Black-hole microstate spectroscopy: ringdown, quasinormal modes, and echoes

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PRD104(2021)6, 066021

Outline

1. Introduction
2. Black hole ringdown
3. Exotic compact object ringdown
4. Numerical method
5. Results
6. Summary & Discussion

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1. Introduction
   - Gravitational wave astronomy
   - Black hole vs Exotic compact object
   - Fuzzball proposal
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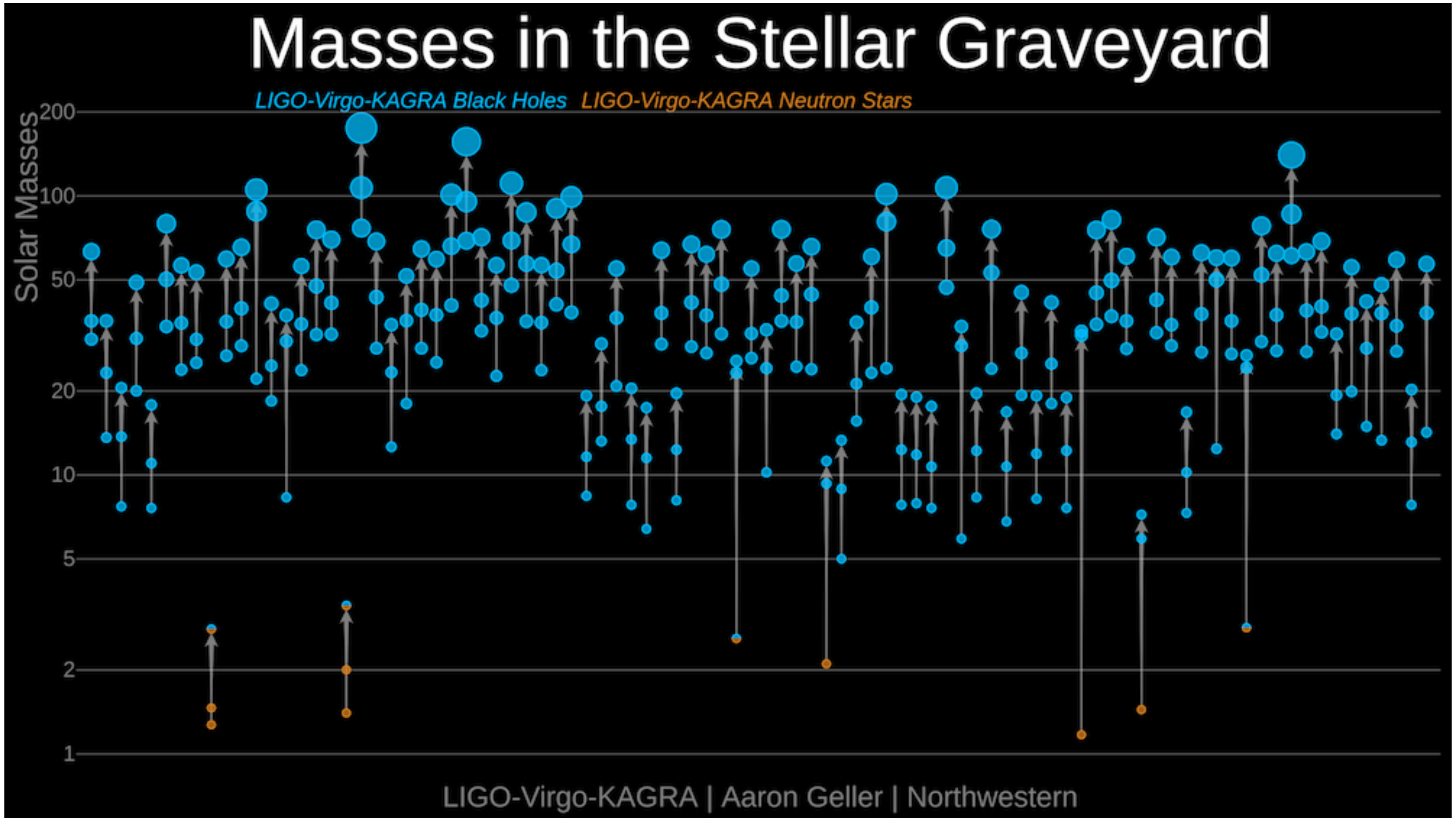
Gravitational wave astronomy



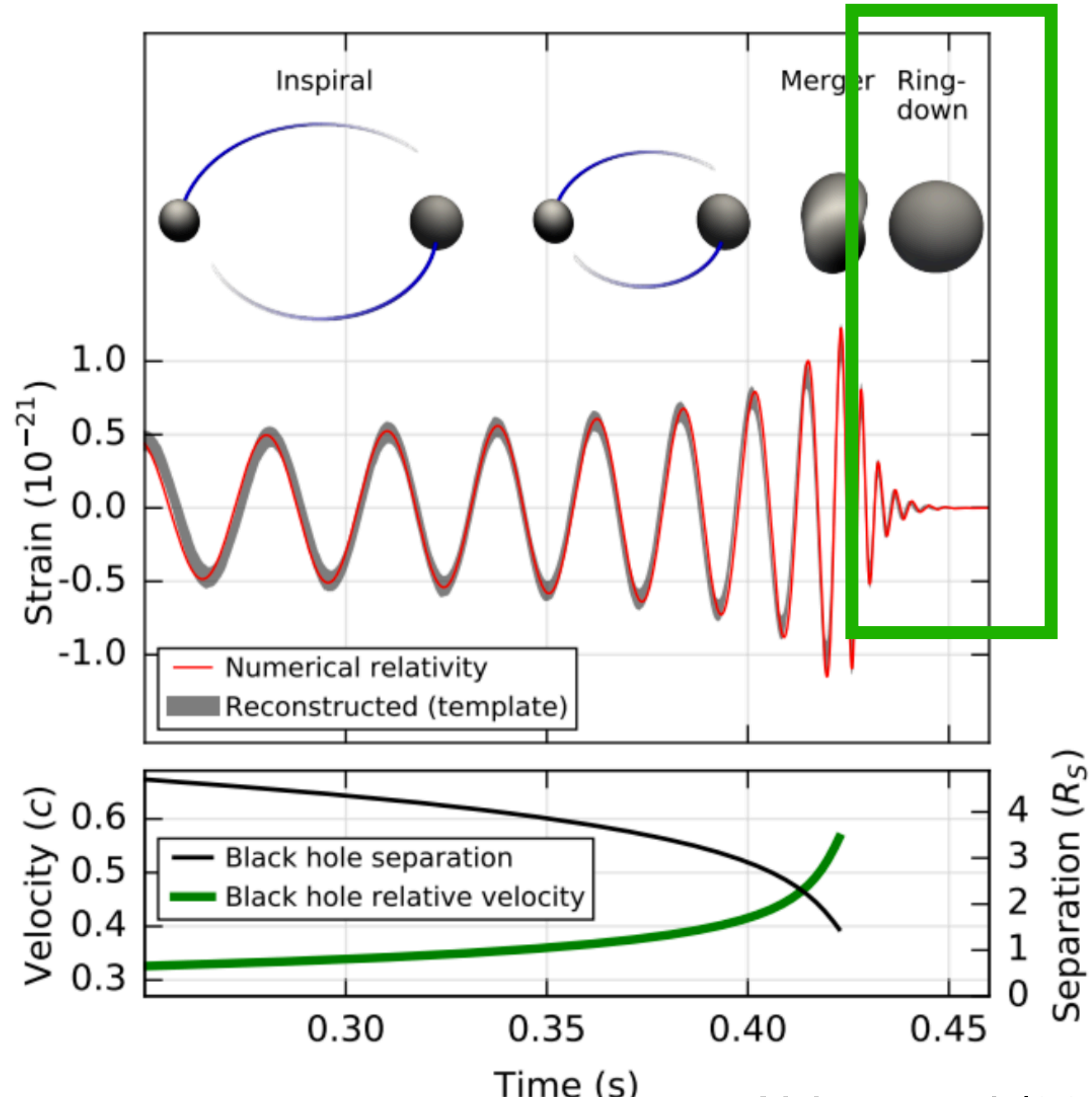
From O3B catalog

- GWs give us hints of ….
  - Origin of black holes/ binaries, primordial black hole, early universe, equation of state of neutron star, BH as a particle detector, test of gravitational theory, et al

GW from binary black hole

- Three different stages
  - Inspiral
  - Merger
  - Ring-down

- How to describe ??.
  - Post-Newtonian
  - BH perturbation theory
  - Numerical relativity
  - … et al



Abbot et al (2016)

GW from binary black hole

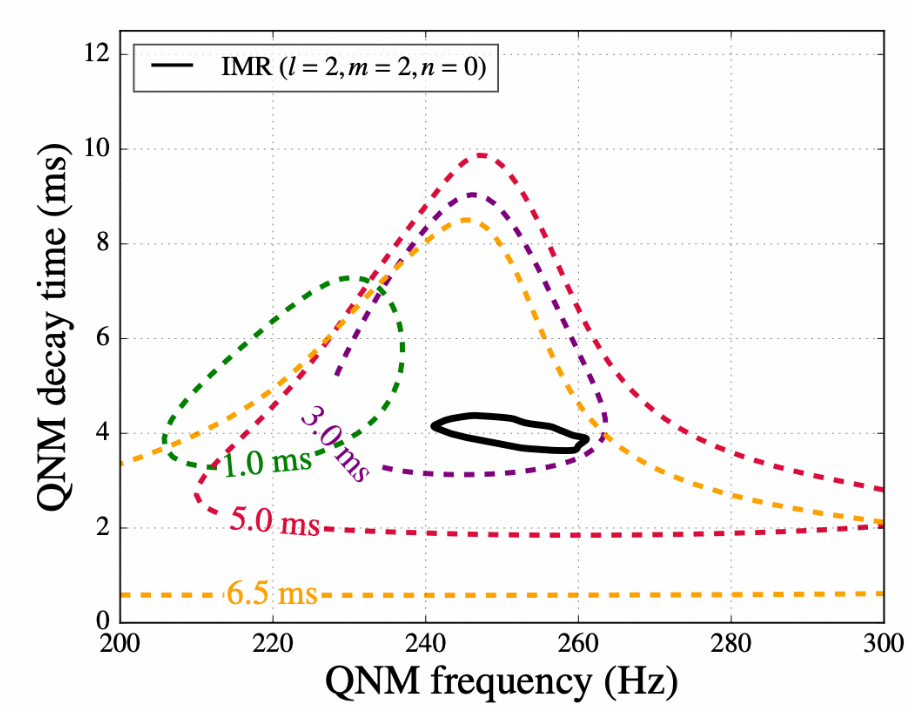
- BH ring down is described by BH quasi-normal mode.

$$\Psi = \sum_{n,l,m} A_{n,l,m} e^{i\omega_{nlm}t}$$

- BH no hair theorem in GR (Carter 1971, Robionson 1975)

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt_{\mathrm{KS}}^2 - \frac{4Mra\sin^2\theta}{\Sigma} dt_{\mathrm{KS}} d\varphi_{\mathrm{KS}} + \frac{4Mr}{\Sigma} dt_{\mathrm{KS}} dr$$
$$+\left(1 + \frac{2Mr}{\Sigma}\right) dr^2 + \Sigma d\theta^2 - 2a\sin^2\theta \left(1 + \frac{2Mr}{\Sigma}\right) dr d\varphi_{\mathrm{KS}} + \frac{\mathscr{F}}{\Sigma}\sin^2\theta d\varphi_{\mathrm{KS}}^2$$

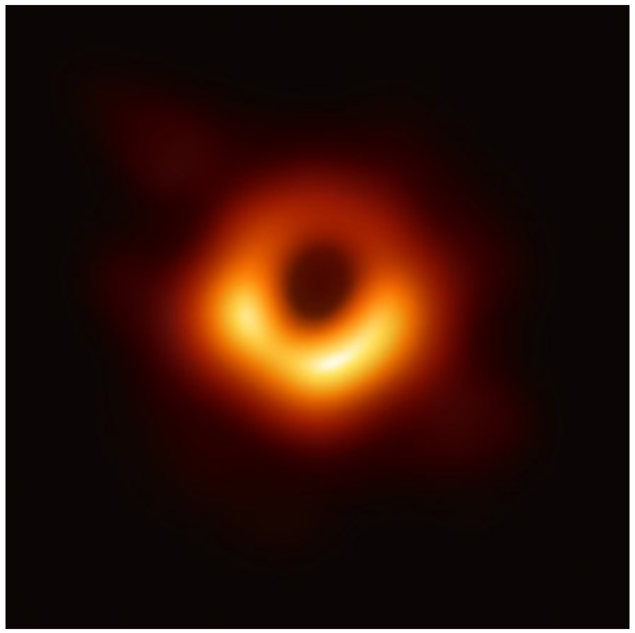
$$\Rightarrow \quad \omega_{nlm}^{\mathrm{QNM}} = \omega_{nlm}^{\mathrm{QNM}}(M, a)$$

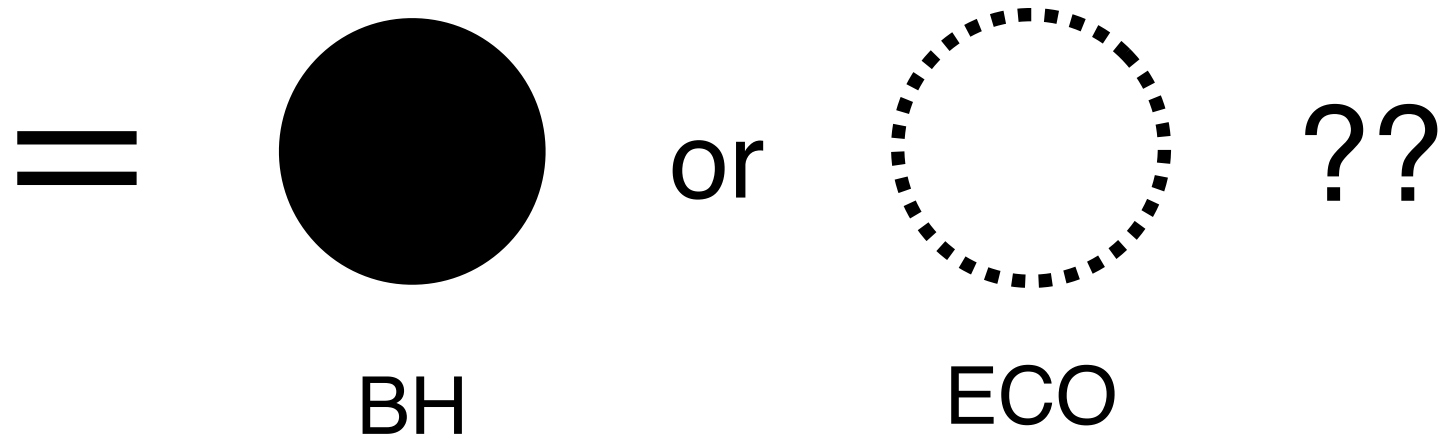


$$\begin{cases} f_{220}^{\mathrm{QNM}} = 251^{+8}_{-8}\ \mathrm{Hz} \\ \tau_{220}^{\mathrm{QNM}} = 4.0^{+0.3}_{-0.3}\ \mathrm{ms} \end{cases}$$

LVC, PRL **116** (2016) 221101

BH vs Exotic compact object





Q. Can we distinguish BH from exotic compact object using ringdown phase or echo ?

- test of gravitational theory
- test of exotic matter

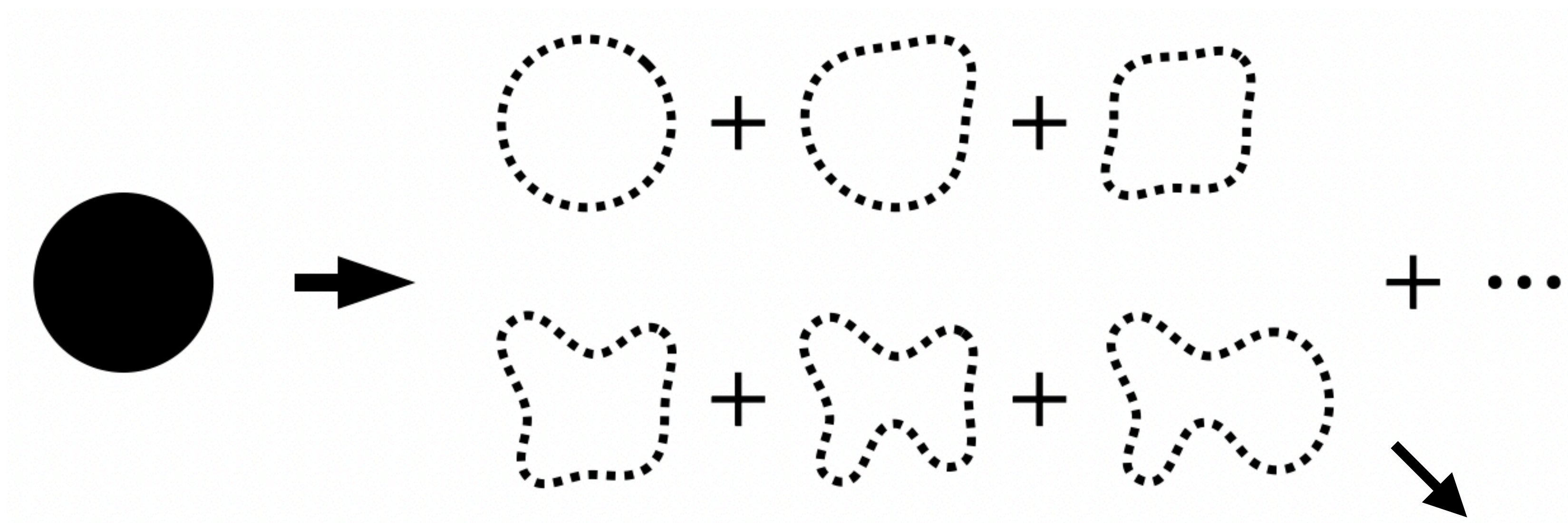
- Exotic compact object (ECO)
  - Boson stars, gravastars, wormholes, firewalls, Fuzzballs et al.

Fuzzball proposal

- BHs in GR have many theoretical problem.
  - How to describe the BH singularity.
  - Information paradox ?
  - What is the origin of the BH entropy ? et al

$$\boxed{S_{\rm BH} = \frac{A}{4}}$$

- Fuzzball proposal (Lunin and Mathur (2002))

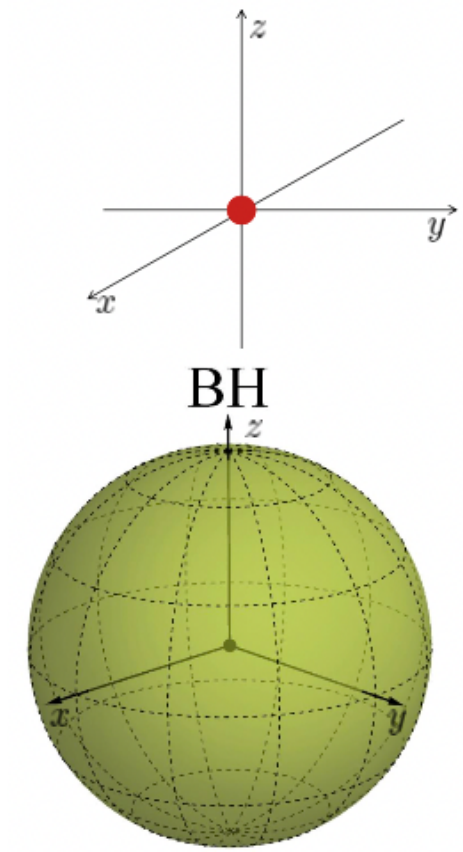


$|\,\mathrm{BH}>$ $\qquad$ $\sum |\,\mathrm{microstate}>$ $\qquad$ BH entropy

- BH horizon
- curvature singularity

- horizonless
- smooth (no singularities in 5-dim)
- same charge as corresponding BH

Fuzzball proposal



- 4-charge BPS BH

$$ds^2 = -f(r)dt^2 + f(r)^{-1}\delta_{ij}dx^i dx^j$$

$$f(r) = \left(H_1 H_2 H_3 H_4\right)^{-1/2} \qquad H_A = 1 + \frac{Q_A}{r}$$

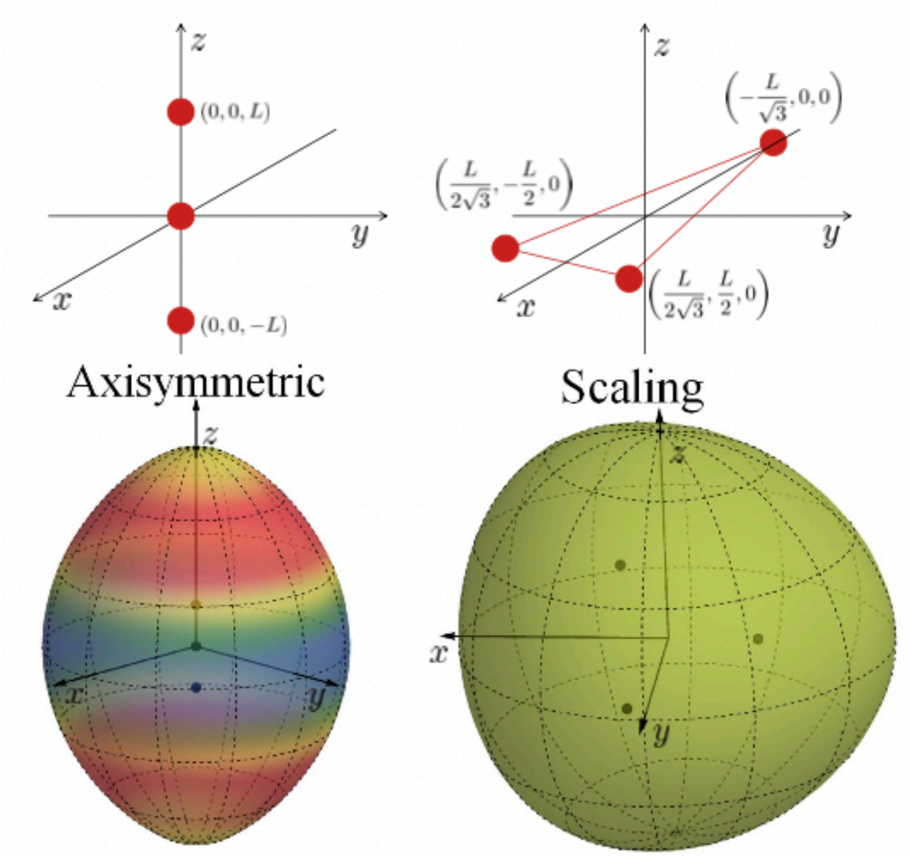
- Microstate geometry see M.Bianch et.al (2017)

$$ds^2 = -e^{2U}(dt^2 + \omega)^2 + e^{-2U}\delta_{ij}dx^i dx^j$$

$$\begin{cases} e^{-4U} = Z_1 Z_2 Z_3 V - \mu^2 V^2 \\ *_3\, d\omega = \dfrac{1}{2}\left(VdW - WdV + K^I dL_I - L_I dK^I\right) \end{cases}$$

$$V = 1 + \sum_{a=1}^{N} \frac{v_a}{|x - x_a|}, \cdots$$

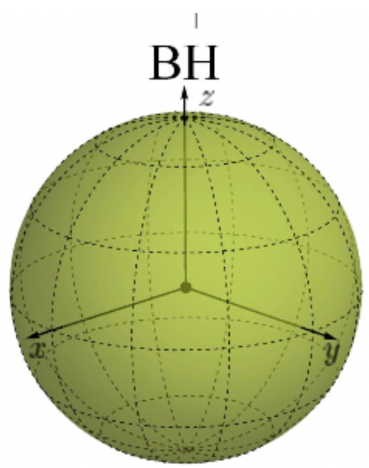
- multi-center
- $Q_A = (Q_0, Q_I)$
- $P^A = (P^0, P^I)$

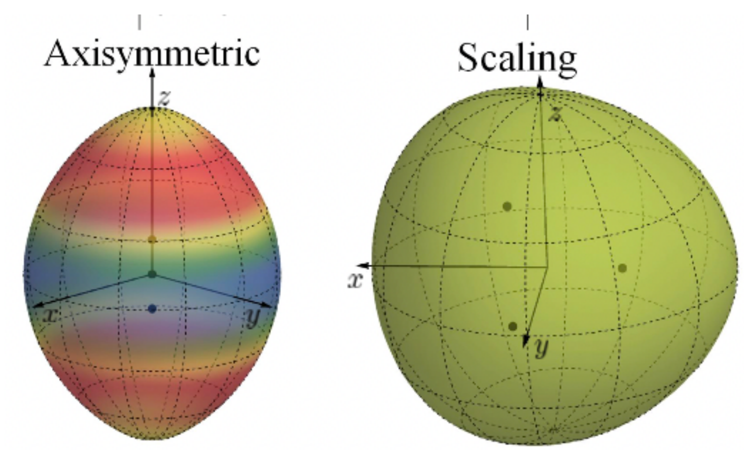


What we want to do

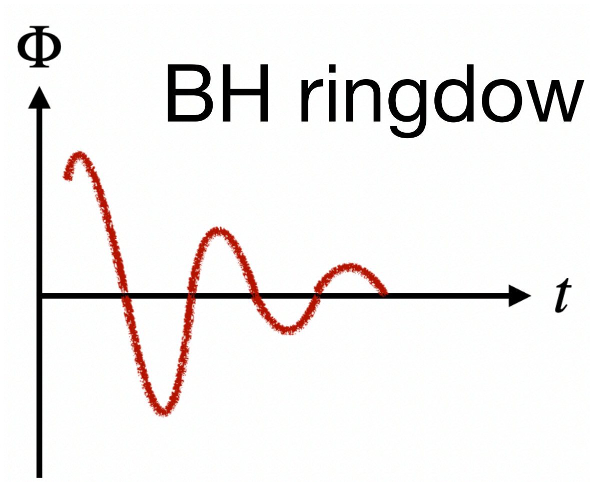
Q. How different is 4-charge BPS BH ringdown from ringdown of corresponding microstate geometries.

$$\boxed{\Box\Phi = 0}$$

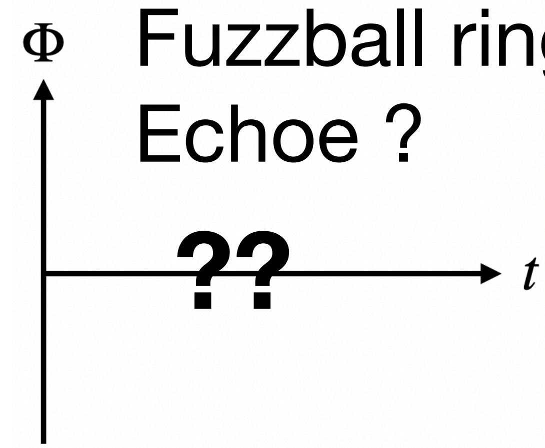




BH ringdown



Fuzzball ringdown ?
Echoe ?



Outline

1. Introduction
2. Black hole ringdown
    - Time domain
    - Frequency domain
    - Computing qusinormal mode
3. Exotic compact object ringdown
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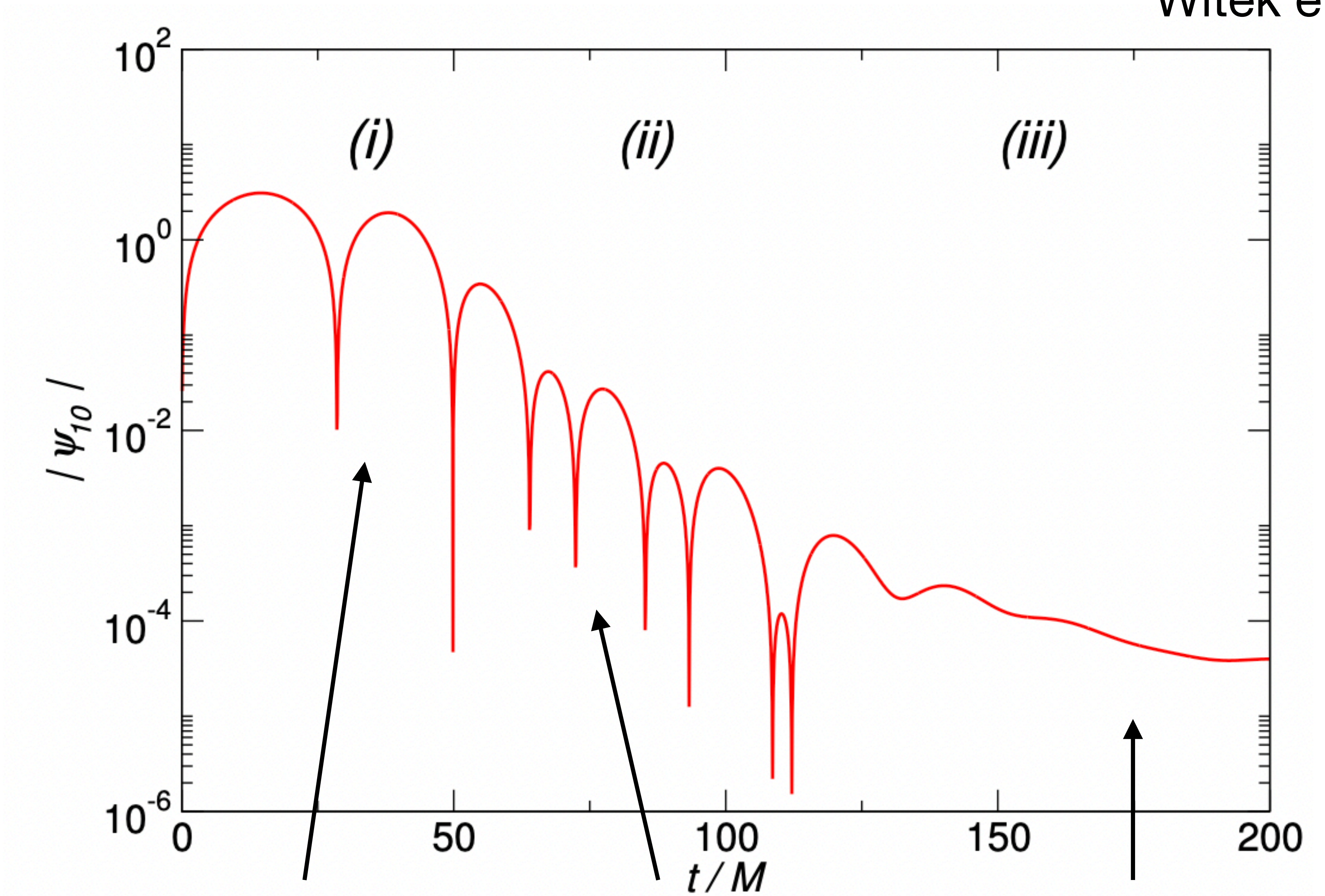
BH Ringdown

3+1 dim evolution of test massless scalar field around BH

Witek et al (2013)



prompt response ring-down power law tail

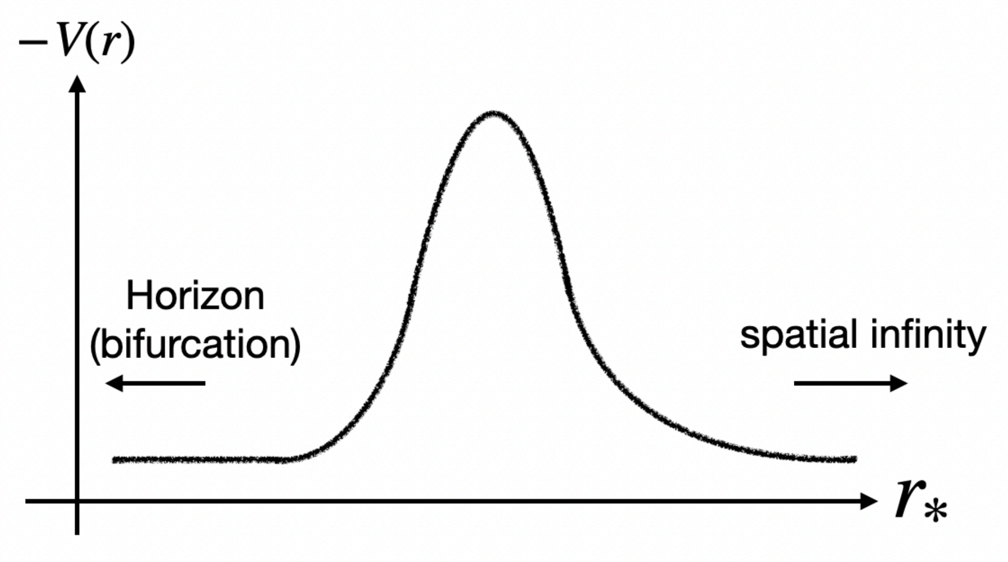
$$\Psi = \sum_{n,l,m} A_{n,l,m} e^{i\omega_{nlm} t}$$

$$\Psi \sim t^{-p}$$

Computing QNM

$$\left(\frac{\partial^2}{\partial r_*^2} + V(r) + \omega^2\right)\Psi = 0$$

$$\Psi \sim e^{-i\omega(t-r_*)}$$



$$\Psi \sim e^{-i\omega(t+r_*)}$$

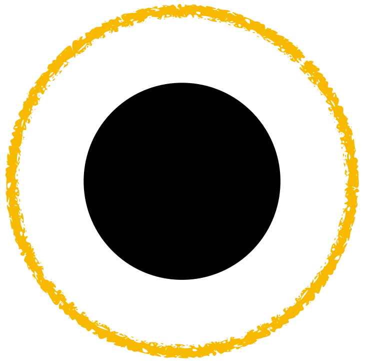
- Direct Integration
- Leaver's method
- WKB approximation

Light ring

$$\begin{cases} \mathrm{Re}(\omega) \sim r_{\mathrm{LR}}^{-1} \\ \mathrm{Im}(\omega) \sim \tau^{-1} \end{cases}$$

r_{LR} : Light ring radius

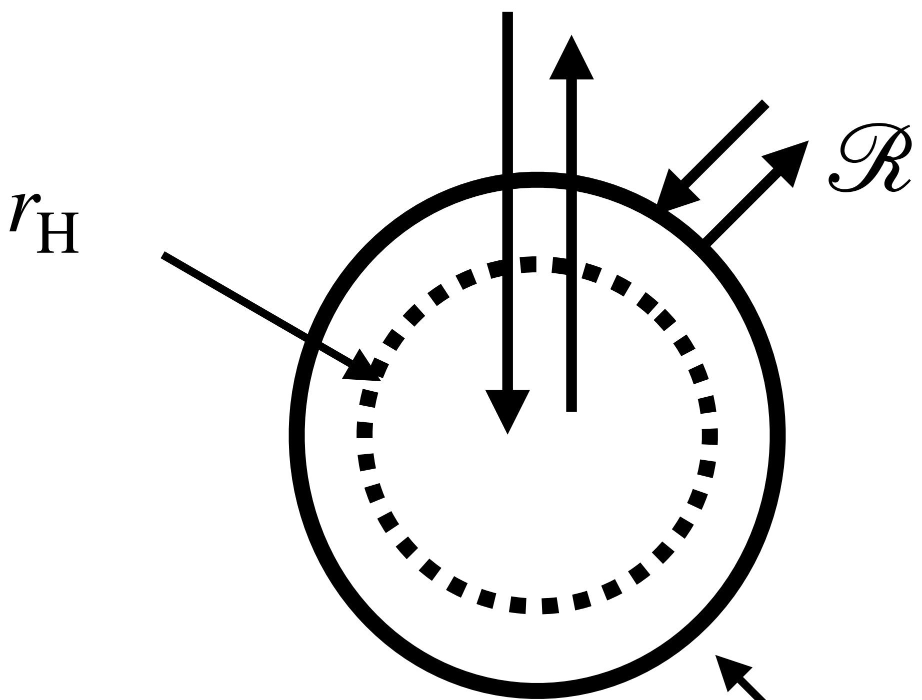
τ : Instability time scale



Outline

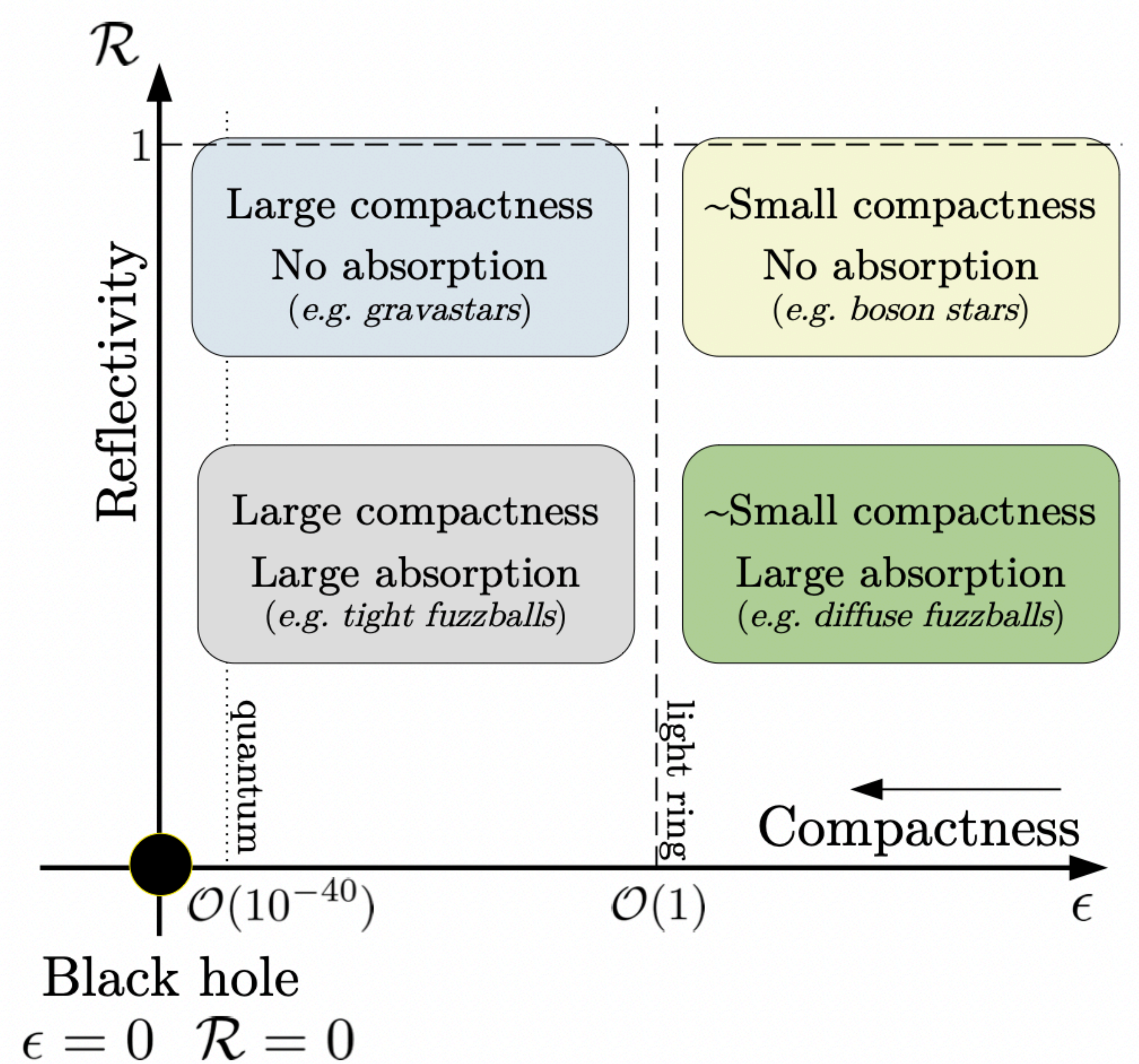
Exotic compact object

- ECO is a theoretical compact object without BH horizon.



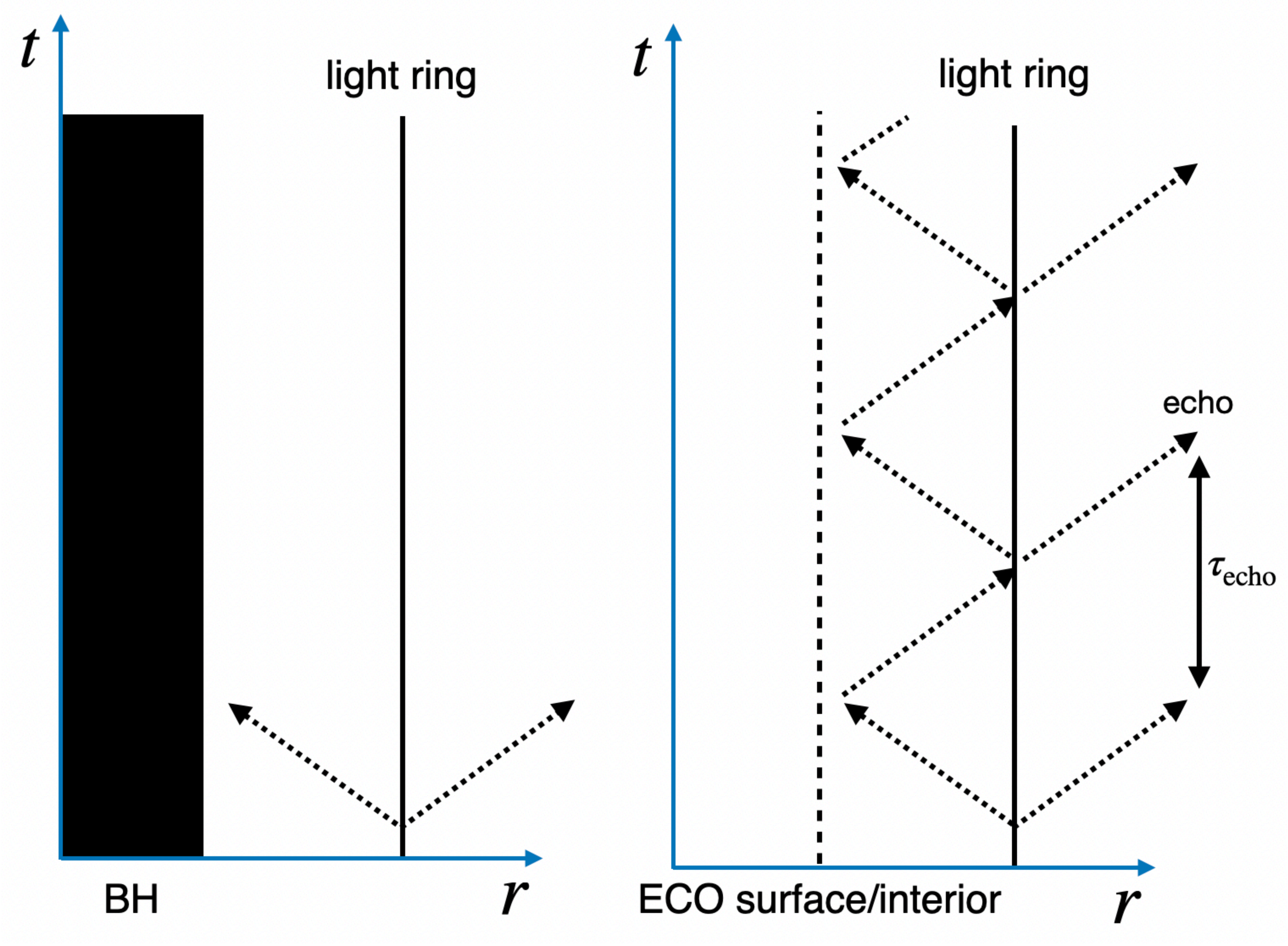
$$r_0 = r_{\mathrm{H}}(1+\epsilon)$$

$$C = \frac{2M}{r_0}$$



From E.Maggio et al (2021)

Black hole vs ECO with light ring



ECO with light ring

- Evolution of test massless scalar field around ECO.

$$ds^2 = -\alpha(r)^2 dt^2 + a(r)^2 dr^2 + r^2 d^2\Omega$$

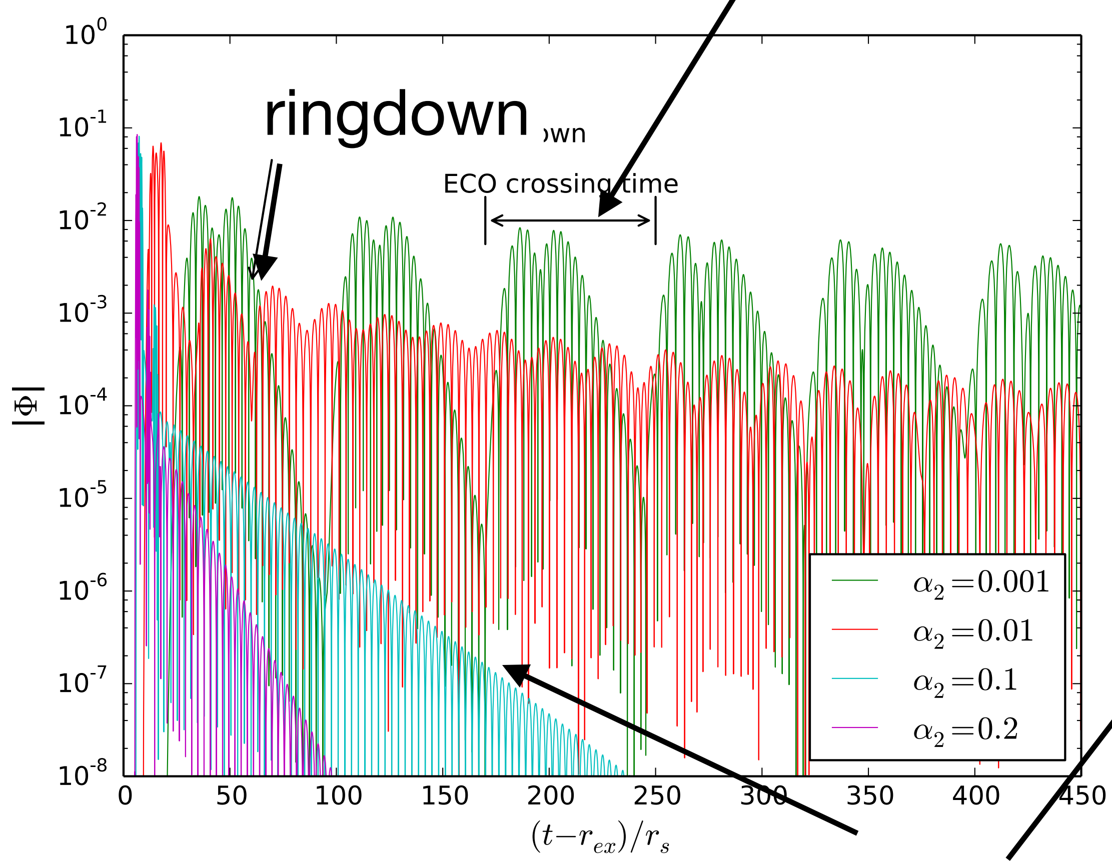
$$\Box \Phi = 0$$

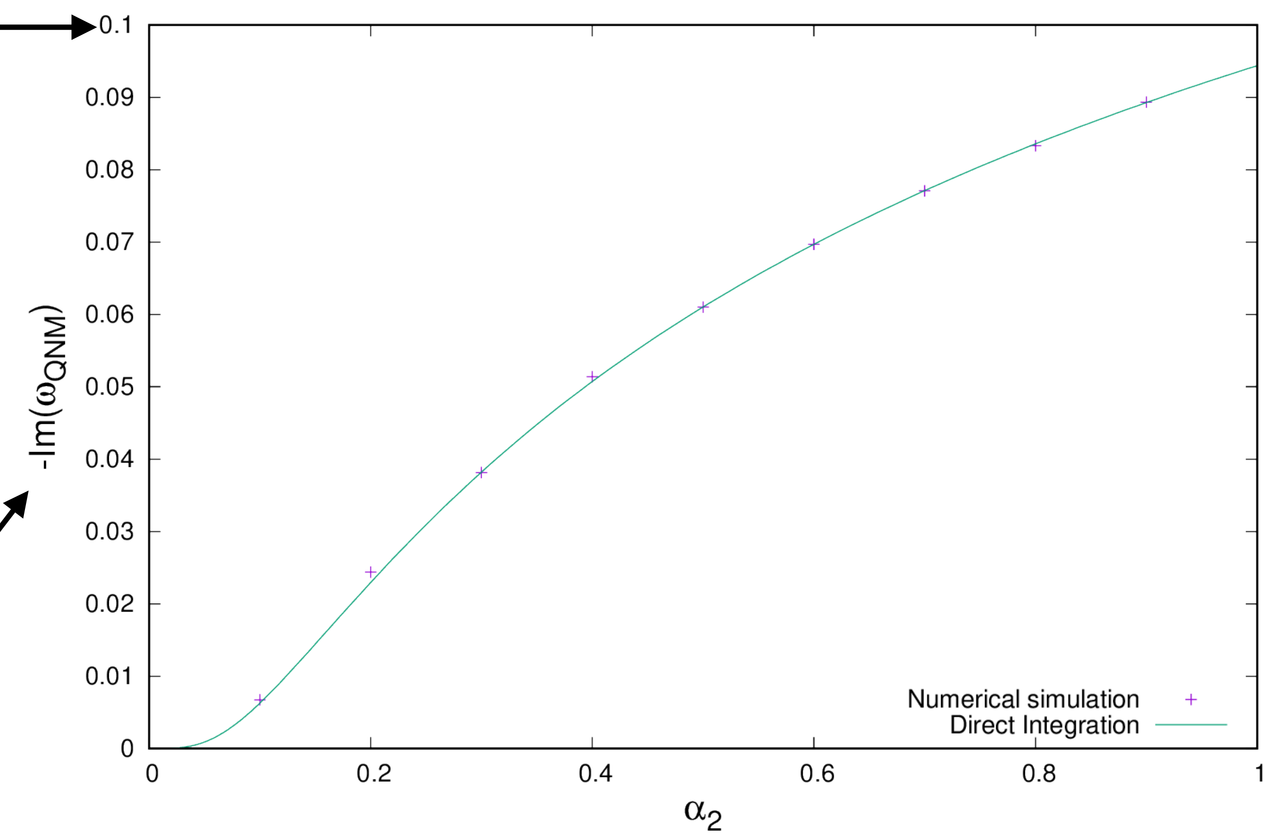
$$\alpha(r)^2 = \begin{cases} \mathrm{Sch.BH} & (2M(1+\epsilon) < r) \\ \alpha_2 & (r < 2M(1-\epsilon)) \end{cases}$$

crossing time

BH QNM

ringdown





QNM

also see E.Maggio et al (2020)

Comments

- Full evolution ??
  - Boson stars merger in GR is possible.

    Sanchis-Gual et al (2019)
  - Other ECO may be difficult.
- Beyond spherical symmetry
  - These pictures are based on spherical symmetry (separability of equation)
  - Non-separable case is more complicated.
- Fuzzball
  - Field eq. is not separable on fuzzball spacetime.
  - It is not obvious if ringdown and echo appear.
  - We need to use 3+1 dim numerical simulation.

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4. Numerical method
   - Numerical code for 4 charge BH
   - 3+1 decomposition
   - Numerical code for fuzzball
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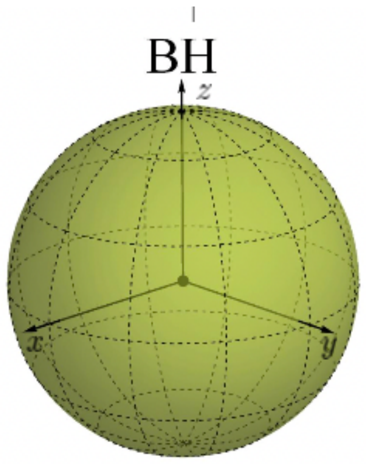
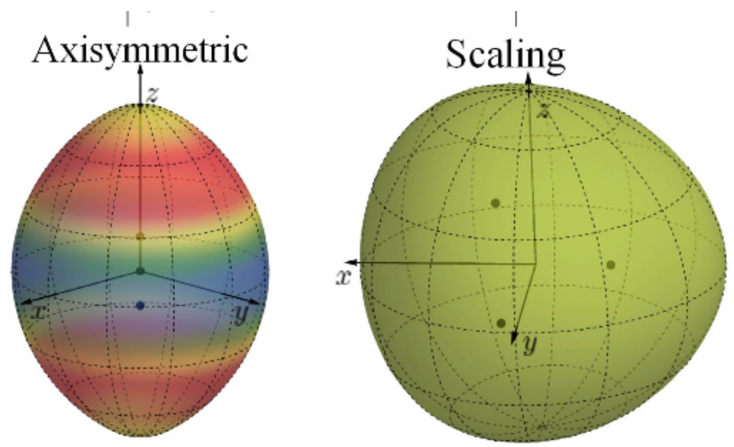
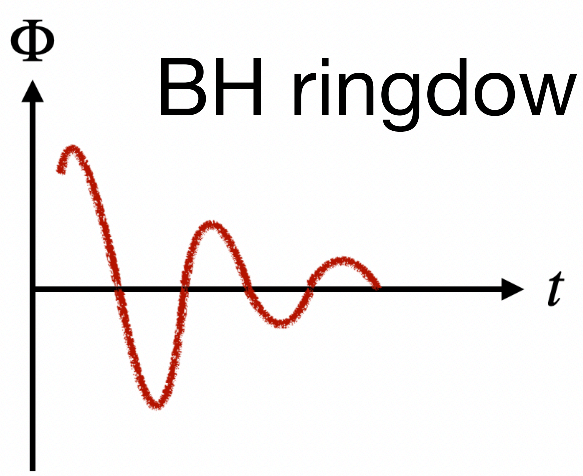
What we want to do

Q. How different is BH ringdown from ringdown of corresponding microstate geometries.

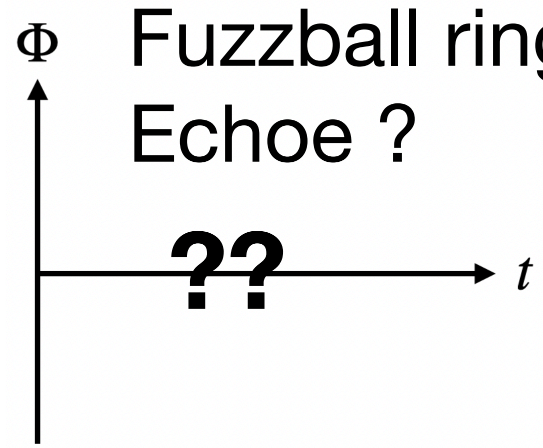
$$\boxed{\Box \Phi = 0}$$





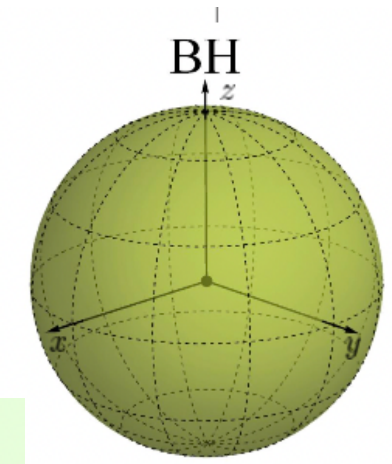


BH ringdown



Fuzzball ringdown ?
Echoe ?

4 charge BH



- metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}\left(dr^2 + r^2 d^2\Omega\right)$$
$$= -B(\varrho)^2 dt^2 + A(\varrho)^2 d\varrho^2 + \varrho^2 d^2\Omega$$

$$f(r) = \left(H_1 H_2 H_3 H_4\right)^{-1/2}$$
$$H_A = 1 + \frac{Q_A}{r}$$
$$M = \frac{Q_1 + Q_2 + Q_3 + Q_4}{4}$$

- evolution equation

$$\Phi = \sum_{l,m} \frac{\sigma_{lm}(t,\varrho)}{\varrho} Y_{lm}(\theta,\phi)$$

$$-\frac{\partial^2}{\partial t^2}\sigma_{lm} + \frac{\partial^2}{\partial \varrho_*^2}\sigma_{lm} - V_{\text{eff}}(\varrho)\sigma_{lm} = 0$$

Effective potential

$$V_{\text{eff}}(\varrho) = B^2\left(\frac{l(l+1)}{\varrho^2} + \frac{1}{A^2\varrho^2}\left(\frac{\partial_\varrho B}{B} - \frac{\partial_\varrho A}{A}\right)\right)$$

$$Q_1 = Q_3, Q_2 = Q_4$$

- initial data

$$\begin{cases} \sigma_{lm}(0,\varrho_*) = e^{-\left(\frac{\varrho_* - \varrho_0}{\sigma}\right)^2} \\ \dot{\sigma}_{lm}(0,\varrho_*) = 0 \end{cases}$$

3+1 decomposition

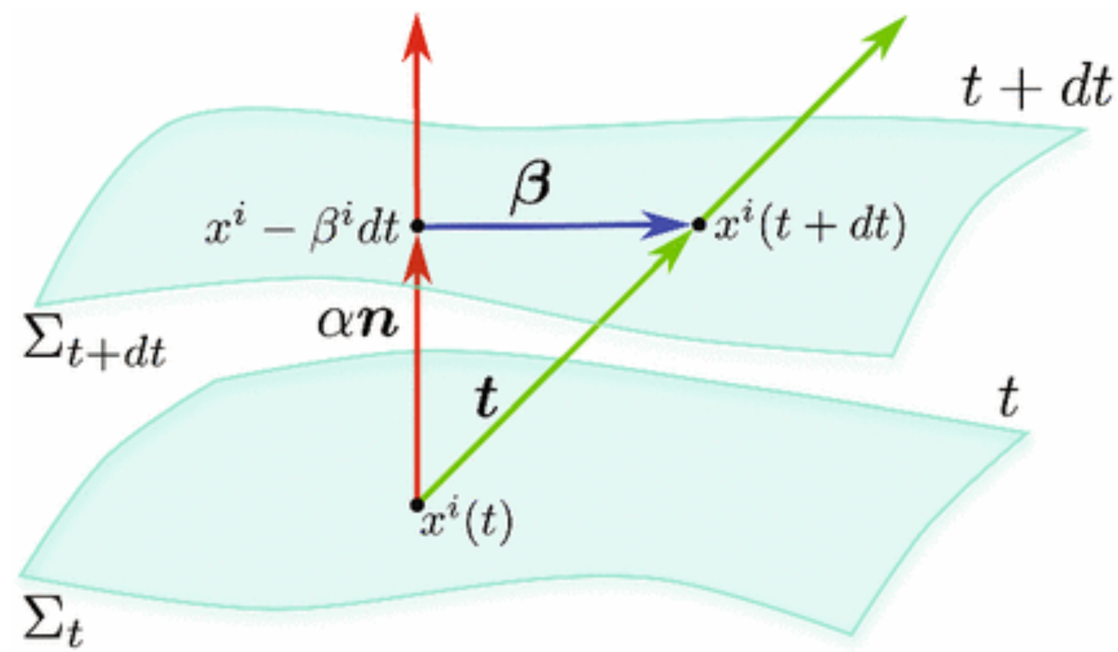
- 4 dim. metric decomposes to lapse, shift, and 3-metric.

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)
\end{aligned}
$$

lapse

3-metric

shift



$$K_{ij} = -\frac{1}{2\alpha}\left(\partial_t - \mathscr{L}_\beta\right)\gamma_{ij}$$:extrinsic curvature

- Klein-Gordon eq.

$$
\Box\Phi = 0 \quad \Longrightarrow \quad
\begin{cases}
(\partial_t - \beta^i \partial_i)\Phi = -2\alpha K_\Phi \\
(\partial_t - \beta^i \partial_i)K_\Phi - \frac{\alpha}{2} D^2\Phi + \alpha K K_\Phi - \frac{1}{2} D^i \alpha D_i \Phi
\end{cases}
$$

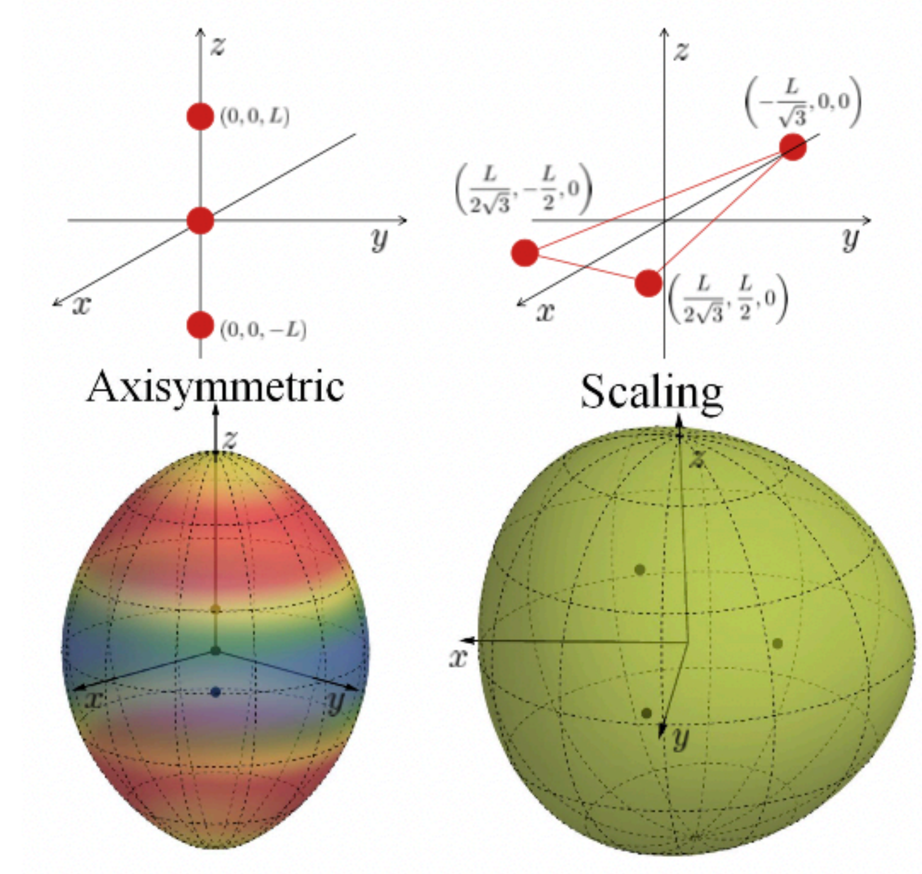
Fuzzball geometry

- Fuzzball metric (3 center)

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U}\sum_{i=1}^{3}\delta_{ij}dx^i dx^j$$

$$\begin{cases} e^{-4U} = Z_1 Z_2 Z_3 V - \mu^2 V^2 \\ *_3\, d\omega = \dfrac{1}{2}\left(VdW - WdV + K^I dL_I - L_I dK^I\right) \end{cases}$$

$$V = 1 + \sum_{a=1}^{N}\frac{v_a}{|x-x_a|}, \cdots \qquad N=3$$

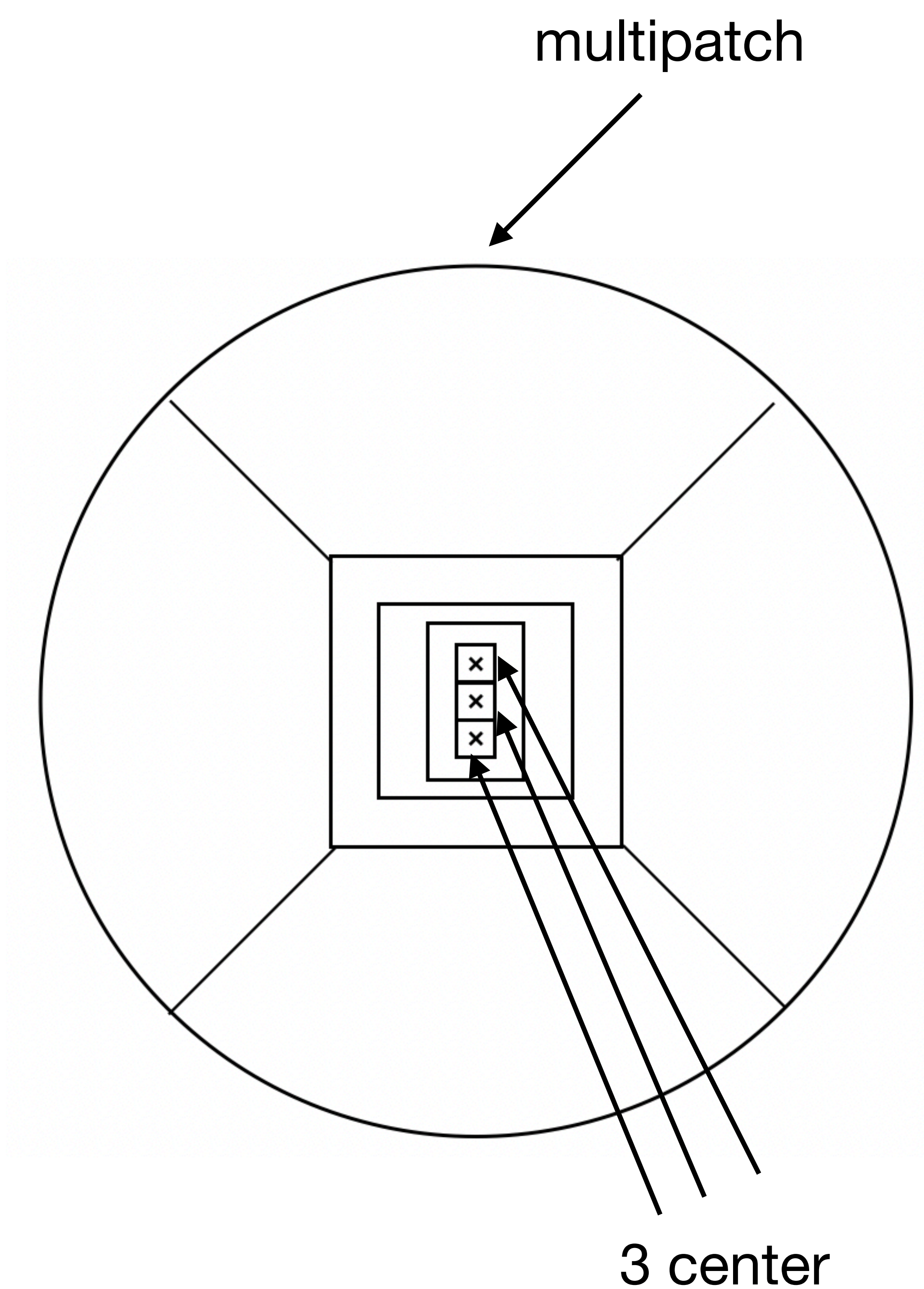


- Geometry is regular in 5 dim. but, singular in 4 dim.

$$\frac{1}{|x-x_a|} \to \frac{\mathrm{erf}\left(\frac{|x-x_a|}{\epsilon}\right)}{|x-x_a|}$$

Einstein ToolKit

- Numerical code (Einstein ToolKit)
  - Einstein Toolkit is software platform for numerical relativity.
  - To resolve the muticenter, we use mesh refinement provided by Carpet.
  - For long simulations, we use multipatch infrastructure provided by Llama.
  - 4th order Runge-Kutta method
  - Spatial derivative is evaluated by 4th order finite differences.
  - 2nd order interpolation in mesh refinement.

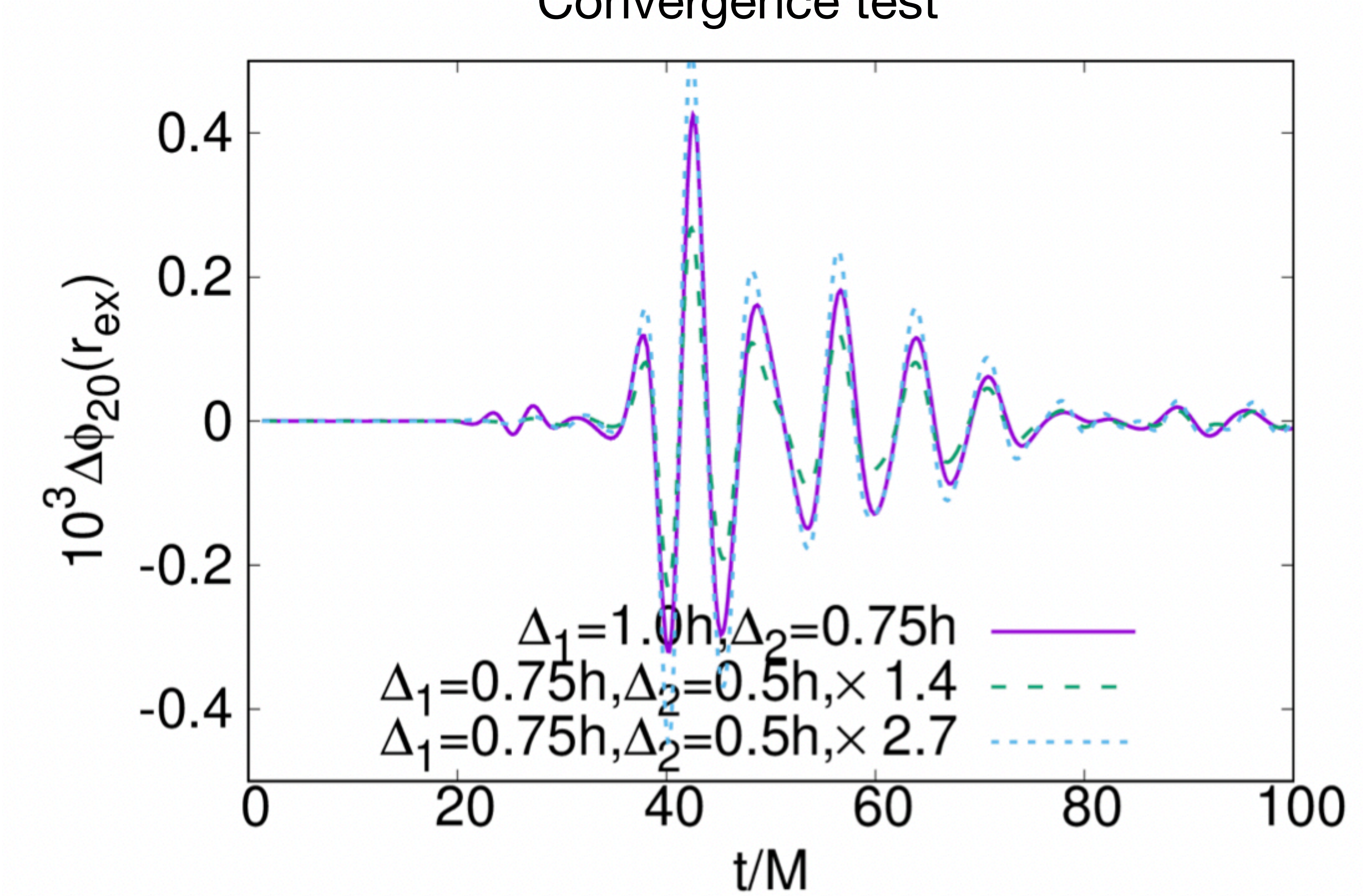


Numerical code

- We extract multipole mode of scalar field during evolution.

$$\Phi = \sum_{l,m} \Phi_{lm}(r,t) Y_{lm}(\theta, \phi)$$

Convergence test



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   - Fuzzball ringdown and echo
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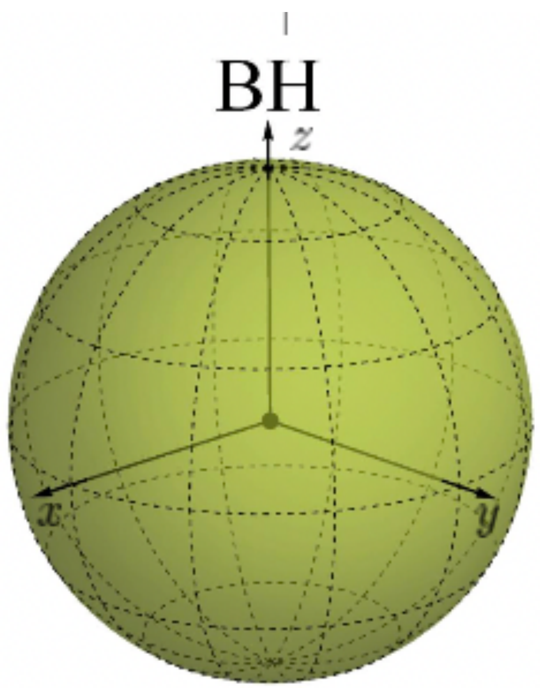
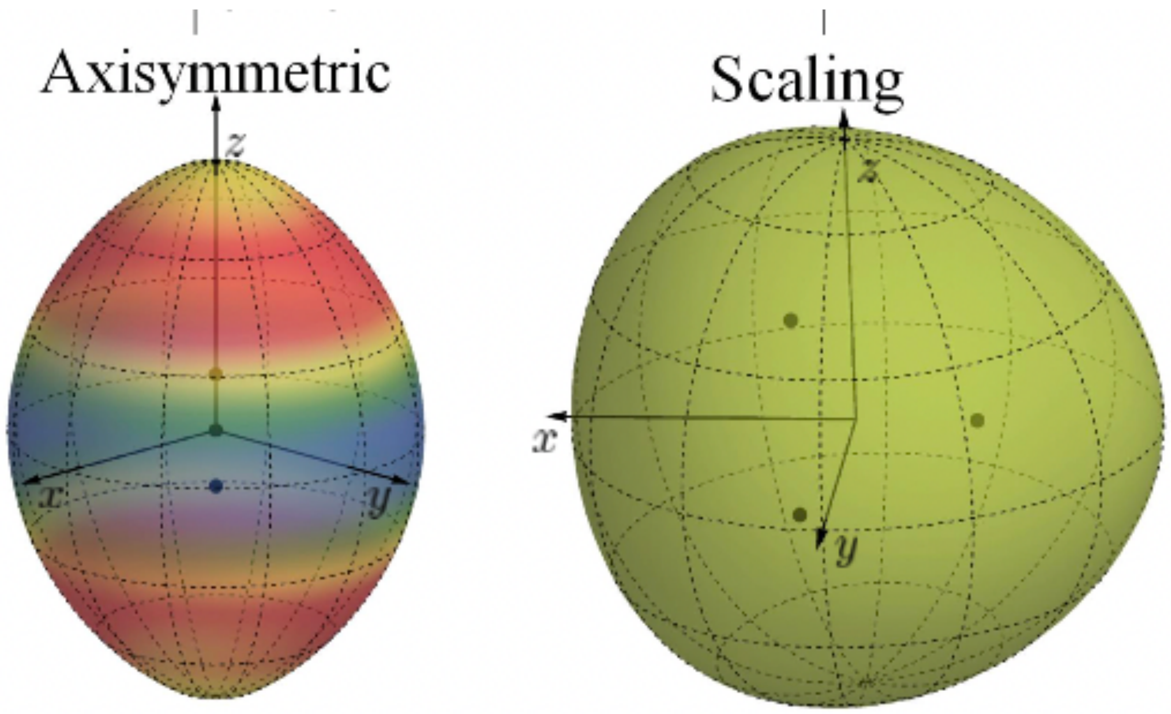


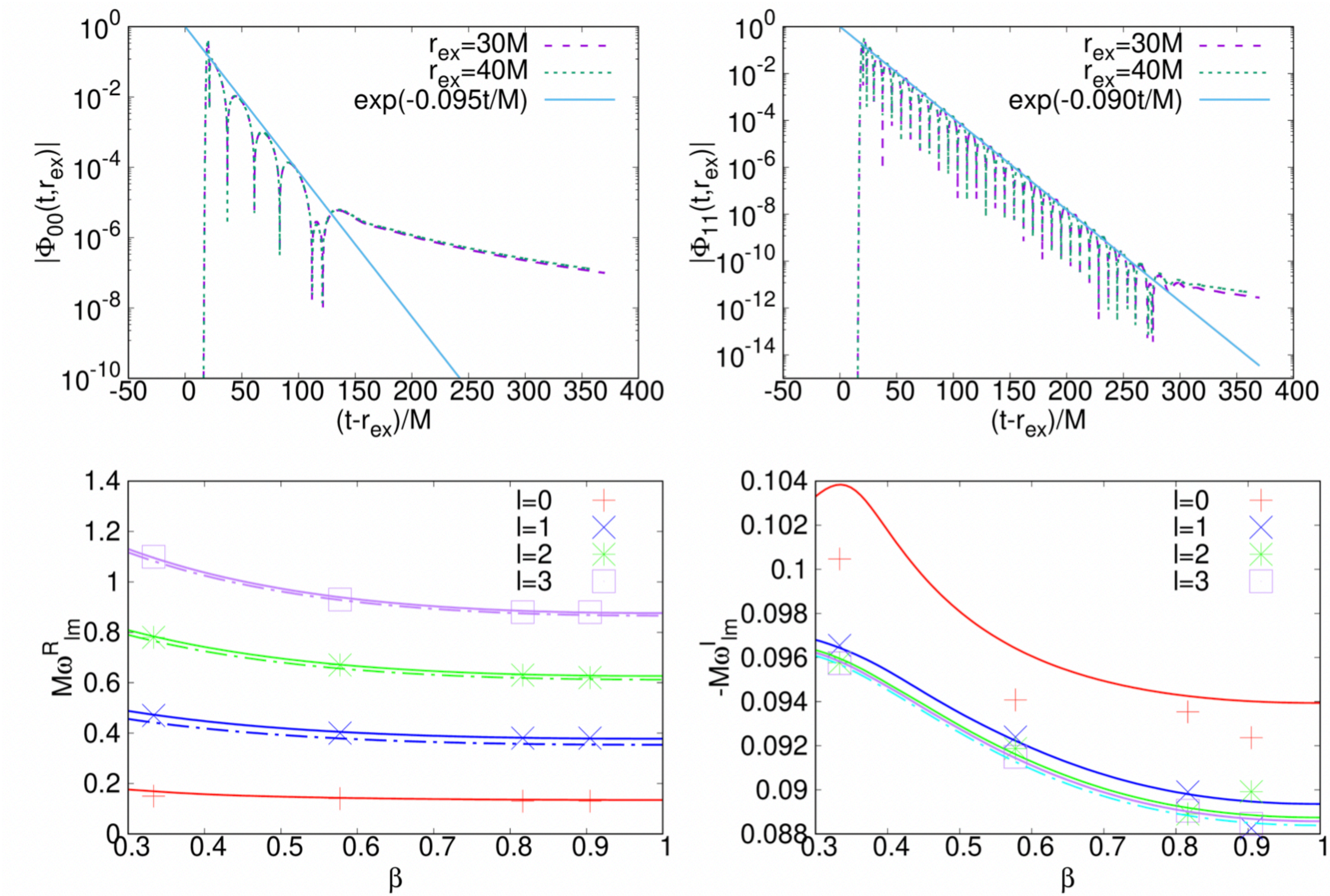
BH



Fuzzball

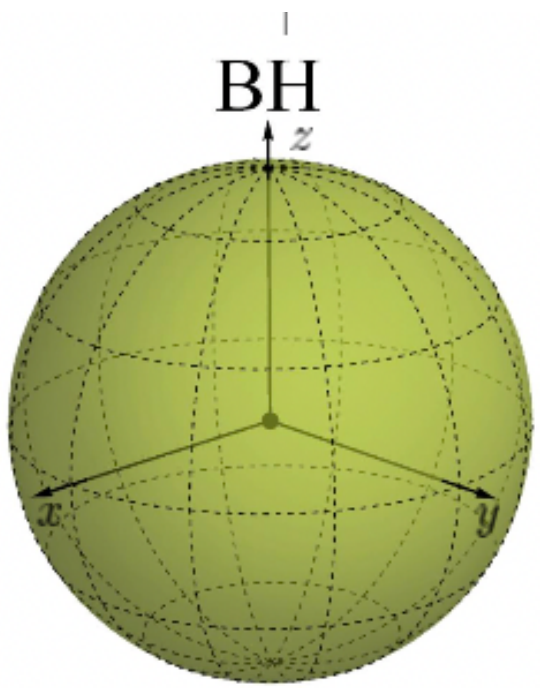
QNM in 4 charge BH

$\beta = \sqrt{Q_2/Q_1}, Q_1 = Q_3, Q_2 = Q_4$

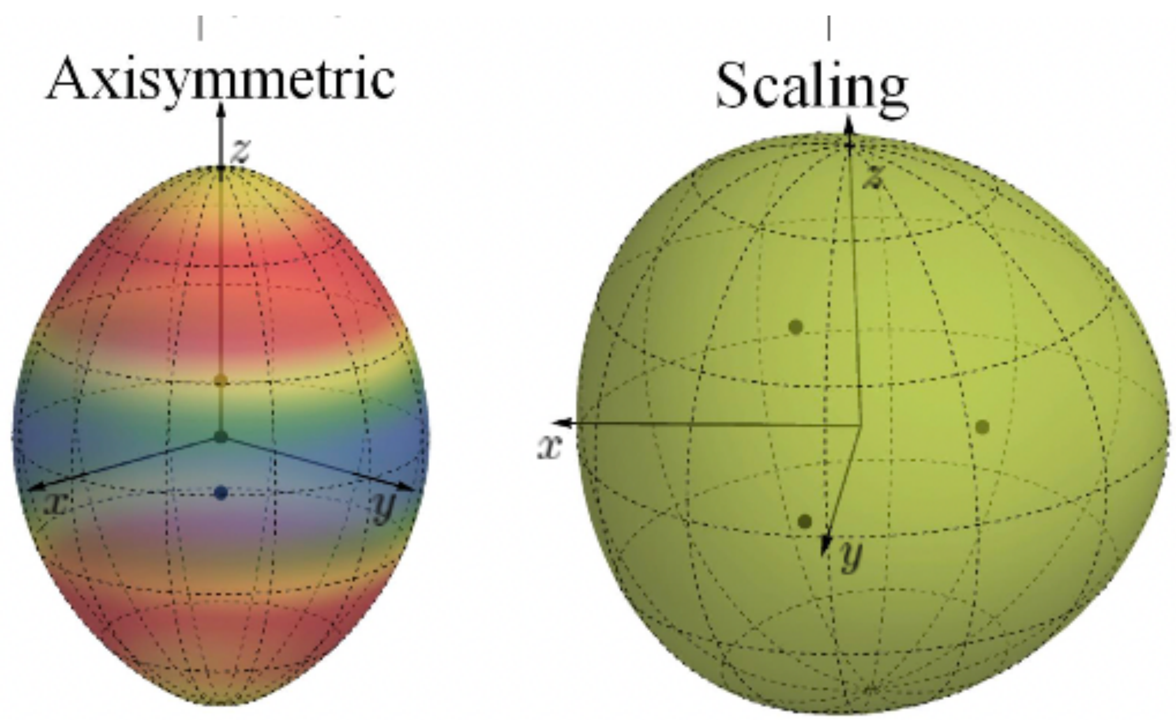


solid line : direct int
dot-dashed line : WKB

$\omega_I M \sim \mathcal{O}(0.1)$



BH



Fuzzball

Light ring around Fuzzball

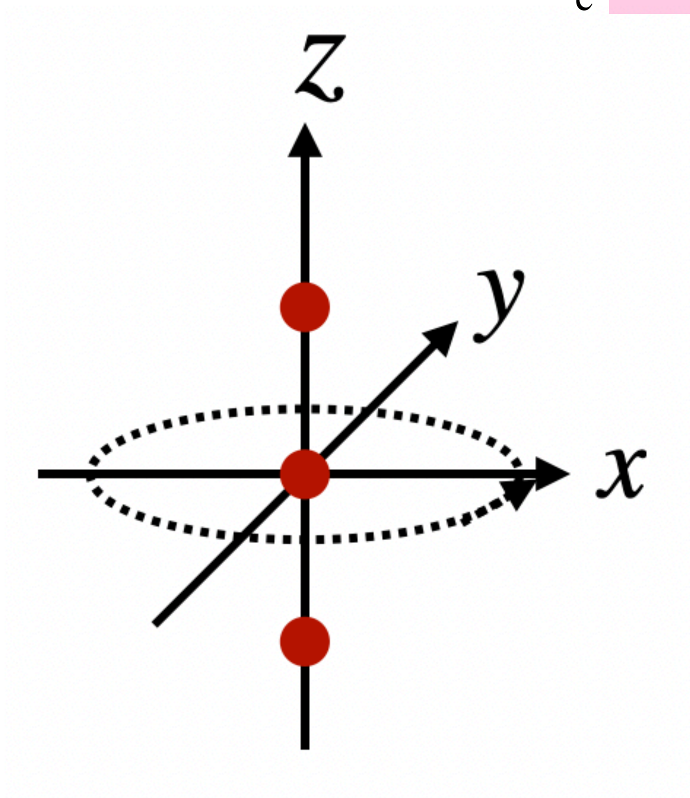
- WKB approximation
  - The unstable photon sphere in axisymmetric fuzzball.

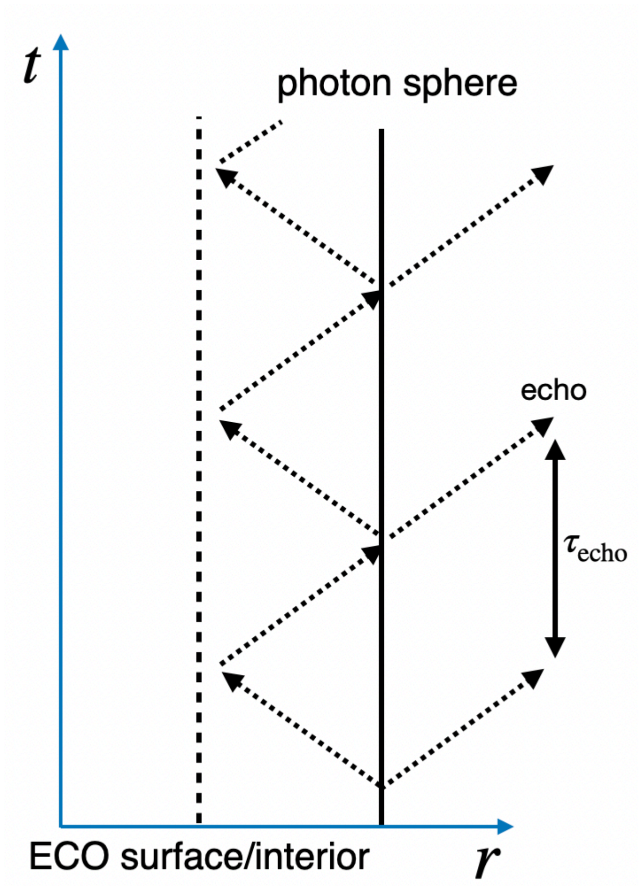
$$\mathrm{Re}\left(\omega_{\mathrm{QNM}}\right) \longleftrightarrow \text{Light ring radius}$$

$$\mathrm{Im}\left(\omega_{\mathrm{QNM}}\right) \longleftrightarrow \text{Instability time scale}$$

  - echo time scale

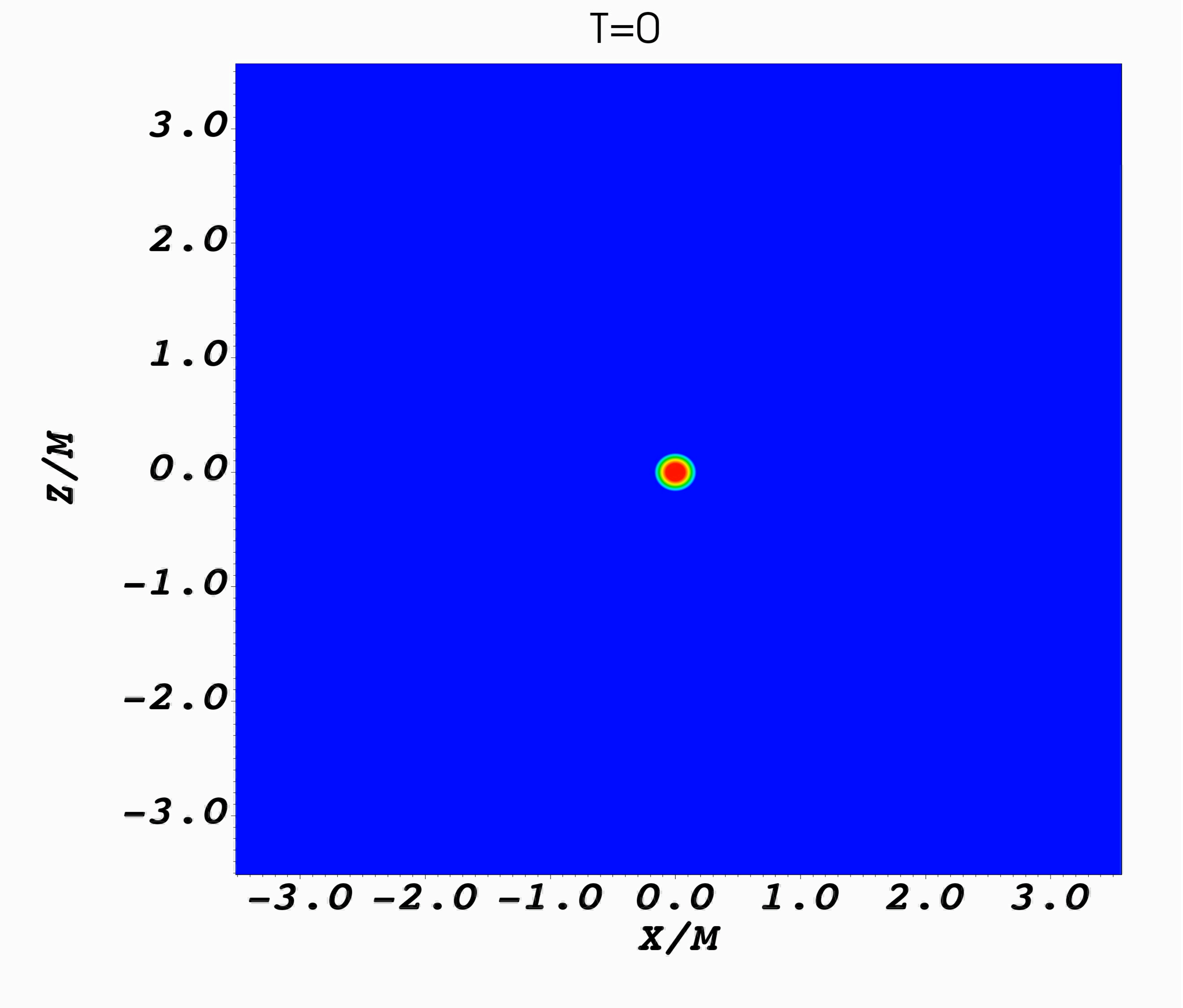
$$\tau_{\mathrm{echo}} = 2\int_{r_{\mathrm{c}}}^{r_{\mathrm{t}}} \frac{dr}{dr/dt}$$





3+1 simulations

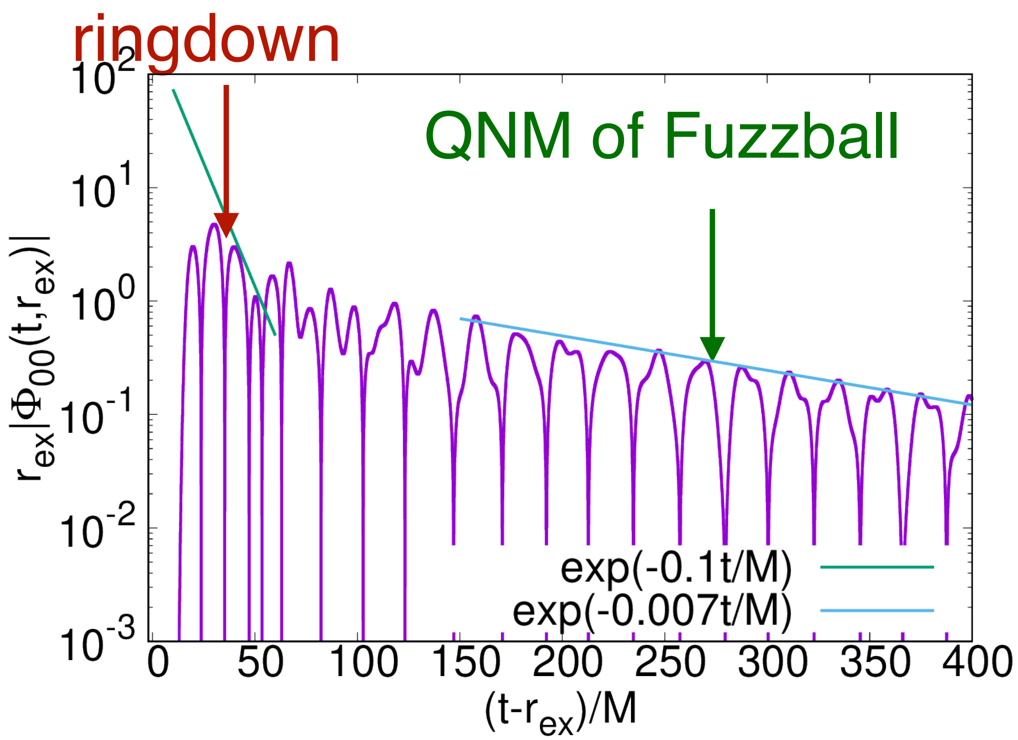
- Axisymmetric Fuzzball

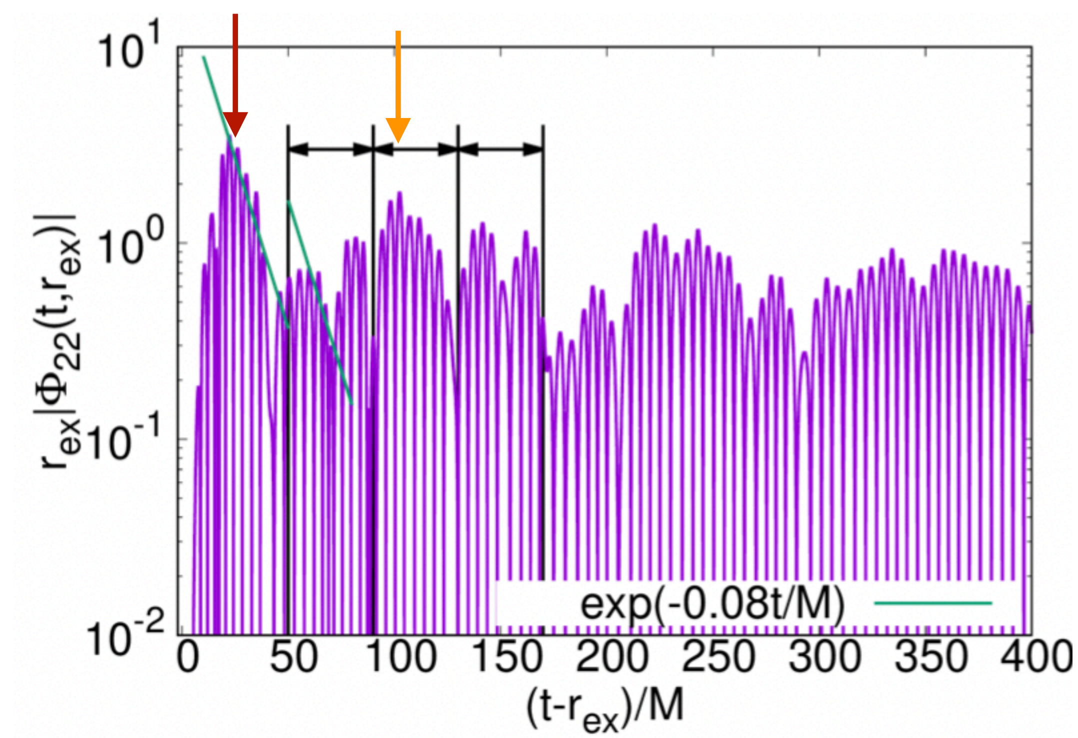


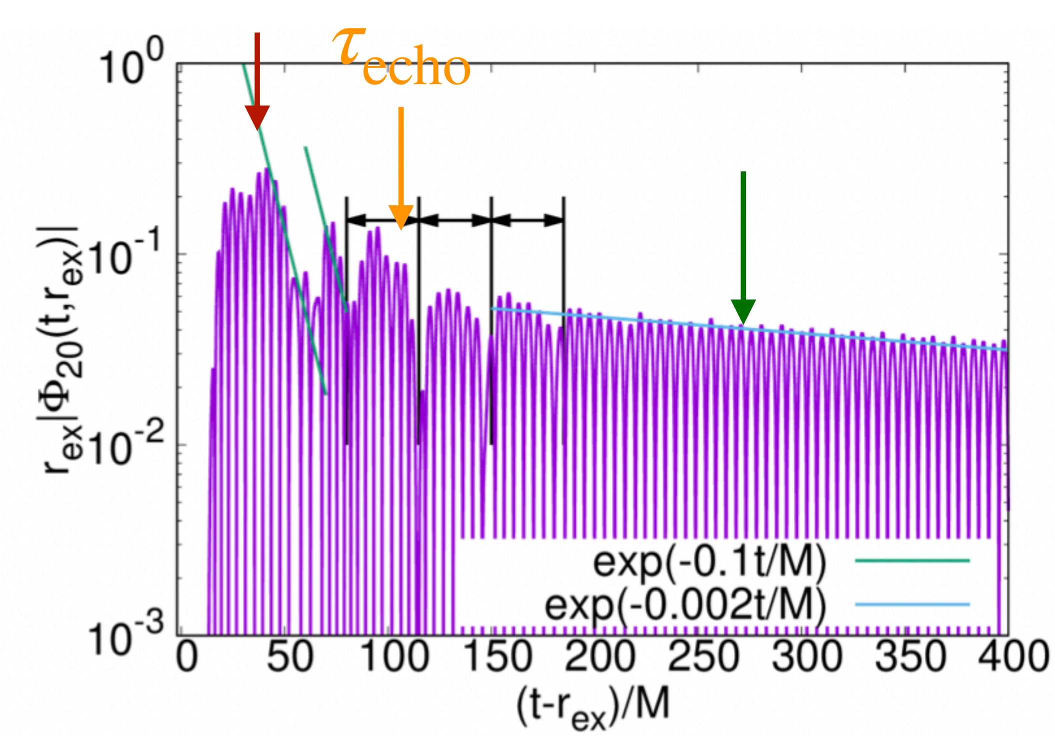
Waveform from Axisymmetric FB

$$\Phi(0,x) \propto e^{-\left(\frac{r-r_0}{\sigma}\right)^2}$$

$$\Phi(0,x) \propto e^{-\left(\frac{r-r_0}{\sigma}\right)^2}\mathrm{Re}(Y_{2,2})$$



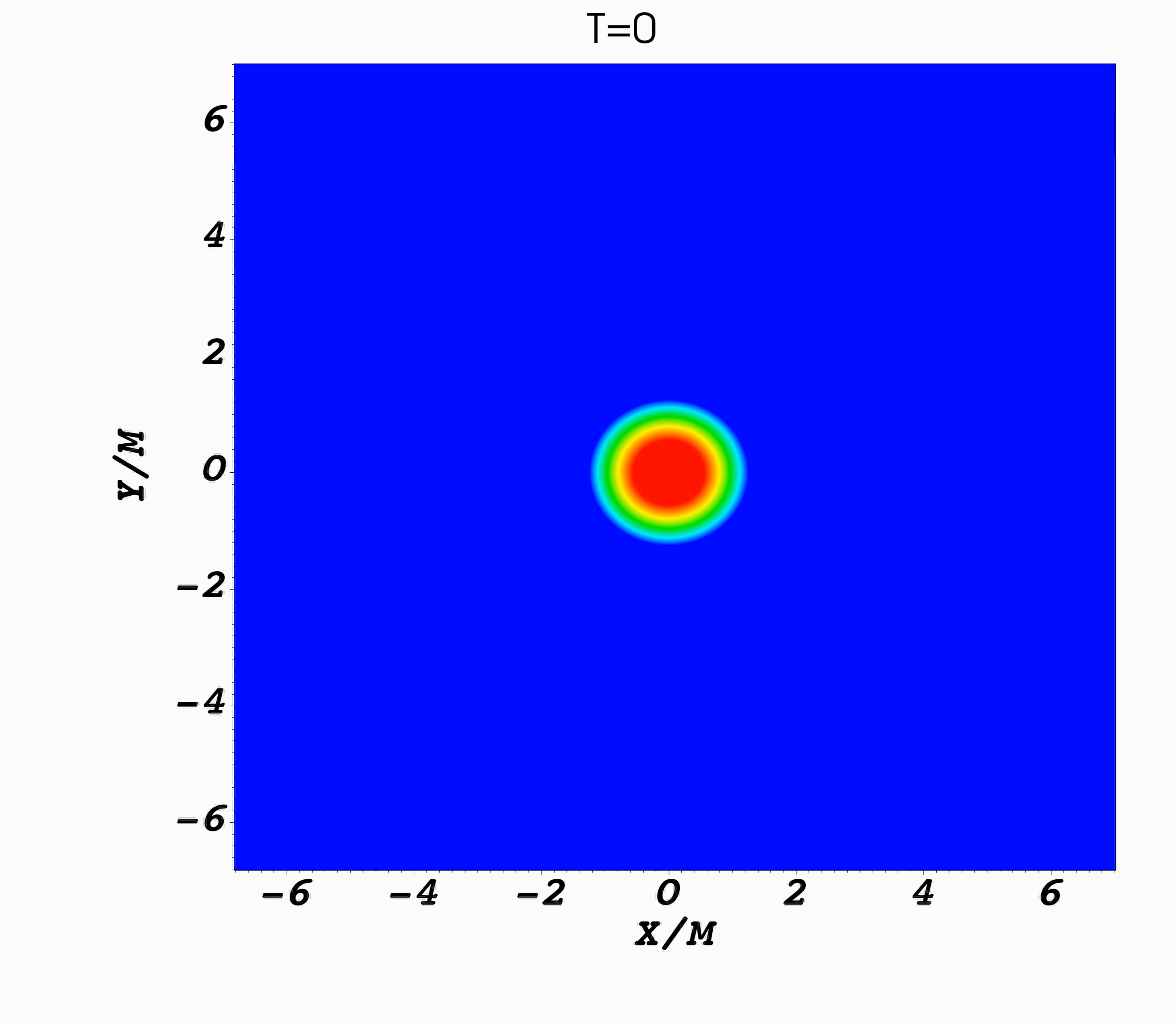




- Ringdown, echo, and QNM appear.
- Ringdown and echo time scale for (l,m)=(2,2) mode are agreement with geodesic approximation.

3+1 simulations

- Scaling Fuzzball

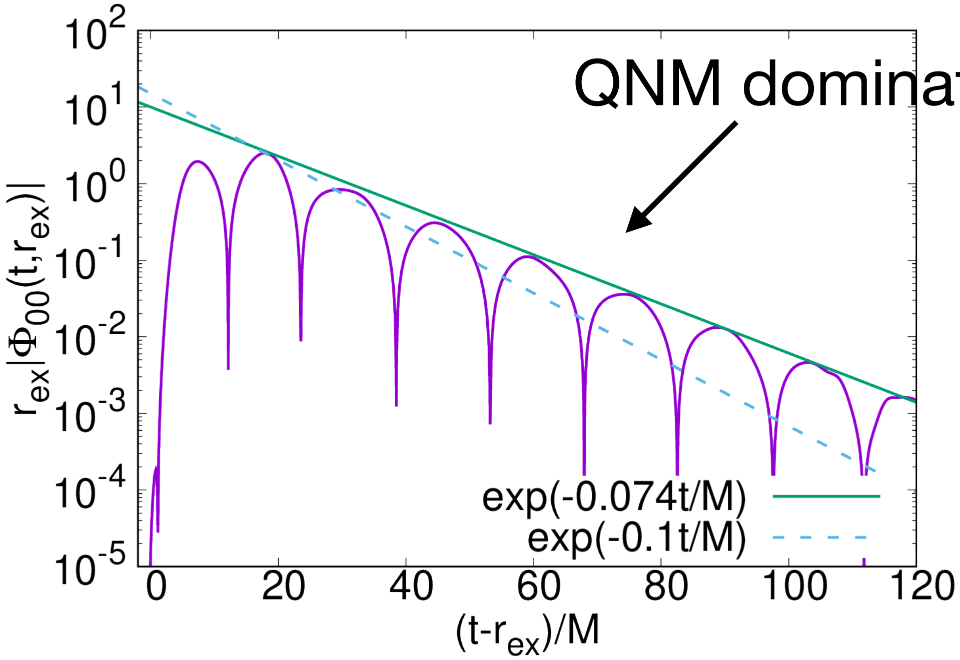


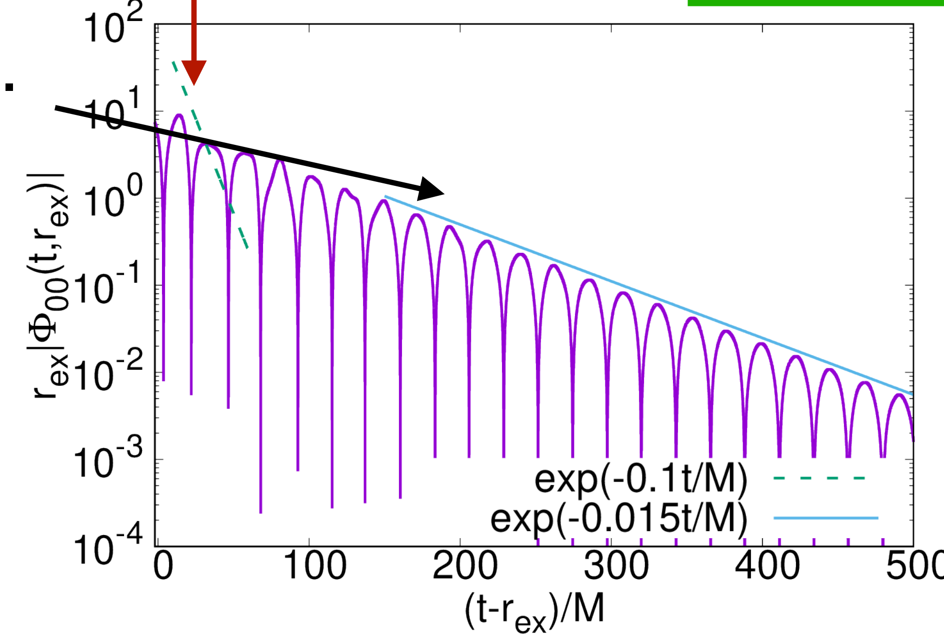
Waveform from Scaling solution

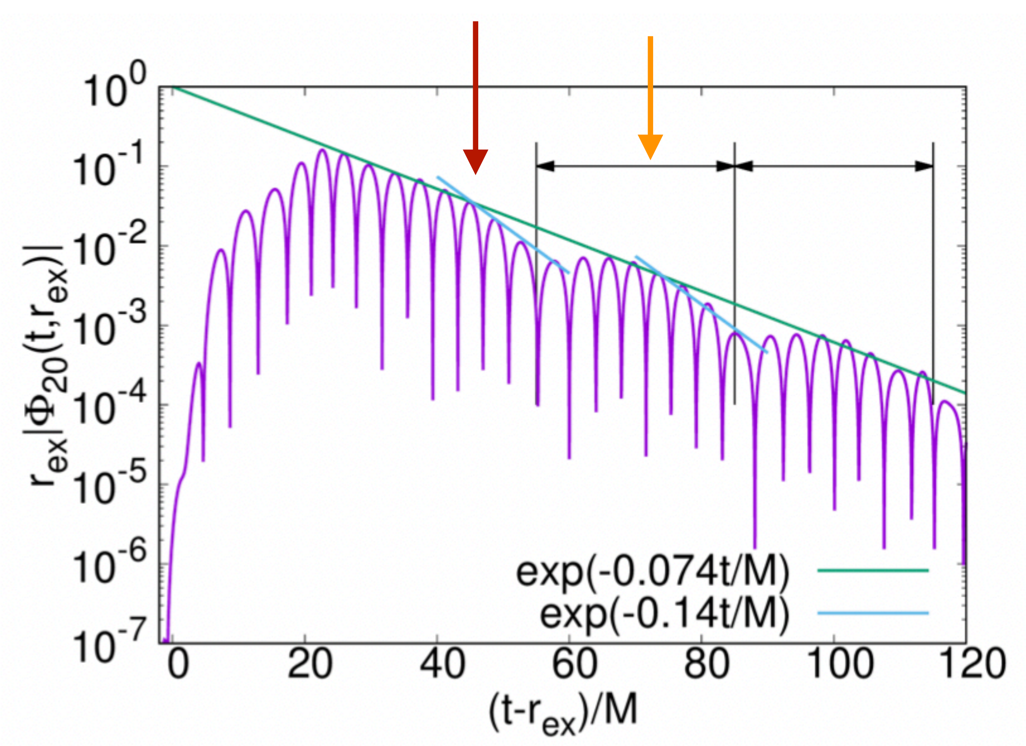
$$\Phi(0,x) \propto e^{-\left(\frac{r-r_0}{\sigma}\right)^2}$$

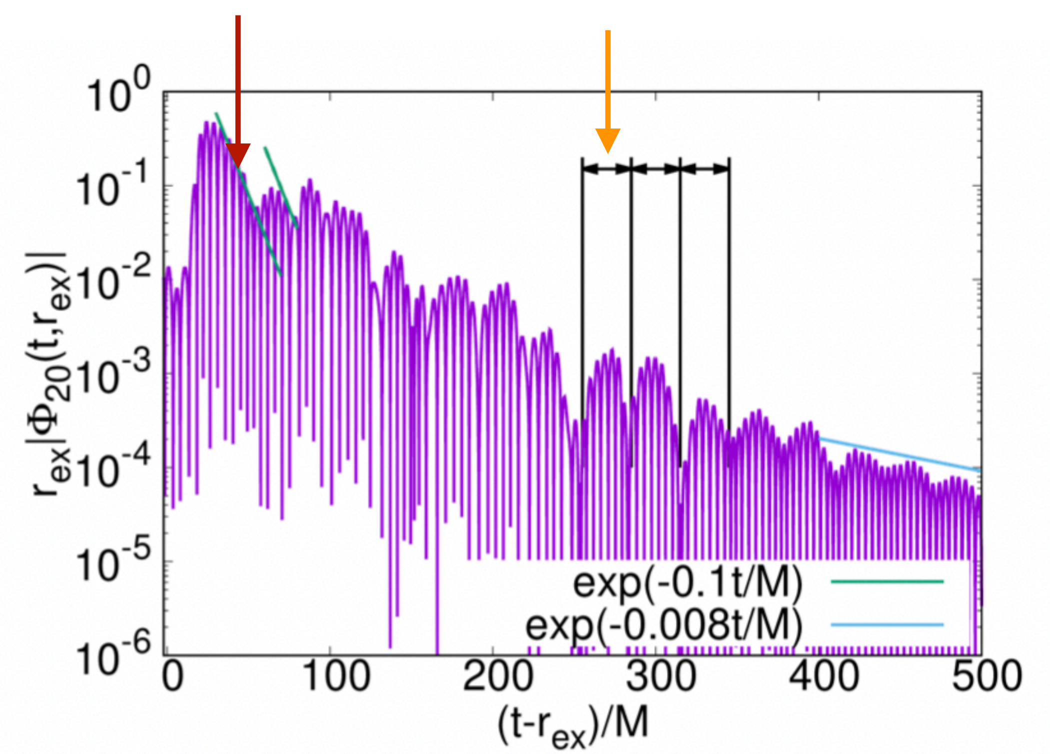
$L = 0.67M$ $L = 0.27M$

QNM dominates soon.



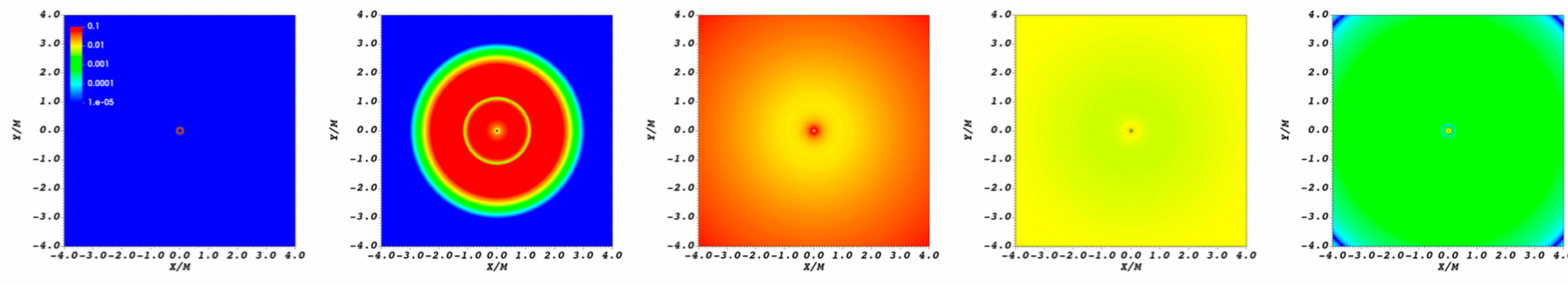




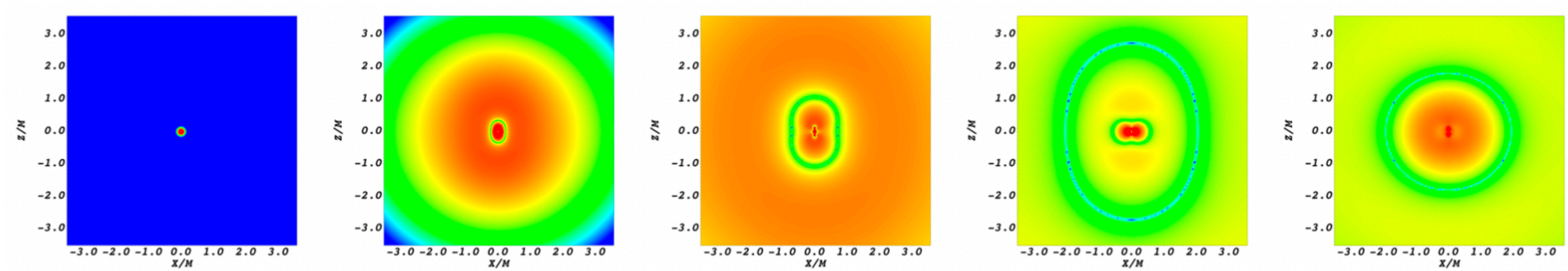


Time evolution for each case

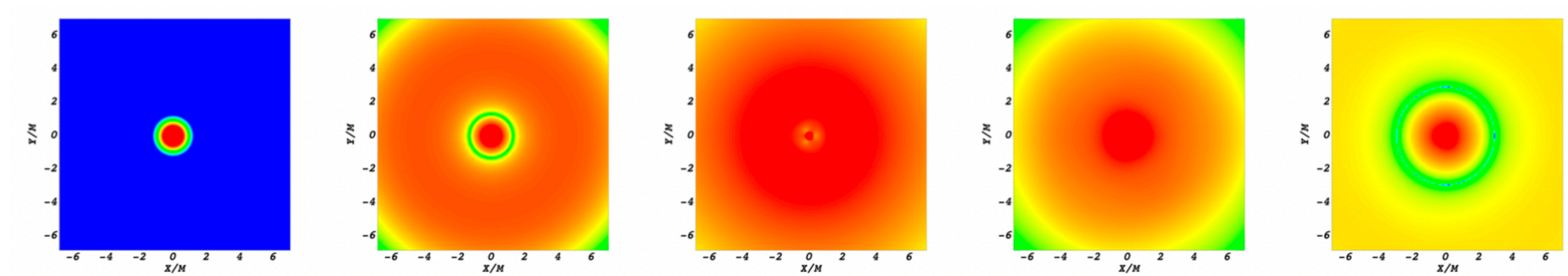
4-charge BH $\longrightarrow$ t



an axisymmetric fuzzball



scaling fuzzball



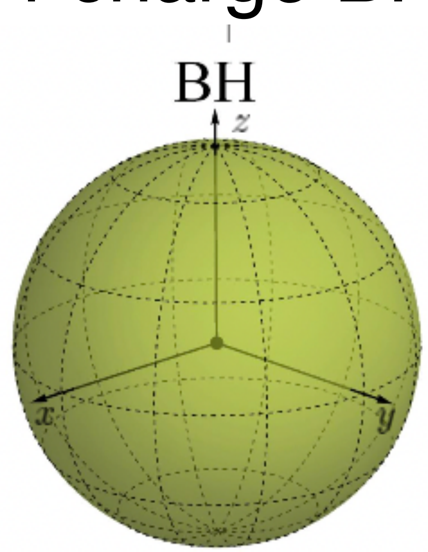
Outline

1. Introduction
2. Black hole ringdown
3. Exotic compact object ringdown
4. Numerical method
5. Results
6. Summary & Discussion

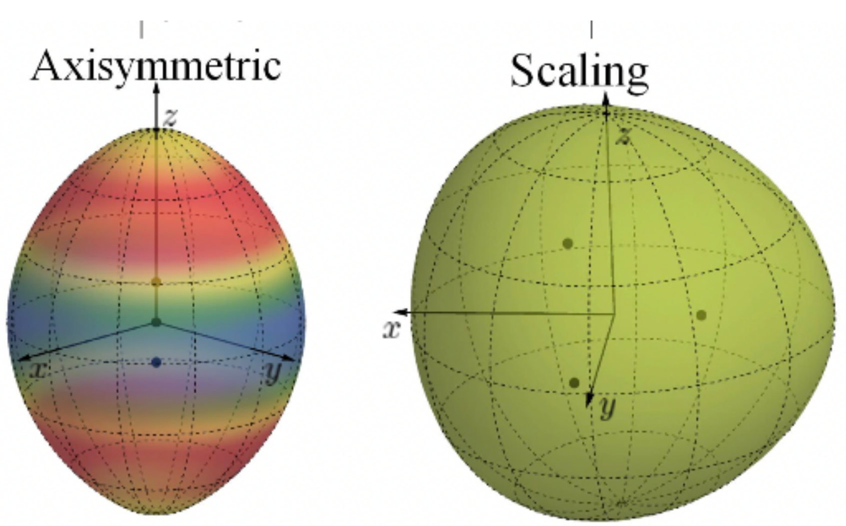
Summary

- 4 charge BH
  - 1+1 dim. simulation of test massless scalar field
  - QNM using direct integration and WKB approximation.
- Fuzzball
  - 3+1 dim. simulations of test massless scalar field
  - ringdown associated with light ring
  - echo
  - QNM of fuzzball
- Fuzzball is the linearly stable (no erogregion).

4 charge BH

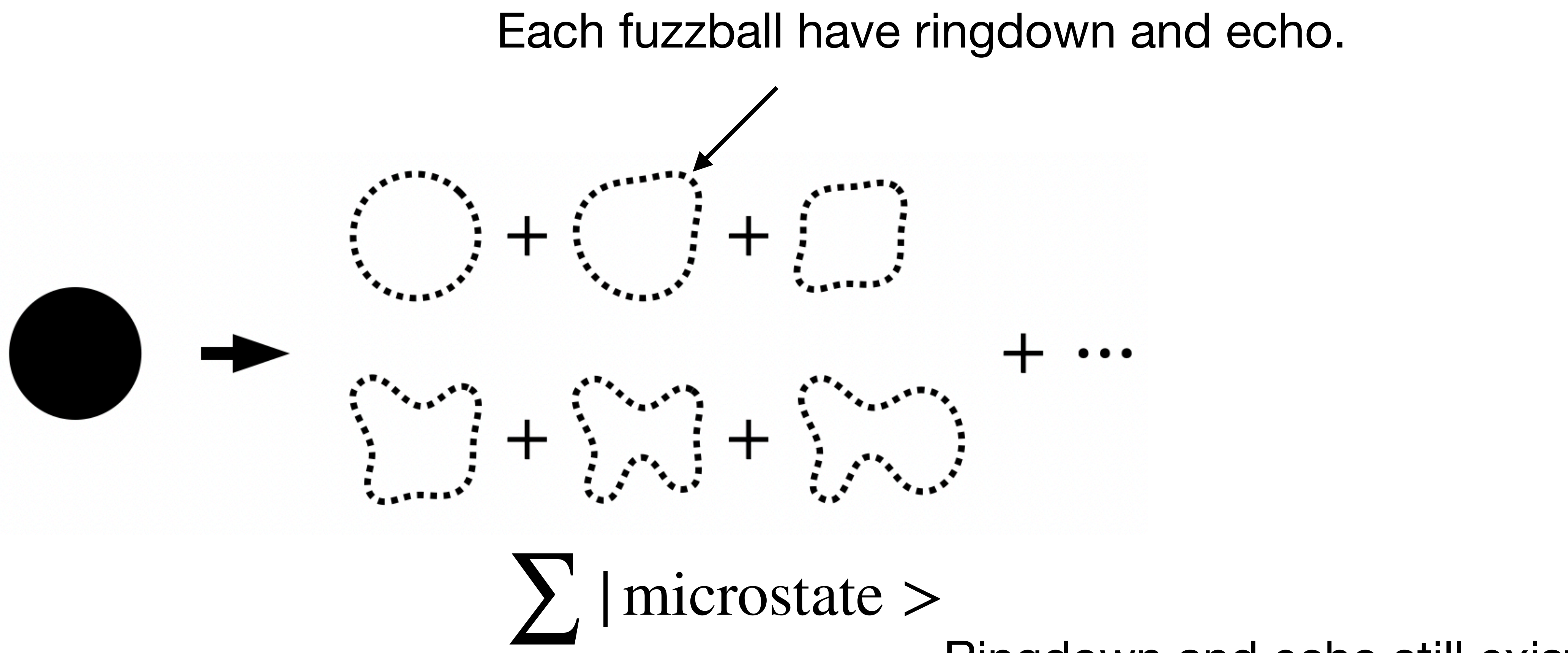


Fuzzball



Discussion

- Physical interpretation ??



- Boundary condition on center ??
  - We regularized the singularities.
  - But, the boundary condition should be determined from 5 dim. picture.

Thank you