Diving into the interior of asymptotically flat hairy BHs and Maximal Warm Holes

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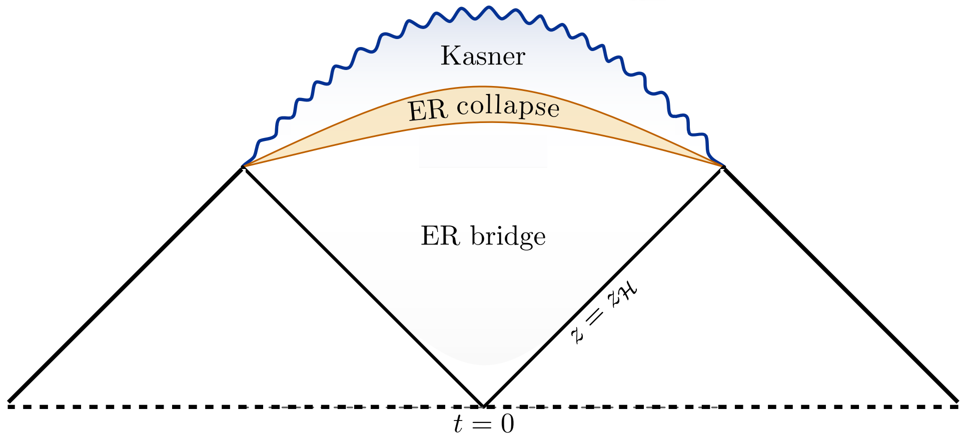


Based on:

OD, Gary Horowitz, Jorge Santos

2109.14633 & 2110.06225

(See also **Henneaux** **2202.04155**)

New horizons for (no-)horizon physics: from gauge to gravity and back,

Galileo Galilei Institute (GGI), Florence

→ Overall Motivation

- **What happens** when we **dive** across an event horizon **into** the **interior of a BH?**

There are classical and quantum challenges (many not yet fully understood):

–Hawking-Penrose **Singularity thms** '70: @ interior of BH, spacetime **ends at a singularity**

–Belinskii-Khalatnikov-Lifshitz (BKL) '70: **How?** General solution near spacelike (cosmological) singularity is intricate & described by a **Kasner cosmology** (possibly w/ **chaotic BKL oscillations**)

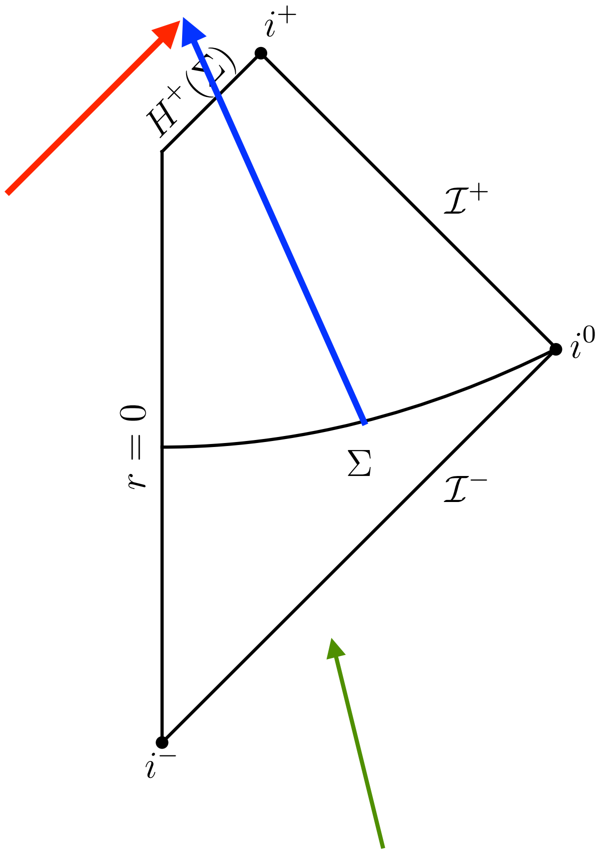
–On other hand, **Weak Cosmic Censorship Conjecture** [Geroch-Horowitz '79]: "**generically**, the maximal development of (asymp flat, geod complete) initial data is an asymp flat spacetime (in particular $\mathcal{I}^+$ **is complete**) that is strongly asymptotically predictable." —> **"naked singularities don't form (from collapse)"**

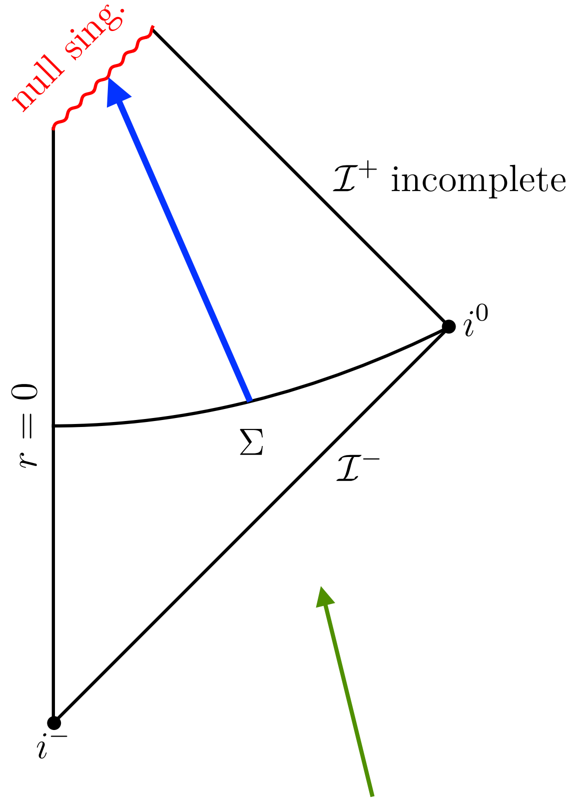
- Geroch '70: For BHs with a **Cauchy horizon** (at which classical predictability breaks down even without large curvatures) **what happens?**

- Penrose '79: **Cauchy horizons are artifacts** of symmetry & do **not** arise from generic initial data: **blueshift instability (a.k.a. mass inflation)** should **produce singularity**.

- **Strong Cosmic Censorship Conjecture** [Penrose '79, Christodoulou '99]: **Generically** (generic asympt flat, complete, initial data Σ) the resulting **solution cannot be extended across a Cauchy horizon** (the maximal Cauchy development of a two-ended Σ is inextendible)

- **Weak CC** is **not** implied by **Strong CC** and the two are **independent:**





violates SCCC, but not WCCC

violates WCCC, but not the SCCC

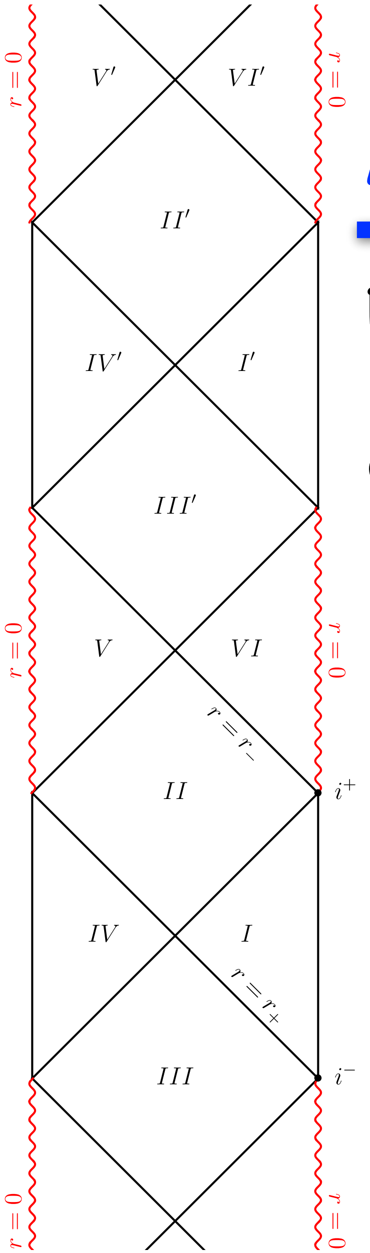
"Cannot predict the future from initial data"

"naked singularity, not clothed by horizon"

→ Motivation: BH interiors in AdS (in short)

Frenkel-Hartnoll-Kruthoff-Shi: [2004.01192]
Santos-Horowitz-Hartnoll-Kruthoff: [2008.12786], [2006.10056]

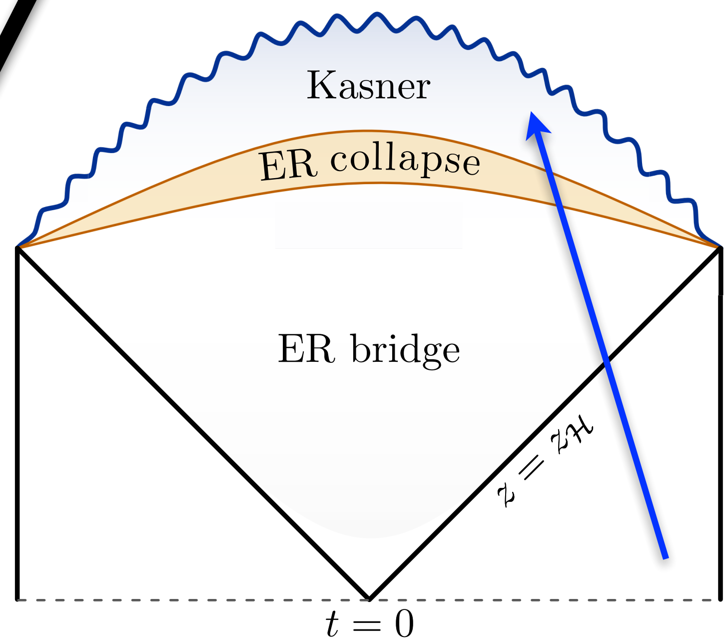
Reissner-Nordström-AdS



Add a charged scalar field →

RN-AdS becomes unstable to formation of scalar condensate

=> Hairy BH forms (Holographic Superconductor)



Dive into the interior of hairy BH:

System **goes through 3 epochs**

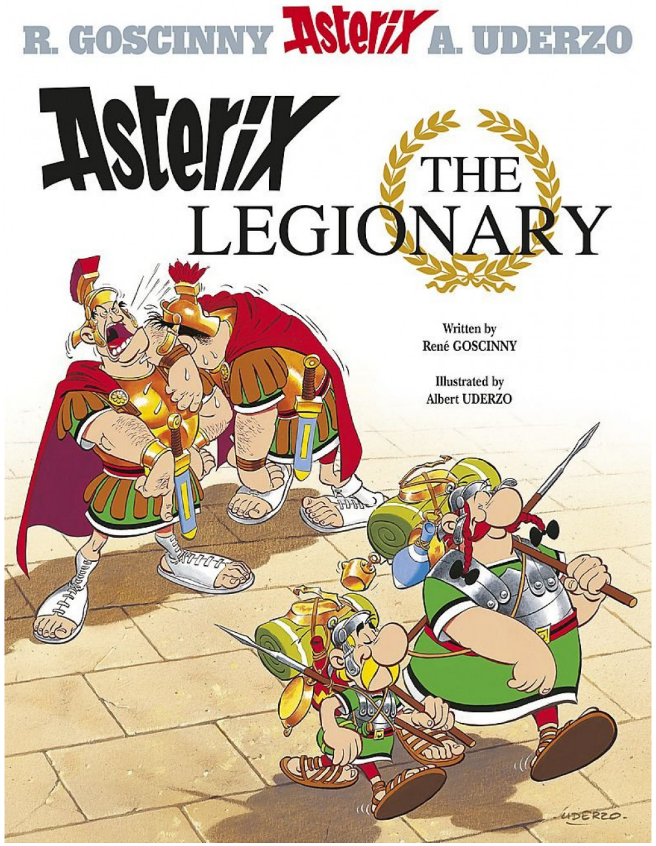
(r is now timelike coord)

& deep in the interior one

approaches <u>not</u> a Cauchy Horizon

but a Kasner (spacelike) singularity

This talk in a nutshell:



Mutatis mutandis,*

is there some **universality** in this physics?

That is to say, do we have similar physics for **asymptotically flat** BHs?

* ***Mutatis mutandis*** is a Medieval Latin phrase meaning "with things changed that should be changed" or "once the necessary changes have been made".

It remains unnaturalized in English and is therefore usually italicized in writing.

→ This talk: Diving into BH interior of an asymp. flat charged hairy BH

1. Start with **Reissner-Nordström**

2. **Add** an (appropriate) **charged scalar field => RN unstable to scalar condensation**

3. **Associated hairy BHs have novel properties: Maximal Warm Holes [Part 1]**

4. **Dive into the interior of the associated hairy BH [Part 2]:**

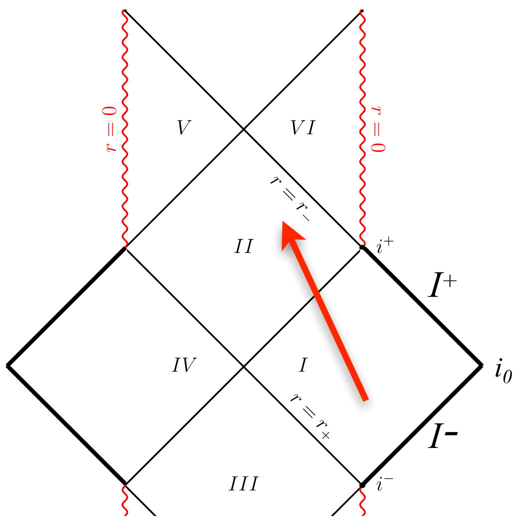
Find that the **system goes through 3 epochs:**

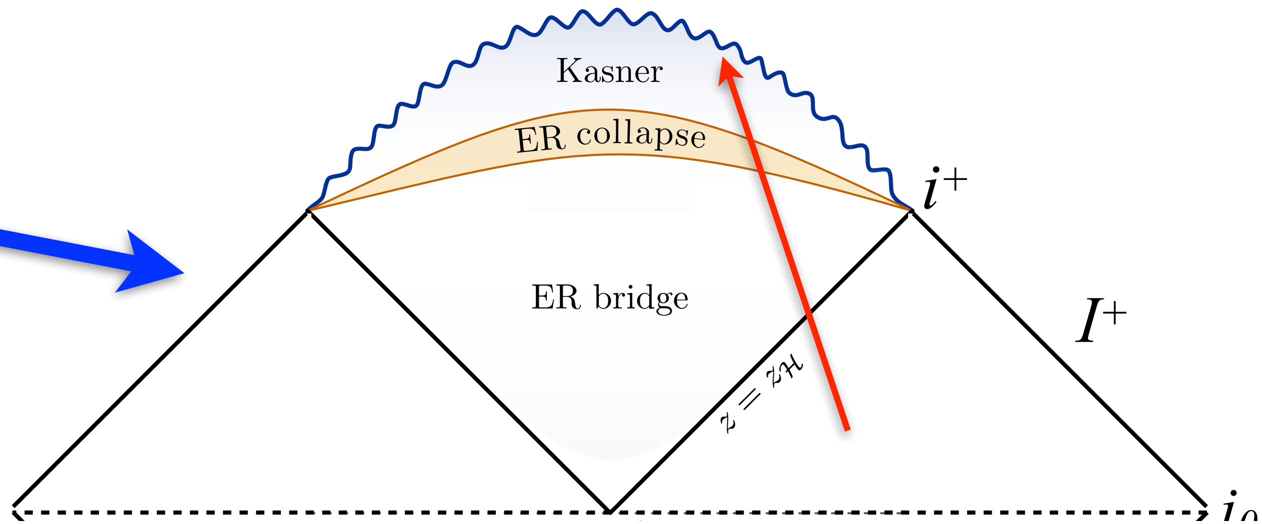
- **Einstein-Rosen bridge collapse epoch**

- **Josephson oscillations epoch**

- **Kasner cosmology epoch**

before a Kasner singularity forms deep in the interior (no Cauchy horizon)





Part 1:

Black Holes of the theory & their properties

Maximal Warm Holes

➙ The theory and its phase diagram of solutions

- **Einstein-Maxwell with charged scalar field & a scalar-Maxwell coupling:**

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - F^2 - 4(\mathcal{D}_a\psi)(\mathcal{D}^a\psi)^\dagger - 4m^2|\psi|^2 - \underbrace{4\alpha F^2|\psi|^2}\right]$$

$$F = \mathrm{d}A$$
$$\mathcal{D} = \nabla - i\,q\,A$$

Scalar-Maxwell coupling:

required to have scalar condensation when Λ=0 & hairy BHs branching from RN

(Theories with self-interacting V also have hairy BHs but they do not branch from RN)

- **Ansatz for solutions** (static, spherically symmetric)

$$\mathrm{d}s^2 = -p(r)\,g(r)^2\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{p(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

$$A = \Phi(r)\,\mathrm{d}t\,, \qquad \psi = \psi^\dagger = \psi(r)$$

- **Reissner-Nordström (RN) solution (no scalar field)**

$$p(r) = p_{\rm RN}(r) \equiv \frac{(r-r_+)(r-r_-)}{r^2}\,, \quad g(r) = 1\,, \quad \text{and} \quad \Phi(r) = \Phi_{\rm RN}(r) \equiv \left(1 - \frac{r_+}{r}\right)\mu$$

$$Q = \mu\,r_+ \text{ and } r_\pm \equiv M \pm \sqrt{M^2 - Q^2}.$$

μ: chemical potential

- Perturb **RN with scalar field:** $\psi(t,r) = \tilde{\psi}(r)\, e^{-i\omega t}$

$$\frac{1}{r^2}\left[r^2 p_{\rm RN}(r)\tilde{\psi}'(r)\right]' + \left\{\frac{[\omega + q\,\Phi_{\rm RN}(r)]^2}{p_{\rm RN}(r)} - m^2 + 2\,\alpha\,\Phi'_{\rm RN}(r)^2\right\}\tilde{\psi}(r) = 0$$

- **If** $q^2\mu^2 \leq m^2$ the scalar field **decays exponentially** at infinity:

$$\psi = \frac{e^{-r\sqrt{m^2-q^2\mu^2}}}{r^{1+\eta}}\left[b + \mathcal{O}(r^{-1})\right],$$

Exponential decay: characteristic of bound states

- **Otherwise** ($q^2\mu^2 > m^2$), scalar field **oscillates** asymptotically => it's not bound to BH. Such solutions would have **infinite energy** => **discard**

- Near horizon of the extremal RN black hole: set $r_- = r_+$ in RN; introduce coord

$$t = \frac{r_+\,\tau}{\lambda}, \quad \text{and} \quad r = r_+(1+\lambda\,\rho) \quad \text{\& take limit} \quad \lambda \to 0$$

=> Solution reduces to the direct product form AdS2 x S2 :

$$ds^2_{\rm AdS_2\times S^2} = L^2_{\rm AdS_2}\left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2}\right) + r_+^2\,\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad A_{\rm AdS_2\times S^2} = \mu_{\rm AdS_2}\,\rho\, d\tau$$

- Full extreme BH is **unstable** if effective scalar mass in AdS2 x S2 **violates AdS2 BF bound:**

$$m^2_{\rm eff}L^2_{\rm AdS_2} \equiv m^2 L^2_{\rm AdS_2} - q^2\mu^2_{\rm AdS_2} < -\frac{1}{4} \qquad \Rightarrow \alpha > \frac{1}{2}\left[\frac{1}{4} + (m^2 - q^2)L^2_{\rm AdS_2}\right]$$

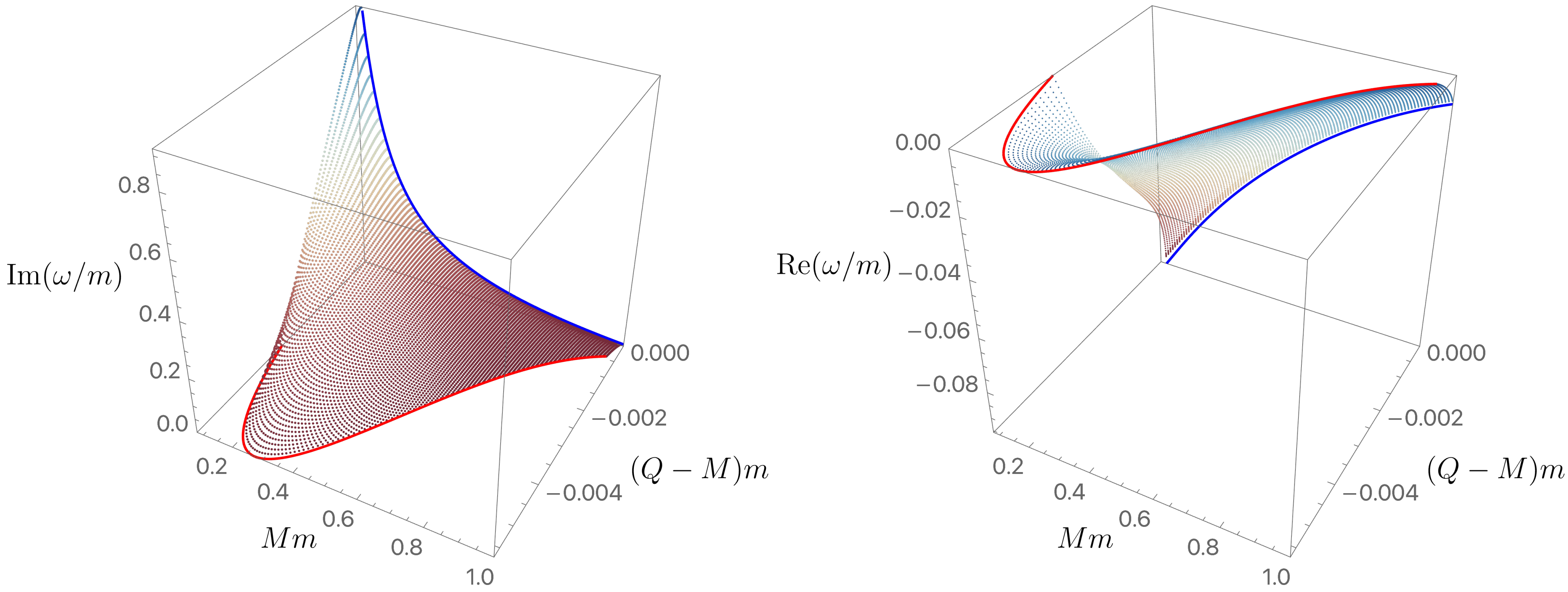
- Conclusion:

Extremal RN has μ=1 so we can have bound states ($q^2 \leq m^2$)

that violate the BF bound if $\alpha > \frac{1}{2}\left[\frac{1}{4} + (m^2 - q^2) L^2_{\mathrm{AdS}_2}\right]$

- **Instability confirmed** by a full numerical analysis:

$$\psi(t,r) = \widetilde{\psi}(r)\, e^{-i\omega t}$$

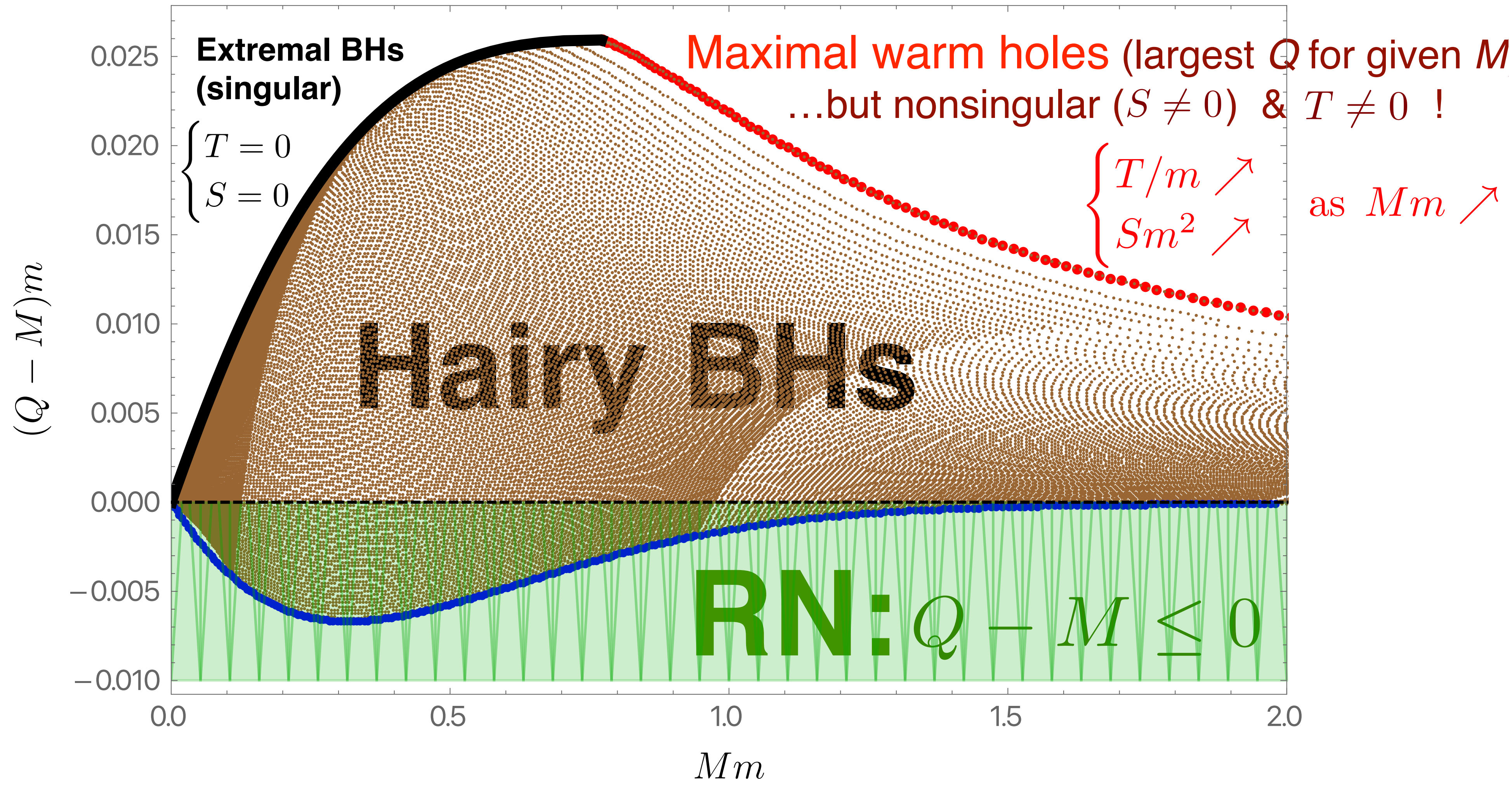


- Plot for α =1, q/m=1/2
- Red: Onset of instability (ω=0). Blue: extremality (Q-M=0)

- **RN are unstable to condensation of bound states if** $q^2\mu^2 \leq m^2$

  => in a phase diagram of static solutions, there should exist **hairy BHs** **bifurcating** from the **RN onset of instability**

- Indeed, that's the case. Phase diagram for α =1, q/m=1:



  - Blue: RN onset = Bifurcation/Merger of RN with hairy BH
  - Red: curve with μ=1 (ie the "last" bound state configuration with $q^2\mu^2 = m^2$)

Why do maximal warm holes exist in this theory but not others?

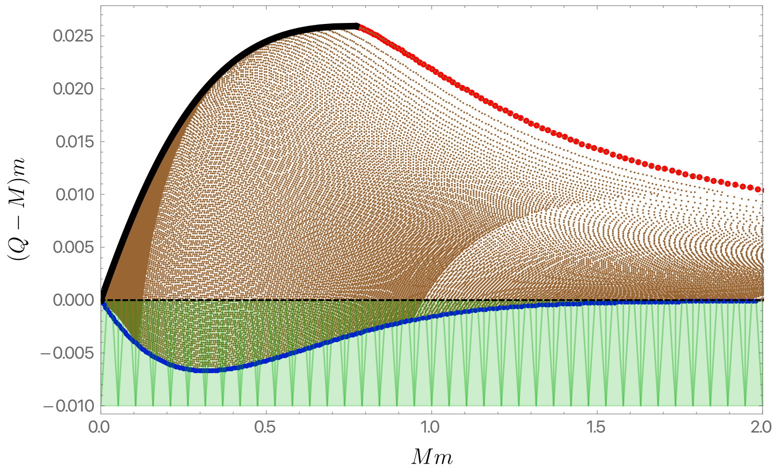
- As one increases Q (for fixed M),

  the region near the horizon behaves as a typical BH with scalar hair

  and wants to become singular.

- **However**, if the M is large enough (M>0.8), before one reaches a singular horizon,

  the asymptotic (bound state) condition $q^2\mu^2 \leqslant m^2$ is **saturated**.



- Since one cannot support scalar hair if this bound is violated

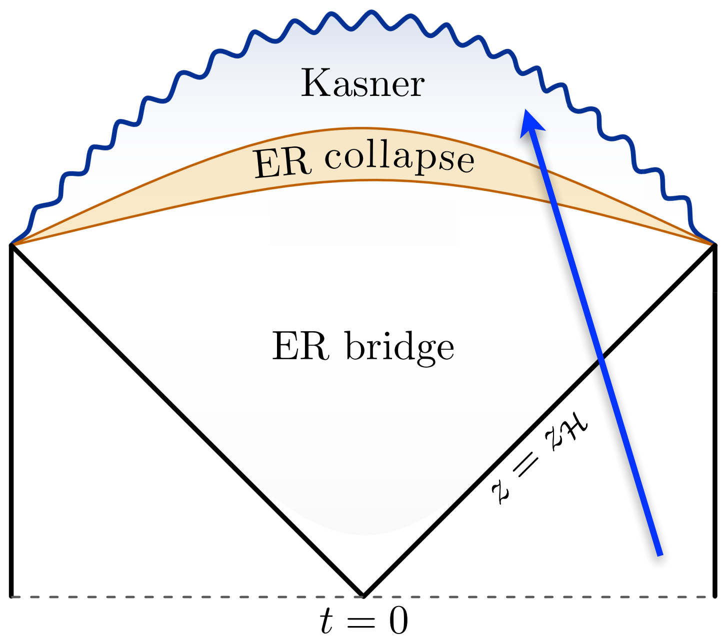
  (& there are no other BHs without hair having Q > M),

  the would be "extremal" BH has T > 0.

This is a new kind of extremal BH that we are calling a maximal warm hole

Part 2:

Diving into the interior of a Hairy BH



→ Theorem: no Inner Horizon in the presence of a scalar field

See also: Santos-Horowitz-Hartnoll-Kruthoff [2008.12786]
Cai-Li-Yang, [2009.05520]

- **Ansatz to study BH interior:**

$$ds^2 = \frac{r_+^2}{z^2}\left[-f(z)e^{-\chi(z)}\frac{dt^2}{r_+^2} + \frac{dz^2}{f(z)} + \frac{dx^2}{1-\kappa\, x^2} + (1-\kappa\, x^2)d\phi^2\right]$$

$$A = \Phi(z)\,dt\,,$$

$$\psi = \psi^\dagger = \psi(z)$$

- From the **EOM**, there is a **quantity** that is conserved **($\partial_z C_1 = 0$):**

$$C_1 = \frac{e^{\frac{\chi}{2}}}{z^2}\left(e^{-\chi} f\right)' - 4e^{\frac{\chi}{2}}\Phi'\,\Phi(1+4\,\alpha\,\psi^2) + 2\kappa\int_{z_{\mathcal{H}}}^{z}\frac{e^{-\frac{\chi(x)}{2}}}{x^2}dx$$

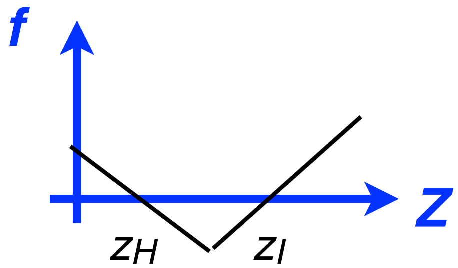
- **Assume** that besides the event horizon there is also an **inner horizon**:

f=0 at horizons & f(z) < 0 for $z_H < z < z_I$ => $f'(z_I) > 0$

(Infinity is at z =0)

- EOM => Φ=0 at horizons

- At event horizon: $C_1 = \frac{e^{\frac{\chi(z_{\mathcal{H}})}{2}}}{z_{\mathcal{H}}^2} f'(z_{\mathcal{H}}) < 0$



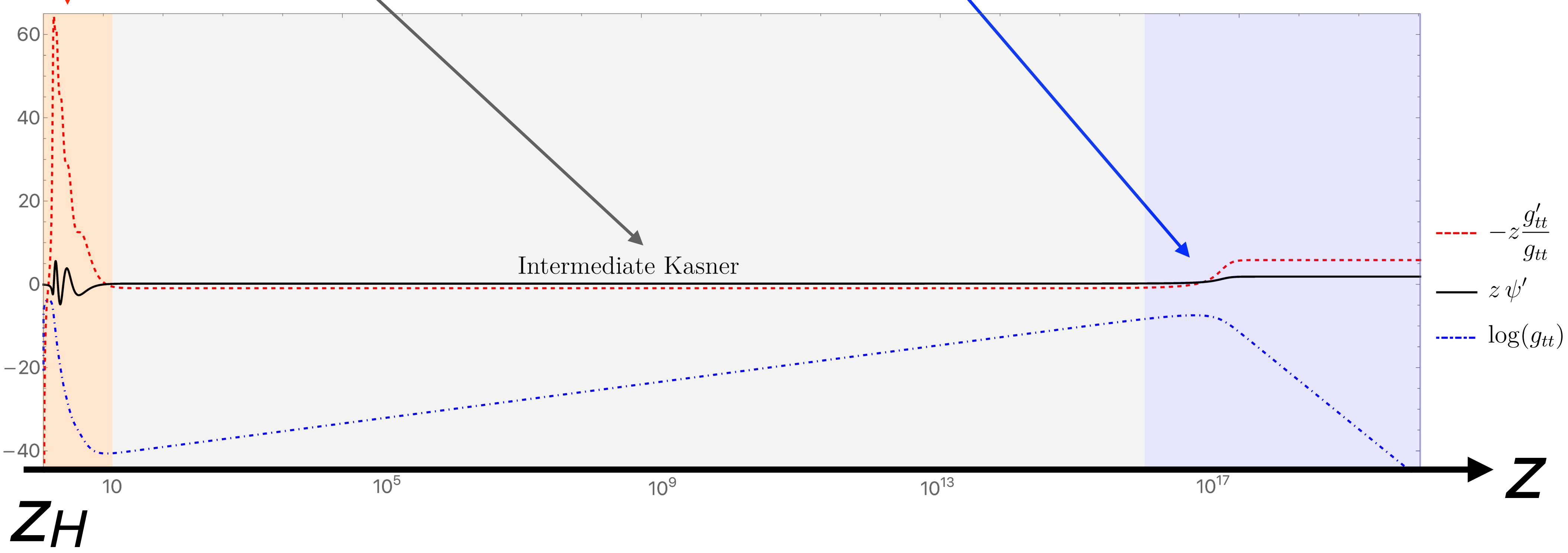
- At inner horizon: $C_1 = \frac{e^{\frac{\chi(z_{\mathcal{I}})}{2}}}{z_{\mathcal{I}}^2} f'(z_{\mathcal{I}}) + 2\kappa\int_{z_{\mathcal{H}}}^{z_{\mathcal{I}}}\frac{e^{-\frac{\chi(x)}{2}}}{x^2}dx > 0$

Since the **constant must be same** this leads to a **contradiction**

=> **NO Inner horizon can be present** (unlike it was **incorrectly assumed**)

➙ So what happens when we dive into the interior of a hairy BH?

- **Would-be Inner horizon z_I** is **replaced** by a **Kasner (spacelike) singularity** as z—> 00
- As z increases, **In-falling observer** experiences 3 epochs :
  1) the **collapse** of the **Einstein-Rosen bridge**;
  2) **Josephson oscillations** of the condensate;
  3) a **Kasner cosmology**, sometimes with **transitions** that change the **Kasner** exponents



The EOM:

$$z^2 e^{-\frac{\chi}{2}}\left[\left(1+4\alpha\psi^2\right)e^{\frac{\chi}{2}}\Phi'\right]' - \frac{2q^2y_+^2\psi^2\Phi}{f} = 0\,,$$

Josephson current
In Maxwell EOM

$$z^2 e^{\frac{\chi}{2}}\left(\frac{e^{-\frac{\chi}{2}}f\psi'}{z^2}\right)' - \left(\frac{y_+^2}{z^2} - \frac{\tilde{q}^2y_+^2e^{\chi}\Phi^2}{f} - 2e^{\chi}z^2\alpha\Phi'^2\right)\psi = 0\,,$$

$$\chi' - 4\,z\left(\frac{\tilde{q}^2y_+^2e^{\chi}}{f^2}\psi^2\Phi^2 + \psi'^2\right) = 0\,,$$

$$e^{\frac{\chi}{2}}z^4\left(\frac{e^{-\frac{\chi}{2}}f}{z^3}\right)' + z^2\kappa - 2y_+^2\psi^2 - \left(1+4\alpha\psi^2\right)e^{\chi}z^4\Phi'^2 = 0\,,$$

$$\tilde{q} \equiv q/m$$

$$y_+ \equiv r_+ m$$

- All three stages can be understood not only numerically but also analytically, despite the highly nonlinear nature of the EOM **(~ like in the BKL original analysis)**

- **Procedure** to **understand analytically** the problem:
  - — We use the **numerics** to find which **terms in EOM are <u>relevant</u>** for the 3 stages
  - — The **<u>remaining</u> terms can be <u>dropped</u>** => mutilated **EOM** can be solved analytically
  - — Finally, we check for **self-consistency** of the procedure.

- The 3 epochs & their approx analytical solns are **cleanly identified for ψ <<1** (eg T~Tc).
  The **scalar field ψ starts small, but** nevertheless **destroys the inner horizon !**

For example, during the ER collapse and Josephson epochs (and often during the Kasner period) one can numerically verify (and a posterior justify) that the **mass terms of ψ** and the **charge term of ψ** in the **Maxwell eqn** can be **dropped** by the time the interesting dynamics kicks.

The EOM:

$$z^2 e^{-\frac{\chi}{2}}\left[\left(1+4\alpha\psi^2\right)e^{\frac{\chi}{2}}\Phi'\right]' - \cancel{\frac{2q^2y_+^2\psi^2\Phi}{f}} = 0\,,$$

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In Maxwell EOM

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→ Epoch 1: collapse of the Einstein-Rosen (ER) bridge

- The <u>linear</u> **vanishing of** g_{tt} towards the would-be inner horizon is **replaced by a rapid collapse to an** <u>**exponentially**</u> **small value!**

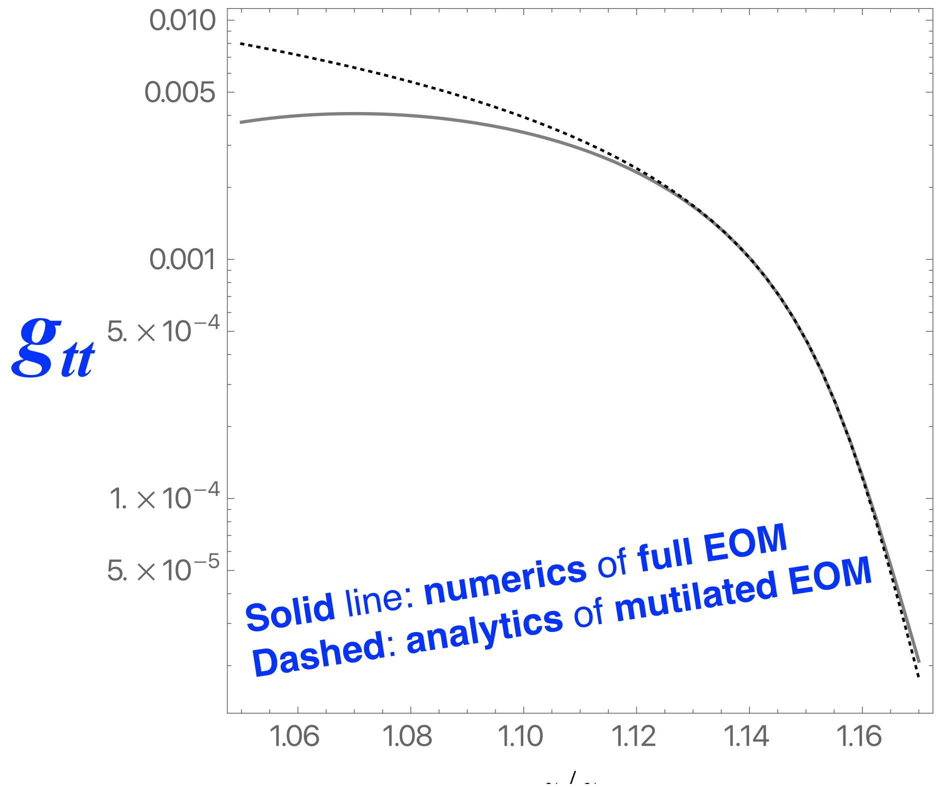
- In the BH interior, g_{tt} sets the measure for the **spatial** t **coord** that runs along the **wormhole** connecting the **two exteriors of the BH**: this is the **Einstein-Rosen bridge.**

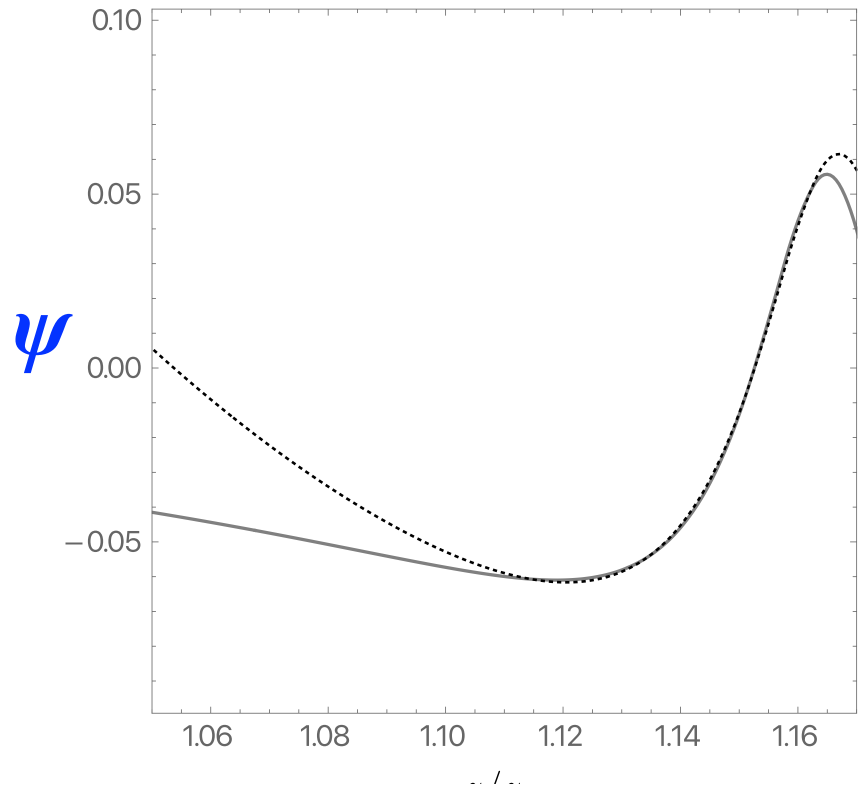
- The **rapid decrease in** g_{tt} can be thought of as a <u>**collapse**</u> of the **ER bridge** for a fixed coord separation Δt.

$$g_{tt} = c_1^2\, W\left(c_1^{-2} e^{-(c_2/c_1)^2\,(z-z_o)}\right) \qquad c_1^2 = \frac{2y_+^2\tilde{q}^2\Phi_o^2\psi_o^2}{z_\star^2(E_o^2 z_\star^2-\kappa)}$$

$$W(x) \equiv ProducLog(x)$$

$$\psi \simeq \psi_o \cos\left(y_+\tilde{q}\Phi_o \int_{z_\star}^{z} \frac{e^{\chi/2}dz}{f} + \varphi_o\right)$$





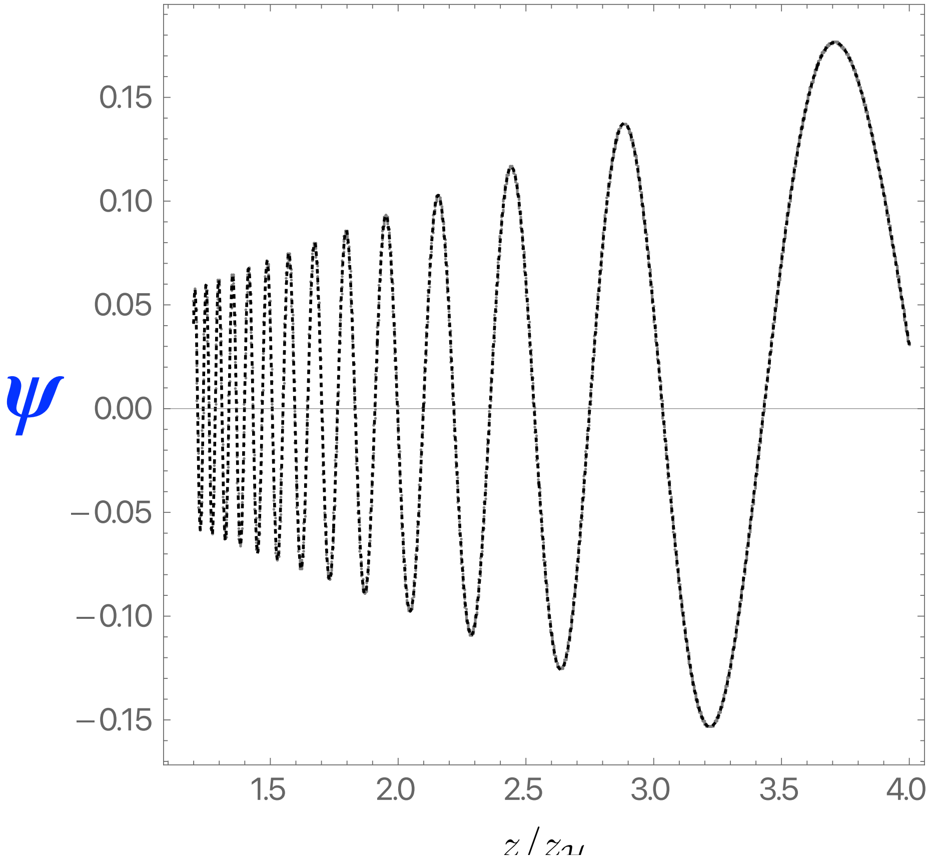
→ Epoch 2: Josephson oscillations of the condensate

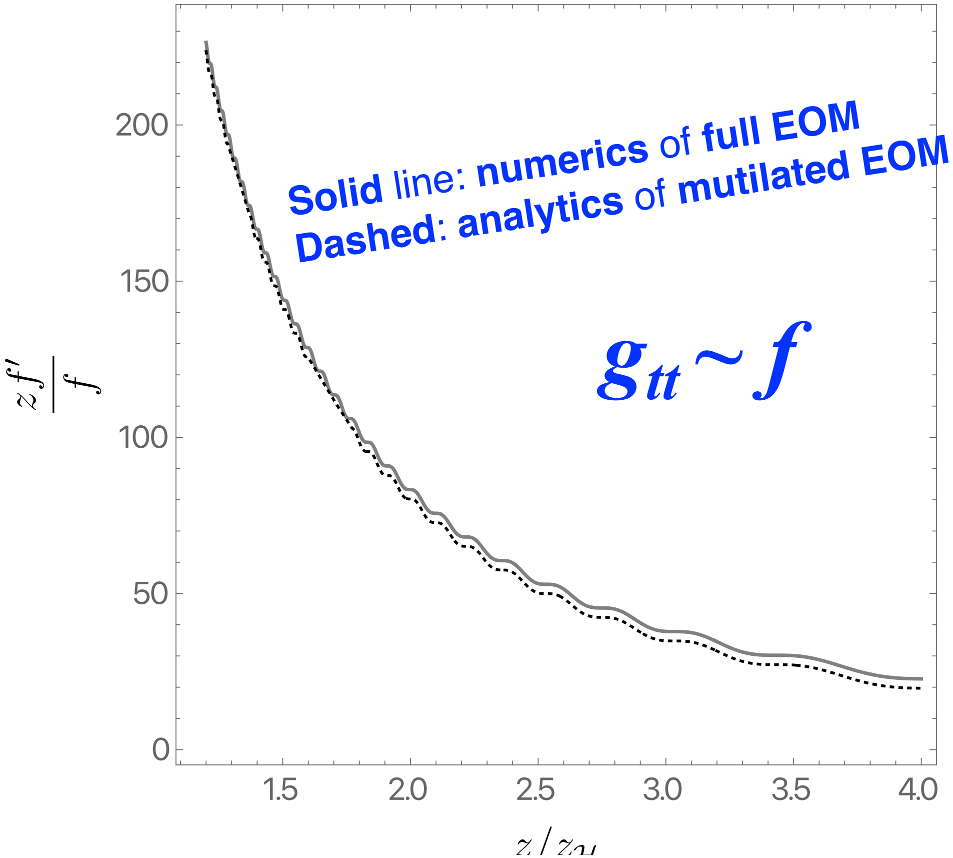
- **After the collapse of the EP bridge**, the **derivative of Max field is small**, $\Phi' \propto e^{-\chi/2} \ll 1$, which can be used to solve the **scalar field ψ** in terms of **Bessel functions:**

$$\psi \simeq c_4 J_0\left(\frac{y_+|\widetilde{q}\Phi_o|c_3}{2z^2}\right) + c_5 Y_0\left(\frac{y_+|\widetilde{q}\Phi_o|c_3}{2z^2}\right)$$

=> **Josephson oscillations**

$$c_{4,5} \simeq \left(\frac{z_\star \pi^2 c_2^2}{32}\,\frac{\psi_o^2}{c_1^2}\right)^{1/4} \sin\left(\frac{c_2\sqrt{z_\star}}{\sqrt{2}}\,\frac{1}{\psi_o c_1} - \varphi_o \pm \frac{\pi}{4}\right)$$





→ Epoch 3: Kasner cosmology and Transitions

- At large z, ψ behaves logarithmically => signals entrance into a **Kasner cosmology** regime:

$$\begin{cases} \psi|_{z>>1} \simeq \beta \ln z \\ \beta = -\frac{4}{\pi} c_5 \end{cases} \Rightarrow \begin{cases} f|_{z>>1} \simeq -f_o z^{3+2\beta^2} \\ \chi|_{z>>1} \simeq 4\beta^2 \ln z \end{cases}$$

=> metric in which all components are **powers of z & ψ~ln z**

- **Maxwell** potential is remains **unimportant** (negligible) while $\beta^2 > 1/2$:

$$\Phi \simeq \Phi_K + \frac{E_K\ z^{1-2\beta^2}}{1+4\alpha\beta^2 \ln^2 z} << 1$$

- Introducing the **proper time** $\tau = \int \sqrt{g_{zz}} \mathrm{d}z \propto z^{-(3/2+\beta^2)}$ $\quad (z=\infty \leftrightarrow \tau = 0)$

the solution takes the **standard (generalised) Kasner form:**

$$\begin{cases} p_t = \frac{2\beta^2-1}{2\beta^2+3}, \\ p_x = \frac{2}{2\beta^2+3}, \\ p_\psi = \frac{2\beta}{2\beta^2+3}. \end{cases}$$

$$ds^2 = -d\tau^2 + \tau^{2p_t} dt^2 + \tau^{2p_x}(dx^2 + d\phi^2), \qquad \psi = p_\psi \ln \tau$$

$$p_t + 2p_x = 1, \qquad p_t^2 + 2p_x^2 + 4p_\psi^2 = 1$$

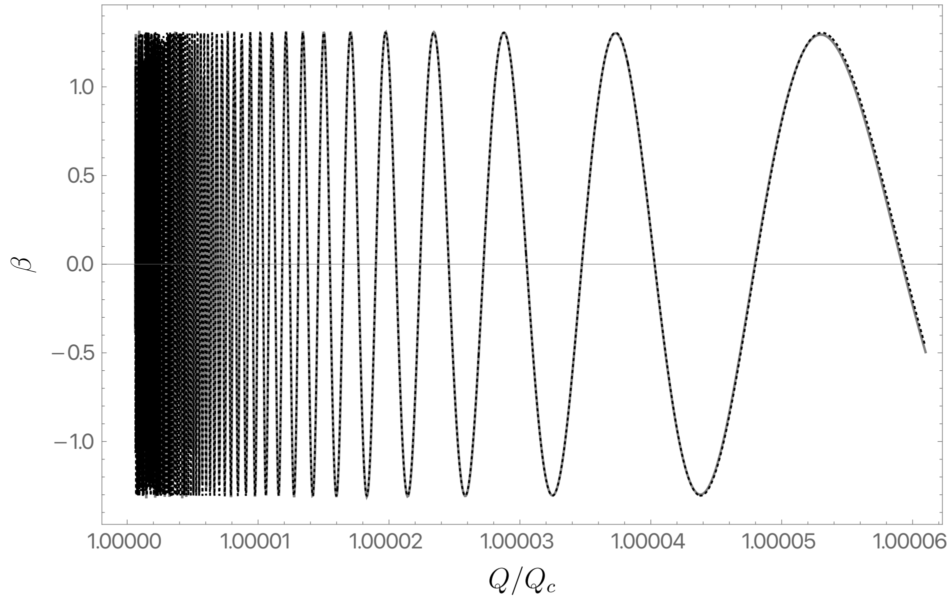
=> **Spacelike curvature singularity** at τ =0 (z=00) [except when p_t=1 <—> β=00].

=> The **would-be inner horizon** is **replaced** by **a (spacelike) Kasner singularity**

- **While $\beta^2 > 1/2$ (Max $\Phi << 1$):** system remains described by **Kasner Cosmology** with **decreasing g_{tt} till the Kasner singularity is reached**

• Parameter β **controls the Kasner exponents** & is an **oscillating function**, well fitted by:

$$\beta = A \sin\left[\frac{B}{Q/Q_c - 1} + C\right]$$



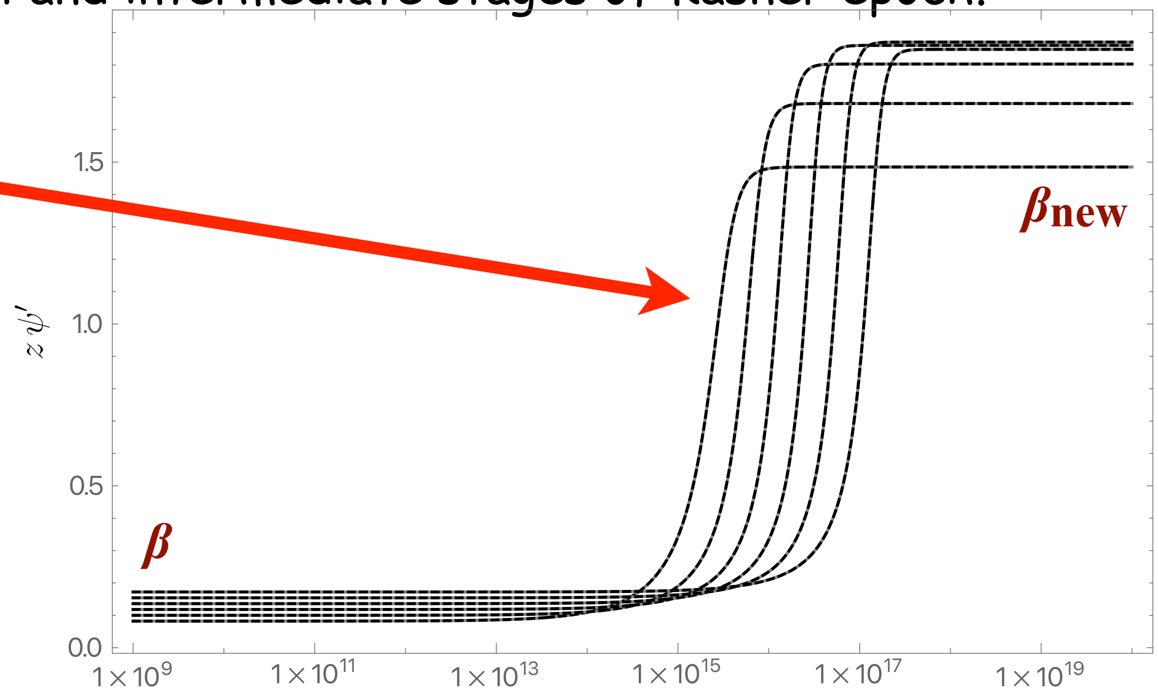
This clearly shows the **extreme sensitivity of the Kasner exponents on Q** near the critical charge Q_c.

• So we **can have** $\beta^2 < 1/2$ at the beginning of the Kasner epoch. **If so** there are new effects.

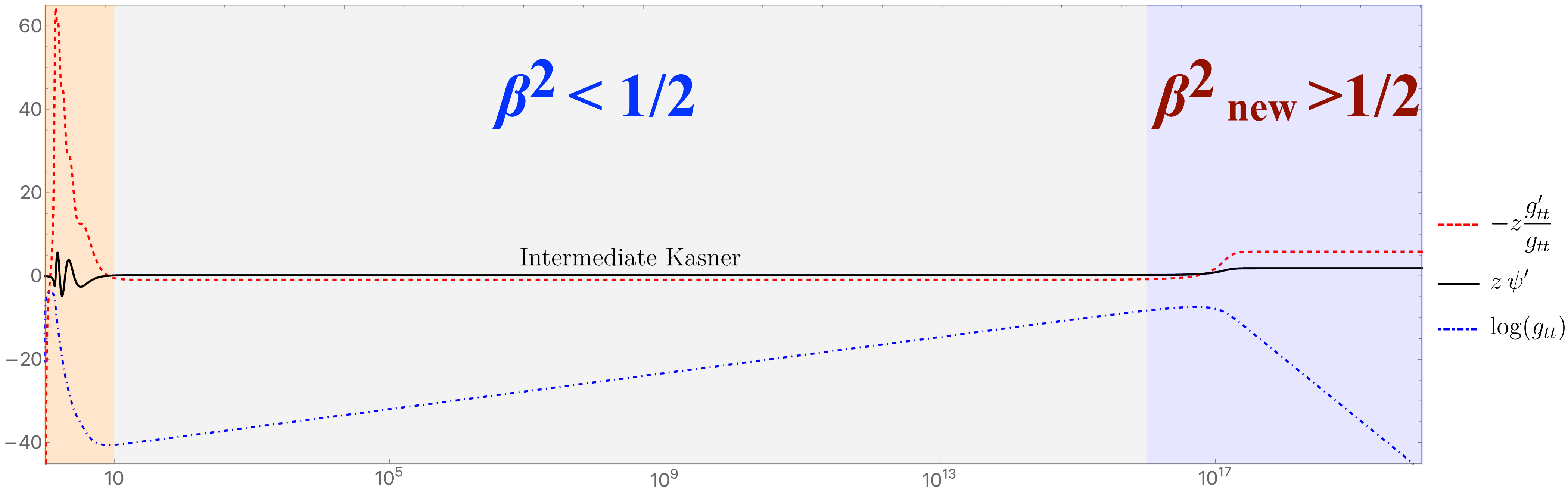
$\beta^2 < 1/2 \Rightarrow g_{tt}$ **increases** during initial and intermediate stages of Kasner epoch.

But **Maxwell Φ is no longer small** and its growth causes a **transition** to a **different** Kasner solution with new exponents $\beta \longrightarrow \beta_{\text{new}}$.

If $\beta^2_{\text{new}} > 1/2$ the system now goes through a new Kasner period with **decreasing** g_{tt} till the Kasner **singularity is reached**



- If $\beta^2{}_{\text{new}} > 1/2$ **after the transition,** the system now goes through a Kasner period with **decreasing** g_{tt} till the Kasner singularity is reached.



- **However if**, after the transition, one **still** has $\beta^2{}_{\text{new}} < 1/2$ the system will go through <u>**new**</u> **Kasner transitions** (generically, a **finite #** of them) till an **ultimate** $\beta^2{}_{\text{final}} > 1/2$ **that finally makes** g_{tt} **->0** as the Kasner sing. is reached. For **fine-tuned initial data**, there can be an **infinite #** of transitions: **chaotic BKL oscillatory behaviour** (Belinskii-Khalatnikov-Lifshitz'70)

- All these findings are in **agreement with the theorem** (**<u>no</u> Inner Horizon when ψ present**):

the presence of a scalar field destroys the possibility of having a Inner Horizon

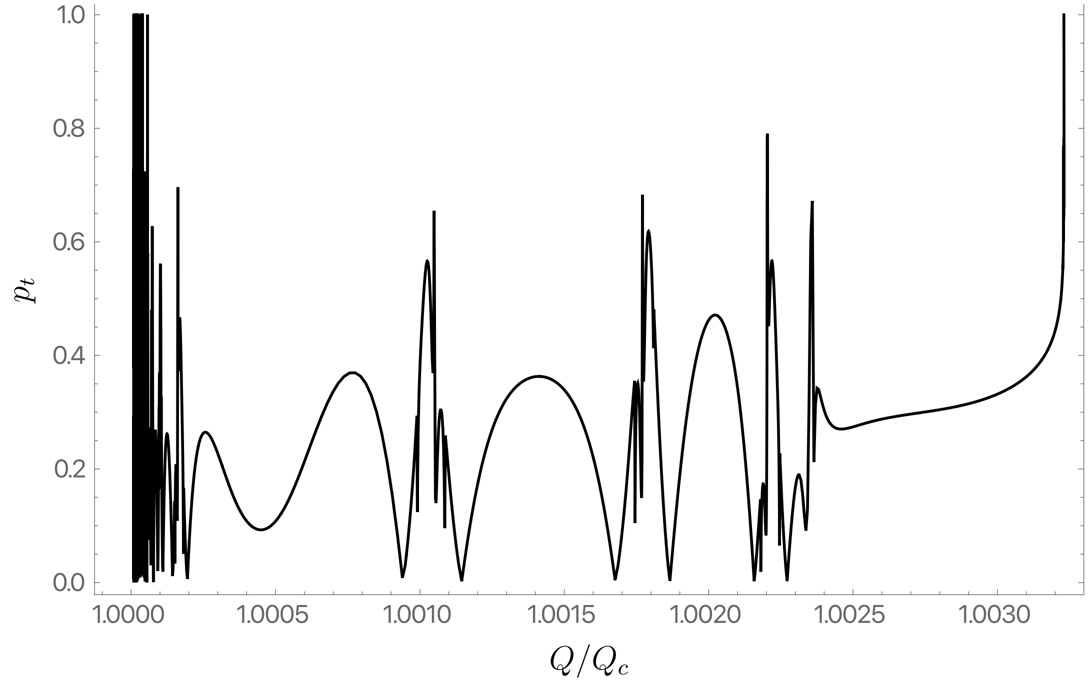


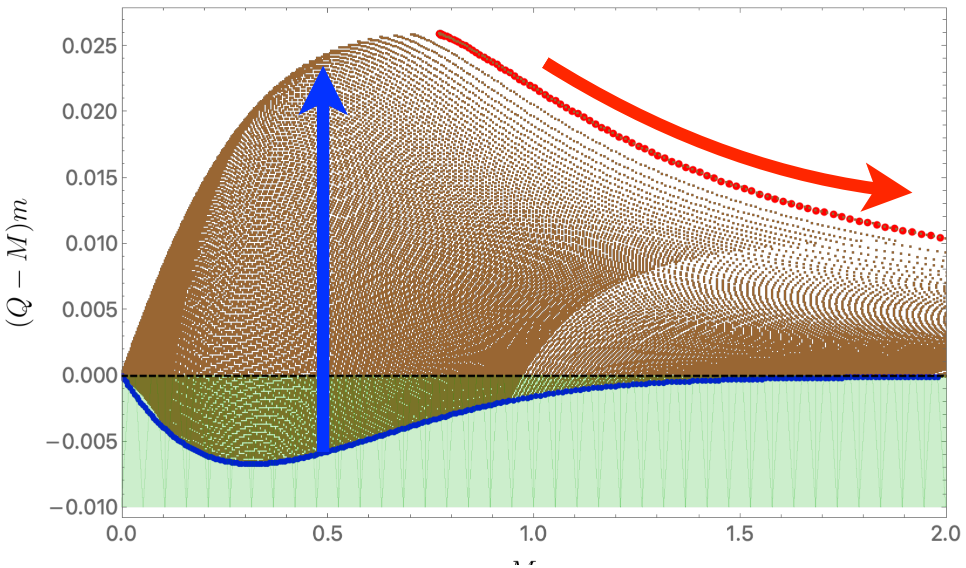
Figure 9: The Kasner exponent p_t as a function of Q/Q_c from the onset of the scalar instability to the maximum charge this black hole can carry. This is for $\alpha = 1$, $q = 5m$ and $mM = 1$. The solution with maximum Q/Q_c is singular.

Reinforce a key property:

- **Extreme sensitivity of the Kasner exponents on Q** near the critical charge Q_c.
- **Spacelike curvature singularity** at τ =0 (z=00)

  [except when p_t=1 <—> β=00

  But this never happens !].



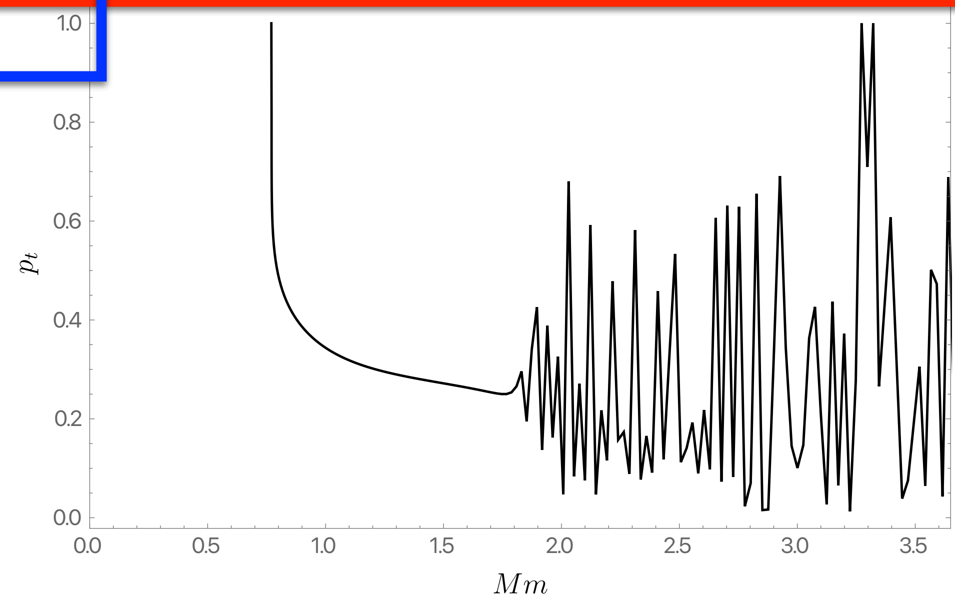


Figure 10: The Kasner exponents inside a family of maximal warm holes (the red curve family of Fig. 1 in [7]). These are maximally charged black holes with $\alpha = 1$, $q = m$. The wiggles on the right are a result of the solution having a charge that approaches Q_c. At

→ Conclusions / main messages

- Hairy BHs can terminate on **non-singular BHs** with **maximal Q** but **non-zero Temperature**:

  maximal warm holes

- We have shown that the **Hairy BH interior** is a complicated place to live:

  — An **in-falling observer** goes through **3 epochs** before approaching a **spacelike Kasner singularity**

  — **Scalar field destroys the would-be Cauchy horizon:**

  it "gets replaced" by a spacelike Kasner singularity

- **Henneaux 2202.04155:**

  BKL dynamics when approaching spacelike singularity has a **cosmological billiard** description:

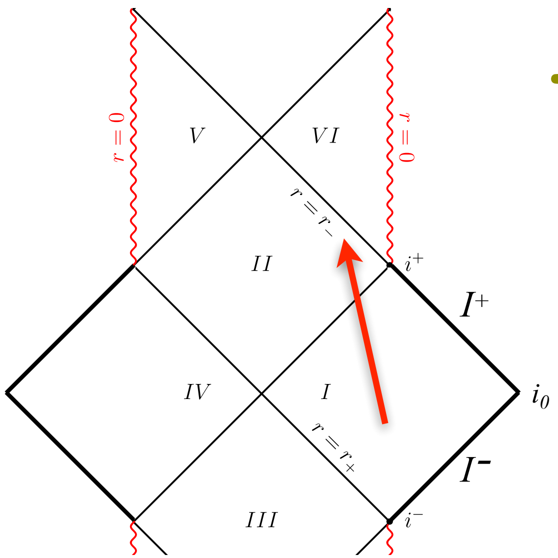
  billiard table in **hyperbolic space where Kasner <-> geodesic motion of a ball**

  & bounces on walls => **transitions** to new Kasner cosmologies.

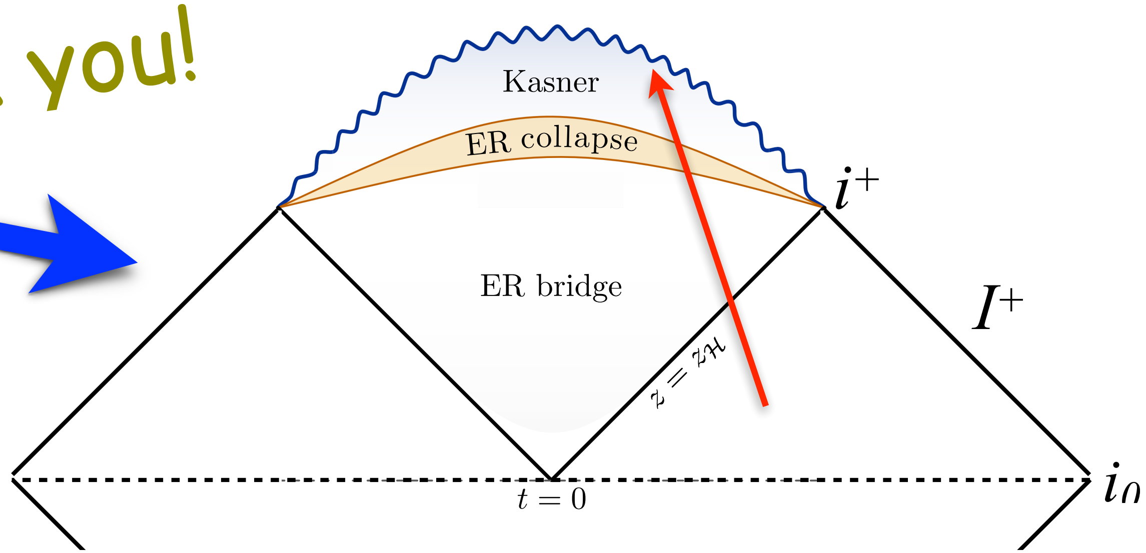
  For our action, for **generic initial conditions** the **volume** of the table is **infinite**

  => **Typically**, system settles into a **single final Kasner cosmology**.

  But exists set of measure zero of initial conditions—> **endless chaotic BKL oscillatory behaviour**



Thank you!



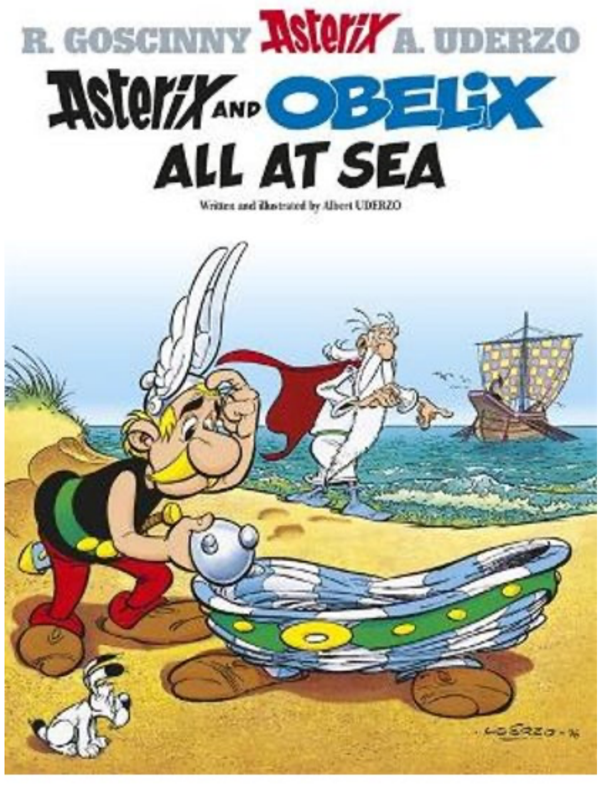
Event at Galileo Galilei Institute

New horizons for **(no-)horizon** physics: from gauge to gravity and back

Today we got rid of one horizon (Cauchy)

... 1/2 work done ...

Grazie mille!

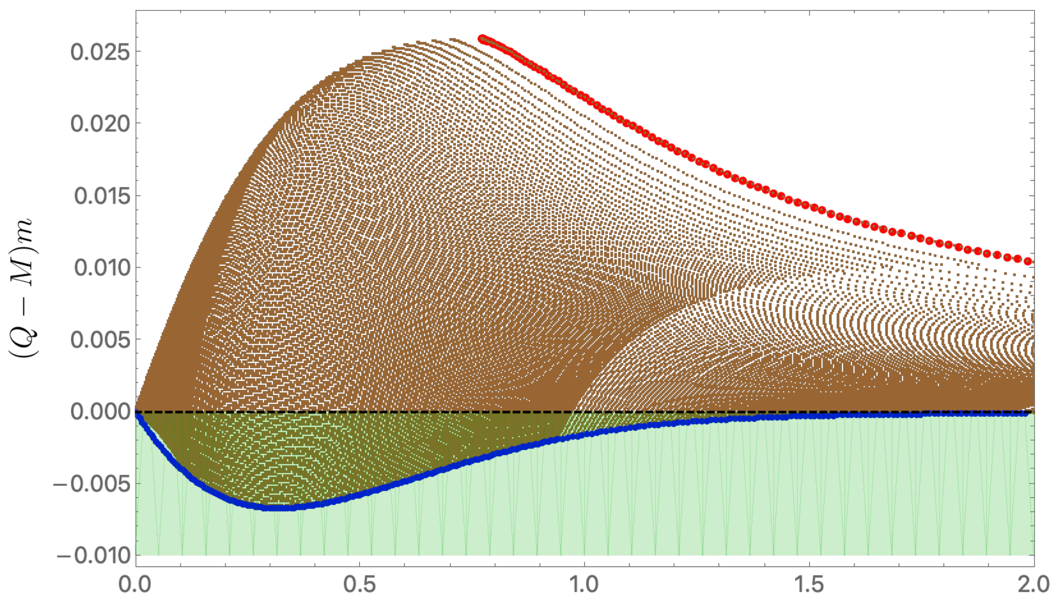


... tomorrow Iosif and Rodolfo just need
to be as efficient with the event horizon

→ Maximal warm holes and the endpoint of Hawking evaporation

- Typically, in a theory without q > m particles, a **near-extremal BH will Hawking radiate** neutral massless particles such as gravitons, photons **and become extremal**.
- Since an **extremal BH has *T*=0**, it is a **stable endpoint** for this process.
- **But maximal warm holes** are smooth BHs with **maximal *Q* and nonzero *T***
  => need **another scenario** for the **endpoint** of their **Hawking evaporation**

- Fix α=1 and q=m. **Hairy BHs** have **T/m<<1:**
  — **charged particles** created by the **Schwinger** mechanism with rate ~ $e^{-\pi m^2/qE}$
  — **neutral photons/gravitons** are produced **thermally**.
- Since **charged particle emission is exp suppressed**, BH should **loose *M* but not *Q*** => **~ vertical line**.
- So for large *Mm*, **Hawking evaporation would appear to end on the red line**.
  But these BHs have **nonzero *T*** => they would appear to **keep radiating**. **This is a puzzle!**

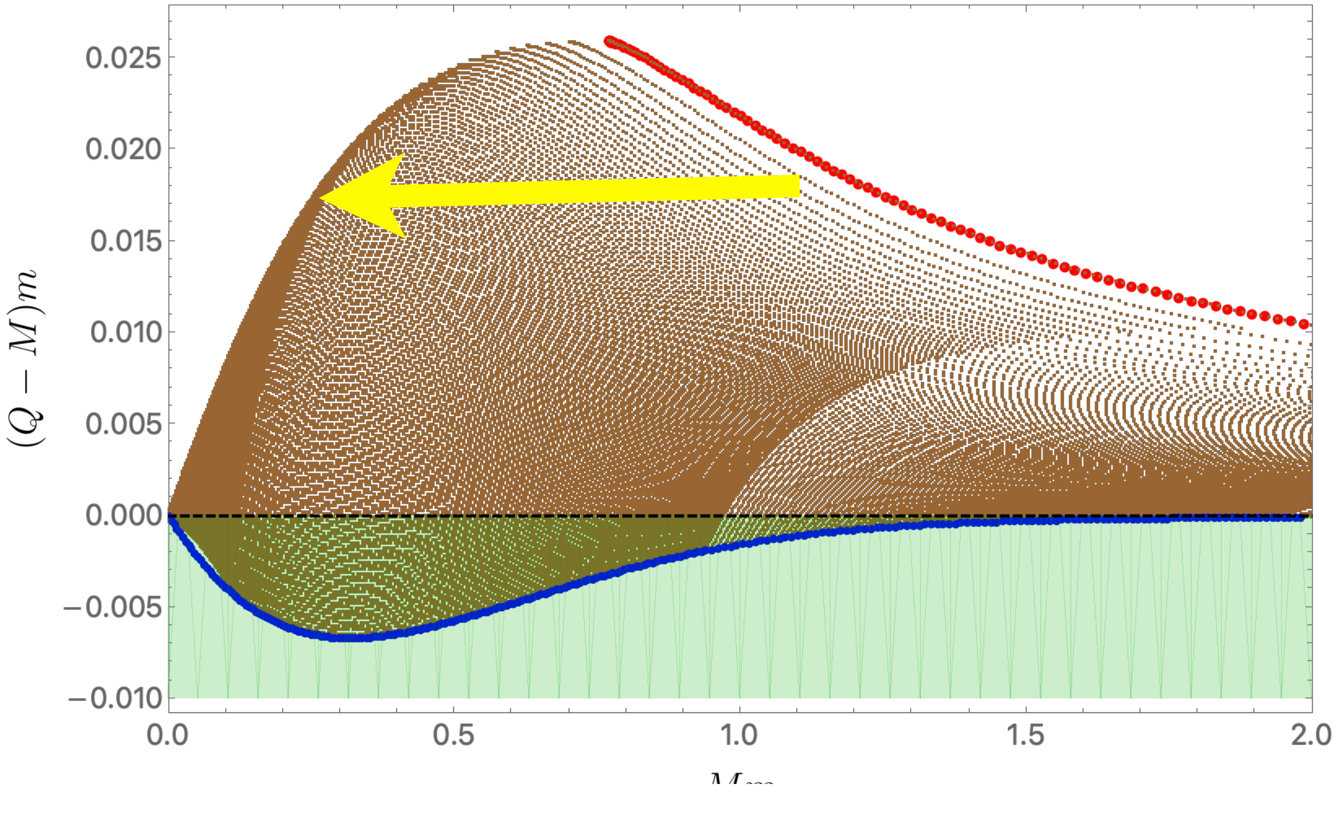


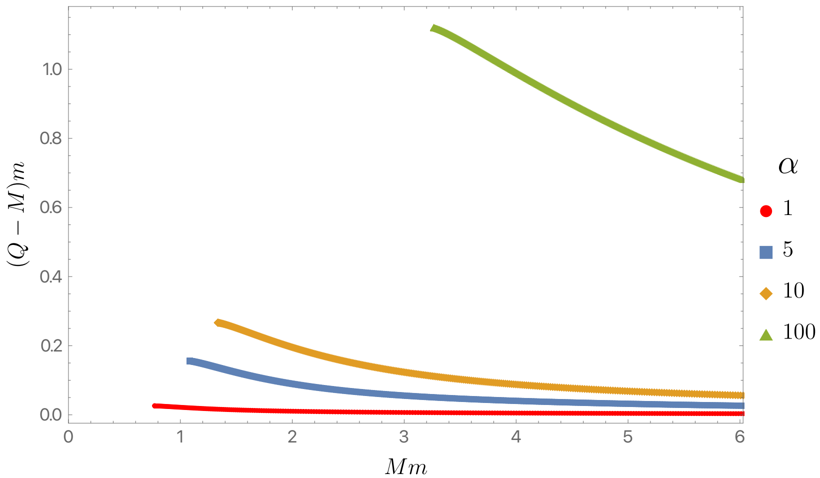
- **Resolution:**
- **Schwinger rate** of charged particle production is ***not*** actually **exp suppressed** but **O(1):**

  q=m, and we checked **E/m ~ O(1)**.

- **In contrast**, for warm holes, T **~** 10^{-3} => **rate of thermal** radiation, $T^4 \sim 10^{-12}$, **is highly suppressed**.

- Thus, **late stages of Hawking radiation** are **dominated** by the **production of *q=m* particles**
  - => ***Q-M*** approximately **constant**
  - => **Hawking radiation causes** the **BH to evolve along a horizontal line** (rather than a vertical line)
  - => **ends in a singular (*S*=0) extremal (*T*=0) solution** as expected.





Maximal warm holes exist
for all scalar-Maxwell couplings α
(above the bound where 2d BF bound
is violated)

Figure 3: Maximal warm holes in theories with $q = m$ and different couplings α. These are all nonsingular $(S > 0)$ black holes with maximum ~~charge and nonzero T.~~ As they approach the solution with minimum mass, $S \to 0$ and $T \to 0$.

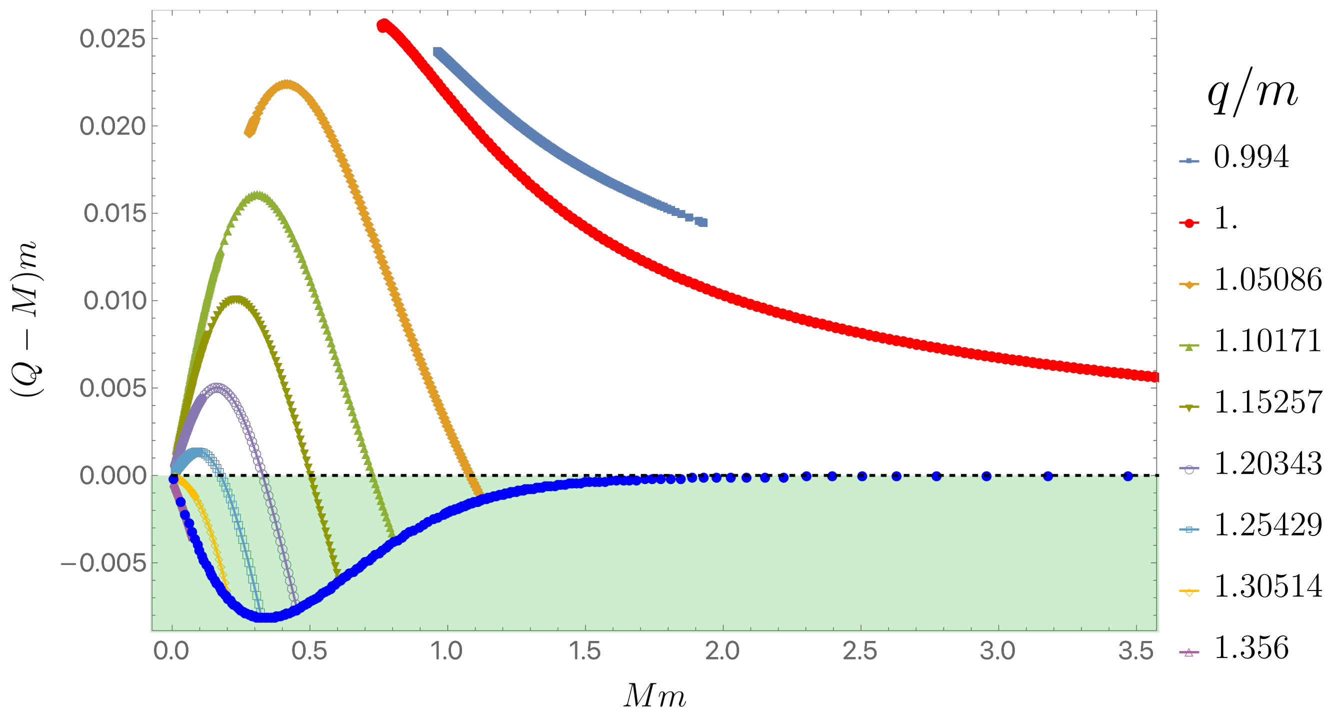


Figure 5: Black holes with $q\mu = m$ as a function of q/m, with $\alpha = 1$. When $Q > M$, these are maximal warm holes. The green shaded region denotes RN black holes, and the bottom blue curve denotes the onset of their instability when $q\mu = m$. For masses outside the range of the maximal warm holes, the extremal black hole is singular.