Black holes in scalar Gauss-Bonnet gravity: quasi-normal modes and gravitational wave emission

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Modified gravity theories

Most of the modifications and extensions of GW which have been proposed so far can be expressed in terms of new fundamental fields (in particular, scalar fields) in the gravitational sector of the theory.

Arguably, the simplest of such modifications are the scalar-tensor theories

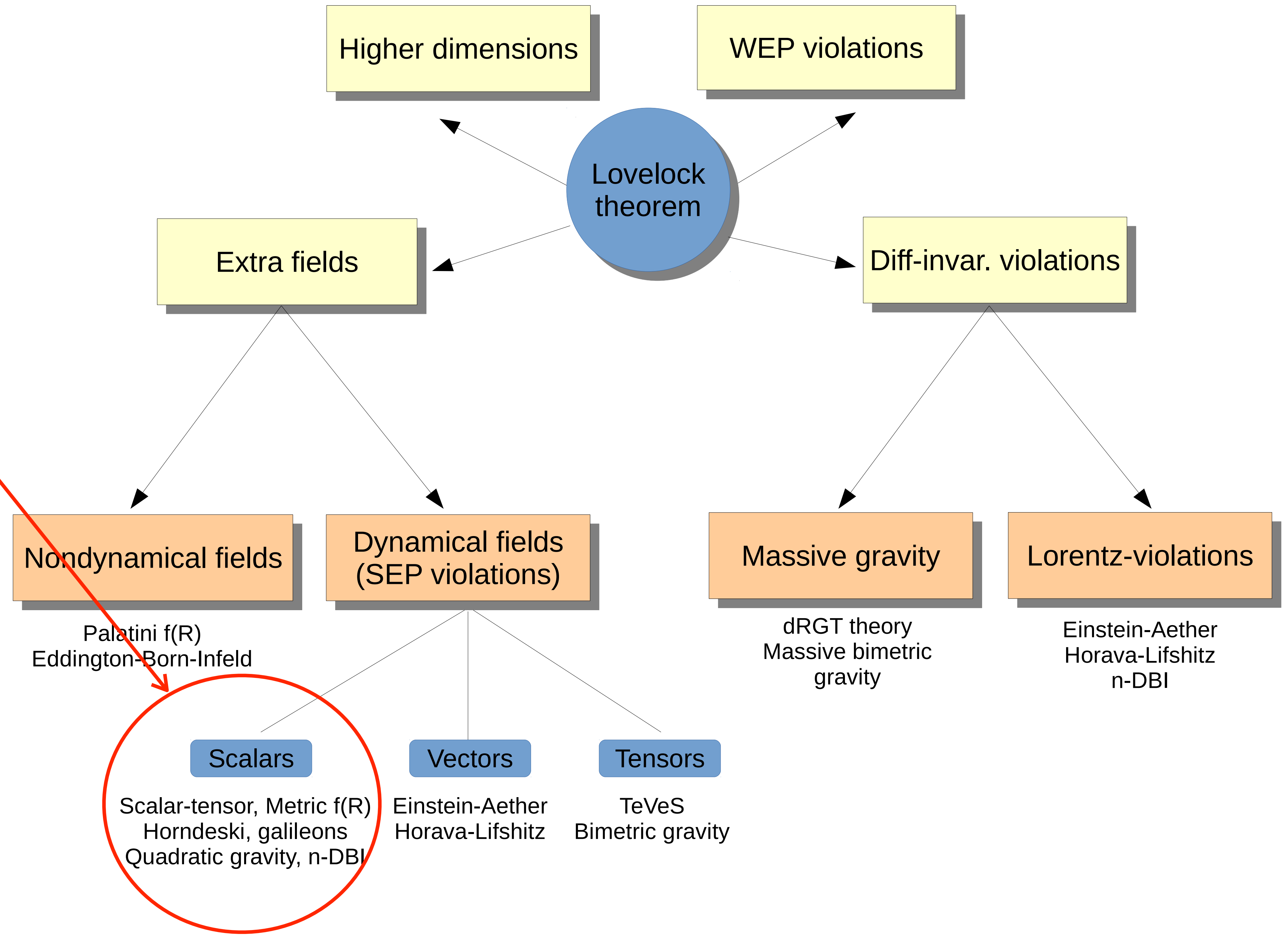
- Simplest ("Bergmann-Wagoner") scalar-tensor theories:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - U(\phi)\right] + S_M[\Psi, g_{\mu\nu}]$$

Note that in these theories *no-hair theorem* applies: BHs as in GR!

- Quadratic gravity: scalar-tensor theories with quadratic curvature terms:

$$S = \frac{1}{16\pi G}\int \sqrt{-g}d^4x\Big[R - 2\nabla_a\phi\nabla^a\phi - V(\phi) + f_1(\phi)R^2 + f_2(\phi)R_{\mu\nu}R^{\mu\nu} + f_3(\phi)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + f_4(\phi)^*RR\Big] + S_{\rm mat}\left[\Psi,\, \gamma(\phi)g_{\mu\nu}\right]\,,$$



Berti et al., CQG 32, 243001, '15

E-H action is the first term in an expansion containing all possible curvature terms, as suggested by low-energy effective string theories.

In general these theories have ghosts and other pathologies, with the exception of a particular combination of the curvature invariants: scalar Gauss-Bonnet gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{GB}f(\phi)\mathcal{R}^2_{GB}\right\} \qquad \left(\mathcal{R}^2_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$$

Modified gravity theories

sGB gravity can also be obtained from the class of Horndeski gravity theories, which are all scalar-tensor theories with second-order-in-time field equations (and thus free from the so-called Ostrogradski instability):

$$S = \int d^4x\sqrt{-g}\,\{K(\phi,X) - G_3(\phi,x)\Box\phi + G_4(\phi,X)R + G_{4,X}\left((\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right)$$

$$+G_5 G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - G_{5,X}\frac{1}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla_\rho\phi)(\nabla^\mu\nabla_\sigma\phi)(\nabla^\rho\nabla^\sigma\phi)\right]\Bigg\}$$

$$+ S_{matter}(\psi_m, g_{\mu\nu}) \qquad (X \equiv \partial_\mu\phi\partial^\mu\phi)$$

In the decoupling limit ($m_s \to 0$ with $m_s^2 M_{pl}$ finite) reduces to *Galileon theory:* scalar field action in flat space with derivative interactions

$$S_{\rm gal}[\pi] = \int d^4x\left\{-\frac{3}{4}(\partial\pi)^2 + \sum_{n=3}^{5} c_n \mathcal{L}_n^{(g)}\left[\frac{1}{\Lambda_3^3}\partial_\mu\partial_\nu\pi\right] + \frac{g_1}{M_{\rm Pl}}\pi T + \frac{g_2}{M_{\rm Pl}\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}\right\}$$

A particular combination of the functions G_i (including G_5) leads to scalar Gauss-Bonnet gravity :

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{GB} f(\phi)\mathcal{R}^2_{GB}\right\} \qquad (\mathcal{R}^2_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

↑ Gauss-Bonnet term: total derivative!

- they are the simplest scalar-tensor gravity theories in which no-hair theorems do not apply (in the case of shift-symmetric theories, sGB gravity are *the only* Horndeski theories violating no-hair theorems! *[Sotiriou & Zhou, PRL '14]*)
- GR deviations appear at large curvatures => no constraints from binary pulsars, need GW
- can arise from string-theory compactifications ($f(\phi)=e^\phi$: Einstein-dilaton Gauss-Bonnet)
- the Gauss-Bonnet term can be seen as an effective-field-theory contribution

but:

- in sGB gravity, scalar field can not have cosmological interpretation (ruled out by GW170817)
- dimensionful coupling constant α_{GB} needs to be at least of order km^2 for observable GW signature (EFT interpretation requires new scale in the theory besides l_P)

sGB gravity theories

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{GB} f(\phi)\mathcal{R}^2_{GB}\right\}$$

Field equations:

$$\Box\phi = -\alpha_{GB} f_{,\phi}\mathcal{R}^2_{GB}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi\right) - \alpha_{GB} g_{\gamma(\mu}g_{\nu)\delta}\varepsilon^{\sigma\delta\rho\chi}\nabla_\tau[\,^*R^{\gamma\tau}{}_{\rho\chi}\partial_\sigma f]$$

Stationary BH solutions:

- if f'(ϕ)≠0 then for each (M,J) there is one stationary BH solution, with non-trivial scalar field configuration (*hairy BH*)
  - scalar field of stationary BHs chacterized by its scalar charge D: $\phi = const. + \frac{D}{r} + O\left(\frac{1}{r^2}\right)$

    for instance, a non-rotating BH has scalar charge $D = \frac{\alpha_{GB}}{2GM} + O\left(\alpha^2_{GB}\right)$
  - stationary BHs only exist for $\zeta = \frac{\alpha_{GB}}{(GM)^2} \leq \zeta^{max} \sim 1$ otherwise the BH becomes a naked singularity *(Sotiriou & Zhou PRD '14)* => minimum mass!
  - examples: (i) Einstein-dilaton Gauss-Bonnet (EdGB) gravity $\alpha_{GB} = \frac{\alpha'}{g}$ $f(\phi) = \frac{1}{4}e^\phi$

    (ii) shift-symmetric Gauss-Bonnet gravity $f(\phi) = \phi$

- if f'(ϕ)=0 for some ϕ, then the Kerr BH with ϕ=const. *is a solution of the field equations*.
  For some values of the mass or angular momentum, scalarized solutions can exist as well, and Kerr BHs naturally grow scalar fields (*spontaneous scalarization*)
  (Silva et al., PRL '18; Doneva et Yazadjiev, PRL '18; Dima et al., PRL '20; Herdeiro et al., PRL '21)

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EdGB gravity

(Mignemi & Stewart PRD '93; Kanti et al. PRD '96; Torii & Maeda, PRD '97; Pani & Cardoso PRD '09; Yunes & Stein PRD '11; Kleinhaus et al. PRL '11)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{GB} f(\phi)\mathcal{R}^2_{GB}\right\} \quad \text{with} \quad \alpha_{GB} = \frac{\alpha'}{g} \quad f(\phi) = \frac{1}{4}e^{\phi}$$

- The GB term arises in low-energy EFT limit of heterotic ST *(Gross & Sloane, NPB '87, Metsaev & Tseytlin NPB '87)*
- $f'(\phi) \neq 0$: for each choice of (M,J), one stationary BH solution with $D=\alpha_{GB}/(2GM)$
- (non-rotating) BHs exist for $\alpha_{GB}/(GM)^2 < 0.619$
- existing bounds: $\sqrt{\alpha_{GB}} \lesssim 5\,\mathrm{km}$ from LIGO/Virgo observations *(Perkins et al., PRD '21)*
- future bounds: ~ one order of magnitude stronger from LISA *(Maselli et al., Nature Astr. '22)*
- for low values of the coupling and thus for small amplitudes of the scalar field, $f(\phi) = \frac{1}{4}e^{\phi} \simeq \frac{1}{4}\phi$: equivalent to shift-symmetric sGB gravity *(Sotiriou & Zhou PRL '14, Barausse & Yagi PRL '15)*

In the rest of the talk I will discuss two recent developments concerning EdGB gravity:

1. QNMs of rotating BHs in EdGB gravity
2. how to detect the scalar charge of EdGB gravity (and other scalar-tensor theories) with LISA

Many other aspects, such as the modelling of inspiral (post-Newtonian expansion) and merger (numerical relativity) not discussed in this talk.

Quasi-normal modes of rotating BHs in EdGB gravity

We know BH QNMs in very few theories, and - until recently - only for non-rotating BHs.
We need examples of QNM modifications in specific theories, to improve searches for GR deviations in GW data from ringdown!
EdGB gravity is now the only example of computation of QNMs of rotating BHs to second order in spin (another example exists at first order)

Perturbation theory:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$
$$\phi = \phi^{(0)} + \delta\phi$$

$\longrightarrow$

$$\Box\phi = -\frac{\alpha_{GB}}{4} e^{\phi}\mathcal{R}^2_{GB}$$
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi\right) - \frac{\alpha_{GB}}{4} g_{\gamma(\mu}g_{\nu)\delta}\varepsilon^{\sigma\delta\rho\chi}\nabla_\tau[{}^*R^{\gamma\tau}{}_{\rho\chi}e^{\phi}\partial_\sigma\phi]$$

Background:
stationary, rotating BH
with (M,J)

$$ds^{(0)\,2} = -A(r,\theta)dt^2 + B^{-1}(r,\theta)dr^2 + C(r,\theta)r^2[d\theta^2 + \sin^2\theta(d\varphi - \hat{\omega}(r,\theta)dt)^2] \quad ; \quad \phi^{(0)}(r,\theta)$$

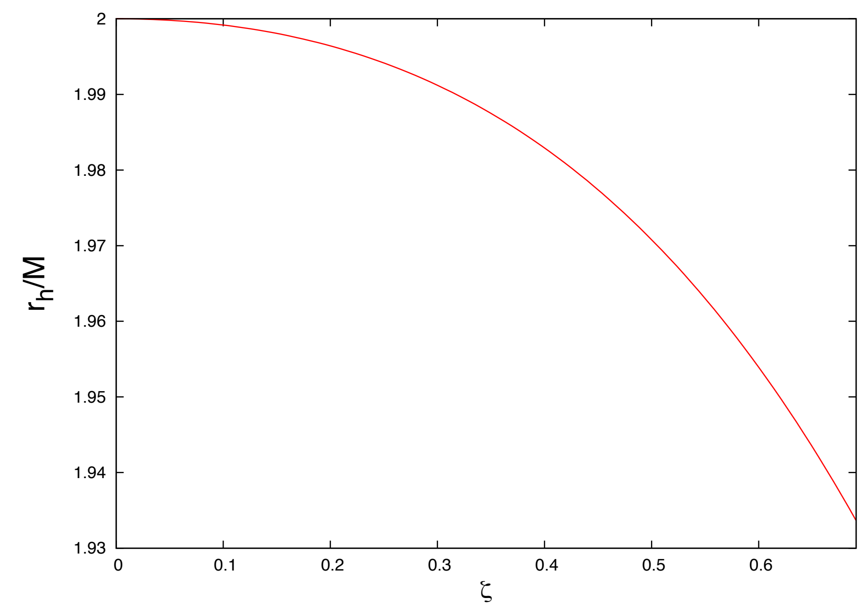
It can be found numerically *(Kanti et al. PRD '96)*, or in terms of polynomial expansions *(Maselli et al., PRD '15)*:
[G=c=1 units]

$$A(r,\theta) = -1 + \frac{2M}{r} + \frac{M^3}{r^3}\left[\chi^2 P_2(\cos\theta) - \frac{\zeta^2}{48} + \ldots\right] + \ldots$$
$$\phi^{(0)}(r,\theta) = \phi_\infty + \frac{M}{r}\left[\frac{1}{2}\zeta + \frac{73}{480}\zeta^2 - \frac{1}{8}\zeta\chi^2 + \ldots\right]\ldots$$

where we have defined
the dimensionless coupling $\zeta = \frac{\alpha_{GB}}{M^2}$

and the dimensionless spin $\chi = \frac{J}{M^2}$

Near the horizon: $A(r,\theta) = \sum_{i=0}^{\infty} a_i (r - r_h)^i$ where $\frac{r_h}{M} = 2 - \frac{1}{2}\chi^2 + \frac{49}{640}\zeta^2 + \ldots$



Note that this is a small coupling and slow rotation expansion: does not converge for rapidly rotating BHs and for ζ~1!

Quasi-normal modes of rotating BHs in EdGB gravity

Then, we expand the perturbations in tensor spherical harmonics, finding a set of axial and polar perturbation functions.

1) Non-rotating BHs *(Blasquez-Salcedo et al., PRD ’16)*

Spherically symmetric background => harmonic components with different values of l,m and with polar and axial parities are *decoupled*

$$\delta\phi(t,r,\theta,\varphi) = \frac{1}{r}\Phi^{lm}(t,r)Y^{lm}(\theta,\varphi)$$

$$h_{\mu\nu}(t,r,\theta,\phi) = \begin{pmatrix} A(r)H_0^{lm}(t,r) & H_1^{lm}(t,r) & h_0^{lm}(t,r)\frac{1}{\sin\theta}\partial_\varphi & h_0^{lm}(t,r)\sin\theta\partial_\theta \\ H_1^{lm}(t,r) & B^{-1}(r)H_2^{lm}(t,r) & h_1^{lm}(t,r)\frac{1}{\sin\theta}\partial_\varphi & h_1^{lm}(t,r)\sin\theta\partial_\theta \\ h_0^{lm}(t,r)\frac{1}{\sin\theta}\partial_\varphi & h_1^{lm}(t,r)\frac{1}{\sin\theta}\partial_\varphi & r^2K^{lm}(t,r) & 0 \\ h_0^{lm}(t,r)\sin\theta\partial_\theta & h_1^{lm}(t,r)\sin\theta\partial_\theta & 0 & r^2\sin^2\theta K^{lm}(t,r) \end{pmatrix} Y^{lm}(\theta,\varphi)$$

Linearized field equations and Fourier transform in time leads to ODEs for Φ^{lm}, H_A^{lm}, h_A^{lm}, K^{lm} which are solved for each l,m with Sommerfeld boundary conditions, leading to the QNM complex frequencies:

$$K' + \frac{\alpha(B-1)e^{\phi_0}}{r^2(\alpha Be^{\phi_0}\phi_0' - r)}\phi_1' + \left(\frac{1}{r} - \frac{A'}{2A}\right)K - i\frac{(\Lambda+1)}{r^2\omega}H_1 + \frac{(\alpha(3B-1)e^{\phi_0}\phi_0' - 2r)}{2r(r-\alpha Be^{\phi_0}\phi_0')}H_2 + \frac{(\alpha(B-1)e^{\phi_0}(rA' + A(2-2r\phi_0')) + Ar^3\phi_0')}{2Ar^3(r-\alpha Be^{\phi_0}\phi_0')}\phi_1 = 0$$

$$H_1' + \left(\frac{2(B-1)B(rA'-2A)}{Ar} - \frac{r(B^2r\phi_0'^2 + 2(B-1)B')}{r-\alpha Be^{\phi_0}\phi_0'} - 4B'\right)\frac{H_1}{4(B-1)B} + \frac{i\omega}{B}H_2 - ir\omega\left(\frac{4B'}{B} + \frac{r^2\phi_0'^2}{r-\alpha Be^{\phi_0}\phi_0'}\right)\frac{K}{4(B-1)} - \frac{i\alpha\omega e^{\phi_0}B'}{Br^2 - \alpha B^2re^{\phi_0}\phi_0'}\phi_1 = 0,$$

$$H_0' + \frac{r(\alpha Be^{\phi_0}A'\phi_0' - 2A)}{2A(r-\alpha Be^{\phi_0}\phi_0')}K' + \left(\frac{2(A-rA')}{r-\alpha Be^{\phi_0}\phi_0'} + 3A'\right)\frac{H_2}{2A} + \frac{\alpha Be^{\phi_0}A'(r\phi_0'-2) - Ar^2\phi_0'}{Ar^2(r-\alpha Be^{\phi_0}\phi_0')}\phi_1 + \frac{\alpha Be^{\phi_0}A'}{Ar^2 - \alpha ABre^{\phi_0}\phi_0'}\phi_1' + \left(\frac{A'}{2A} - \frac{1}{r}\right)H_0 + \frac{i\omega}{A}H_1 = 0,$$

$$H_2(2A - 3\alpha B^2e^{\phi_0}A'\phi_0') - BrK'(A'(r-3\alpha Be^{\phi_0}\phi_0') + 2A) + K(\alpha Be^{\phi_0}\phi_0'(2r\omega^2 - \Lambda A') + 2A\Lambda - 2r^2\omega^2) + \frac{\phi_1(\alpha e^{\phi_0}(B(2r\omega^2 - A'((1-3B)r\phi_0' + 3B + 2\Lambda + 1)) - 2r\omega^2) - ABr^2\phi_0')}{r^2} + \phi_1'\left(\frac{\alpha B(3B-1)e^{\phi_0}A'}{r} + ABr\phi_0'\right) - \frac{2AH_0(\Lambda+1)(r-\alpha Be^{\phi_0}\phi_0')}{r} + ABH_0'(2r - \alpha(3B-1)e^{\phi_0}\phi_0') + 2iBH_1\omega(2r - \alpha(3B-1)e^{\phi_0}\phi_0') = -16\pi ABr^2A_{lm},$$

$$H_2 - \frac{AH_0(4B'(r-\alpha Be^{\phi_0}\phi_0') + Br^2\phi_0'^2)}{2(B-1)(\alpha Be^{\phi_0}A'\phi_0' - 2A)} + \frac{\alpha e^{\phi_0}\phi_1(AA'B' - B((A')^2 - 2AA''))}{Ar(\alpha Be^{\phi_0}A'\phi_0' - 2A)} = 0,$$

$$\frac{H_2(4\alpha Be^{\phi_0}\phi_0'((B-1)(\Lambda+1) - rB') + 4r(B(rB' + B - \Lambda - 3) + \Lambda + 2) + B^2r^3{\phi_0}'^2)}{4(B-1)Br^2(r-\alpha Be^{\phi_0}\phi_0')} + \frac{K'\left(\frac{r(B^2(-r)\phi_0'^2 - 2(B-1)B')}{r-\alpha Be^{\phi_0}\phi_0'} - 4B' + \frac{8(B-1)B}{r}\right)}{4(B-1)B} + \frac{\Lambda K\left(\frac{4B'}{B} + \frac{r^2\phi_0'^2}{r-\alpha Be^{\phi_0}\phi_0'}\right)}{4(B-1)r} + \frac{\phi_1'(Br^3\phi_0' - \alpha e^{\phi_0}(B(3rB' + 4(B-1)r\phi_0' - 4B + 4) - rB'))}{2Br^3(r-\alpha Be^{\phi_0}\phi_0')} + \frac{\phi_1(2\alpha e^{\phi_0}((2\Lambda+1)rB' + B(3rB' + 4(B-1)r\phi_0' - 4B + 4)) - r^2(4rB' + B(r\phi_0'(r\phi_0'+2) + 4) - 4))}{4Br^4(r-\alpha Be^{\phi_0}\phi_0')} + \frac{H_2'(\alpha(3B-1)e^{\phi_0}\phi_0' - 2r)}{2r(r-\alpha Be^{\phi_0}\phi_0')} + \frac{\alpha(B-1)e^{\phi_0}\phi_1''}{r^2(\alpha Be^{\phi_0}\phi_0' - r)} + K'' = 0$$

$$\frac{d^2\phi_1}{dr_*^2} + \left(\frac{BrA'(r\phi_0'+1) + A(rB'(r\phi_0'+1) + 2Br(r\phi_0'' + 2\phi_0') + 4(\Lambda+1))}{2r^2} - \omega^2\right)\phi_1 + \alpha B^2e^{\phi_0}A'K'' - \frac{\alpha A(B-1)Be^{\phi_0}H_0''}{r} - \frac{H_0'(\alpha e^{\phi_0}(2(B-1)BA' + A(3B-1)B') + ABr^2\phi_0')}{2r} - \frac{BH_2'(\alpha(3B-1)e^{\phi_0}A' + Ar^2\phi_0')}{2r} + \alpha H_2e^{\phi_0}(B^2r(A')^2 + A(-B(2BrA'' + A'(3rB' + 2\Lambda + 2) - 2r\omega^2) - 2r\omega^2)) + \frac{\alpha AH_0(\Lambda+1)e^{\phi_0}B'}{r^2} + \frac{BK'(\alpha e^{\phi_0}(A(2BrA'' + A'(3rB' + 4B)) - Br(A')^2) + 2A^2r^2\phi_0')}{2Ar} - iH_1\left(\frac{\alpha(3B-1)\omega e^{\phi_0}B'}{r} + Br\omega\phi_0'\right) + \frac{\alpha Ke^{\phi_0}(AB'(2r\omega^2 - \Lambda A') + B\Lambda((A')^2 - 2AA''))}{2Ar} - \frac{2i\alpha(B-1)B\omega e^{\phi_0}H_1'}{r} = 0.$$

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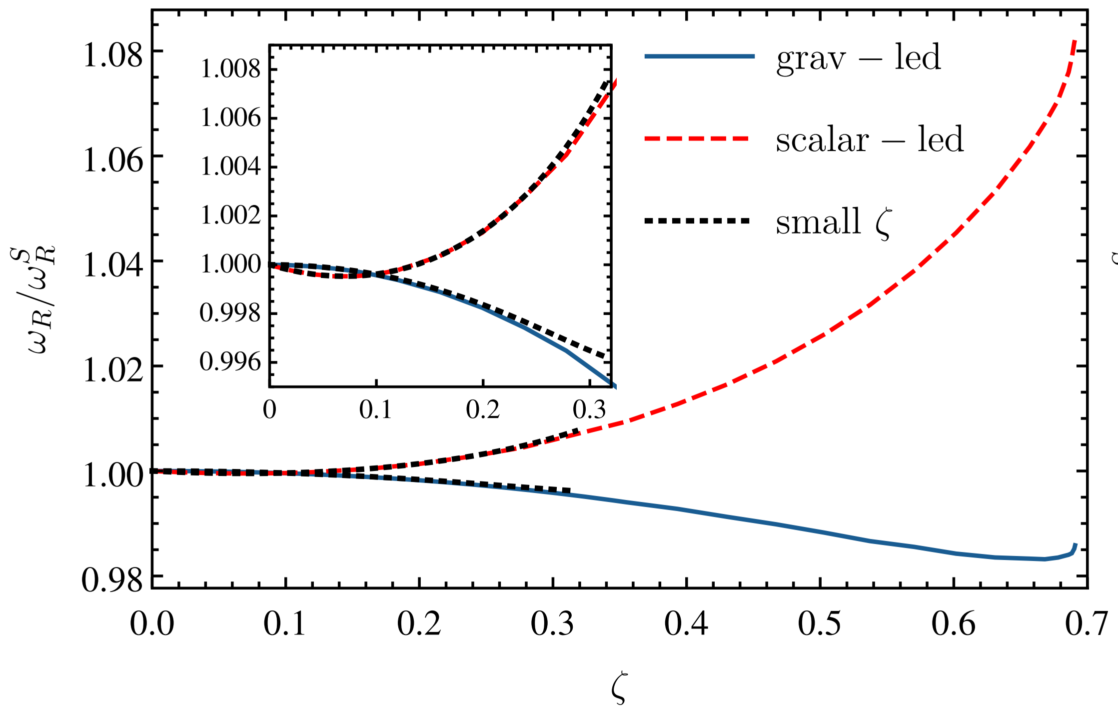
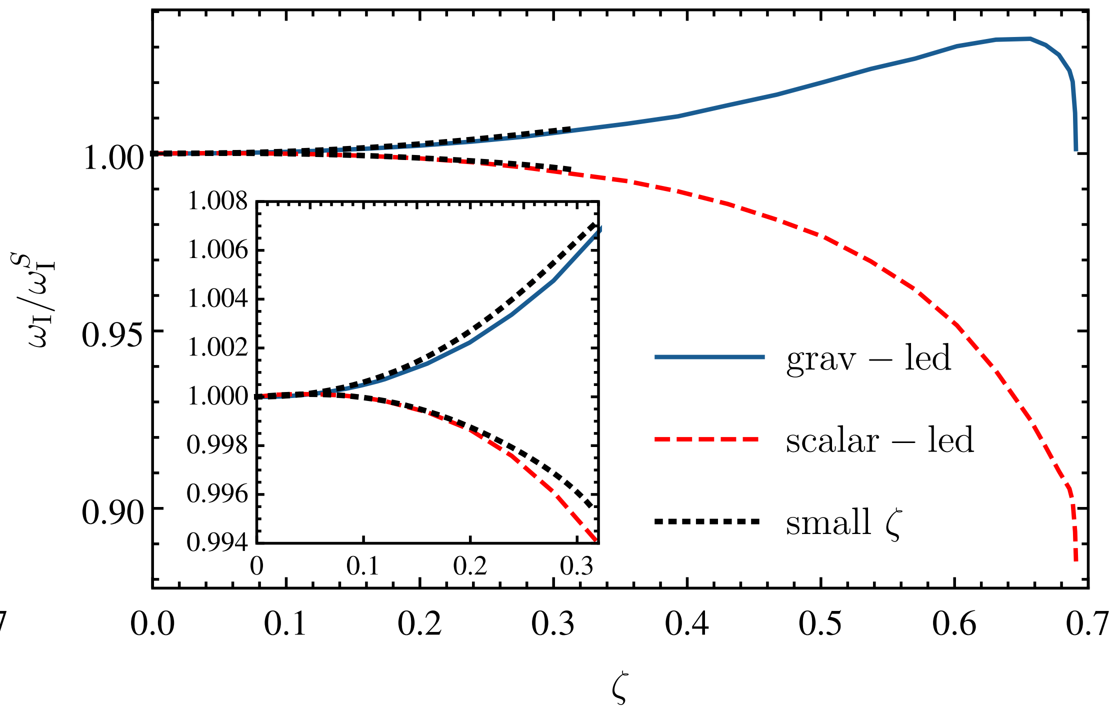
Linearized field equations and Fourier transform in time leads to ODEs for Φ^{lm}, H_A^{lm}, h_A^{lm}, K^{lm} which are solved for each l,m with Sommerfeld boundary conditions, leading to the QNM complex frequencies:

- two classes of gravitational QNMs: scalar-led and gravitational-led
- $O(\zeta^2)$ shifts of the gravitational and scalar QNM frequencies of GR





Quasi-normal modes of rotating BHs in EdGB gravity

Then, we expand the perturbations in tensor spherical harmonics, finding a set of axial and polar perturbation functions.

2) Rotating BHs at **first order** in the spin *(Pierini & Gualtieri, PRD '21)*

[for a similar computation in a different theory (DCS gravity) see Wagle et al.'21]

Non spherical background => couplings between perturbations with different parities and with different l's!

General structure of the equations:

$$\mathcal{P}_{lm} + \chi\, m\, \bar{\mathcal{P}}_{lm} + \chi(Q_{lm}\tilde{\mathcal{A}}_{l-1\,m} + Q_{l+1\,m}\tilde{\mathcal{A}}_{l+1\,m}) + O(\chi^2) = 0$$

$$\mathcal{A}_{lm} + \chi\, m\, \bar{\mathcal{A}}_{lm} + \chi(Q_{lm}\tilde{\mathcal{P}}_{l-1\,m} + Q_{l+1\,m}\tilde{\mathcal{P}}_{l+1\,m}) + O(\chi^2) = 0$$

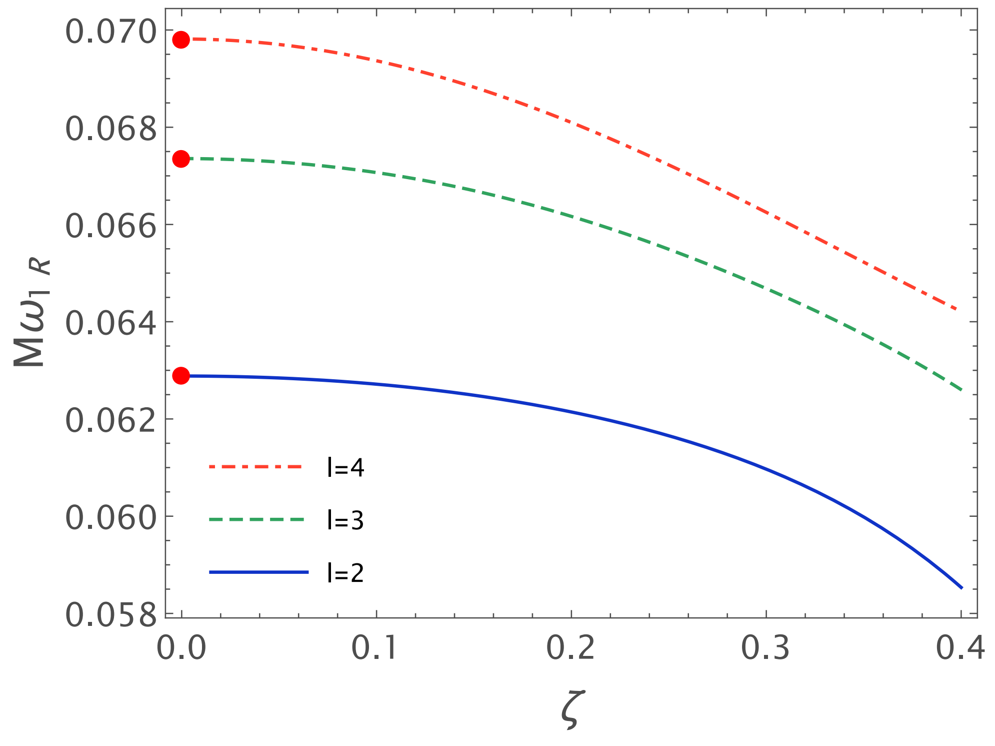
$\mathcal{P}_{lm},\ \mathcal{A}_{lm},$ etc: combinations of the polar and axial perturbation functions and their derivatives, with background-dependent coefficients

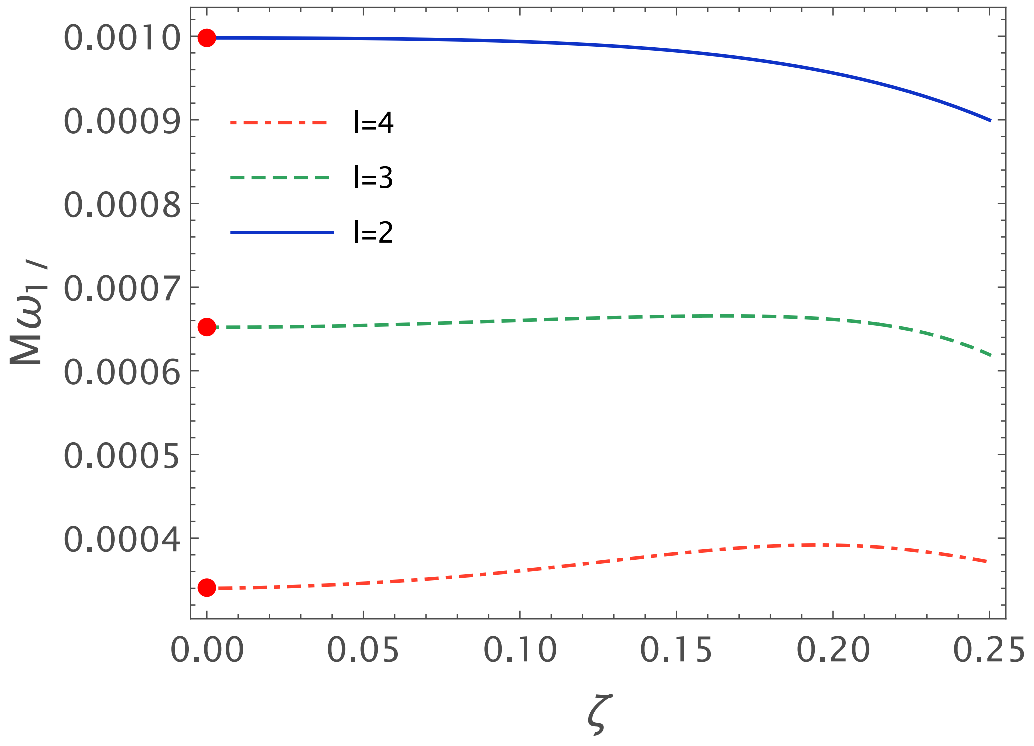
In practice, it can be shown that at first order in the spin the couplings do not affect the QNMs!

Numerical integration of the ODEs with Sommerfeld boundary conditions similar to that of GR (new terms in ζ in the equations, coupling with scalar field)

QNM complex frequencies: $$\omega^{nlm}(\chi,\zeta) = \omega_0^{nl}(\zeta) + \chi\, m\omega_1^{nl}(\zeta) + \mathcal{O}(\chi^2)$$

These results suggest that the GR deviations in the QNMs are significanty magnified by rotation, but first-order computation only accurate for very low value of the spin, and may miss important effects only arising at second order.





Quasi-normal modes of rotating BHs in EdGB gravity

Then, we expand the perturbations in tensor spherical harmonics, finding a set of axial and polar perturbation functions.

3) Rotating BHs at **second order** in the spin *(Pierini & Gualtieri, in preparation)*

Couplings have to be included in the QNM computation! (see e.g. *Pani, IJMPA '13*)

$$\mathcal{P}_\ell + \chi m \bar{\mathcal{P}}_\ell + \chi^2 \hat{\mathcal{P}}_\ell + m^2 \chi^2 \bar{\bar{\mathcal{P}}}_\ell + \chi \left(Q_\ell \tilde{\mathcal{A}}_{\ell-1} + Q_{\ell+1} \tilde{\mathcal{A}}_{\ell+1} \right) + O(\chi^3) = 0$$

$$\mathcal{A}_{\ell+1} + \chi m \bar{\mathcal{A}}_{\ell+1} + \chi \; Q_{\ell+1} \tilde{\mathcal{P}}_\ell + m \; \chi^2 \; Q_{\ell+1} \check{\mathcal{P}}_\ell + O(\chi^3) = 0$$

$$\mathcal{A}_{\ell-1} + \chi m \bar{\mathcal{A}}_{\ell-1} + \chi \; Q_\ell \tilde{\mathcal{P}}_\ell + m \; \chi^2 \; Q_\ell \check{\mathcal{P}}_\ell + O(\chi^3) = 0$$

In the numerical integration we have to solve at the same time equations with different parities and different values of l:

$$\frac{d}{dr}\Xi + P\Xi = 0$$ where $$\Xi_\ell = \{H1_\ell, K_\ell, \phi_\ell, \phi'_\ell, h0_{\ell-1}, h1_{\ell-1}, h0_{\ell+1}, h1_{\ell+1}\}$$ and P is an 8x8 matrix.

Each perturbation function satisfies boundary conditions

$$A^{lm}_{\rm in} e^{-ik_{\rm H} r_*} + A^{lm}_{\rm out} e^{ik_{\rm H} r_*} \qquad (r \to r_{\rm h})$$

$$A^{lm}_{\rm in} e^{-i\omega r_*} + A^{lm}_{\rm out} e^{i\omega r_*} \qquad (r \to \infty)$$

where

$$k_{\rm H} = \omega - m\Omega_{\rm H}$$

$$\Omega_{\rm H} = -\lim_{r\to r_{\rm h}} \frac{g_{t\varphi}}{g_{\varphi\varphi}}$$

We find 8 solutions sayisfying outgoung or ingoing b.c., forming a matrix $X^{nlm}(\omega^{nlm})$, and impose $\det X^{nlm}(\omega^{nlm}) = 0$

finding the QNMs $$\omega^{nlm}(\chi, \zeta) = \omega_0^{nl}(\zeta) + \chi\, m \omega_1^{nl}(\zeta) + \chi^2[\omega_{2a}^{nl}(\zeta) + m^2 \omega_{2b}^{nl}(\zeta)] + \mathcal{O}(\chi^3)$$

Quasi-normal modes of rotating BHs in EdGB gravity

Then, we expand the perturbations in tensor spherical harmonics, finding a set of axial and polar perturbation functions.

3) Rotating BHs at **second order** in the spin *(Pierini & Gualtieri, in preparation)*

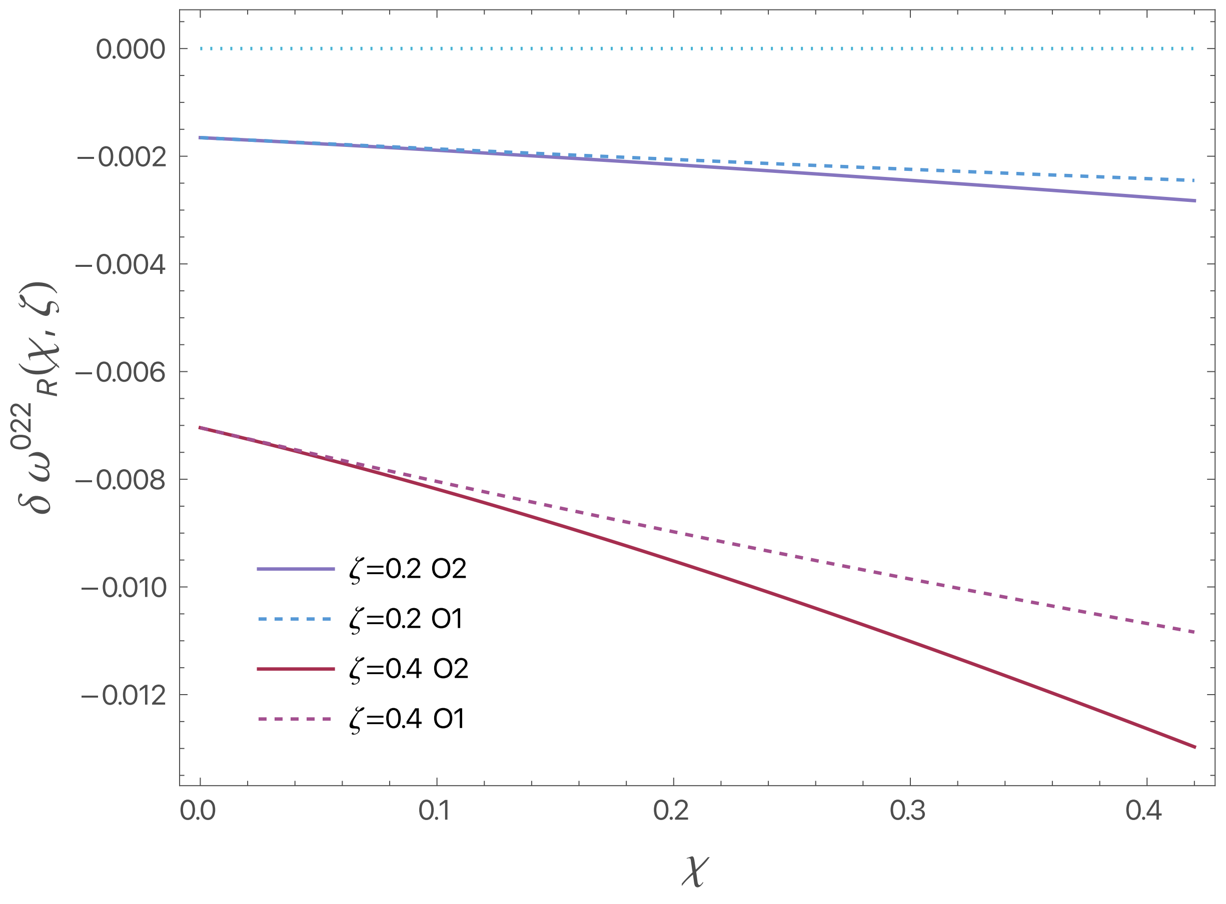
Couplings have to be included in the QNM computation! (see e.g. *Pani, IJMPA '13*)

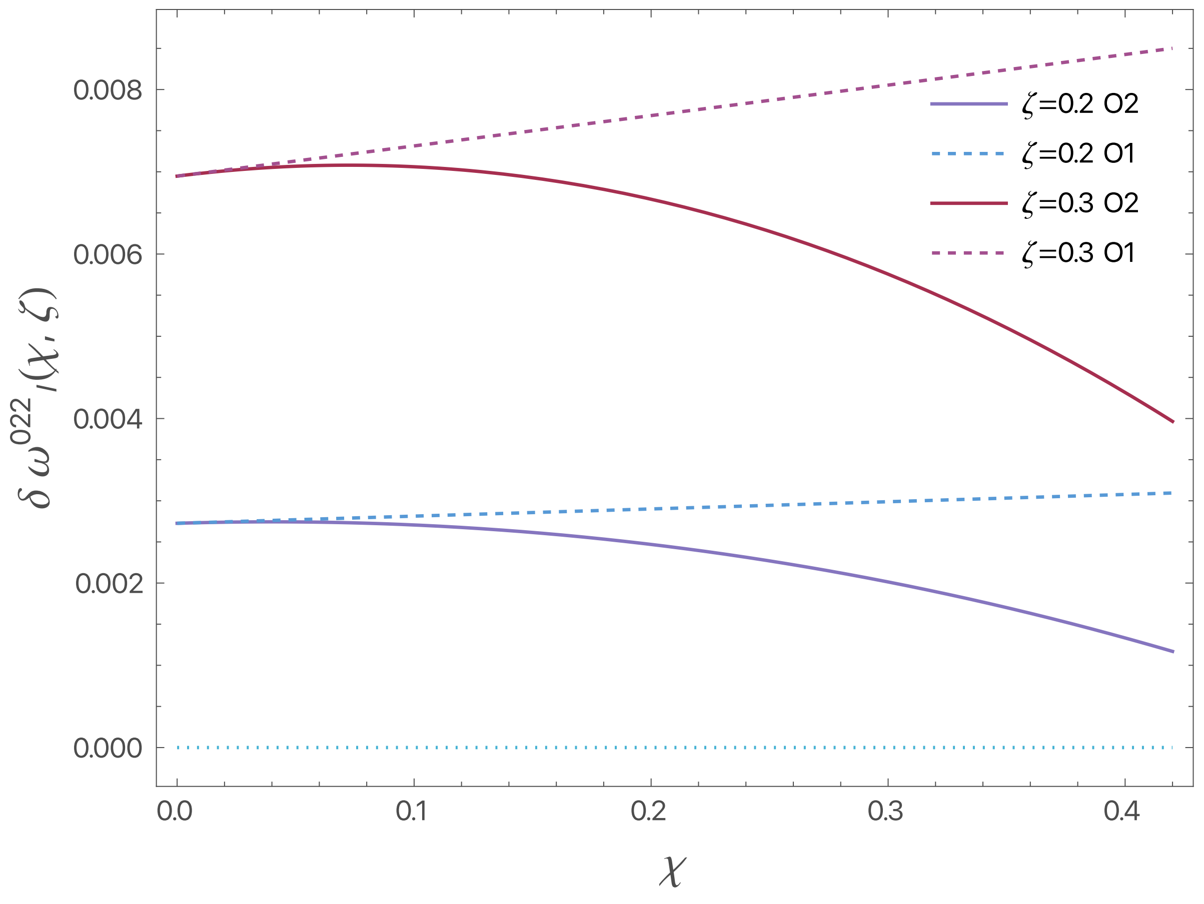
$$\mathcal{P}_\ell + \chi m \bar{\mathcal{P}}_\ell + \chi^2 \hat{\mathcal{P}}_\ell + m^2 \chi^2 \bar{\bar{\mathcal{P}}}_\ell + \chi \left(Q_\ell \tilde{\mathcal{A}}_{\ell-1} + Q_{\ell+1} \tilde{\mathcal{A}}_{\ell+1} \right) + O(\chi^3) = 0$$

$$\mathcal{A}_{\ell+1} + \chi m \bar{\mathcal{A}}_{\ell+1} + \chi \; Q_{\ell+1} \tilde{\mathcal{P}}_\ell + m \; \chi^2 \; Q_{\ell+1} \check{\mathcal{P}}_\ell + O(\chi^3) = 0$$

$$\mathcal{A}_{\ell-1} + \chi m \bar{\mathcal{A}}_{\ell-1} + \chi \; Q_\ell \tilde{\mathcal{P}}_\ell + m \; \chi^2 \; Q_\ell \check{\mathcal{P}}_\ell + O(\chi^3) = 0$$

Results:





Including second order in spin, we find that the deviations are similar to those obtained up to first order, but larger!

These results can be used in the data analysis of the ringdown signal from LIGO/Virgo and ET
(see Carullo's talk; Vellucci et al., in preparation)

How to detect a scalar charge with LISA

(Maselli et al., PRL '20; Maselli et al., Nature Astronomy, '22)

Let us consider a compact object (a BH or a NS) with mass M, surrounded by a massless scalar field.
Sufficiently far away from the body, in a stationary configuration:

$$\phi = const. + d\frac{GM}{r} + O\left(\frac{1}{r^2}\right)$$

the scalar charge d (which is not a conserved, Noether charge) is a measure of how much scalar field the object carries.
For a BH in EdGB gravity, $d=2\zeta+O(\zeta^2)$ => a measurement of d is a measurement of α_{GB}.
For other theories it is still a measurement of the fundamental coupling.
If, instead, the compact body is a NS, it measures the coupling of the field (which can be DM in GR!) with ordinary matter.

The main effect of a scalar charge is dipole emission, which does not occur in GR!
In a post-Newtonian expansion, it dominates over quadrupolar emission at large distances:

$$\dot{E}_{GW} = \dot{E}_{GR}\left[1 + \frac{5}{96}(d_2 - d_1)^2\left(\frac{GM}{r}\right)^{-1}\right]$$

(Barausse et al., PRL '16, Yunes et al. PRD '21)

If the scalar field has a (very small) mass m:

$$\phi = const. + \frac{D}{r}e^{-mr} + \ldots$$

$$\dot{E}_{GW} = \dot{E}_{GR}\left[1 + \frac{5}{96}(d_2 - d_1)^2\left(\frac{\omega^2 - m^2}{\omega^2}\right)^{3/2}\Theta(\omega - m)\left(\frac{GM}{r}\right)^{-1}\right]$$

the effect is similar at distances smaller than the Compton length of the scalar field => this applies to ultralight dark matter as well!
(*Barsanti et al., PRD in preparation*)

Summarizing: even a small scalar charge can significantly affect the waveform, accelerating the inspiral.

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Best bounds from LIGO/Virgo observations from (comparable mass) binaries: $d \lesssim 10^{-1}$.

These bounds are theory dependent: for EdGB *(Perkins et al., PRD '21)* the phase shift is

$$\delta\phi_{-1} = -\frac{5}{168}\frac{d^2}{\eta^{9/5}}\frac{(m_1^2 s_2 - m_2^2 s_1)^2}{m^4} \qquad (s_i = 1 + \text{spin corrections})$$

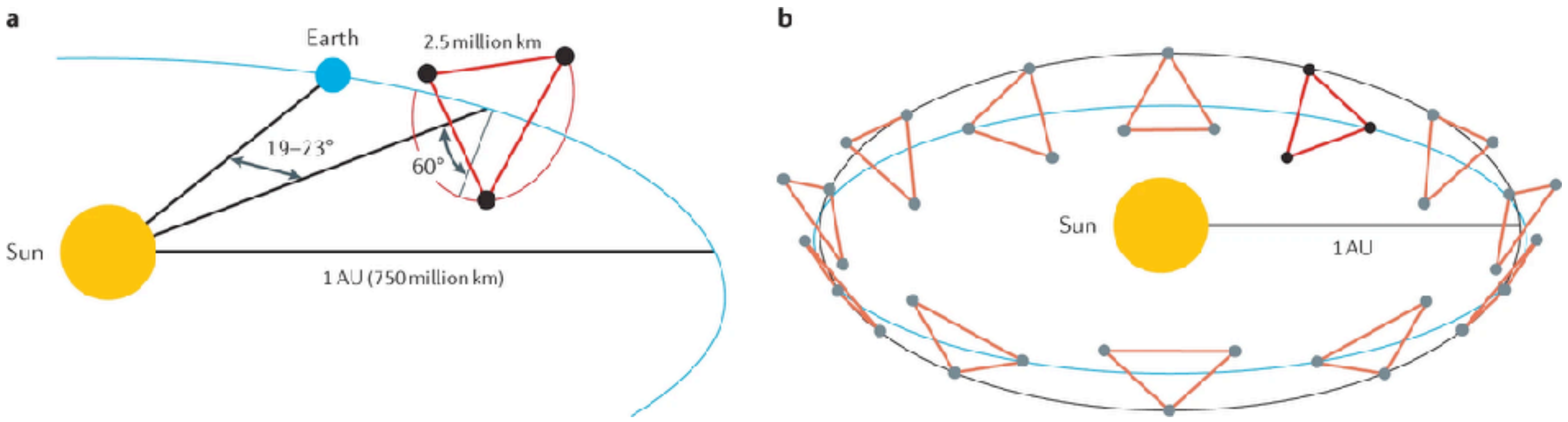
but for other theories they are different.

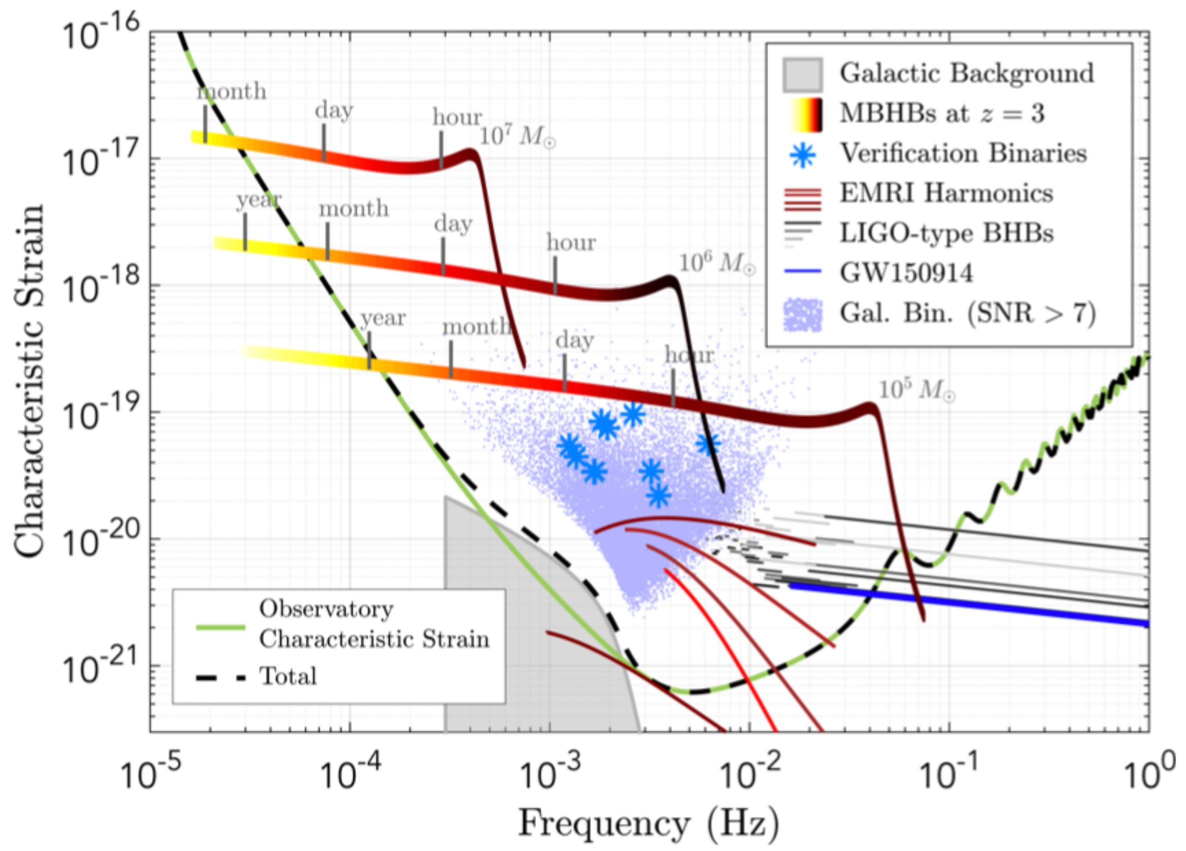
As I am going to show, better probes for the scalar charge are binary systems with very, very small mass-ratio, observed by LISA.

How to detect a scalar charge with LISA

(Maselli et al., PRL '20; Maselli et al., Nature Astronomy, '22)

The Laser Interferometer Space Antenna (LISA), to be launched by ESA in 2037,
will detect GWs in the [10^{-3}-10^{-1}]Hz band, observing BH binary coalescences with signal-to-noise ratios up to thousands.



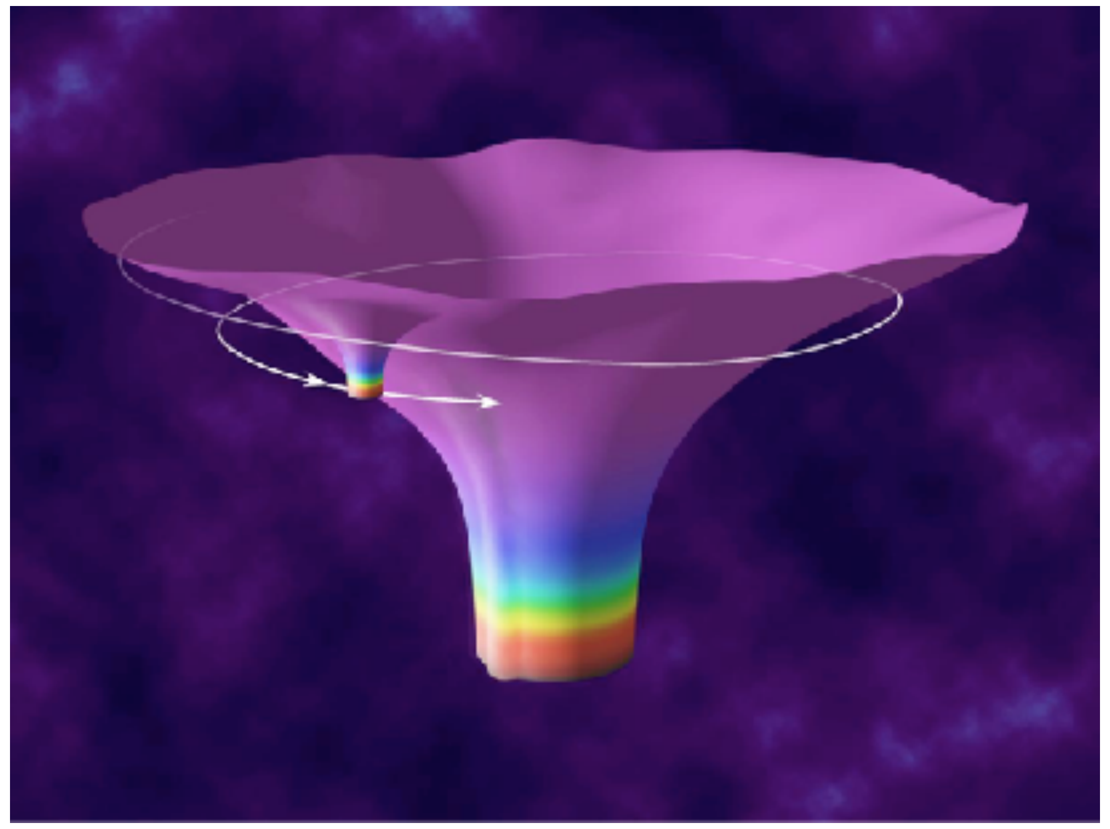


The most promising LISA sources for fundamental physics are the Extreme Mass-Ratio Inspirals (EMRIs)
between a stellar mass compact object (a BH or a NS with m_p from few to tens of $M_\odot$) and a supermassive BH with $M \sim 10^6$-10^7 $M_\odot$

EMRIs are expected to complete ~10^4-10^5 cycles while in the LISA observation band,
thus allowing extremely accurate tests and measurements!

EMRI will perform a mapping of the BH spacetime, revealing even tiny evidence of new physics!

Since $q=m_p/M<<1$, they can be studied using perturbation theory



How to detect a scalar charge with LISA

(Maselli et al., PRL '20; Maselli et al., Nature Astronomy, '22)

Remarkably, a scalar field coupled with gravity would affect the EMRI waveform in a theory-independent way!

Consider a very general action of the form:

$$S[\mathbf{g}, \phi, \Psi] = S_0[\mathbf{g}, \phi] + \alpha S_{\rm coupling}[\mathbf{g}, \phi] + S_{\rm m}[\mathbf{g}, \phi, \Psi]$$

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi G}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)$$

non-minimal coupling between metric tensor and scalar field;

coupling can be dimensionful: $[\alpha]=(G \cdot \text{mass})^n$

example: EdGB, n=2

action of matter fields

We assume that the gravity theory belongs to one of these two classes:

1. Those which satisfy no-hair theorems, i.e. stationary BHs described by Kerr metric: most scalar-tensor theories, ϕ as fundamental field in GR

2. Those which evade no-hair theorems but have a dimensionful coupling α, with n≥1
   (e.g. when ϕ is coupled with curvature invariants, as in scalar Gauss-Bonnet gravity)

=> any correction of the SMBH depends on $\zeta \equiv \frac{\alpha}{M^n} = q^n \frac{\alpha}{m_{\rm p}^n} \ll 1$ because $q = \frac{m_{\rm p}}{M} \ll 1\,,\quad \frac{\alpha}{m_{\rm p}^n} < 1$

(hereafter G=1 for simplicity)

=> in both cases 1 and 2, the SMBH spacetime is well approximated by the Kerr metric

This greatly simplifies the description of EMRIs!

How to detect a scalar charge with LISA

(Maselli et al., PRL ’20; Maselli et al., Nature Astronomy, ’22)

Skeletonized approach (*Eardley ApJ ’75, Damour & Esposito Farese PRD ’92*)
to describe the motion of the inspiralling body with mass m_p onto a SMBH with mass $M \gg m_p$:

The inspiralling body is treated as a point particle, replacing the matter action S_m with the *particle action*

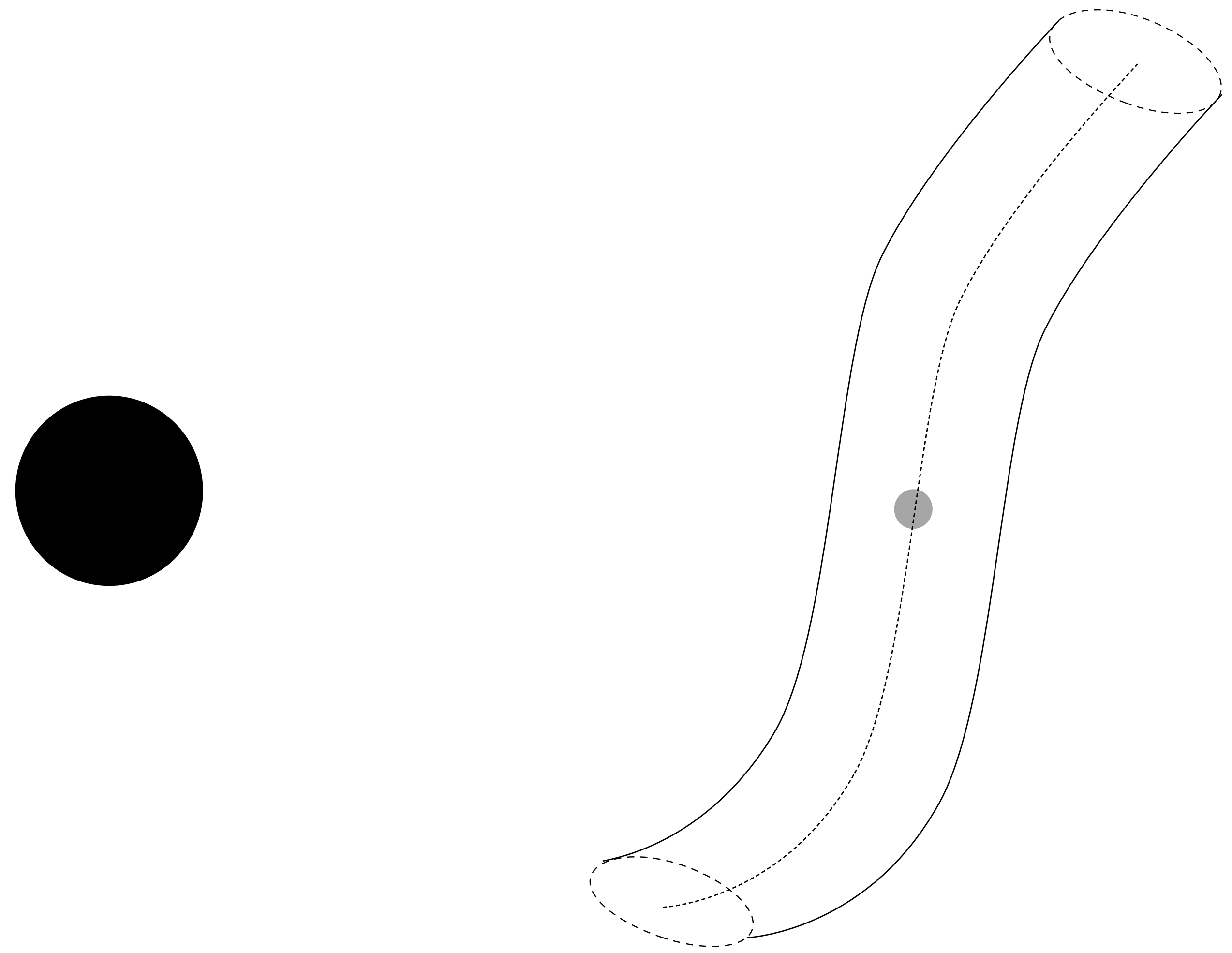
$$S_p = -\int m(\varphi) ds = -\int m(\varphi) \sqrt{g_{\mu\nu} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda}} d\lambda$$

y_p^ν : wordline of the (center of mass of the) particle

$m(\varphi)$: scalar function depending on the scalar field at the location of the particle

$$l \ll L$$

lengthscale of skeletonized body (“particle”) — lengthscale of exterior spacetime



The motion of the particle can be studied using spacetime perturbation theory, expanding the field equations in the mass ratio q<<1.

Scalar field affects the motion of the particle, but it *does not* affect the background spacetime, which is described by the Kerr metric

How to detect a scalar charge with LISA

(Maselli et al., PRL ’20; Maselli et al., Nature Astronomy, ’22)

Field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \int m_{\rm p} \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_{p\,\mu}}{d\lambda} \frac{dy_{p\,\nu}}{d\lambda} d\lambda$$

same as in GR

$$\Box\phi = -4\pi d \, m_{\rm p} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

This term is universal (in the wide class of theories considered); it depends on the theory only through the value of the scalar charge!

The scalar field sourced by the charge releases energy at infinity, speeding up the inspiral and thus affecting the gravitational waveform.

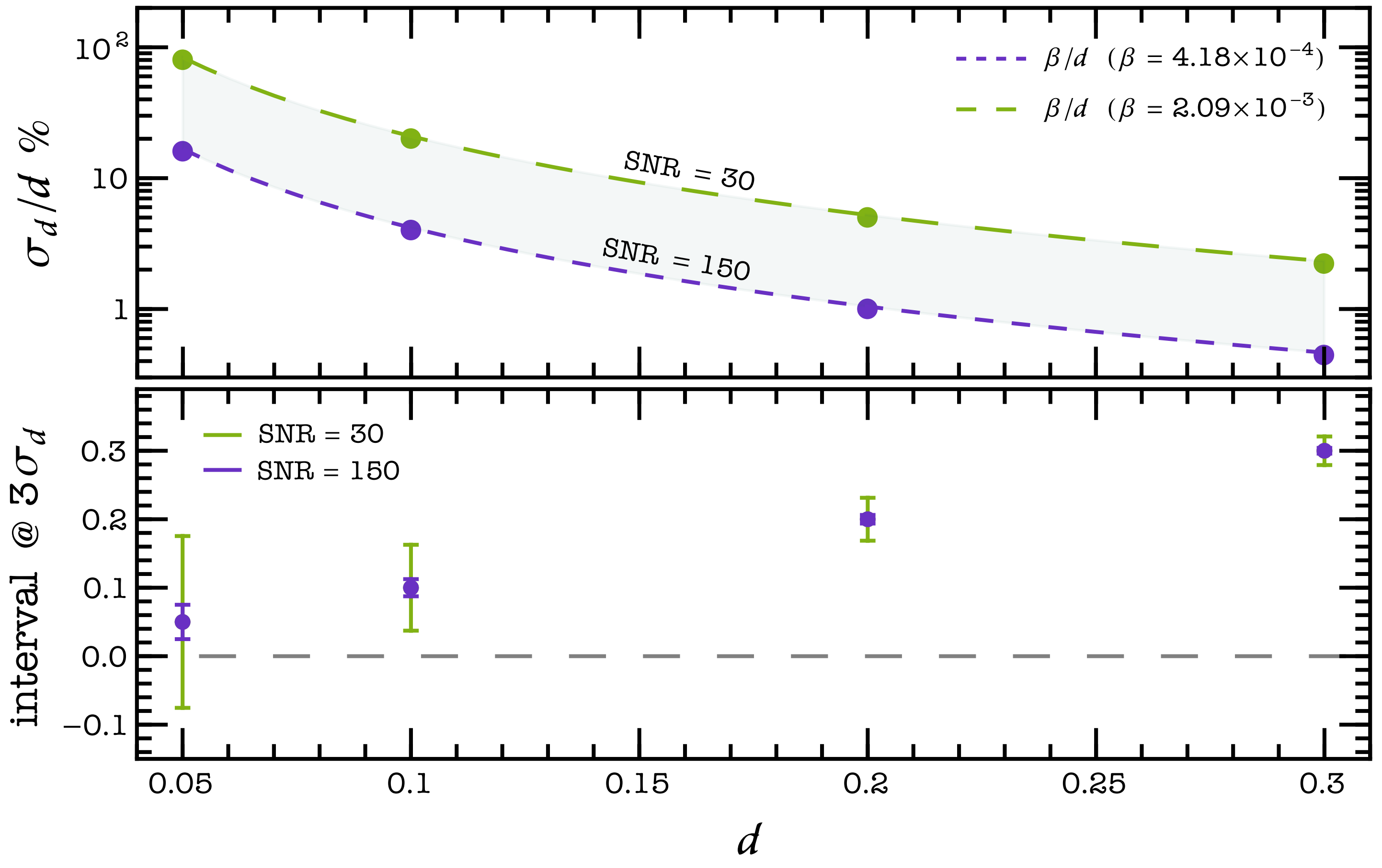
$$\dot{E}_{\rm orb} = -\dot{E}_{\rm grav} - \dot{E}_{\rm scal}$$

Thus, when LISA will detect the GW signal from an EMRI, it will be possible to set bounds (or measure!) the scalar charge of the particle.

We have performed an analysis of the parameter space (masses, spin, orientation, separation, etc., and the scalar charge d) for circular equatorial EMRIs, using a Fisher-matrix approach to compute statistical errors and correlations:

LISA can measure scalar charges as small as 0.05.
These results can be improved by stacking more observations.

Preliminary results *(Barsanti et al., in preparation)* indicate similar results for non-circular orbits and for massive scalar fields, for $m_s \lesssim 10^{-16}$eV



Conclusions

- Scalar Gauss-Bonnet gravity theories such as EdGB gravity are a promising case-study of GR deviations: they have fundamental physics motivations, seem to be mathematically consistent, and predict BHs with scalar charge.

- The computation of QNMs of rotating BHs in modified theories of gravity is techincally challenging; today only two such computations exist at $O(\chi)$, and just one at $O(\chi^2)$ (EdGB gravity).

- The computation of QNMs of rotating BHs in EdGB gravity shows that rotation signifincantly enhances the effects of GR deviation. This computation can be used as a test-bed for the search of beyond-GR effects in GW signals

- Present and future GW detectors are sensitive to the scalar charge of compact objects. This quantity encodes beyond-GR effects and its signature may be enhanced due to dipolar emission, which is not present in GR.

- Extreme mass-ratio inspirals observed by LISA are a very promising GW source to detect the scalar charge of compact objects, if it exists. In this case, the leading effect of the scalar charge in the GW signal does not depend on the detail of the underlying gravity theory.