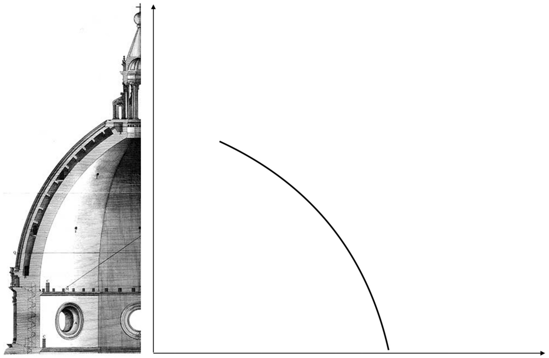
Mar 28, 2022 - Apr 01, 2022

Phase transitions in particle physics

www.ggi.infn.it/ptpp



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Overview on Chiral Perturbation Theory @ Finite Temperature

Giovanni Villadoro

The Abdus Salam International Centre for Theoretical Physics

What is Chiral Perturbation Theory ?

=> Low Energy Asymptotic Expansion of QCD

$$\sum_n c_{n,0} \left(\frac{E}{\Lambda}\right)^n + \sum_{n,m} c_{n,m} \left(\frac{E}{\Lambda}\right)^n e^{-b_m \Lambda/E} + \ldots$$

Complementary to the Asymptotic High-Energy Expansion in

$$\sum_n \frac{a_{n,0}}{[\log(E/\Lambda)]^n} + \sum_{n,m} \frac{a_{n,m}}{[\log(E/\Lambda)]^n} \left(\frac{\Lambda}{E}\right)^m + \ldots \qquad \Leftrightarrow \qquad \sum_n \tilde{a}_{n,0}\, \alpha(E)^n + \sum_{n,m} \tilde{a}_{n,m}\, \alpha(E)^n\, e^{-\beta_0 m/\alpha(E)} + \ldots$$

Remark of Asymptotic (Divergent) Expansions:

@ each order in $\epsilon = \frac{E}{\Lambda}$ $\mathcal{O}_N = \sum_n^N c_n \epsilon^n$ $|\mathcal{O} - \mathcal{O}_N| = o(\epsilon^{N+1})$

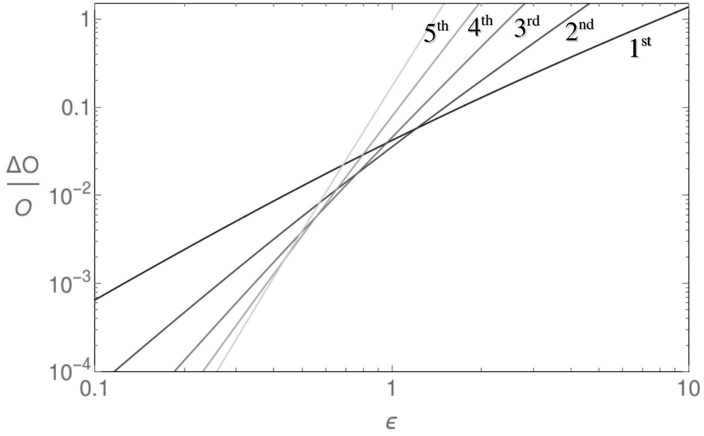
However in realistic cases ε is fixed by the problem (e.g. $\epsilon = \frac{m_\pi^2}{m_K^2}$)

@ fixed ϵ $|\mathcal{O} - \mathcal{O}_N| \gtrsim e^{-a/\epsilon}$ for any *N*. Best @ $N = N_{opt} \simeq 1/\epsilon$

Paradoxically

as $\epsilon \to 1$ $N_{opt} \to 1$

LO better than NLO better than NNLO …



Chiral Perturbation Theory

Weinberg
Callan Coleman Wess Zumino,
Gasser Leutwyler

$$\frac{SU(N_f)_L \times SU(N_f)_R}{SU(N_f)_V}$$

$$\mathcal{L} = \frac{f_\pi^2}{4}\left[\langle \partial_\mu U \partial^\mu U^\dagger \rangle + 2B_0 \langle U M_q^\dagger + M_q U^\dagger \rangle\right]$$

$$U = e^{i\Pi/f_\pi}$$

$$\Pi(x) = \vec{\pi}(x) \cdot \vec{\sigma}$$

$$B_0 = -\langle \bar{q} q \rangle / f_\pi^2$$

$$\Lambda = \min\{4\pi f_\pi \,, 2m_K \,,\; m_{\eta'} \,, m_\rho \;\; ...\}$$

$$\epsilon_{1\pi} \simeq \frac{m_\pi^2}{m^2_{\rho,\eta',2K}} \sim 2 \div 4\%$$

$$\epsilon_{2\pi} \simeq \frac{s = (0.3 \div 0.5 \text{ GeV})^2}{m^2_{\rho,\eta',2m_K}} \sim 10 \div 40\%$$

$$\epsilon_{1K} \simeq \frac{m_K^2}{m^2_{K^\star}, (4\pi f_\pi)^2} \sim 30\%$$

Chiral Perturbation Theory @ Finite Temperature

capture analytic low-T dependence of QCD

equilibrium/static properties

Imaginary Time formalism

partition function →

pressure, energy/entropy densities,
equation of state, condensates
(<qbar q>, top. susc., …)

spatial correlators →

screening length, decay constants …

dynamical/time dependent properties

Real-Time formalism (Schwinger-Keldysh)

time dependent correlators →

dispersion/absorption corrections
thermalization rates
...

Equilibrium/static properties

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LIGHT QUARKS AT LOW TEMPERATURES ☆

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HADRONS BELOW THE CHIRAL PHASE TRANSITION

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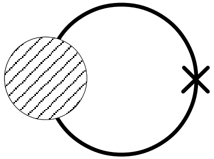
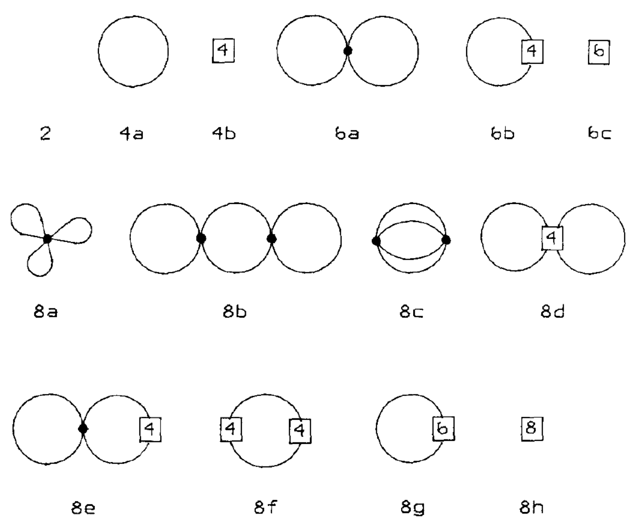


$$K_n(m_\pi\,\beta) \sim \begin{cases} T^n & T \gg m_\pi \\ T^{1/2} e^{-m_\pi/T} & T \ll m_\pi \end{cases}$$



$$P = \tfrac{3}{2}T^4 h_0(\tau) + 4\pi a T^4 [h_1(\tau)]^2 + \pi T^8 h(\tau) b_{\text{eff}}(\tau) + O(T^{10}),$$

$m \to 0$

$$P = \tfrac{1}{30}\pi^2 T^4 \left(1 + \frac{T^4}{36F^4}\ln\frac{\Lambda_p}{T} + O(T^6)\right)$$

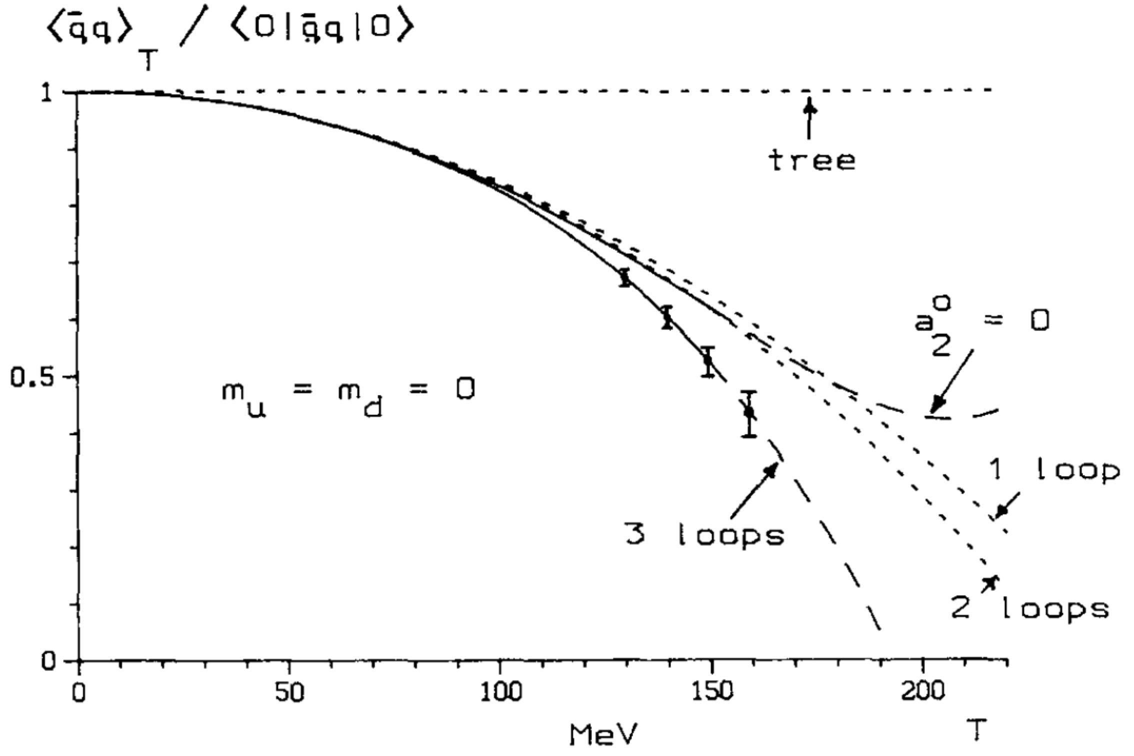
$$s = \frac{\partial P}{\partial T}, \quad u = Ts - P, \quad c_v = \frac{\partial u}{\partial T} = T\frac{\partial s}{\partial T}$$

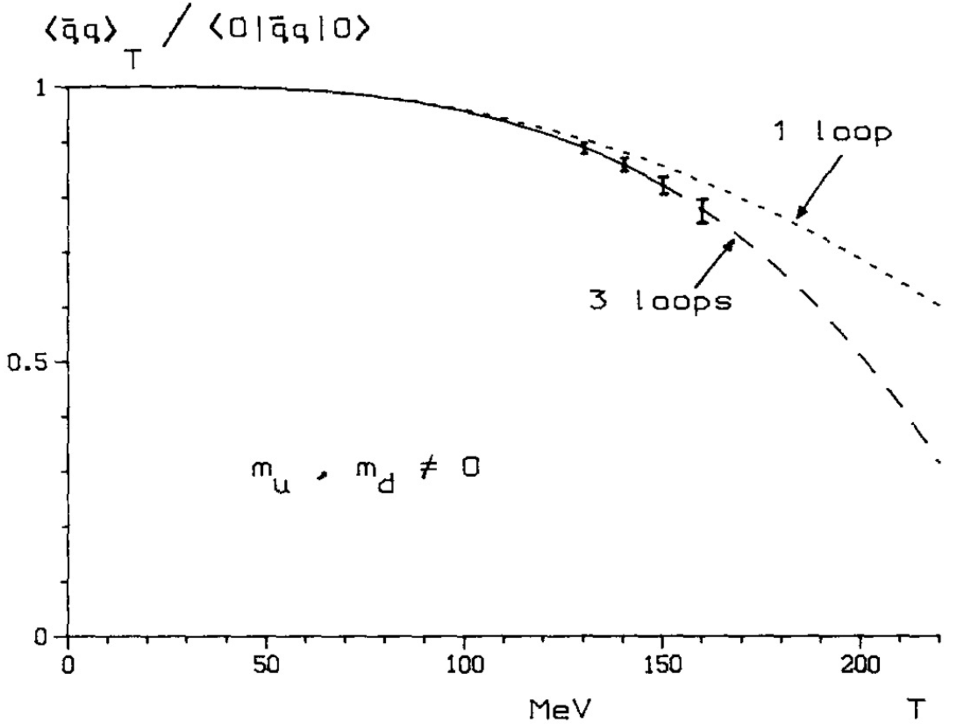
$$\begin{cases} u = \tfrac{1}{10}\pi^2 T^4 \left[1 + \dfrac{T^4}{108F^4}\left(7\ln\dfrac{\Lambda_p}{T} - 1\right) + O(T^6)\right], \\ s = \tfrac{2}{15}\pi^2 T^3 \left[1 + \dfrac{T^4}{144F^4}\left(8\ln\dfrac{\Lambda_p}{T} - 1\right) + O(T^6)\right], \\ c_v = \tfrac{2}{5}\pi^2 T^3 \left[1 + \dfrac{T^4}{432F^4}\left(56\ln\dfrac{\Lambda_p}{T} - 15\right) + O(T^6)\right]. \end{cases}$$

note $\epsilon = \dfrac{T^2}{(2f_\pi)^2}$

rather than $\dfrac{T^2}{(4\pi f_\pi)^2}, \; \dfrac{T^2}{m_\rho^2}, \ldots$

$$\langle \bar{q}q \rangle \overset{m \to 0}{=} \langle 0|\bar{q}q|0\rangle \left(1 - \frac{T^2}{8F^2} - \frac{T^4}{384F^4} - \frac{T^6}{288F^6} \ln\frac{\Lambda_q}{T} + O(T^8)\right)$$

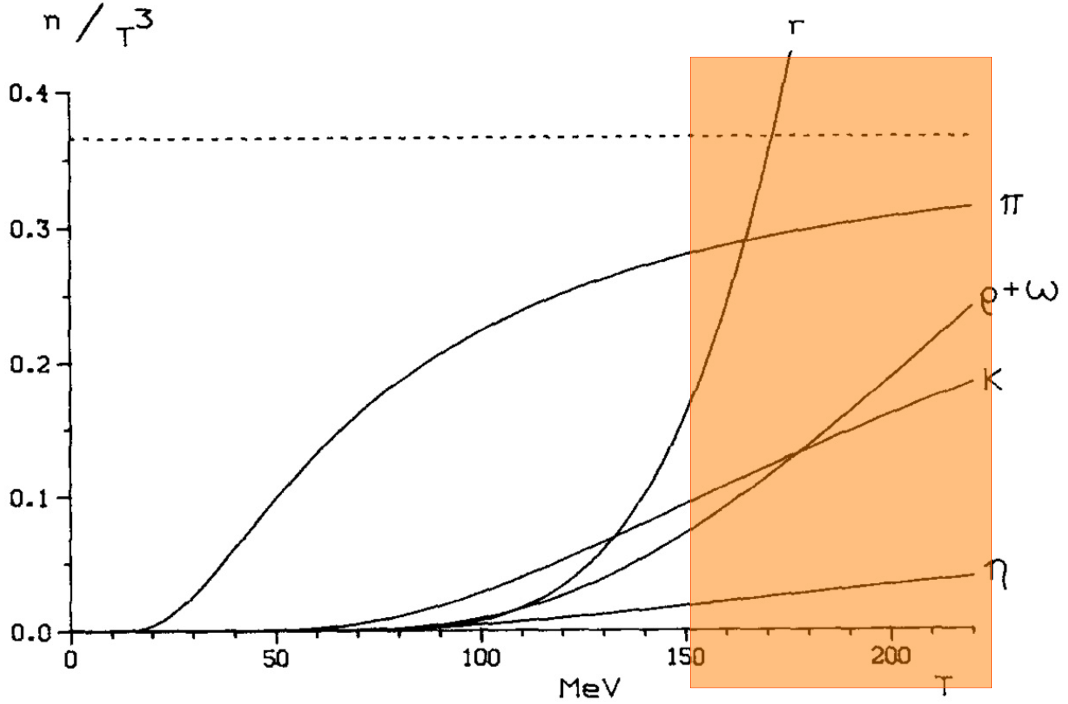


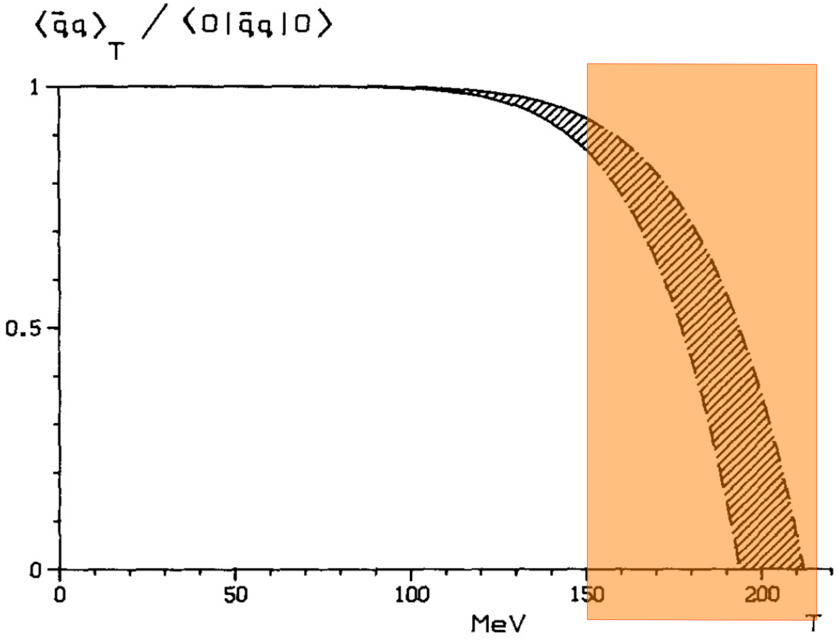


$$\langle j_5^\mu(\vec{x}) j_5^\nu(0)\rangle \sim f_\pi(T)^2 e^{-m_\pi(T)|\vec{x}|}$$

$$f_\pi(T) = f_\pi\left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2} + o(T^4)\right)\,, \qquad m_\pi(T) = m_\pi\left(1 + \frac{1}{48}\frac{T^2}{f_\pi^2} + o(T^4)\right)$$

Effects from heavy states





Topological Susceptibility and θ-dependence

The QCD axion, precisely

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$$\frac{\chi_{top}(T)}{\chi_{top}} \overset{\text{NLO}}{=} \frac{m_\pi^2(T) f_\pi^2(T)}{m_\pi^2 f_\pi^2} = \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle}$$

$T \to 0$

$$F(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\theta}{2}\right)}$$

$$= \langle \bar{q}q \rangle \sqrt{m_u + m_d + 2 m_u m_d \cos\theta}$$

$T \to \infty$

$$\sim C \left(\frac{T_c}{T}\right)^\beta \cos(\theta) \qquad \beta = 7 + n_f/3$$

$$\frac{F(\theta)_T}{F(\theta)} = 1 + \frac{3}{2} \frac{T^4}{f_\pi^2 m_\pi^2(\theta)} J_0\left[\frac{m_\pi^2(\theta)}{T^2}\right]$$

$$J_0[\xi] \equiv -\frac{1}{\pi^2} \int_0^\infty dq\, q^2 \log\left(1 - e^{-\sqrt{q^2+\xi}}\right)$$

QCD and instantons at finite temperature

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Range of validity

Naively χPT seems good up to $T \sim 2 f_\pi \sim 200$ MeV

$T_c \sim 150$ MeV deconfinement $\rightarrow$ no mesons? (crossover …)

As $T \rightarrow T_c$ $\langle \bar{q} q \rangle \rightarrow 0$ $\rightarrow$ pions strongly coupled ($f_\pi \rightarrow 0$)

$$\epsilon = \frac{T^2}{(2f_\pi)^2} \rightarrow \frac{T^2}{(2f_\pi(T))^2} \rightarrow$$ breaking @ $T \sim f_\pi \leq T_c$

Range of validity

(EFT of QFT) @ Finite T $\neq$ EFT of (QFT @ Finite T)

$$\sim e^{-\Lambda/T} \qquad \Lambda \stackrel{?}{=} \Lambda_{QCD}\,, T_c\,, \ldots$$

analogous to Debye mass in QED $\rightarrow$ Euler-Heisenberg

Dynamical Properties

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ON THE MEAN FREE PATH OF PIONS IN HOT MATTER

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Pion propagation at finite temperature

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Pion Dynamics at Finite Temperature

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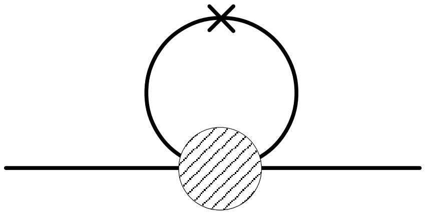
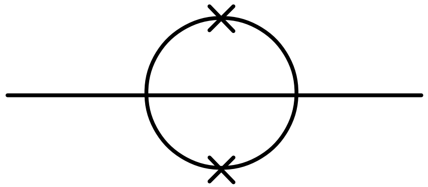
pole condition in pion propagator:

$$(p^0)^2=\mathbf{p}^2+M_\pi^2+\Sigma_R^T(p^0,\mathbf{p}) \quad \rightarrow \quad p^0=\omega(p)-\frac{i}{2}\gamma(p)$$

$2\pi\, n_B(|p^0|, T)\, \delta(p^2 - m_\pi^2)$



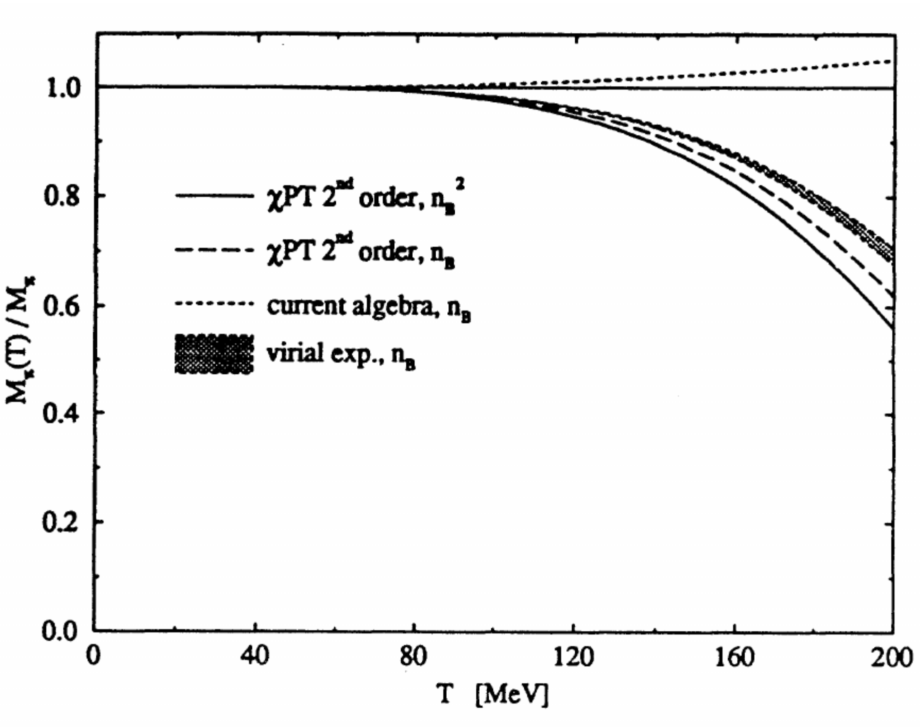
$$\Sigma_R^T(\omega_p, \mathbf{p}) = -\int \frac{d^3q}{(2\pi)^3 2\omega_q^T} n_B(\omega_q^T) T_{\pi\pi}(s) - \frac{1}{2}\int \frac{d^3q_1}{(2\pi)^3 2\omega_{q_1}} \frac{d^3q_2}{(2\pi)^3 2\omega_{q_2}} n_B(\omega_{q_1}) \times n_B(\omega_{q_2}) \hat{T}^{R}_{\pi\pi\pi}(p, q_1, q_2) \,,$$

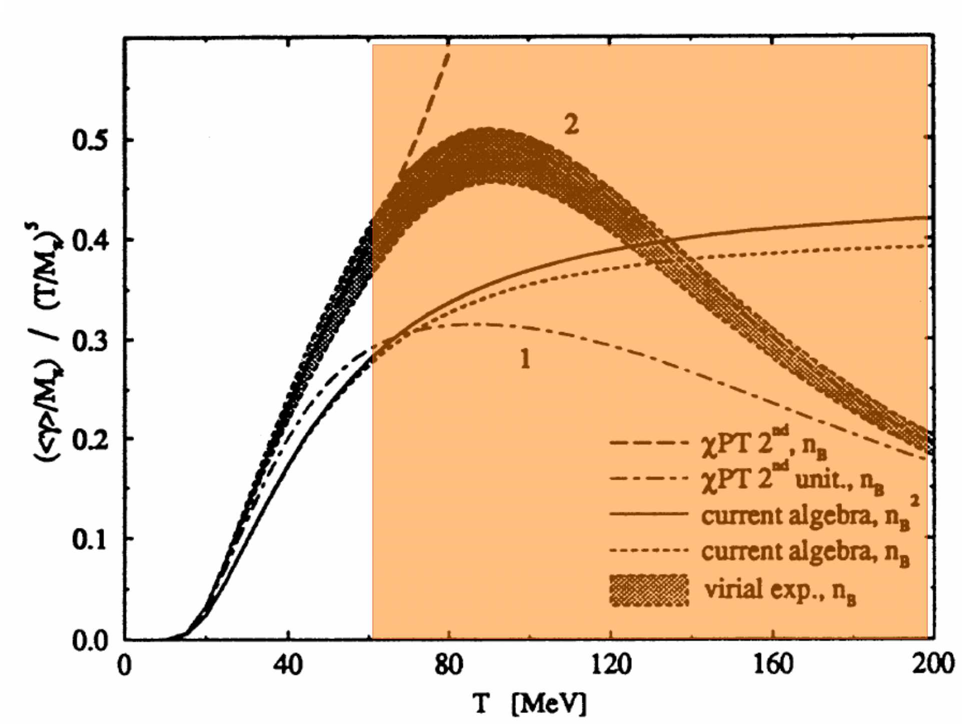


$$\gamma(p) = \frac{1}{4\omega_p} \frac{1}{1+n_B(\omega_p)} \int \frac{d^3q_1}{(2\pi)^3 2\omega_{q_1}} \frac{d^3q_2}{(2\pi)^3 2\omega_{q_2}} \frac{d^3q_3}{(2\pi)^3 2\omega_{q_3}} (2\pi)^4 \delta^{(4)}(p+q_1+q_2+q_3) \times n_B(\omega_{q_1})[1+n_B(\omega_{q_2})][1+n_B(\omega_{q_3})] |T_{\pi\pi}(s,t)|^2 \,,$$

$$\sim \frac{1}{\omega_p} \int \frac{d^3q}{(2\pi)^3 2\omega_q} n_B(\omega_q) \sqrt{s(s-4M_\pi^2)}\, \sigma_{\pi\pi}(s) \qquad \sim \frac{T^5}{12 f_\pi^4}$$

LO
massless limit





worse convergence in real time:
π-π scattering with total √s
~ 2(m_π+2T) ~ 0.6 GeV for T ~ 60 MeV

Breakdown of chiral perturbation theory for the axion hot dark matter bound

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Conclusions

- ChPT provides low-T asympt. expansion of QCD
- Many observables computed at higher order
- Expansions breaks at $T \sim f_\pi \sim T_c$ (as naively expected)
- How close to T_c depends on observable (extra care needed for real-time obs.)
- Since $m_\pi \sim f_\pi$ thermal corrections fastly converge at low T

Thank you!