Nucleon partonic structure: concepts and measurements

Part 5: GPDs and exclusive processes

M. Diehl

Deutsches Elektronen-Synchroton DESY

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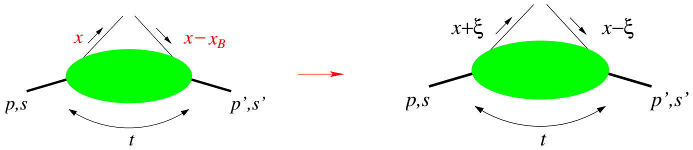
GPDs: definition and properties



$$F^q = \int \frac{dz^-}{4\pi}\, e^{ixP^+z^-} \left\langle p', s' \right| \bar{q}(-\tfrac{1}{2}z)\, W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\gamma^+ q(\tfrac{1}{2}z) \left| p, s \right\rangle_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$$

- kinematic variables:

  x, ξ momentum fractions w.r.t. $P = \frac{1}{2}(p+p')$

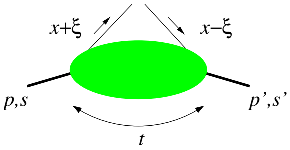
  $\xi = (p-p')^+/(p+p')^+$ plus-momentum transfer

  in DVCS: $\xi = x_B/(2-x_B)$, x integrated over

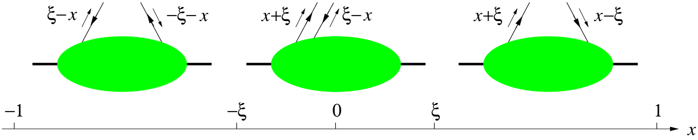
  t can trade for transverse momentum transfer $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p}$

  $$t = -\frac{4\xi^2 m^2 + \boldsymbol{\Delta}^2}{1-\xi^2}$$

GPDs: definition and properties



- nonzero for $-1 \leq x \leq 1$
- $|x| > \xi$ similar to parton densities
  correlation $\psi^*_{x-\xi}\,\psi_{x+\xi}$ instead of probability $|\psi_x|^2$
  $|x| < \xi$ coherent emission of $q\bar{q}$ pair
- regions related by Lorentz invariance
  spacelike partons incoming in some frames, outgoing in others



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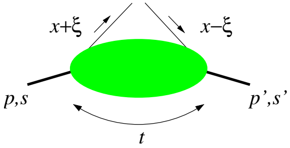

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$$= H^q\, \bar{u}(p', s')\gamma^+ u(p, s) + E^q\, \bar{u}(p', s')\, \frac{i}{2m_p} \sigma^{+\alpha}(p'-p)_\alpha\, u(p, s)$$

- proton spin structure:

  $H^q \leftrightarrow s = s'$ for $p = p'$ recover usual densities:

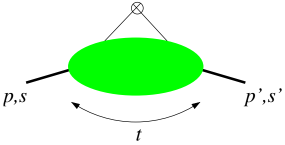
  $$H^q(x, \xi = 0, t = 0) = \begin{cases} q(x) & x > 0 \\ -\bar{q}(-x) & x < 0 \end{cases}$$

  $E^q \leftrightarrow s \neq s'$ decouples for $p = p'$

- similar definitions for polarized quarks $\tilde{H}^q, \tilde{E}^q$ and for gluons

  $H^g(x, \xi = 0, t = 0) = xg(x)$ for $x > 0$

GPDs: definition and properties



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- Mellin moments: $\int dx\, x^n \;\to$ local operator $\to$ form factors
- can be calculated in lattice QCD
- $\int dx \;\to$ vector current $\bar{q}(0)\gamma^+ q(0)$

  $\sum_q e_q \int dx\, H^q(x, \xi, t) = F_1(t)$ Dirac f.f.

  $\sum_q e_q \int dx\, E^q(x, \xi, t) = F_2(t)$ Pauli f.f.

Localizing partons: impact parameter

- states with definite light-cone momentum p^+
  and transverse position (impact parameter):

$$|p^+, \boldsymbol{b}\rangle = \frac{1}{(2\pi)^2} \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\,\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$$

  formal: eigenstates of 2 dim. position operator

- can exactly localize proton in 2 dimensions
  no limitation by Compton wavelength
- and stay in frame where proton moves fast
  $\rightsquigarrow$ parton interpretation
- different from localization in 3 spatial dimensions
  well-known for form factors; also for GPDs

Belitsky, Ji, Yuan '03; Brodsky et al. '06

Localizing partons: impact parameter

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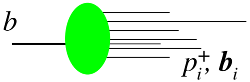
formal: eigenstates of 2 dim. position operator

- $\boldsymbol{b}$ is center of momentum of the partons in proton



$$\boldsymbol{b} = \frac{\sum_i p_i^+ \boldsymbol{b}_i}{\sum_i p_i^+} \qquad (i = q, \bar{q}, g)$$

consequence of Lorentz invariance: transverse boosts

$$k^+ \to k^+ \qquad \boldsymbol{k} \to \boldsymbol{k} - k^+ \boldsymbol{v}$$

nonrelativistic analog: Galilei invariance $\xrightarrow{\text{Noether}}$ center of mass

Impact parameter GPDs

for simplicity take $\xi = 0$ ($\xi \neq 0$ and $s \neq s'$ later)

- operator $\mathcal{O} = \int \frac{dz^-}{4\pi}\, e^{ixP^+z^-}\ \bar{q}(-\tfrac{1}{2}z)\, W[-\tfrac{1}{2}z,\, \tfrac{1}{2}z]\gamma^+ q(\tfrac{1}{2}z)\big|_{z^+=0,\, \boldsymbol{z}=\boldsymbol{0}}$

$\langle p^+, \vec{b}' \,|\, \mathcal{O} \,|\, p^+, \vec{b} \rangle =$

$$\delta^{(2)}(\vec{b}' - \vec{b}) \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{i\vec{b}\vec{\Delta}}\ H(x, \xi = 0, t = -\vec{\Delta}^2)$$

$\rightsquigarrow$ Exercise 8 in Barbara Pasquini's lectures

Impact parameter GPDs

for simplicity take $\xi = 0$ ($\xi \neq 0$ and $s \neq s'$ later)

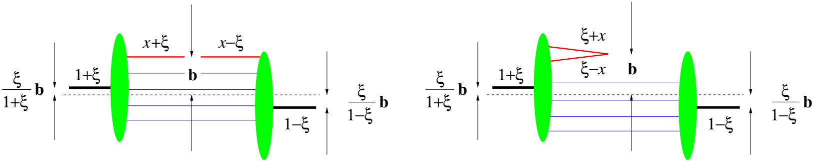
- $q(x, b^2) = (2\pi)^{-2} \int d^2\boldsymbol{\Delta}\, e^{-i\boldsymbol{b}\boldsymbol{\Delta}}\, H^q(x, \xi = 0, t = -\boldsymbol{\Delta}^2)$
  gives distribution of quarks with
  - longitudinal momentum fraction x
  - transverse distance b from proton center
- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2b\, b^2\, q(x, b^2)}{\int d^2b\, q(x, b^2)} = 4\, \frac{\partial}{\partial t} \log H(x, \xi = 0, t) \Big|_{t=0}$$

- integrated over x $\rightsquigarrow$ form factor

$$\langle b^2 \rangle = \frac{\int dx \int d^2b\, b^2\, q(x, b^2)}{\int dx \int d^2b\, q(x, b^2)} = 4\, \frac{\partial}{\partial t} \log F_1(t) \Big|_{t=0}$$

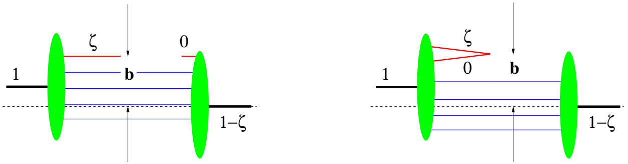
Impact parameter GPDs: $\xi \neq 0$



DGLAP region ERBL region

- Fourier transf. w.r.t. $\boldsymbol{\Delta}$ $t = -\frac{4\xi^2 m^2 + \boldsymbol{\Delta}^2}{1-\xi^2}$
- hadron center of momentum shifts because of plus-momentum transfer
- key observable: t dependence of cross sections at given ξ

Impact parameter GPDs: $\xi \neq 0$



- especially simple for $x = \xi$

  change to asymmetric variables: $\xi = \frac{\zeta}{2-\zeta}$ and $t = -\frac{\zeta^2 m_p^2 + \boldsymbol{\Delta}^2}{1-\zeta}$
- Fourier transf. w.r.t. $\boldsymbol{\Delta}$

  $\rightsquigarrow$ distance of struck parton from spectator system

in following concentrate on $\xi = 0$

Apples, oranges, and other fruit

form factor	distribution	$\langle b^2 \rangle$
F_1^p	$\sum_q e_q \, (q - \bar{q})$	$(0.66\,\mathrm{fm})^2$
G_E^p		$(0.71\,\mathrm{fm})^2 = (0.66\,\mathrm{fm})^2 + \dfrac{\kappa_p}{m_p^2}$
G_A	$\Delta u + \Delta\bar{u} - (\Delta d + \Delta\bar{d}\,)$	$(0.52$ to $0.54\,\mathrm{fm})^2$

- in form factor integral parton distributions have average $x \sim 0.2$
- generalized gluon dist. at $x = 10^{-3}$ $\rightsquigarrow$ $\langle b^2 \rangle = (0.57$ to $0.60\,\mathrm{fm})^2$
  from J/Ψ photoproduction at HERA

note:

$$4 \, \frac{\partial}{\partial t} \log G(t) \Big|_{t=0} = \text{squared impact parameter}$$

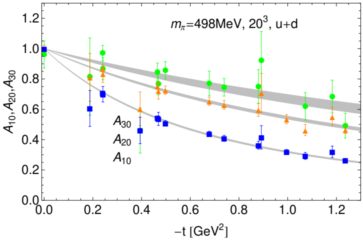
$$6 \, \frac{\partial}{\partial t} \log G(t) \Big|_{t=0} = \text{squared radius}$$

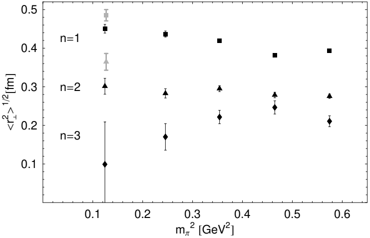
numbers: G_E and F_1 from Particle Data Group; G_A from Bernard et al. ’01

Lattice calculations

- results for GPD moments

$$A_{n,0}(t) = \int dx\, x^{n-1} H(x, \xi = 0, t) = \int d^2\boldsymbol{b}\; e^{i\boldsymbol{b}\boldsymbol{\Delta}} \int dx\, x^{n-1} q(x, b^2)$$





black: $L^3 = 28^3$, grey: $L^3 = 20^3$

LHPC Collaboration, arXiv:0705.4295

- steeper t slope for larger n

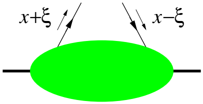
  $\rightsquigarrow$ decrease of $\langle b^2 \rangle_x$ with x

Evolution

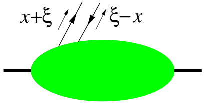
- for non-singlet combinations (e.g. $q - \bar{q}$ or $u - d$)

$$\mu^2 \frac{d}{d\mu^2} H^{\mathrm{NS}}(x,\xi,t) = \int dx'\, V^{\mathrm{NS}}(x,x',\xi)\, H^{\mathrm{NS}}(x',\xi,t)$$

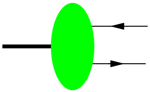
- for singlet $\sum_q (q + \bar{q})$: matrix equation for mixing with gluon GPD
- same evolution for E (independent of proton spin)



generalization of DGLAP
evolution to $\xi \neq 0$
recover usual DGLAP for $\xi = 0$



ERBL evolution as for
meson distribution amplitudes



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- same evolution for E (independent of proton spin)
- evolution local in t (take $-t \ll \mu^2$ to be safe)
  Fourier trf $\rightsquigarrow$ evolution local in b (take $1/\mu \ll b$ to be safe)
- for $\xi = 0$: $q(x, b^2)$ fulfills usual DGLAP evolution equation

$$\mu^2 \frac{d}{d\mu^2} q_{\mathrm{NS}}(x,b^2) = \int_x^1 \frac{dz}{z} \, P_{\mathrm{NS}}\Big(\frac{x}{z}\Big) q_{\mathrm{NS}}(z,b^2)$$

Evolution

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- evolution local in t (take $-t \ll \mu^2$ to be safe)
  Fourier trf $\rightsquigarrow$ evolution local in b (take $1/\mu \ll b$ to be safe)
- average impact parameter

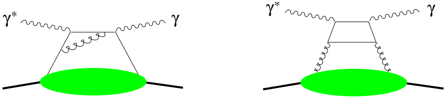
$$\langle b^2 \rangle_x = \frac{\int d^2b \, b^2 \, q(x,b^2)}{\int d^2b \, q(x,b^2)}$$

evolves as

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\frac{1}{q_{\mathrm{NS}}(x)} \int_x^1 \frac{dz}{z} \, P_{\mathrm{NS}}\Big(\frac{x}{z}\Big) \, q_{\mathrm{NS}}(z) \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

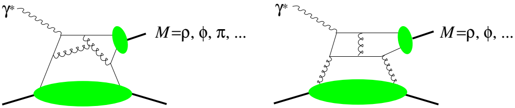
Key processes involving GPDs

- deeply virtual Compton scattering (DVCS)

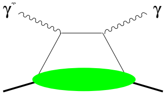


also: $\gamma p \to \gamma^* p$ with $\gamma^* \to \ell^+\ell^-$ (timelike CS)
$\gamma^* p \to \gamma^* p$ (double DVCS)

- meson production: large Q^2 or heavy quarks



DVCS amplitudes and GPDs



- twist-two amplitudes involve 4 four GPDs per parton
  - H, E: unpolarized quark/gluon
  - $\tilde{H}, \tilde{E}$: long. pol. quark/gluon
- for photon helicity conserving amplitudes write

$$e^{-2}\mathcal{A}(\gamma^* p \to \gamma p) = \bar{u}(p')\gamma^+ u(p)\, \mathcal{H} + \bar{u}(p')\, \tfrac{i}{2m_p}\sigma^{+\alpha}(p'-p)_\alpha\, u(p)\, \mathcal{E}$$
$$+ \bar{u}(p')\gamma^+\gamma_5 u(p)\, \widetilde{\mathcal{H}} + \bar{u}(p')\, \tfrac{(p'-p)^+}{2m_p}\gamma_5\, u(p)\, \widetilde{\mathcal{E}}$$

  - Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$ depend on ξ, t, Q^2
  - representation holds for any Q^2, not only at twist two
- at leading twist and LO in α_s

$$\mathcal{H} = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right] H^q(x, \xi, t)$$

same kernels for E, different set for $\widetilde{H}, \widetilde{E}$

Why DVCS?

- theoretical accuracy at NNLO
- very close to inclusive DIS
  power corrections empirically not too large, in part computed

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- only quark flavour combination $\frac{4}{9}u + \frac{1}{9}d + \frac{1}{9}s$
  with neutron target in addition $\frac{4}{9}d + \frac{1}{9}u + \frac{1}{9}s$
- gluons only through Q^2 dependence
  via LO evolution, NLO hard scattering
  most promonent at small x, ξ

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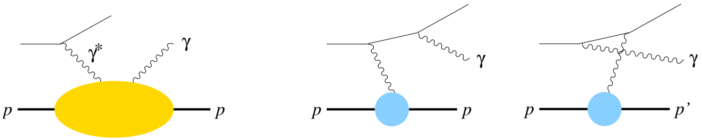
• useful to get information from meson production

- e.g. $\mathcal{A}_{\rho^0} \propto \frac{2}{3}(u+\bar{u}) + \frac{1}{3}(d+\bar{d}) + \frac{3}{4}g$
  $\mathcal{A}_{\phi} \propto \frac{1}{3}(s+\bar{s}) + \frac{1}{4}g$
- but theory description more difficult
  meson wave function, larger corrections in $1/Q^2$ and α_s
- J/Ψ production: directly sensitive to gluons

Deeply virtual Compton scattering

- competes with Bethe-Heitler process at amplitude level



- analogy with optics:
  - DVCS ∼ diffraction experiment
  - BH ∼ reference beam with known phase

Deeply virtual Compton scattering

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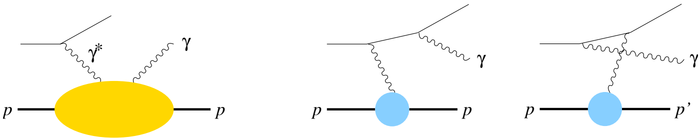
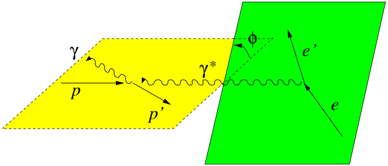


- cross section for $\ell p \to \ell \gamma p$

$$\frac{d\sigma_{\mathrm{VCS}}}{dx_B\, dQ^2\, dt} : \frac{d\sigma_{\mathrm{BH}}}{dx_B\, dQ^2\, dt} \sim \frac{1}{y^2}\frac{1}{Q^2} : \frac{1}{t} \qquad\qquad y = \frac{Q^2}{x_B\, s_{\ell p}}$$

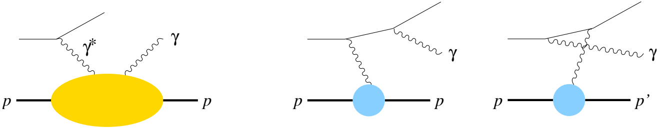
- $1/Q^2$ and $1/t$ from photon propagators
  $1/y^2$ from vertex $e \to e\gamma^*$
- small y: σ_{VCS} dominates $\rightsquigarrow$ high-energy collisions
  moderate to large y: get VCS via interference with BH
  $\rightsquigarrow$ separate $\mathrm{Re}\,\mathcal{A}(\gamma^* p \to \gamma p)$ and $\mathrm{Im}\,\mathcal{A}(\gamma^* p \to \gamma p)$



- filter out interference term using cross section dependence on
  - beam charge e_ℓ
  - azimuth ϕ
  - beam polarization P_ℓ
  - target polarizaton S_L, S_T, ϕ_S

- general structure:

$$d\sigma(\ell p \to \ell \gamma p) \sim d\sigma^{BH} + e_\ell \, d\sigma^{I} + d\sigma^{C}$$



Summary of part 5

- generalised parton distributions:
  extend factorisation concept to exclusive processes
- factorisation of amplitude instead of cross section
- gives access to transverse spatial distribution of partons

Notes

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