Love Numbers and Magnetic Susceptibility of Charged Black Holes

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based on [Phys. Rev. D 105, 044026] *with Vitor Cardoso*

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CENTRA CENTRO MULTIDISCIPLINAR DE ASTROFÍSICA TÉCNICO LISBOA

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- *Study the static response of charged BH's in arbitrary* D

Charged Black Holes in Higher D

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- Theory :

$$S[g,\mathcal{A}] = \frac{1}{2\kappa^2}\int d^D x\sqrt{g}R - \frac{1}{4}\int d^D x\sqrt{g}\mathcal{F}^2$$

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$$G_{AB} = \kappa^2 T_{AB}, \qquad d\star\mathcal{F} = 0, \qquad T_{AB} = \mathcal{F}_{AC}\mathcal{F}_B{}^C - \frac{1}{4}g_{AB}\mathcal{F}^2$$

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- Static, Spherically Symmetric, Electrically Charged BH's :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_n^2, \qquad \mathcal{F} = E_0(r)dt\wedge dr$$

Charged Black Holes in Higher D

- Solution :

$$f(r) = 1 - \frac{2M}{r^{n-1}} + \frac{Q^2}{r^{2n-2}}, \qquad E_0(r) = \frac{q}{r^n}, \qquad Q^2 = \frac{\kappa^2 q^2}{n(n-1)}$$

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- Hawking Temperature

$$T_H = \frac{n-1}{4\pi r_+^n} \left(r_+^{n-1} - r_-^{n-1} \right)$$

Perturbation Theory in Higher D

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$$ds^2 = g_{AB} dx^A dx^B = g_{ab}(y) dy^a dy^b + r^2(y) \gamma_{ij}(z) dz^i dz^j$$

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- Perturbations :

$$\delta\mathcal{A} = \delta\mathcal{A}_A dx^A = \delta\mathcal{A}_a dy^a + \delta\mathcal{A}_i dz^i$$
$$h = h_{AB}dx^A dx^B = h_{ab}dy^a dy^b + 2h_{ai}dy^a dz^i + h_{ij}dz^i dz^j$$

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- Scalar sector $\mathcal{S} = \{h_{ab}, h_a, h_L, h_T^{(0)}; \delta\mathcal{A}_a, a\}$

Perturbation Theory in Higher D

- Project on corresponding harmonics

$$\mathcal{T} = T(y)\mathbb{T}_{ij}, \quad \mathcal{V} = V(y)\mathbb{V}_i, \quad \mathcal{S} = S(y)\mathbb{S}$$

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- Decoupled sets of PDE's on $\mathcal{N}^m$ for $\{T, V, S\}$
- Construct gauge-invariant variables within each sector
- Derive decoupled master equations within each sector
- Want to study *static* solutions

Perturbation Theory in Higher D

- Static perturbations are governed by Fuchsian equations

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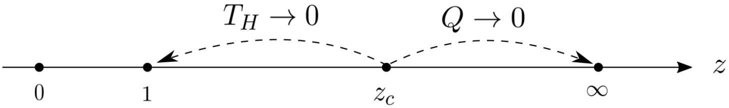
$$\boxed{\Psi''(z) + \left(\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\eta}{z-z_c}\right)\Psi'(z) + \frac{\alpha\beta\,(z-h)}{z(z-1)\,(z-z_c)}\Psi(z) = 0}$$



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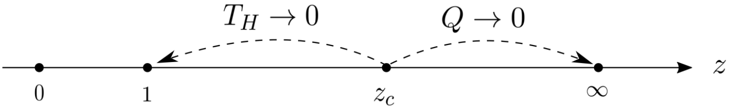

$$\boxed{z(1-z)\Psi''(z) + [c-(a+b+1)z]\,\Psi'(z) - ab\Psi(z) = 0}$$

Hypergeometric

Perturbation Theory in Higher D

$$\Psi(z) = A\Psi_{resp}(z) + B\left(z^{-2l-1}\Psi_{tidal}(z) + R\Psi_{resp}(z)\ln z\right), \quad l = \frac{L}{D-3}$$

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$$Unless\,:\, \Psi(z) = B\left(k\Psi_{resp}(z) + z^{-2l-1}\Psi^{(N)}_{tidal}(z)\right) \to k = 0$$

Tensor Static Response

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Equivalent to massless test scalar field

$$h_{ij} = 2r^2 H_T \mathbb{T}_{ij}, \;\; H_T(z) = r(z)^{-n/2} z^{\frac{2l(n-1)+n-2}{2(n-1)}} \Psi(z), \;\; z = \left(\frac{r_+}{r}\right)^{n-1}$$

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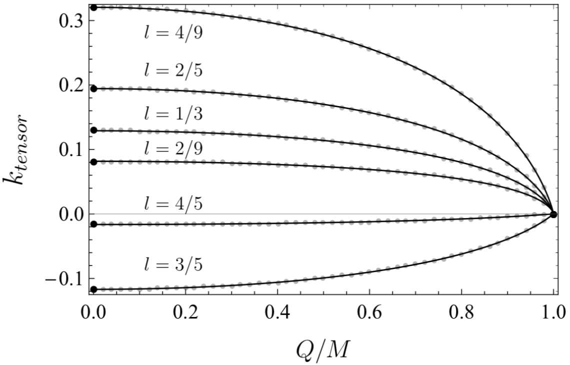

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- Decoupled ("master") variables :

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- Defined only up to their amplitudes

$$(a_+, b_+) = \left(\frac{Q m_V}{\Delta + M\left(n^2 - 1\right)} \sigma_{(+)}, \frac{Q}{q} \sigma_{(+)} \right) \quad (\forall \sigma_{(+)})$$

$$(a_-, b_-) = \left(\sigma_{(-)}, -\frac{2\kappa^2 q}{\Delta + M\left(n^2 - 1\right)} \sigma_{(-)} \right) \quad (\forall \sigma_{(-)})$$

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$$h_{ai} \sim \left(k_- + \frac{2(n-1)n\sin\epsilon}{\tilde{\Delta} + n^2 - 1} \mathbf{\Theta} k_+ + \left(1 + \frac{2(n-1)n\sin\epsilon}{\tilde{\Delta} + n^2 - 1} \mathbf{\Theta} \right) z^{-2l-1} + ... \right) \mathbb{V}_i$$

$$\delta A_i \sim \left(k_+ - \frac{m_V \sin\epsilon}{\tilde{\Delta} + n^2 - 1} \frac{k_-}{\mathbf{\Theta}} + \left(1 - \frac{m_V \sin\epsilon}{\tilde{\Delta} + n^2 - 1} \frac{1}{\mathbf{\Theta}} \right) z^{-2l-1} + ... \right) \mathbb{V}_i$$

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Vector Static Response

- $Q = 0$: In general $\neq 0$, but [Hui et.al. '21]

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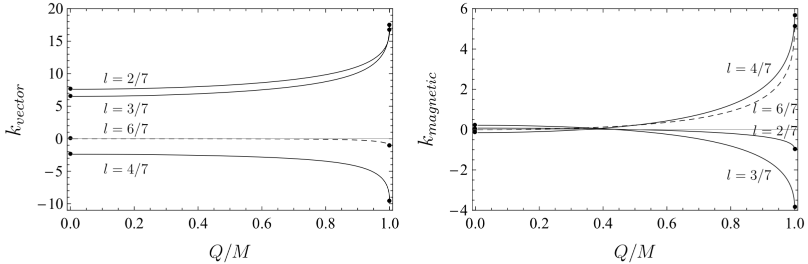
- For $Q \neq 0$, new modes polarise

Vector Static Response

$$Q \neq 0 \quad \& \quad T_H \neq 0 \quad (D = 10)$$



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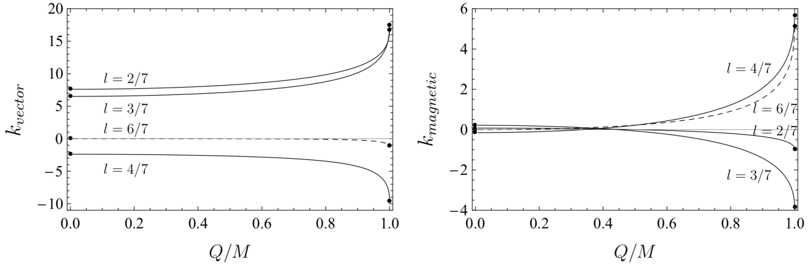

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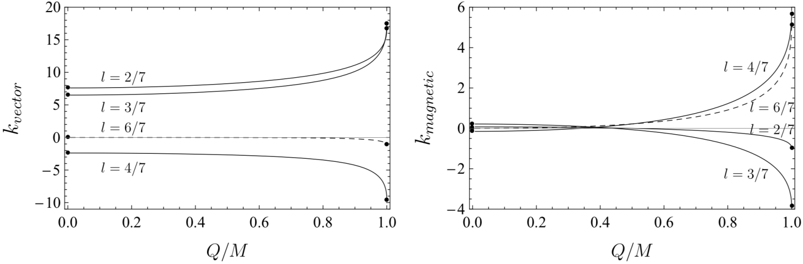

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(Apply recent progress by [Bonelli+ '22])

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Thank you !

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- The conductor condition is $E(R) = 0$:

$$\alpha_E = -R^3$$

- Electric polarisability is an intrinsic property.