$$\langle p', s' \mid T^{\mu\nu}_{B} \mid p, s \rangle = \frac{P^{\{\mu}\gamma^{\nu\}}}{2} A(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\alpha}\Delta_\alpha}{4M} B(t)$$

$$+ \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} D(t)$$

$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

GORDON IDENTITY

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{(p'+p)^\mu}{2M} + i\sigma^{\mu\alpha}\frac{(p'_\alpha - p_\alpha)}{2M} \right] u(p)$$

$$\langle p', s' | T^{\mu\nu}_{B} | p, s \rangle = \bar{u}(p') \left\{ \frac{P^\mu P^\nu}{M} A(t) + i \frac{P^{\{\mu} \sigma^{\nu\}\alpha} \Delta_\alpha}{4M} \left(A(t) + B(t) \right) \right.$$

$$\left. + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} D(t) \right\}$$

$$\langle P | \hat{P}^\nu | P \rangle = \langle P | \int d^3x \, T^{0\nu}(x) | P \rangle$$

$$= \lim_{\Delta \to 0} \langle P + \frac{\Delta}{2} | \, | \int d^3x \, T^{0\nu}(x) | P - \frac{\Delta}{2} \rangle$$

$$T^{0\nu}(x) = e^{-i \hat{P} \cdot \bar{x}} \, T^{0\nu}(0) \, e^{i \hat{P} \cdot \bar{x}}$$

$$= \lim_{\Delta \to 0} \int d^3x \, e^{-i \vec{\Delta} \cdot \bar{x}} \langle P + \frac{\Delta}{2} | T^{0\nu}(0) | P - \frac{\Delta}{2} \rangle$$

$$= \lim_{\Delta \to 0} (2\pi)^3 \delta(\vec{\Delta}) \langle P + \frac{\vec{\Delta}}{2} | T^{0\nu}(0) | P - \frac{\vec{\Delta}}{2} \rangle$$

$$= A(0) P^{\nu} \underbrace{(2P^0)(2\pi)^3 \delta(0)}_{\langle P | P \rangle}$$

$$\langle P | \hat{P}^{\nu} | P \rangle = A(0) P^{\nu} \langle P | P \rangle = P^{\nu}$$

$$A(0) = A_q(0) + A_g(0) = 1$$

$$P^{\mu} = (M, 0, 0, 0) \qquad S^{\mu} = (0, 0, 0, 1)$$

$$J^3 = \frac{1}{2} \int d^3x \, \varepsilon_{3jk} J^{0jk} = \int J^{012} d^3x \qquad \langle P, \frac{1}{2} | J^3 | P, \frac{1}{2} \rangle = J \langle P, \frac{1}{2} | P, \frac{1}{2} \rangle$$

$$J^{\mu\alpha\beta} = x^{\alpha} T^{\mu\beta} - x^{\beta} T^{\mu\alpha}$$

$$\langle P, \tfrac{1}{2} | J^{3} | P, \tfrac{1}{2} \rangle = \langle P, \tfrac{1}{2} | \int d^3x \left[x^1 T^{02} - x^2 T^{01} \right] | P, \tfrac{1}{2} \rangle$$

$$= \epsilon_{ij3} \langle P, \tfrac{1}{2} | \int d^3x \; x^i \, T^{0j} \; | P, \tfrac{1}{2} \rangle$$

$$= \lim_{\Delta \to 0} \epsilon_{ij3} \langle P + \tfrac{\Delta}{2}, \tfrac{1}{2} | \int d^3x \; x^i \, T^{0j}(x) | P - \tfrac{\Delta}{2}, \tfrac{1}{2} \rangle$$

$$= \lim_{\Delta \to 0} \epsilon_{ij3} \langle P + \tfrac{\Delta}{2}, \tfrac{1}{2} | \int d^3x \; x^i \, e^{-i \bar{x} \cdot \bar{\Delta}} \langle P + \tfrac{\Delta}{2}, \tfrac{1}{2} | T^{0j}(0) | P - \tfrac{\Delta}{2} \rangle$$

$$= \lim_{\Delta \to 0} \epsilon_{ij3} \left[i \frac{\partial}{\partial \Delta^i} (2\pi)^3 \, \delta(\bar{\Delta}) \right] \langle P + \tfrac{\Delta}{2}, \tfrac{1}{2} | T^{0j}(0) | P - \tfrac{\Delta}{2} \rangle$$

$$= \lim_{\Delta\to 0} \epsilon_{ij3} (2\pi)^3 \delta(\vec{\Delta}) \Big[-i \frac{\partial}{\partial \Delta^i} \big[A(t)+B(t)\big] \bar{u}(p') P^{\{0}\, i\sigma^{j\}\alpha} \frac{\Delta_\alpha}{4M}$$

+ TERMS INDEPENDENT OF Δ

+ TERMS QUADRATIC IN $\Delta \Big]$

$$= \epsilon_{ij3} (2\pi)^3 \delta(0) \left(+\frac{1}{4M}\right) \bar{u}(p') (-) \Big[\underset{\substack{\downarrow \\ M}}{P^0}\, \sigma^{ji} + \cancel{p^j \sigma^{0i}} \Big] u(p) \big(A(0)+B(0)\big)$$

$$\longrightarrow \epsilon_{ij3}\sigma^{ji} = -2\sigma^{12}$$

$$= (2\pi)^3 \delta(0) \frac{1}{4M} 2M \bar{u}(p', \tfrac{1}{2}) \sigma^{12} u(p, \tfrac{1}{2})$$

$$= \frac{1}{2} \big[A(0) + B(0)\big] \langle p | p \rangle = J \langle p | p \rangle$$

$$J = \frac{1}{2} \big[A(0) + B(0)\big]$$

DISPERSION RELATIONS FOR GPDs

$$\text{Re}\, A_2(\nu,t) = A_2(0,t) + \frac{2}{\pi}\nu^2 P\int_{\nu_0}^{\infty} \text{Im}\, A_2(\nu',t)\,\frac{d\nu'}{\nu'(\nu'^2-\nu^2)}$$

$$\nu = \frac{Q^2}{4M\xi} \qquad \nu' = \frac{Q^2}{4Mx} \qquad d\nu' = -\frac{Q^2}{4M}\frac{1}{x^2}\,dx \qquad \nu_0 = \frac{Q^2}{4M}$$

$$\frac{Q^2}{4M} = k$$

$$\text{Re}\, A_2(\nu,t) = A_2(0,t) + \frac{2}{\pi}\frac{\cancel{k^2}}{\xi^2} P\int_0^1 \text{Im}\, A_2(x,t)\,\frac{\cancel{k}}{x^2}\,\frac{dx}{\frac{\cancel{k}}{x}\left(\frac{\cancel{k^2}}{x^2}-\frac{\cancel{k^2}}{\xi^2}\right)}$$

$$\text{Re}\, A_2(\xi,t) = A_2(0,t) + \frac{2}{\pi} P\int_0^1 dx \quad \text{Im}\, A_2(x,t)\,\frac{1}{x\,(\xi^2/x^2-1)}\,dx$$

$$\frac{2}{x}\,\frac{1}{\frac{\xi^2}{x^2}-1} = \frac{2x}{\xi^2-x^2} = -\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

$$\mathrm{Re}\,A_2(\xi,t) = \underset{\displaystyle\Delta(t)}{\underset{\big\downarrow}{A_2(\eta,t)}} - \frac{1}{\pi}\,\mathcal{P}\int_0^1 dx\ \ \mathrm{Im}\,A_2(x,t)\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

$$A_2 = -\int_0^1 dx\ \ E^+\left[\frac{1}{x-\xi+i\varepsilon}+\frac{1}{x+\xi-i\varepsilon}\right]$$

$$\mathrm{Im}\,A_2 = \pi\ E^+(x,x,t) \qquad \mathrm{Re}\,A_2(\xi,t) = -\mathcal{P}\int_0^1 dx\ \overset{(x,\xi,t)}{E^+}\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

$$\Delta(t) = -\mathcal{P}\int_0^1 dx\ \left[E^+(x,\xi,t) - E^+(x,x,t)\right]\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

- USE ξ- INDEPENDENCE OF $\Delta(t)$ ⟶ TAKE $\xi = 0$

$$\Delta(t) = -P \int_0^1 dx \left[E^{+}(\underset{x}{x}, 0, t) - E^{+}(x, x, t) \right] \frac{2}{x}$$

- TIME REVERSAL INVARIANCE $\quad E(x, \xi, t) = E(x, -\xi, t)$

$$\int_0^1 dx \, \frac{2}{x} \left[E(x, 0, t) \underbrace{- E(-x, 0, t)}_{\hookrightarrow \int_{-1}^{0} dx \, \frac{2}{x} E(x, 0, t)} \right] = \int_{-1}^{1} dx \, \frac{2}{x} E(x, 0, t)$$

$$\int_0^1 dx \; \frac{2}{x}\Big[E(x,x,t) - \underbrace{E(-x,x,t)}\Big] = \int_{-1}^{1} dx \; \frac{2}{x} \; E(x,x,t)$$

$$\int_{-1}^{0} dx \; \frac{2}{x} \; E(x,-x,t) \overset{TR}{=} \int_{-1}^{0} dx \, \frac{2}{x} \, E(x,x,t)$$

$$\Delta(t) = -P\int_{-1}^{1} dx \; \Big[E(x,0,t) - E(x,x,t)\Big]\frac{2}{x}$$

$$E(x,\xi,t) = E_{DD} + E_D$$

$$\frac{E_D(x,\xi,t)}{x} = -\delta(x)\;\frac{1}{N_f}\int_{-1}^{1} dz \; \frac{D(z,t)}{z}$$

$$\frac{E_D(x,x,t)}{x} = -S(x)\,\frac{1}{N_f}\int_{-1}^{1} dz\,\frac{D(z,t)}{z(1-z)}$$

$$\Delta(t) = \frac{2}{N_f}\int_{-1}^{1} dz\; D(z,t)\underbrace{\left[1/z - \frac{1}{z(1-z)}\right]}_{-\frac{1}{1-z}}$$

$$\Delta(t) = -\frac{2}{N_f}\int_{-1}^{1} dz\,\frac{D(z,t)}{1-z}$$

→ POLYAKOV-WEISS TERM

$D(t)$ OF E $\qquad$ $D(t=0) = \frac{4}{5} d_1$