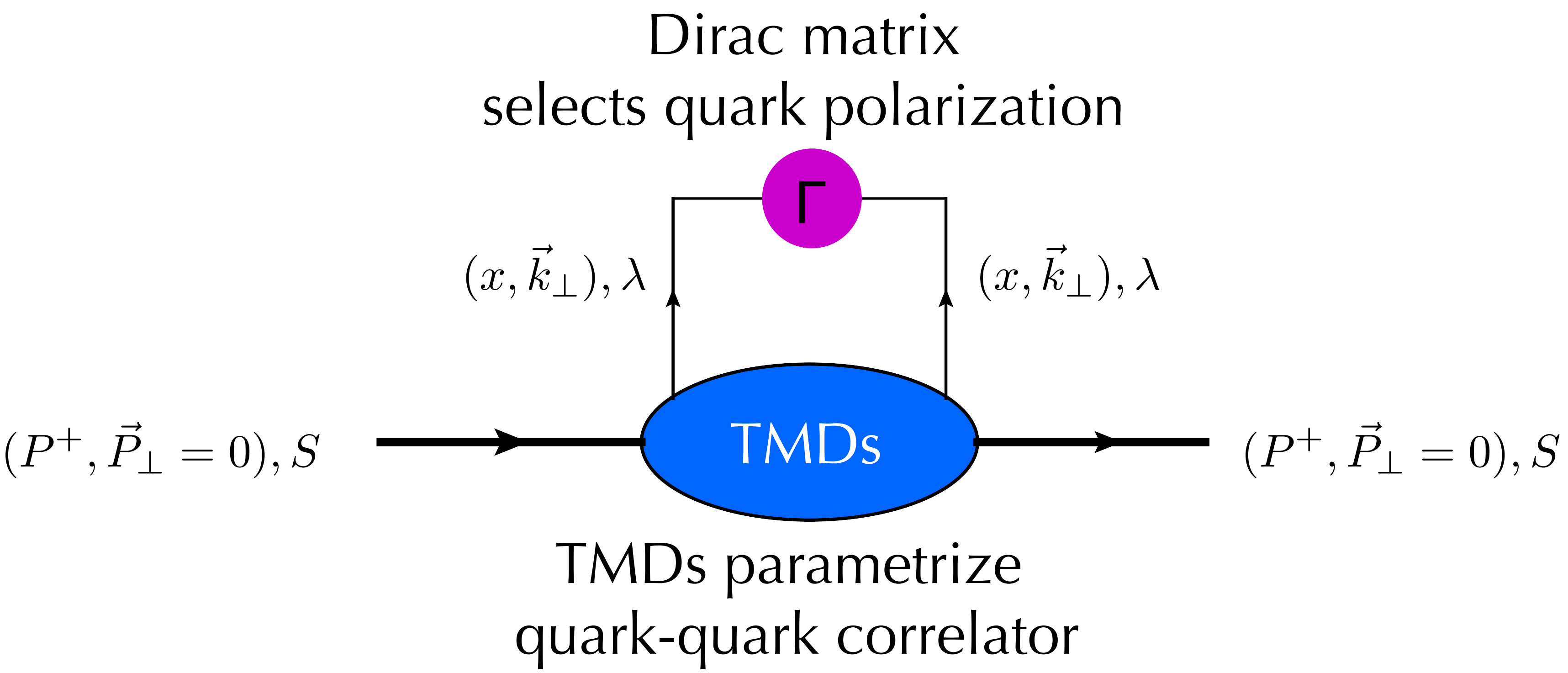
$$F_{UT,T}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$$

- transverse parton momenta of TMDs and FFs are convoluted
  (convolutions may contain additional powers of transverse parton momenta)

Bacchetta et. al, JHEP 0702 (2007) 93

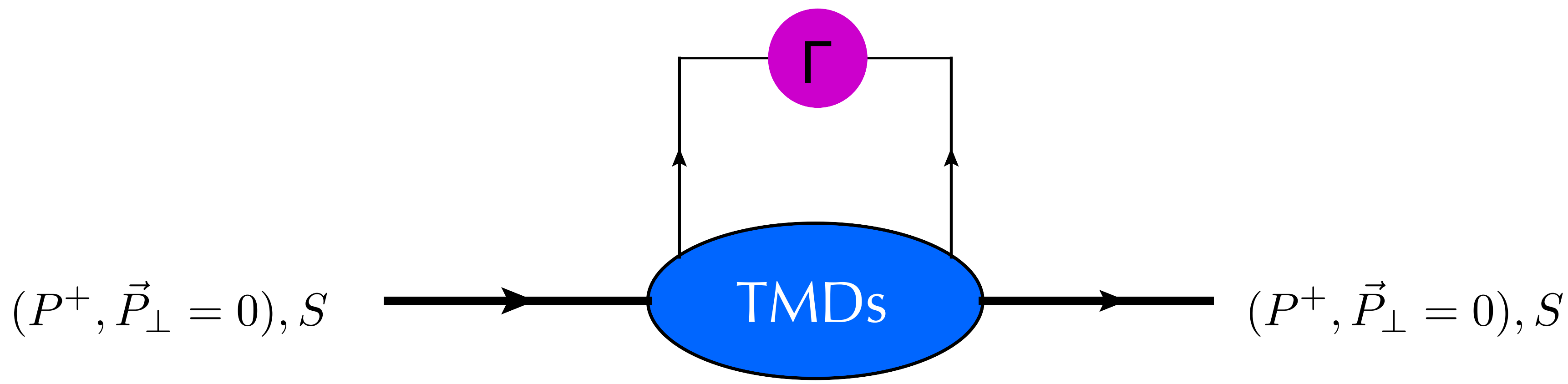
TMDs - PDFs of Quarks



$$\Phi_{ij}(x, k_\perp, S) = \int \frac{dz^- d^2 z_\perp}{2(2\pi)^3} e^{ik\cdot z} \langle p, S|\bar{\psi}_j(-\frac{z}{2})\, \mathcal{W}[-\frac{z}{2}, \frac{z}{2}]\psi_i(\frac{z}{2})|p, S\rangle\Big|_{z^+=0}$$

$$\Phi^{[\Gamma]} = \frac{1}{2}\mathrm{Tr}[\Phi\Gamma]$$

TMD-PDFs of quarks at leading twist



- leading-twist: $\Gamma = (\gamma^+,\, \gamma^+\gamma_5,\, i\sigma^{i+}\gamma_5)$
- spin four-vector: $S = \left(\frac{\Lambda\, P^+}{M}, -\frac{\Lambda\, P^-}{M}, \vec{S}_\perp\right) \qquad S^2 = \Lambda^2 - \vec{S}_\perp^{\,2} = -1 \qquad P\cdot S = 0$

$$
\begin{aligned}
\Phi^{q[\gamma^+]}(x,\vec{k}_\perp) &= f_1^q - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{M} f_{1T}^{\perp\, q} \\
\Phi^{q[\gamma^+\gamma_5]}(x,\vec{k}_\perp) &= \lambda\,\Lambda\, g_1^q + \frac{\lambda\, \vec{k}_\perp\cdot\vec{S}_\perp}{M} g_{1T}^{\perp\, q} \\
s_\perp^i\, \Phi^{q[i\sigma^{i+}\gamma_5]}(x,\vec{k}_\perp) &= \vec{s}_\perp\cdot\vec{S}_\perp\, h_1^q + \frac{\Lambda\, \vec{k}_\perp\cdot\vec{s}_\perp}{M} h_{1L}^{\perp\, q} - \frac{\epsilon_\perp^{ij} k_\perp^i s_\perp^j}{M} h_1^{\perp\, q} \\
&\quad + \frac{1}{2M^2}\left(2\vec{k}_\perp\cdot\vec{s}_\perp\, \vec{k}_\perp\cdot\vec{S}_\perp - \vec{k}^{\,2}\, \vec{s}_\perp\cdot\vec{S}_\perp\right) h_{1T}^{\perp\, q}
\end{aligned}
$$

- TMDs depend on x and $\vec{k}_\perp^{\,2}$

Partonic interpretation

$$\mathcal{O}^{\Gamma}_{\lambda'\lambda} = \int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \bar{\psi}^q_{\lambda'}(-\frac{z}{2})\, \Gamma\, \psi^q_{\lambda}(\frac{z}{2})\, e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)}$$

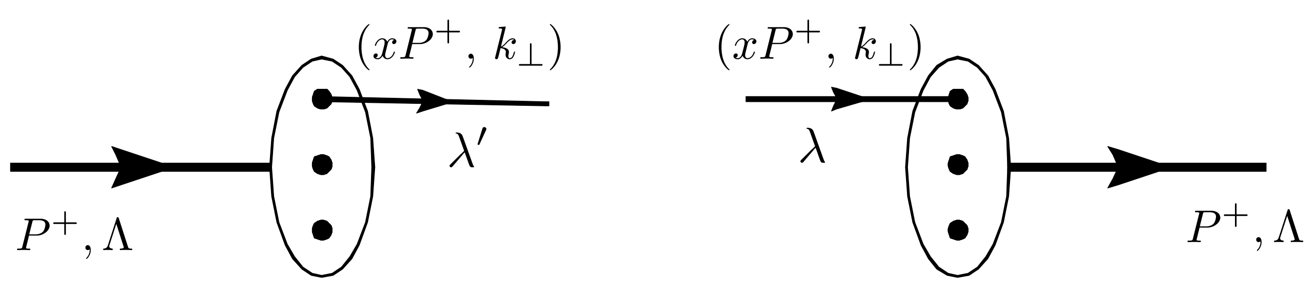
$\Longrightarrow$ insert Fourier expansion of quark field

$$\Gamma = \begin{cases} \gamma^+ & \Rightarrow \quad \psi^\dagger_{+,\uparrow}\psi_{+,\uparrow} + \psi^\dagger_{+,\downarrow}\psi_{+,\downarrow} \\ \gamma^+\gamma_5 & \Rightarrow \quad \psi^\dagger_{+,\uparrow}\psi_{+,\uparrow} - \psi^\dagger_{+,\downarrow}\psi_{+,\downarrow} \\ i\sigma^{i+}\gamma_5 & \Rightarrow \quad \psi^\dagger_{+,s_i}\psi_{+,s_i} - \psi^\dagger_{+,-s_i}\psi_{+,-s_i} \end{cases}$$

quark-number density

quark-helicity density

transverse-spin density

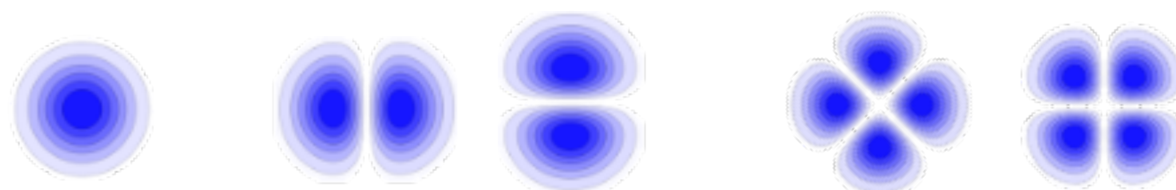


- Density interpretation spoiled by QCD effects (radiative corrections)

TMDs and their probabilistic interpretation

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$



unpolarized
target and partons

deformation due to
spin-spin and spin-orbit correlations

- TMDs in black survive transverse-momentum integration
- TMDs in red are T-odd (change sign in SIDIS and DY processes)
- TMDs in blue require OAM transfer
- No effects for U/L and L/U polarizations due to parity invariance

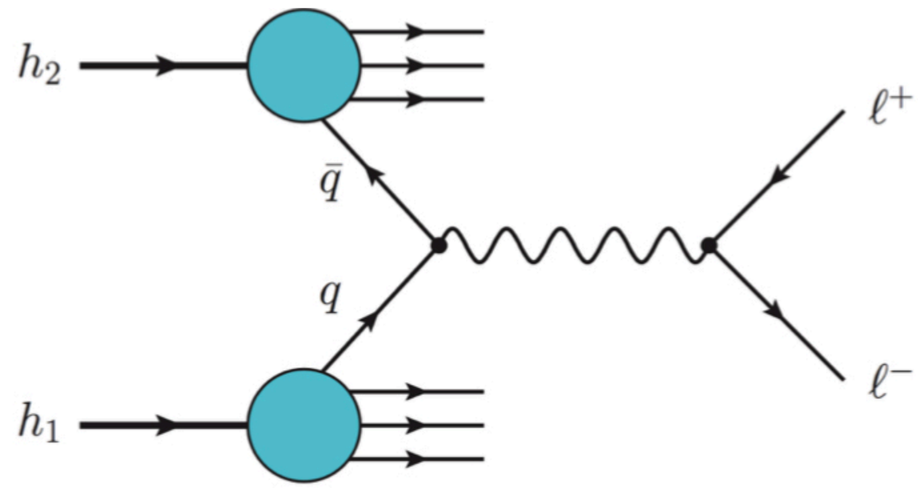
Quark fragmentation functions

final hadron pol. \ quark polarization	U	L	T
U	D_1		$H_1^\perp$
L		G_1	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_1,\ H_{1T}^\perp$

- Same interpretation as for TMDs, but with the role of quark and hadron interchanged
- FFs in red are T-odd

Drell-Yan process at tree level

$$h_1 + h_2 \to l^+ + l^- + X$$



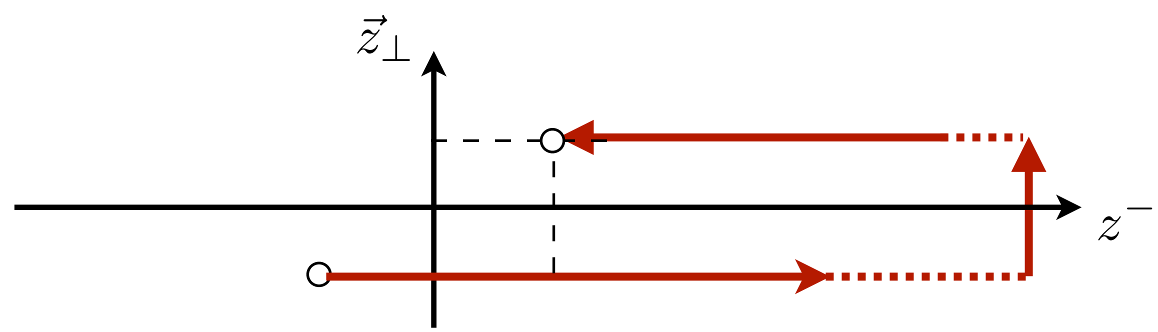
- Hadronic tensor at tree level, for low $\vec{q}_\perp$ of gauge boson

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{k}_{a\perp}\, d^2\vec{k}_{b\perp}\, \delta^{(2)}(\vec{k}_{a\perp} + \vec{k}_{b\perp} - \vec{q}_\perp)$$
$$\times \mathrm{Tr}\left[\Phi^q(x_a, \vec{k}_{a\perp}, P_1, S_1)\, \gamma^\mu\, \Phi^{\bar{q}}(x_b, \vec{k}_{b\perp}, P_2, S_2)\, \gamma^\nu\right]\Big|_{\substack{k_a^+ = x_B P_1^+ \\ k_b^+ = x_b P_2^+}}$$

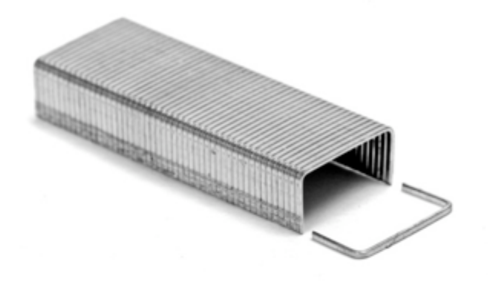
- It involves two TMD-PDFs
- transverse parton momenta are convoluted
- longitudinal momentum fractions fixed by the kinematics
- cross section parametrised by 48 structure functions
  [Arnold, Metz, Schlegel, PRD 79 (2009) 489]

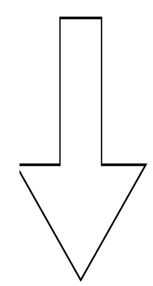
Need of the Gauge-Link

$$\Phi^{[\Gamma]}(x, \vec{k}_\perp, P, S) = \frac{1}{2} \int \frac{dz^- d^2 \vec{z}_\perp}{(2\pi)^3} \, e^{ik\cdot z} \langle P, S | \bar{\psi}(-\frac{z}{2}) \, \Gamma \, \boxed{\text{Gauge Link}} \, \psi(\frac{z}{2}) | P, S \rangle \Big|_{z^+=0}$$

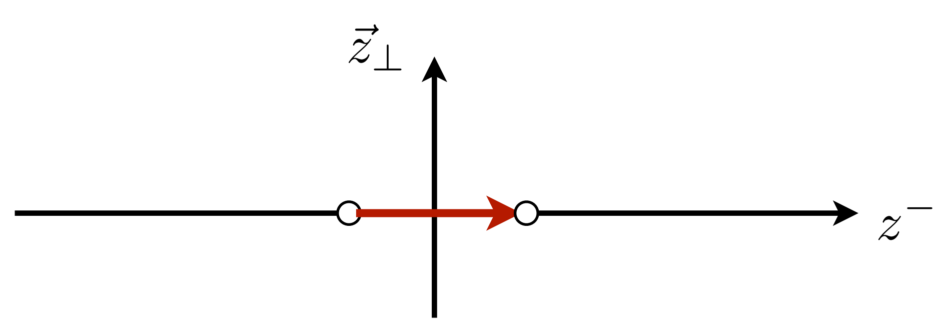


The staple gauge-link





$k_\perp$ integration



Light-front wave function representation

Proton state

Probability Amplitude for the N,β Fock state

$$|(P^+, \vec{P}_\perp), \Lambda\rangle = \sum_{N,\beta} [dx]_N [d\vec{k}_\perp]_N \Psi^\Lambda_{N,\beta}(x_i, \vec{k}_{\perp\, i}) |N, \beta; (x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp\, i}), \lambda_i\rangle$$

Light-front wave functions

Internal variables: $x_i = \dfrac{p_i^+}{P^+}$ $\qquad \sum_{i=1}^N x_i = 1 \qquad \sum_{i=1}^N \vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz} \Psi^\Lambda_{\lambda_1 \ldots \lambda_N} = \lambda_i \Psi^\Lambda_{\lambda_1 \ldots \lambda_N}$$

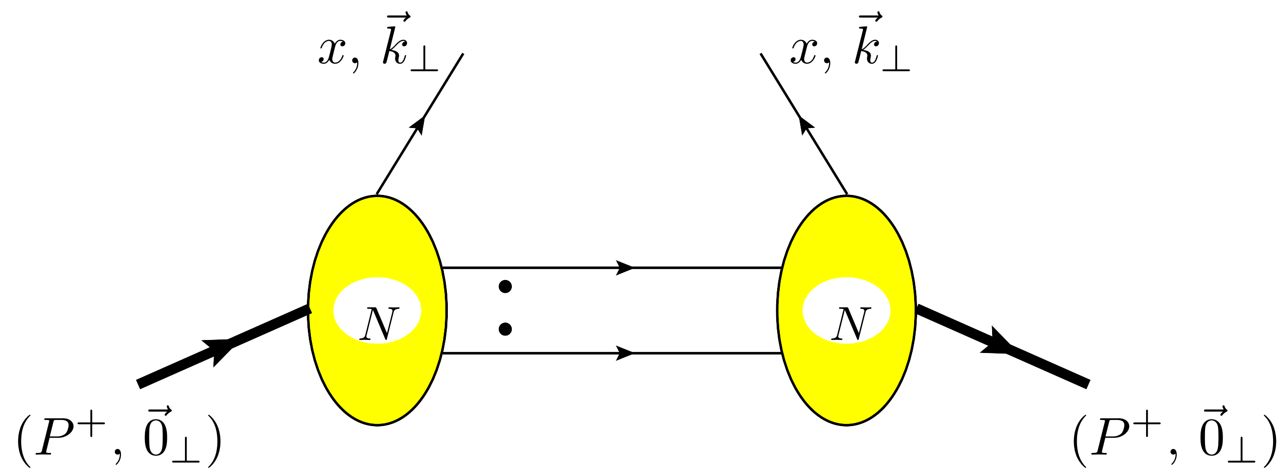
Eigenstates of total OAM

$$\hat{L}_z \Psi^\Lambda_{\lambda_1 \ldots \lambda_N} = \ell_z \Psi^\Lambda_{\lambda_1 \ldots \lambda_N}$$

$$\boxed{\Lambda = \sum_{i=1}^N \lambda_i + \ell_z}$$

$A^+ = 0$ **gauge**

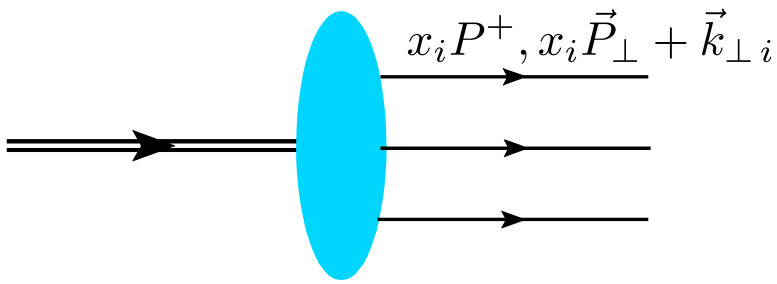
Light-Front Wave Function Overlap Representation



$$\text{TMDs} \sim \sum_N \int [dx]_N \, |\Psi_N(k_N)|^2 \delta(\ldots)$$ probability density in 3D momentum space

$$\text{PDFs} \sim \sum_N \int [d^3k]_N \, |\Psi_N(k_N)|^2 \delta(\ldots)$$ probability density in 1D momentum space

Quark-OAM: partial wave decomposition of LFWF



$$| P, \Lambda\rangle = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\ \Psi^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i})\ \frac{\varepsilon^{ijk}}{\sqrt{6}}\, u^\dagger_{i\lambda_1}(1) u^\dagger_{j\lambda_2}(2) d^\dagger_{k\lambda_3}(3) |\, 0\rangle$$

$\downarrow$

LCWF: eigenstate of OAM

J_z^q	$\rightarrow$	$(\uparrow\uparrow\uparrow)_{LC} = \frac{3}{2}$	$(\uparrow\uparrow\downarrow)_{LC} = \frac{1}{2}$	$(\uparrow\downarrow\downarrow)_{LC} = -\frac{1}{2}$	$(\downarrow\downarrow\downarrow)_{LC} = -\frac{3}{2}$
$L_z^q = \frac{1}{2} - J_z^q$	$\rightarrow$	$L_z^q = -1$	$L_z^q = 0$	$L_z^q = 1$	$L_z^q = 2$

${}^{L_z}\langle P, \uparrow | P, \uparrow\rangle^{L_z}$:probability to find the proton in a state with eigenvalue of OAM L_z

$\Downarrow$

$$\mathcal{L}_z = \sum_{L_z} L_z\ {}^{L_z}\langle P, \uparrow | P, \uparrow\rangle^{L_z}$$

$\downarrow$

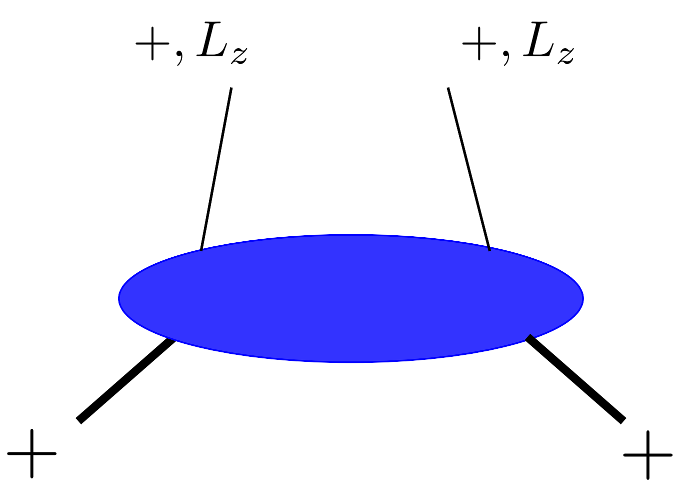
squared of LFWFs

$A^+ = 0$ **gauge**

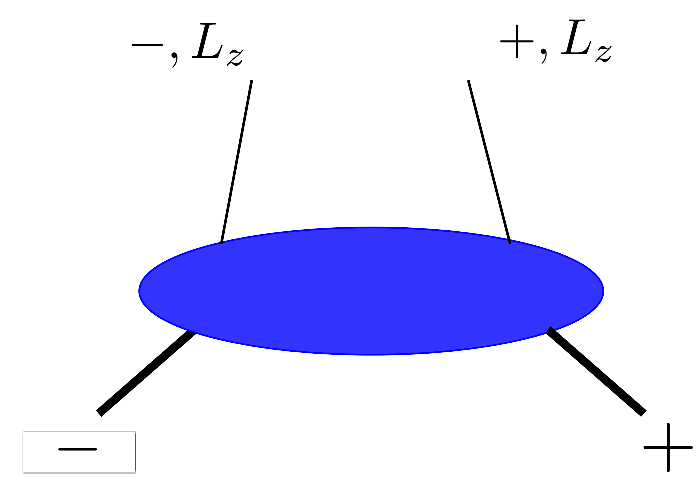
OAM decomposition of T-even TMDs

$\Delta J = \Delta J^q + \Delta L_z^q$ total angular momentum conservation

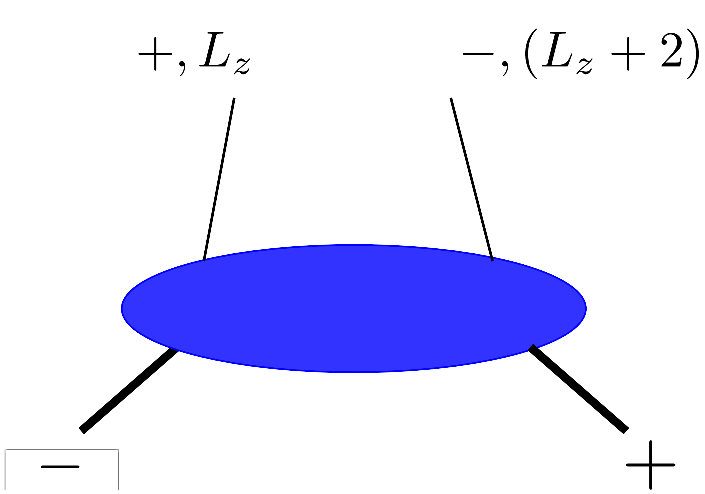
f_1, g_{1L}



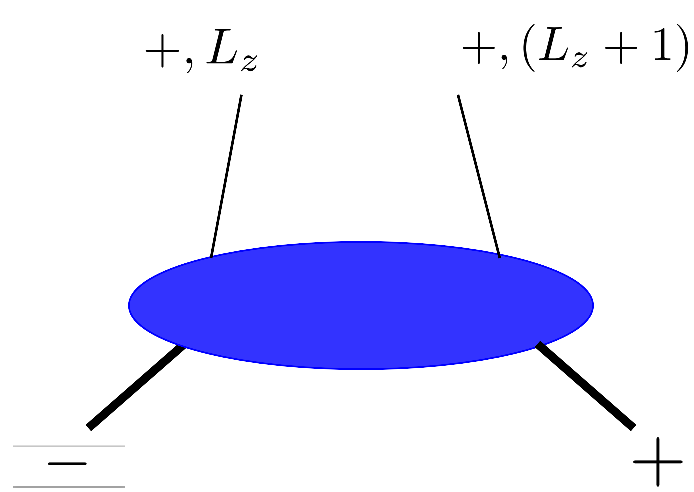
h_1



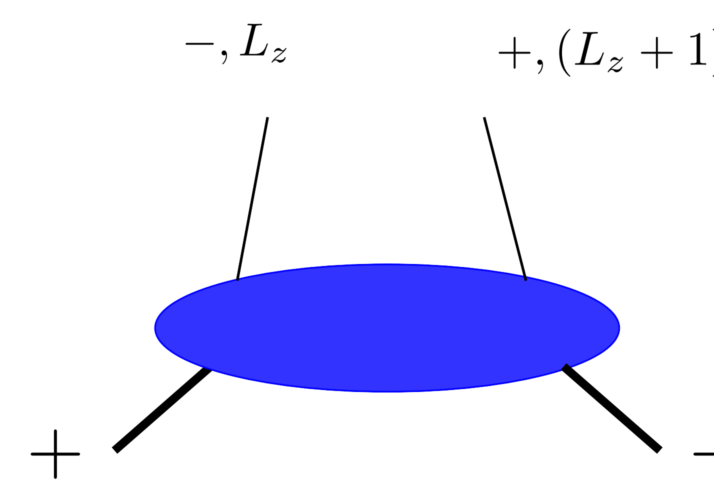
$h_{1T}^{\perp}$



g_{1T}

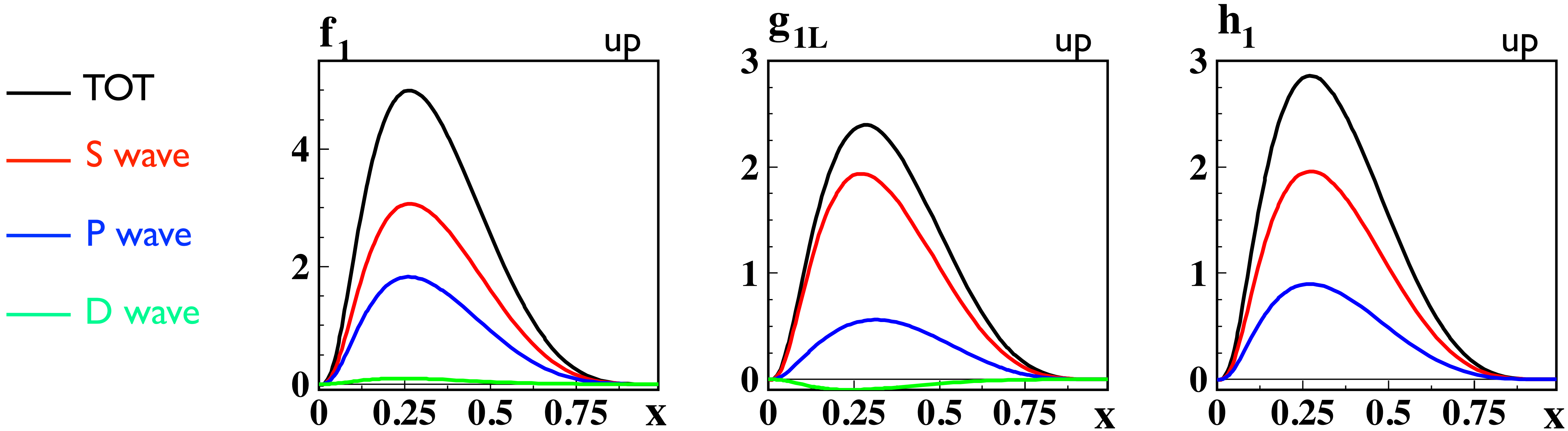


$h_{1L}^{\perp}$

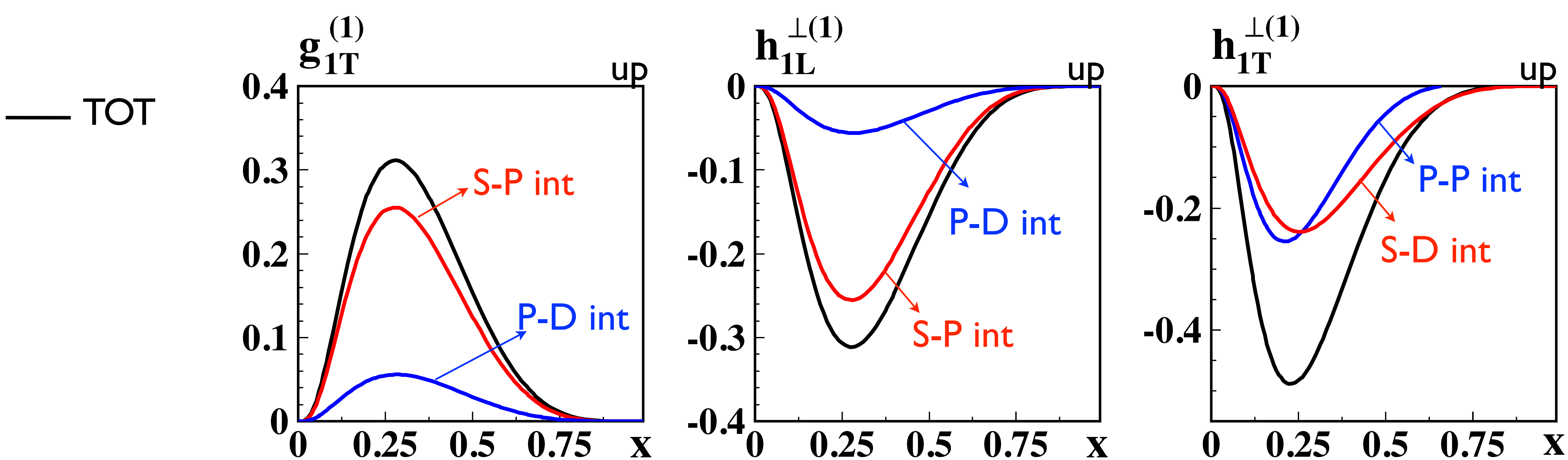


OAM content of TMDs

Model results with light-front wave functions fitted to nucleon electromagnetic form factors

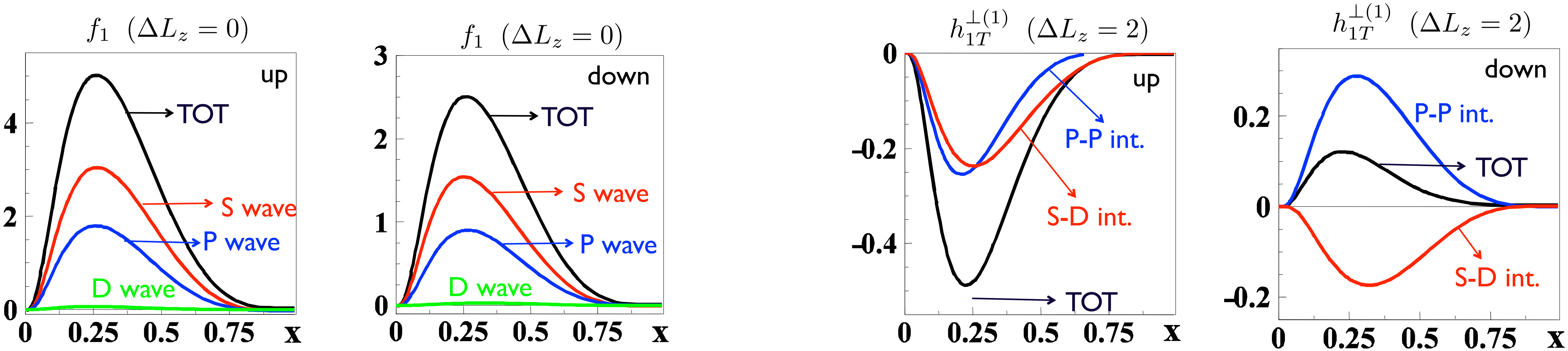


$j^{(1)}(x) = \int d^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, j(x, k_\perp^2)$

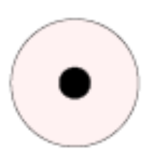


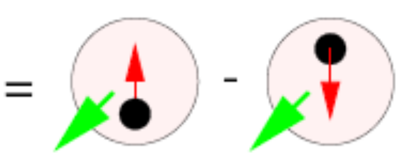
BP, Cazzaniga, Boffi, PRD78 (2008)

OAM content of TMDs in observables

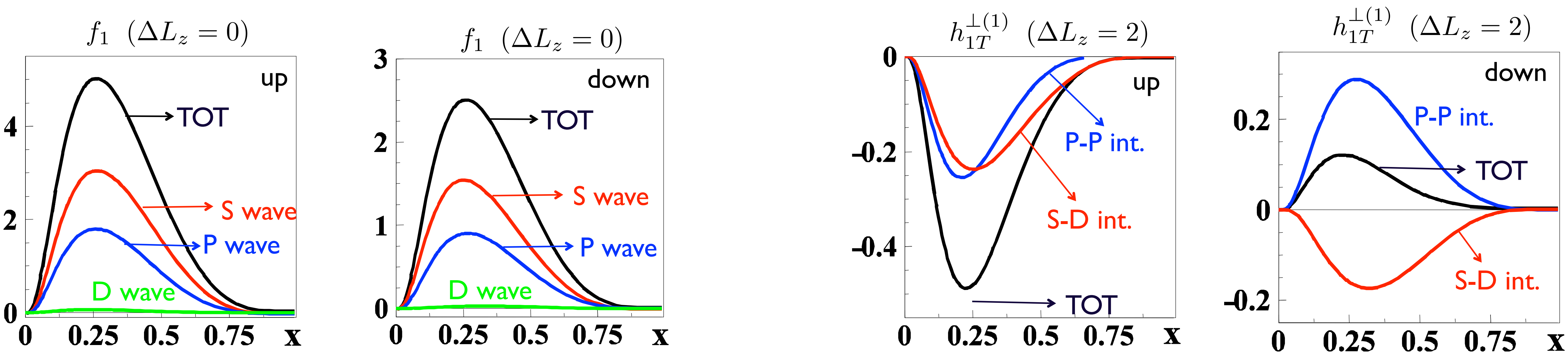


"pretzelosity"

f_1 = 

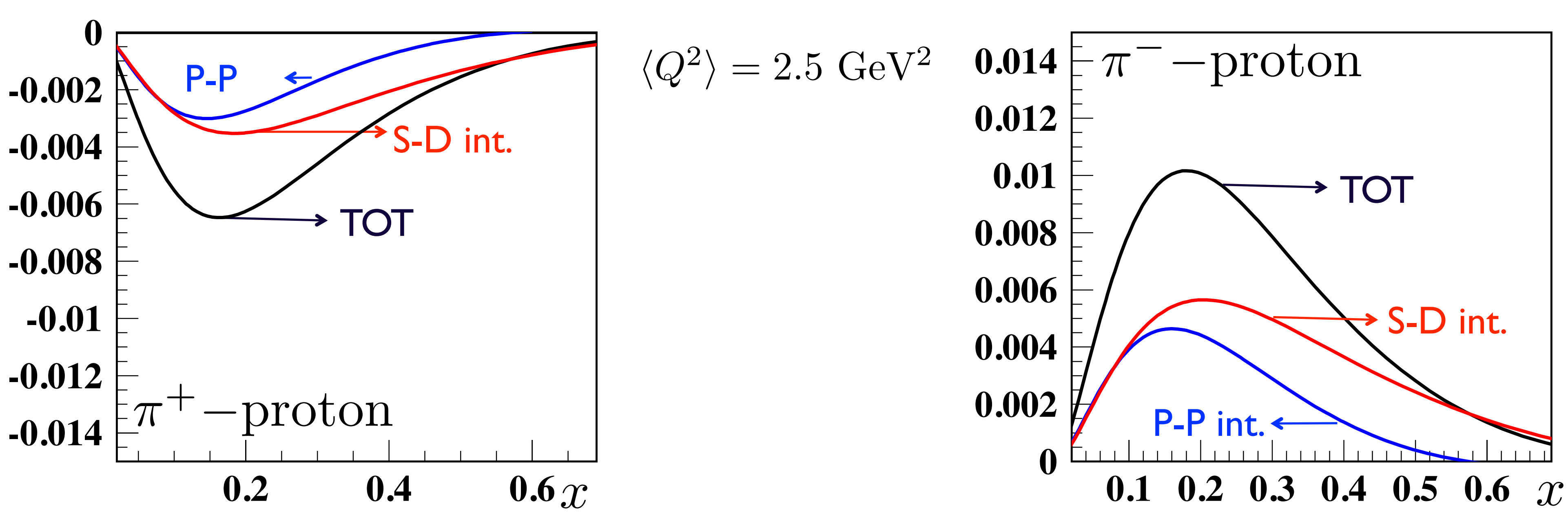
$h_{1T}^{\perp}$ = 

OAM content of TMDs in observables



✦ Effects on SIDIS observables

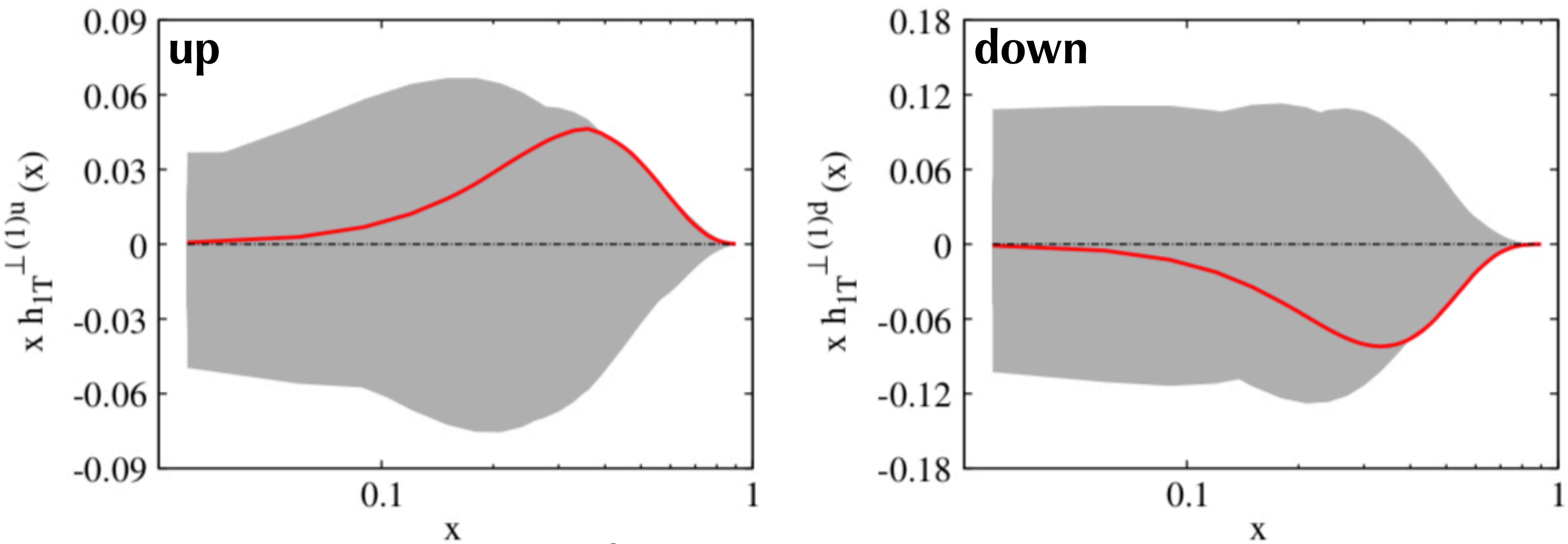
$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



Boffi, Efremov, BP, Schweitzer, PRD79(2009)

First attempt to extract pretzelosity from data

Lefky, Prokudin, PRD91(2015) 034010

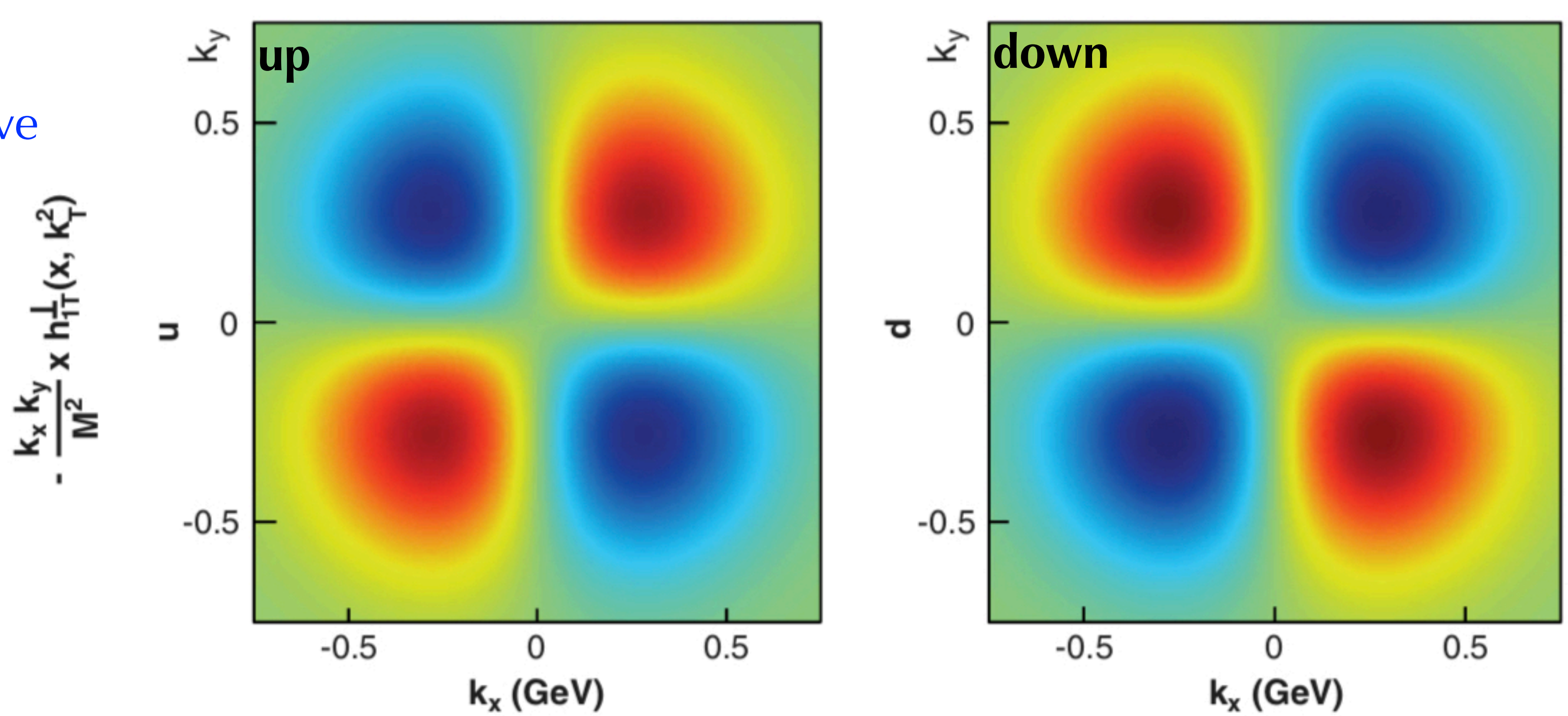


$Q^2 = 2.4\ \mathrm{GeV}^2$

convolution in SIDIS with $P^3_{h,\perp}$ factor $\longrightarrow$ suppressed for $\langle P_{h,\perp}\rangle < 1\,\mathrm{GeV}$

red: positive

blue: negative

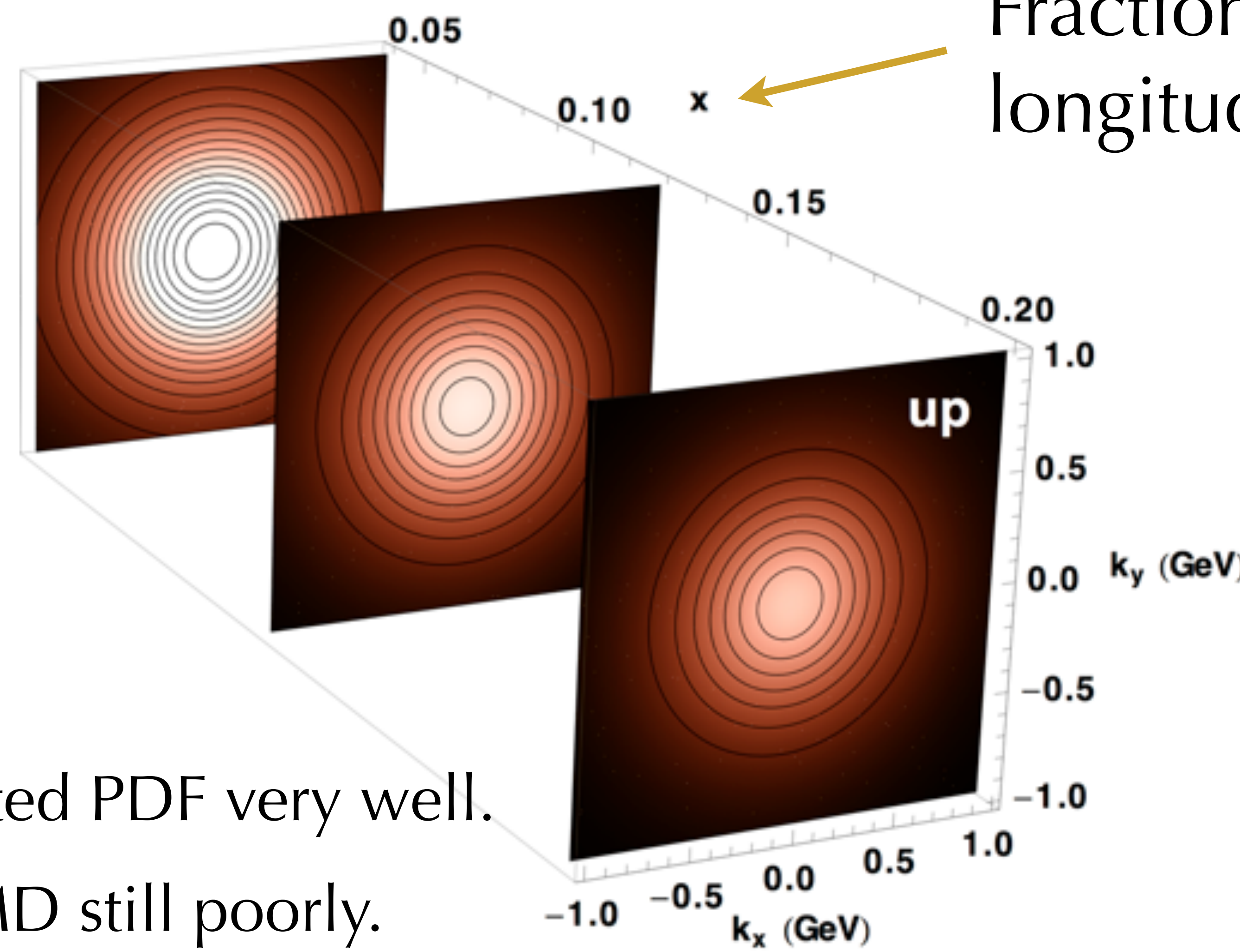


$x = 0.1$

future measurements will be very important to clarify the sign and size of the pretzelosity

The unpolarized TMD f_1

Fraction of longitudinal momentum



Transverse momentum

We know the integrated PDF very well.

We aim to the the TMD still poorly.

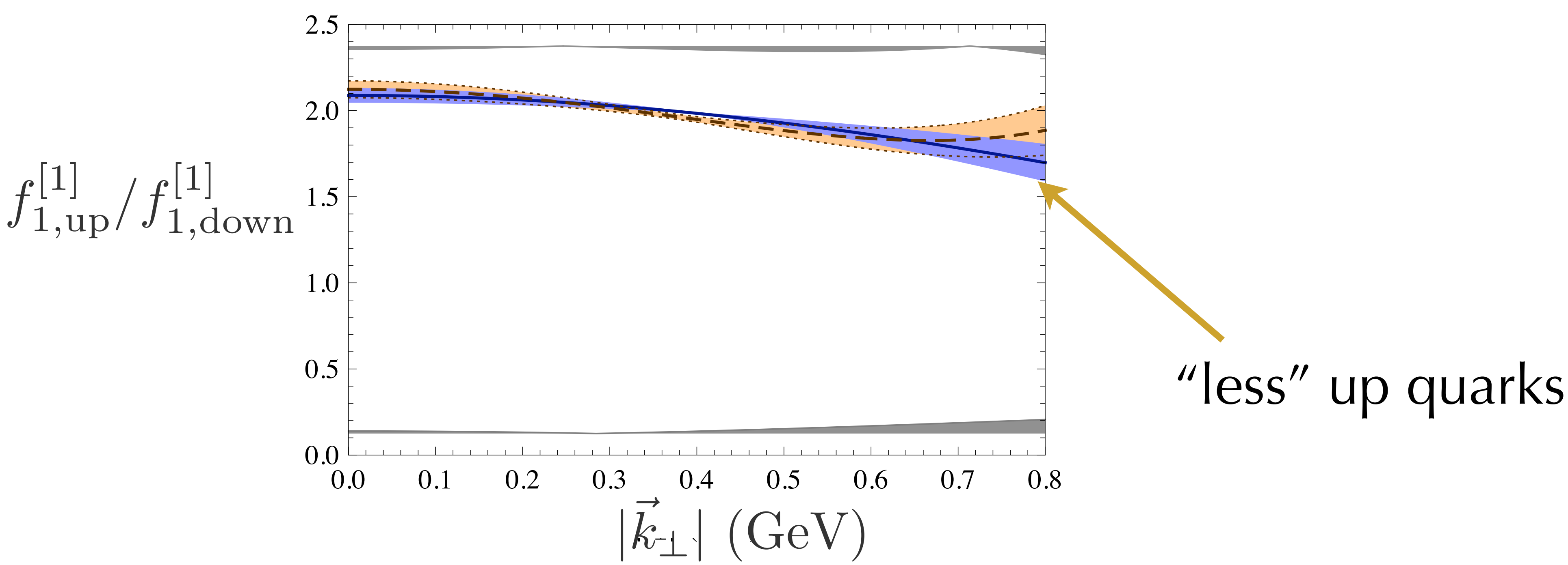
Correlation between x and $k_\perp$
Flavor dependence of TMDs

Flavor structure of TMDs: indications from lattice QCD

$$f_{1,\,q}^{[1]}(\vec{k}_\perp^2) = \int_0^1 \mathrm{d}x\,\left(f_{1,\,q}(x,\vec{k}_\perp^2) - f_{1,\,\bar{q}}(x,\vec{k}_\perp^2)\right)$$

number of quarks as function of transverse momentum

$$\frac{\int d^2\vec{k}_\perp^{\,2}\, f_{1,\mathrm{up}}^{[1]}(\vec{k}_\perp^{\,2})}{\int d^2\vec{k}_\perp^{\,2}\, f_{1,\mathrm{down}}^{[1]}(\vec{k}_\perp^{\,2})} = \frac{n_{\mathrm{up}}}{n_{\mathrm{down}}} = 2$$

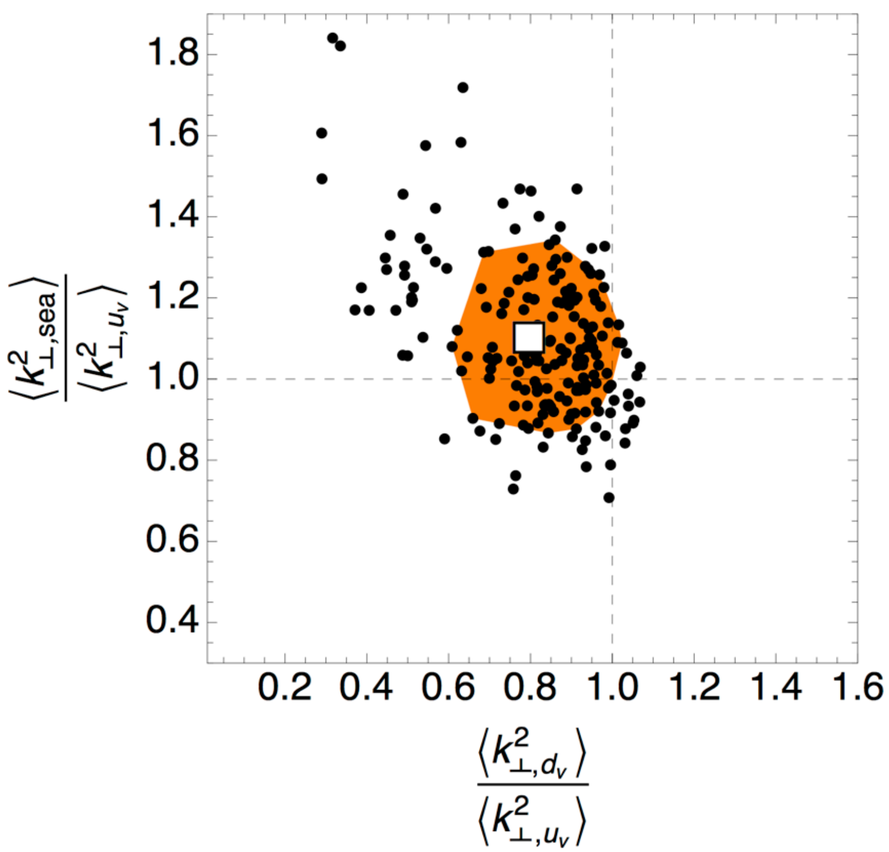


Pioneering lattice-QCD studies hint at a down distribution being wider than up

*Musch, Hagler, Negele, Schaefer, PRD**83** (2011) 094507*

Flavor structure of TMDs: indications from data

Ratio of width of sea / width of up valence



Ratio width of down valence/ width of up valence

fit to SIDIS multiplicities from HERMES:

$$\langle k^2_{\perp,d_v}\rangle < \langle k^2_{\perp,u_v}\rangle < \langle k^2_{\perp,sea}\rangle$$

Signori,et.al., JHEP 1311 (13)

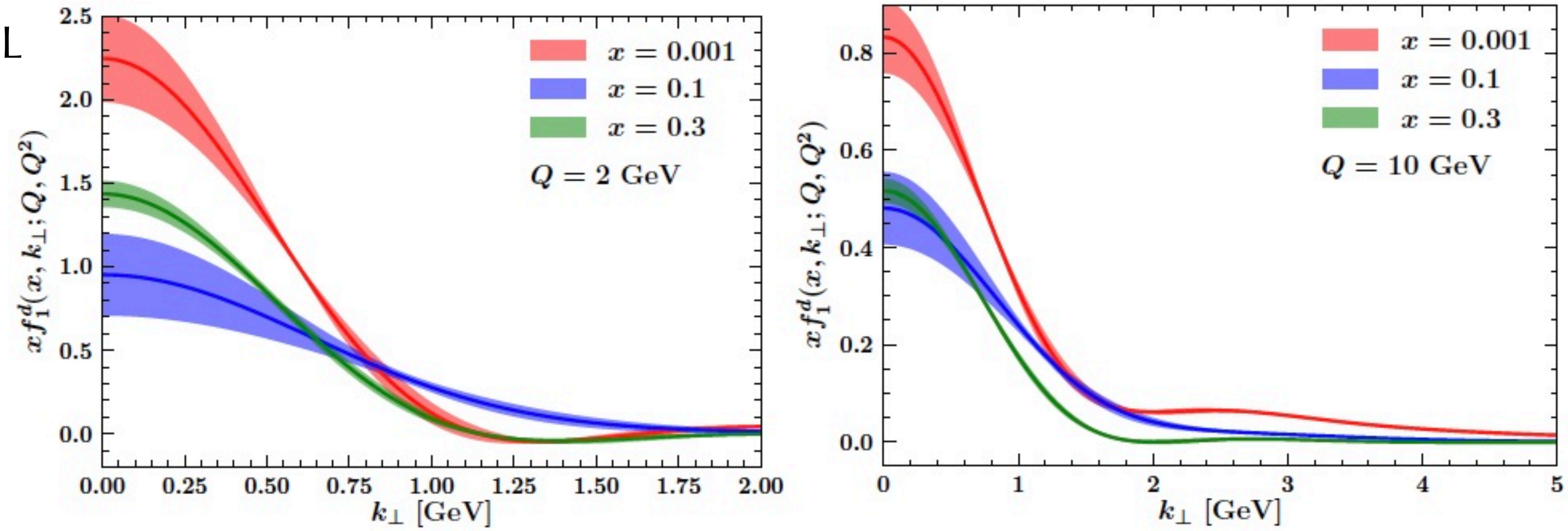
There is room for flavour dependence, but we do not control it well

Quark unpolarized TMD extractions

	Framework	HERMES	COMPASS	DY	Z Production	N of points
Pavia 2016 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL	✗	✗	✓	✓	457
Pavia 2019 arXiv:1912.07550	NNNLL	✗	✗	✓	✓	353
SV 2020 arXiv:1912.06532	NNNLL	✓	✓	✓	✓	1039
MAP 2022 in progress	NNNLL	✓	✓	✓	✓	>1500

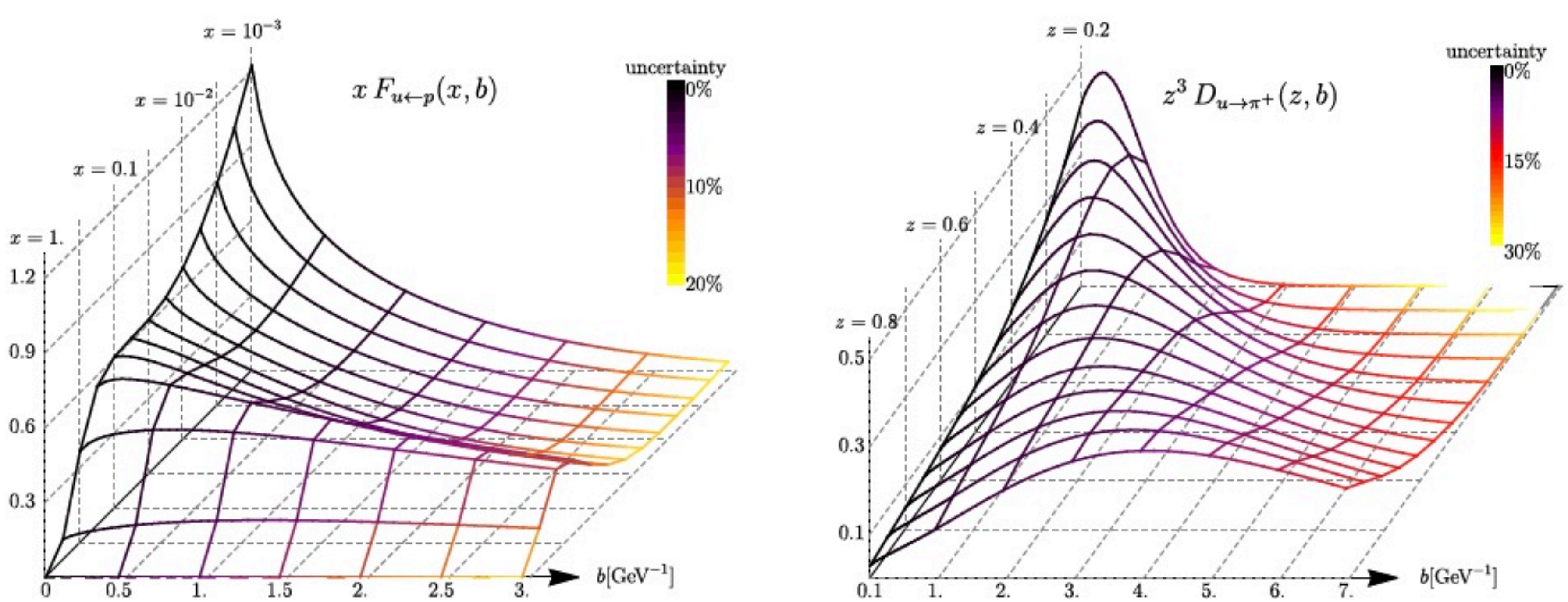
Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$

DY data at NNNLL



Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, JHEP 07 (2020) 117

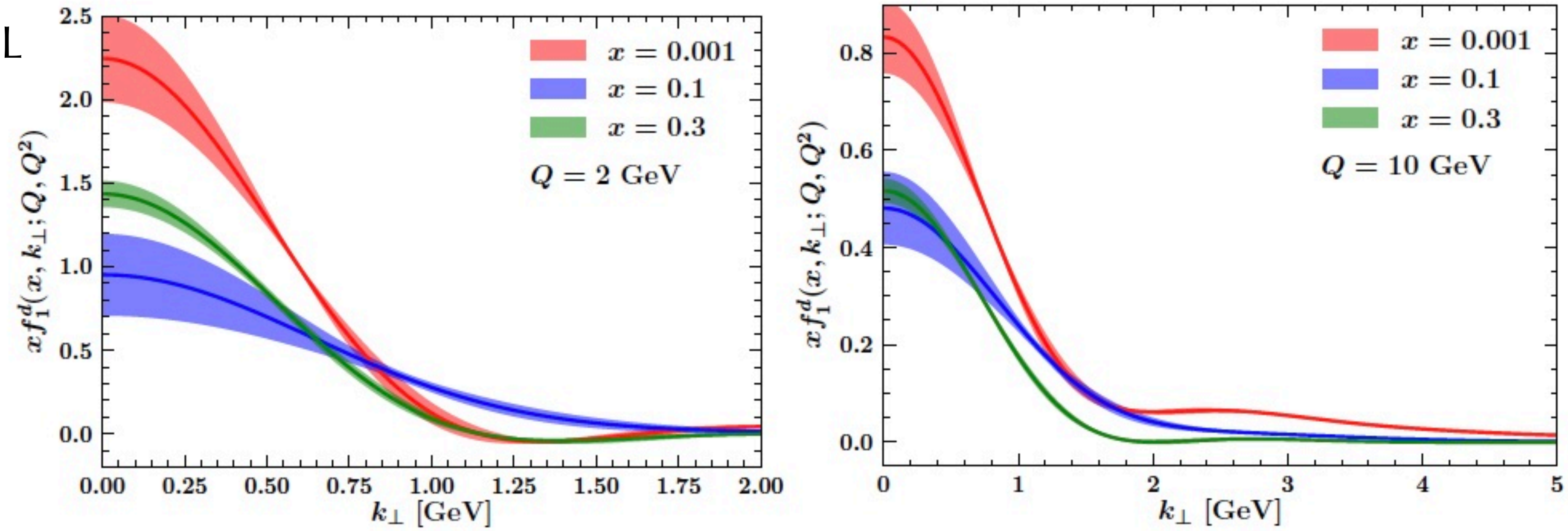
DY+ SIDIS data
at NNNLL



Scimemi, Vladimirov, JHEP 06 (2020) 137

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$

DY data at NNNLL

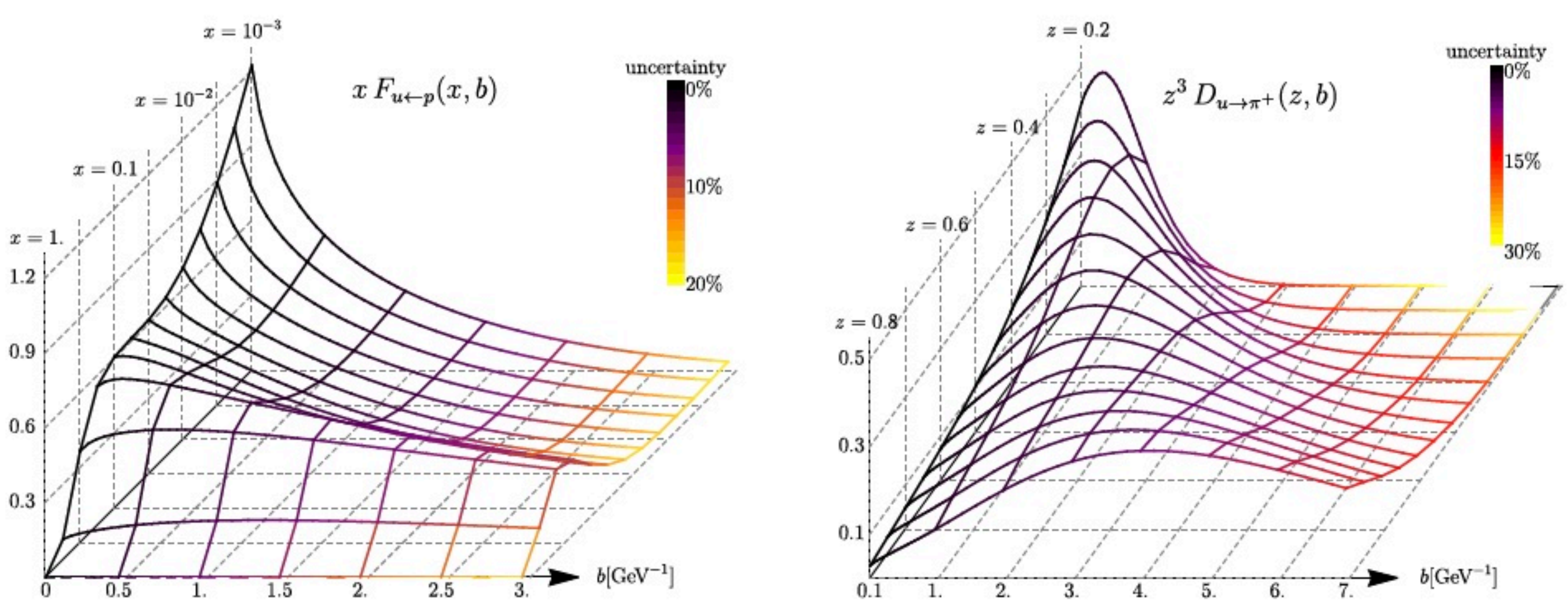


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, JHEP 07 (2020) 117

Open issues:

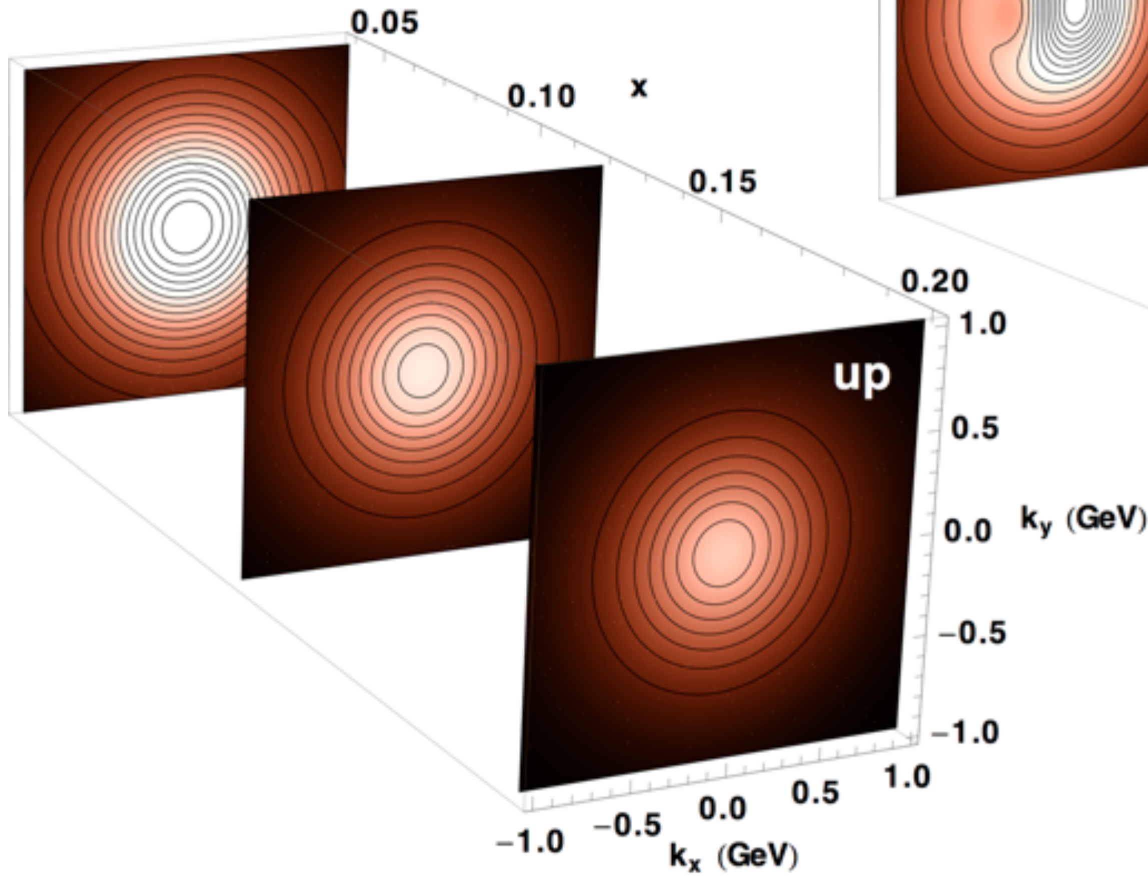
- Flavor dependence and more flexible functional forms
- Different choices in implementation of TMD formalism
- More data needed to test the formalism and functional form of parametrizations
- Improvements on the knowledge of the fragmentation functions

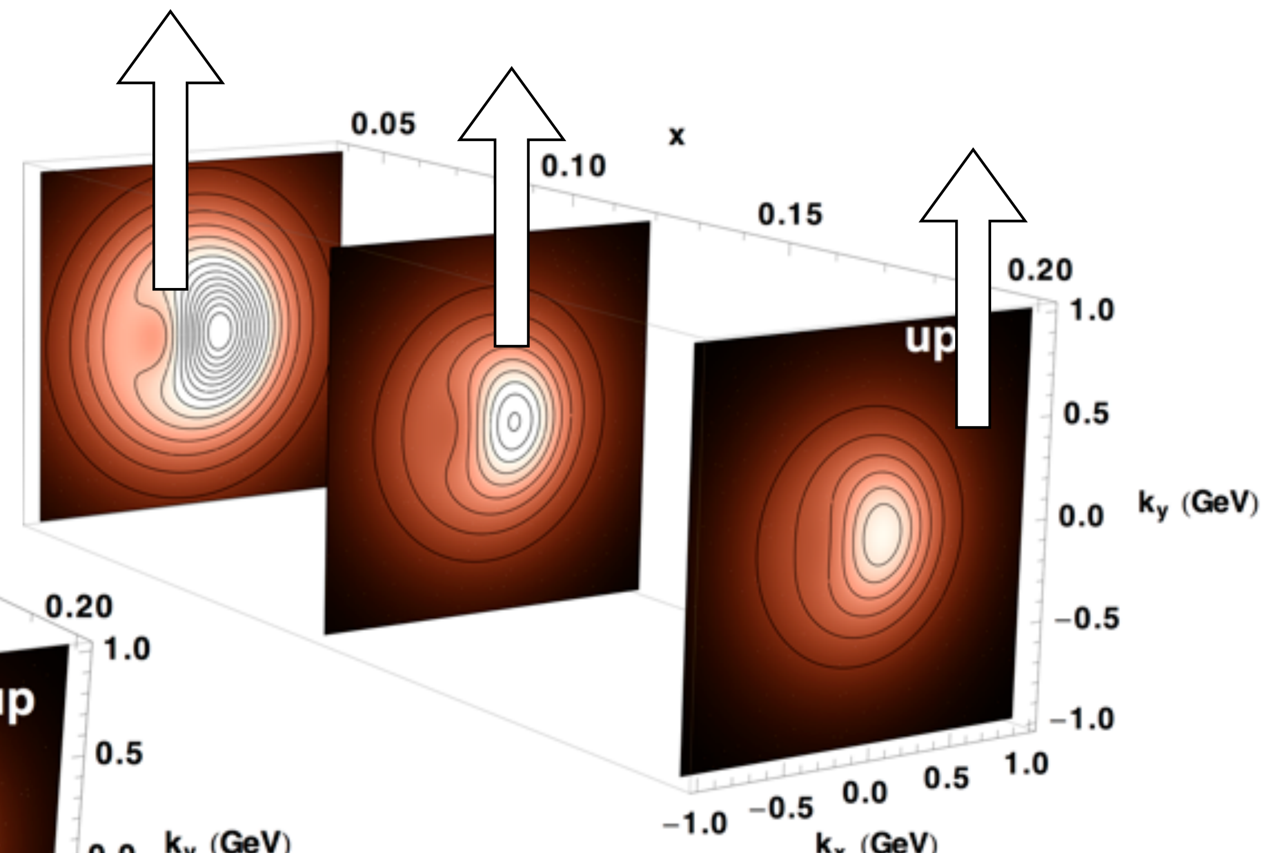
DY+ SIDIS data at NNNLL



Scimemi, Vladimirov, JHEP 06 (2020)137

Adding the spin





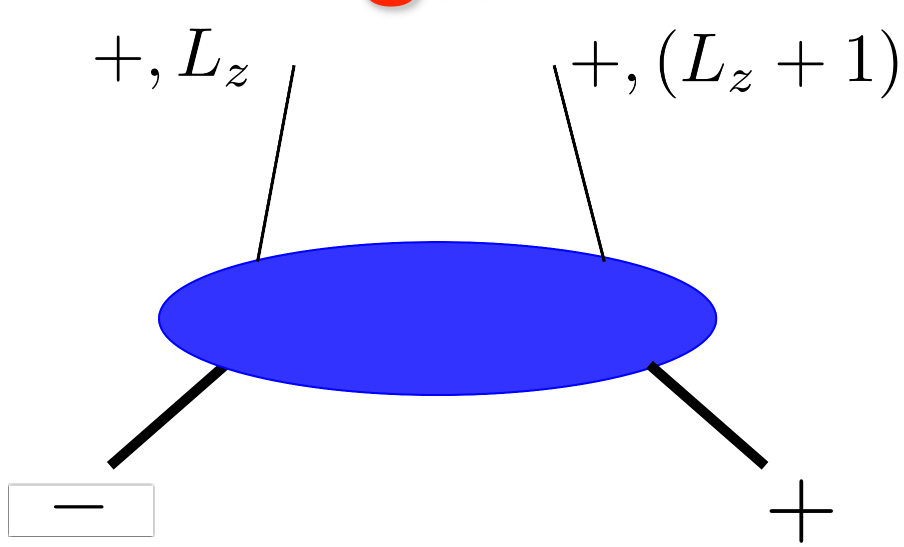
correlation between x, $k_\perp$ and spin
information on parton OAM

correlation between x and $k_\perp$

The Worm-Gear functions

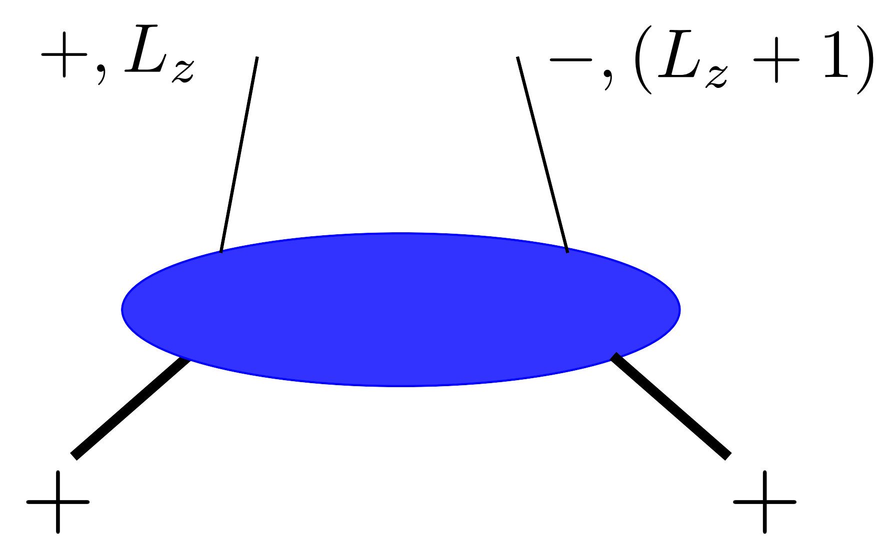
Another way to access orbital angular momentum
information without final state interactions:
Worm gear functions

g_{1T}



longitudinally pol. quarks
in transversely pol. nucleon

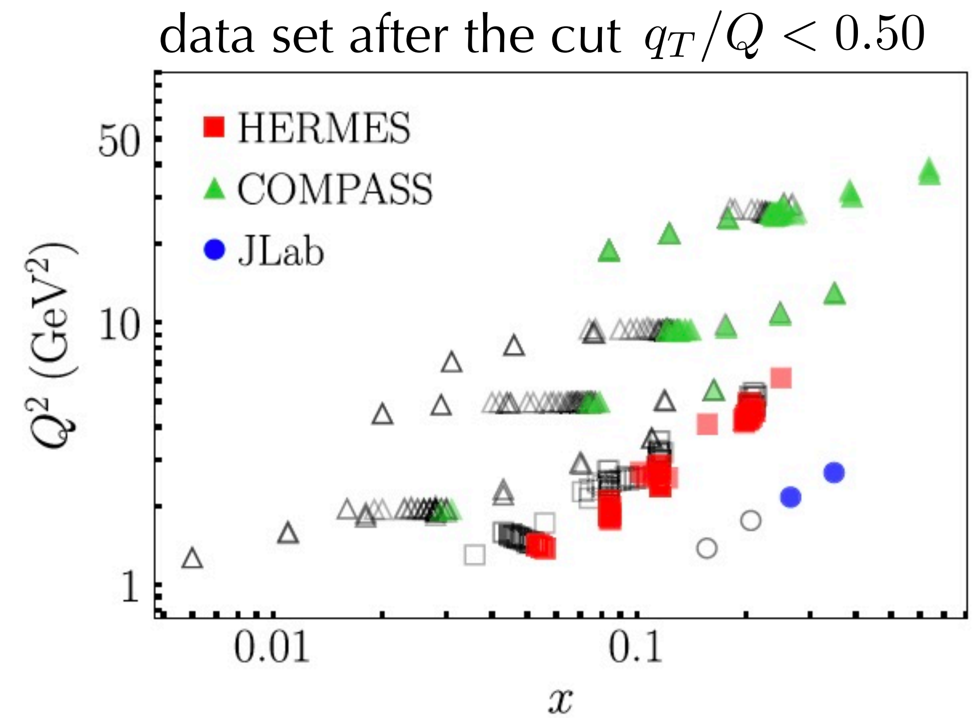
$h_{1L}^{\perp}$



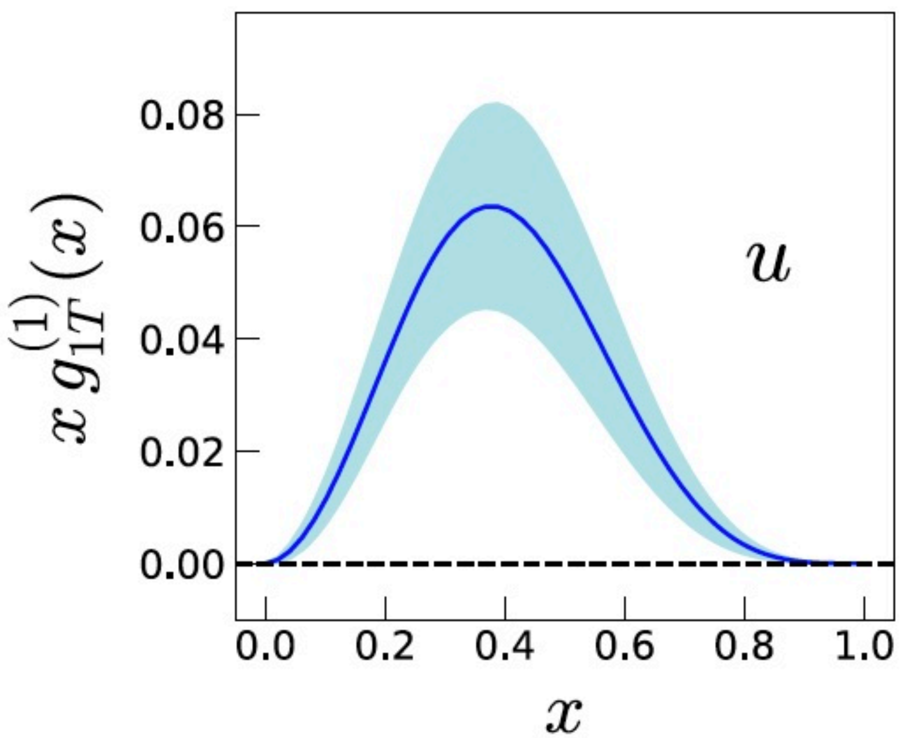
transversely pol. quarks
in longitudinal pol. nucleon

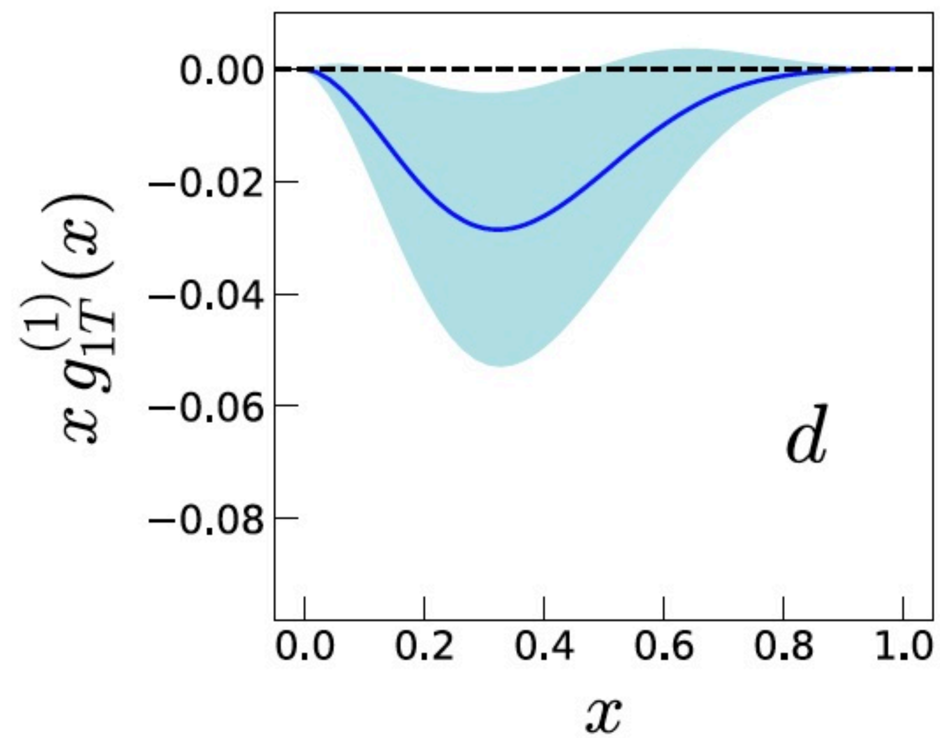
First extraction of g_{1T}

Bhattacharya et al, PRD105, 034007 (2022)



$$g_{1T}^{(1)} = \int \mathrm{d}^2\vec{k}_\perp \left(\frac{k_\perp^2}{2M^2}\right) g_{1T}(x, \vec{k}_\perp^2)$$



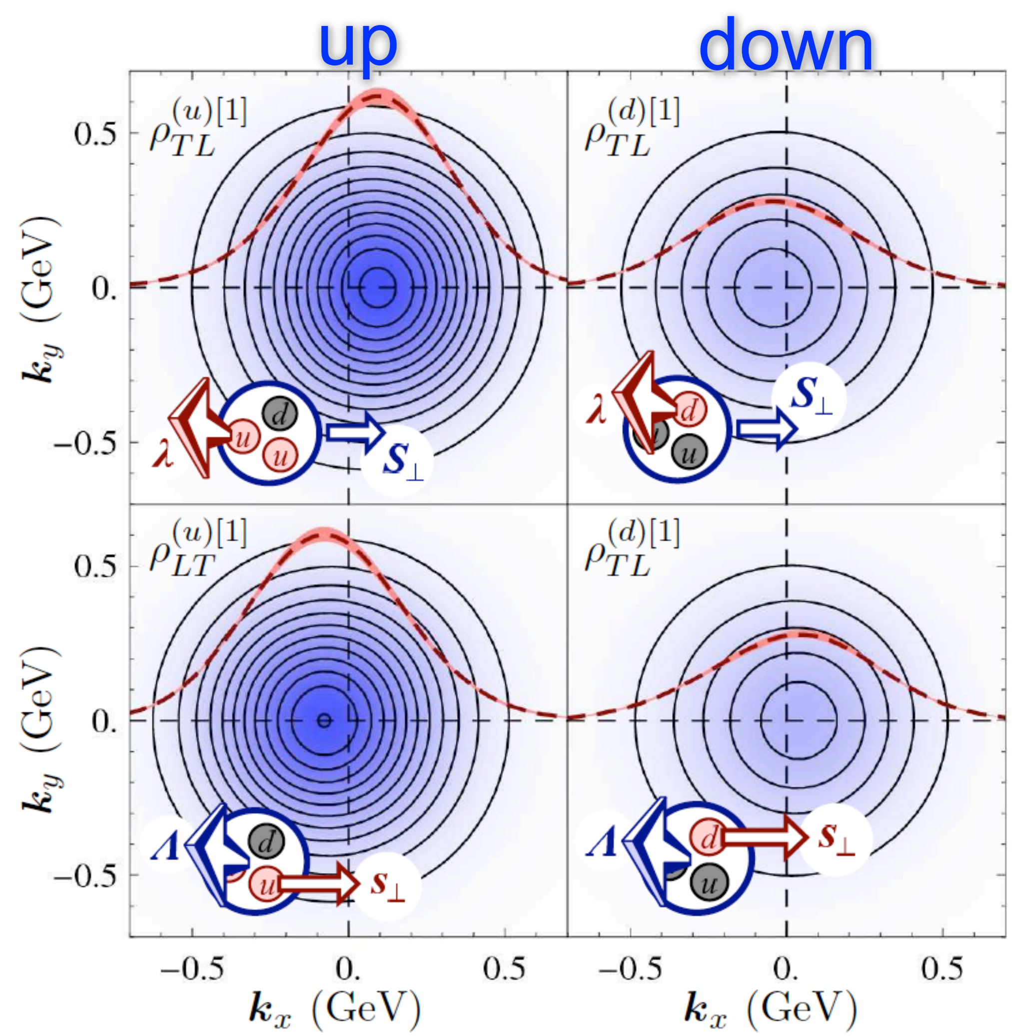


Worm-gear shift $\langle k_x \rangle_{TL}$ compatible with lattice results

Pioneering lattice QCD studies

Musch, Hagler, Negele, Schaefer, Europhysics Lett. ***88*** *(2009) 61001*

$\langle k_x^u \rangle = 67(5)$ MeV **g1T**



$\langle k_x^d \rangle = -30(5)$ MeV

$\langle k_x^u \rangle = -60(5)$ MeV **$h_{1L}^\perp$**

$\langle k_x^d \rangle = 16(5)$ MeV

$$\langle k_x^q \rangle_{g_{1T}} \approx -\langle k_x^q \rangle_{h_{1L}^\perp}$$

~~g_{1T}, $h_{1L}^\perp$ ⟷ IPDs~~

genuine effect of intrinsic transverse momentum of quarks!
not counterpart in impact parameter space distributions

Relations among T-even TMDs

[Avakian, Efremov, Schweitzer, Yuan, 2008]
[Lorcé, Pasquini, 2011]

	Linear Relations		Quadratic Relations	
*=SU(6) Flavor dependent $D^u = \frac{2}{3}$, $D^d = -\frac{1}{3}$	$D^1\, f_1^q + g_{1L}^q = 2\, h_1^q$	* * *		
Flavor independent	$g_{1T}^q = -h_{1L}^{\perp q}$	* * * ● ● ●	$2\, h_1^q\, h_{1T}^{\perp q} = -(g_{1T}^q)^2$	* * ● ● ●
	$g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}$	* * * ● ● ●		

Bag [Jaffe, Ji 1991); Signal (1997); Barone & al. (2002); Avakian & al., (2008-2010)]

χQSM [Lorcé, Pasquini, Vanderhaeghen (2011)]

LFQM [Pasquini & al. (2008)]

S Diquark [Ma & al. (1996-2009); Jakob & al. (1997); Bacchetta & al. (2008)]

AV Diquark [Ma & al. (1996-2009); Jakob & al. (1997); Bacchetta & al. (2008)]

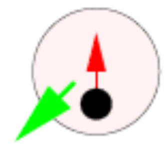
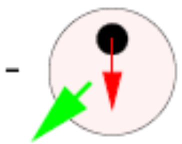
Cov. Parton [Efremov & al. (2009)]

Quark Target [Meissner & al. (2007)]

Common assumptions:

- No gluons
- Independent quarks

Quark OAM from pretzelosity

$h_{1T}^{\perp} =$  -  "pretzelosity"

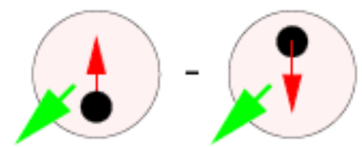
model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

Quark OAM from pretzelosity

$h_{1T}^{\perp} =$  "pretzelosity"

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$

chiral even and charge even

$\Delta L_z = 0$

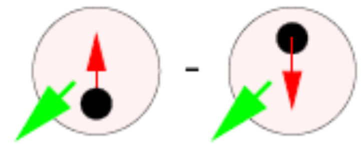
$h_{1T}^{\perp}$

chiral odd and charge odd

$|\Delta L_z| = 2$

no operator identity
relation at level of matrix elements of operators

Quark OAM from pretzelosity

$h_{1T}^{\perp}$ =  "pretzelosity"

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

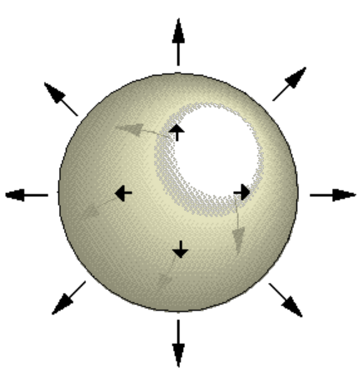
first derived in LF-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\lvert\Delta L_z\rvert = 2$

no operator identity
relation at level of matrix elements of operators

$\Downarrow$



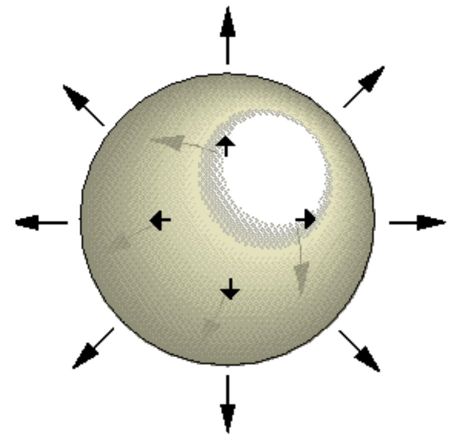
valid in all quark models with spherical symmetry in the rest frame

[Lorcé, BP, PLB 710 (2012) 486]

Common assumptions :

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame
- SU(6) symmetry

spherical symmetry
in the rest frame



rest frame

$$|\vec{0}, \sigma\rangle$$

zero OAM

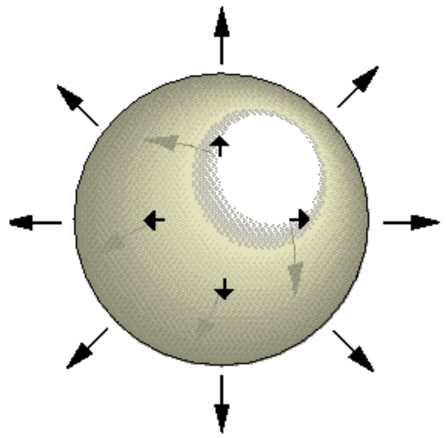
the quark distribution does not depend
on the direction of polarization

[Lorcé, BP, PLB 710 (2012) 486]

Common assumptions :

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame
- SU(6) symmetry

spherical symmetry
in the rest frame



rest frame

$|\vec{0}, \sigma\rangle$

zero OAM

the quark distribution does not depend
on the direction of polarization

Light-cone boost $\longrightarrow$

infinite-momentum frame

$|\vec{k}, \lambda\rangle_{LF}$

NON-zero OAM

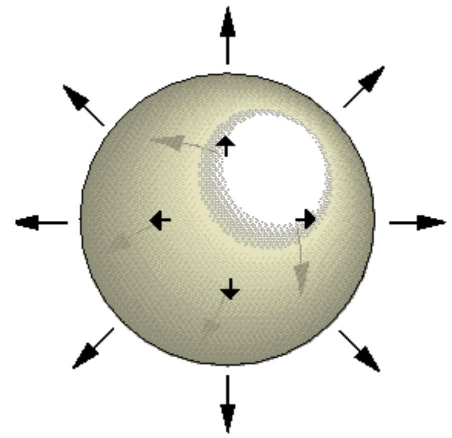
LC polarizations of quark and nucleon
are NOT all independent

[Lorcé, BP, PLB 710 (2012) 486]

Common assumptions :

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame
- SU(6) symmetry

spherical symmetry
in the rest frame



rest frame

$|\vec{0},\sigma\rangle$

zero OAM

the quark distribution does not depend
on the direction of polarization

Light-cone boost

infinite-momentum frame

$|\vec{k},\lambda\rangle_{LF}$

NON-zero OAM

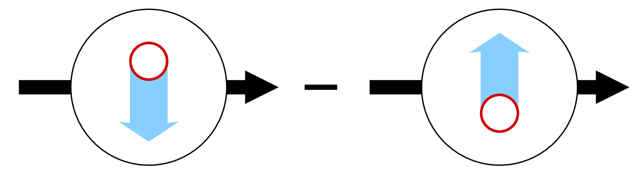
LC polarizations of quark and nucleon
are NOT all independent

relations
among polarized TMDs

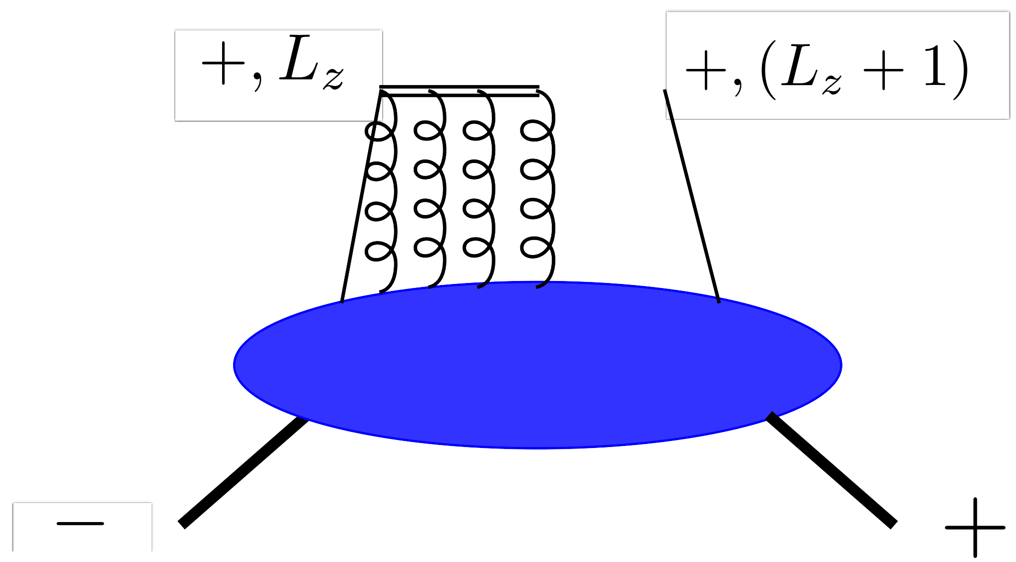
[Lorcé, BP, PLB 710 (2012) 486]

Sivers function

$f_{1T}^{\perp} =$



unpolarized quarks in $\perp$ pol. nucleon



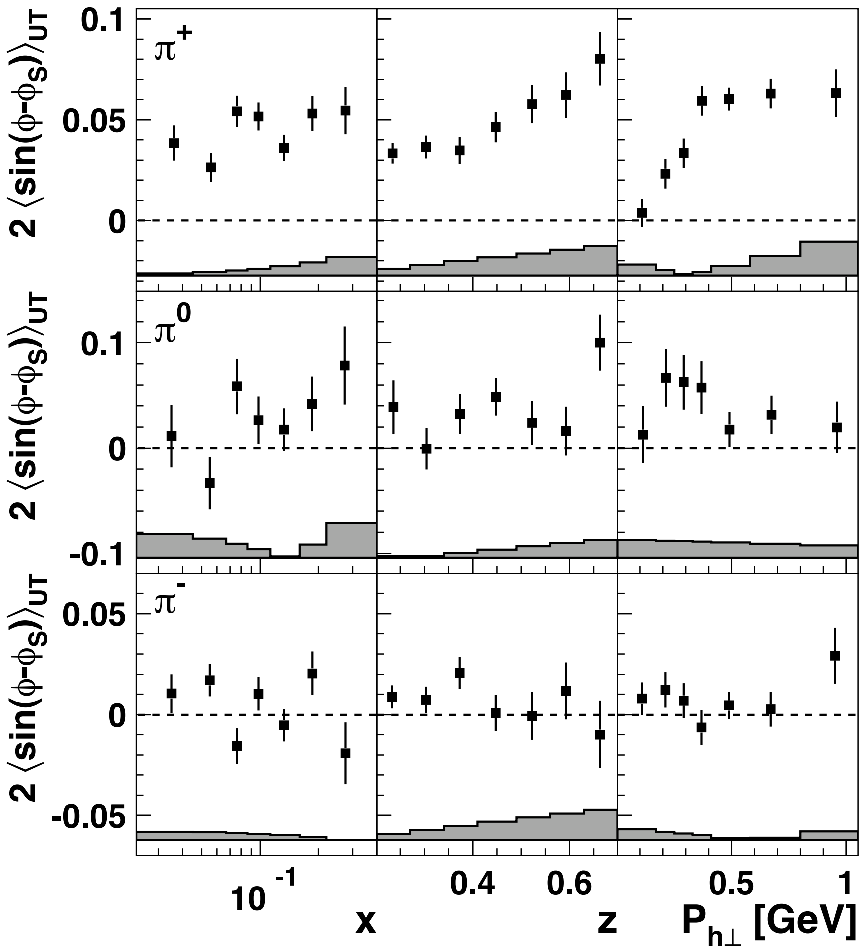
the helicity mismatch requires orbital angular momentum (OAM)

non trivial correlation between quark OAM and nucleon transverse spin

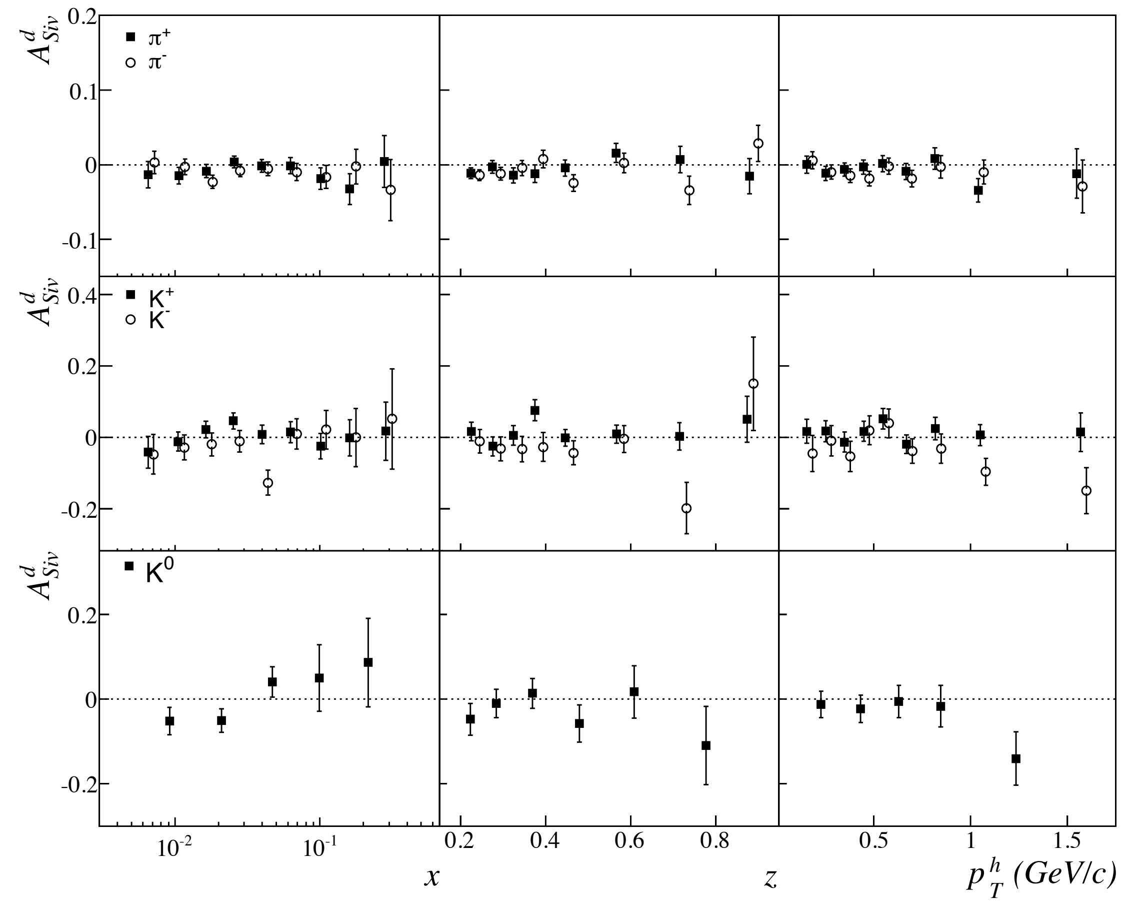
non-zero ONLY with final-state interaction

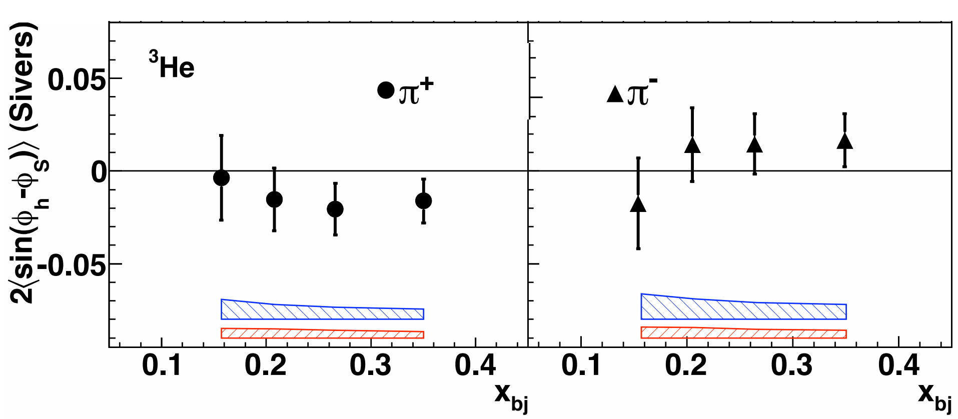
Sivers effect has been measured in SIDIS

*PRL**103** (09) 152002*



*PL B**673** (09) 127*



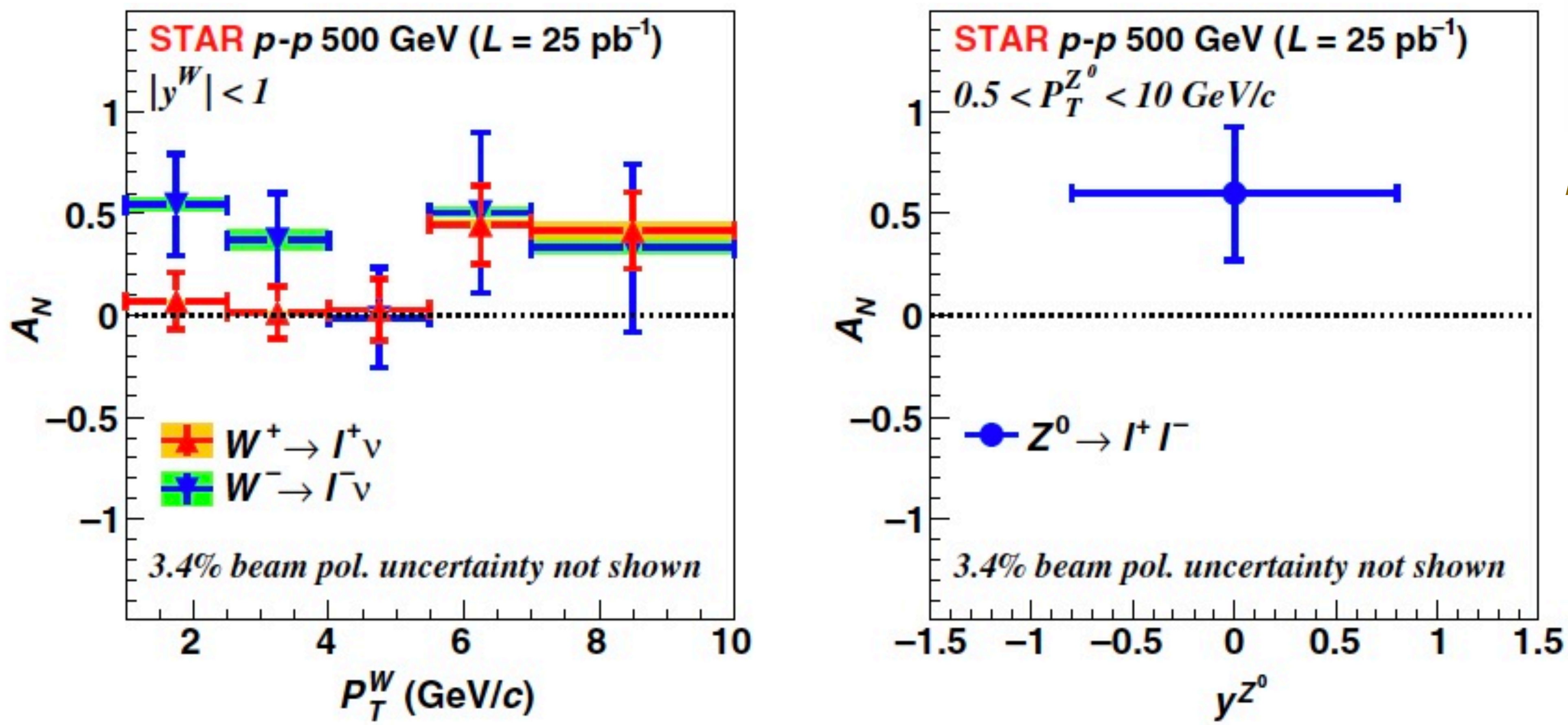


Jefferson Lab

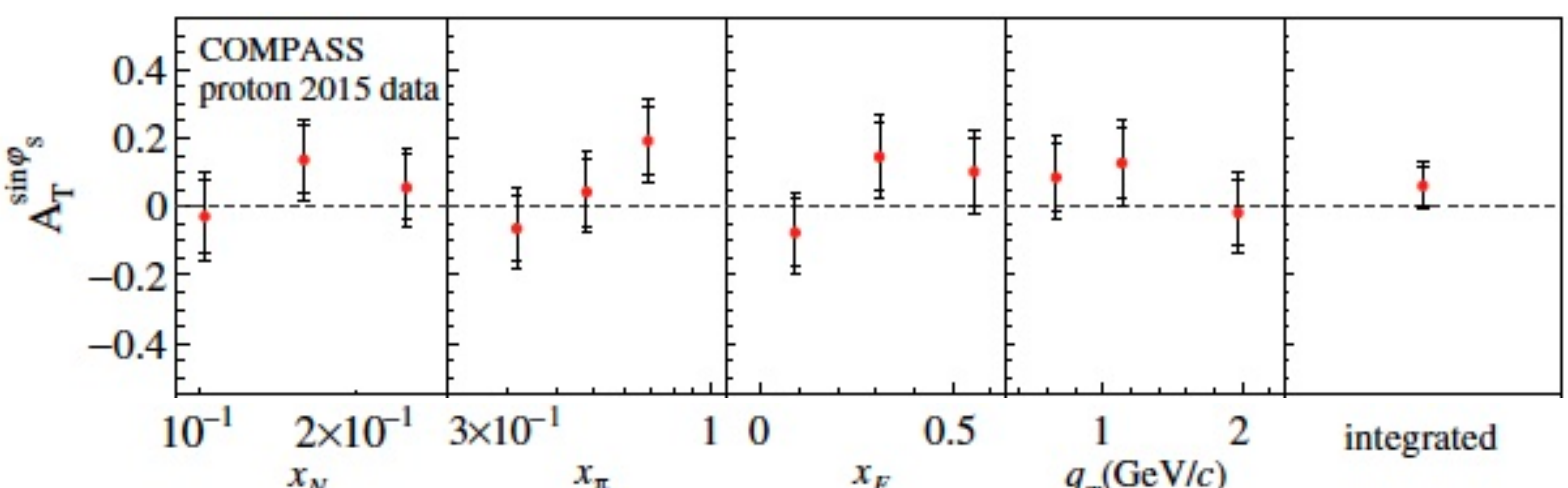
Hall A

*PRL**107** (2011) 072003*

Sivers effect has been measured in DY and $W^{\pm}/Z^0$ production

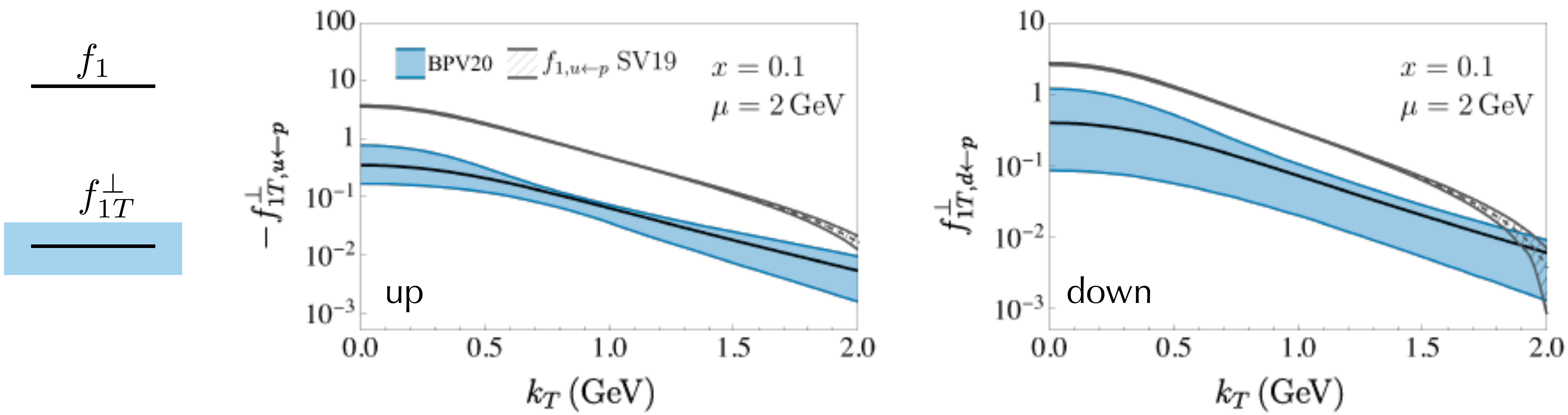


PRL 116 (2016) 132301

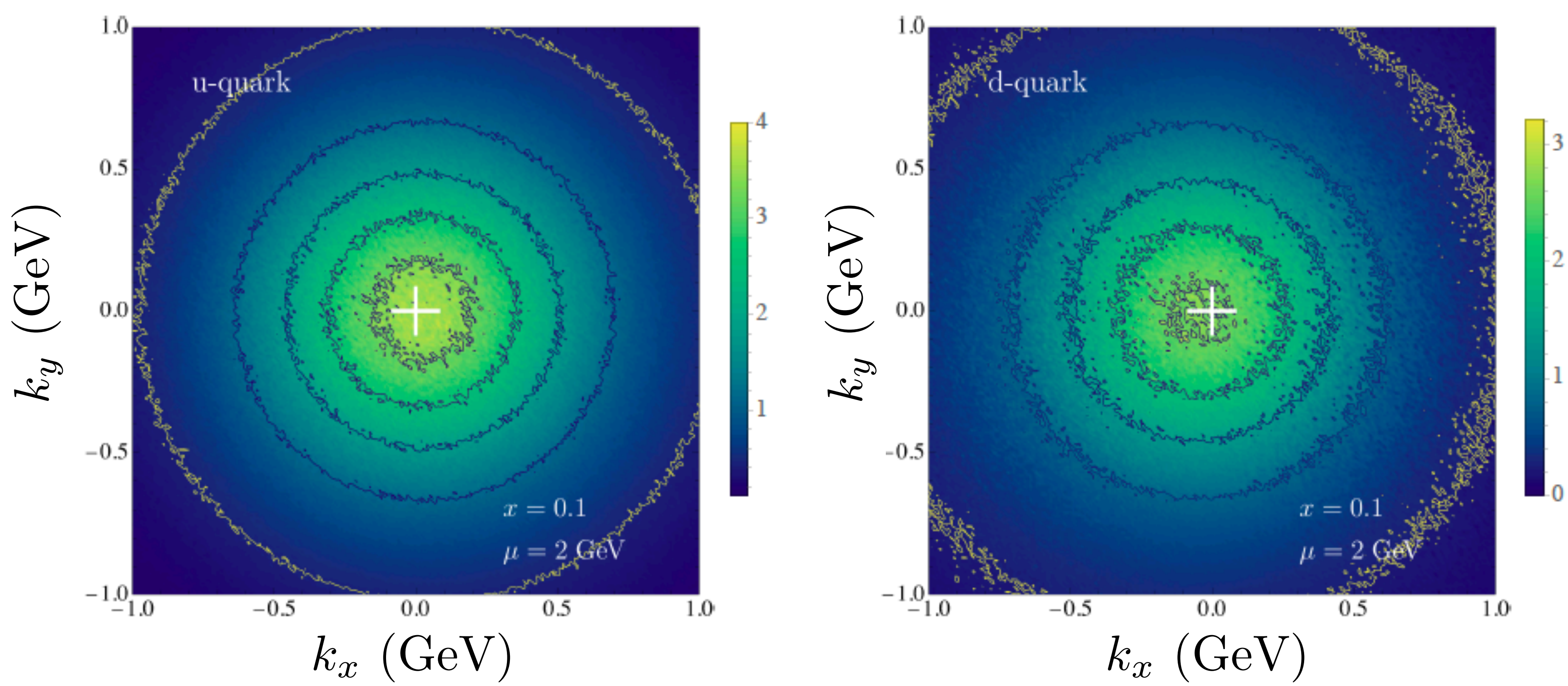


PRL 119(2017)112002

Global fit to SIDIS, DY, $W^{\pm}/Z$ boson production



$$\rho_{UT_y}(x, \vec{k}_\perp, S_y) = f_1(x, k_\perp) - \frac{k_x}{M} f_{1T}^{\perp}(x, k_\perp)$$

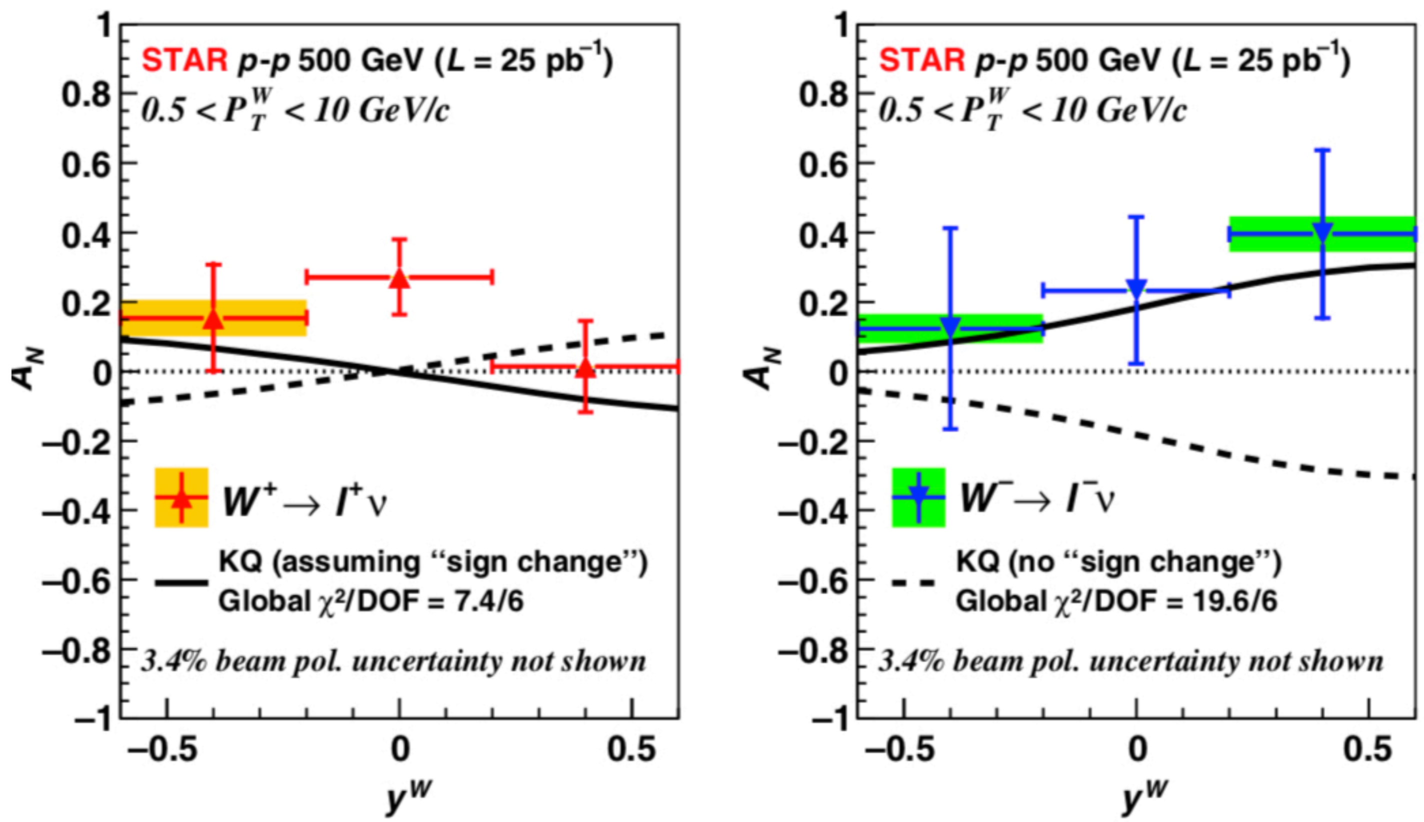


M. Bury, A. Prokudin, A. Vladimirov, JHEP 05 (2021) 151

First hints of sign change

$p^{\uparrow}p \to W^{\pm}/Z$

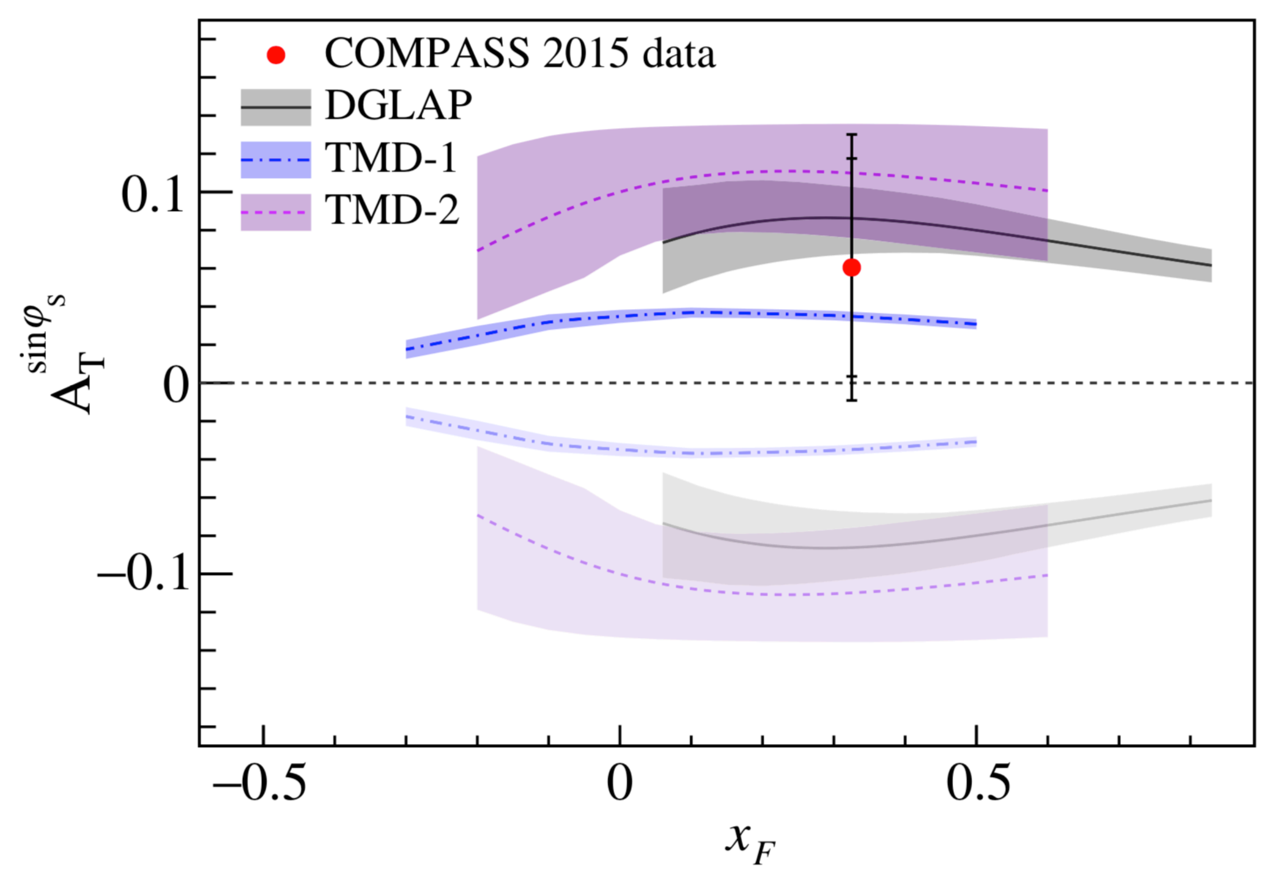
@ RHIC-STAR Coll.
PRL 116(2016)132301



sign change

no sign change

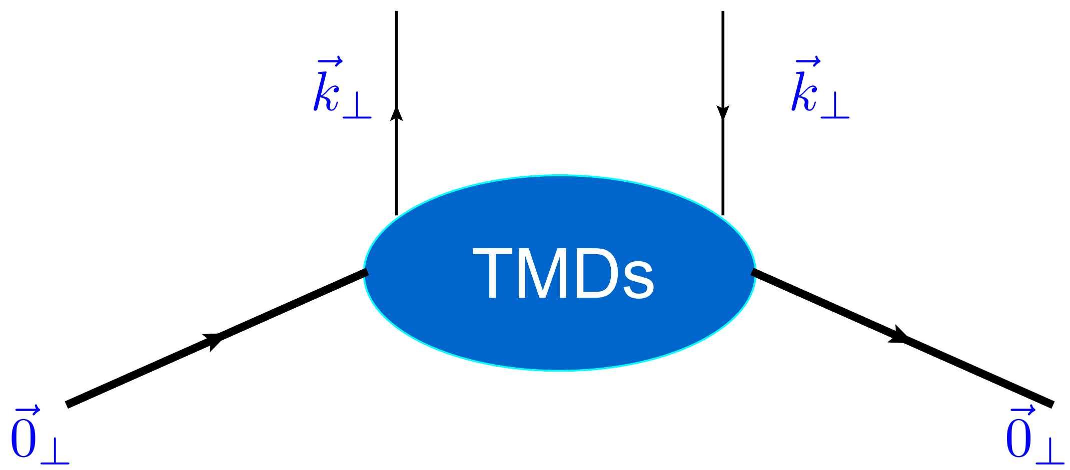
Drell-Yan $\pi p \to \mu\mu X$
@ COMPASS
PRL 119(2017)112002

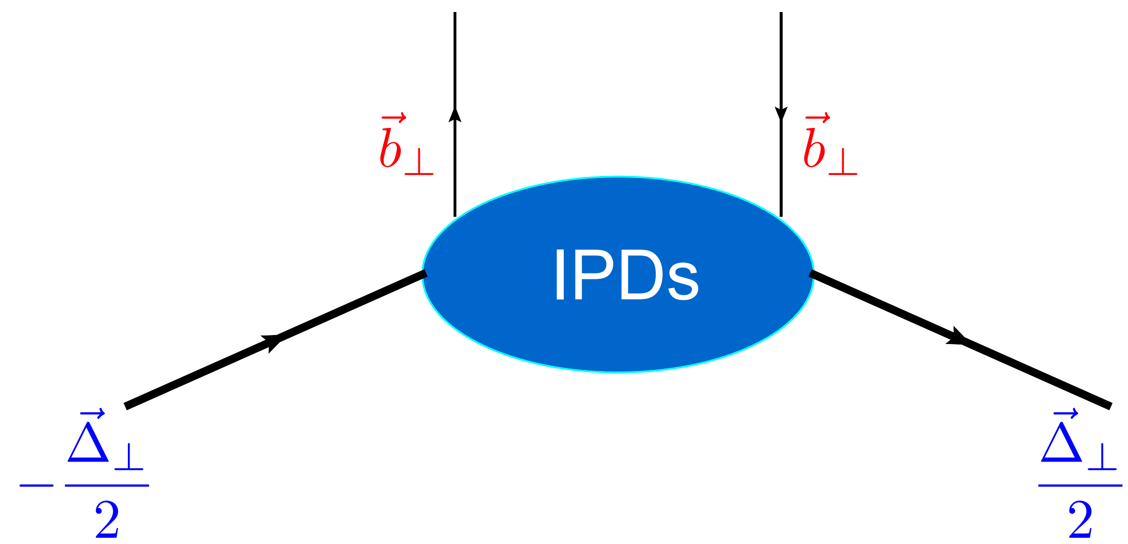


sign change

no sign change

TMDs vs IPDs





$$\int d^2 z_\perp e^{-i\vec{z}_\perp \cdot \vec{k}_\perp} \langle 0_\perp | \bar{\psi}(-\frac{\vec{z}_\perp}{2}) \ldots \psi(\frac{\vec{z}_\perp}{2}) | \vec{0}_\perp \rangle$$

$$\int d^2 \Delta_\perp e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \langle -\frac{\vec{\Delta}_\perp}{2} | \bar{\psi}(\vec{b}_\perp) \ldots \psi(\vec{b}_\perp) | \frac{\vec{\Delta}_\perp}{2} \rangle$$

(longitudinal components are not shown)

difference of transverse position	average of transverse momenta	average position	difference of transverse momenta

$\vec{z}_\perp \longleftrightarrow \vec{k}_\perp$ $\qquad$ $\vec{b}_\perp \longleftrightarrow \vec{\Delta}_\perp$

$\longrightarrow$ see Lectures of Markus Diehl

TMDs vs IPDs

ρ_{XY} X = proton pol
Y = quark pol

correlations in $\vec{k}_\perp, \Lambda, \vec{s}_\perp$

$$\rho_{LT}(x,\vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, s^i_\perp\, k^i_\perp \frac{1}{M} h^\perp_{1L}]$$

$\Lambda s^i b^i_\perp$
time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \vec{S}_\perp, \lambda$

$$\rho_{TL}(x,\vec{k}_\perp) = \frac{1}{2}[f_1 + \lambda\, S^i_\perp\, k^i_\perp \frac{1}{M} g^\perp_{1T}]$$

$S^i \lambda b^i_\perp$
time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \Lambda, \lambda$

$$\rho_{LL}(x,\vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\,\lambda\, g_{1L}]$$

correlations in $\vec{b}_\perp,\ \Lambda,\ \lambda$

$$\tilde{\rho}_{TL}(x,\vec{b}_\perp) = \frac{1}{2}[H + \Lambda\,\lambda\,\tilde{H}]$$

correlations in $\vec{k}_\perp, \vec{S}_\perp, \vec{s}_\perp$

$$\rho_{TT}(x,\vec{k}_\perp) = \frac{1}{2}[f_1 + S^i_\perp\, s^i_\perp\, h_1 + S^i_\perp\,(2k^i k^j - k^2_\perp \delta^{ij})\, s^j_\perp \frac{1}{2M^2} h^\perp_{1T}]$$

correlations in $\vec{b}_\perp,\ \vec{S}_\perp,\ \vec{s}_\perp$

$$\tilde{\rho}_{TT}(x,\vec{b}_\perp) = \frac{1}{2}[H - S^i_\perp\, s^i_\perp(\, H_T - \frac{1}{4M^2}\Delta_b\,\tilde{H}_T) + S^i_\perp\,(2b^i b^j - b^2\delta^{ij})\, s^j_\perp \frac{1}{M^2}\tilde{H}''_T]$$

Diehl, Haegler, EPJ C44 (2005) 87

TMDs vs IPDs

$$\rho_{XY} \quad \begin{array}{l} X = \text{proton pol} \\ Y = \text{quark pol} \end{array}$$

correlations in $\vec{k}_\perp, \Lambda, \vec{s}_\perp$

$$\rho_{LT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, s_\perp^i\, k_\perp^i\, \frac{1}{M} h_{1L}^\perp]$$

$\Lambda \cancel{s^i} b_\perp^i$

time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \vec{S}_\perp, \lambda$

$$\rho_{TL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \lambda\, S_\perp^i\, k_\perp^i\, \frac{1}{M} g_{1T}^\perp]$$

$S^i \lambda b_\perp^i$

time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \Lambda, \lambda$

$$\rho_{LL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, \lambda\, g_{1L}]$$

correlations in $\vec{b}_\perp$, Λ, λ

$$\tilde{\rho}_{TL}(x, \vec{b}_\perp) = \frac{1}{2}[H + \Lambda\, \lambda\, \tilde{H}]$$

correlations in $\vec{k}_\perp, \vec{S}_\perp, \vec{s}_\perp$

$$\rho_{TT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + S_\perp^i\, s_\perp^i\, h_1 + S_\perp^i\, (2k^i k^j - k_\perp^2 \delta^{ij})\, s_\perp^j\, \frac{1}{2M^2} h_{1T}^\perp]$$

correlations in $\vec{b}_\perp$, $\vec{S}_\perp$, $\vec{s}_\perp$

$$\tilde{\rho}_{TT}(x, \vec{b}_\perp) = \frac{1}{2}[H - S_\perp^i\, s_\perp^i\, (\,H_T - \frac{1}{4M^2} \Delta_b\, \tilde{H}_T) + S_\perp^i\, (2b^i b^j - b^2 \delta^{ij})\, s_\perp^j\, \frac{1}{M^2} \tilde{H}_T''\,]$$

Diehl, Haegler, EPJ C44 (2005) 87

TMDs vs IPDs

$$\rho_{XY} \quad \begin{array}{l} X = \text{proton pol} \\ Y = \text{quark pol} \end{array}$$

correlations in $\vec{k}_\perp, \Lambda, \vec{s}_\perp$

$$\rho_{LT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, s^i_\perp\, k^i_\perp \frac{1}{M} h^\perp_{1L}]$$

$\cancel{\Lambda s^i b^i_\perp}$

time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \vec{S}_\perp, \lambda$

$$\rho_{TL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \lambda\, S^i_\perp\, k^i_\perp \frac{1}{M} g^\perp_{1T}]$$

$\cancel{S^i \lambda b^i_\perp}$

time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \Lambda, \lambda$

$$\rho_{LL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, \lambda\, g_{1L}]$$

correlations in $\vec{b}_\perp,\ \Lambda,\ \lambda$

$$\tilde{\rho}_{TL}(x, \vec{b}_\perp) = \frac{1}{2}[H + \Lambda\, \lambda\, \tilde{H}]$$

correlations in $\vec{k}_\perp, \vec{S}_\perp, \vec{s}_\perp$

$$\rho_{TT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + S^i_\perp\, s^i_\perp\, h_1 + S^i_\perp\, (2k^i k^j - k^2_\perp \delta^{ij})\, s^j_\perp \frac{1}{2M^2} h^\perp_{1T}]$$

correlations in $\vec{b}_\perp,\ \vec{S}_\perp,\ \vec{s}_\perp$

$$\tilde{\rho}_{TT}(x, \vec{b}_\perp) = \frac{1}{2}[H - S^i_\perp\, s^i_\perp (\, H_T - \frac{1}{4M^2} \Delta_b\, \tilde{H}_T) + S^i_\perp\, (2b^i b^j - b^2 \delta^{ij})\, s^j_\perp \frac{1}{M^2} \tilde{H}''_T]$$

Diehl, Haegler, EPJ C44 (2005) 87

TMDs vs IPDs

ρ_{XY} $X =$ proton pol

$Y =$ quark pol

correlations in $\vec{k}_\perp, \vec{S}_\perp$

$$\rho_{TU}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + S_\perp^i \epsilon^{ij} k_\perp^j \frac{1}{M} f_{1T}^\perp]$$

correlations in $\vec{b}_\perp, \vec{S}_\perp$

$$\rho_{TU}(x, \vec{k}_\perp) = \frac{1}{2}[H - S_\perp^i \epsilon^{ij} b_\perp^j \frac{1}{M} \frac{\partial}{\partial b_\perp^2} E]$$

correlations in $\vec{k}_\perp, \vec{s}_\perp$

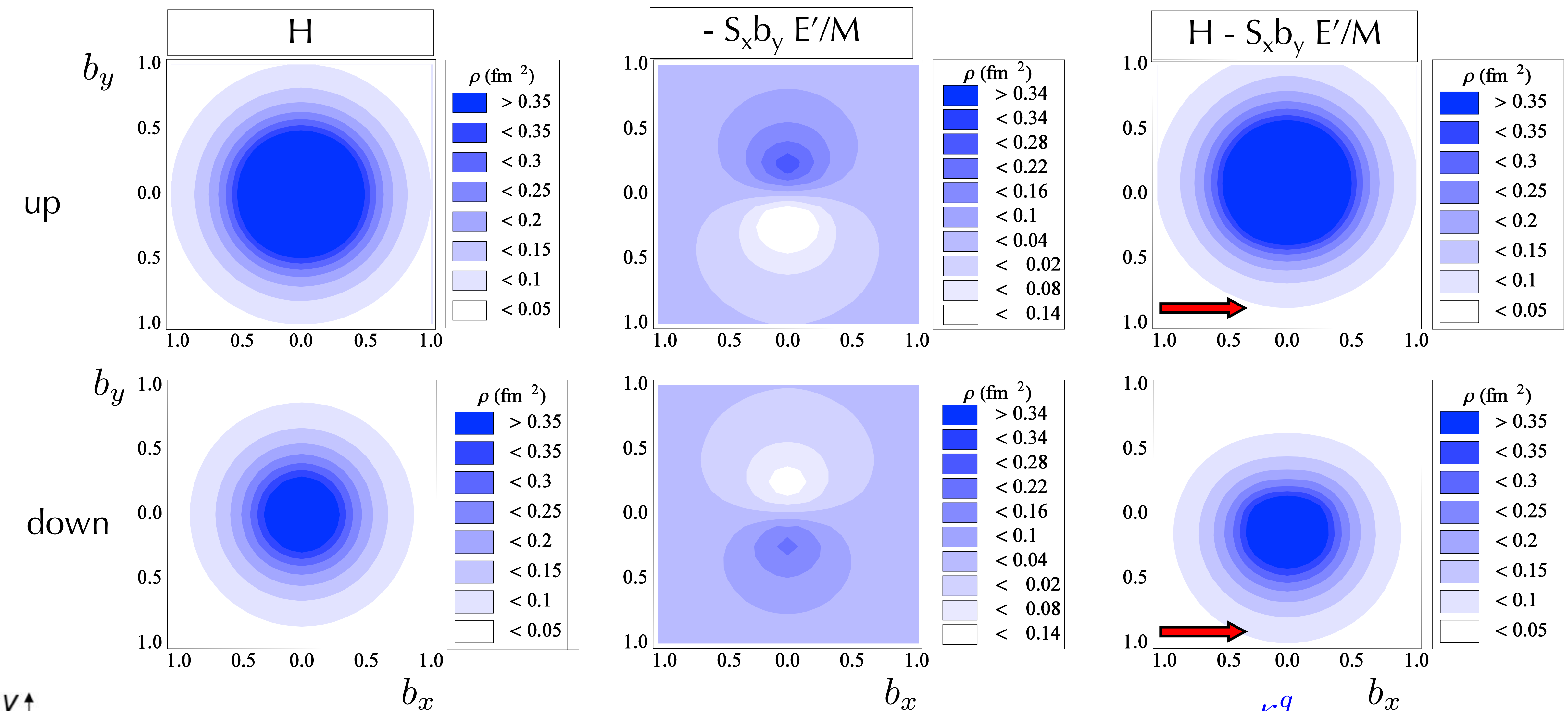
$$\rho_{UT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + s_\perp^i \epsilon^{ij} k_\perp^j \frac{1}{M} h_1^\perp]$$

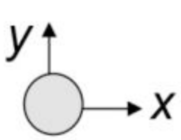
correlations in $\vec{b}_\perp, \vec{s}_\perp$

$$\rho_{UT}(x, \vec{k}_\perp) = \frac{1}{2}[H - s_\perp^i \epsilon^{ij} b_\perp^j \frac{1}{M} (E_T' + 2\tilde{H}_T')]$$

IPD for unpolarized quarks in a transversely pol. Proton

$$\int dx \frac{1}{2} \left[\mathcal{H}(x, \vec{b}_\perp^2) + S^i \epsilon_\perp^{ij} b_\perp^j \frac{1}{M} \left(\mathcal{E}(x, \vec{b}_\perp^2) \right)' \right]$$





$$d_y^q = \frac{\kappa^q}{2M}$$

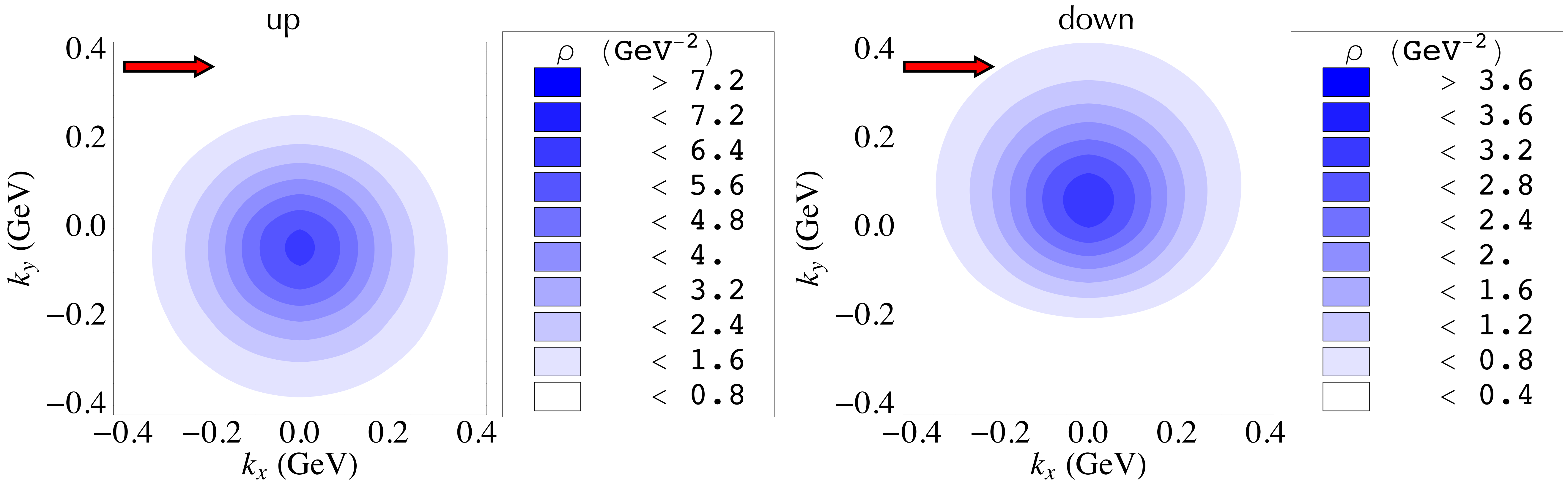
average distortion: ⊥ flavor dipole moment

$$\kappa_u^p = 1.86, \ \kappa_d^p = -1.57 \quad \Rightarrow \quad |d_y^q| \sim 0.1 - 0.2\,\mathrm{fm}$$

[B.P., Boffi, PLB, 2007]

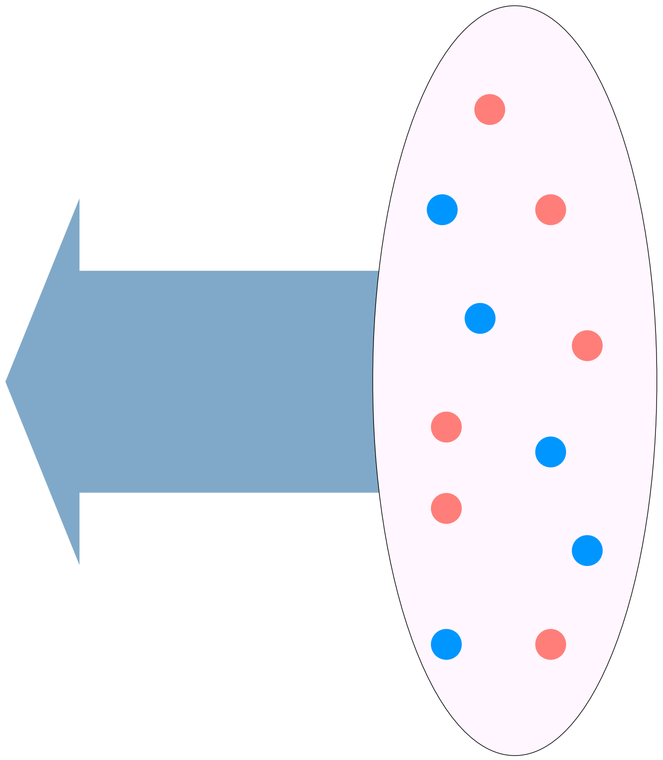
TMDs for unpolarized quarks in a transversely pol. proton

$$\int dx \frac{1}{2}\left[f_1(x,\vec{k}_\perp^{\,2}) - S^i \epsilon_\perp^{ij} k_\perp^j \frac{1}{M}\, f_{1T}^\perp(x,\vec{k}_\perp^{\,2})\right]$$



Model relation TMD ⟷ GPD

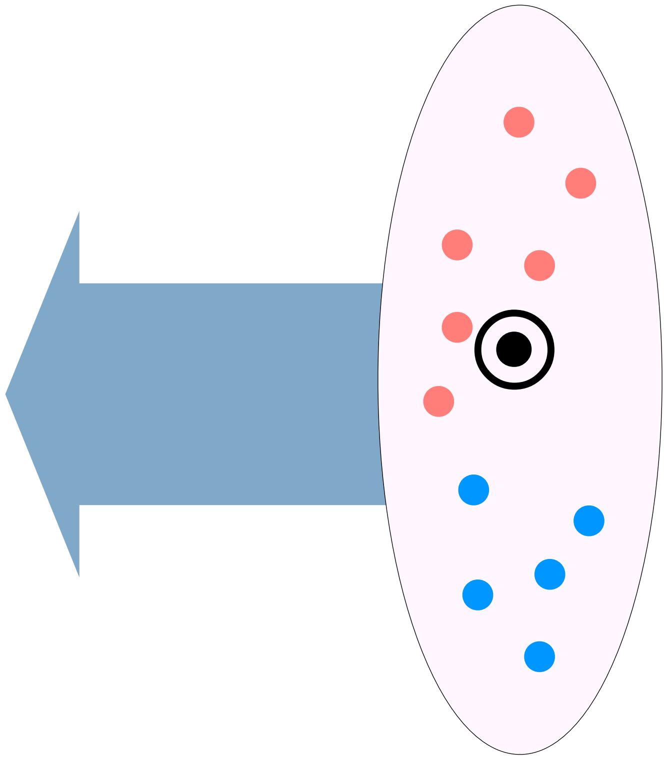
unpolarized quark in unpolarized



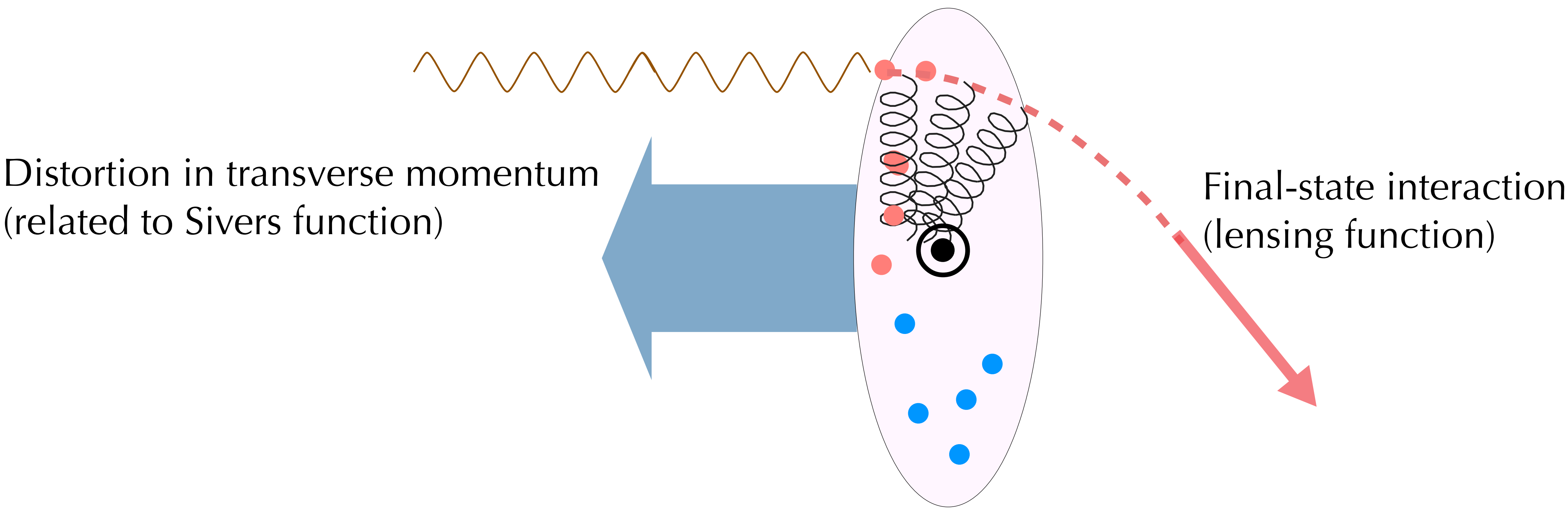
Model relation TMD ⟷ GPD

unpolarized quark in transversely pol. nucleon

Distortion in impact parameter
(related to GPD E)

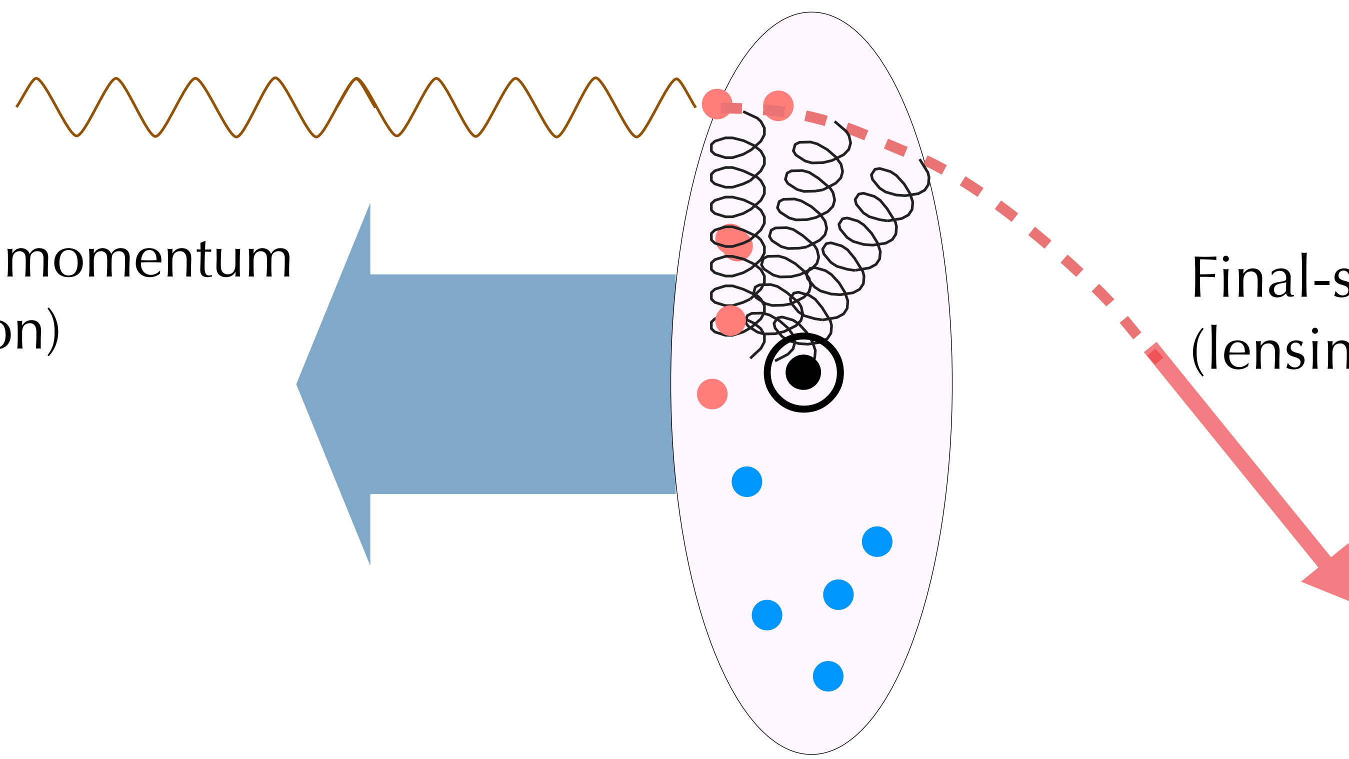


Model relation TMD ⟷ GPD



Model relation TMD $\longleftrightarrow$ GPD

Distortion in transverse momentum
(related to Sivers function)



Final-state interaction
(lensing function)

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, \underset{\substack{\uparrow \\ \text{Sivers function}}}{f_{1T}^{\perp q}(x,\vec{k}_T^2)} \simeq \int d^2\vec{b}_T\, \underset{\substack{\uparrow \\ \text{Lensing function}}}{\mathcal{I}^{q,i}(x,\vec{b}_T)}\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \Big(\underset{\substack{\uparrow \\ \text{F.T. of E(x,0,t)}}}{\mathcal{E}^q(x,\vec{b}_T^2)}\Big)'$$

*Burkardt, PRD **66** (2002) 114005*

- Relation valid only in restricted class of models, as, for example, the scalar-diquark model

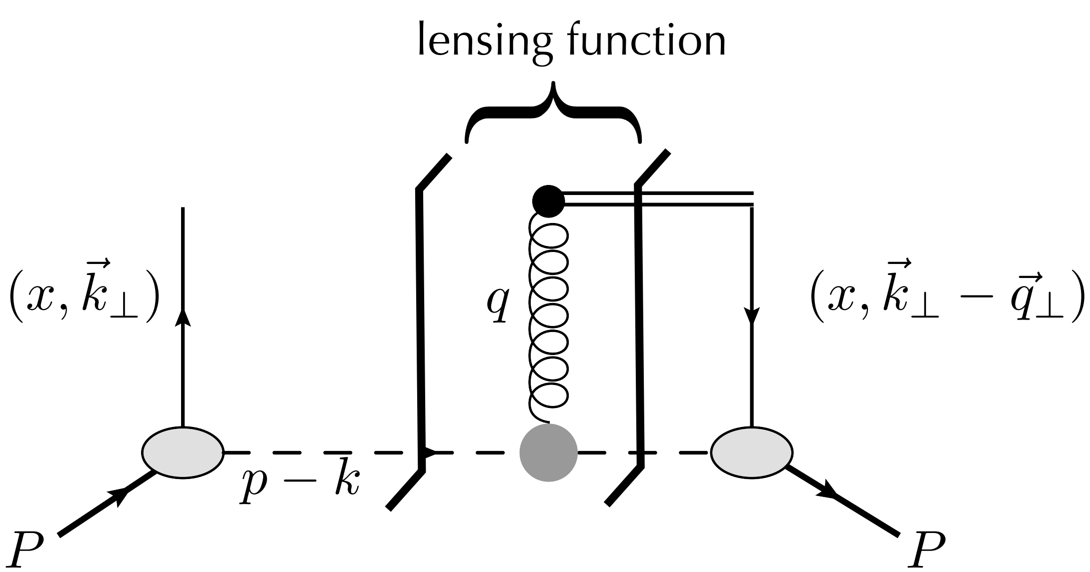
BP, Rodini, Bacchetta, Phys. Rev. D100, 054039 (2019)

Model results

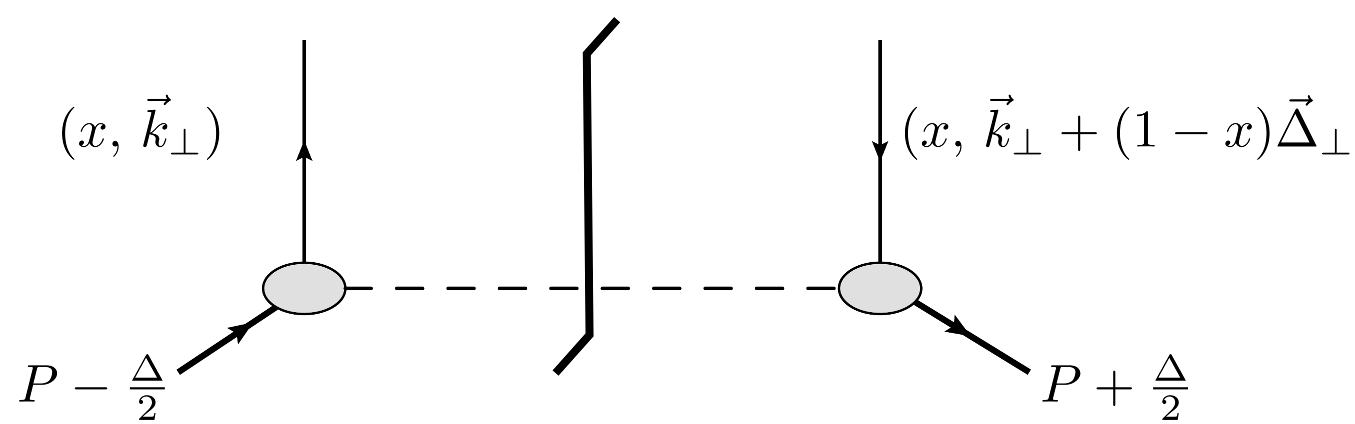
Sivers effect = Lensing function $\otimes$ IPD

Scalar diquark model:
- two-particle system (one active quark and a scalar spectator)
- perturbative coupling between Wilson line and spectator $\longrightarrow$ no-helicity flip of the spectator

TMD



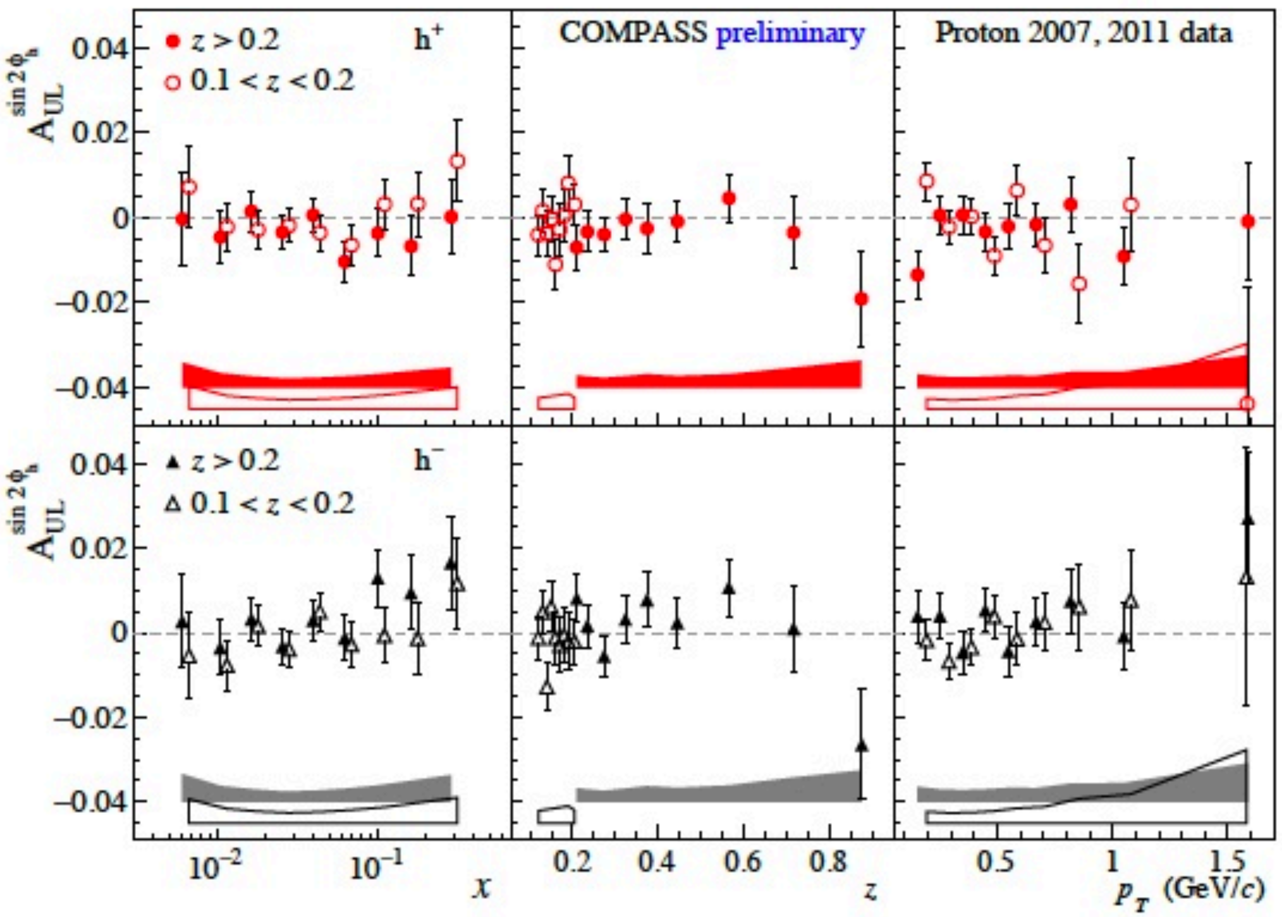
IPD



It is violated when considering coupling with the gauge boson that are not helicity conserving (e.g., axial diquark model) or for bound system with more than two constituents

BP, Rodini, Bacchetta, PRD 100, 054039 (2019)

Limitations of existing data/facilities



- sample data for $A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$

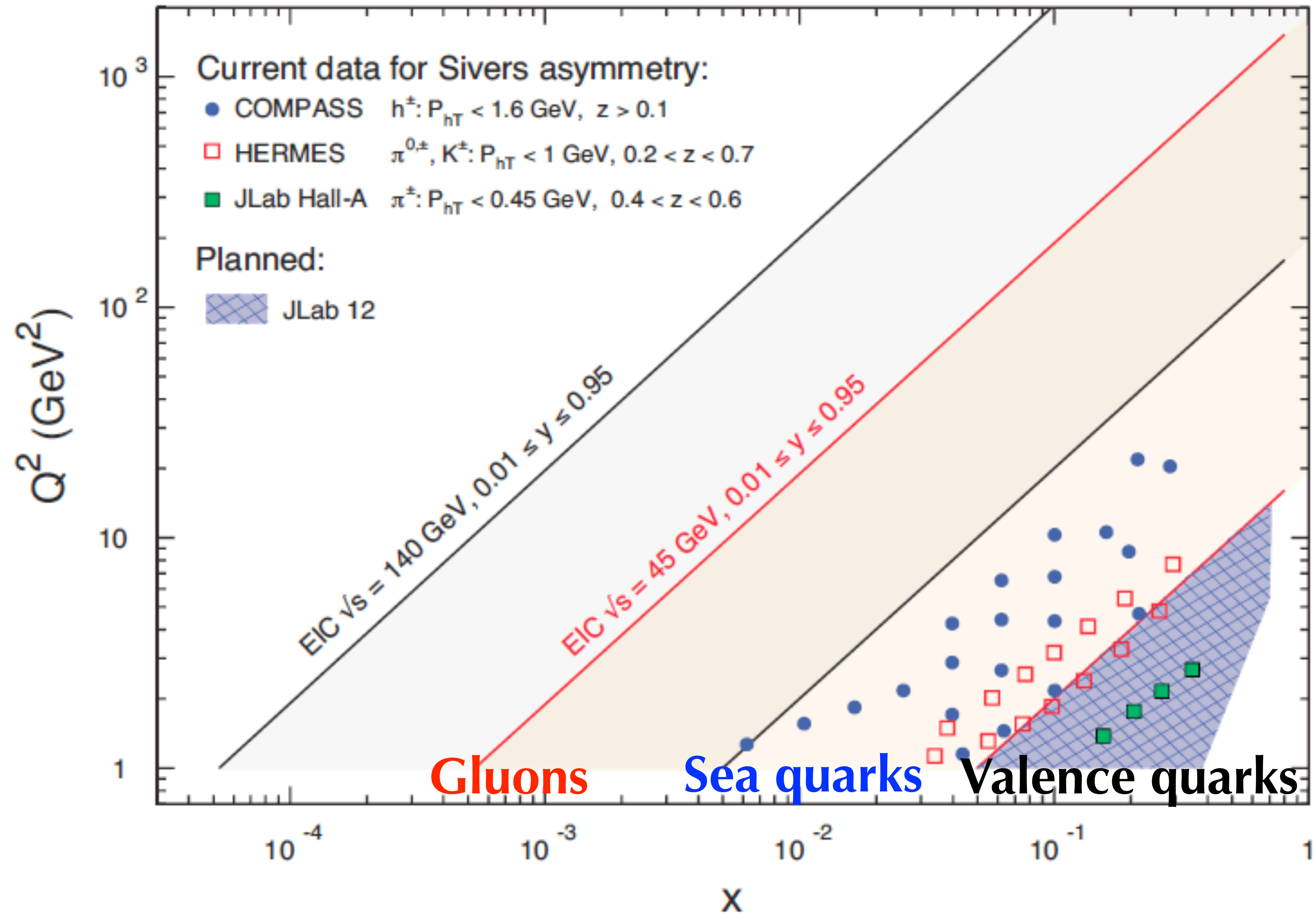
- models predict small effects

- data basically only allow conclusion that effect is compatible with zero

Existing data/facilities often suffer from one or more of the following:

- lack of data precision (due to lack of machine luminosity)
- lack of kinematical coverage
- lack of polarization
- limited detector capabilities

Paste, present and future TMD measurements

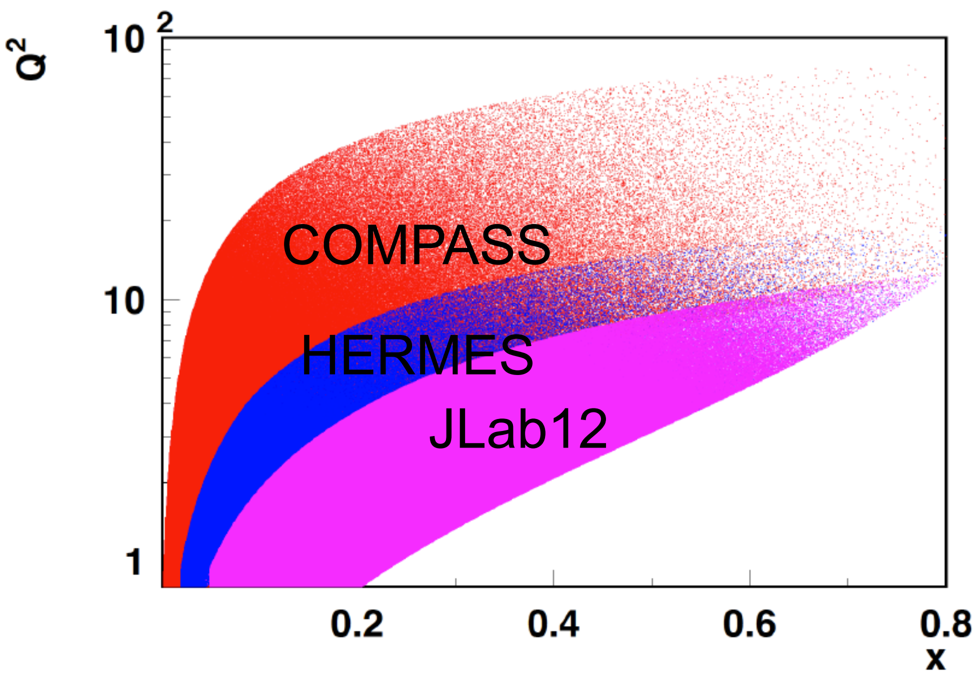


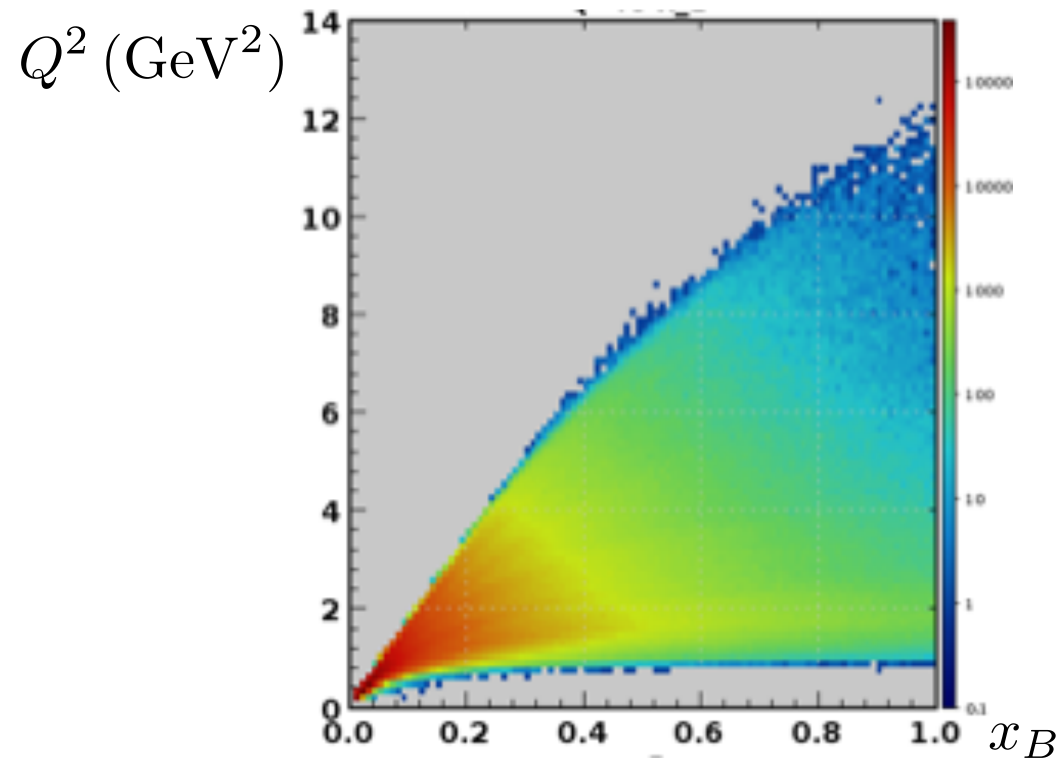
EIC

- multidimensional binning
- high Q^2 reach
- large range in transverse momentum

Accardi et al., The Electron Ion Collider: the next QCD Frontier, EPJA52 (2016) 268

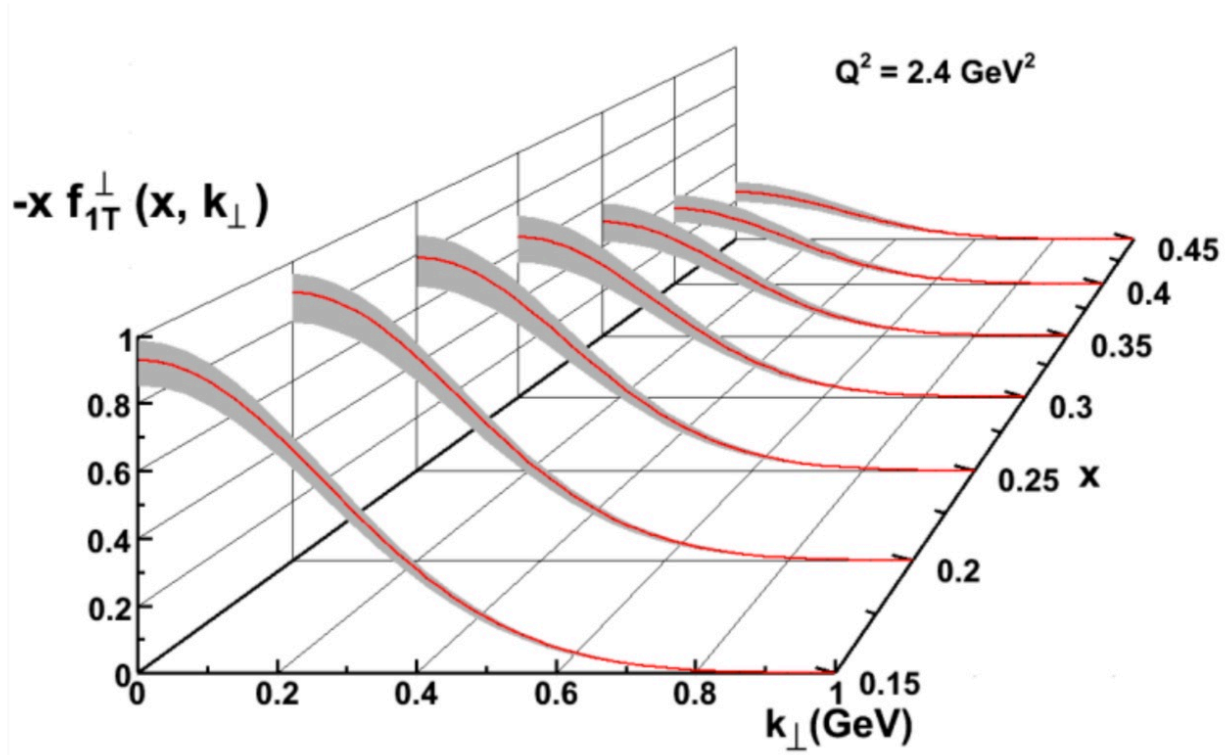
JLab12 SIDIS program





- JLab12 program very important to constrain TMD distributions at large x_B
- complementary measurements with different targets
- Hall B: large acceptance (CLAS), unpolarized and polarized H e D targets; cross sections, single and double-spin asymmetries; start kaon SIDIS program with RICH detector
- Hall C: SHMS + HMS, precision magnetic spectrometer setup, unpolarized target; L/T separation in SIDIS, precision cross section of π^+ and π^-, and K^+ and K^-
- Hall A: forward large acceptance (SOLID), longitudinal and transversely polarized ^3He target; pion and kaon run; access to neutron structure at high x_B and Q^2

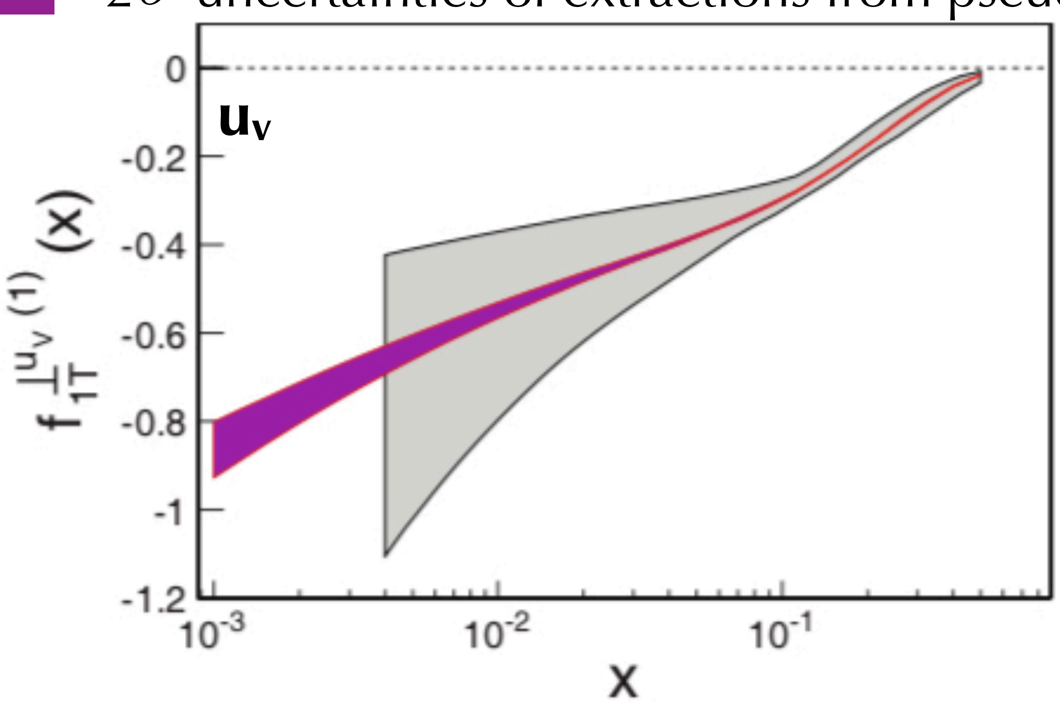
Sivers function at JLab12 and EIC

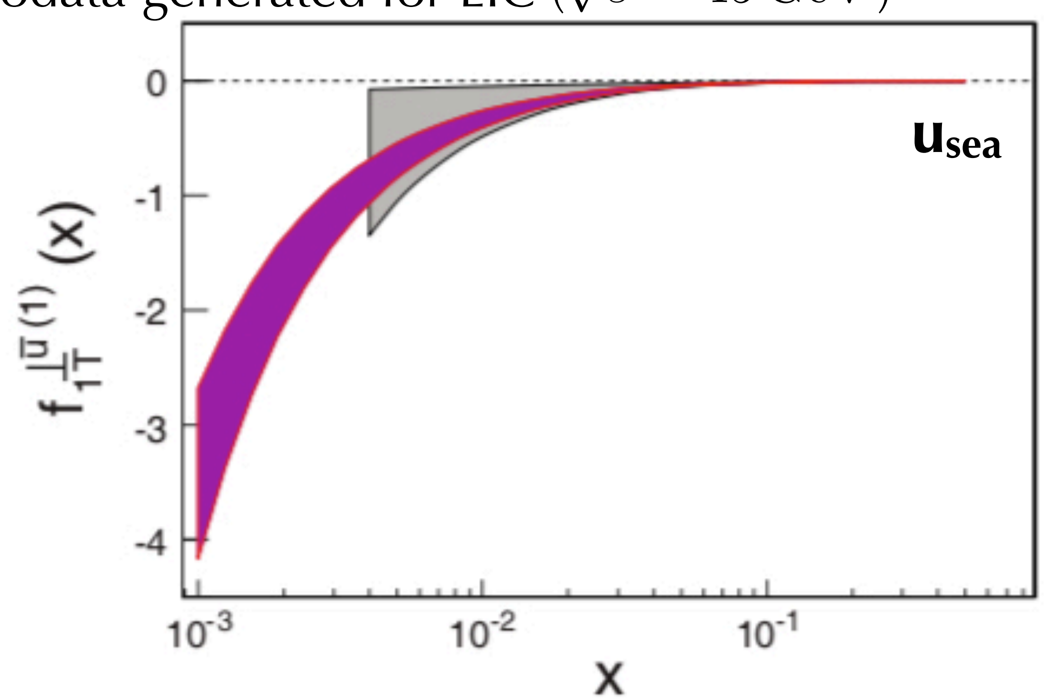


Dudek et al., Physics opportunities with the 12 GeV JLab upgrade at JLab, EPJA48(2012) 187

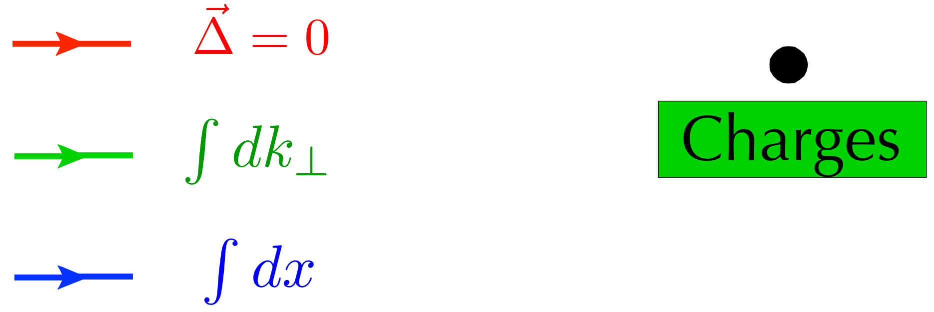
2σ uncertainties of extractions from currently available data

2σ uncertainties of extractions from pseudodata generated for EIC ($\sqrt{s} = 45\,\mathrm{GeV}$)

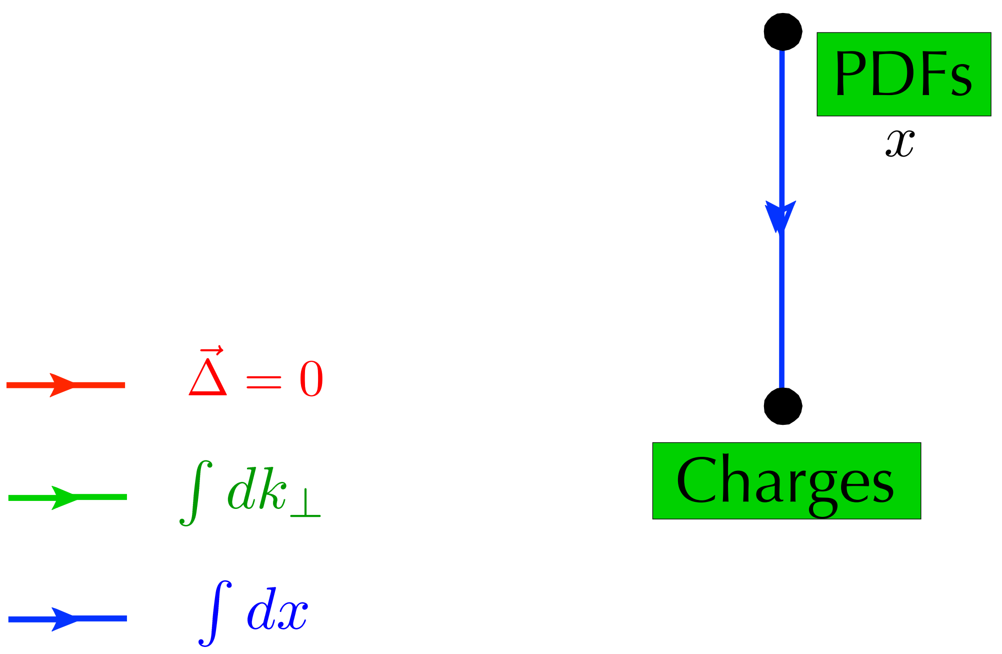




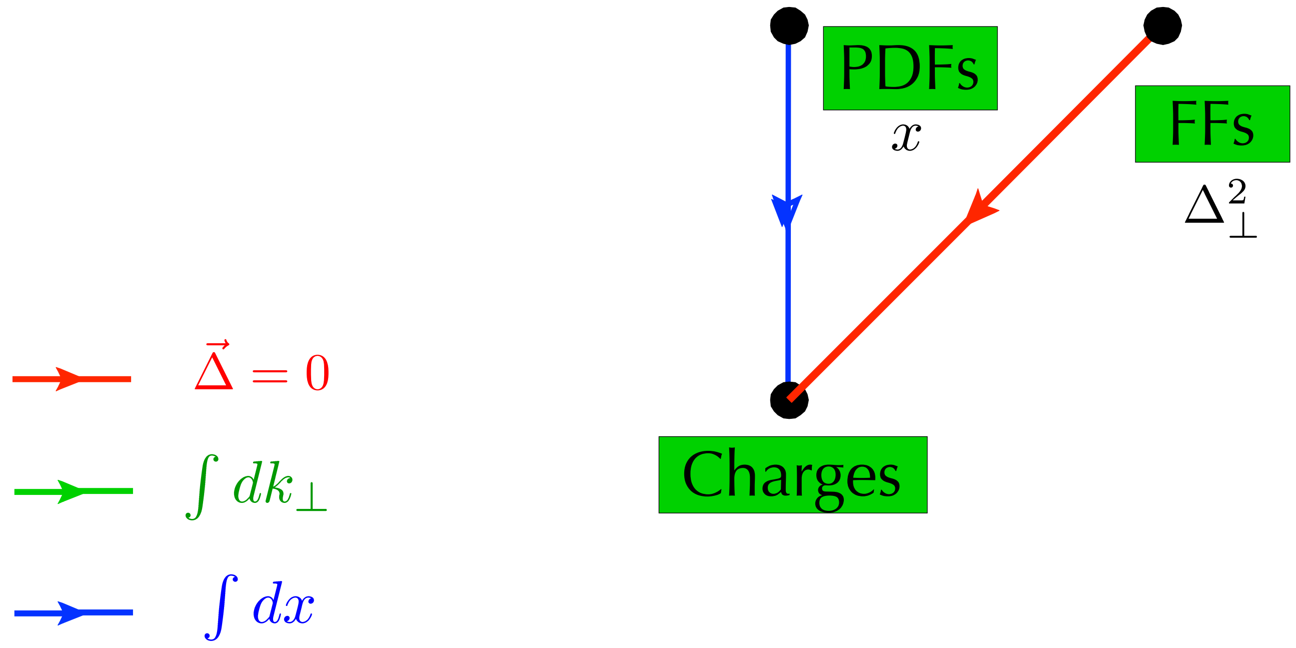
Accardi et al., The Electron Ion Collider: the next QCD Frontier, EPJA52 (2016) 268



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



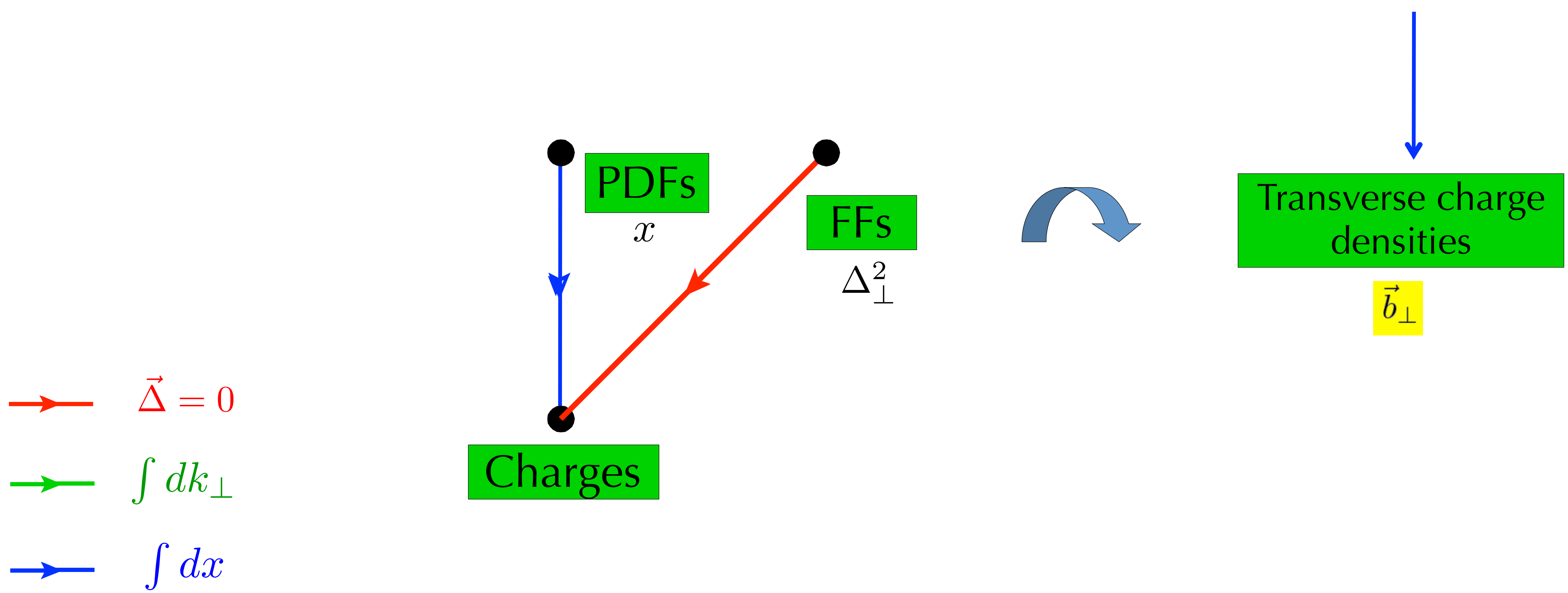
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier transform

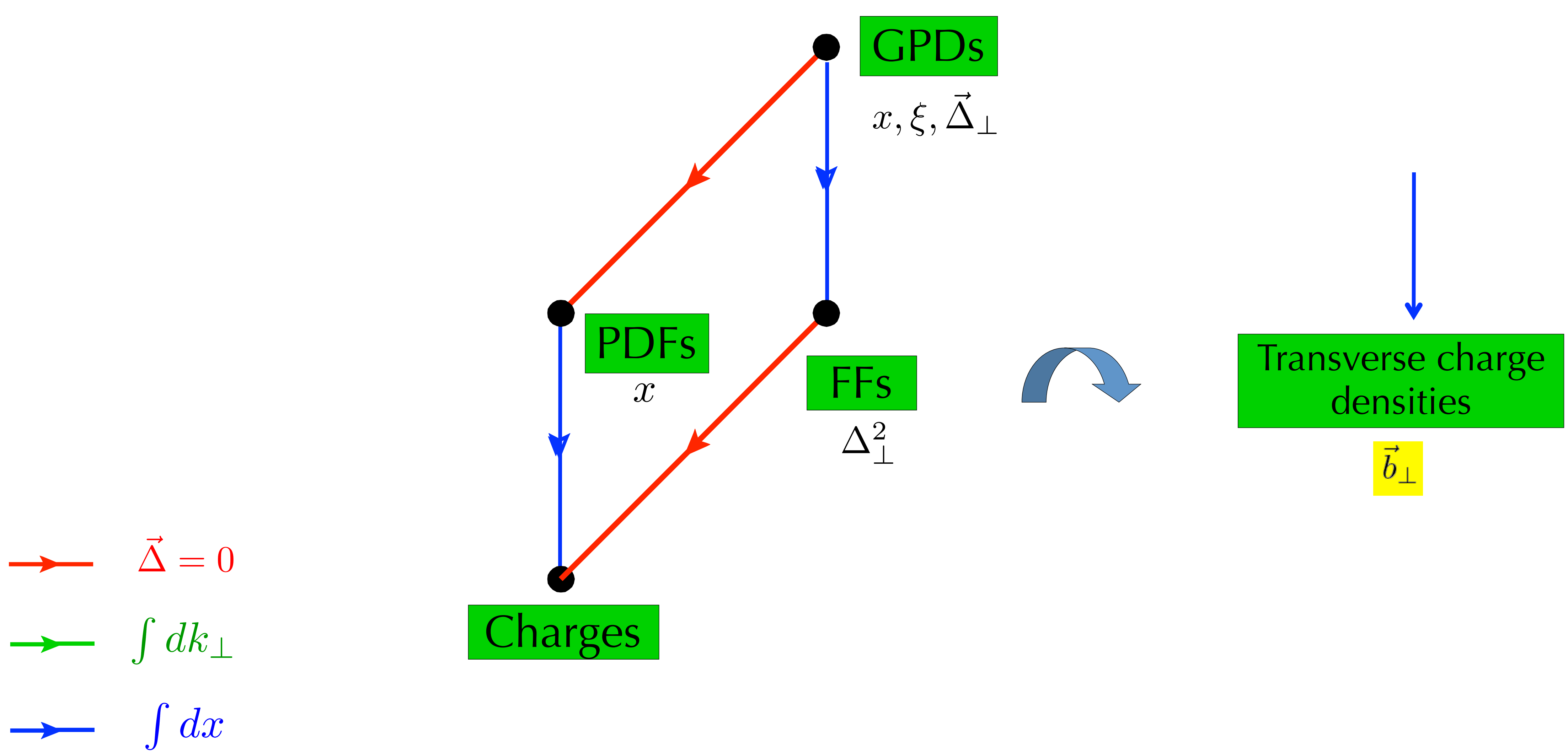
$\Delta_\perp \leftrightarrow b_\perp$



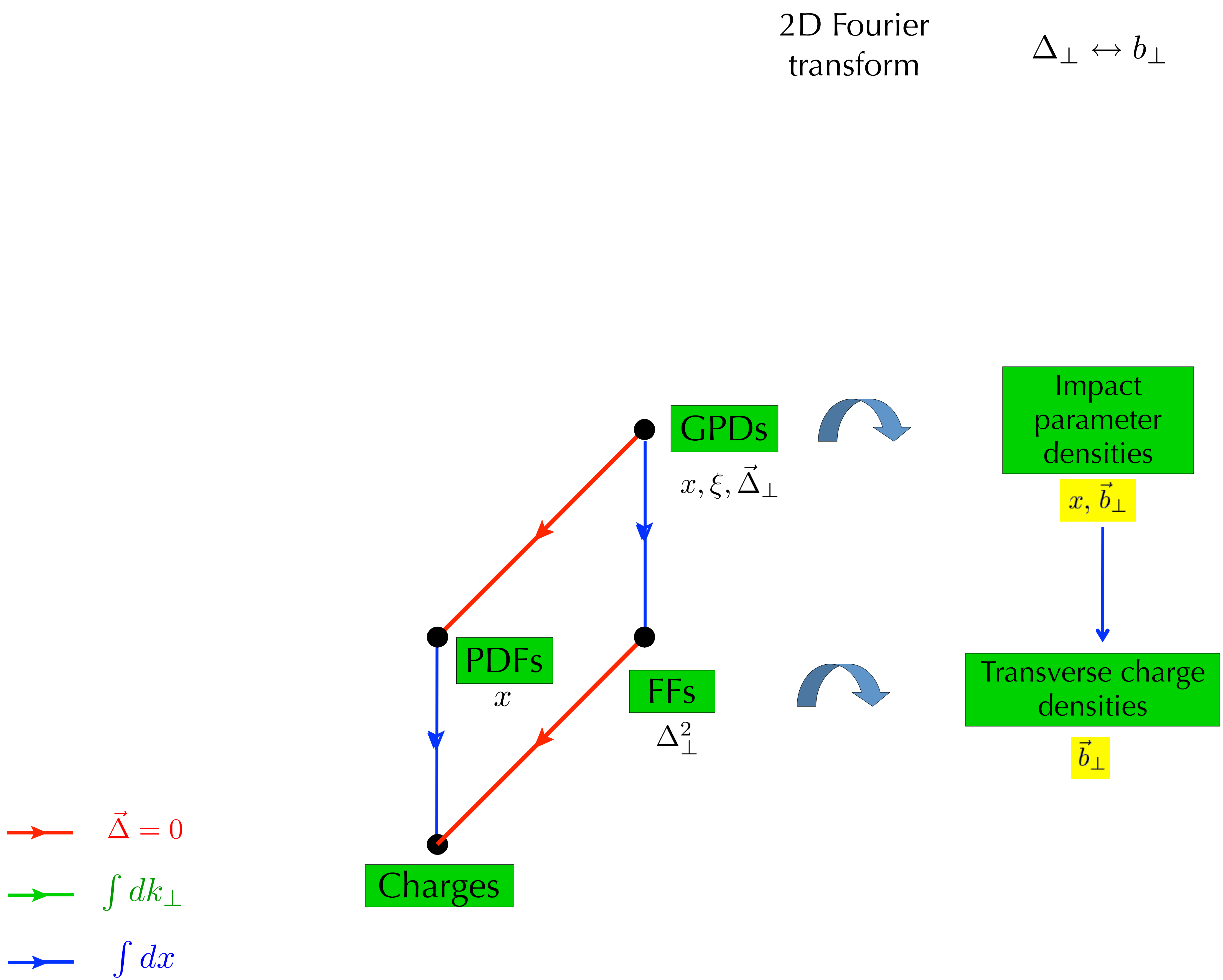
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

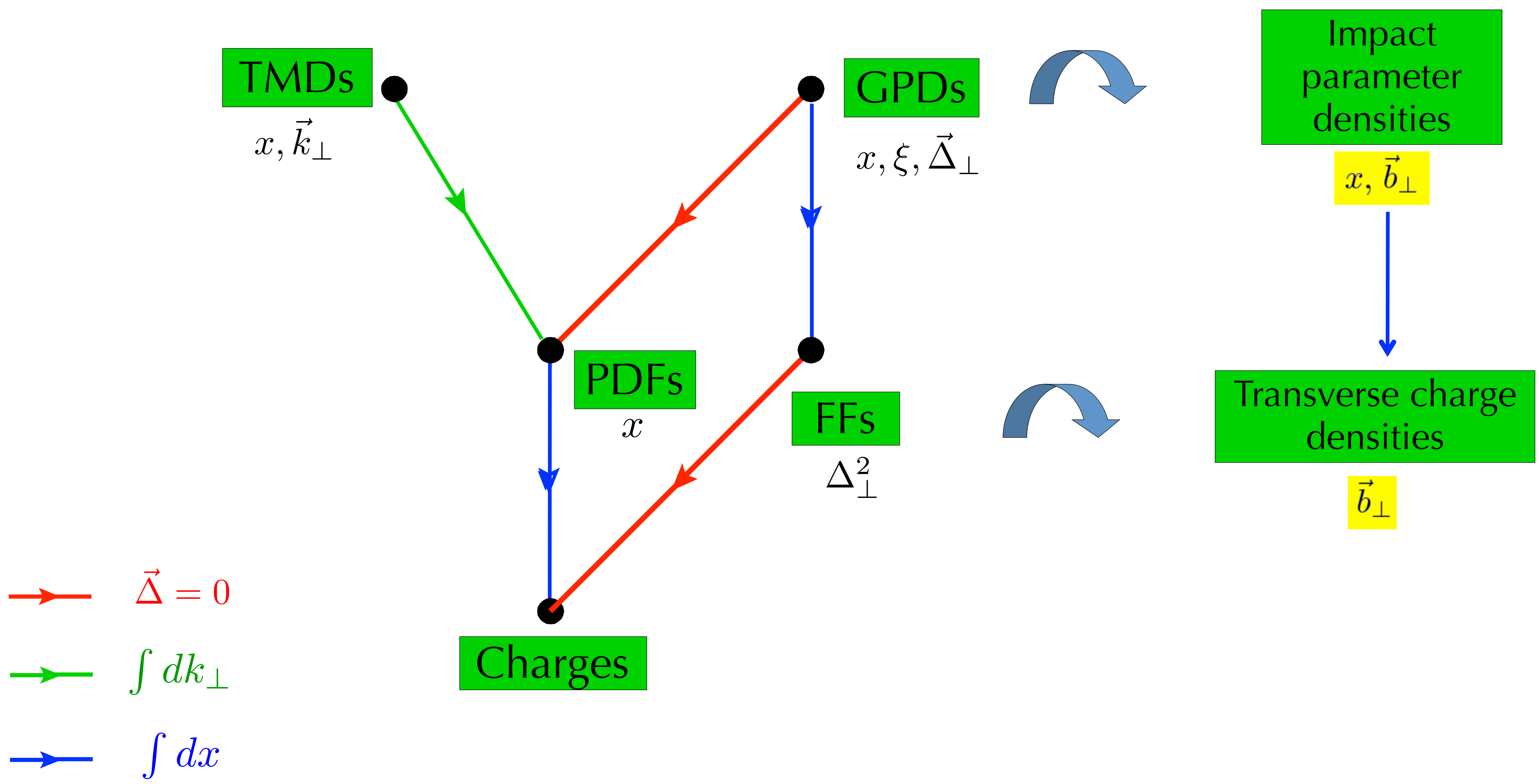


Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

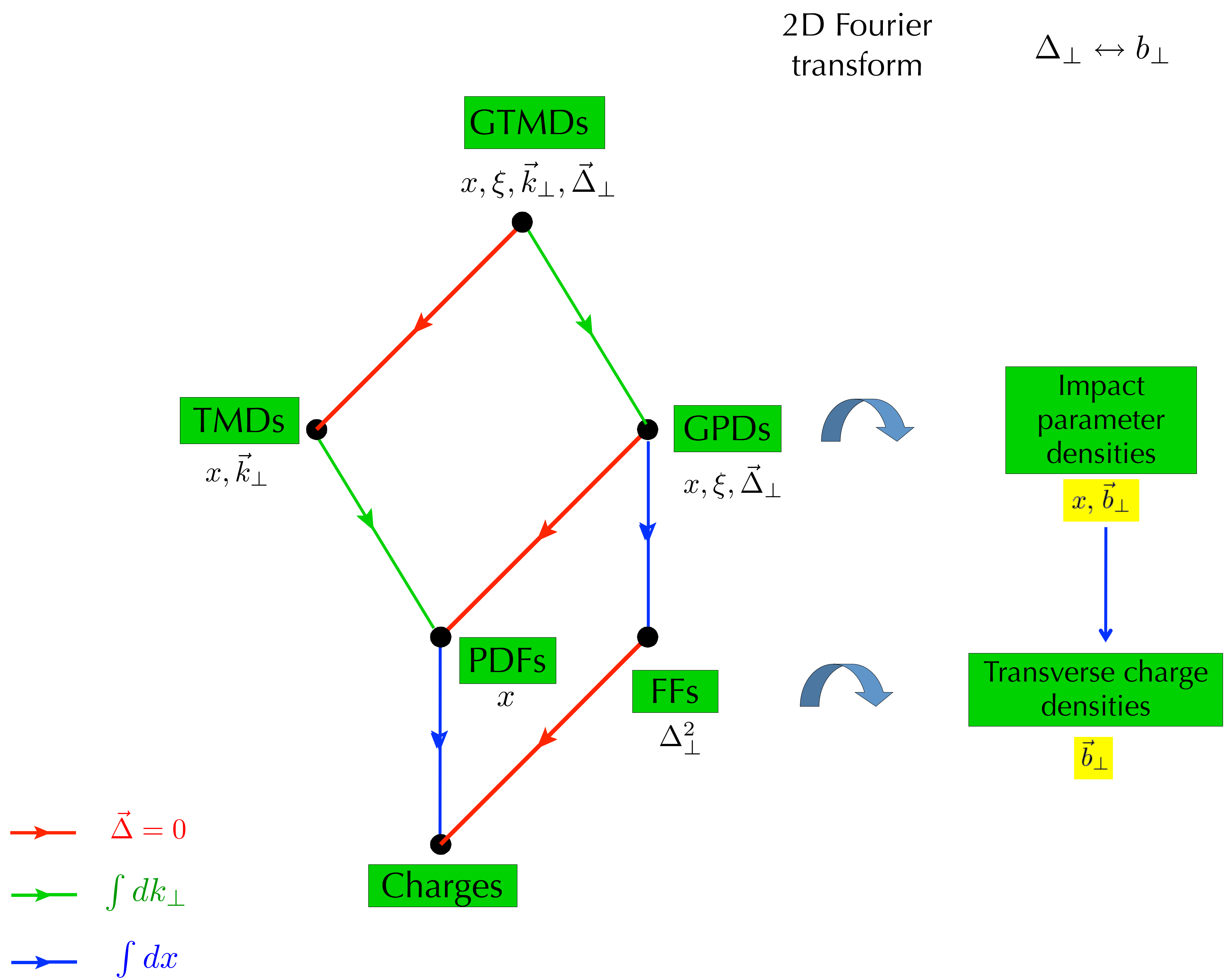


Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

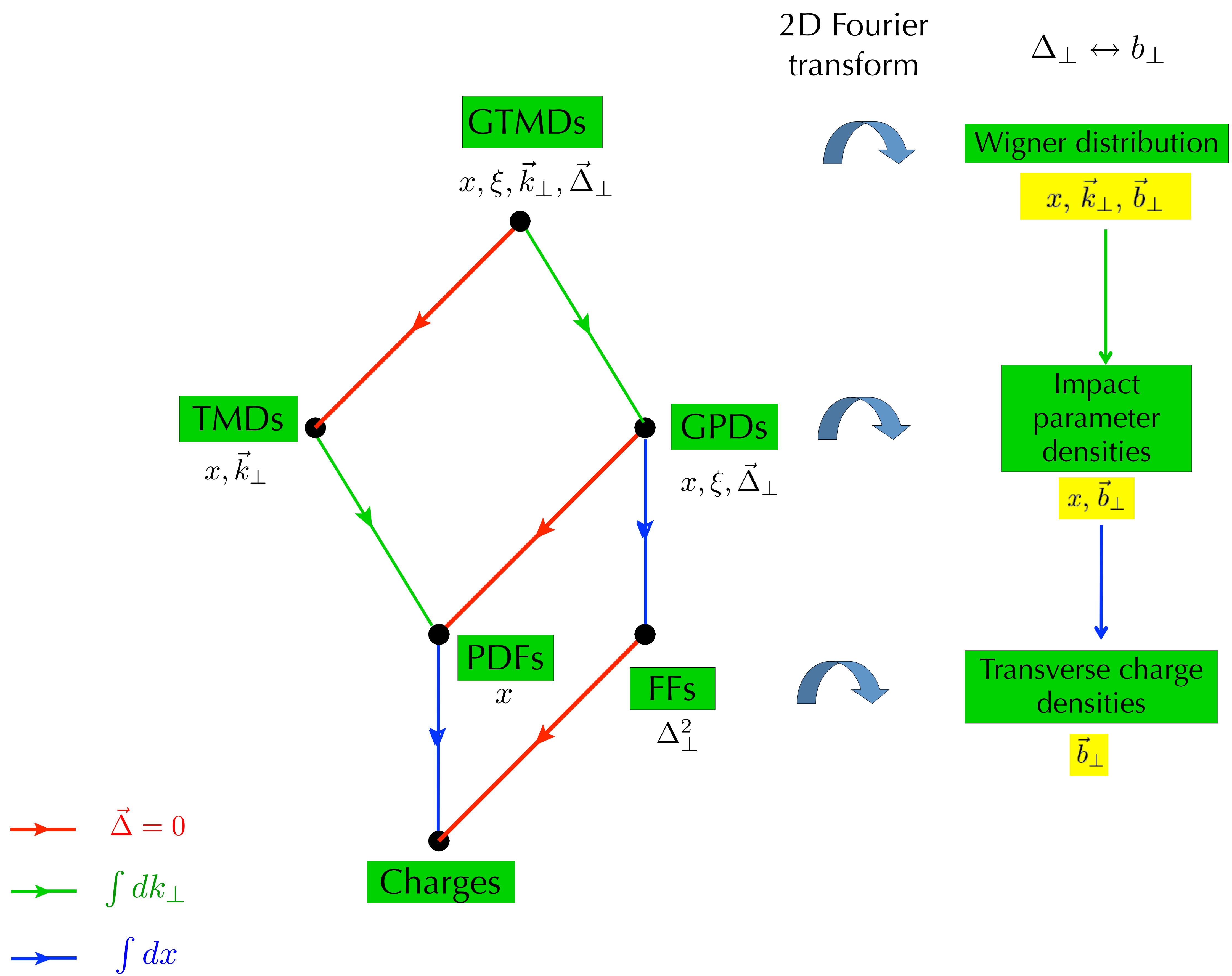
2D Fourier transform $\Delta_\perp \leftrightarrow b_\perp$



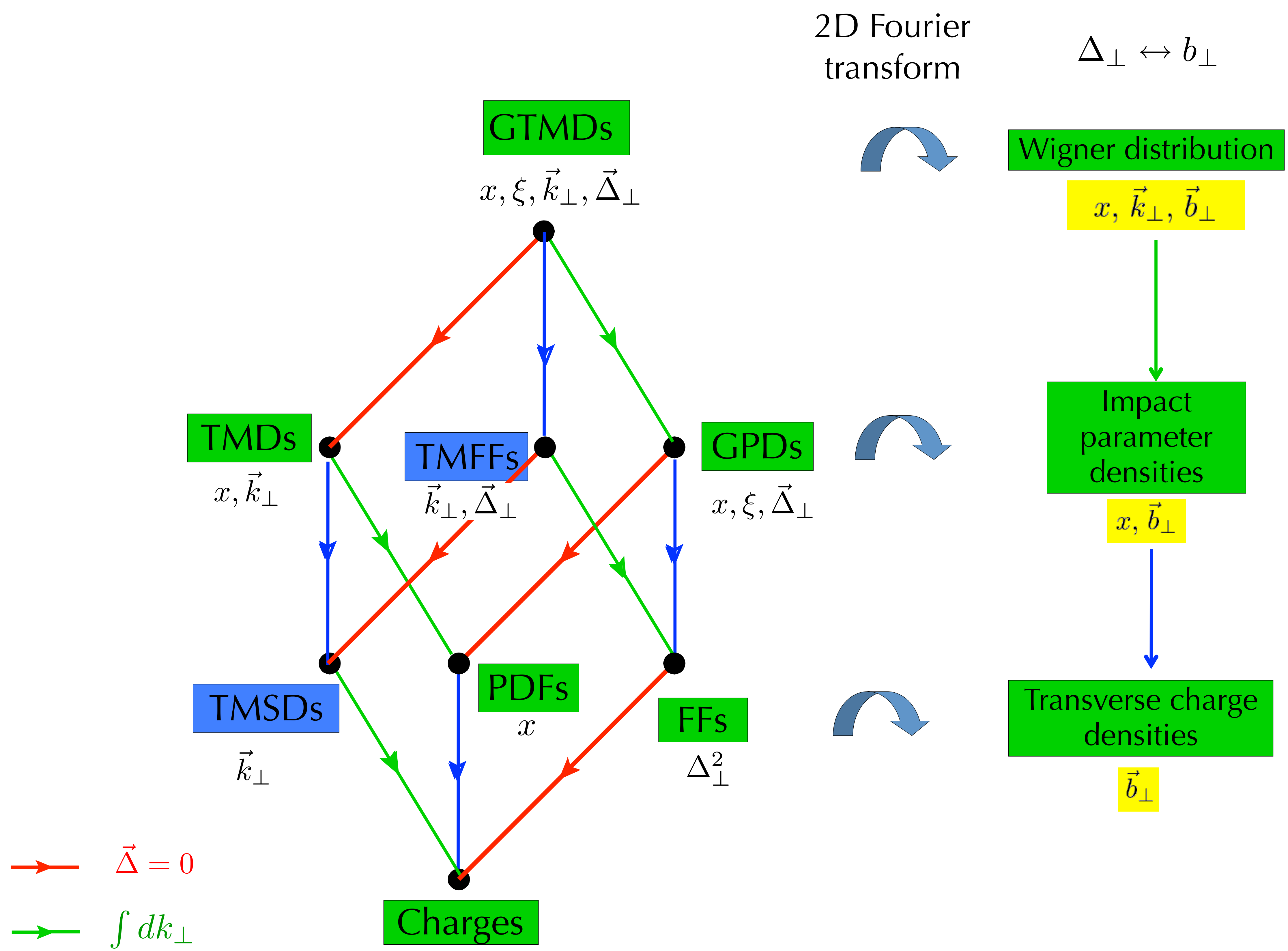
Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

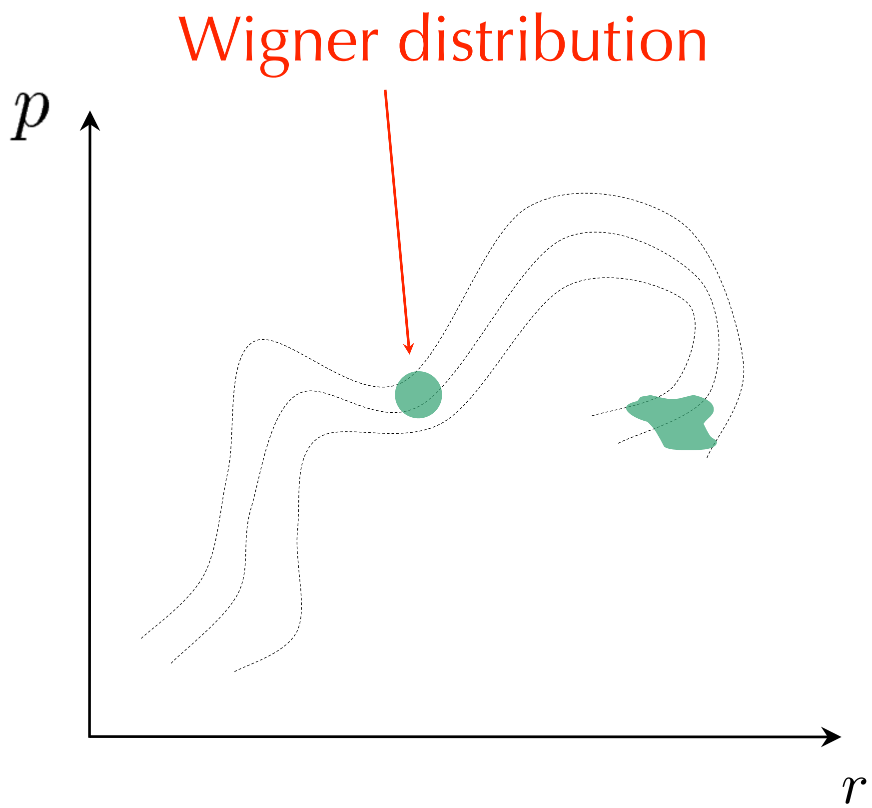


Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

Phase-Space Distributions in Quantum-Mechanics

Wigner (1932)
Moyal (1949)

Quantum Mechanics



$$
\begin{aligned}
\rho_W(r,k) &= \int \frac{\mathrm{d}z}{2\pi} e^{-ikz} \psi^*(r-\frac{z}{2})\psi(r+\frac{z}{2}) \\
&= \int \frac{\mathrm{d}\Delta}{2\pi} e^{-i\Delta r} \phi^*(k+\frac{\Delta}{2})\phi(k-\frac{\Delta}{2})
\end{aligned}
$$

Position-space density

$$|\psi(r)|^2 = \int \mathrm{d}k\, \rho_W(r,k)$$

Momentum-space density

$$|\phi(k)|^2 = 2\pi \int \mathrm{d}r\, \rho_W(r,k)$$

Quantum average

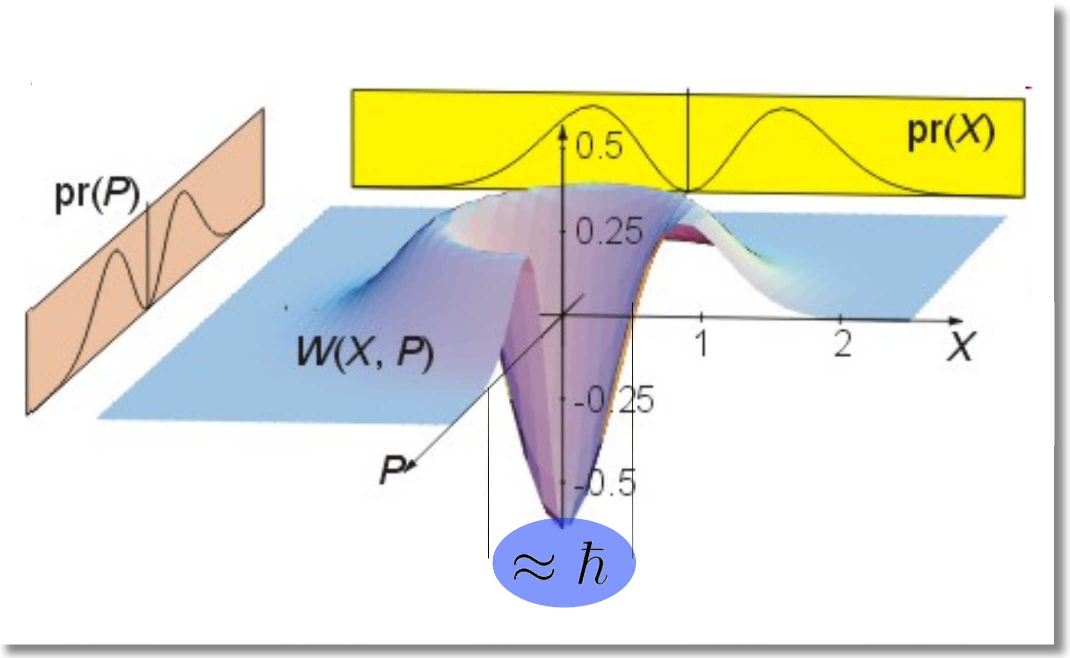
$$\langle \hat{O} \rangle = \int \mathrm{d}r\, \mathrm{d}k\, O(r,k)\, \rho_W(r,k)$$

Wigner distributions $(x, \vec{b}_\perp, \vec{k}_\perp)$

- Extend the concept of classical phase-space density

- Phase-space distributions of partons inside the nucleon

- Quasi-probabilistic interpretation



Heisenberg's uncertainty relation

$\Rightarrow$ Quasi-probabilistic interpretation $\xrightarrow{\hbar \to 0}$ classical density

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

Quark Wigner operator

$$\widehat{W}^{[\Gamma]}(\vec{r},k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r} - \frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\vec{r} + \frac{z}{2})$$

Wilson line

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

Quark Wigner operator $$\widehat{W}^{[\Gamma]}(\vec{r},k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z}\bar{\psi}(\vec{r}-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\vec{r}+\frac{z}{2})$$

Wilson line

Fixed light-front time $$z^+ = 0 \quad \Longleftrightarrow \quad \int \mathrm{d}k^-$$

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

Quark Wigner operator
$$\widehat{W}^{[\Gamma]}(\vec{r},k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r}-\frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\vec{r}+\frac{z}{2})$$

Wilson line

Fixed light-front time
$$z^+ = 0 \quad \Longleftrightarrow \quad \int \mathrm{d}k^-$$

Wigner distributions in the Breit frame
$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r}, k^+, \vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \langle \frac{\vec{\Delta}}{2}, \Lambda' | \widehat{W}^{[\Gamma]}(0, k^+, \vec{k}_\perp) | -\frac{\vec{\Delta}}{2}, \Lambda \rangle$$

3+3 D

no semi-classical interpretation

Ji (2003)
Belitsky, Ji, Yuan (2004)

Wigner Distributions in QFT

Dirac matrix

$\sim$ quark polarization

Quark Wigner operator $$\widehat{W}^{[\Gamma]}(\vec{r},k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \bar{\psi}(\vec{r}-\frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\vec{r}+\frac{z}{2})$$

Wilson line

Fixed light-front time $$z^+ = 0 \quad \Longleftrightarrow \quad \int \mathrm{d}k^-$$

Wigner distributions in the Breit frame $$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r},k^+,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\langle \frac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\frac{\vec{\Delta}}{2},\Lambda\rangle$$

3+3 D

no semi-classical interpretation

Ji (2003)
Belitsky, Ji, Yuan (2004)

Wigner distributions in the Drell-Yan frame
$(\Delta^+ = 0)$

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp,k^+,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\langle p^+,\frac{\vec{\Delta}_\perp}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|p^+,-\frac{\vec{\Delta}_\perp}{2},\Lambda\rangle$$

2+3 D

semi-classical interpretation

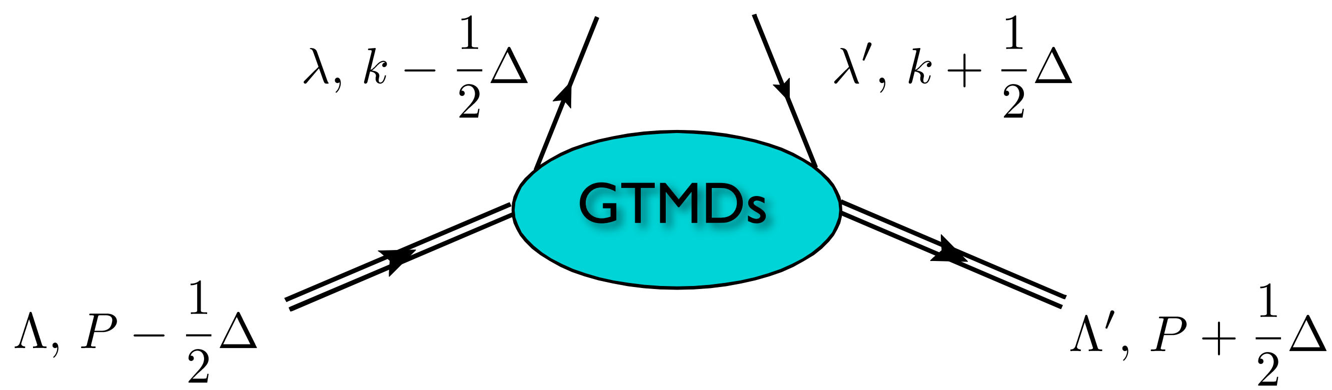
GTMDs

Lorcè, BP (2011)
Lorcè, BP, Xiong, Yuan (2012)

Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



Quark polarization

$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}^q_{\lambda'}(-\frac{z}{2})\, \Gamma\, \mathcal{W}(-\frac{z}{2}, \frac{z}{2}|n) \psi^q_\lambda(\frac{z}{2}) | P, \Lambda\rangle e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)}}$$

Nucleon polarization

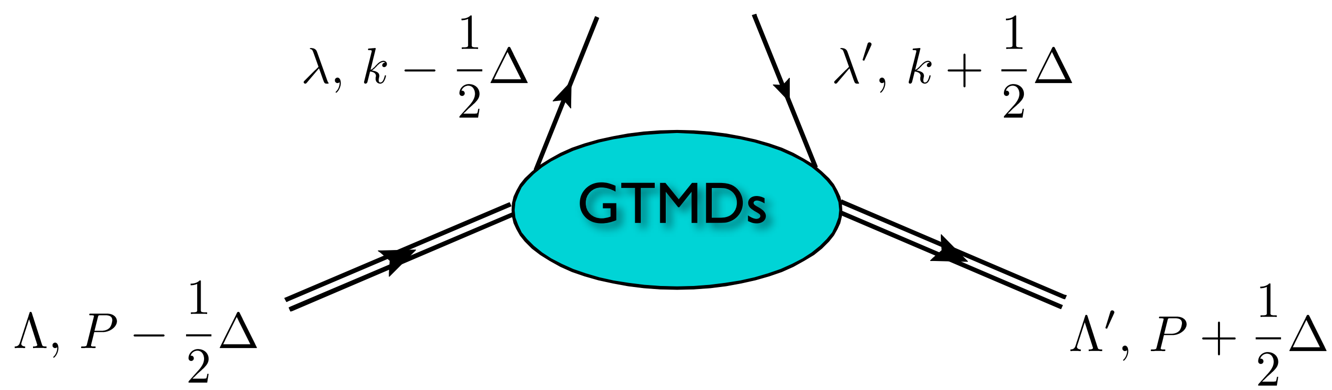
4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$$

Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda}=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_{\lambda}(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations $\longleftrightarrow$ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

x : average fraction of quark longitudinal momentum

ξ : fraction of longitudinal momentum transfer

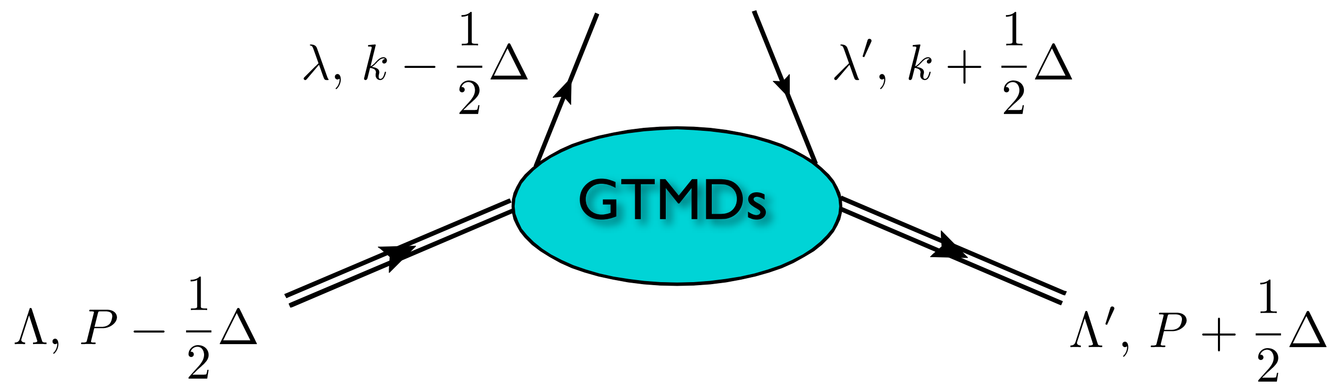
$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$: nucleon transverse momentum

Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



Quark polarization

$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}}$$

Nucleon polarization

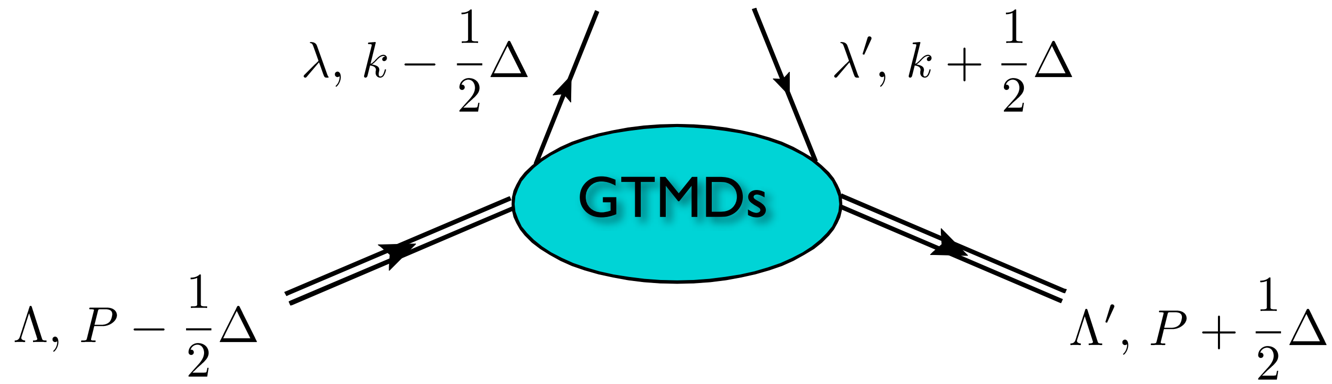
4 X 4 =16 polarizations $\longleftrightarrow$ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}^q_{\lambda'}(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}|n) \psi^q_\lambda(\tfrac{z}{2}) | P, \Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations $\longleftrightarrow$ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$$

$\downarrow$ Fourier transform $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$ at $\xi = 0$

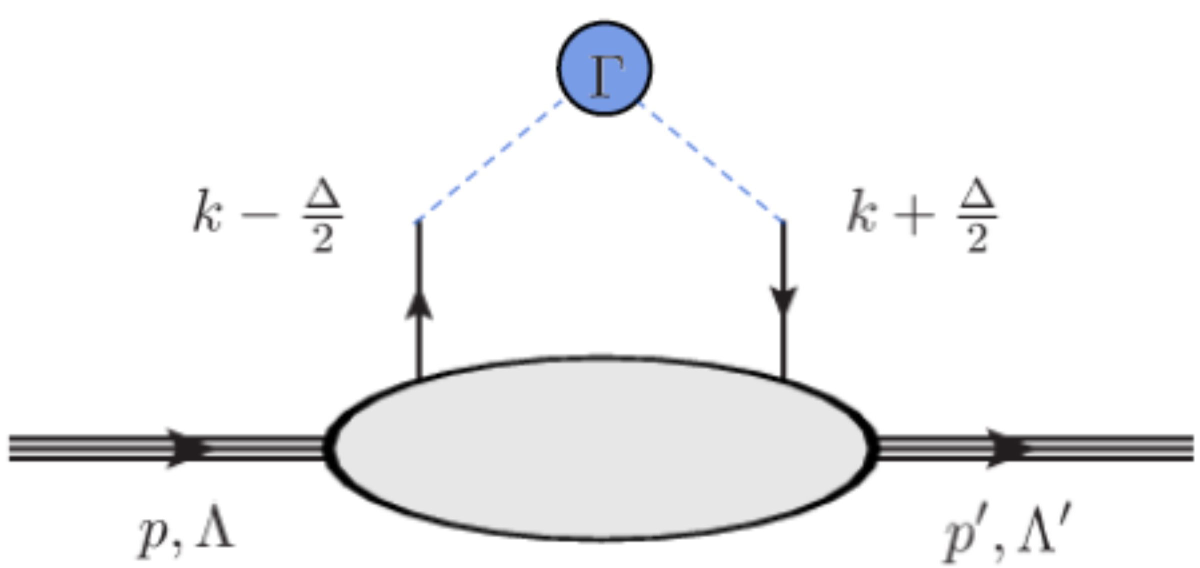
$\tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x, \xi, \vec{k}_\perp, \vec{b}_\perp)$ 32 real Wigner distributions

Transverse phase-space distributions

★ Twist-2: $\boxed{\Gamma_{\rm twist-2} = \gamma^+,\, \gamma^+\gamma_5,\, i\sigma^{j+}\gamma_5}$

quark polarization: **U** **L** **T**

★ Nucleon polarization: **U** **L** **T**



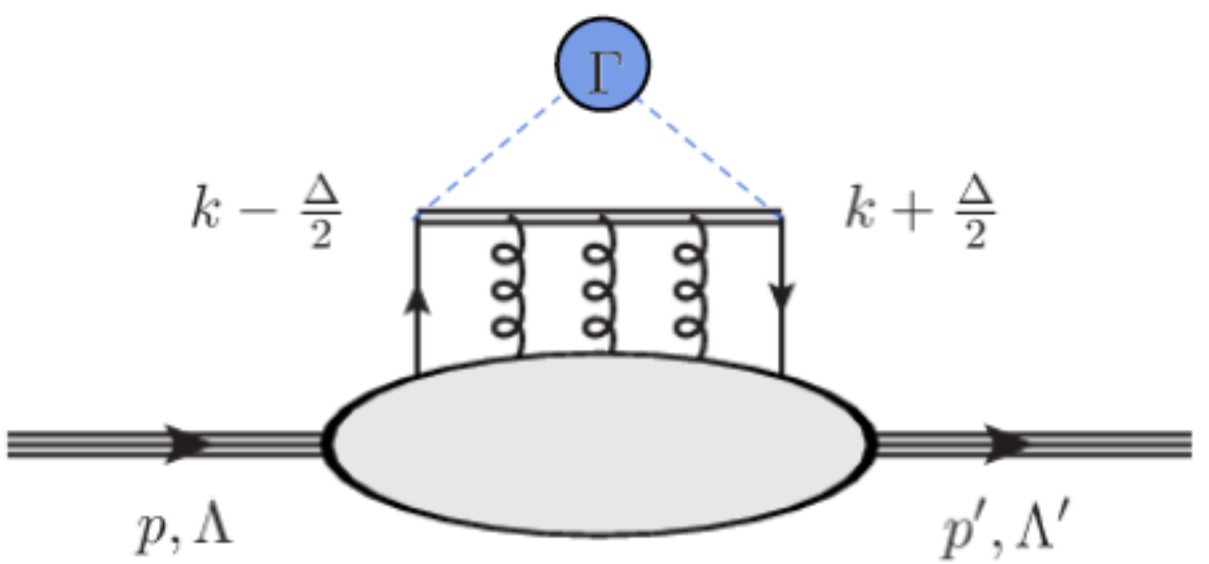
Transverse phase-space distributions

★ Twist-2: $\boxed{\Gamma_{\mathrm{twist}-2} = \gamma^+,\, \gamma^+\gamma_5,\, i\sigma^{j+}\gamma_5}$

quark polarization: **U** **L** **T**

★ Nucleon polarization: **U** **L** **T**

★Gauge link: T-even and T-odd functions

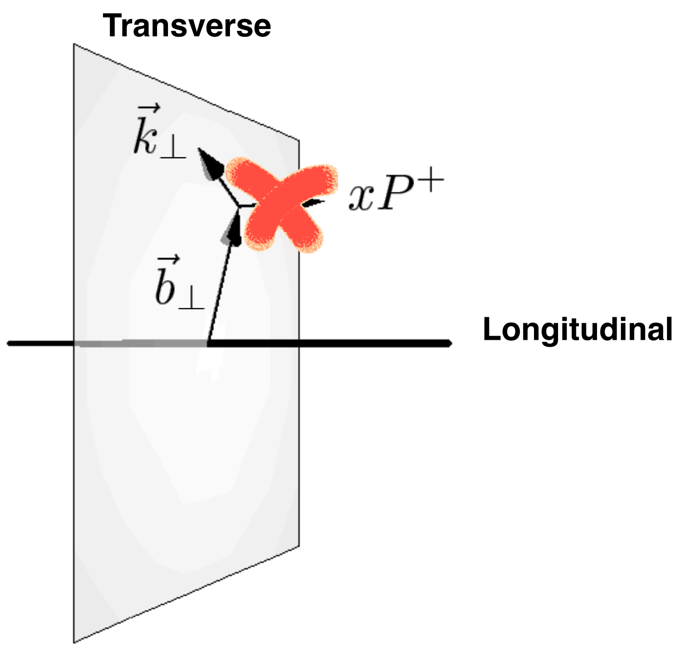


Transverse phase-space distributions

★ Twist-2: $\boxed{\Gamma_{\mathrm{twist}-2} = \gamma^+,\, \gamma^+\gamma_5,\, i\sigma^{j+}\gamma_5}$

quark polarization: **U** **L** **T**

★ Nucleon polarization: **U** **L** **T** → 16 complex GTMDs → 32 real Wigner Distributions



Transverse Phase-Space distributions

$$\rho_X(\vec{k}_\perp, \vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x, \vec{k}_\perp, \vec{b}_\perp) \quad X = UU,\, UL, UT,\, LU,\, \ldots$$

Angular Correlations

$$\rho_{\vec{S}\vec{S}^q} = \rho_{UU} + S_L\,\rho_{LU} + S_L^q\,\rho_{UL} + S_L S_L^q\,\rho_{LL} + S_T^i\,(\rho_{T^iU} + S_L^q\,\rho_{T^iL}) + S_T^{qi}\,(\rho_{UT^i} + S_L\,\rho_{LT^i}) + S_T^i S_T^{qj}\,\rho_{T^iT^j}$$

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

$\boxed{\xi = 0}$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T,\ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

Angular Correlations

$$\rho_{\vec{S}\vec{S}^q} = \rho_{UU} + S_L\,\rho_{LU} + S_L^q\,\rho_{UL} + S_L S_L^q\,\rho_{LL} + S_T^i\,(\rho_{T^iU} + S_L^q\,\rho_{T^iL}) + S_T^{qi}\,(\rho_{UT^i} + S_L\,\rho_{LT^i}) + S_T^i S_T^{qj}\,\rho_{T^iT^j}$$

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1\rangle$	$\langle S_L^q \ell_L^q\rangle$	$\langle S_x^q \ell_x^q\rangle$	$\langle S_y^q \ell_y^q\rangle$
L	$\langle S_L \ell_L^q\rangle$	$\langle S_L S_L^q\rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q\rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q\rangle$
T_x	$\langle S_x \ell_x^q\rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q\rangle$	$\langle S_x S_x^q\rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q\rangle$
T_y	$\langle S_y \ell_y^q\rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q\rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q\rangle$	$\langle S_y S_y^q\rangle$

$\boxed{\xi = 0}$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\cancel{\tilde{E}_T}$
T	E	$\cancel{\tilde{E}}$	$H_T,\ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp\ \text{fixed}} \longrightarrow \text{2+2 dimensions}\,(\vec{b}_\perp,\vec{k}_\perp)$$

Multipole decomposition

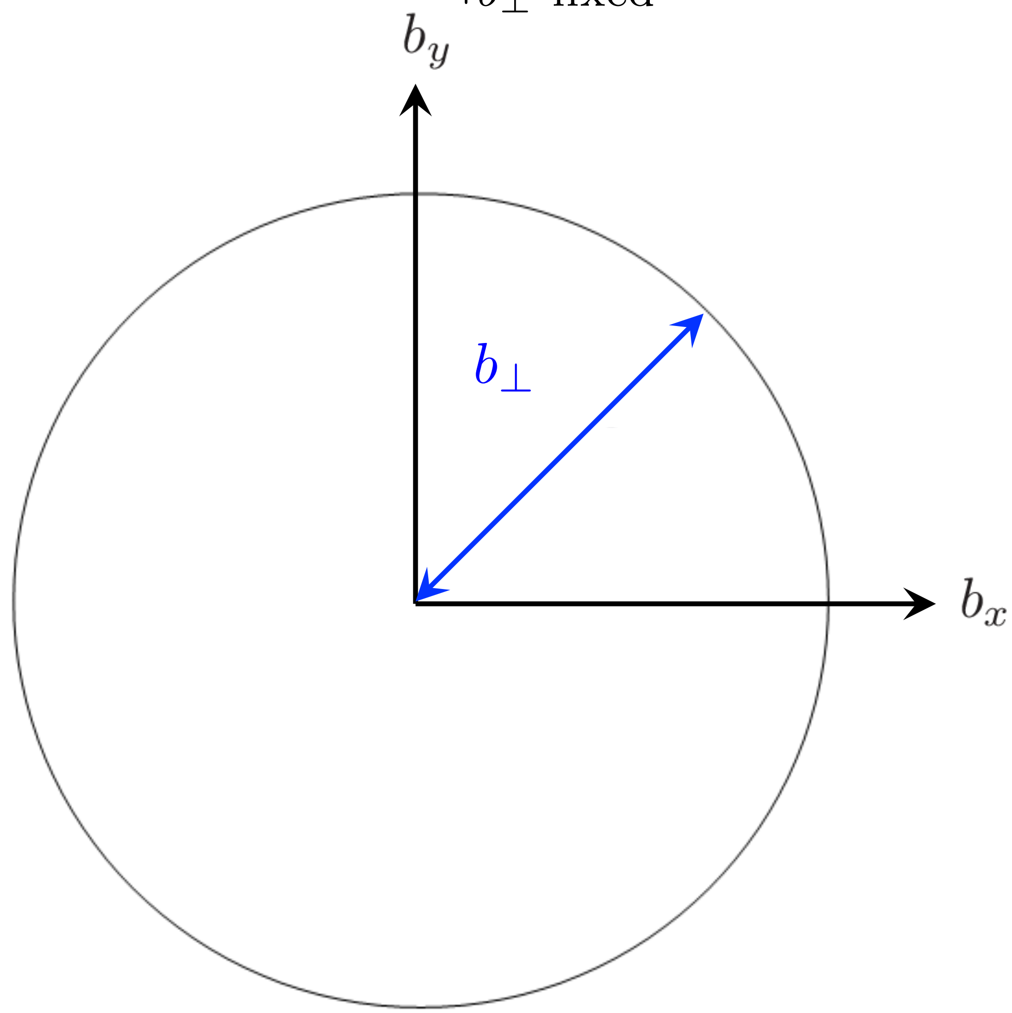
$$\rho_X = \sum_{m_k,m_b} \rho_X^{(m_k,m_b)}$$

using PT symmetries

Lorcé, BP, PRD 96, 2016

Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\,\rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp\ \text{fixed}}$$ $\longrightarrow$ 2+2 dimensions $(\vec{b}_\perp,\vec{k}_\perp)$



Multipole decomposition

$$\rho_X = \sum_{m_k,m_b} \rho_X^{(m_k,m_b)}$$

using PT symmetries

Lorcé, BP, PRD 96, 2016

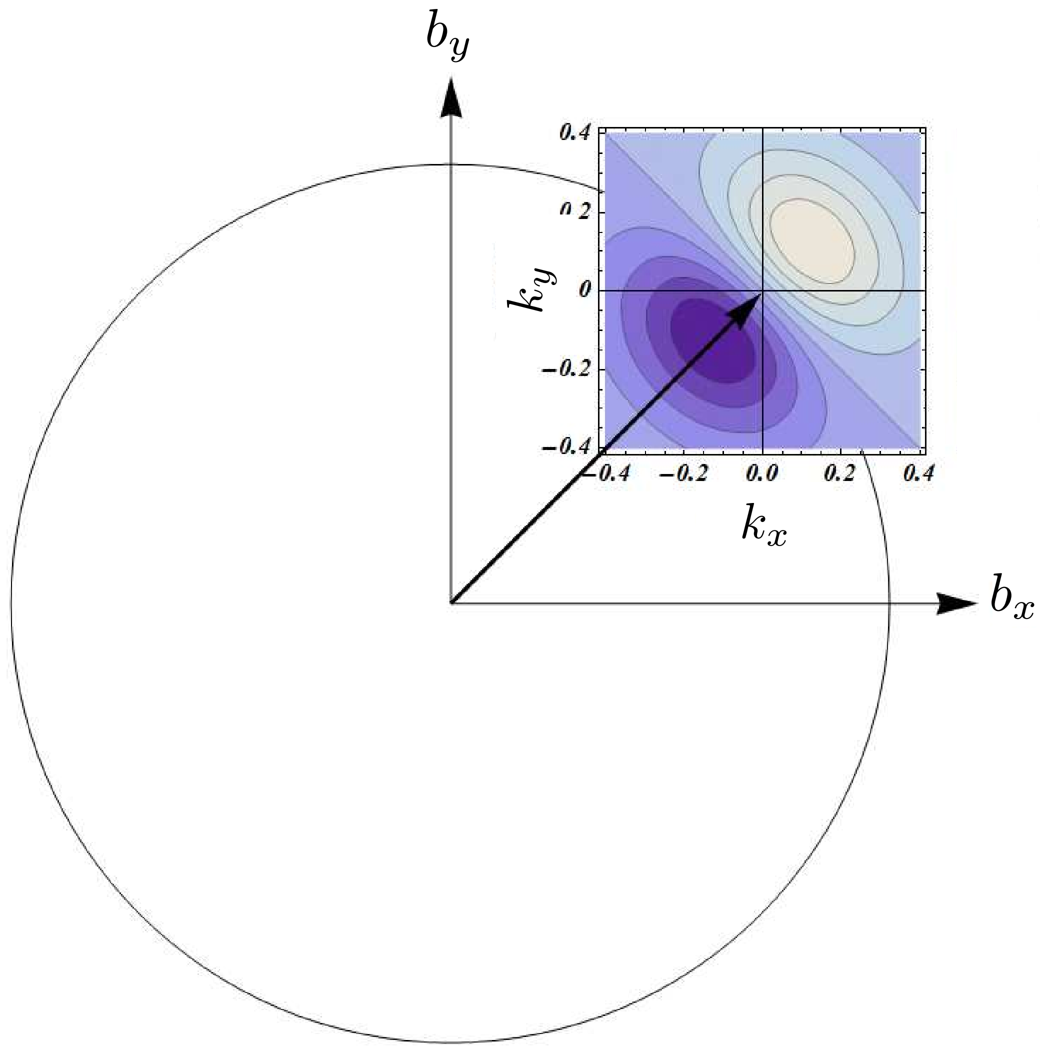
Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\,\rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp\ \text{fixed}} \longrightarrow \text{2+2 dimensions}\,(\vec{b}_\perp,\vec{k}_\perp)$$

Multipole decomposition

$$\rho_X = \sum_{m_k,m_b} \rho_X^{(m_k,m_b)}$$

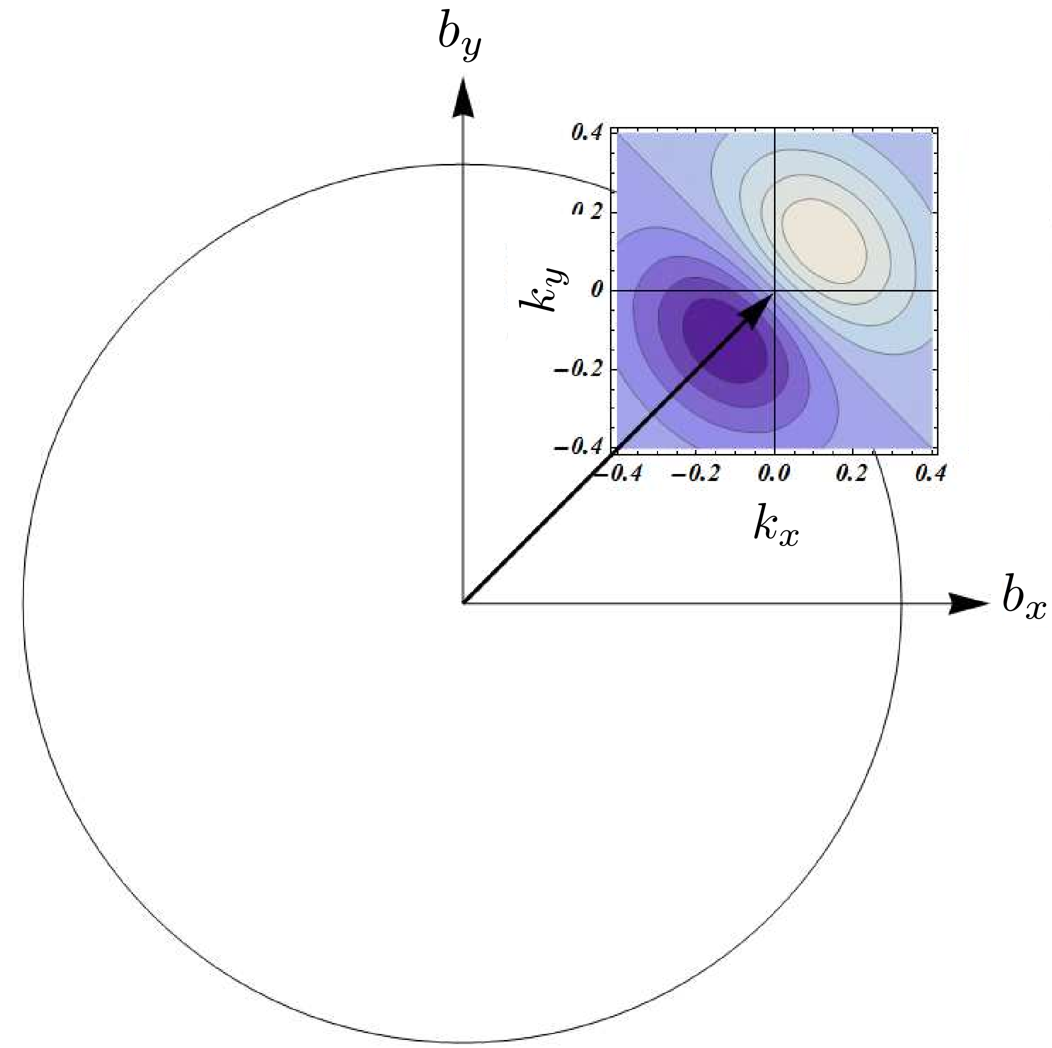
using PT symmetries



Lorcé, BP, PRD 96, 2016

Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\,\vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x,\vec{k}_\perp,\vec{b}_\perp;\hat{P}=\vec{e}_z,\eta=+1)\big|_{\vec{b}_\perp \text{ fixed}} \longrightarrow \text{2+2 dimensions}\,(\vec{b}_\perp,\vec{k}_\perp)$$

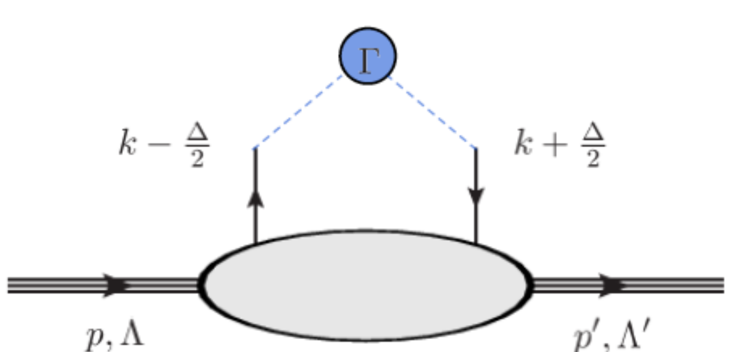


Multipole decomposition

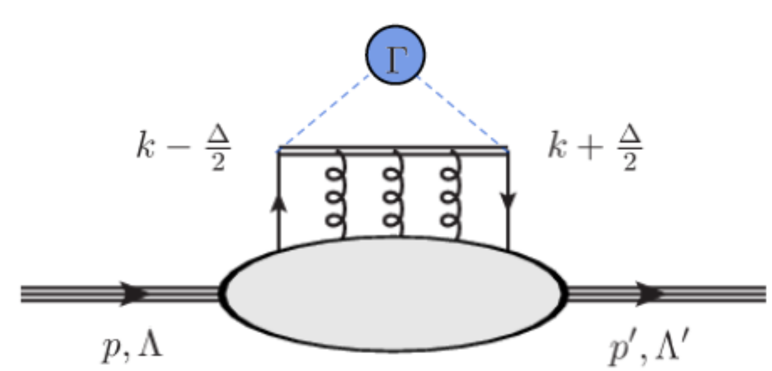
$$\rho_X = \sum_{m_k,m_b} \rho_X^{(m_k,m_b)}$$

using PT symmetries

ρ_X^e T-even



ρ_X^o T-odd

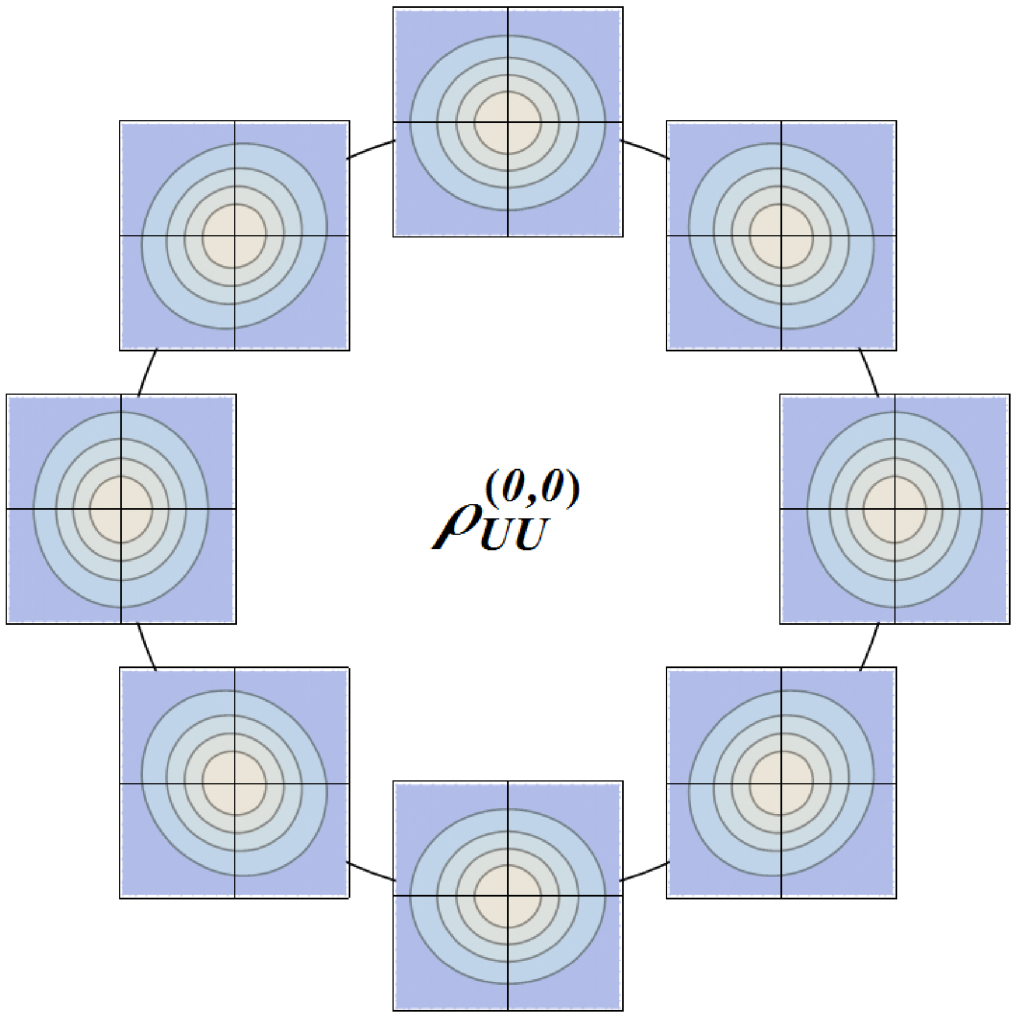


Lorcé, BP, PRD 96, 2016

UU

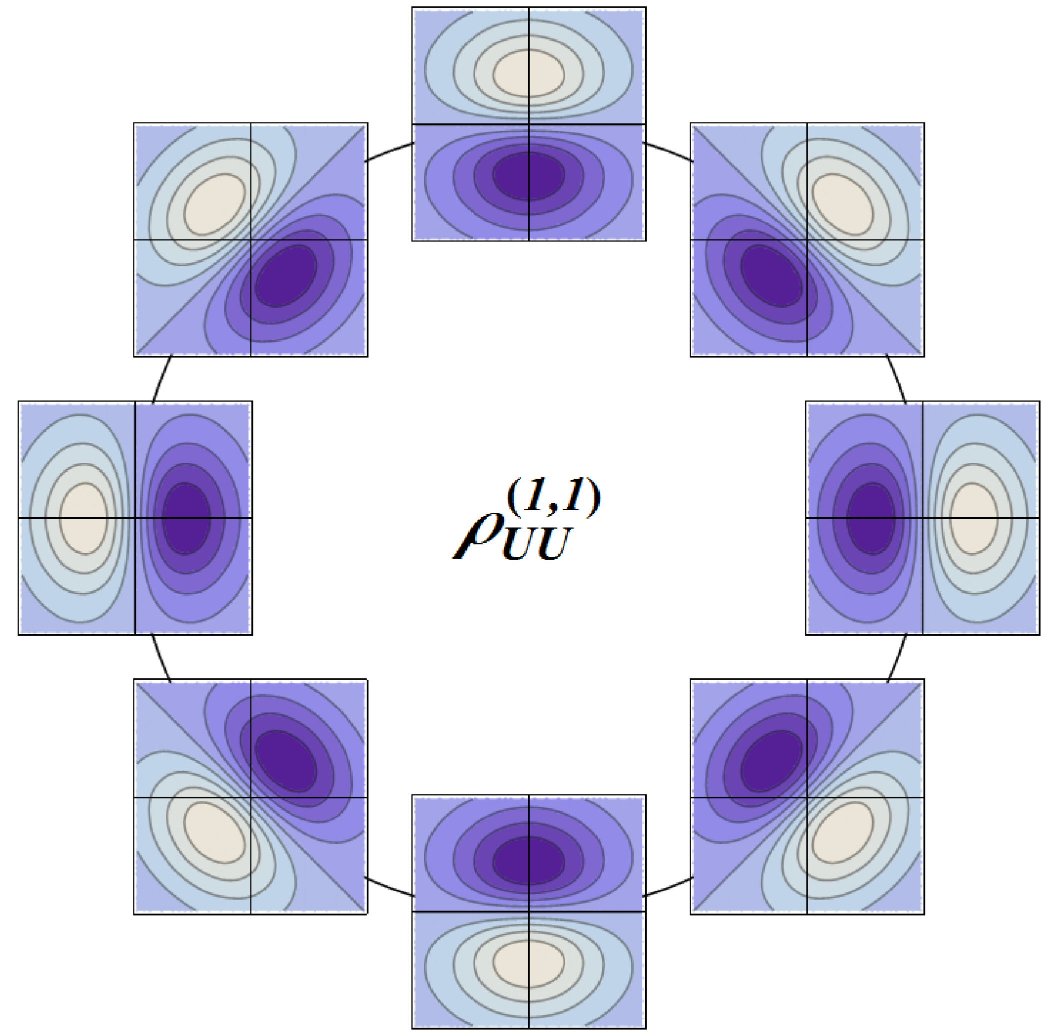
Unpolarized quarks in unpolarized proton

$\Re e[F_{11}]$



naive time-reversal even

$\Im m[F_{11}]$



naive time-reversal odd

Integral over $k_\perp$ ➔ GPD (monopole)

Integral over $b_\perp$ ➔ TMD (monopole)

no counterpart in the GPD and TMD cases

polar flow $(\vec{k}_\perp \perp \vec{b}_\perp)$ preferred over radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

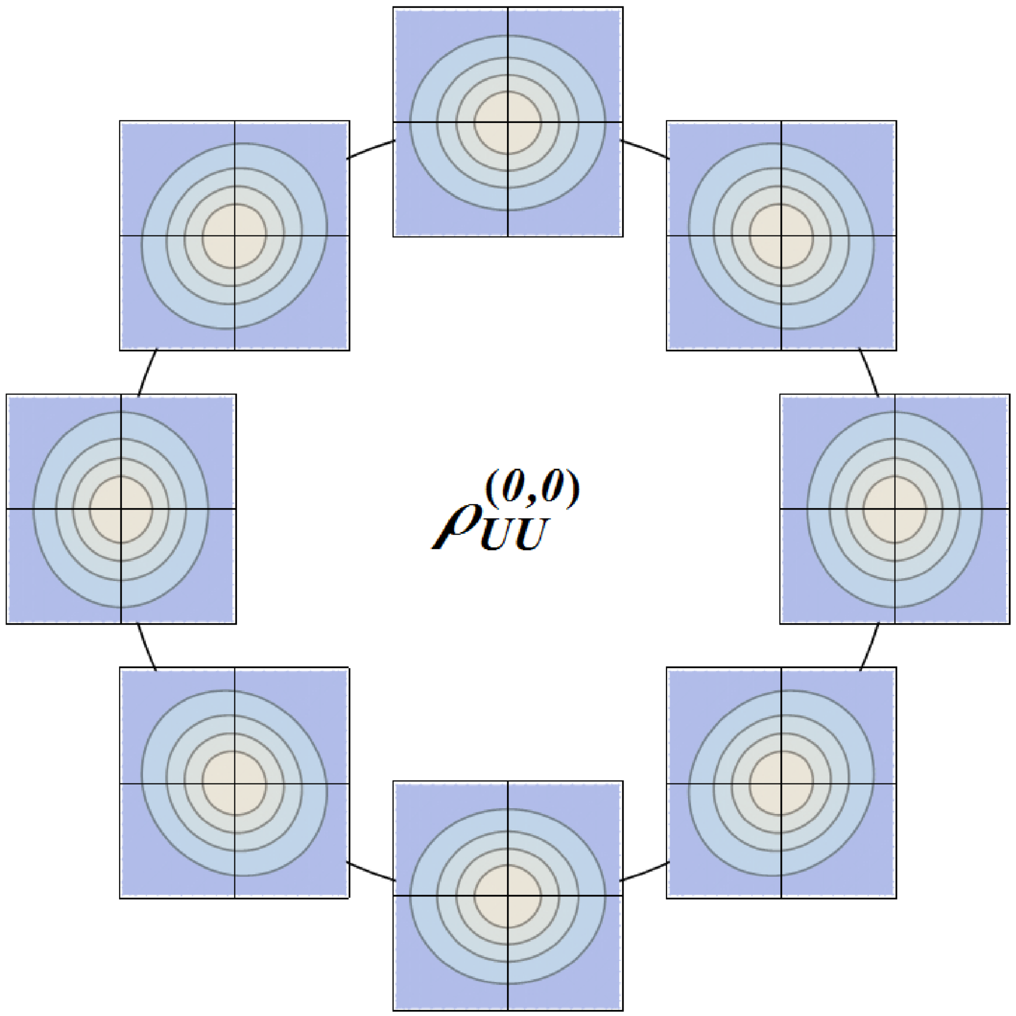
bottom-up symmetry ➔ no net OAM

net radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

due to initial/final state interactions

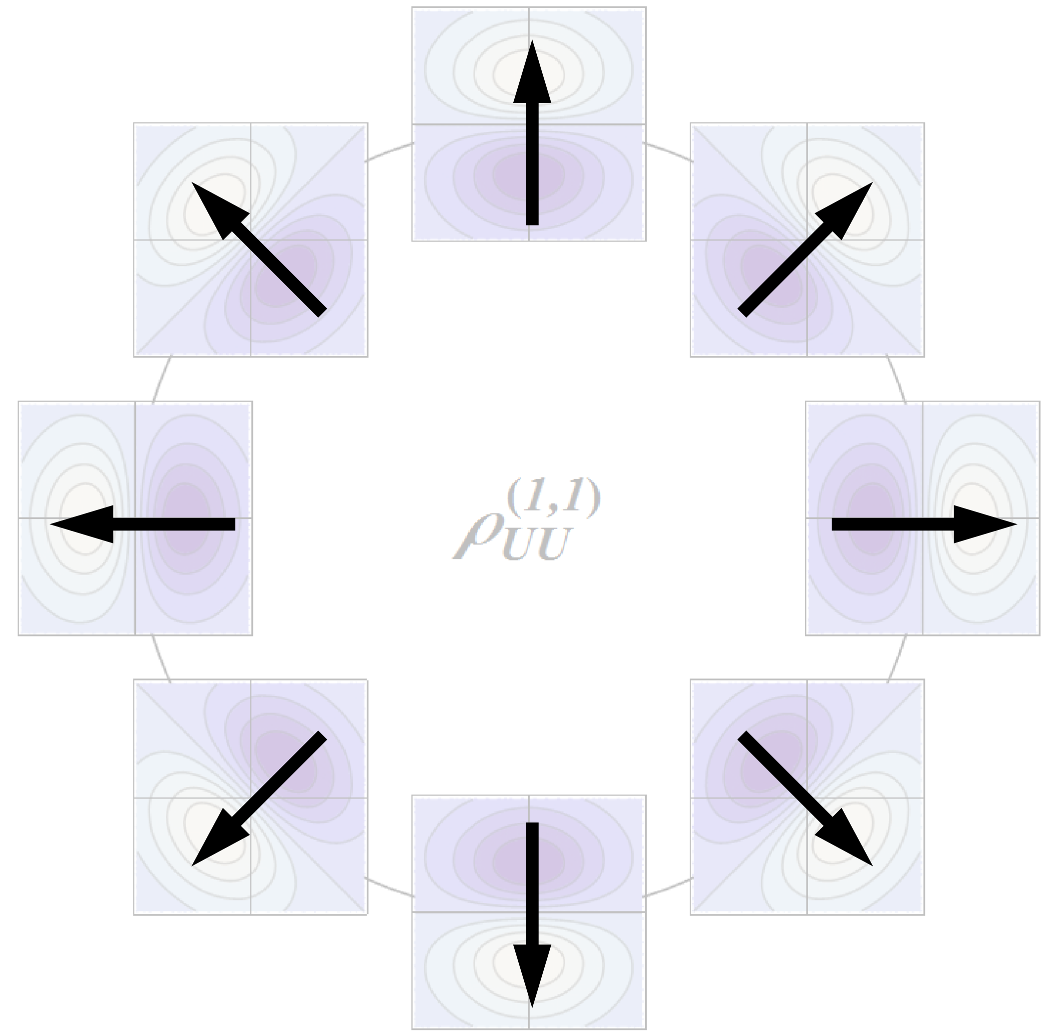
Unpolarized quarks in unpolarized proton

$\Re e[F_{11}]$



naive time-reversal even

$\Im m[F_{11}]$



naive time-reversal odd

Integral over $k_\perp$ ➔ GPD (monopole)

Integral over $b_\perp$ ➔ TMD (monopole)

no counterpart in the GPD and TMD cases

polar flow $(\vec{k}_\perp \perp \vec{b}_\perp)$ preferred over radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

bottom-up symmetry ➔ no net OAM

net radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$

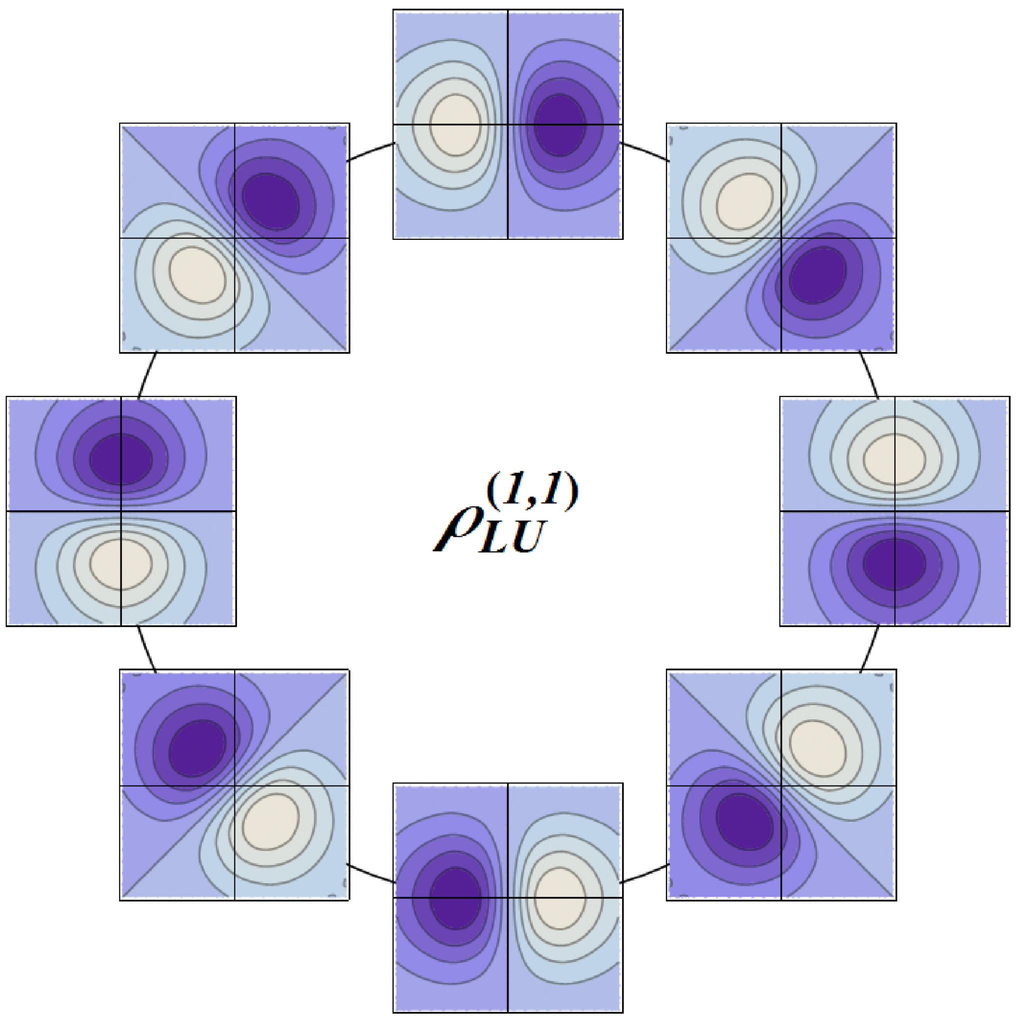
due to initial/final state interactions

LU

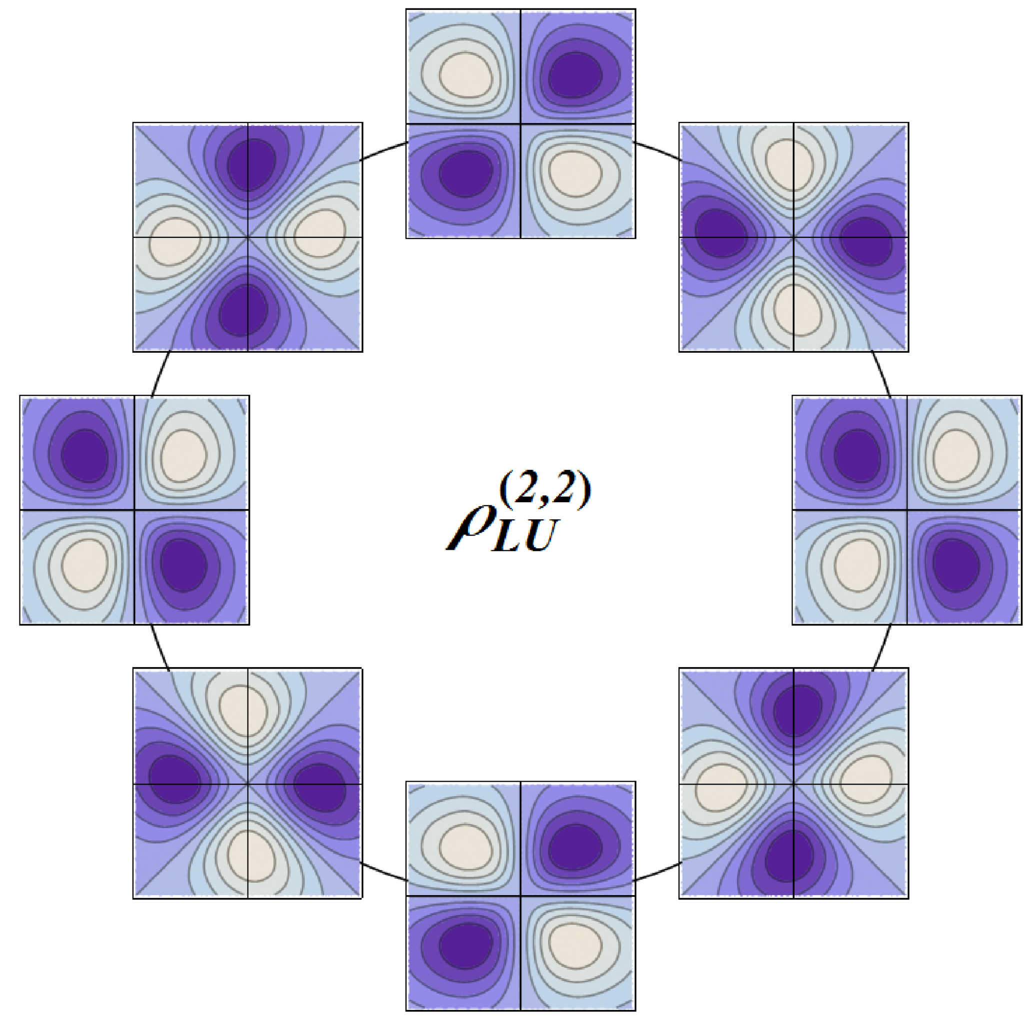
Unpolarized quarks in Longitudinally pol. proton

unique information from GTMDs

$\Re e[F_{14}]$



$\Im m[F_{14}]$



naive time-reversal even

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

orbital flow → net OAM correlated S_z with

naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z

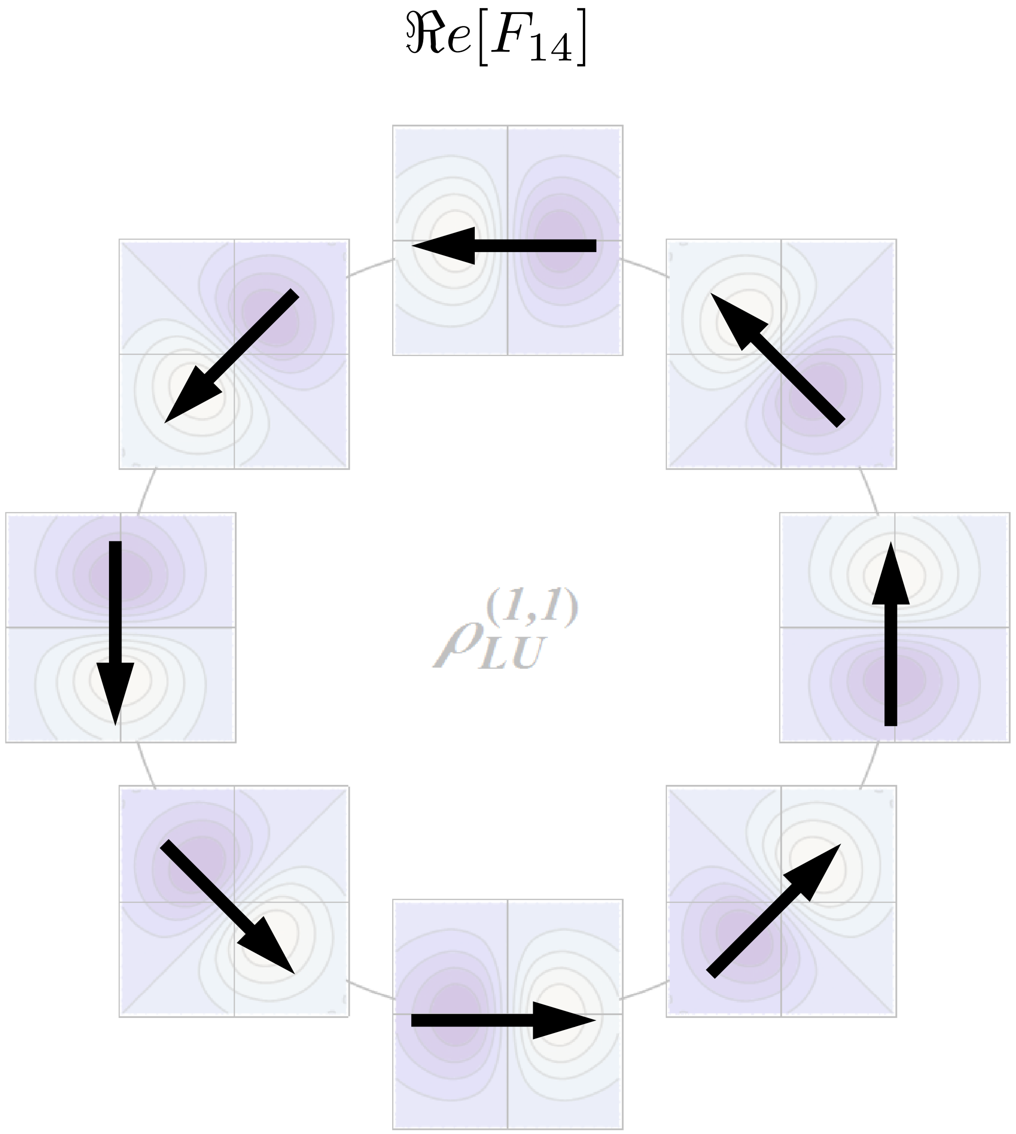
with no-net quark flow

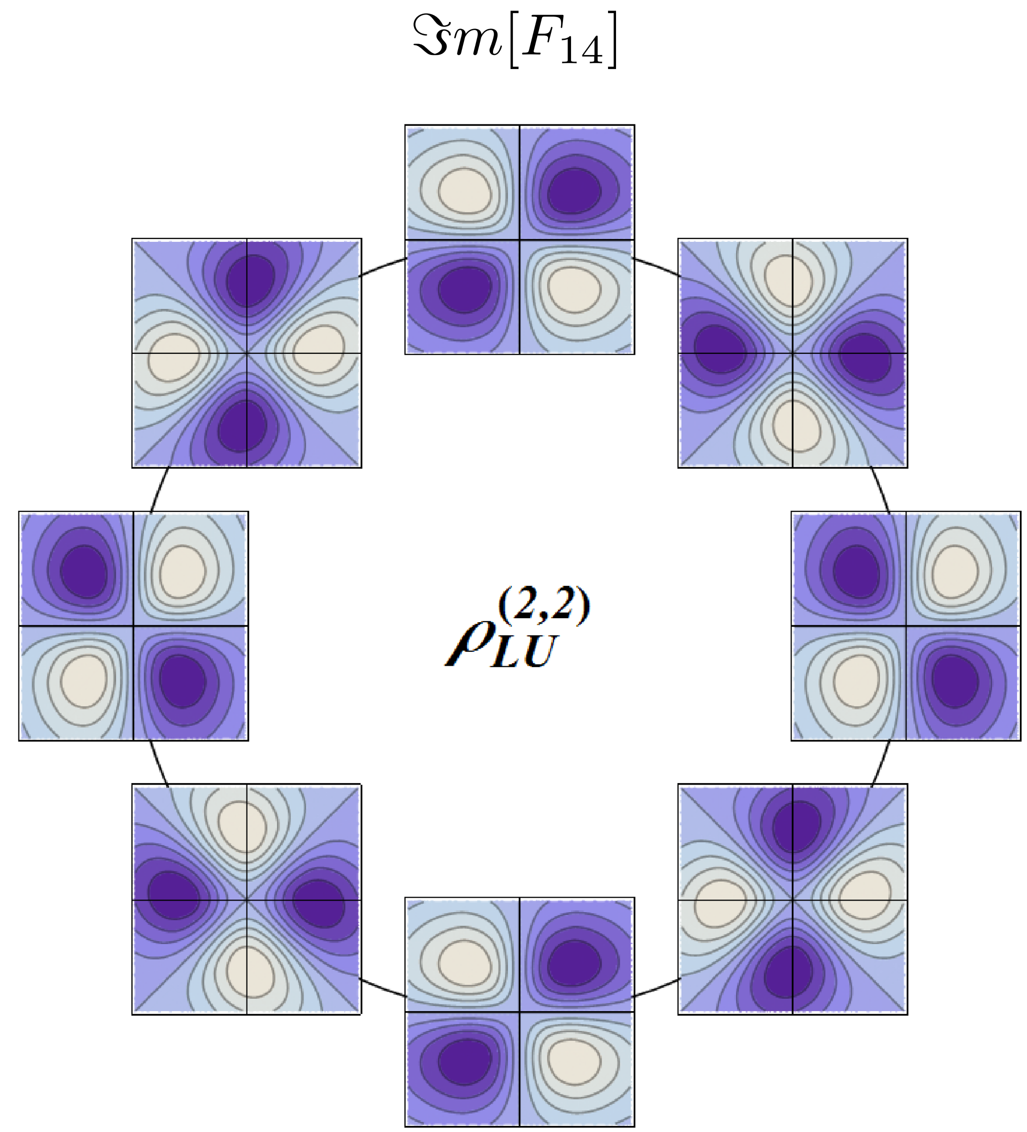
Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Unpolarized quarks in Longitudinally pol. proton

LU

unique information from GTMDs





naive time-reversal even

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

orbital flow → net OAM correlated S_z with

naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z
with no-net quark flow

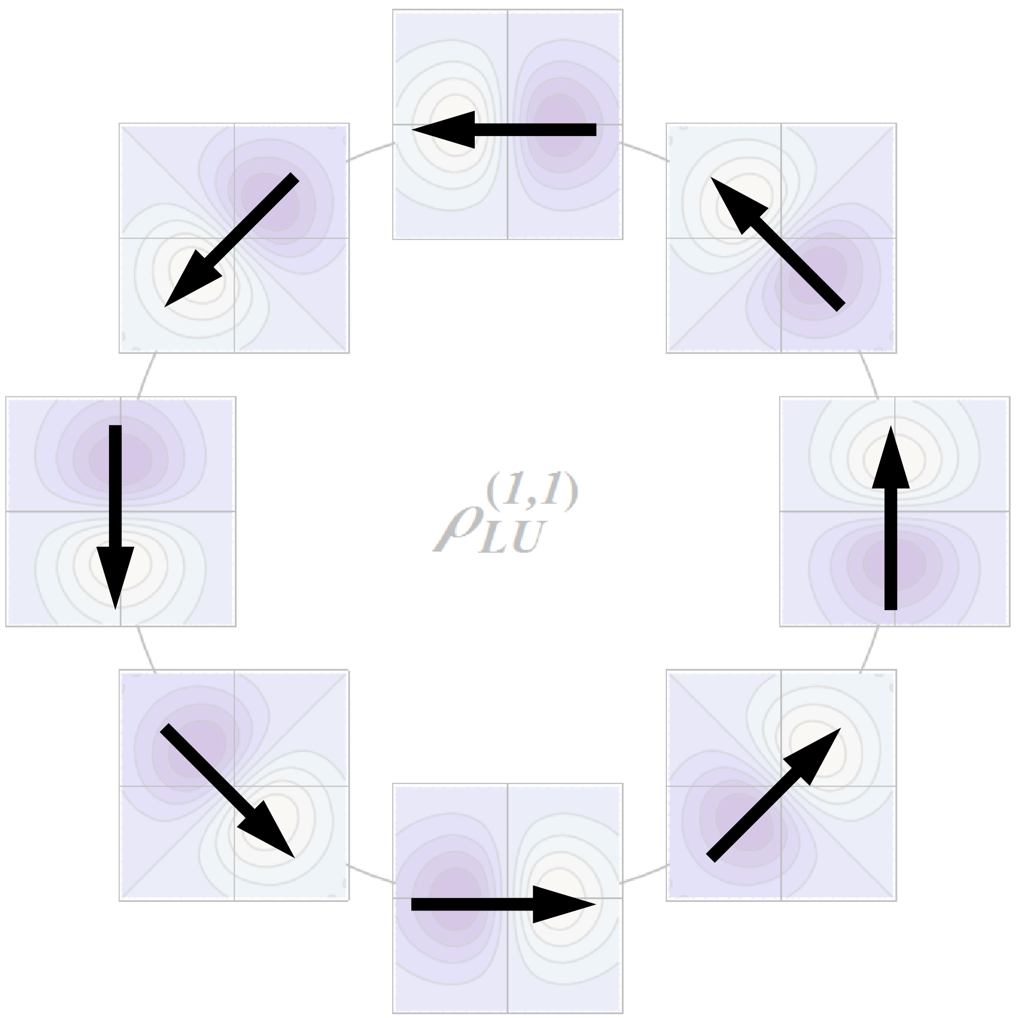
Lorcé, BP, PRD 96, 2016; PRD 84, 2011

LU

Unpolarized quarks in Longitudinally pol. proton

unique information from GTMDs

$\Re e[F_{14}]$

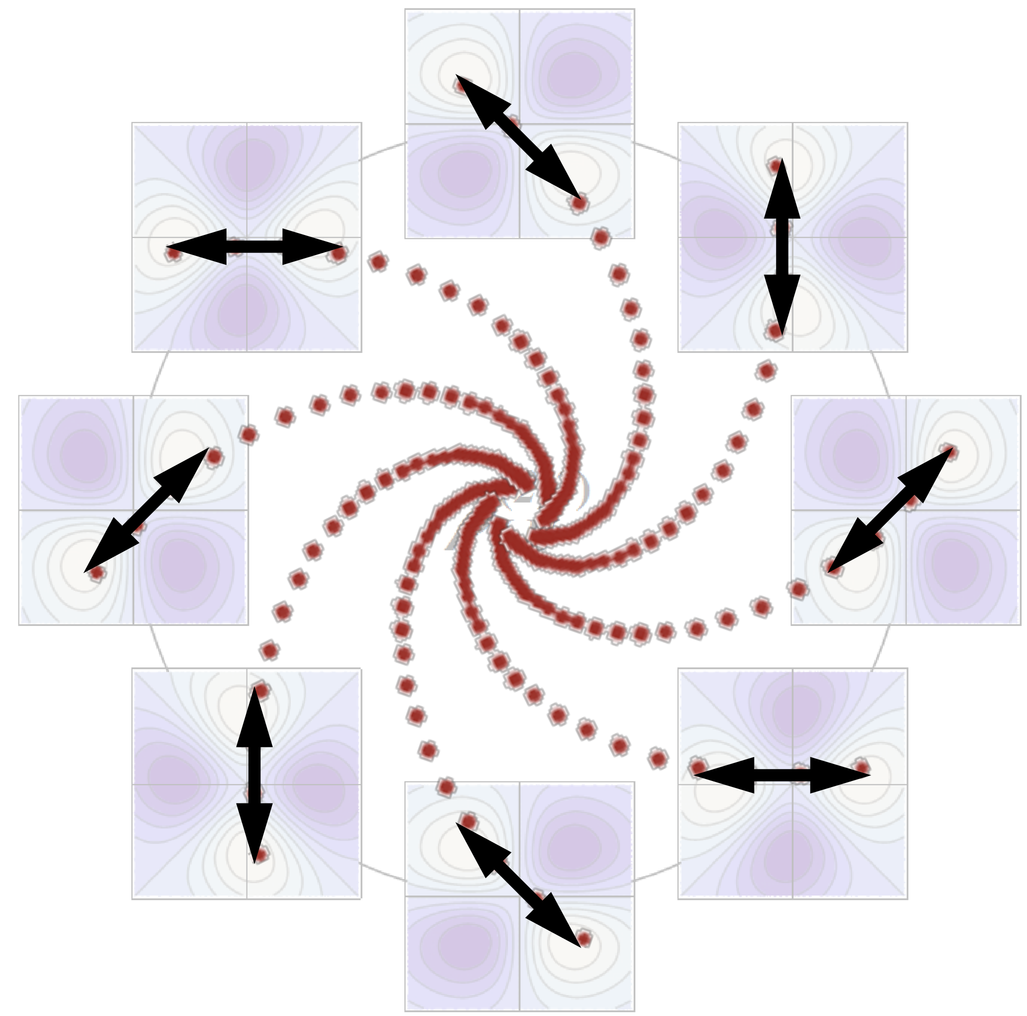


naive time-reversal even

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

orbital flow → net OAM correlated with S_z

$\Im m[F_{14}]$



naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z
with no-net quark flow

Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp) \rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp \, \vec{k}_\perp \, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp)\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\, \vec{k}_\perp\, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

- intuitive definition of OAM
- mutually orthogonal components of quark position and momentum
  $\longrightarrow$ no conflict with uncertainty principle
- the integrand $l_z^q(x)$ represents the OAM density
- same equation for both Jaffe-Manohar (staple-like link) and Ji (straight link) OAM
- equation holds also for gluon OAM
- it can be calculated in LQCD *Engelhardt, PRD95 (2017) 094505*

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp)\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\, \vec{k}_\perp\, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorcé, BP, PRD 84 (2011) 014015

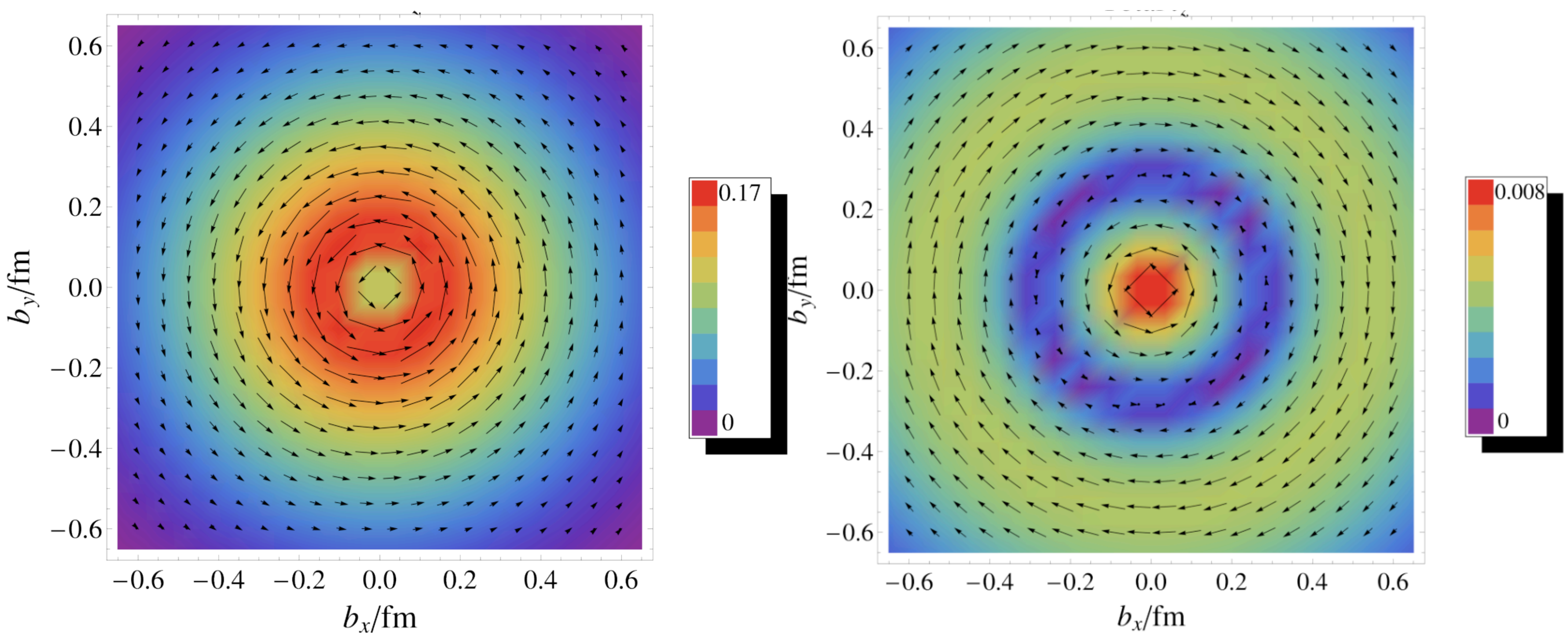
Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp) \rangle = \int \mathrm{d}x \mathrm{d}\vec{k}_\perp \, \vec{k}_\perp \, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$



Proton spin

u-quark OAM

d-quark OAM

Lorcé, BP, PRD 84 (2011) 014015

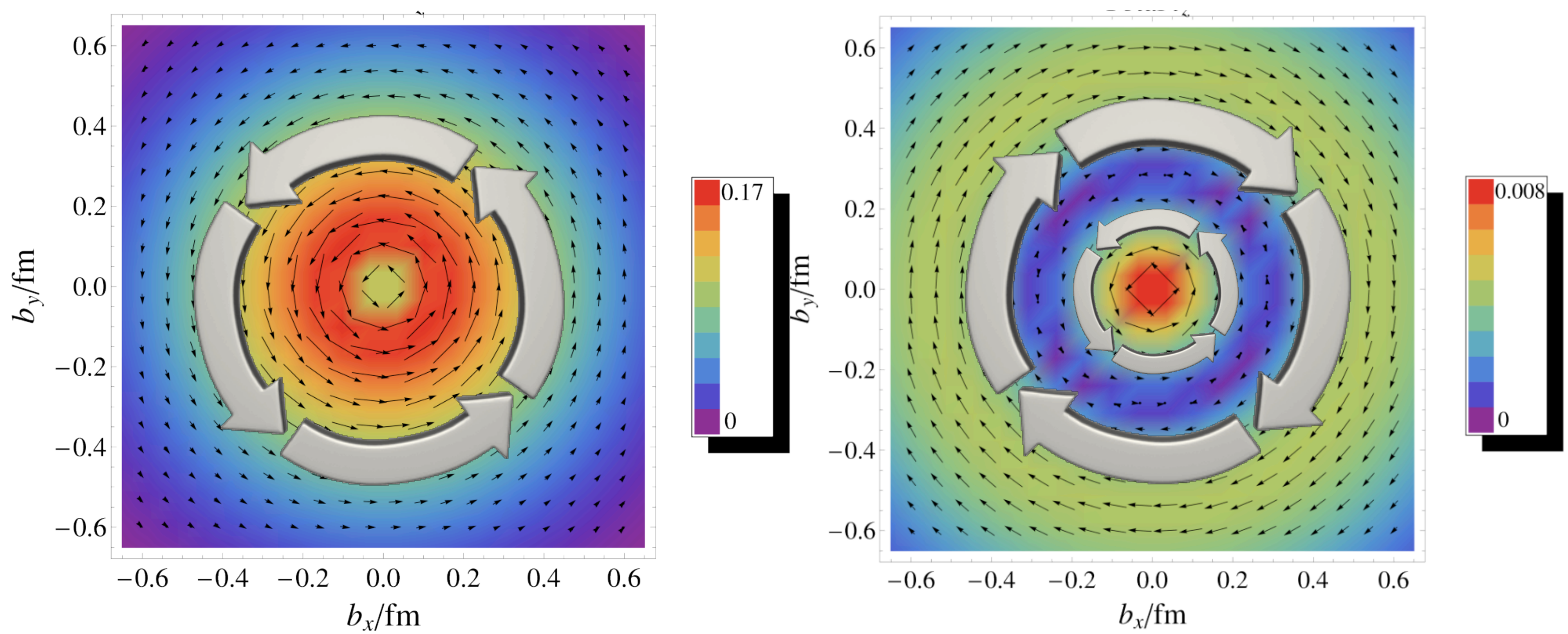
Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp) \rangle = \int \mathrm{d}x \mathrm{d}\vec{k}_\perp \, \vec{k}_\perp \, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$



$\longrightarrow$ Proton spin

$\longrightarrow$ u-quark OAM

$\longleftarrow$ d-quark OAM

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

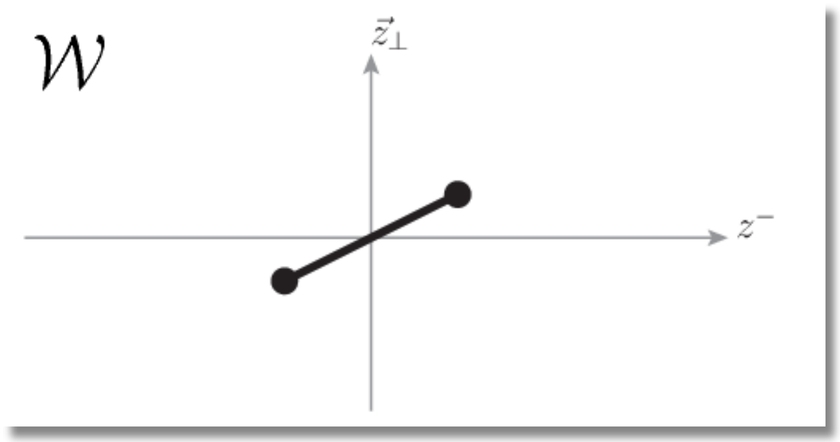
Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

$$\ell_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

[Lorcé, BP (2011)]
[Lorcé, BP, Xiong, Yuan(2011)]

Kinetic

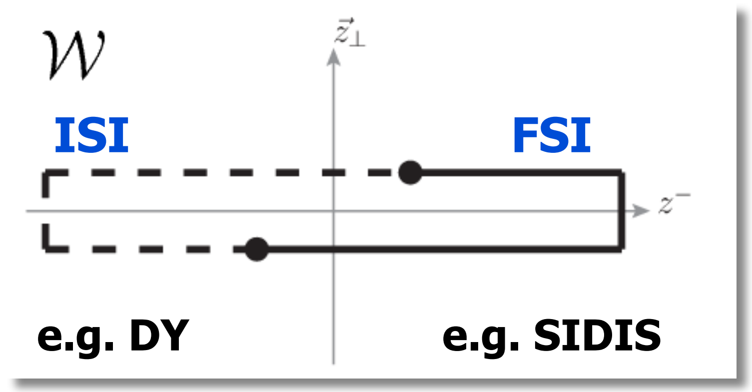
[Ji (1997)]



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

Canonical

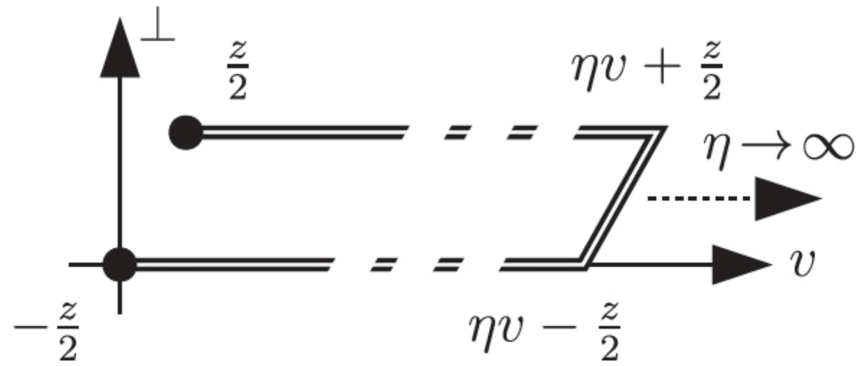
[Jaffe, Manohar (1990)]



[Hatta (2012)]

difference between the two definitions can be interpreted as
the change in the quark OAM as the quark leaves the target in a DIS experiment
[M. Burkardt (2013)]

Lattice calculation



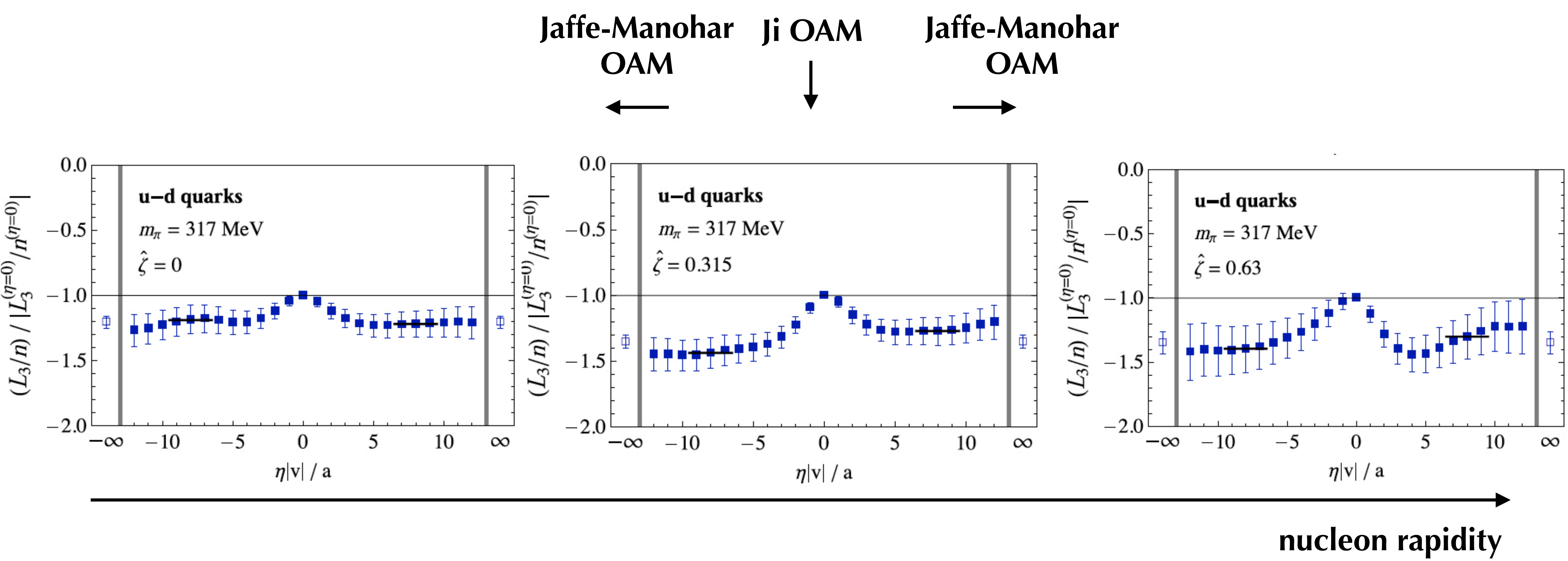
Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar limit $\eta \to \infty$

Staple direction off the light-cone

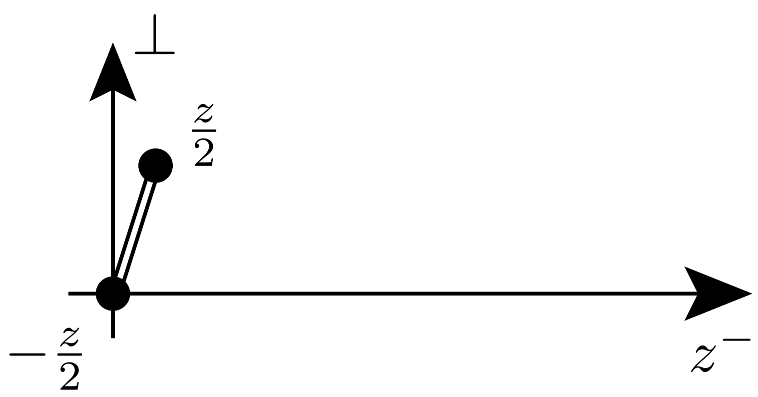
light-cone limit for $\hat{\zeta} = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{|P^2|}} \to \infty$

M. Engelhardt, Phys. Rev. D95, 094505 (2017)

M. Engelhardt et al., PRD102, 074505 (2020)



Lattice calculation



Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar limit $\eta \to \infty$

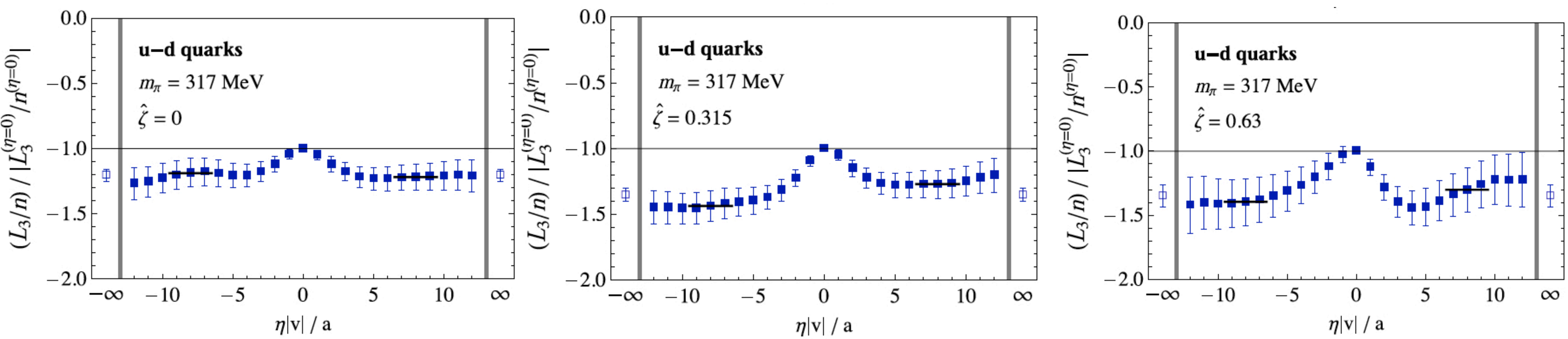
Staple direction off the light-cone

light-cone limit for $\hat{\zeta} = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{|P^2|}} \to \infty$

M. Engelhardt, Phys. Rev. D95, 094505 (2017)

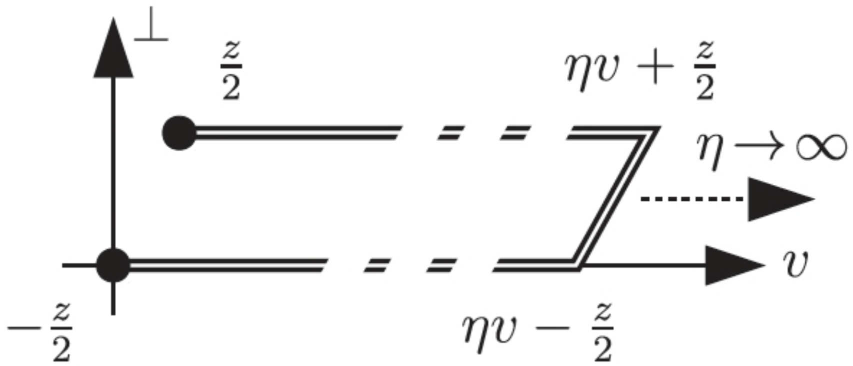
M. Engelhardt et al., PRD102, 074505 (2020)

Jaffe-Manohar OAM ← **Ji OAM** ↓ **Jaffe-Manohar OAM** →



nucleon rapidity

Lattice calculation



Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar limit $\eta \to \infty$

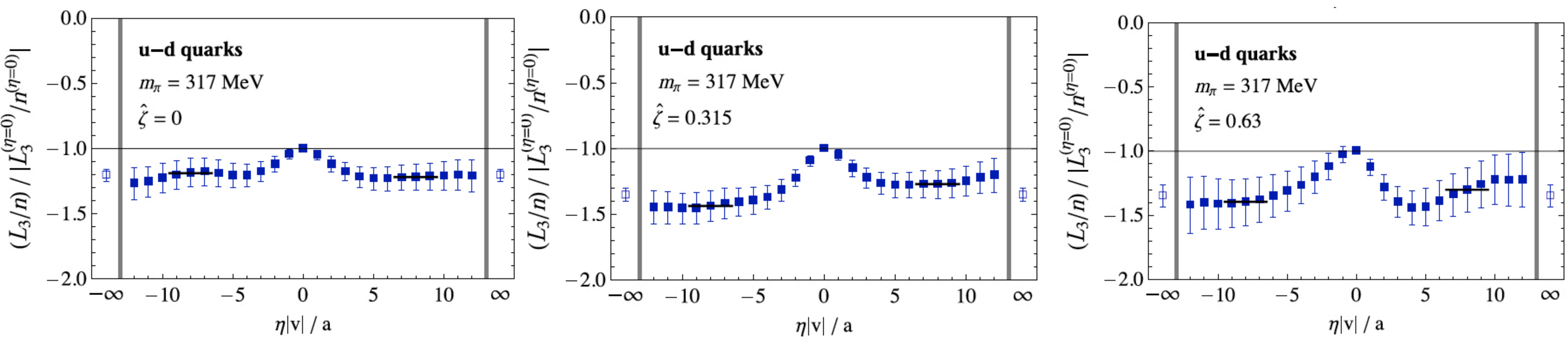
Staple direction off the light-cone

light-cone limit for $\hat{\zeta} = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{|P^2|}} \to \infty$

M. Engelhardt, Phys. Rev. D95, 094505 (2017)

M. Engelhardt et al., PRD102, 074505 (2020)

Jaffe-Manohar OAM ← **Ji OAM** ↓ **Jaffe-Manohar OAM** →

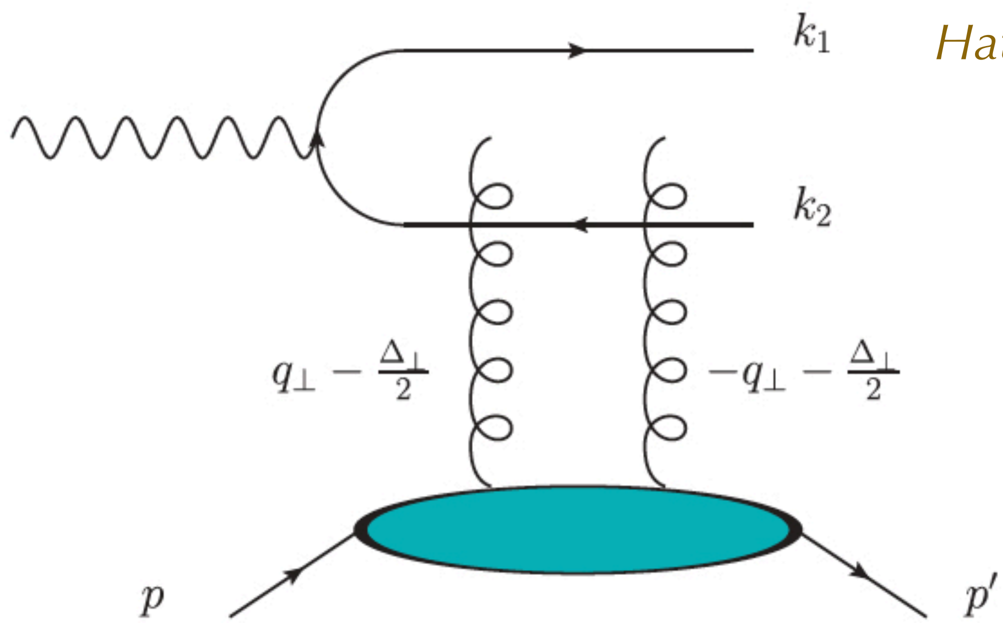


nucleon rapidity

Observables for GTMDs and Wigner functions

Diffractive Exclusive back-to-back dijet production in ℓN / ℓA collisions

Hatta, Xiao, Yuan, PRL 116 (2016) 202301



$$\vec{\Delta}_\perp \approx -(\vec{k}_{\perp,1} + \vec{k}_{\perp,2}) \qquad \vec{k}_\perp \sim \vec{P}_\perp = \frac{(\vec{k}_{\perp,1} - \vec{k}_{\perp,2})}{2} \qquad |\vec{P}_\perp| \gg |\vec{k}_{\perp,1} + \vec{k}_{\perp,2}|$$

- Reconstruction of full dijet kinematics and measure the azimuthal modulations in the angle between $\vec{\Delta}_\perp$ and $\vec{P}_\perp$

- At small x: sensitivity to gluon GTMDs

- Estimates in the CGC effective field theory suggest that modulations are maximum some tens of percent level *Mäntysaari, Mueller, Schenke, PRD99 (2019) 074004; Boer, Setyadi, PRD104 (20121) 074006*

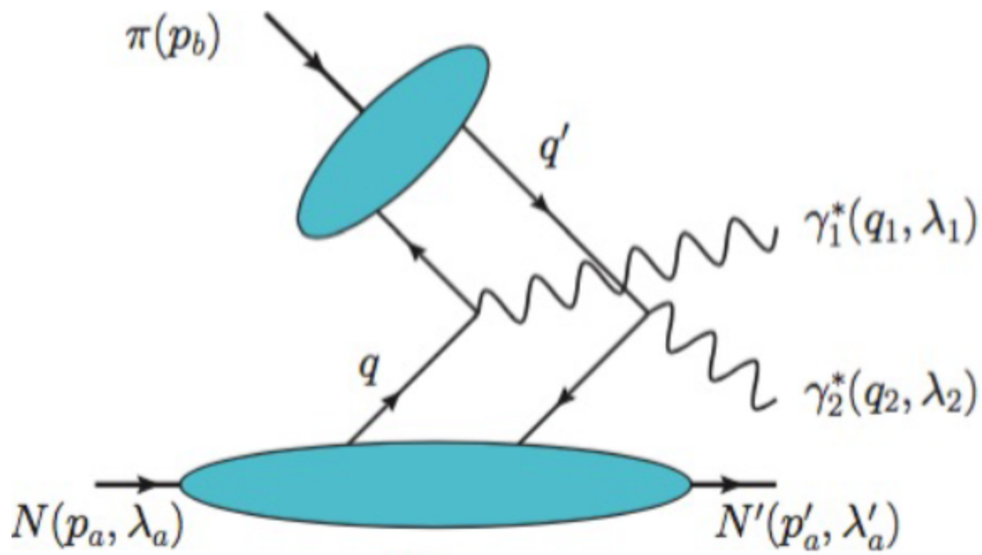
- With proton polarization one may access $F_{1,4}^g$

Hatta, Nakagawa, Xiao, Yuan, Zhao, PRD 95 (2017) 114032; Ji, Yuan, Zhao, PRL 118 (2017) 192004

Observables for GTMDs and Wigner functions

Exclusive pion-nucleon double Drell-Yan (quark GTMDs)

Bhattacharya, Metz, Zhou, PLB 771 (2017) 396

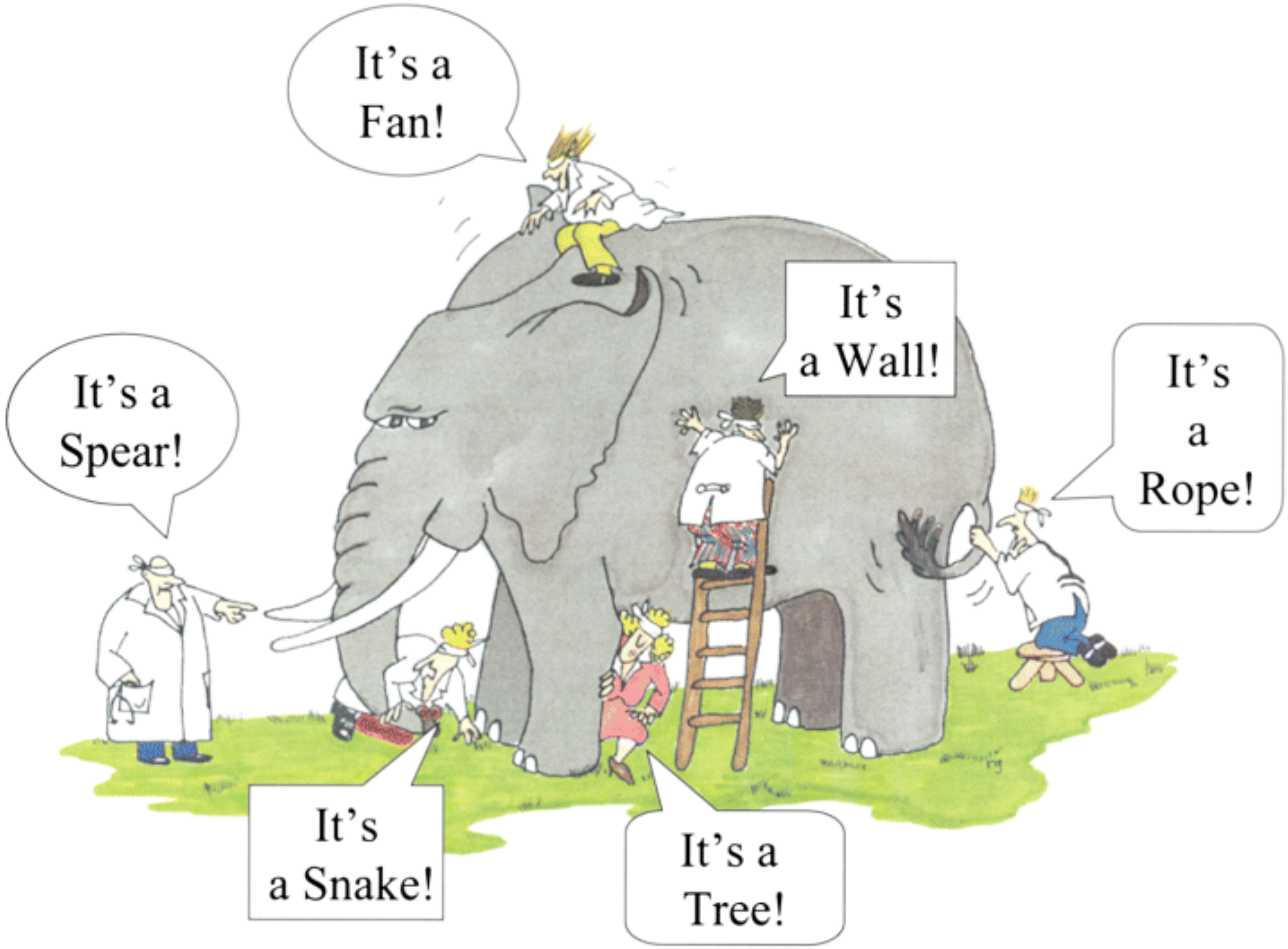


- At present, the only known process that is sensitive to quark GTMDs

- In leading order is sensitive to ERBL region only

- Low count rate (amplitude $T \sim \alpha_{\rm em}^2$)

The blind men and the elephant



Different observables in different kinematical regimes
need to talk to each other
to reconstruct the full picture of the nucleon