Resummed lattice QCD equation of state at finite baryon density: strangeness neutrality and beyond

arXiv:2202.05574, [Borsanyi:2022qlh]

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BERGISCHE UNIVERSITÄT WUPPERTAL

WB collaboration

BMW collaboration

The sign problem

The QCD partition function:

$$Z(V, T, \mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \;\det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U)e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

Dealing with the sign problem

- (Sign) Reweighting techniques
- Canonical ensemble
- Lefshetz Thimble
- Density of state methods
- Dual variables
- Complex Langevin
- ...

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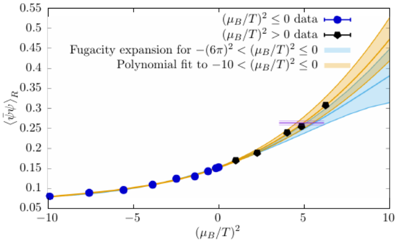

[Borsanyi:2021hbk]

Dealing with the sign problem

- (Sign) Reweighting techniques
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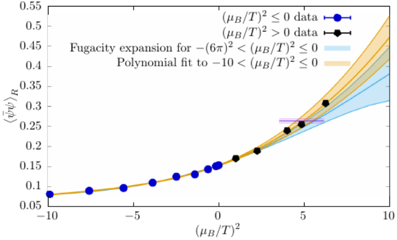
- (Taylor) expansion
- Imaginary μ



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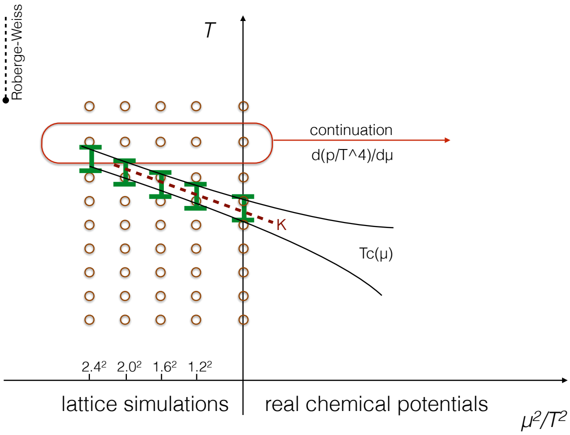
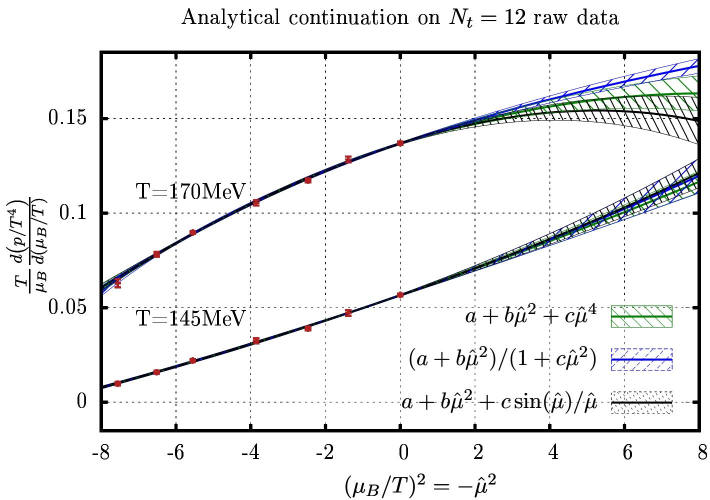
Analytic continuation from imaginary chemical potential



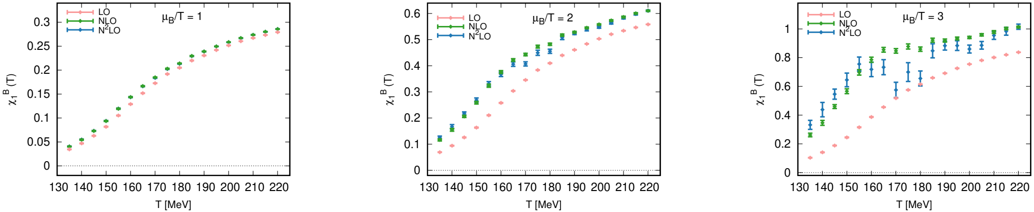
Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- [Borsanyi:2018grb]
- [Borsanyi:2020fev]
- [Bellwied:2021nrt]
- . . .

Different functions



Trouble with the equation of state



[Borsanyi:2021sxv], [Borsanyi:2018grb], $N_t = 12$

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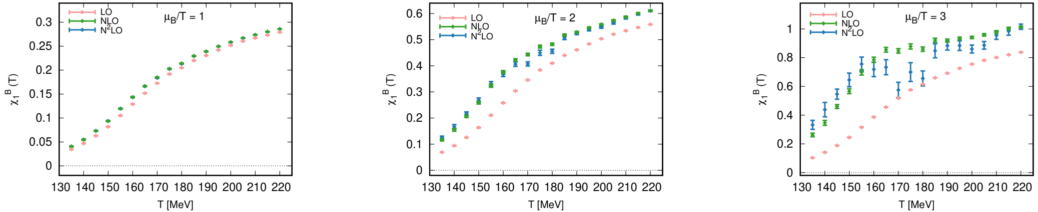

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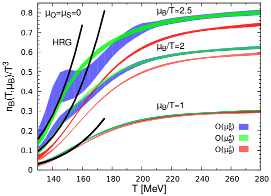

[Bazavov:2017dus]
Taylor method
$N_t = 6, 8, 12, (16)$ (2nd Order)
$N_t = 6, 8$ (4th and 6th Order)

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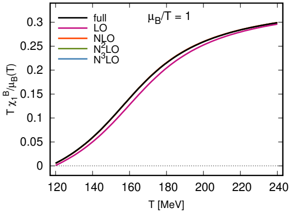

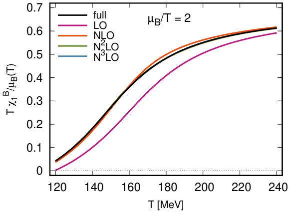

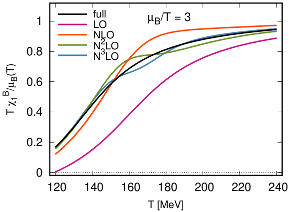

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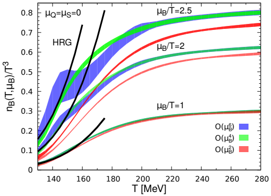

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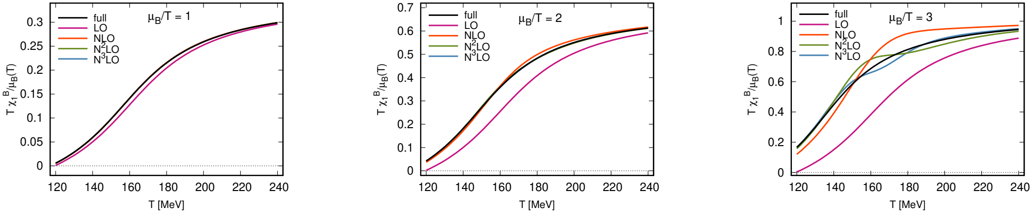

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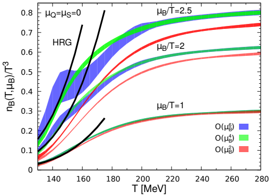

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$N_t = 6, 8, 12, (16)$ (2nd Order)
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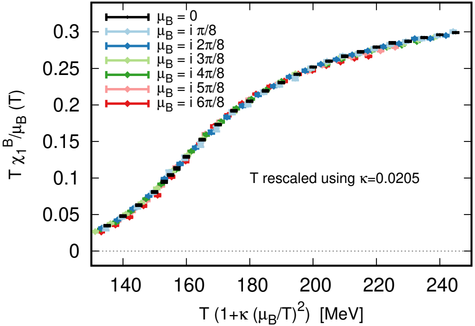
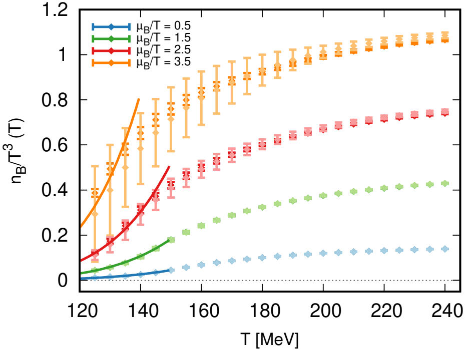
- extrapolation at fixed T cross the transition line
- potential for other expansion

Results at $\mu_S = 0$

Find a different extrapolation scheme for extrapolating to higher μ_B.





- [Borsanyi:2021sxv]
- $N_t = 10, 12, 16$

Strangeness Neutrality

Enforcing the conditions $\mu_Q = 0$ and $\chi_1^S = 0$:

$$\frac{\mathrm{d}\mu_S}{\mathrm{d}\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S}.$$

On this line, total derivatives with respect to the baryochemical potential read

$$\frac{\mathrm{d}}{\mathrm{d}\hat{\mu}_B} = \frac{\partial}{\partial\hat{\mu}_B} + \frac{\mathrm{d}\hat{\mu}_S}{\mathrm{d}\hat{\mu}_B}\frac{\partial}{\partial\hat{\mu}_S} = \frac{\partial}{\partial\hat{\mu}_B} - \frac{\chi_{11}^{BS}}{\chi_2^S}\frac{\partial}{\partial\hat{\mu}_S}.$$

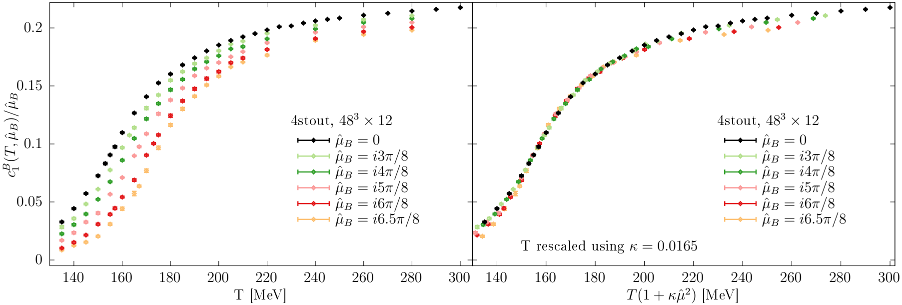
For the pressure we get:

$$c_n^B(T, \hat{\mu}_B) \equiv \left.\frac{\mathrm{d}^n\hat{p}(T, \hat{\mu}_B)}{\mathrm{d}\hat{\mu}_B^n}\right|_{\substack{\mu_Q=0 \\ \chi_1^S=0}}.$$

The net baryon density is given by:

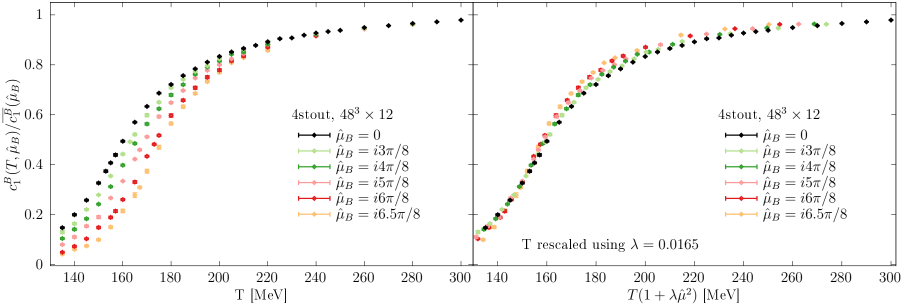
$$c_1^B(T, \hat{\mu}_B) = \chi_1^B - \frac{\chi_{11}^{BS}}{\chi_2^S}\chi_1^S = \chi_1^B$$

c_1^B



This rescaling will break down at large $T \longrightarrow$ rescaling with SBL

c_1^B



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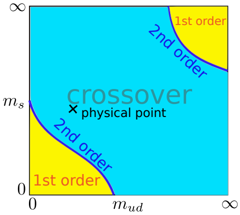
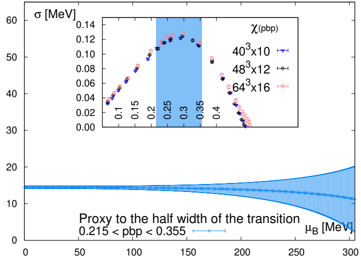
Why does the rescaling work?

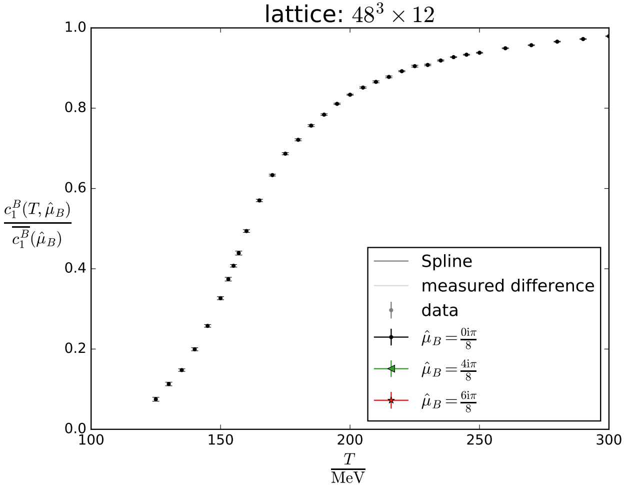
- It is an observation that it works
- It could be related to the critical scaling in the chiral limit
- If the universal contribution to EoS is large $\to$ single scaling variable
- If strength of transition is strongly Influenced by light quark masses $\to$ curves keep there shape
- Fits with the observation of constant width of the transition





[Borsanyi:2020fev]

Measuring the shift



c_1^B: net baryon density

$\overline{c_1^B}$: SBL of net baryon density

$$\Pi(T,\hat{\mu}_B,N_\tau) = \frac{T'(T,\hat{\mu}_B,N) - T}{T\hat{\mu}_B}$$

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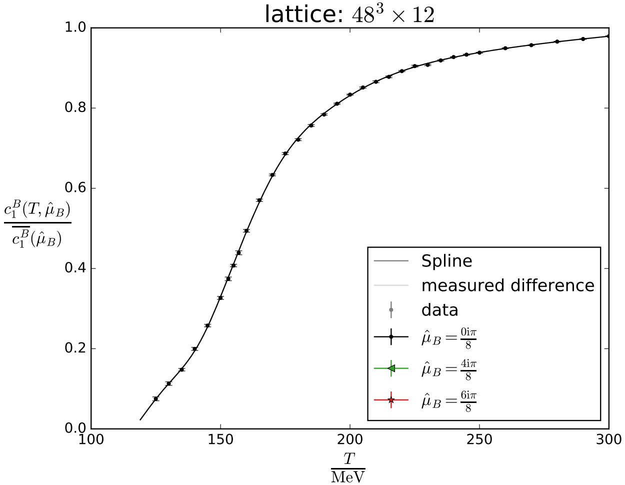

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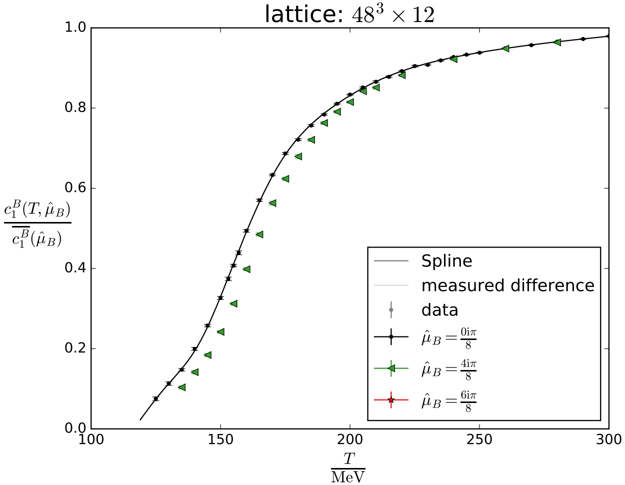

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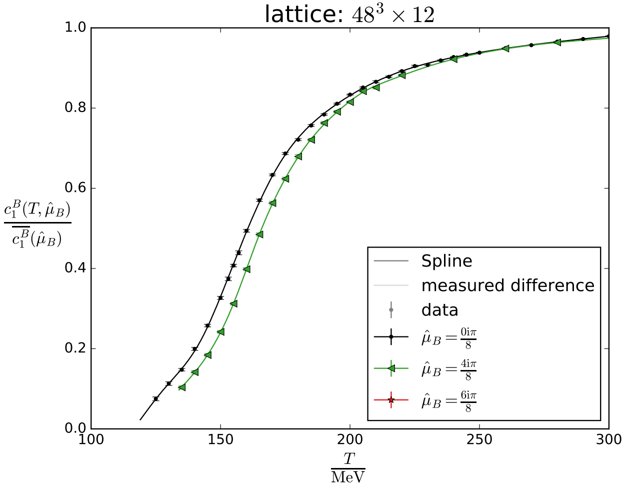

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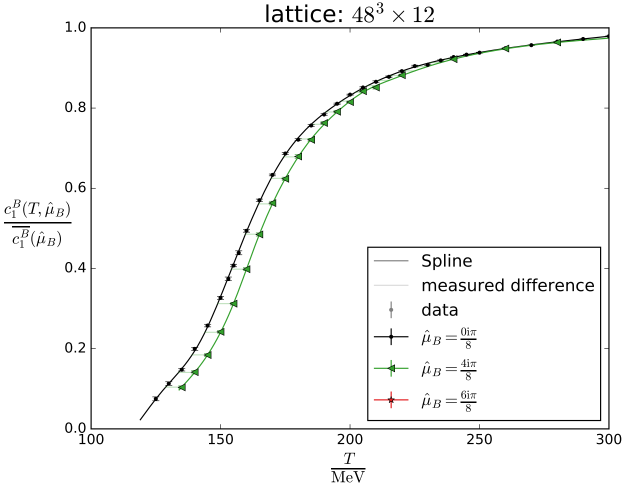

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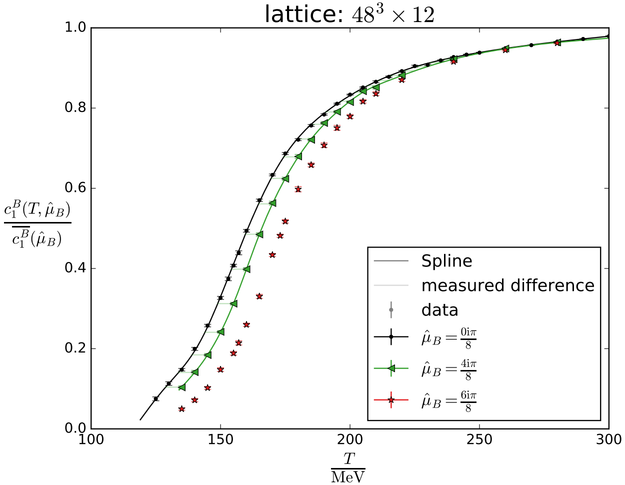

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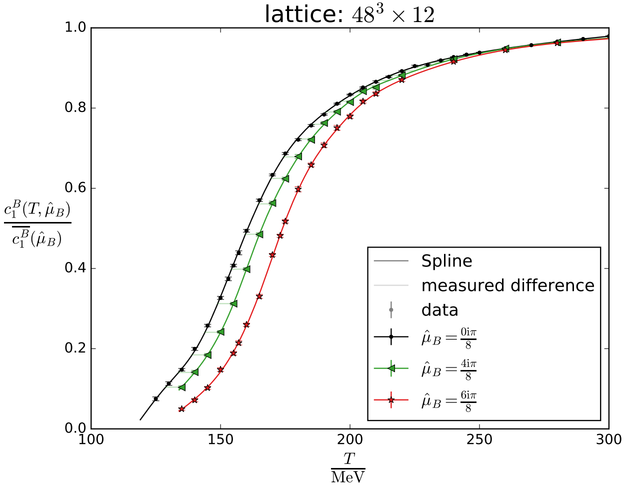

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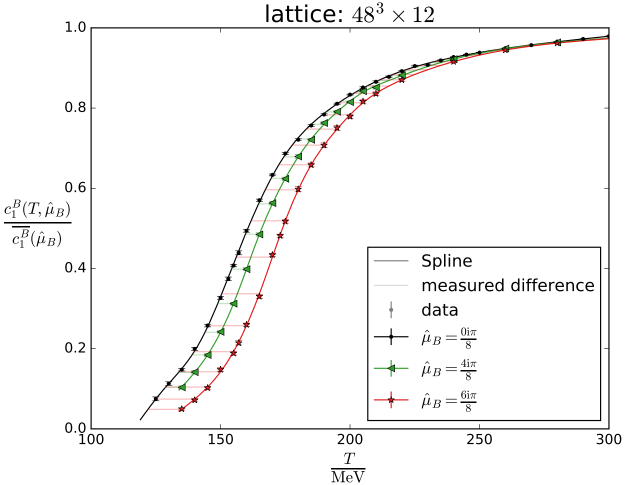

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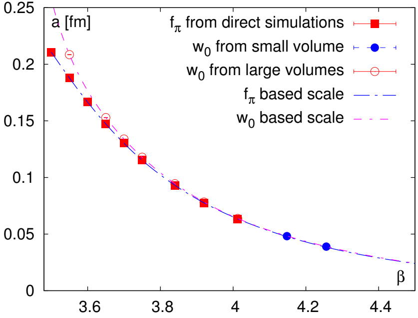
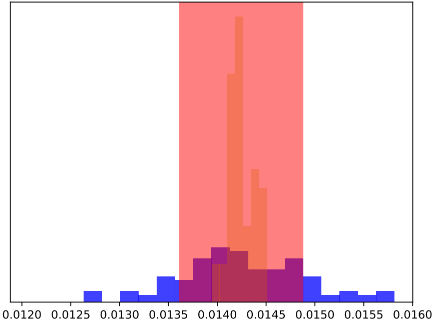
Lattice Setup



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$
- Continuum estimate from lattice sizes: $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5, (5.5), 6$ and 6.5
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Systematic Errors

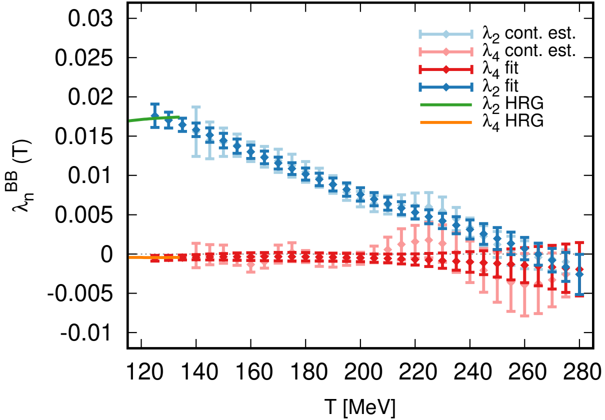
- 3 different sets of spline node points at μ_B=0
- 2 different sets of spline node points at finite imaginary μ_B
- w_0 or f_π based scale setting
- 2 different chemical potential ranges in the global fit: $\hat{\mu}_B \leq 5.5$ or $\hat{\mu}_B \leq 6.5$
- 2 functions for the chemical potential dependence of the global fit: linear or parabola
- including the coarsest lattice, $N_\tau = 8$, or not, in the continuum extrapolation.



histogram for λ_2 at $T = 155$ MeV

In total we perform 96 Fits. We weight every result with a $Q > 0.01$ uniformly

The expansion coefficients



$$\Pi(T, \hat{\mu}_B, N_\tau) = \frac{T'(T, \hat{\mu}_B, N) - T}{T\hat{\mu}_B}$$

$$\Pi(T, \hat{\mu}_B, N_\tau) = \lambda_2^A + \lambda_4^A \hat{\mu}_B^2 + \lambda_6^A \hat{\mu}_B^4 + \frac{1}{N_\tau^2}\left(\alpha^A + \beta^A \hat{\mu}_B^2 + \gamma^A \hat{\mu}_B^4\right)$$

We make a fit to calculate derivatives and constrain it with the HRG.

Thermodynamics

$$\frac{p(T,\hat{\mu}_B)}{T^4} = \frac{p(T,0)}{T^4} + \int_0^{\hat{\mu}_B} c_1^B(T,\hat{\mu}'_B)\mathrm{d}\hat{\mu}'_B,$$

with

$$c_1^B(T,\hat{\mu}_B) = c_2^B(T',0)\frac{\overline{c_1^B}(\hat{\mu}_B)}{\overline{c_2^B}(0)},$$

and $\frac{p(T,0)}{T^4}$ from [Borsanyi:2013bia] The entropy density is defined as $s = \left.\frac{\partial p}{\partial T}\right|_{\mu_B,\mu_S}$, which can be rewritten in terms of dimensionless quantities as:

$$\hat{s} = 4\hat{p} + T\left.\frac{\partial \hat{p}}{\partial T}\right|_{\mu_B} = 4\hat{p} + T\left.\frac{\partial \hat{p}}{\partial T}\right|_{\hat{\mu}_B} - \hat{\mu}_B\chi_1^B,$$

where $\hat{s} \equiv \frac{s}{T^3}$ and we took into account the difference between derivatives at fixed μ_B versus at fixed $\hat{\mu}_B$.

Thermodynamics II

By noticing that on the strangeness neutral line

$$\frac{\mathrm{d}\hat{p}(T,\hat{\mu}_B,\hat{\mu}_S(T,\hat{\mu}_B))}{\mathrm{d}T} = \chi_1^S \frac{\partial \hat{\mu}_S}{\partial T} + \frac{\partial \hat{p}}{\partial T} = \frac{\partial \hat{p}(T,\hat{\mu}_B,\hat{\mu}_S(T,\hat{\mu}_B))}{\partial T},$$

we can write the logarithmic temperature derivative of the pressure as:

$$T\left.\frac{\partial \hat{p}(T,\hat{\mu}_B)}{\partial T}\right|_{\hat{\mu}} = T\frac{\partial \hat{p}(T,0)}{\partial T}$$

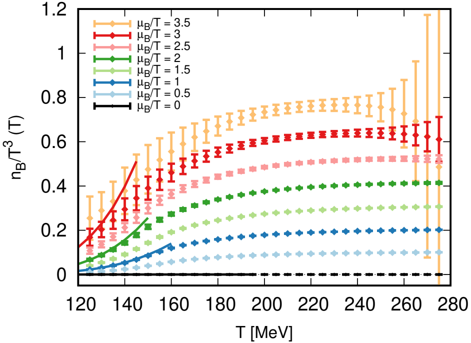
$$+\frac{1}{2}\int_0^{\hat{\mu}_B^2} T \left.\frac{\mathrm{d}c_2^B(T',0)}{\mathrm{d}T'}\right|_{T'} \times \left[1+\lambda_2^{BB}y+\lambda_4^{BB}y^2+T\left(\frac{\mathrm{d}\lambda_2^{BB}}{dT}y+\frac{\mathrm{d}\lambda_4^{BB}}{dT}y^2\right)\right] dy$$

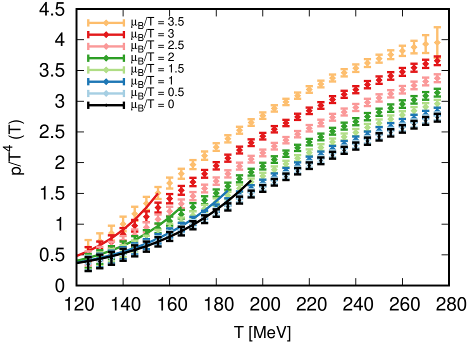
where $\frac{\mathrm{d}c_2^B(T)}{\mathrm{d}T}$ is calculated at $\mu_B = 0$ and $T' = T\left(1+\lambda_2^{BB}y+\lambda_4^{BB}y^2\right)$
Given the pressure and the entropy, the dimensionless energy density is given by:

$$\hat{\epsilon} = \hat{s} - \hat{p} + \hat{\mu}_B \chi_1^B,$$

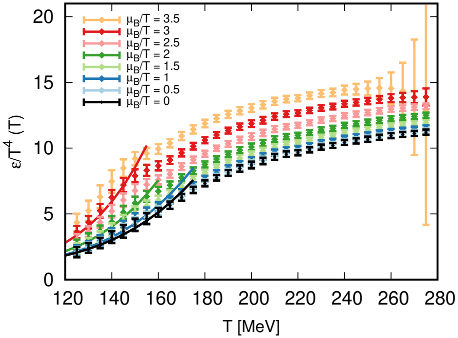
where $\hat{\epsilon} = \frac{\epsilon}{T^4}$.

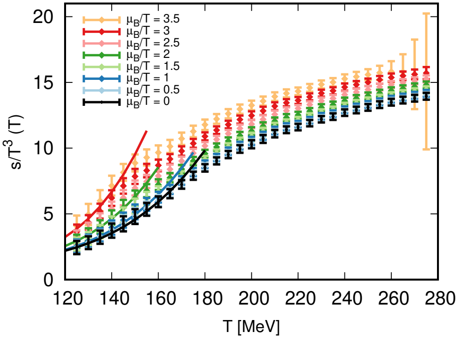
Results at $n_S = 0$ and $\mu_Q = 0$ I





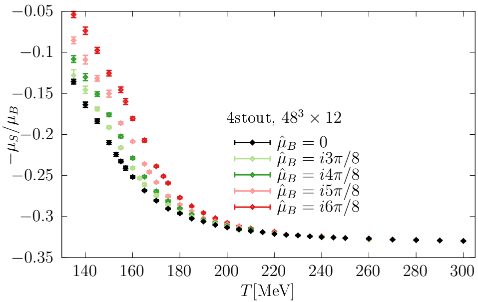
Results at $n_S = 0$ and $\mu_Q = 0$ II

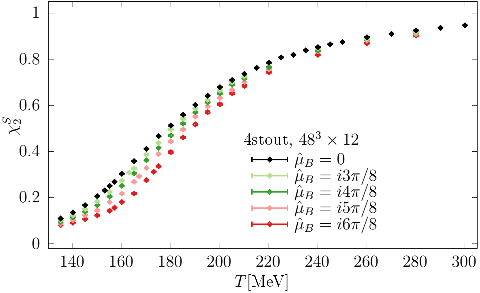




More strangeness

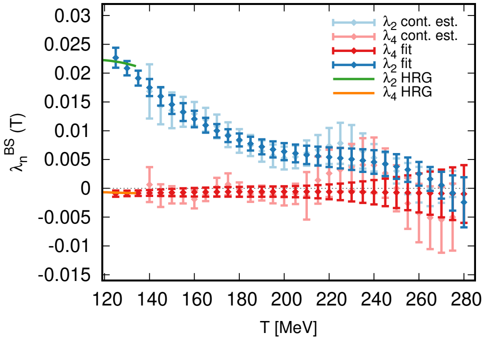
Two more observables:

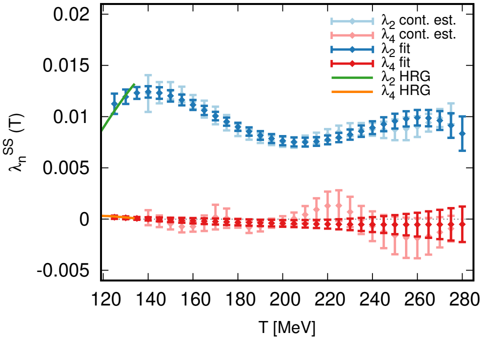




More strangeness

Two more expansion:





Beyond strangeness neutrality

$$\Delta\hat{\mu}_S \equiv \hat{\mu}_S - \hat{\mu}_S^\star,$$

the dimensionless strangeness and baryon densities become:

$$\chi_1^S(\hat{\mu}_S) \approx \chi_2^S(\hat{\mu}_S^\star)\Delta\hat{\mu}_S$$
$$\chi_1^B(\hat{\mu}_S) \approx \chi_1^B(\hat{\mu}_S^\star) + \chi_{11}^{BS}(\hat{\mu}_S^\star)\Delta\hat{\mu}_S,$$

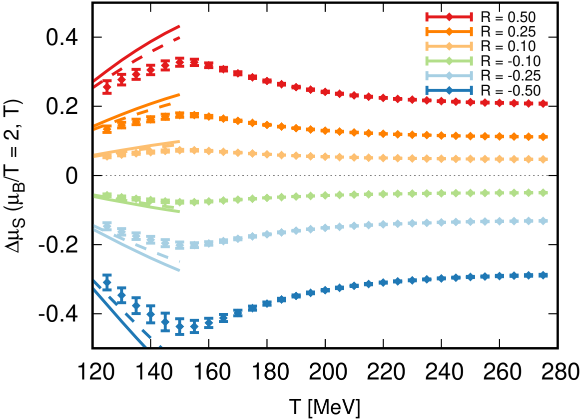
where we only kept the linear leading order terms in $\Delta\hat{\mu}_S$. We will express thermodynamic quantities in terms of the strangeness-to-baryon fraction:

$$R = \frac{\chi_1^S}{\chi_1^B} = \frac{\chi_2^S(\hat{\mu}_S^\star)\Delta\hat{\mu}_S}{\chi_1^B(\hat{\mu}_S^\star)\Delta\hat{\mu}_S + \chi_{11}^{BS}(\hat{\mu}_S^\star)}.$$

Inverting this equation we get:

$$\Delta\hat{\mu}_S = \frac{R\hat{\chi}_1^B(\hat{\mu}_S^\star)}{\chi_2^S(\hat{\mu}_S^\star) - R\chi_{11}^{BS}(\hat{\mu}_S^\star)}.$$

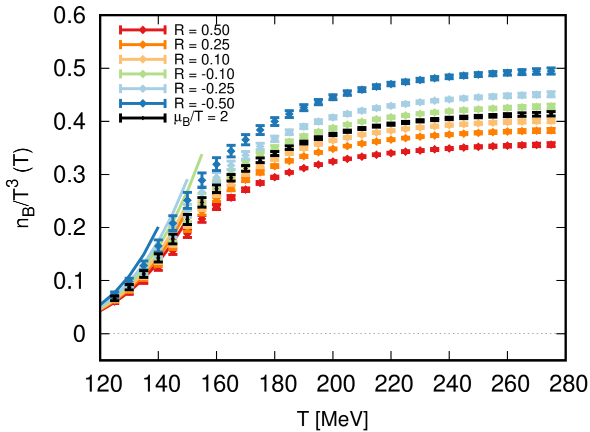
Beyond strangeness neutrality



$$R = \frac{\chi_1^S}{\chi_1^B}$$

$$\Delta\hat{\mu}_S = \frac{R\hat{\chi}_1^B(\hat{\mu}_S^\star)}{\chi_2^S(\hat{\mu}_S^\star) - R\chi_{11}^{BS}(\hat{\mu}_S^\star)}$$

Strange Baryon density

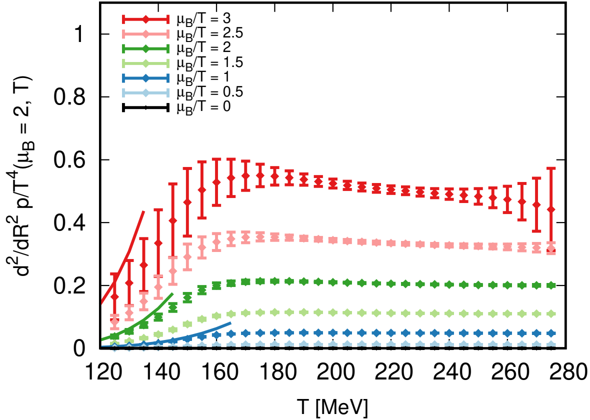


Expanding the baryon density:

$$\frac{\chi_1^B(T,\hat{\mu}_B,R)}{\chi_1^B(T,\hat{\mu}_B,R=0)} \approx 1 + R\frac{\chi_{11}^{BS}(T,\hat{\mu}_B,R=0)}{\chi_2^S(T,\hat{\mu}_B,R=0)}$$

where all quantities on the right hand side are along the strangeness neutral line.

Strange Pressure



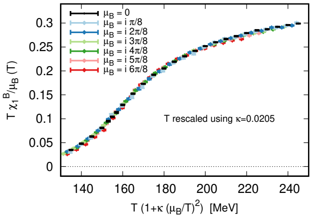
At the strangeness neutral line the $\mathcal{O}(R)$ correction of the pressure vanishes. The leading order correction gives:

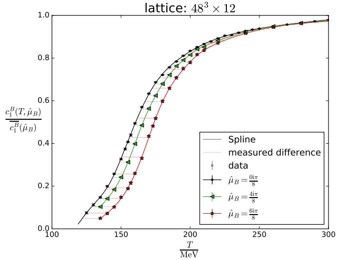
$$\hat{p}(T, \hat{\mu}_B, R) \approx \hat{p}(T, \hat{\mu}_B, R) + \frac{1}{2}\frac{\mathrm{d}^2\hat{p}}{\mathrm{d}R^2}(T, \hat{\mu}_B)\, R^2,$$

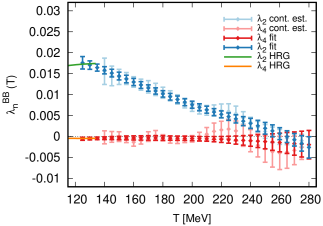
where

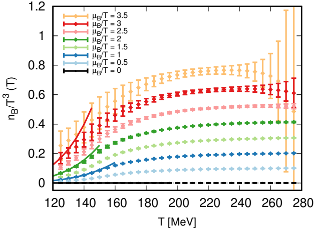
$$\frac{\mathrm{d}^2\hat{p}}{\mathrm{d}R^2}(T, \hat{\mu}_B) = \frac{\left(\chi_1^B(T, \hat{\mu}_B)\right)^2}{\chi_2^S(T, \hat{\mu}_B)}.$$

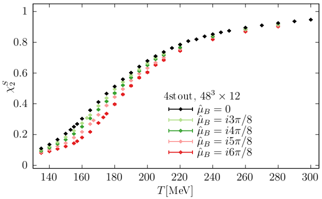
Summary

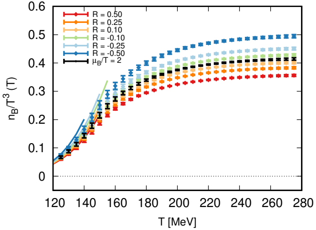

μB = 0
μB = i π/8
μB = i 2π/8
μB = i 3π/8
μB = i 4π/8
μB = i 5π/8
μB = i 6π/8
T rescaled using κ=0.0205
T χ1B/μB (T)
T (1+κ (μB/T)2) [MeV]


lattice: 48³ × 12
Spline
measured difference
data
c1B(T, μ̂B) / c1B(μ̂B)
T / MeV

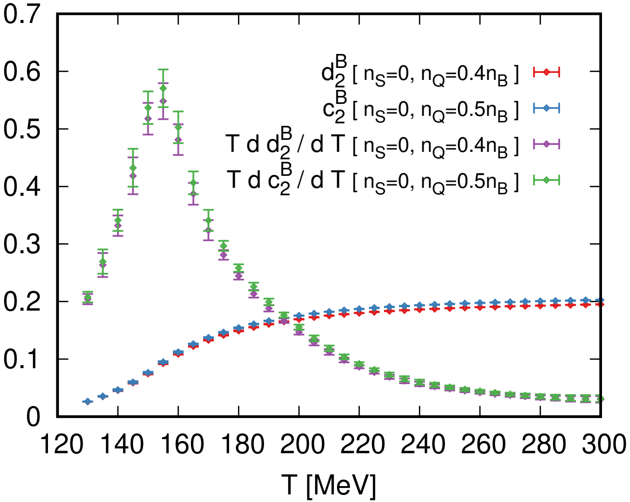

λ2 cont. est.
λ4 cont. est.
λ4 fit
λ2 fit
λ2 HRG
λ4 HRG
λnBB (T)
T [MeV]


μB/T = 3.5
μB/T = 3
μB/T = 2.5
μB/T = 2
μB/T = 1.5
μB/T = 1
μB/T = 0.5
μB/T = 0
nB/T3 (T)
T [MeV]


4stout, 48³ × 12
μ̂B = 0
μ̂B = i3π/8
μ̂B = i4π/8
μ̂B = i5π/8
μ̂B = i6π/8
χ2S
T[MeV]


R = 0.50
R = 0.25
R = 0.10
R = -0.10
R = -0.25
R = -0.50
μB/T = 2
nB/T3 (T)
T [MeV]

μ_Q


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
120 140 160 180 200 220 240 260 280 300
T [MeV]
d_2^B [n_S=0, n_Q=0.4n_B]
c_2^B [n_S=0, n_Q=0.5n_B]
T d d_2^B / d T [n_S=0, n_Q=0.4n_B]
T d c_2^B / d T [n_S=0, n_Q=0.5n_B]

κ vs. λ