Nucleon partonic structure: concepts and measurements Part 7: Double parton scattering

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Hadron-hadron collisions

standard description based on factorisation formulae

cross sect = parton distributions \times parton-level cross sect



• factorisation formulae are for inclusive cross sections $pp \rightarrow Y + X$ where Y = produced in parton-level scattering, specified in detail X = summed over, no details

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Hadron-hadron collisions

standard description based on factorisation formulae

 $\mathsf{cross}\ \mathsf{sect} = \mathsf{parton}\ \mathsf{distributions} \times \mathsf{parton-level}\ \mathsf{cross}\ \mathsf{sect}$



• factorisation formulae are for inclusive cross sections $pp \rightarrow Y + X$ where Y = produced in parton-level scattering, specified in detail X = summed over, no details

- also have interactions between "spectator" partons their effects cancel in inclusive cross sections thanks to unitarity but they affect the final state X
- Spectator interactions can be soft (low p_T scattering) → underlying event or hard → multiparton interactions
- here: concentrate on hard double parton scattering (DPS)

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Single vs. double hard scattering

 \blacktriangleright example: prod'n of two gauge bosons, transverse momenta $m{q}_1$ and $m{q}_2$



single scattering:

 $|m{q}_1|$ and $|m{q}_1|\sim$ hard scale Q^2 $|m{q}_1+m{q}_2|\ll Q^2$



double scattering: both $|{m q}_1|$ and $|{m q}_1| \ll Q^2$

• for transv. mom. $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{single}}}{d^2 \boldsymbol{q}_1 \, d^2 \boldsymbol{q}_2} \sim \frac{d\sigma_{\text{double}}}{d^2 \boldsymbol{q}_1 \, d^2 \boldsymbol{q}_2} \sim \frac{1}{Q^4 \Lambda^2}$$

but single scattering populates larger phase space:

$$\sigma_{
m single} \sim rac{1}{Q^2} \ \gg \ \sigma_{
m double} \sim rac{\Lambda^2}{Q^4}$$

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Single vs. double hard scattering

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single scattering:

 $|m{q}_1|$ and $|m{q}_1|\sim$ hard scale Q^2 $|m{q}_1+m{q}_2|\ll Q^2$



double scattering: both $|{\bm q}_1|$ and $|{\bm q}_1| \ll Q^2$

double scattering favored at small x (high energies): densities for two partons rise faster than for single parton

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A numerical example

gauge boson pair production



single scattering: $qq \rightarrow qq + W^+W^+$

suppressed by α_s^2





J Gaunt et al, arXiv:1003.3953

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A numerical example

gauge boson pair production



single scattering: $qq \rightarrow qq + W^+W^+$ suppressed by α_s^2



CMS 77.4 fb⁻¹ (13 TeV) H-Observed Predictions: stat PYTHIA8 (CP5) syst - Factorization approach total stat syst $1.96 \pm 0.74 \ (\pm 0.54 \ , \pm 0.51) \ pb$ μ-μ- + e-μ- $\mu^{+}\mu^{+} + e^{+}\mu^{+}$ 1.36 ± 0.46 (± 0.33 , ± 0.32) pb 1.41 ± 0.40 (± 0.28 , ± 0.28) pb $u^{\pm}u^{\pm} + e^{\pm}u^{\pm}$ Inclusive ones www (pb)

CMS, arXiv:1909.06265

measurement

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Feynman graphs: momentum vs. distance



large (plus or minus) momenta of partons $x_i p$, $\bar{x}_i \bar{p}$ fixed by final state exactly as for single hard scattering

- ► transverse parton momenta not the same in amplitude \mathcal{A} and in \mathcal{A}^* cross section $\propto \int d^2 \mathbf{r} F(x_i, \mathbf{k}_i, \mathbf{r}) F(\bar{x}_i, \bar{\mathbf{k}}_i, -\mathbf{r})$
- ► Fourier trf. to impact parameter: $F(x_i, k_i, r) \rightarrow F(x_i, k_i, y)$ cross section $\propto \int d^2 y F(x_i, k_i, y) F(\bar{x}_i, \bar{k}_i, y)$
- interpretation: y = transv. dist. between two scattering partons
 = equal in both colliding protons

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DPS cross section: collinear factorisation



$$\frac{d\sigma_{\text{double}}}{dx_1 \, d\bar{x}_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \, \hat{\sigma}_1 \, \hat{\sigma}_2 \int d^2 \boldsymbol{y} \, F(x_1, x_2, \boldsymbol{y}) \, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

$$C = \text{combinatorial factor}$$

C = combinatorial factor $\hat{\sigma}_i = \text{ parton-level cross sections}$ $F(x_1, x_2, y) = \text{ double parton distribution (DPD)}$ y = transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation no semi-classical approximation required
- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F

Paver, Treleani 1982, 1984; Mekhfi 1985, ..., MD, Ostermeier, Schäfer 2012

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DPS cross section: TMD factorisation



for measured transv. momenta

$$\frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, d^2 \boldsymbol{q}_1 \, dx_2 \, d\bar{x}_2 \, d^2 \boldsymbol{q}_2} = \frac{1}{C} \, \hat{\sigma}_1 \, \hat{\sigma}_2$$
$$\times \int \frac{d^2 \boldsymbol{z}_1}{(2\pi)^2} \, \frac{d^2 \boldsymbol{z}_2}{(2\pi)^2} \, e^{-i(\boldsymbol{z}_1 \boldsymbol{q}_1 + \boldsymbol{z}_2 \boldsymbol{q}_2)} \int d^2 \boldsymbol{y} \, F(x_i, \boldsymbol{z}_i, \boldsymbol{y}) \, F(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y})$$

•
$$F(x_i, \boldsymbol{z}_i, \boldsymbol{y}) = \text{double-parton TMDs}$$

 $oldsymbol{z}_i =$ Fourier conjugate to parton transverse mom. $oldsymbol{k}_i$

operator definition as for TMDs: schematically have

$$F(x_i, \boldsymbol{z}_i, \boldsymbol{y}) = \frac{\mathcal{FT}}{z_i^- \to x_i p^+} \langle p | \bar{q} \left(-\frac{1}{2} z_2 \right) \Gamma_2 q \left(\frac{1}{2} z_2 \right) \bar{q} \left(y - \frac{1}{2} z_1 \right) \Gamma_1 q \left(y + \frac{1}{2} z_1 \right) | p \rangle$$

- to be completed by renormalisation, Wilson lines, soft factors
- essential for studying factorisation, scale and rapidity dependence

M Buffing, MD, T Kasemets 2017

• analogous def for collinear distributions $F(x_i, y)$

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Pocket formula

- make simplest possible assumptions
- if two-parton density factorises as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1) f(x_2) G(\boldsymbol{y})$$

where $f(x_i) = usual PDF$

if assume same G(y) for all parton types then cross sect. formula turns into

$$\frac{d\sigma_{\text{double}}}{dx_1 \, d\bar{x}_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \frac{d\sigma_1}{dx_1 \, d\bar{x}_1} \frac{d\sigma_2}{dx_2 \, d\bar{x}_2} \frac{1}{\sigma_{\text{eff}}}$$

with $1/\sigma_{\rm eff} = \int\! d^2 {\bm y} \; G({\bm y})^2$

→ scatters are completely independent

- used in bulk of phenomenological estimates
- underlies modeling of MPI in Pythia, Herwig, ... (with refinements)
- fails if any of the above assumptions is invalid or if original cross sect. formula misses important contributions

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Experimental investigations (incomplete)



other channels:

- double open charm C + C with $C = D^0, D^+, D_s^+, \Lambda_c^+$ LHCb 2012
- $W + J/\Psi$, $Z + J/\Psi$ ATLAS 2014, 2015
- $\Upsilon + \Upsilon$ (estimate $\sigma_{\text{eff}} \approx 2.2 \div 6.6 \,\text{mb}$)
- same-sign WW (LHC run 2)

M. Diehl

CMS 2016

CMS 2017

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Parton correlations

if neglect correlations between two partons

$$F(x_1, x_2, \boldsymbol{y}) = \int d^2 \boldsymbol{b} f(x_2, \boldsymbol{b}) f(x_1, \boldsymbol{b} + \boldsymbol{y})$$

where $f(x_i, b) = \text{impact parameter dependent single-parton density}$

and if neglect correlations between x and y of single parton

$$f(x_i, \boldsymbol{b}) = f(x_i) F(\boldsymbol{b})$$

then $F(x_1, x_2, \boldsymbol{y}) = f(x_1) f(x_2) \int d^2 \boldsymbol{b} F(\boldsymbol{b}) F(\boldsymbol{b} + \boldsymbol{y})$



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Parton correlations

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and if neglect correlations between x and y of single parton

$$f(x_i, \boldsymbol{b}) = f(x_i) F(\boldsymbol{b})$$

then $F(x_1, x_2, y) = f(x_1) f(x_2) \int d^2 b F(b) F(b + y)$

• information on f(x, b) from study of GPDs and elastic form factors \oplus measurements of double parton scattering

ightarrow complete independence between two partons disfavored

cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003-04; Blok et al 2013

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Parton correlations

at certain level of accuracy expect correlations between

- \blacktriangleright x_1 and x_2 of partons
 - most obvious: energy conservation $\Rightarrow x_1 + x_2 \leq 1$
 - significant x₁ x₂ correlations found in quark models Chang, Manohar, Waalewijn 2012; Rinaldi et al 2013–20; Broniowski
 - et al 2013–19: Kasemets, Mukherjee 2016: Courtoy et al 2019–20
- x_i and y

even for single partons see correlations between x and b distribution

- HERA results on $\gamma p \rightarrow J/\Psi p$ give $\langle b^2 \rangle \propto \text{const} + 4\alpha' \log(1/x) \text{ with } 4\alpha' \approx (0.16 \text{ fm})^2$ for gluons with $x \sim 10^{-3}$
- lattice calculations of x^0 , x^1 , x^2 moments \rightarrow strong decrease of $\langle b^2 \rangle$ with x above ~ 0.1

plausible to expect similar correlations in double parton distributions even if two partons not uncorrelated

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Consequence for multiple interactions

- \blacktriangleright indications for decrease of $\langle {m y}^2 \rangle$ with x
- if interaction 1 produces high-mass system
 - \rightarrow have large x_1, \bar{x}_1
 - ightarrow smaller $oldsymbol{y}~
 ightarrow$ more central collision
 - \rightarrow secondary interactions enhanced

Frankfurt, Strikman, Weiss 2003 studies in Pythia: Corke, Sjöstrand 2011; Blok, Gunnellini 2015



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Colour structure

quark lines in amplitude and its conjugate can couple to color singlet or octet:

$${}^{1}F \to (\bar{q}_{2} \mathbb{1}q_{2}) \ (\bar{q}_{1} \mathbb{1}q_{1}) \qquad {}^{8}F \to (\bar{q}_{2} t^{a} q_{2}) \ (\bar{q}_{1} t^{a} q_{1})$$

- ⁸F describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)
- for two-gluon dist's more color structures
- color correlations suppressed by Sudakov logarithms

... but not necessarily negligible for moderately hard scales Manohar, Waalewijn arXiv:1202:3794

 $U = \mathsf{Sudakov} \ \mathsf{factor}, \ Q = \mathsf{hard} \ \mathsf{scale}$





Mekhfi 1988

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Double parton scattering: ultraviolet problem

$$\frac{d\sigma_{\text{DPS}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \ \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 \boldsymbol{y} \ F(x_1, x_2, \boldsymbol{y}) \ F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



 \blacktriangleright for $\pmb{y} \ll 1/\Lambda\,$ can compute

$$F(x_1,x_2,oldsymbol{y})\sim rac{1}{oldsymbol{y}^2}$$
 splitting fct \otimes usual PDF

gives strong correlations in colour and spin



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Double parton scattering: ultraviolet problem

$$\frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, dx_2 \, d\bar{x}_2} = \frac{1}{C} \, \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 \boldsymbol{y} \, F(x_1, x_2, \boldsymbol{y}) \, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



• for $oldsymbol{y} \ll 1/\Lambda$ can compute

$$F(x_1,x_2,oldsymbol{y})\sim rac{1}{oldsymbol{y}^2}$$
 splitting fct \otimes usual PDF

gives UV divergent cross section $\propto \int d^2 y/y^4$ in fact, formula not valid for $|y| \sim 1/Q$

 problem also for two-parton TMDs UV divergences logarithmic instead of quadratic



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... and more problems



 double counting problem between double scattering with splitting (1v1) and single scattering at loop level

> MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012 Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012 already noted by Cacciari, Salam, Sapeta 2009

also have graphs with splitting in one proton only: "2v1"

$$\sim \int d^2 \boldsymbol{y} / \boldsymbol{y}^2 \, imes F_{\text{int}}(x_1, x_2, \boldsymbol{y})$$

B Blok et al 2011-13

J Gaunt 2012

B Blok, P Gunnellini 2015



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A consistent solution

MD, J. Gaunt, K. Schönwald 2017



► regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2 \boldsymbol{y} \ \Phi^2(\nu y) \ F(x_1, x_2, \boldsymbol{y}) \ F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$

- $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
- cutoff scale $\nu \sim Q$

• $F(x_1, x_2, y)$ has both splitting and 'intrinsic' contributions analogous regulator for transverse-momentum dependent DPDs

keep definition of DPDs as operator matrix elements cutoff in y does not break symmetries that haven't already been broken

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A consistent solution

MD, J. Gaunt, K. Schönwald 2017



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- $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
- cutoff scale $\nu \sim Q$
- $F(x_1, x_2, y)$ has both splitting and 'intrinsic' contributions analogous regulator for transverse-momentum dependent DPDs
- full cross section: $\sigma = \sigma_{DPS} \sigma_{sub} + \sigma_{SPS}$
 - subtraction σ_{sub} to avoid double counting: = σ_{DPS} with F computed for small y in fixed order perturb. theory much simpler computation than σ_{SPS} at given order

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A consistent solution

MD, J. Gaunt, K. Schönwald 2017



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- $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
- cutoff scale $\nu \sim Q$
- $F(x_1, x_2, y)$ has both splitting and 'intrinsic' contributions analogous regulator for transverse-momentum dependent DPDs

• full cross section: $\sigma = \sigma_{DPS} - \sigma_{sub (1v1 + 2v1)} + \sigma_{SPS} + \sigma_{tw2 \times tw4}$

- subtraction σ_{sub} to avoid double counting: = σ_{DPS} with F computed for small y in fixed order perturb. theory much simpler computation than σ_{SPS} at given order
- can also include twist 2 × twist 4 contribution and double counting subtraction for 2v1 term

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Subtraction formalism at work



 $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$

$$\begin{array}{ll} \bullet \mbox{ for } y \sim 1/Q \mbox{ have } \sigma_{\rm DPS} \approx \sigma_{\rm sub} \\ \mbox{ because pert. computation of } F \mbox{ gives good approx. at considered order} \\ \Rightarrow \mbox{ } \sigma \approx \sigma_{\rm SPS} \mbox{ dependence on } \Phi(\nu y) \mbox{ cancels between } \sigma_{\rm DPS} \mbox{ and } \sigma_{\rm sub} \end{array}$$

► for
$$y \gg 1/Q$$
 have $\sigma_{sub} \approx \sigma_{SPS}$
because DPS approximations work well in box graph
 $\Rightarrow \sigma \approx \sigma_{DPS}$ with regulator fct. $\Phi(\nu y) \approx 1$

- same argument for 2v1 term and $\sigma_{tw2 \times tw4}$ (were neglected above)
- subtraction formalism works order by order in perturb. theory
 J. Collins, Foundations of Perturbative QCD, Chapt. 10

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DGLAP evolution

define DPDs as matrix elements of renormalised twist-two operators:

 $F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p | \mathcal{O}_1(\boldsymbol{0}; \mu_1) \mathcal{O}_2(\boldsymbol{y}; \mu_2) | p \rangle \qquad f(x; \mu) \sim \langle p | \mathcal{O}(\boldsymbol{0}; \mu) | p \rangle$ $\Rightarrow \text{ separate DGLAP evolution for partons 1 and 2:}$

$$\frac{\partial}{\partial \log \mu_i^2} F(x_i, \boldsymbol{y}; \mu_i) = P \underset{x_i}{\otimes} F \qquad \text{for } i = 1, 2$$

- ▶ DGLAP logarithm from strongly ordered region $|q_1| \ll |k| \sim |q_2| \ll Q_2$ repeats itself at higher orders (ladder graphs)
- resummed by DPD evolution in σ_{DPS} if take ν ~ μ₁ ~ Q₁, μ₂ ~ Q₂ and appropriate initial conditions (correct small y limit)
- ► can enhance DPS region over SPS region $|q_1| \sim |q_2| \sim Q_{1,2}$ which dominates by power counting

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Double counting in TMD factorisation for DPS



left and right box can independently be collinear or hard:

 \rightsquigarrow DPS, DPS/SPS interference and SPS

get nested double counting subtractions

M Buffing, MD, T Kasemets 2017

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Summary

- multiparton interactions ubiquitous in hadron-hadron collisions multiple hard scattering often suppressed, but not necessarily
 - in specific kinematics
 - for multi-differential cross sections, high-multiplicity final states
- double hard scattering depends on detailed hadron structure
 - transverse spatial distribution
 - different correlation and interference effects
- short-distance singularity and double counting with single scattering: have consistent solution using cutoff and subtraction terms
- subject of high interest for
 - control over final states at LHC
 - understanding QCD dynamics and hadron structure

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