The Abdus Salam International Centre for Theoretical Physics

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Hidden Symmetries of Gravity

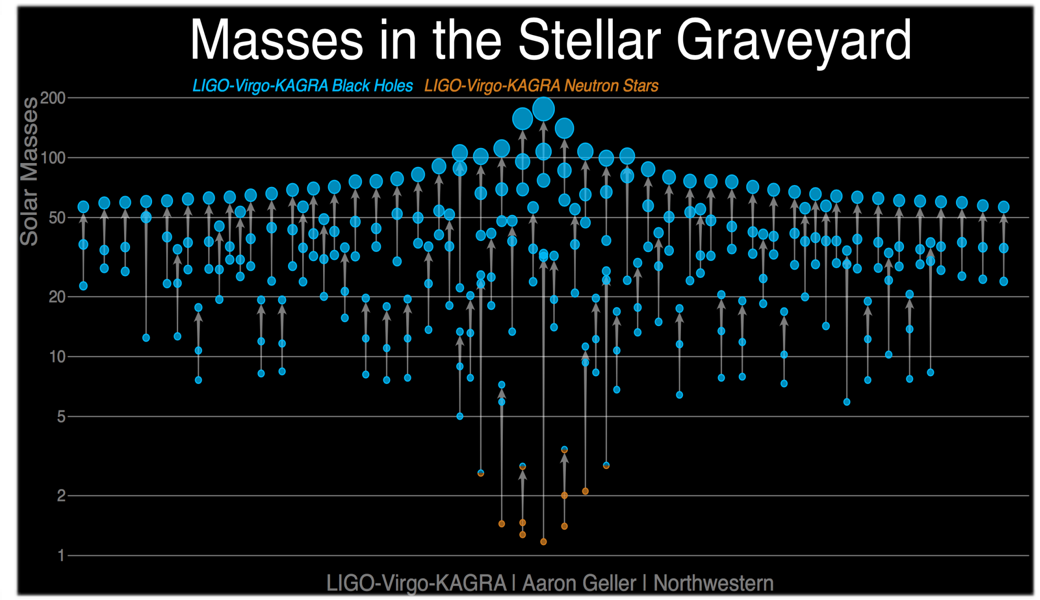
based on
arXiv: 2010.00593, 2010.00595, 2105.01069, 2111.02072, 2203.08832
and other works in progress in collaboration with
L. Hui, A. Joyce, R. Penco, A. Podo, A. Solomon, E. Trincherini

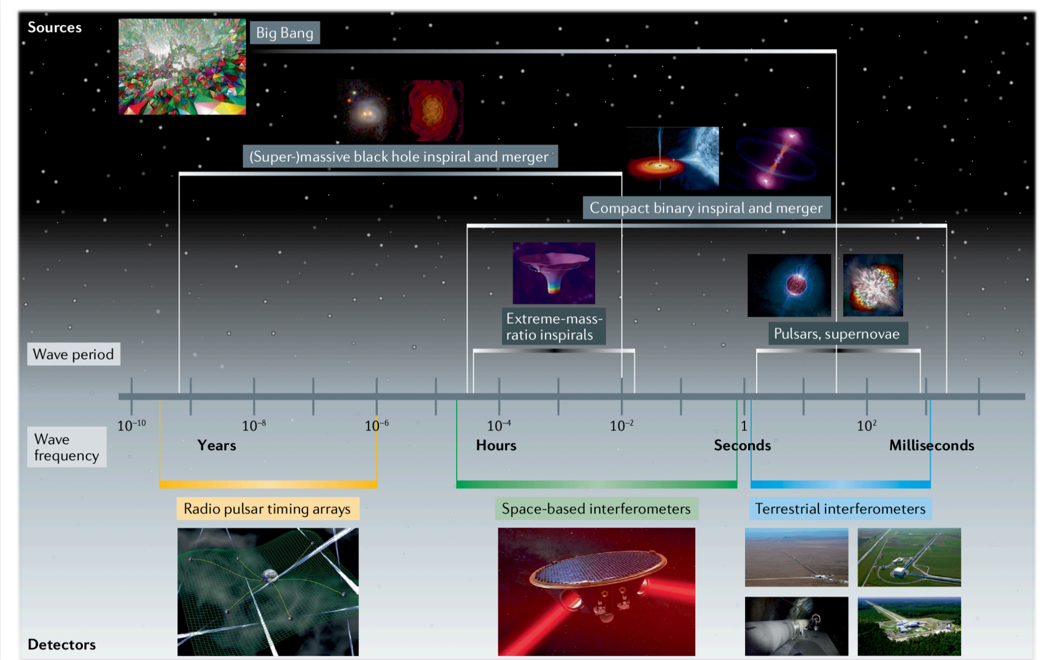
Gravitational-wave astronomy

- The past five years have witnessed a revolution in astronomy: direct detection of gravitational waves.

- The ever-increasing number of GW observations of merging binary systems is providing us with a unique opportunity to test General Relativity in the strong field regime, shed light on the fundamental aspects of gravity and black holes, probe the fundamental nature of astrophysical compact objects.

- Extraordinary scientific potential of upgraded detectors and future facilities.





[Nature Reviews Physics, 3, 344–366 (2021)]

- We are witnessing the dawn of the era of precision physics with gravitational waves.
[Berti et al. '15], [Barack et al. '18], [Cardoso and Pani '19], [Baibhav et al. '19], [Barausse et al. '20], [Perkins, Yunes and Berti '20], [Bailes et al. '21], [Berti et al. '22]…

Symmetries of black holes

- At the theoretical level, *symmetries* can help us shed light on the fundamental aspects of black holes and gravity, and constrain broad classes of theories beyond GR in a model-independent way.

Symmetries of black holes

- Black hole perturbation theory has a long history starting from the work of Regge and Wheeler and Zerilli.
- Interestingly, recent investigations suggest the subject has depths yet to be plumbed.

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Outline

I will focus mainly on two observables: the Love numbers (LNs) and the quasi-normal modes (QNMs).

I. Ladder symmetries of black holes and the vanishing of the Love numbers.
II. More on symmetries and dualities.
III. EFT of BH perturbations.

I.

Ladder Symmetries of Black Holes

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Hidden symmetries of black holes

- "The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."
(S. Chandrasekhar, in "*The mathematical theory of black holes*")

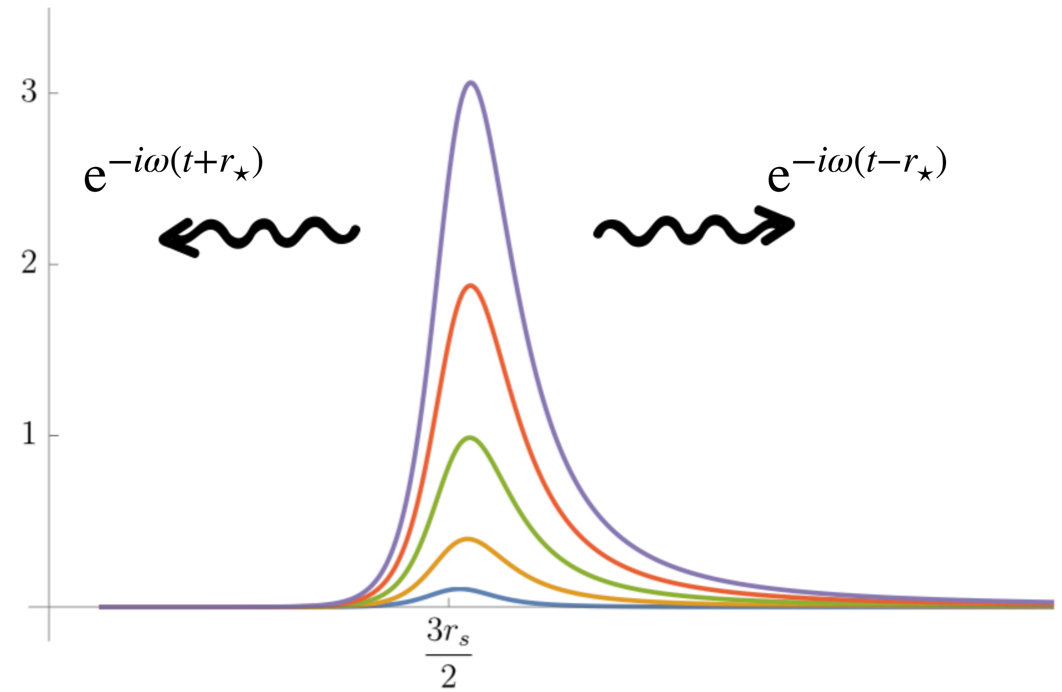
- Black holes are among the simplest and most robust objects in nature: uniquely determined by their mass and spin (and charge).

- This *simplicity* is inherited by the perturbations.

- Some aspects of this *simplicity* are well understood in terms of (hidden) symmetries of General Relativity.

Hidden symmetries of black holes

- For black holes in GR, one notable result that follows from symmetries is isospectrality, i.e. the degeneracy of the quasi-normal mode (QNM) spectra of the two d.o.f. in a gravitational wave.
- Isospectrality has been known to follow from a duality of the linearized equations of motion (a.k.a. Chandrasekhar relation) since the 1970s [Chandrasekhar '75].



- Symmetry behind the vanishing of the Love numbers unclear until last year.

Static response and tidal deformability

- The Love numbers are the coefficients encoding the (static) tidal deformability of a compact object (analogous to the electric and magnetic susceptibilities in EM).

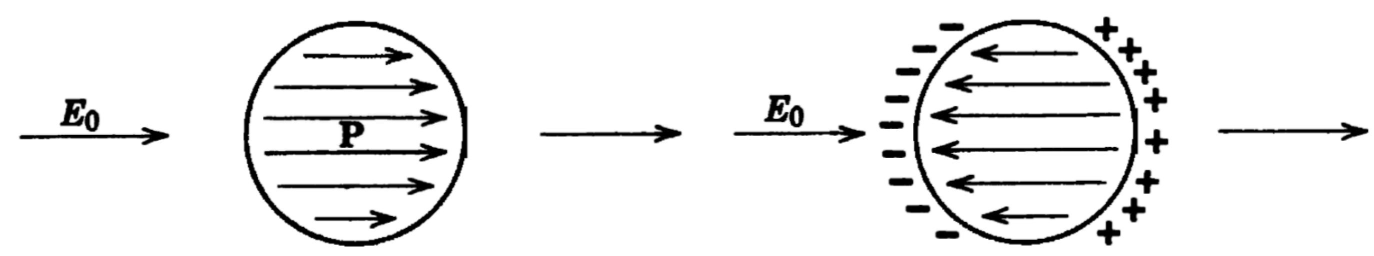


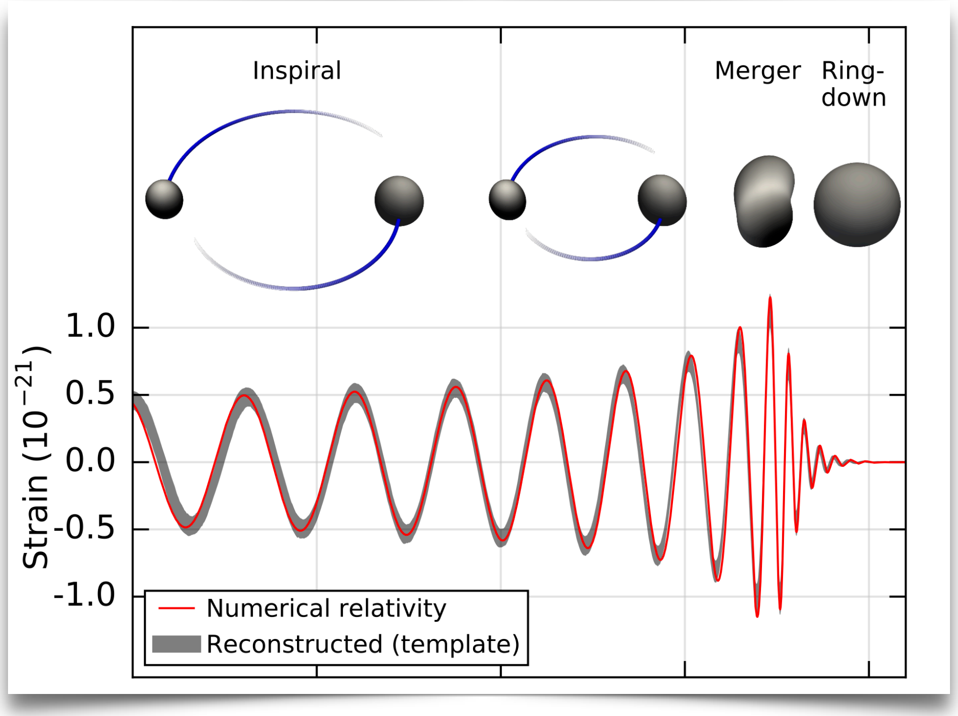
Figure 4.7 Dielectric sphere in a uniform field E_0, showing the polarization on the left and the polarization charge with its associated, opposing, electric field on the right.

- In EM we solve $\vec{\nabla}^2\Phi = 0$:

$$\Phi_{\text{ext}} = \sum_{\ell} A_\ell \left[r^\ell + k_\ell r^{-\ell-1}\right] P_\ell(\cos\theta)\,, \qquad \Phi_{\text{int}} = \sum_{\ell} B_\ell r^\ell P_\ell(\cos\theta)\,.$$

- The boundary condition at $r = +\infty$ fixes A_ℓ, while k_ℓ and B_ℓ are determined by regularity conditions across the surface (continuity of $\vec{E}_{\parallel}$ and $\vec{D}_{\perp}$).
- For instance, if $\vec{E}_0 = A_1\hat{z}$, one finds $k_{\ell=1} = -\dfrac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} r_0^3$ (ϵ_0 and ϵ are the vacuum and dielectric permittivities).
- k_ℓ are the coeffs. of the induced response.

Tidal Love numbers



- Tidal deformability affects the dynamics during the inspiral.
- An alteration in the phase of the GW signal can be used to constrain the tidal deformability of the objects.
- A novel and important channel to test GR and compact objects in the strong-field regime.
- The explicit calculation in GR (in *D*=4) shows that $k_\ell = 0$ for a black hole, as opposed to other types of compact objects.
[Fang and Lovelace '05], [Binnington and Poisson '09], [Damour and Nagar '09], [Kol and Smolkin '11].

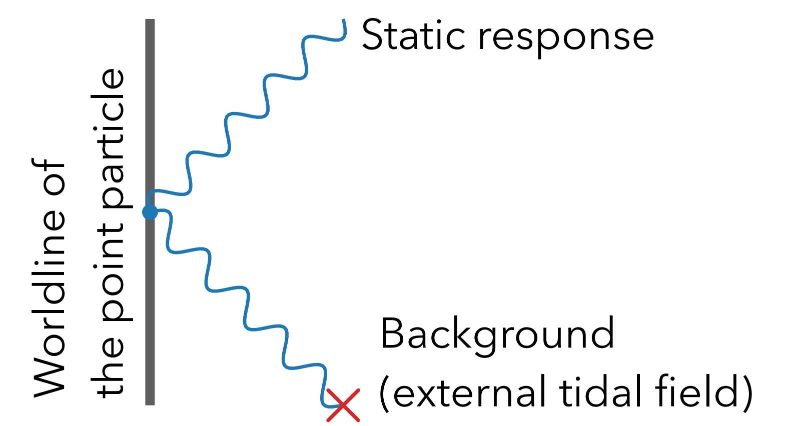
Vanishing Love Numbers

- A conceptually clean way to define the (conservative) LNs is in terms of the worldline effective action [Goldberger and Rothstein ’04, ’05, …], [Kol and Smolkin ’11], [Porto ‘16].
- At distances large compared to the characteristic size of an object, there is an effective description where the object is modeled as a point particle. Corrections due to the object’s finite size and its internal structure are encoded in higher-derivative operators in the effective theory.
- Let’s consider e.g. a scalar field around a BH:†

$$S = -\frac{1}{2}\int \mathrm{d}^4x\,(\partial\phi)^2 - M\int \mathrm{d}\tau + \int \mathrm{d}\tau\Big[-g\phi + \sum_{\ell=0}^{\infty}\frac{\lambda_\ell}{2\ell\,!}\Big(\partial_{(a_1}\cdots\partial_{a_\ell)_T}\phi\Big)^2\Big]\,.$$

- λ_ℓ are the LN coefficients.
- One generically expects: $\lambda_\ell \sim \mathcal{O}(1)r_s^{2\ell-1}$ and to find (classical) RG running.
- After matching with the UV result: $\lambda_\ell = 0$ in D=4 and no running.
- Generically non-zero in D>4 [Kol and Smolkin ’11], [Hui, Joyce, Penco, LS and Solomon ’21].

† $(\partial\phi)^2$ is the quadratic Lagrangian for the scalar in the bulk; τ is the coordinate that parameterizes the particle’s worldline.



Vanishing Love Numbers

- The vanishing of the Love numbers is a naturalness puzzle from an effective field theory perspective. [Rothstein '14], [Porto '16]

$$S = -\frac{1}{2}\int \mathrm{d}^4x\,(\partial\phi)^2 - M\int \mathrm{d}\tau + \int \mathrm{d}\tau\left[-g\phi + \sum_{\ell=0}^{\infty}\frac{\lambda_\ell}{2\ell\,!}\left(\partial_{(a_1}\cdots\partial_{a_\ell)_T}\phi\right)^2\right]$$

- Looks like something that can very likely follow from a symmetry in the theory.

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Ladder symmetries of black holes

- In [2105.01069] we showed that the vanishing of the Love numbers is the consequence of linearly realized symmetries governing static perturbations around black holes.
- The same symmetries are also connected to well-known 'no-hair' theorems of black holes in general relativity.
- Underlying the symmetries is a ladder structure that can be used to construct the full tower of solutions, and derive their general properties:
  (1) solutions that decay with radius spontaneously break the symmetries, and must diverge at the horizon;
  (2) solutions regular at the horizon respect the symmetries, and take the form of a finite polynomial that grows with radius.

  Property (1) is consistent with no hair; (1) and (2) combined imply that the Love numbers vanish.

An illustrative case: massless scalar field on Schwarzschild

- Starting point: $\partial_r(\Delta\partial_r\phi_\ell) - \ell(\ell+1)\phi_\ell = 0\,, \qquad \Delta \equiv r(r-r_s)\,, \qquad \phi(r,\theta,\varphi) = \sum_{\ell m}\phi_\ell(r)Y_{\ell m}(\theta,\varphi).$

  $H_\ell\phi_\ell = 0$ where $H_\ell \equiv -\Delta(\partial_r(\Delta\partial_r) - \ell(\ell+1))\,.$

- Define operators: $D_\ell^+ \equiv -\Delta\partial_r + \dfrac{\ell+1}{2}(r_s - 2r)\,, \qquad D_\ell^- \equiv \Delta\partial_r + \dfrac{\ell}{2}(r_s-2r)\,,$

  $$D_{\ell+1}^-D_\ell^+ - D_{\ell-1}^+D_\ell^- = \frac{(2\ell+1)r_s^2}{4}\,.$$

- "Commutators": $H_{\ell+1}D_\ell^+ = D_\ell^+H_\ell\,, \qquad H_{\ell-1}D_\ell^- = D_\ell^-H_\ell\,,$

  i.e., $D_\ell^\pm$ are *ladder operators*; e.g., $H_\ell\phi_\ell = 0 \quad \Rightarrow \quad D_\ell^\pm\phi_\ell \propto \phi_{\ell\pm1}\,.$

- Implication: $\phi_0 =$ constant is a good $\ell = 0$ solution

  climb the ladder to get: $\phi_\ell = D_{\ell-1}^+\cdots D_0^+\phi_0 \sim 1 + \ldots + r^\ell$,

  which (1) is regular at the horizon and (2) matches tidal field b.c.

- Symmetry: Define recursively $Q_\ell \equiv D_{\ell-1}^+Q_{\ell-1}D_\ell^-\,, \quad Q_0 \equiv \Delta\partial_r$

  which satisfy $[Q_\ell, H_\ell] = 0\,.$

  $\delta\phi_\ell = Q_\ell\phi_\ell$ is a (*horizontal*) symmetry of the scalar action,

  with *conserved currents* $P_\ell \equiv \Delta\partial_r(D_1^-\cdots D_\ell^-\phi_\ell), \quad \partial_rP_\ell = 0\,.$

- Use conserved currents to connect asymptotics:

  $\phi_\ell \sim r^\ell$ as $r \to +\infty \quad \leftrightarrow \quad \phi_\ell \sim$ const. as $r \to r_s$ and $P_\ell = 0\,.$

  $\phi_\ell \sim r^{-(\ell+1)}$ as $r \to +\infty \quad \leftrightarrow \quad \phi_\ell \sim \log(r - r_s)$ as $r \to r_s$ and $P_\ell \neq 0\,.$

An illustrative case: massless scalar field on Schwarzschild

[Hui, Joyce, Penco, LS and Solomon '21]

$$\phi_\ell \sim r^\ell \quad \text{as} \quad r \to +\infty \qquad \leftrightarrow \qquad \phi_\ell \sim \text{const.} \quad \text{as} \quad r \to r_s \quad \text{and} \quad P_\ell = 0\,.$$
$$\phi_\ell \sim r^{-(\ell+1)} \quad \text{as} \quad r \to +\infty \qquad \leftrightarrow \qquad \phi_\ell \sim \log(r - r_s) \quad \text{as} \quad r \to r_s \quad \text{and} \quad P_\ell \neq 0\,.$$

- The vanishing of the Love numbers follows from two facts: (1) the purely decaying solution ($\sim 1/r^{\ell+1}$ at large r) is divergent at the horizon, and (2) the solution that is regular at the horizon is a finite polynomial going as $\sim 1 + r + \ldots + r^\ell$.

- The growing branch respects the symmetry, while the decaying branch spontaneously breaks the symmetry.

- Fact (1) is consistent with the no-hair theorem (a black hole cannot sustain static, scalar profile that decays at infinity [Bekenstein '72]).

From Schwarzschild to AdS, with Love

[Hui, Joyce, Penco, LS and Solomon '21]

- The symmetry has a geometric origin: it arises from the (E)AdS isometries of a dimensionally reduced black hole spacetime.

  Let's consider a static scalar ϕ in a Schwarzschild background,

$$S = \frac{1}{2}\int \mathrm{d}\theta \mathrm{d}\varphi \mathrm{d}r \sqrt{g}\, \phi \,\Box\, \phi\,, \qquad \mathrm{d}s^2 = \mathrm{d}r^2 + \Delta\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right).$$

After a Weyl rescaling, the metric becomes EAdS_3 with

$$\tilde{g}_{ij} = \Omega^2 g_{ij}\,, \qquad \tilde{\phi} = \Omega^{-\frac{1}{2}}\phi\,, \qquad \text{where} \qquad \Omega \equiv L^2/\Delta\,,$$

$$S = \frac{1}{2}\int \mathrm{d}^3x\sqrt{\tilde{g}}\left(\tilde{\phi}\,\tilde{\Box}\,\tilde{\phi} + \frac{r_s^2}{4L^4}\tilde{\phi}^2\right), \qquad \mathrm{d}\tilde{s}^2 = \mathrm{d}r_\star^2 + \frac{4L^4}{r_s^2}\sinh^2\!\left(\frac{r_\star r_s}{2L^2}\right)\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

where $\mathrm{d}r_\star = (L^2/\Delta)\mathrm{d}r$. The space has 6 Killing vectors: 3 rotations and 3 translations (or "boosts"). The translation that mixes $r_\star$ and θ acts on the original ϕ as

$$\delta\phi = -2\Delta\cos\theta\partial_r\phi + (r_s - 2r)\partial_\theta(\sin\theta\,\phi)$$

or, equivalently,

$$\delta\phi_\ell = c_{\ell+1}D^-_{\ell+1}\phi_{\ell+1} - c_\ell D^+_{\ell-1}\phi_{\ell-1}\,.$$

From Schwarzschild to AdS, with Love

[Hui, Joyce, Penco, LS and Solomon '21]

- At large r, $\delta\phi$ reduces to a SCT, $\delta\phi = c_i\big(x^i - \vec{x}^{\,2}\partial^i + 2x^i\vec{x}\cdot\vec{\partial}\,\big)\phi$.
- We claim that this is the sought-after infrared symmetry that forbids Love number (and hair) couplings in the point-particle effective action.

Hidden symmetries at finite frequency

[Hui, Joyce, Penco, LS and Solomon '22]

- In binary systems, the induced deformation in a compact object, as well as the perturbing tidal field of the companion, is never exactly static.
  Is it possible to extend the ladder symmetries beyond the static limit?

- The scalar action is

$$S = \frac{1}{2}\int \mathrm{d}t\mathrm{d}r\mathrm{d}\Omega_{S^2}\left[\frac{r^4}{\Delta}(\partial_t\phi)^2 - \Delta(\partial_r\phi)^2 + \phi\,\nabla^2_{\Omega_{S^2}}\phi\right].$$

- Define the *near-zone* approximation by replacing $(r^4/\Delta)\partial_t^2\phi$ with $(r_s^4/\Delta)\partial_t^2\phi$.

- This has the virtue of preserving the correct singularity as $r \to r_s$, while still accurately capturing the dynamics at larger r, as long as $\omega r \ll 1$.

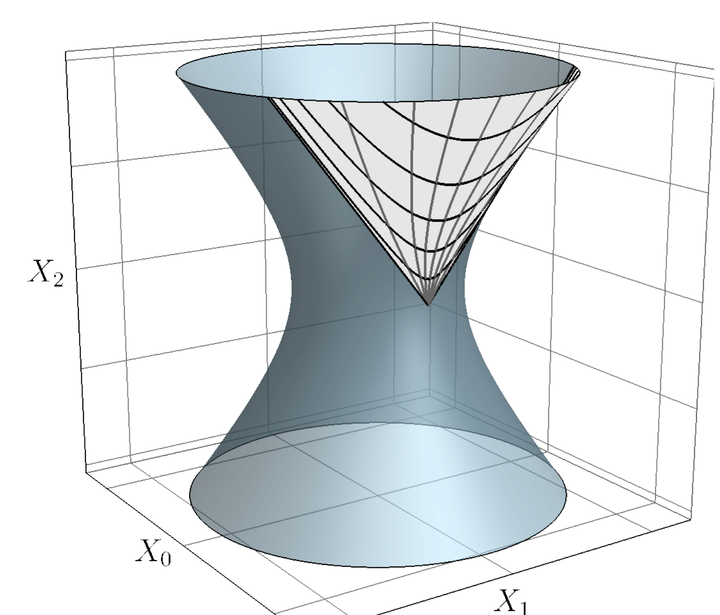
Hidden symmetries at finite frequency

[Hui, Joyce, Penco, LS and Solomon '22]

- In this limit, the scalar action is the same as that of a massless scalar minimally coupled to an *effective near-zone metric*:

$$\mathrm{d}s^2_{\text{near-zone}} = -\frac{\Delta}{r_s^2}\mathrm{d}t^2 + \frac{r_s^2}{\Delta}\mathrm{d}r^2 + r_s^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right).$$

- This metric has the following main properties:
_ only extension of dynamics at finite ω that retains the (static) ladder generators;
_ is a conformally-flat $\mathrm{AdS}_2 \times S^2$ spacetime ($\Rightarrow$ 6 KVs + 9 CKVs).



Hidden symmetries at finite frequency

[Hui, Joyce, Penco, LS and Solomon '22]

- The 6 KVs and 9 CKVs are:

$$T = 2r_s\,\partial_t$$
$$L_\pm = e^{\pm t/2r_s}(2r_s\,\partial_r\sqrt{\Delta}\partial_t \mp \sqrt{\Delta}\partial_r)$$
$$J_{23} = \partial_\varphi$$
$$J_{12} = \cos\varphi\,\partial_\theta - \cot\theta\sin\varphi\,\partial_\varphi$$
$$J_{13} = \sin\varphi\,\partial_\theta + \cot\theta\cos\varphi\,\partial_\varphi$$

$$J_{01} = -\frac{2\Delta}{r_s}\cos\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\sin\theta\,\partial_\theta$$
$$J_{02} = -\cos\varphi\left[\frac{2\Delta}{r_s}\sin\theta\,\partial_r + \frac{\partial_r\Delta}{r_s}\left(\frac{\tan\varphi}{\sin\theta}\partial_\varphi - \cos\theta\partial_\theta\right)\right]$$
$$J_{03} = -\sin\varphi\left[\frac{2\Delta}{r_s}\sin\theta\,\partial_r - \frac{\partial_r\Delta}{r_s}\left(\frac{\cot\varphi}{\sin\theta}\partial_\varphi + \cos\theta\partial_\theta\right)\right]$$
$$K_\pm = e^{\pm t/2r_s}\frac{\sqrt{\Delta}}{r_s}\cos\theta\left(\frac{r_s^3}{\Delta}\partial_t \mp \partial_r\Delta\partial_r \mp 2\tan\theta\partial_\theta\right)$$
$$M_\pm = e^{\pm t/2r_s}\cos\varphi\left[\frac{r_s^2}{\sqrt{\Delta}}\sin\theta\partial_t \mp \frac{\sqrt{\Delta}\partial_r\Delta\sin\theta}{r_s}\partial_r \pm \frac{2\sqrt{\Delta}}{r_s}\cos\theta\partial_\theta \mp \frac{2\sqrt{\Delta}}{r_s}\frac{\tan\varphi}{\sin\theta}\partial_\varphi\right]$$
$$N_\pm = e^{\pm t/2r_s}\sin\varphi\left[\frac{r_s^2}{\sqrt{\Delta}}\sin\theta\partial_t \mp \frac{\sqrt{\Delta}\partial_r\Delta\sin\theta}{r_s}\partial_r \pm \frac{2\sqrt{\Delta}}{r_s}\cos\theta\partial_\theta \pm \frac{2\sqrt{\Delta}}{r_s}\frac{\cot\varphi}{\sin\theta}\partial_\varphi\right]$$

- Different perspective on the vanishing of the LNs proposed by [Charalambous, Dubovsky and Ivanov '21].
- This unifies the different sets of symmetries.
- Only T, J_{ij} and J_{0i} remain good symmetries in the static limit ($\omega = 0$).
- J_{01} recovers precisely the ladders: $\delta\phi = \xi^\mu\partial_\mu\phi + \frac{1}{4}\nabla_\mu\xi^\mu\phi$, or equivalently $\delta\phi_\ell = c_{\ell+1}D^-_{\ell+1}\phi_{\ell+1} - c_\ell D^+_{\ell-1}\phi_{\ell-1}$, $D^+_\ell \equiv -\Delta\partial_r + \frac{\ell+1}{2}(r_s - 2r)$ and $D^-_\ell \equiv \Delta\partial_r + \frac{\ell}{2}(r_s - 2r)$.

Hidden symmetries at finite frequency

[Hui, Joyce, Penco, LS and Solomon '22]

- The ladder generators are useful because they allow to recursively define an on-shell conserved charge P_ℓ at each ℓ (*horizontal charge*).

- Evaluating P_ℓ on the solution with the correct boundary condition at the horizon, one recovers the induced multipoles,

$$P_\ell \propto i\omega \prod_{k=1}^{\ell} (k^2 + 4\omega^2 r_s^2)$$

without the need of the explicit solution or any analytic continuation.

Ladder in Kerr: static limit

[Hui, Joyce, Penco, LS and Solomon '21]

- The previous algebraic ladder structure has a direct analogue in a Kerr background:
$$\mathrm{d}s^2 = -\frac{\rho^2 - r_s r}{\rho^2}\mathrm{d}t^2 - \frac{2ar_s r\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 + \frac{(r^2+a^2)^2 - a^2\Delta\sin^2\theta}{\rho^2}\sin^2\theta\mathrm{d}\varphi^2$$
with $\rho^2 \equiv r^2 + a^2\cos^2\theta$ and $\Delta \equiv r^2 - rr_s + a^2$.
- The static Klein-Gordon eq., $\partial_r(\Delta\partial_r\phi_\ell) + \frac{a^2m^2}{\Delta}\phi_\ell - \ell(\ell+1)\phi_\ell = 0$, has both *ladder* and *horizontal* symmetries.
- The ladder symmetries $D^\pm_\ell$ descend from a CKV of the 3D-static metric:
$$\mathrm{d}s_K^2 = \frac{\rho^2 - rr_s}{\Delta}\left(\mathrm{d}r^2 + \Delta\mathrm{d}\theta^2 + \frac{\Delta^2\sin^2\theta}{\rho^2 - rr_s}\mathrm{d}\varphi^2\right).$$
- $\xi^\mu = (0, \Delta\cos\theta, \frac{1}{2}(2r - r_s)\sin\theta, 0)$ is the CKV that induces
$\delta\phi = \xi^\mu\partial_\mu\phi + \frac{1}{2}(2r - r_s)\cos\theta\,\phi \quad \Rightarrow \quad \delta\phi_\ell = c_{\ell+1}D^-_{\ell+1}\phi_{\ell+1} - c_\ell D^+_{\ell-1}\phi_{\ell-1}$,
- The conserved charges P_ℓ associated with the horizontal symmetries, evaluated for the "growing branch", are non-zero (and imaginary), unlike in the Schwarzschild case:
$$P_\ell \propto iq\prod_{k=1}^{\ell}\left(k^2 + 4q^2\right), \qquad q \equiv \frac{am}{r_+ - r_-},$$
which reproduces the dissipative response [Le Tiec and Casals '20].

Ladder in Kerr: near-zone regime

[Hui, Joyce, Penco, LS and Solomon '22]

- Different near-zone approximations at finite ω for the Kerr metric.
- Some examples are: [Maldacena and Strominger '97], [Castro, Maloney, and Strominger '10], [Lowe and Skanata '11], [Charalambous, Dubovsky, and Ivanov '21].
- We are again interested in the near zone that retains the ladder symmetries of the static regime, i.e.,

$$\partial_r(\Delta\partial_r\phi_\ell) + \left[\frac{r_s^2 r_+^2}{\Delta}(\omega - m\Omega_+)^2 - \ell(\ell+1)\right]\phi_\ell = 0\,, \qquad \Omega_+ \equiv \frac{a}{r_s r_+}\,, \quad \Delta \equiv (r - r_-)(r - r_+).$$

- This is the equation of motion of a massless scalar propagating in the effective near-zone metric

$$\mathrm{d}s^2_{\text{near-zone}} = -\frac{\Delta - a^2\sin^2\theta}{r_s r_+}\mathrm{d}t^2 - 2a\sin^2\theta\,\mathrm{d}t\mathrm{d}\varphi + \frac{r_s r_+}{\Delta}\mathrm{d}r^2 + r_s r_+\mathrm{d}\Omega^2_{S^2}\,.$$

- This takes the form of the Schwarzschild near zone after a coordinate redefinition.
$\Rightarrow$ It is $\mathrm{AdS}_2 \times S^2$ with 6 KVs and 9 CKVs (which include our previous ladder generators).

Ladder in Spin: From Scalar to Vector and Tensor

[Hui, Joyce, Penco, LS and Solomon '21, '22]

- Consider for simplicity the static limit, $\omega = 0$.
- Ladder operators in the spin, $E_s^\pm$, raise and lower s in the Teukolsky equation ($E_s^\pm \phi_\ell^{(s)} = \phi_\ell^{(s\pm1)}$),

$$\partial_r\Big(\Delta\partial_r\phi_\ell^{(s)}\Big) + s(2r - r_s)\partial_r\phi_\ell^{(s)} + \Big(\frac{a^2m^2 + is(2r - r_s)am}{\Delta} - (\ell - s)(\ell + s + 1)\Big)\phi_\ell^{(s)} = 0\,,$$

- Allow to extend the previous results from the scalar to vector and tensor fields.

- $E_s^\pm$ are related to what are known as Teukolsky–Starobinsky identities. In Chandrasekhar's notation,

$$\phi^{(-1)} = \Delta\mathscr{D}_0^\dagger\mathscr{D}_0^\dagger\Delta\phi^{(1)}, \quad \phi^{(1)} = \mathscr{D}_0\mathscr{D}_0\phi^{(-1)}, \quad \phi^{(-2)} = \Delta^2\mathscr{D}_0^\dagger\mathscr{D}_0^\dagger\mathscr{D}_0^\dagger\mathscr{D}_0^\dagger\Delta^2\phi^{(2)}, \quad \phi^{(2)} = \mathscr{D}_0\mathscr{D}_0\mathscr{D}_0\mathscr{D}_0\phi^{(-2)},$$

where $\mathscr{D}_0 \equiv \partial_r + i[am - \omega(r^2 + a^2)]/\Delta$.

The new twist we are adding is: in the static limit, we can truncate these operations, enabling us to increment s by unity, $E_s^\pm\phi_\ell^{(s)} = \phi_\ell^{(s\pm1)}$.

II.

More on Symmetries and Dualities

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More hidden symmetries and dualities

- Isospectrality and the vanishing of the Love numbers are fragile properties of gravity that are usually broken in theories beyond GR.
  [Cardoso, Kimura, Maselli, Berti, Macedo and McManus '19], [Cardoso and Pani '19]

- Studying symmetries of gravity beyond GR and massless spin-2 perturbations:
  (1) allows us to better understand why black holes are so robust and GR so special;
  (2) in order to test GR with GW observations, it is important to understand whether there are other theories that share the same properties and yield similar predictions for the observables.

- Hints of isospectrality for partially massless spin-2 perturbations–analogously to massless spin-2 perturbations in GR. [Brito, Cardoso and Pani '13]

Partially massless spin-2 perturbations in S(A)dS

[Rosen and LS '20]

- Partially massless fields are special irreducible representations of massive spinning particles on Einstein spacetimes.
- For SdS, when $m^2 = \frac{2\Lambda}{3}$ (in D=4), the massive spin-2 field inherits a gauge symmetry that allows to remove the helicity-0 component.
- In D=4, the number of propagating d.o.f. is 4 instead of 5: 2 even and 2 odd.

$$\frac{d^2}{dr_\star^2}\vec{h}_\pm + \mathbb{W}_\pm \cdot \vec{h}_\pm = 0\,, \qquad \mathbb{W}_{\pm\,ij} = \omega^2\delta_{ij} - \mathbb{V}_{\pm\,ij}\,.$$

- For generic ℓ, the equations are coupled and proving isospectrality amounts to finding a duality transformation relating $\vec{h}_+$ and $\vec{h}_-$.
- One can quite easily find a solution for $\ell = 1$ [Rosen and LS '20]

$$h_+ = f(r)\partial_r h_- + F(r)h_-\,, \qquad F = \frac{18f(r)}{9r - \Lambda r^3} + \frac{2}{9}r_s\Lambda\,, \qquad f(r) = 1 - \frac{r_s}{r} - \frac{\Lambda r^2}{3}\,.$$

- The symmetry is regular and finite at the horizons $\Rightarrow$ isospectrality (only in SdS).
- This is another example of even-odd duality, different from the Chandrasekhar relation in GR.

Partially massless spin-2 perturbations in SdS

[Rosen and LS '20]

- How about higher-dimensional SdS spacetimes?
- Isospectrality for massless spin-2 perturbations is known to be broken in D>4 [Konoplya '03]. The same is true for partially massless fields [Rosen and LS '20].
- Consider, e.g., the extremal or the eikonal limit. The equations admit analytic solutions for the QNMs:

$$\frac{\omega}{\kappa} = -i\left(n+\frac{1}{2}\right) + \sqrt{\frac{\bar{r}^2 U(\bar{r})}{D-3} - \frac{1}{4}}\,, \qquad n = 0, 1, 2\ldots$$

where $\kappa = \Lambda(r_c - r_b)/(D-2)$ and

$$U(\bar{r}) = \begin{cases} m^2 + \dfrac{\ell(\ell+D-3) - D + 2}{\bar{r}^2} - \dfrac{D-4}{\bar{r}^2} & \text{(even)}\,, \\ m^2 + \dfrac{\ell(\ell+D-3) - D + 2}{\bar{r}^2} & \text{(odd)}\,, \\ m^2 + \dfrac{\ell(\ell+D-3)}{\bar{r}^2} & \text{(tensor)}\,. \end{cases}$$

with $m^2 = \frac{2\Lambda}{D-1}$.

Dualities and Love numbers

- Dualities like the Chandrasekhar relation have implications not only for the QNMs but also for the LNs.

- Let's take the Chandrasekhar relation in GR:
$$h_+ = f(r)\partial_r h_- + \left(\frac{3r_s(r_s - r)}{r^2(2\lambda r + 3r_s)} - \frac{2\lambda(\lambda+1)}{3r_s}\right)h_- \,.$$
where $f = 1 - \frac{r_s}{r}$ and $\lambda \equiv (\ell - 1)(\ell + 2)/2$.

- In the asymptotic regions ($r_\star \to \pm\infty$), $h_+ = \text{constant} \times h_-$. This implies that $\lambda_\ell^{(E)}$ and $\lambda_\ell^{(B)}$ are related nontrivially.

- In particular, since the constant is finite, whenever $\lambda_\ell^{(E)}$ vanishes, $\lambda_\ell^{(B)}$ must be zero as well, and viceversa.

- In D>4, there are cases in which $\lambda_\ell^{(B)} = 0$, but $\lambda_\ell^{(E)} \neq 0$ [Hui, Joyce, Penco, LS and Solomon '20].
A symmetry may exist, but it diverges at the boundary.

III.

Effective Field Theory of BH perturbations

Black holes beyond GR

- One main goal of gravitational-wave astronomy is to test General Relativity.
- Different ways to test GR:
  (1) Null test: are two numbers (mass and spin) sufficient to reconcile with observations?
  (2) Concrete models help guide our thinking on the form of possible deviations and the associated model testing.
  (3) Parametrize the deviations from general relativistic expectations in a model-independent way:
    _ this can take the form of a "phenomenological parametrization", or
    [Loutrel, Yunes and Pretorius '14], [Cardoso et al. '19], [McManus et al. '19], [Glampedakis and Silva '19], [Maselli et at. '19]…
    _ parametrization at the level of an Effective Field Theory (EFT).
    [Endlich et al. '17], [Cardoso et al '18], [Franciolini, Hui, Penco, LS and Trincherini '18], [Hui, Podo, LS and Trincherini '21]…
- Clear signatures of new physics at the level, e.g., of the QNM spectrum include: the presence of extra modes in addition to the GW polarizations, the shifting of the ringdown frequencies, the breaking of isospectrality, the presence of 'echoes' in the GW signal.

Black holes beyond GR

- Usually hard in more phenomenological parametrizations to see which variations of the parameters correspond to theories respecting physical principles like locality, Lorentz invariance, the equivalence principle, etc.
- A parametrization at the level of an EFT makes instead more transparent which types of deformations correspond to theories that respect physical principles, and makes it easier to connect UV theories with observations through a systematic matching procedure.
- Mapping between the two approaches can be very complicated.

EFT for BH perturbations

Strategy:

- Fix the low-energy degrees of freedom:
  (1) if only h_+ and $h_\times$, include powers of the Riemann tensor;
  [Endlich et al. '17], [Cardoso, Kimura, Maselli, Senatore '18], [Cano et al. '20]…
  (2) possibility of additional (scalar, vector, …) modes.
  [Franciolini, Hui, Penco, LS and Trincherini '18], [Hui, Podo, LS and Trincherini '21]

- Include all possible operators, up to a certain order in derivatives, compatible with the symmetries.

EFT for BH perturbations

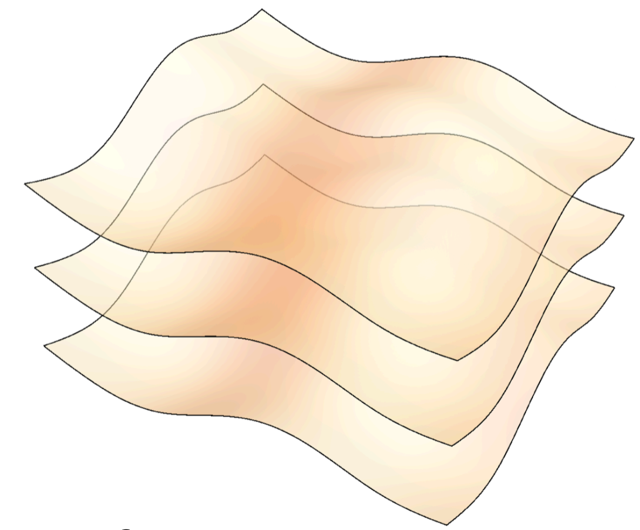
- I'll focus on case (2):
  [Franciolini, Hui, Penco, LS and Trincherini '18], [Hui, Podo, LS and Trincherini '21]

$$S_{\mathrm{EFT}} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\Phi)^2 + \alpha\, M_{\mathrm{Pl}}\Phi\mathscr{G}_{\mathrm{GB}} + c\frac{(\partial\Phi)^2\,\Box\,\Phi}{\Lambda^3} + c_{n,m}\frac{\nabla^n\Phi^m}{\Lambda^{n+m-4}} + \dots\right]$$

- $\alpha \lesssim (1.7\,\mathrm{km})^2$ [Witek et al. '18], [Perkins et al. '19, '21]
- $\Lambda \sim (M_{\mathrm{Pl}}/\alpha)^{1/3}$ for all the leading interactions involving the scalar Φ to become strong at the same scale.
- Under these conditions, $(\partial\Phi)^2\,\Box\,\Phi$ can induce *order unity* changes to the observables compared to $\Phi\mathscr{G}_{\mathrm{GB}}$ (that is, compared e.g. to [Sotiriou and Zhou '14], [Blázquez-Salcedo '16]).
- An EFT is a convenient model-independent framework that allows to catch them all in a single shot.

EFT for BH QNMs in scalar-tensor theories

[Franciolini, Hui, Penco, LS and Trincherini '18]
[Hui, Podo, LS and Trincherini '21]



- The BH is equipped with an extra scalar d.o.f. (hair)

$$\Phi(x) = \bar{\Phi}(r) + \pi(x) \, .$$

- In the unitary gauge ($\pi = 0$), the most general action for GR + scalar field is

$$S = \int d^4x \sqrt{-g}\, \mathscr{L}\left(g_{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}, R_{\mu\nu\rho\sigma}, g^{rr}, K_{\mu\nu}, \nabla_\mu ; r\right) .$$

(π is the goldstone boson that nonlinearly realizes spatial translations.)

$$\begin{aligned} S = \int d^4x \sqrt{-g} \Bigg[& \frac{M_{Pl}^2}{2} R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}_{\mu\nu} K^{\mu\nu} \\ & + M_2^4(r)(\delta g^{rr})^2 + M_3^3(r)\delta g^{rr}\delta K + M_4^2(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu} + M_5^2(r)(\partial_r \delta g^{rr})^2 + M_6^2(r)(\partial_r\delta g^{rr})\delta K \\ & + M_7(r)\bar{K}_{\mu\nu}(\partial_r \delta g^{rr})\delta K^{\mu\nu} + M_8^2(r)(\partial_a \delta g^{rr})^2 + M_9^2(r)(\delta K)^2 + M_{10}^2(r)\delta K_{\mu\nu}\delta K^{\mu\nu} + M_{11}(r)\bar{K}_{\mu\nu}\delta K \delta K^{\mu\nu} \\ & + M_{12}(r)\bar{K}_{\mu\nu}\delta K^{\mu\rho}\delta K^{\nu}{}_{\rho} + \lambda(r)\bar{K}_{\mu\rho}\bar{K}^{\rho}{}_{\nu}\delta K \delta K^{\mu\nu} + M_{13}^2(r)\delta g^{rr}\delta\hat{R} + M_{14}(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta\hat{R}^{\mu\nu} + \ldots \Bigg] . \end{aligned}$$

EFT for BH QNMs in scalar-tensor theories

[Franciolini, Hui, Penco, LS and Trincherini '18]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}_{\mu\nu} K^{\mu\nu} \right.$$
$$+ M_2^4(r)(\delta g^{rr})^2 + M_3^3(r)\delta g^{rr}\delta K + M_4^2(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu} + M_5^2(r)(\partial_r \delta g^{rr})^2 + M_6^2(r)(\partial_r \delta g^{rr})\delta K$$
$$+ M_7(r)\bar{K}_{\mu\nu}(\partial_r \delta g^{rr})\delta K^{\mu\nu} + M_8^2(r)(\partial_a \delta g^{rr})^2 + M_9^2(r)(\delta K)^2 + M_{10}^2(r)\delta K_{\mu\nu}\delta K^{\mu\nu} + M_{11}(r)\bar{K}_{\mu\nu}\delta K \delta K^{\mu\nu}$$
$$\left. + M_{12}(r)\bar{K}_{\mu\nu}\delta K^{\mu\rho}\delta K^{\nu}{}_{\rho} + \lambda(r)\bar{K}_{\mu\rho}\bar{K}^{\rho}{}_{\nu}\delta K \delta K^{\mu\nu} + M_{13}^2(r)\delta g^{rr}\delta \hat{R} + M_{14}(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta \hat{R}^{\mu\nu} + \ldots \right].$$

- The coefficients $M_i(r)$ encode all possible deviations from GR for BH with scalar hair.
- If the scalar-matter coupling is gravitational (or bigger), the most relevant signature is the scalar mode itself.
- If the scalar-matter coupling is absent or very weak, potential signatures include:
  _ modified QNMs while preserving isospectrality;
  _ breaking of isospectrality;
  _ even-odd mixing (if pseudo-scalar).
- `Tunings' among the couplings allow to recover isospectrality.
  [Hui, Podo, LS and Trincherini '21], [Cardoso et al. '19]

EFT for BH QNMs in scalar-tensor theories

- Generalized to slowly-rotating black holes in [Hui, Podo, LS and Trincherini '21].

- How to connect deviations from the GR QNM spectrum to the theory?
  _ 'Phenomenological' parametrization: reconstruct *finite*-series potential from modified GR QNMs. [Volkel, Franchini and Barausse '22]
  _ EFT approach: `WKB'-like approach can connect the deviations in the QNM spectrum to (derivatives of) the effective couplings computed at the light ring. [Franciolini, Hui, Penco, LS and Trincherini '18]

EFT for BH QNMs in scalar-tensor theories

- In theories beyond GR the equations for the perturbations are often coupled:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r_\star^2} + \omega^2 - \mathbb{V}(r)\right) \cdot \overrightarrow{h} = 0\,.$$

- A generalized `WKB'-like approach to estimate the QNMs consists in expanding around the maxima of the eigenvalues of the potential matrix [Hui, Podo, LS and Trincherini, in preparation].

Conclusions

Conclusions and open directions

- Symmetries and EFTs are key tools that can shed light on the fundamental aspects of gravity and compact objects, and constrain broad classes of theories beyond GR.

- Isospectrality and the vanishing of the Love numbers in GR are examples of properties that follow from (hidden) symmetries in the theory.

Conclusions and open directions

Some open questions:

_ Consequences of the new symmetry generators for other observables (e.g., the QNMs)?

_ The symmetry generators involving $e^{\pm t/2r_s}$ act as `ladders in ω'. They generate solutions outside the near-zone regime. Still, they reproduce the QNM spectrum at large *n* (e.g. [Bertini, Cacciatori and Klemm '11]). Can we make this robust by finding ladder symmetries at *any* ω?

_ $D_\ell^\pm$ are related to SCTs on flat space. How about the *horizontal* symmetry?
(See also [Achour, Livine, Mukohyama and Uzan '22] for a recent development on the *horizontal* symmetry.)

_ Isospectrality of partially massless field on Kerr spacetime?

_ Characterize more systematically how different operators in the EFT affect the observables.

_ Can we use (nonlinearly realized) symmetries to say something about the nonlinear regime in the merger?