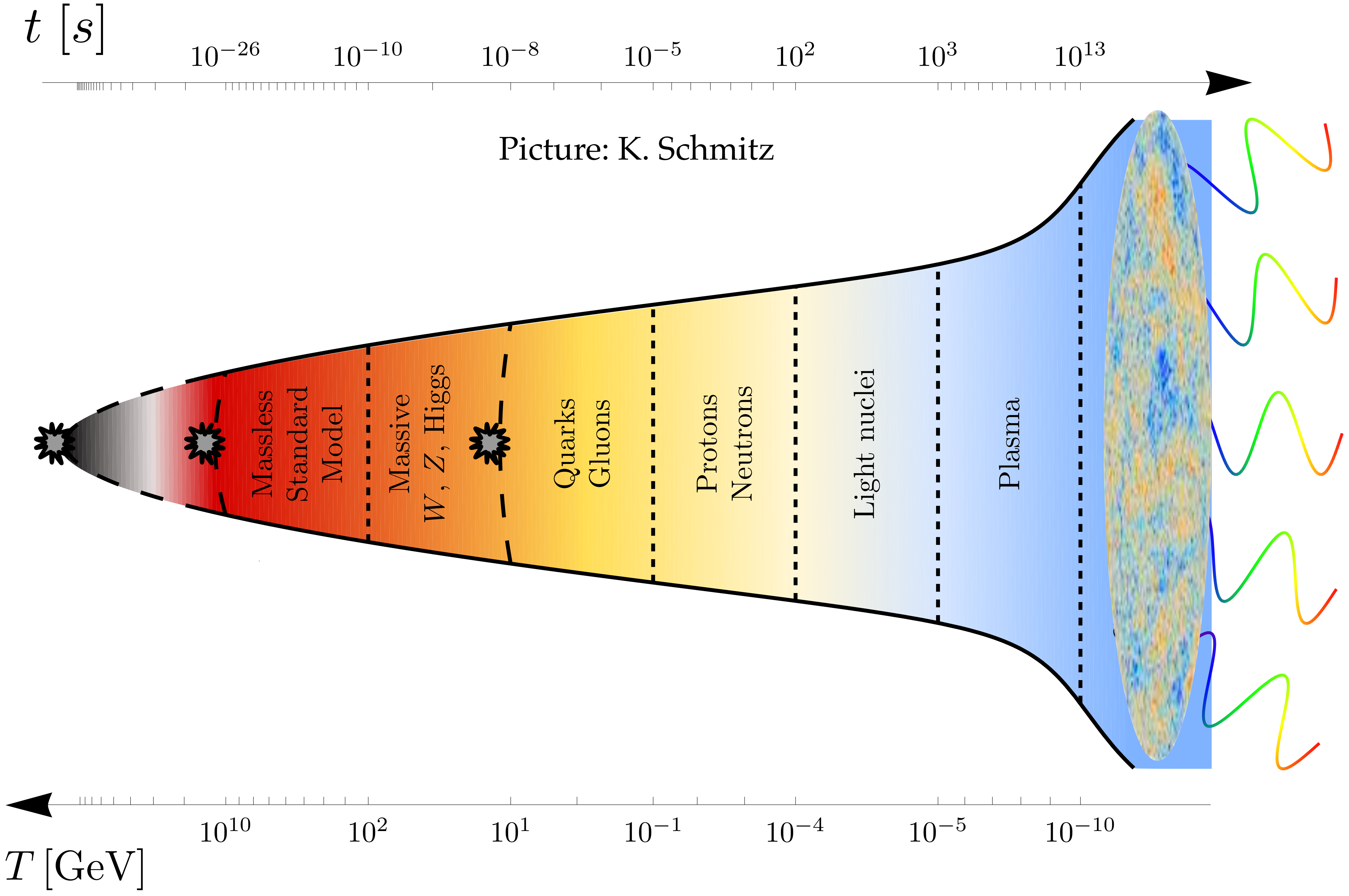
# Thermal Field Theory and Cosmology

Jacopo Ghiglieri, SUBATECH, Nantes

Phase transitions in particle physics, GGI Firenze, March 29 2022

# Rates in the early universe

- Over the long thermal history, many phenomena enter and/or leave equilibrium
  - DM candidates
  - Mechanisms for baryogenesis
  - Thermal relics
  - ….




- governed by rates (production, equilibration, interaction, nucleation…) competing with the Hubble rate

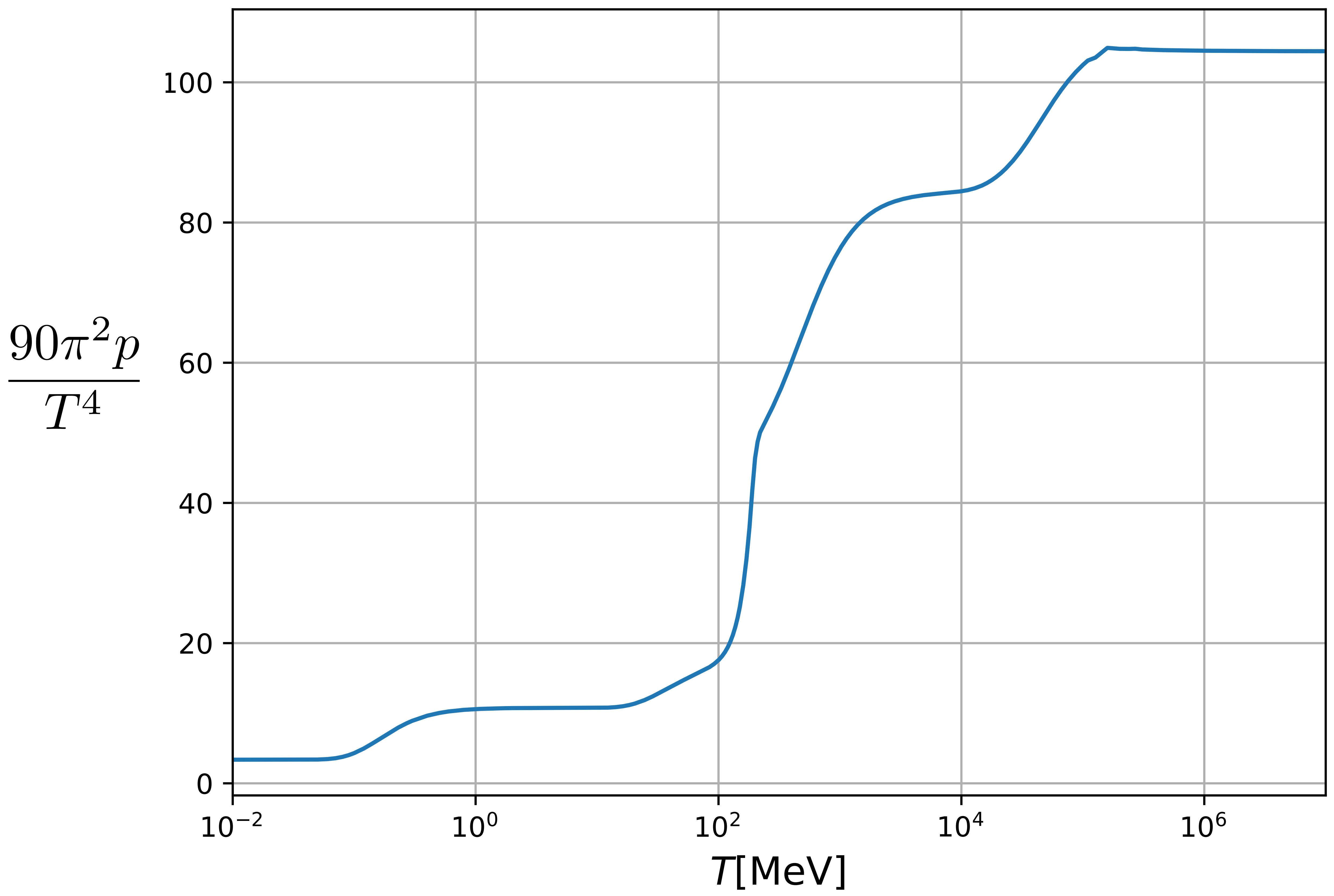
# In this talk

- Thermodynamics: phase transitions, the Hubble rate itself,…
- Defining and computing (some of) these rates using modern Thermal Field Theory (TFT) techniques
  - Slowly-varying modes over a fast background
  - Massless states: the example of gravitational waves
  - Massive states: the example of right-handed neutrinos and NLO corrections

# Thermodynamics and phase transitions

# Thermodynamics

- The Hubble rate is proportional to the energy density $H = \sqrt{\frac{8\pi e}{3m_{\text{Pl}}^2}} \sim \frac{T^2}{m_{\text{Pl}}}$
- Many "transitions" in the SM
- How to compute them? And why are they so interesting?
- A short tale of phase transitions and gravitational waves




Hindmarsh **2008.09136**
Laine Meyer (2015)

# Baryogenesis

- Need to satisfy Sakharov's conditions
  - B violation
  - C and CP violation
  - Deviations from thermal equilibrium

# Electroweak baryogenesis

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# Electroweak baryogenesis

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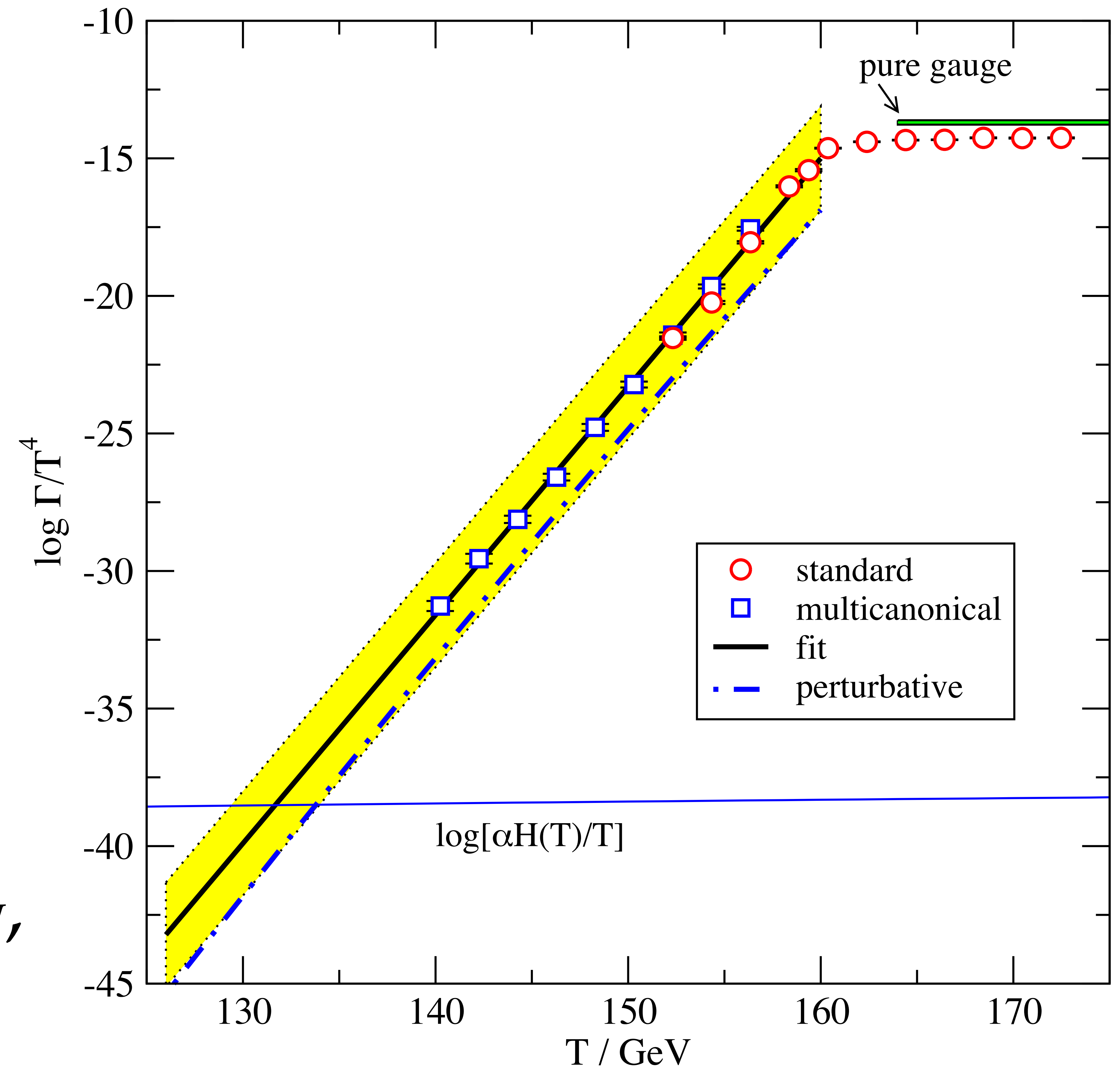
- Feynman rules always conserve B, but sphaleron processes violate B (and conserve B-L)
  Non-perturbative solutions, in equilibrium at $T > T_{EW}$, exponentially suppressed below. Decouple at $T \sim 130$ GeV D'Onofrio Rummukainen Tranberg **PRL113** (2014)



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- The CKM phase violates CP

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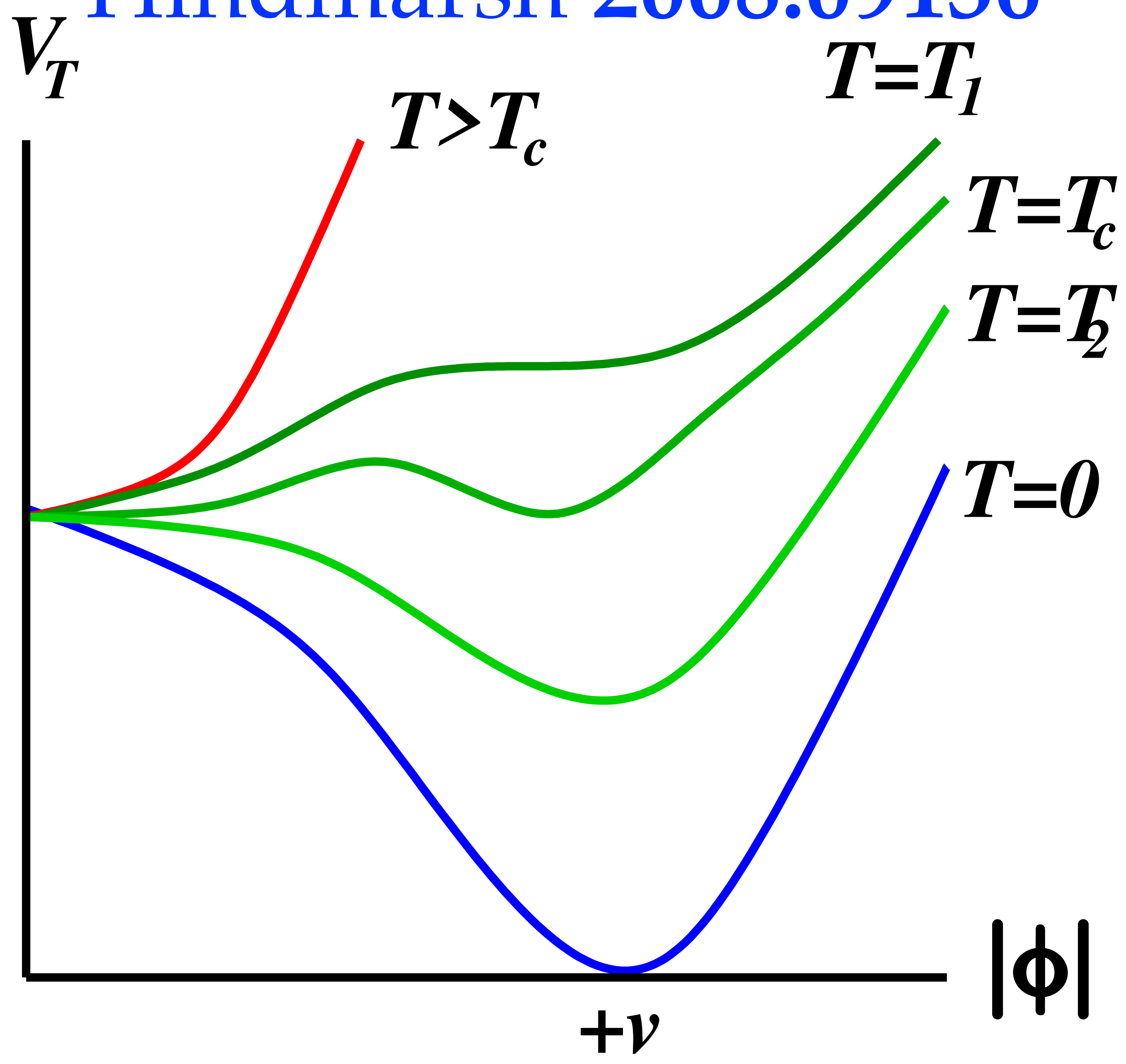


- The CKM phase violates CP
- A strong first order phase transition is needed. Sphaleron rate suppressed in bubbles of the broken phase nucleating within the symmetric phase
- Bubble dynamics would also create a gravitational wave signature, potentially observable by LISA

# Electroweak baryogenesis

- Need to satisfy Sakharov's conditions
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  - Deviations from thermal equilibrium

- Not enough CP violation in the SM
- No phase transition in the SM for $M_H$>72 GeV, but crossover
  Gurtler Hilgenfritz Schiller, Laine Rummukainen, Csidor Fodor Heitger (1997-99)
- Both issues can be addressed in many BSM models

# Electroweak baryogenesis

- Need to satisfy Sakharov's conditions
  - B violation
  - C and CP violation
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- Review on phase transitions and GWs: Hindmarsh Lüben Lumma Pauly **2008.09136**

# A weakly coupled plasma

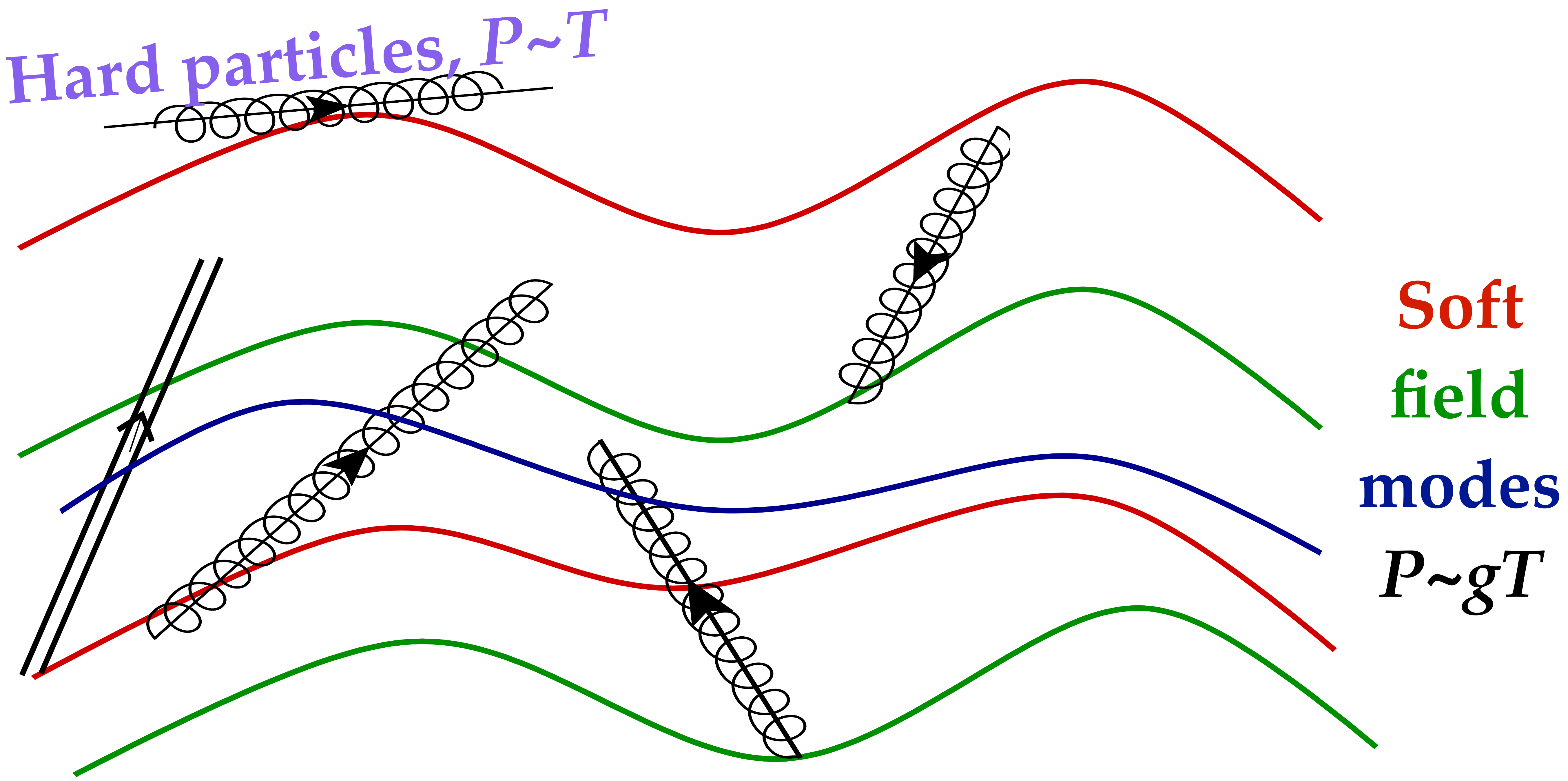



Figure by D. Teaney

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

# A weakly coupled plasma

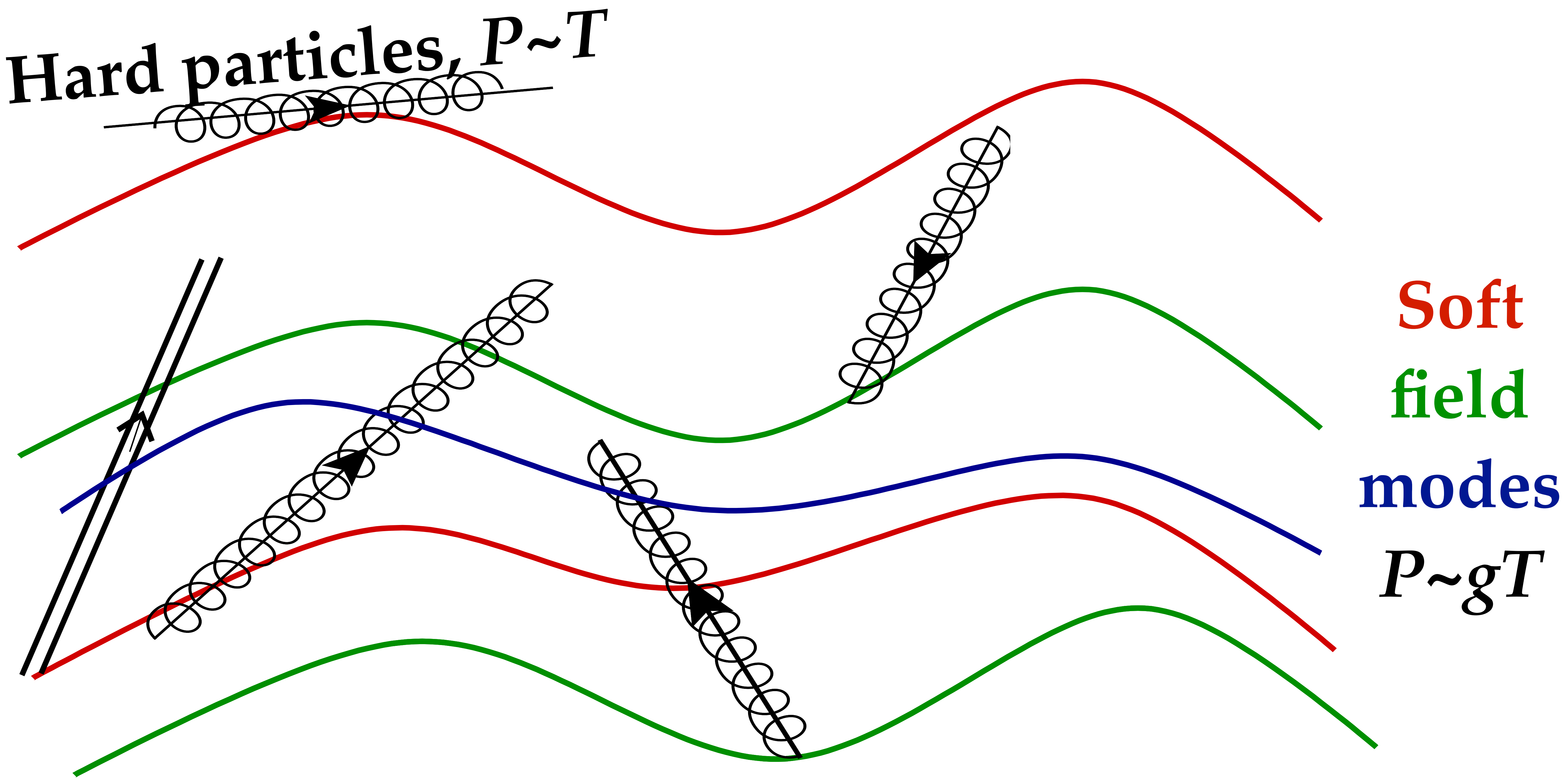



Figure by D. Teaney

- The bosonic soft fields have large occupation numbers ⇒ they can be treated classically

$$\frac{1}{e^{\omega/T}-1} \overset{\omega\ll T}{\approx} \frac{T}{\omega} \gg 1$$

# A weakly coupled plasma

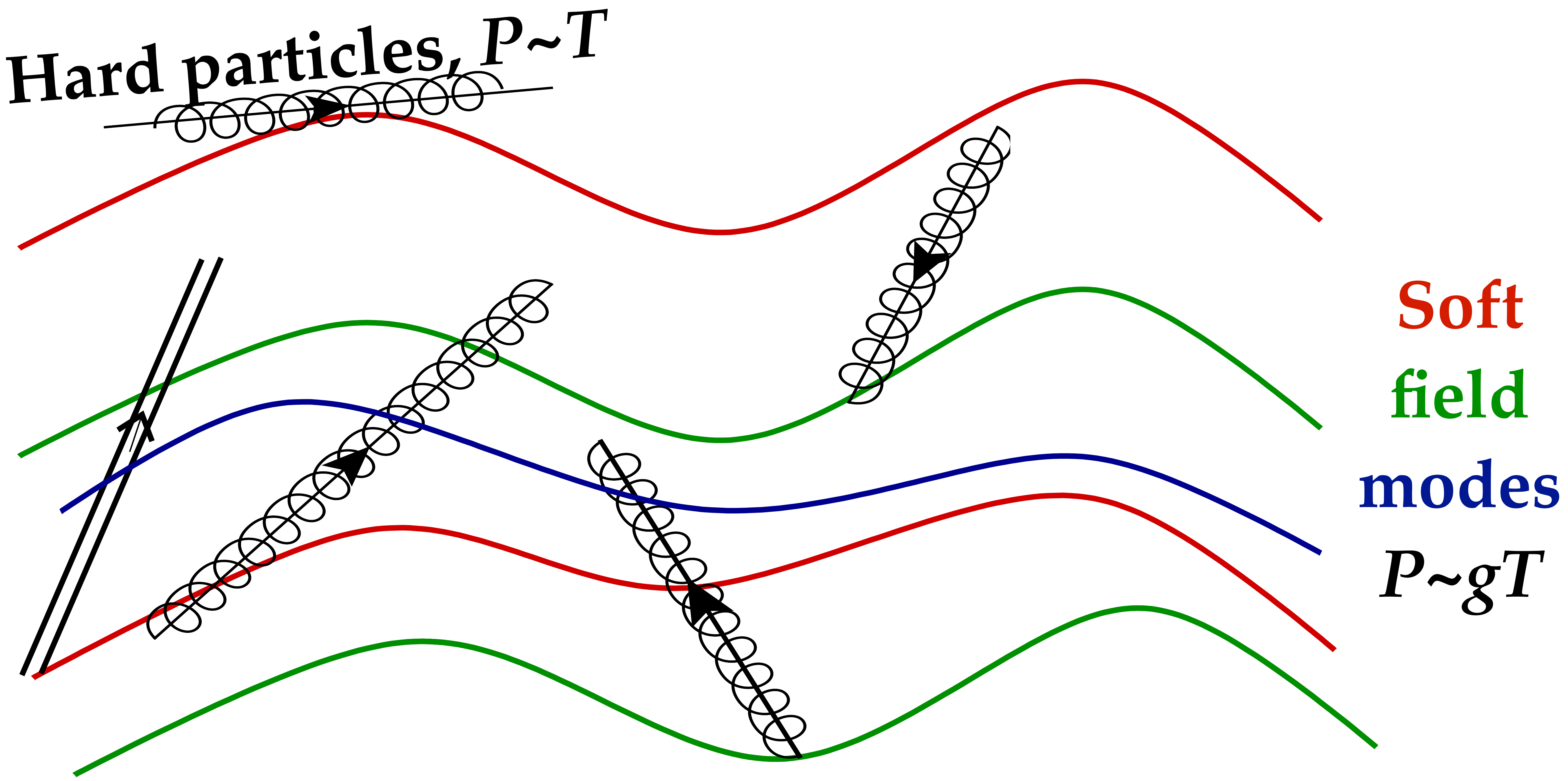



Figure by D. Teaney

- Their loop expansion is $g^2T/\omega$ ($g$ the gauge couplings, top Yukawa and $\sqrt{\lambda}$)
- It breaks down for bosons with $m \lesssim g^2T$. These are the magnetic modes of the gauge bosons and the Higgs when $gv \lesssim g^2T$.
- Need non-perturbative input for the phase transition!

# An EFT approach

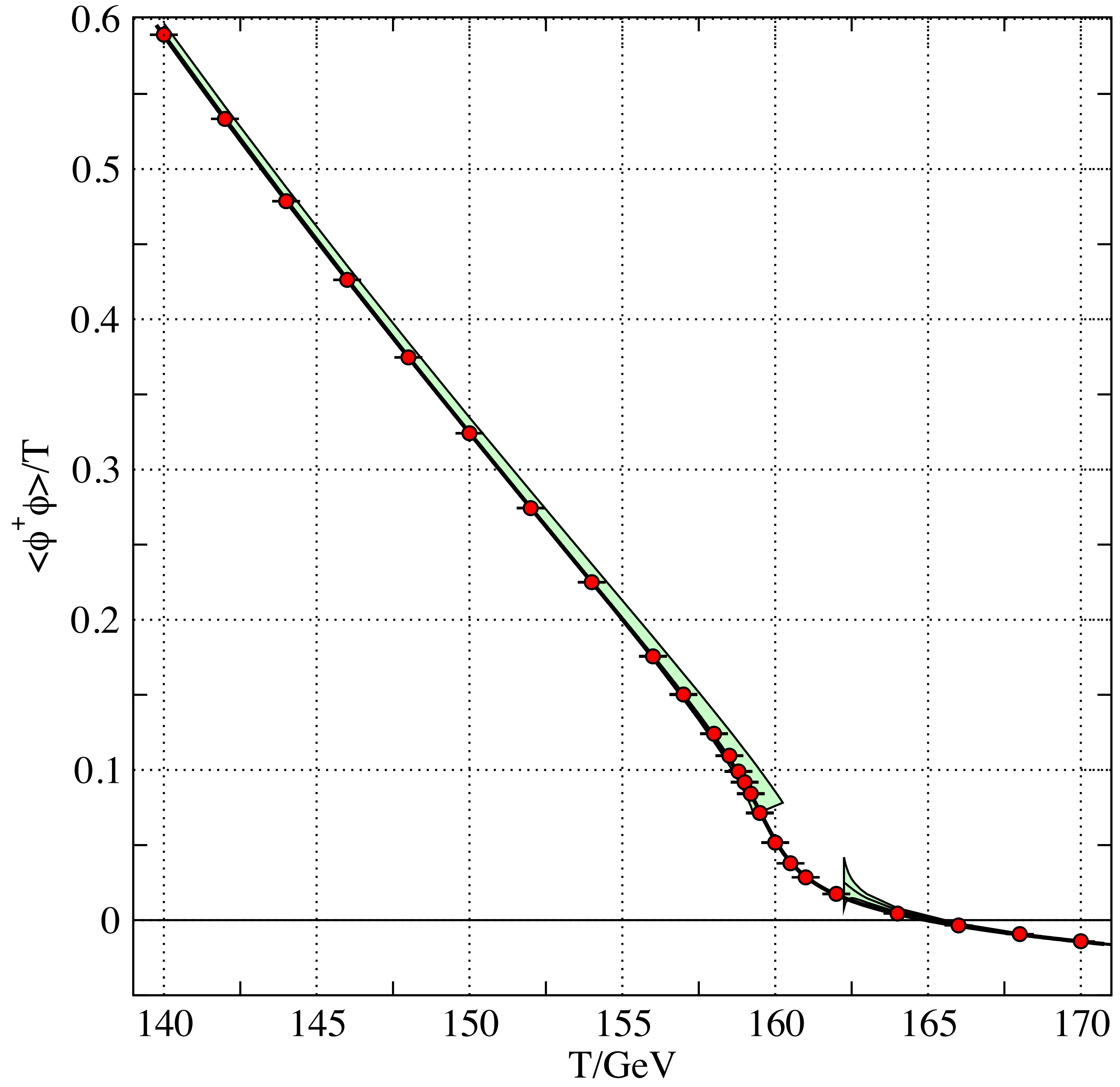
- In the Matsubara formalism, frequencies are discrete. $2\pi nT$ for bosons, $\pi(2n+1)T$ for fermions.
- Non-zero Matsubara modes are *hard modes*, $q\sim T$, can be integrated out perturbatively. Dimensional reduction to 3D theory (EQCD for QCD)
- Electric modes of gauge bosons are *soft modes*, $q\sim gT$, can also be integrated out (perturbatively or not)
- Remains: zero modes of scalars and spatial gauge bosons (MQCD for QCD) No problems on the lattice (no chiral fermions)

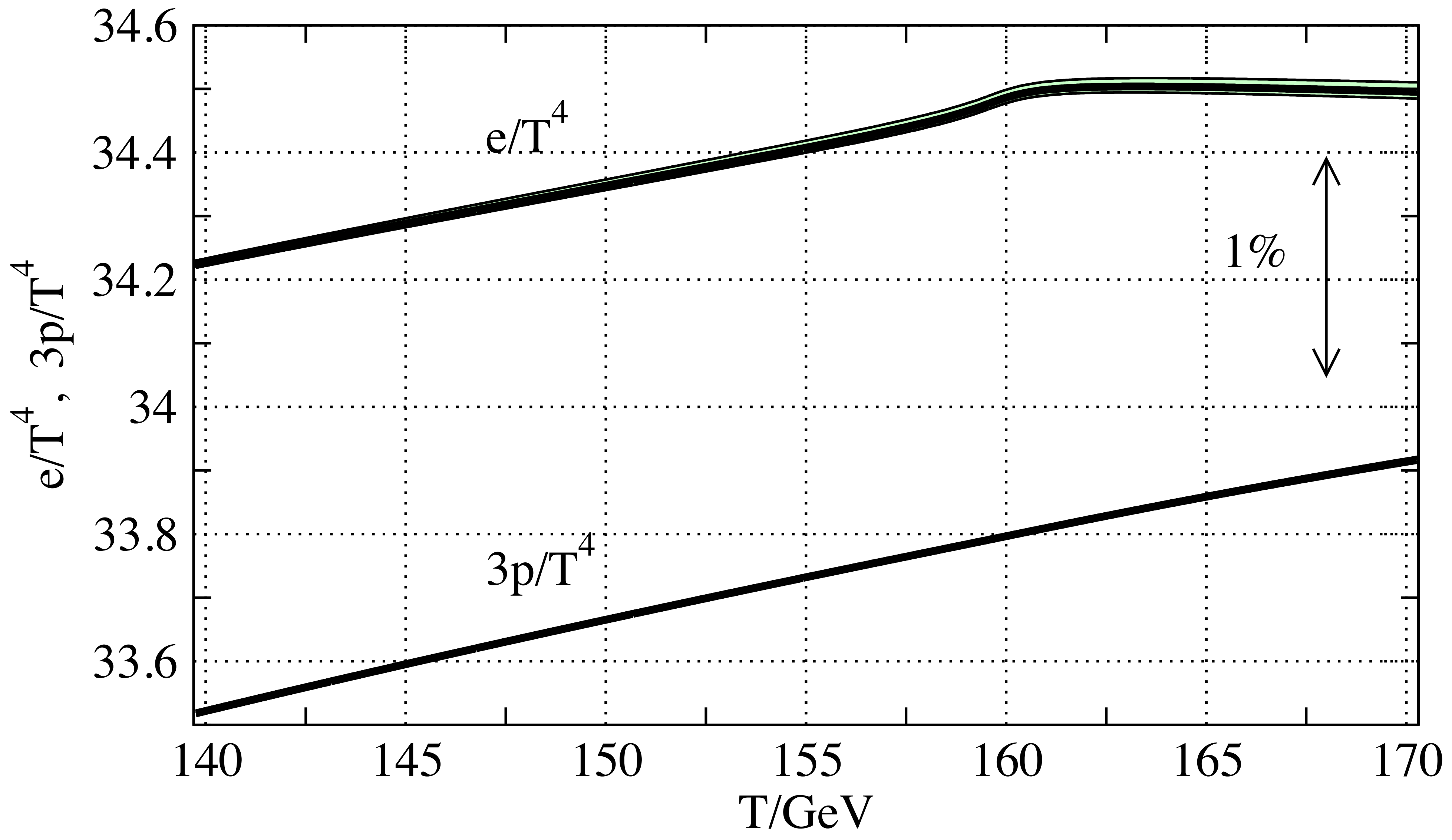
$$L = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}B_{ij}B_{ij} + (D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

Laine Kajantie Rummukainen Shaposhnikov (1995-97) Braaten Nieto (1995-97)

# An EFT approach

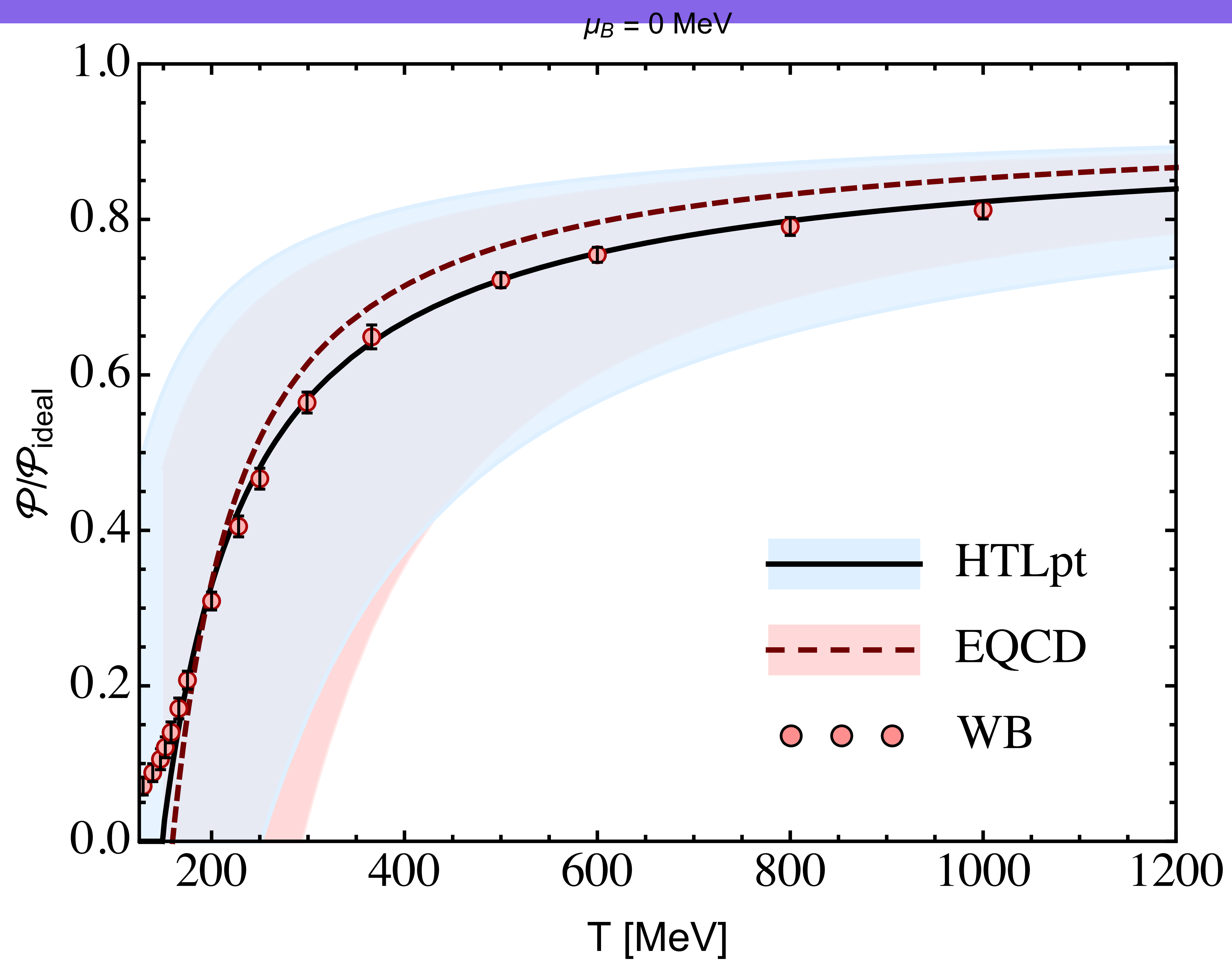
- State of the art for the SM at $M_H$=125 GeV. Lattice D'Onofrio Rummukainen (2015), pert thy Laine Meyer (2015)

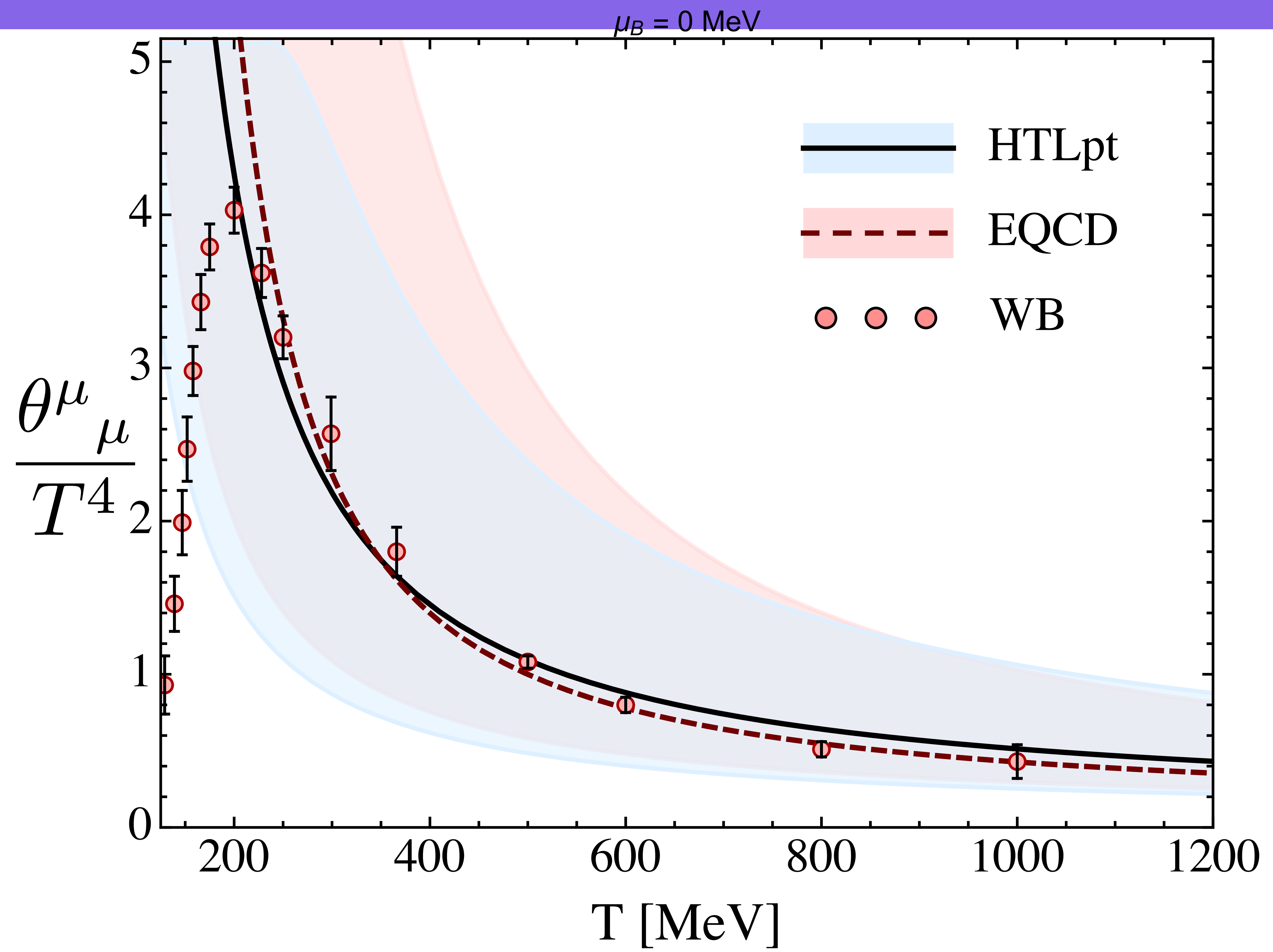



- Narrow non-perturbative window for the SM. Thermodynamics at the 1% level. Below the ideal gas result $e$=106.75 $\pi^2/30\ T^4 \approx 35.1\ T^4$

# An EFT approach




*Review*: JG Kurkela Strickland Vuorinen **Phys. Rep. 880** (2020)
*Lattice:* Budapest-Wuppertal, Borsanyi *et al* **JHEP1011** (2010)

- Very different from QCD transition: here all but a handful of dofs are weakly-coupled

# An EFT approach

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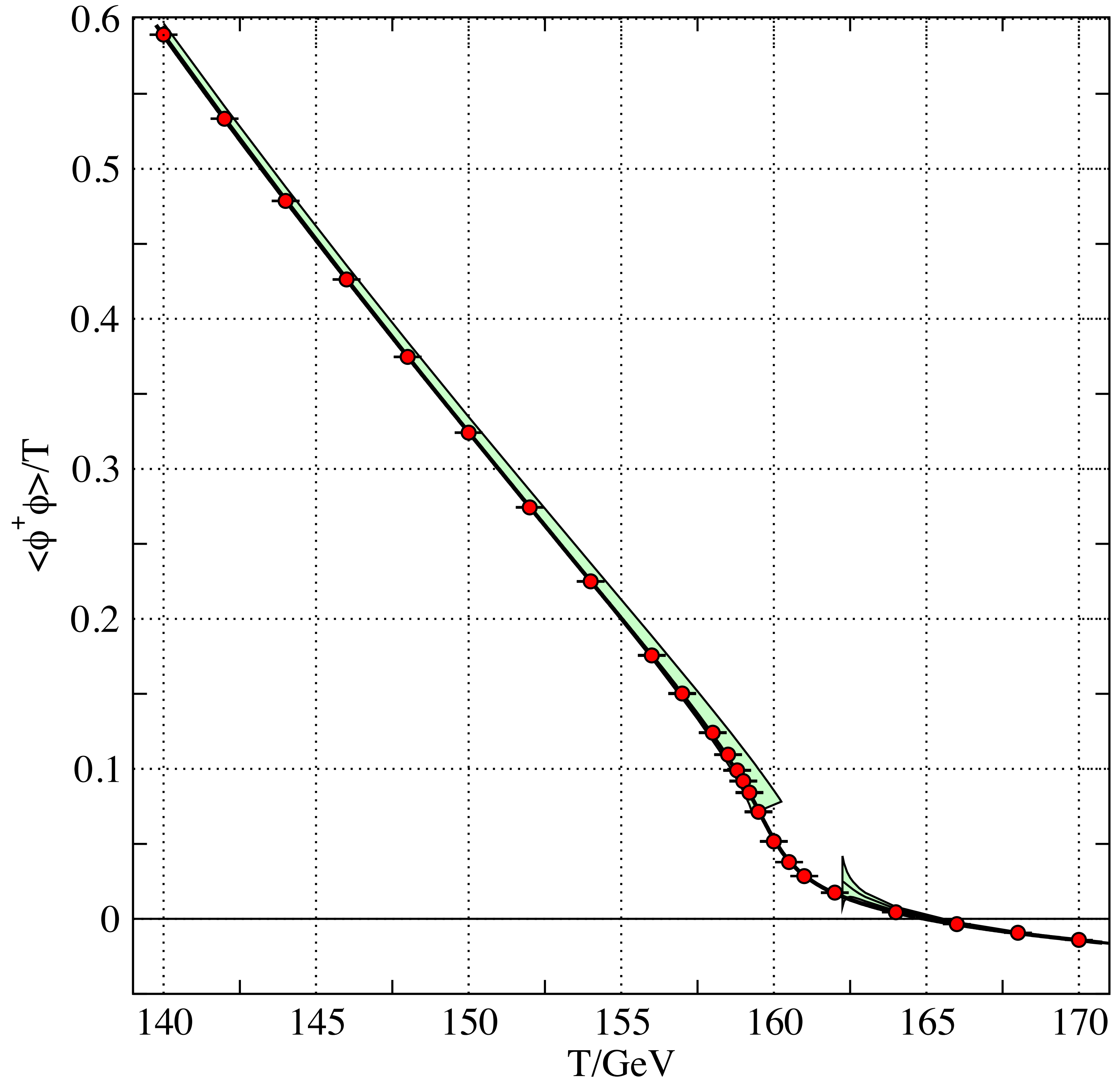
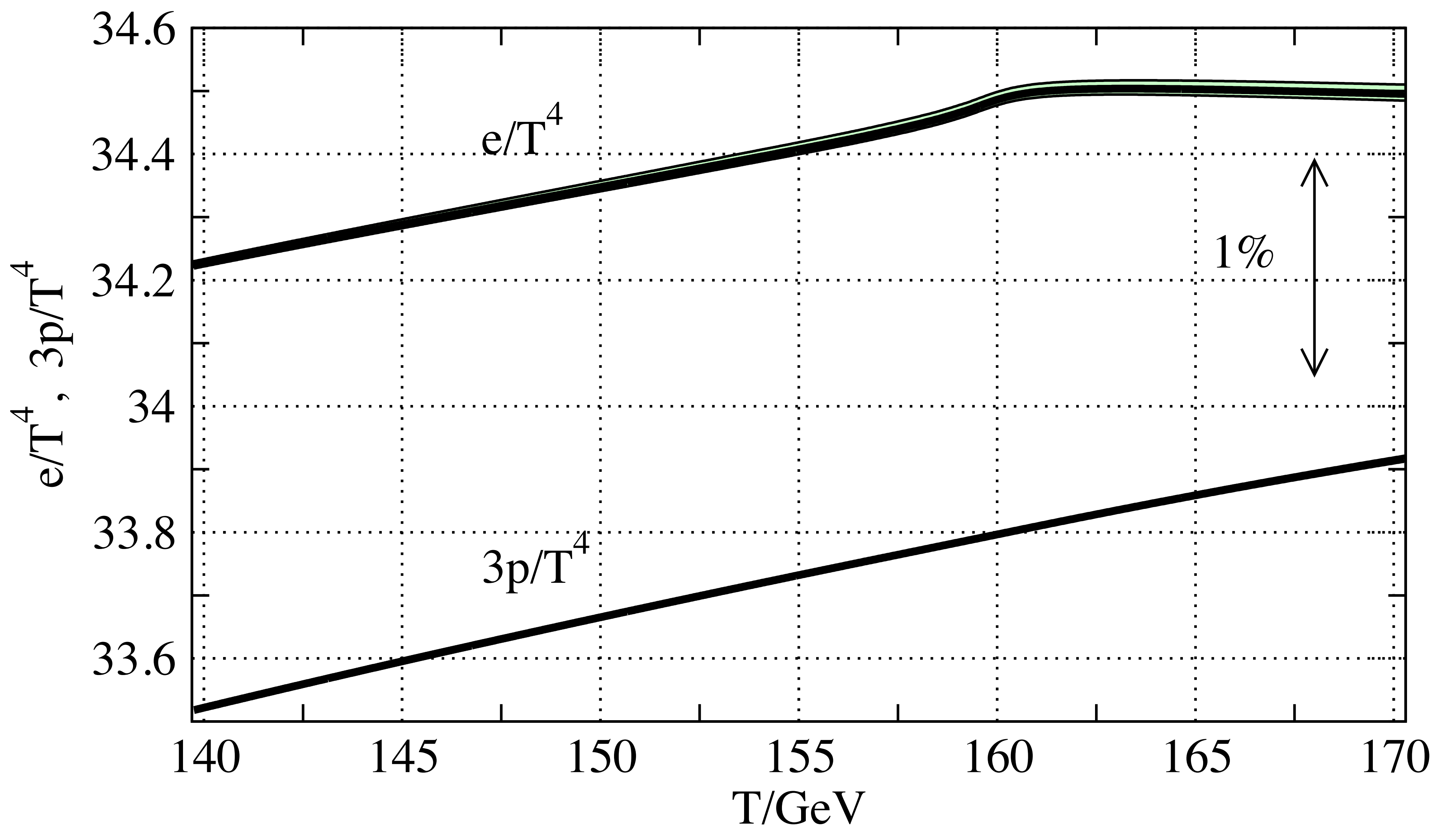


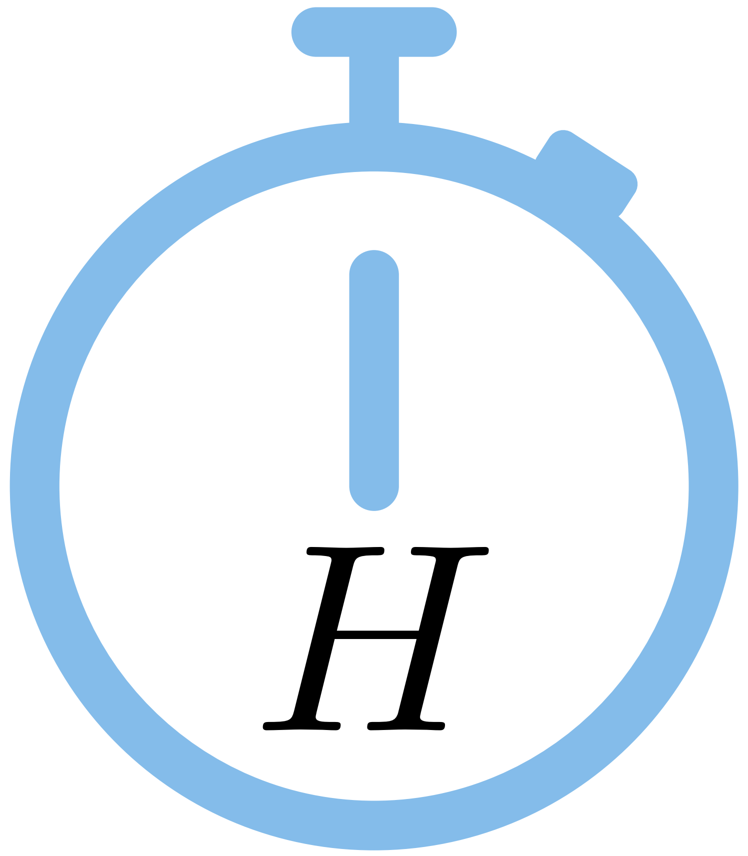




- Very active research in adapting existing lattice measurements or performing new ones for BSM scenarios who promise phase transitions and GW signatures

# Production and interaction rates

# General approach

- Factor the system into "fast" and "slow" modes, and integrate out the former to obtain evolution eqs. for the latter



# Production and equilibration

- A particle $\phi$ is weakly coupled (coupling $h$) to an equilibrated bath with its internal couplings $g$ $\quad \mathcal{L} = \mathcal{L}_\phi + h\phi J + \mathcal{L}_{\text{bath}}$
  $J$ built of bath fields, one can prove to first order in $h$ and **all orders** in $g$

$$\dot{f}_\phi(t, \mathbf{k}) = \Gamma(k) \left[ f_{\text{eq}}(k^0) - f_\phi(t, \mathbf{k}) \right] + \mathcal{O}(h^4)$$

$$\Gamma(k) = \frac{h^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [J(X), J(0)] \rangle$$

Bödeker Sangel Wörmann **PRD93** (2016)

- Single-particle phase-space distribution: $f(t,\mathbf{k})$, sensible only for sufficiently weakly interacting particles

- For conserved charges, equations for the density $n$ can similarly be defined with no quasiparticle assumptions Bödeker Laine (2014)

# Production and equilibration

$$\dot{f}_\phi(t,\mathbf{k}) = \Gamma(k)\left[f_{\rm eq}(k^0) - f_\phi(t,\mathbf{k})\right] + \mathcal{O}(h^4) \qquad \Gamma(k) = \frac{h^2}{2k^0}\int d^4X e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- The derivation is based (and relies) on a **separation of timescales** between production/equilibration and the plasma dynamics
- **All-order proof** of the equivalence of production and equilibration rates, $\Gamma_{\rm prod}$=$\Gamma(k)\, f_{\rm eq}(k^0)$. Goes beyond previous statements based on detailed balance in a leading-order Boltzmann approach.
- When doing perturbative expansions, Boltzmann expressions are recovered where applicable (LO). Higher orders are possible and natural in this form
- Easier to include non-perturbative input in this framework if needed. See P. Schicho's talk on Wednesday for QCD and heavy ions

# Production and equilibration

$$\dot{f}_\phi(t,\mathbf{k}) = \Gamma(k)\left[f_{\rm eq}(k^0) - f_\phi(t,\mathbf{k})\right] + \mathcal{O}(h^4) \qquad \Gamma(k) = \frac{h^2}{2k^0}\int d^4X e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- Applications of this TFT result to heavy ions (e.g. photon production, thermalisation) and cosmology. Not in this talk:
  - Non-equilibrium Kadanoff-Baym equations yield similar results Drewes (2010) Drewes Mendizabal Weniger (2013) Garny Hohenegger Kartavtsev (2010-13)
  - Cases where $f(t,\mathbf{k}) \gg 1$ (e.g. bosonic fields during reheating) and classical non-perturbative methods are used COSMOLATTICE Figueroa Florio Torrenti Valkenburg (2020)

# Production and equilibration

$$\dot{f}_\phi(t,\mathbf{k}) = \Gamma(k)\left[f_{\rm eq}(k^0) - f_\phi(t,\mathbf{k})\right] + \mathcal{O}(h^4) \qquad \Gamma(k) = \frac{h^2}{2k^0}\int d^4X e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- When using these equations in cosmology, the l.h.s is modified to include Hubble expansion

$$\dot{f}_\phi(t,\mathbf{k}) \to (\partial_t - H\mathbf{k}\cdot\nabla_\mathbf{k})f_\phi(t,\mathbf{k})$$

  and often (number, energy) densities are the quantity of interest, e.g. $n_\phi = \int_\mathbf{k} f_\phi$

$$\dot{n}_\phi + 3Hn_\phi = \int_\mathbf{k}\Gamma(k)[f_{\rm eq}(k^0) - f_\phi(t,k)]$$

- If **scale separation** is present and $g \ll 1$, perturbative expansion of $\Gamma(k \gtrsim T)$ can reproduce standard Boltzmann. But **quasiparticle picture is not necessary**!

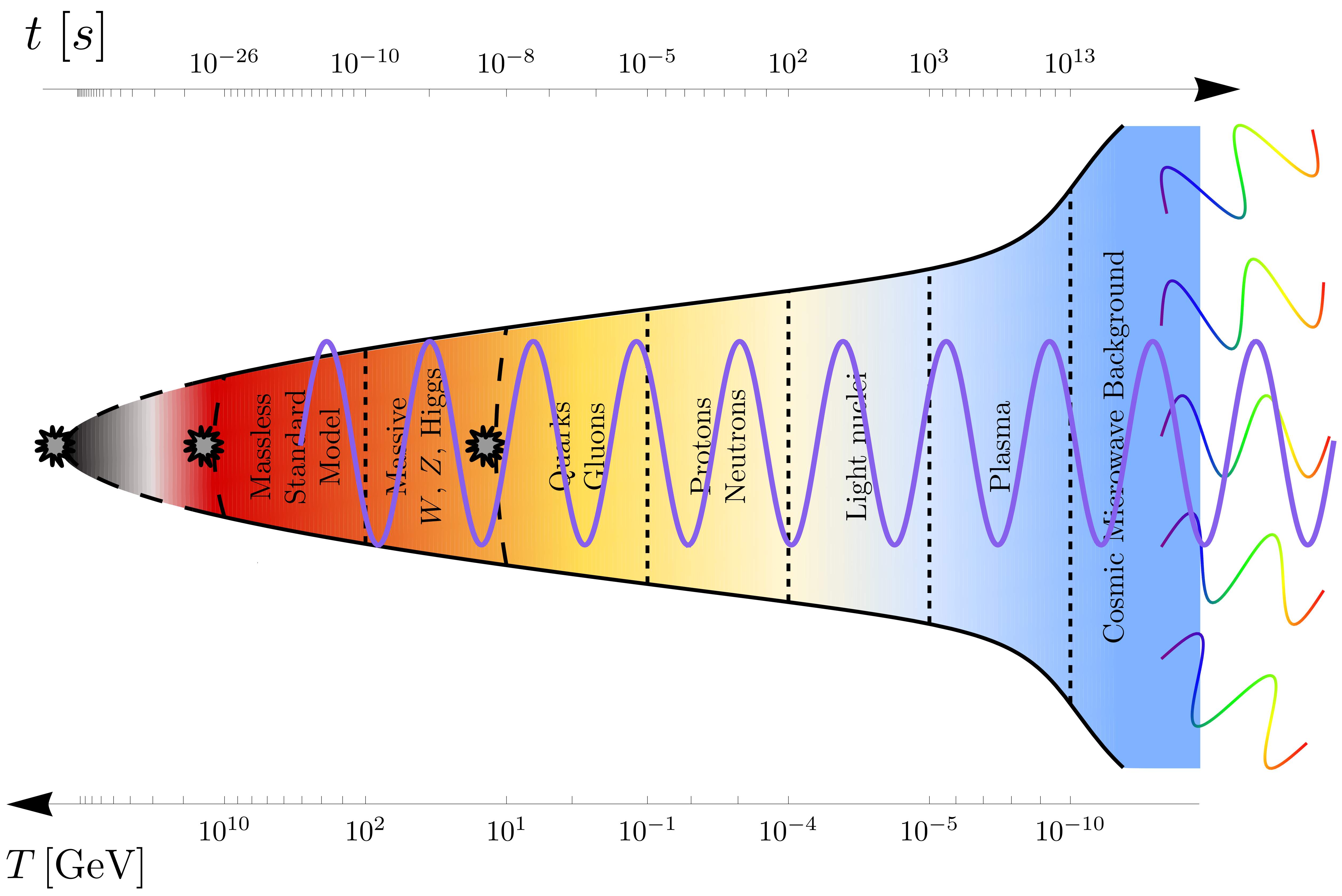
# Massless particles: gravitational waves

JG Laine **JCAP1507** (2015) JG Jackson Laine Zhu **JHEP2007** (2020)
Ringwald Schütte-Engel Tamarit **JCAP2103** (2021)

# Gravitational waves in the early universe

Many potential sources of GWs

- Inflation
- Reheating
- Phase transitions
- …




All model-dependent and/or speculative to a degree

Review: Caprini Figueroa *Class. Quant. Grav.* 35 (2018)

# Gravitational waves from equilibrium

- GWs can be produced from eq. too. Weinberg
- Now $J \propto T^{\mu\nu}/m_{\rm Pl}$, so as long as $T_{\rm max} < m_{\rm Pl}$ the GW-plasma coupling is indeed weak: freeze-in production over the history of the early universe?
- By the previous arguments: $\dot{f}_{\rm GW}(t,\mathbf{k}) = \Gamma(k)\left[n_{\rm B}(k) - f_{\rm GW}(t,\mathbf{k})\right] + \mathcal{O}\left(\frac{1}{m_{\rm Pl}^4}\right)$

$$\Gamma(k) = \frac{8\pi}{k\, m_{\rm Pl}^2}\int d^4X\, e^{ik(t-z)}\langle[T_{12}(X), T_{12}(0)]\rangle$$

- $\Gamma(k)$ also determines the absorption rate of previously emitted GWs from other sources

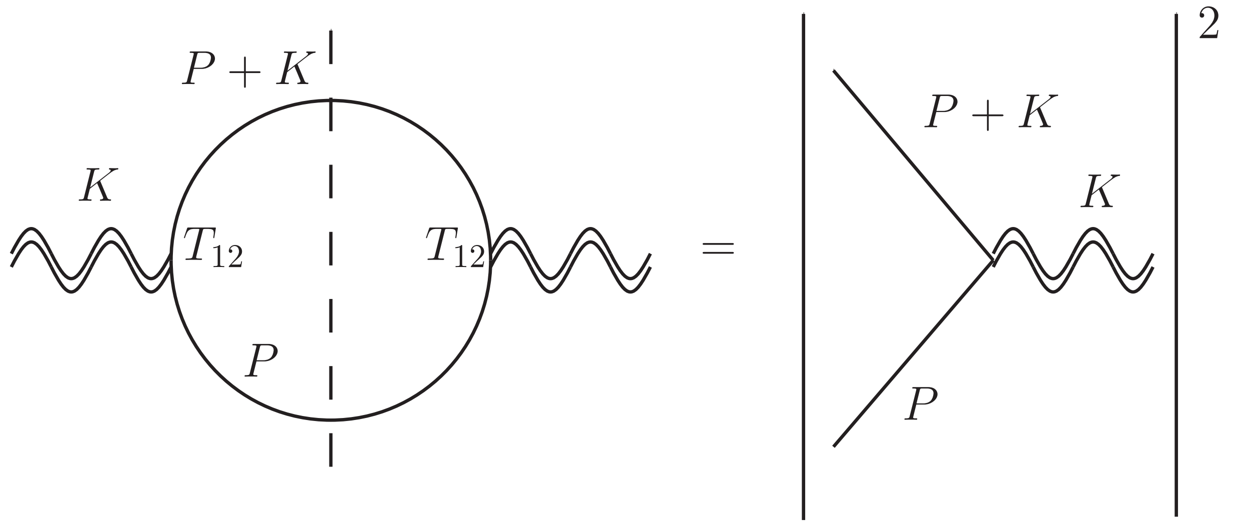
  Baym Patil Pethick **PRD96** (2017) Flauger Weinberg **PRD99** (2019)

JG Laine **JCAP1507** (2015)

# Leading order for *k*~*T*

$$\Gamma(k) = \frac{8\pi}{k\, m_{\rm Pl}^2} \int d^4X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle$$

- What it means: cut two-point function with thermal propagators. A naive example: at LO $T_{12}$ is bilinear in the fields of the QFT, so




$$\sim \int d^4P\, n(p^0) n(p^0+k^0) \left|\mathcal{M}\right|^2 \delta(P^2)\delta((P+K)^2)$$

thermal distribution functions x matrix element x on-shell kinematics

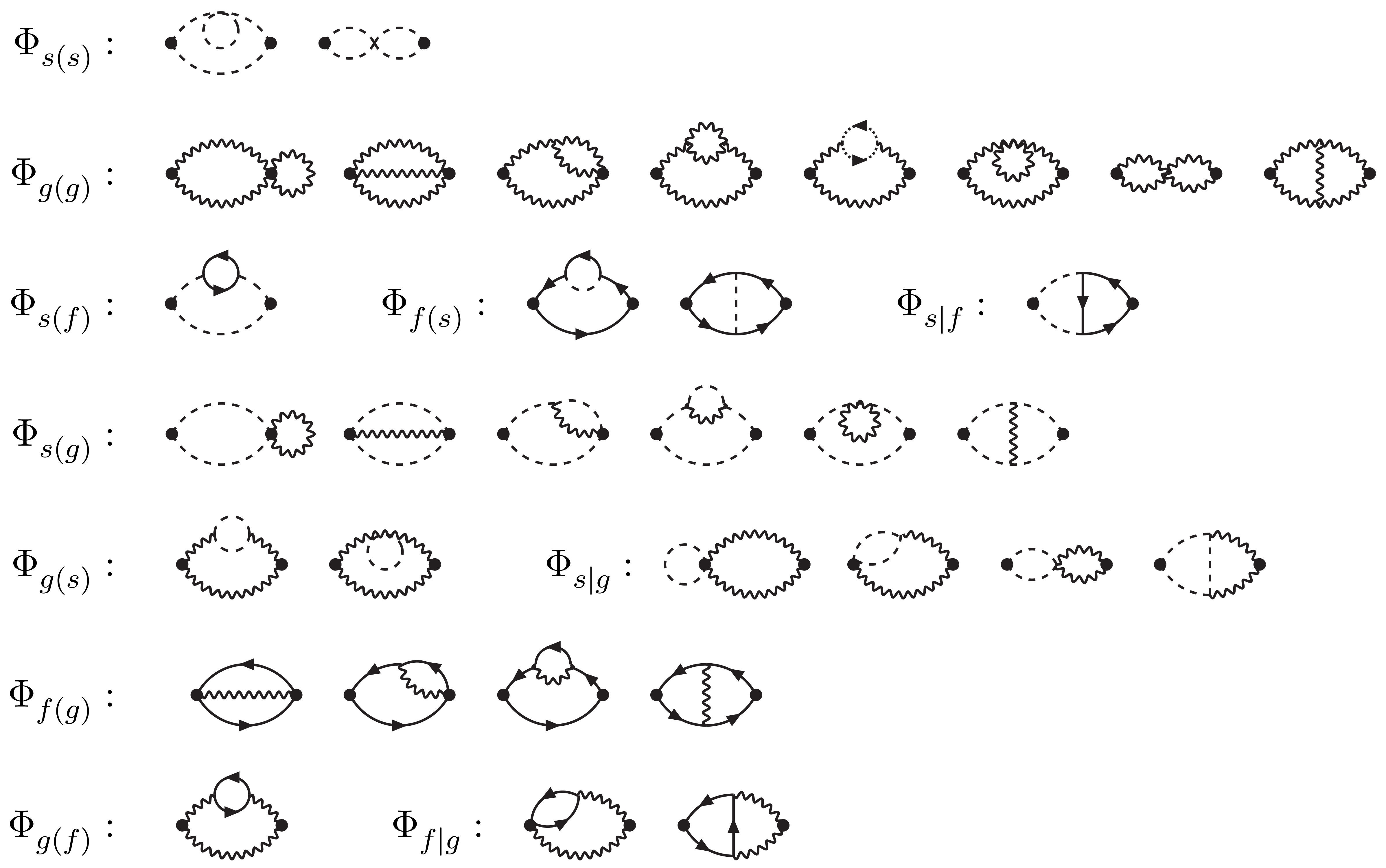
- Kinematically forbidden: need extra scatterings
- A complete LO calculation for *k*~*T* requires all 2↔2 scatterings between SM particles yielding a graviton

# Leading order for *k*~*T*

$$\Gamma(k) = \frac{8\pi}{k\, m_{\rm Pl}^2} \int d^4X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle$$

- Work from this def, compute all two-loop graphs in the SM for the *TT* correlator and take the cuts

- Powerful method to get thermal spectral functions at thermal frequencies and nonzero virtualities too
  Laine Zhu Jackson *et al* (2010-20)



# Leading order for *k*~*T*

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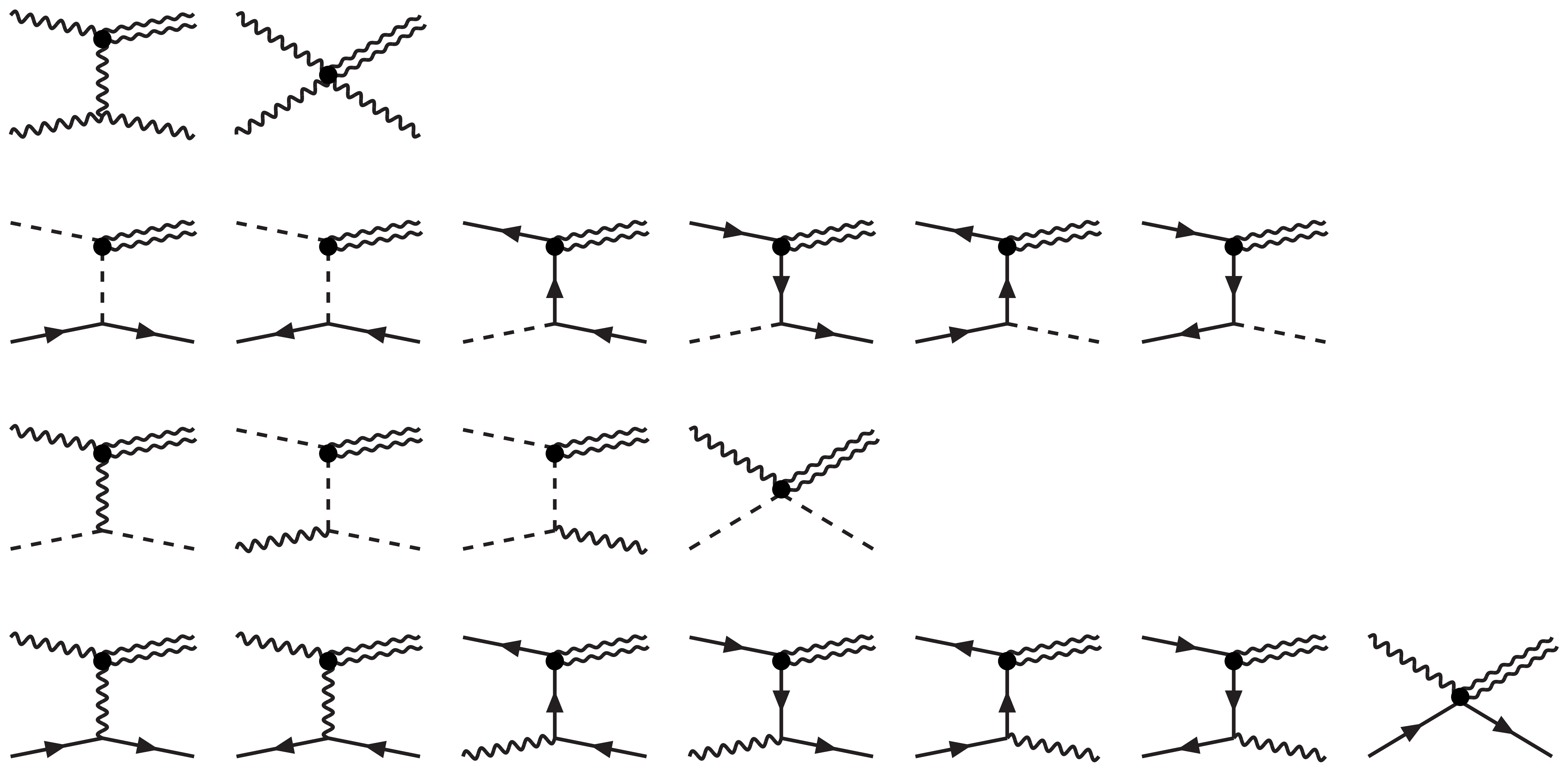
- Work from this def, compute all two-loop graphs in the SM for the TT correlator and take the cut
- Cutting the two-loop diagrams gives rise to the squares of these diagrammatic structures (crossings not shown)

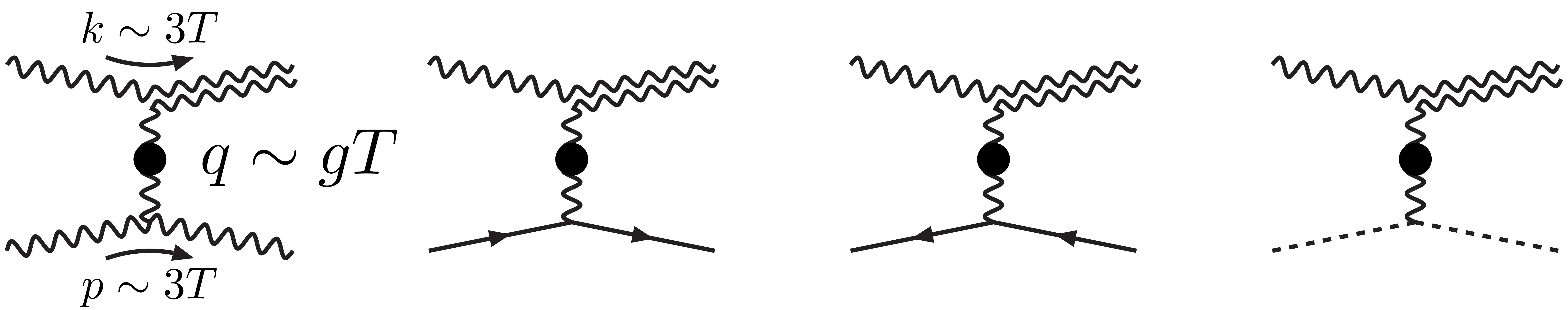


# Leading order for *k*~*T*

- Hence, at LO for *k*~*T*, equivalence with kinetic theory

$$\dot{f}_{\rm GW}(t,\mathbf{k}) = \Gamma(k)\, n_{\rm B}(k) = \frac{1}{8k}\int d\Omega_{2\to 2} \sum_{abc} \left|\mathcal{M}^{ab}_{cG}(\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}_1,\mathbf{k})\right|^2 f_a(p_1)\, f_b(p_2)\, [1 \pm f_c(k_1)]$$

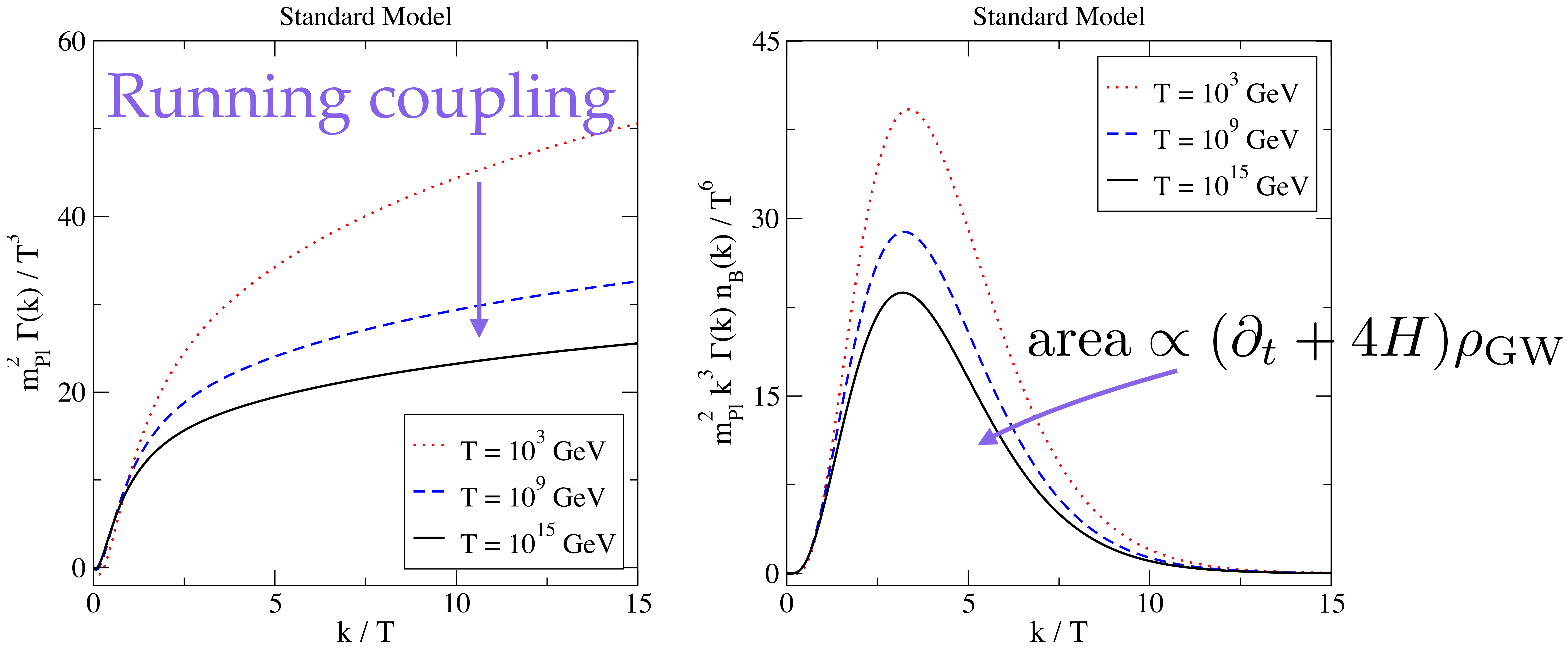
- The phase space integration runs over log-IR divergent soft gauge boson exchanges



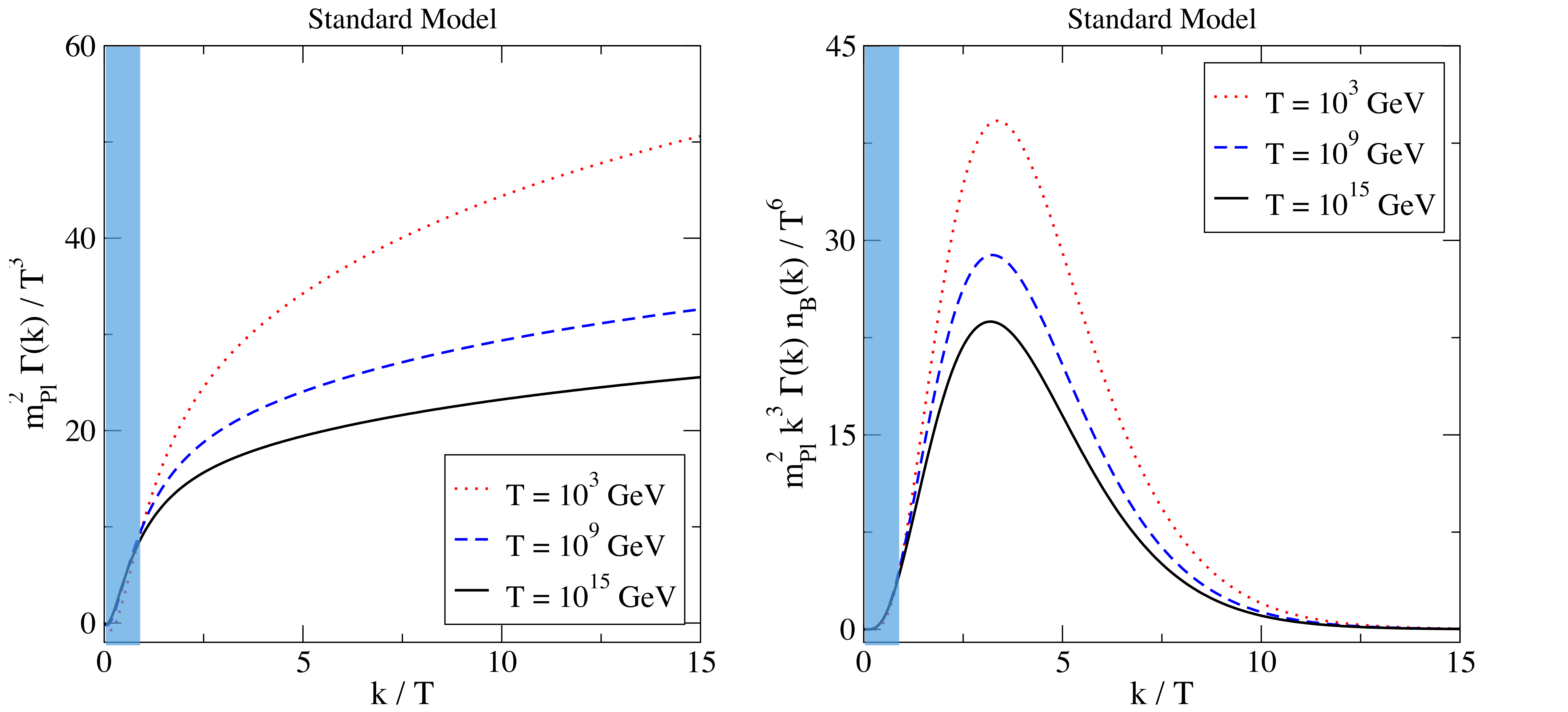

- Sensitivity to **collectivity**: screening, plasma oscillations and Landau damping. Treated by Hard Thermal Loop resummation: based on recent developments in TFT we implement a well-behaved subtraction and replacement with the HTL resummed evaluation JG Laine (2015-16)

# Leading order for $k{\sim}T$

$$\frac{\mathrm{d}\,\rho_{\rm GW}}{\mathrm{d}t\,\mathrm{d}k} = \frac{k^3\Gamma(k)n_{\rm B}(k)}{\pi^2} = \frac{k^3 T n_B(k)}{\pi^2 m_{\rm Pl}^2}\left\{\sum_{i=1}^{3} d_i\, m_{{\rm D}i}^2 \ln\left(1+\frac{4k^2}{m_{{\rm D}i}^2}\right) + g^2T^2\chi\left(\frac{k}{T}\right)\right\}$$



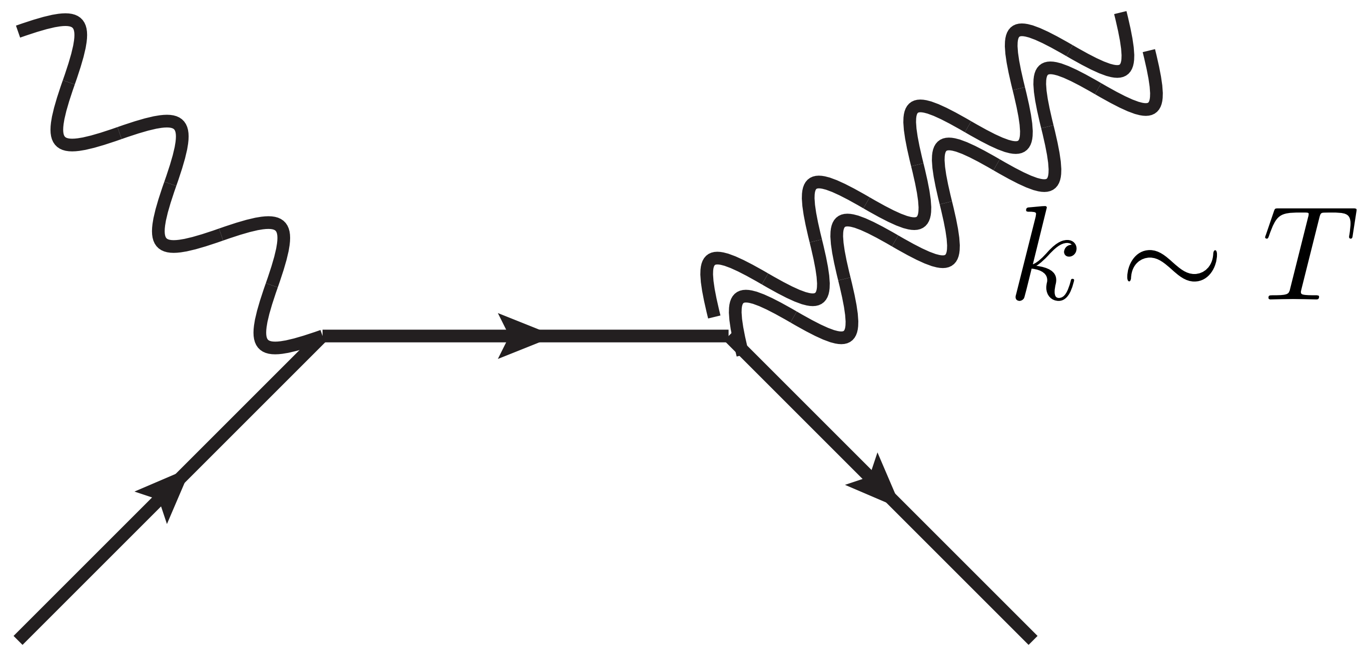

JG Jackson Laine Zhu **JHEP2007** (2020)




JG Jackson Laine Zhu **JHEP2007** (2020)

- The rate is valid for $k \gtrsim T$. At smaller $k$ our rate is not LO correct, but extrapolates to $k$=0 better than what was happening in similar calculations for gravitino and axion production
  (e.g. Pradler Steffen **PRD75** (2007), Rychkov Strumia **PRD75** (2007))

# Going to the IR




😎 Well-defined vacuum-like particle external states, at most HTL internally

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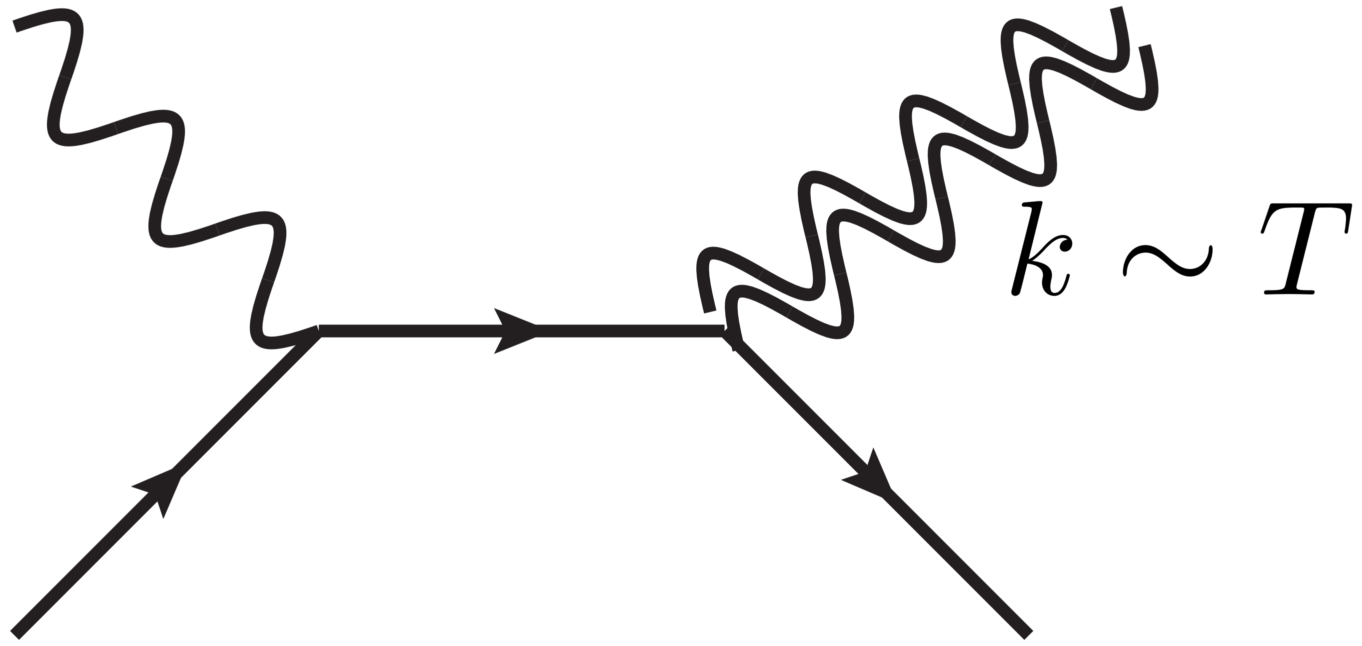


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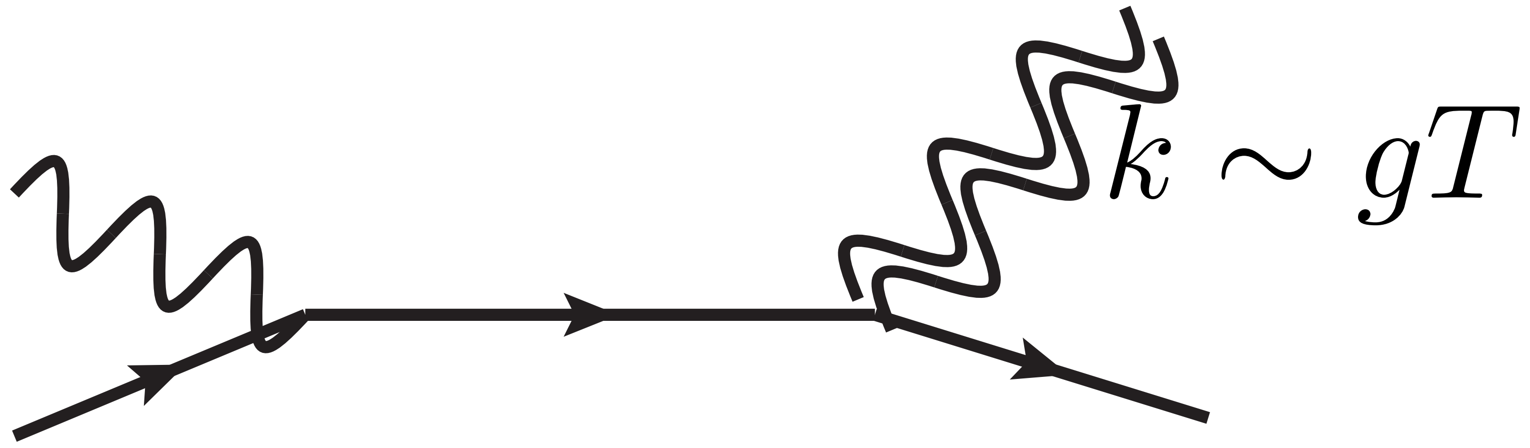
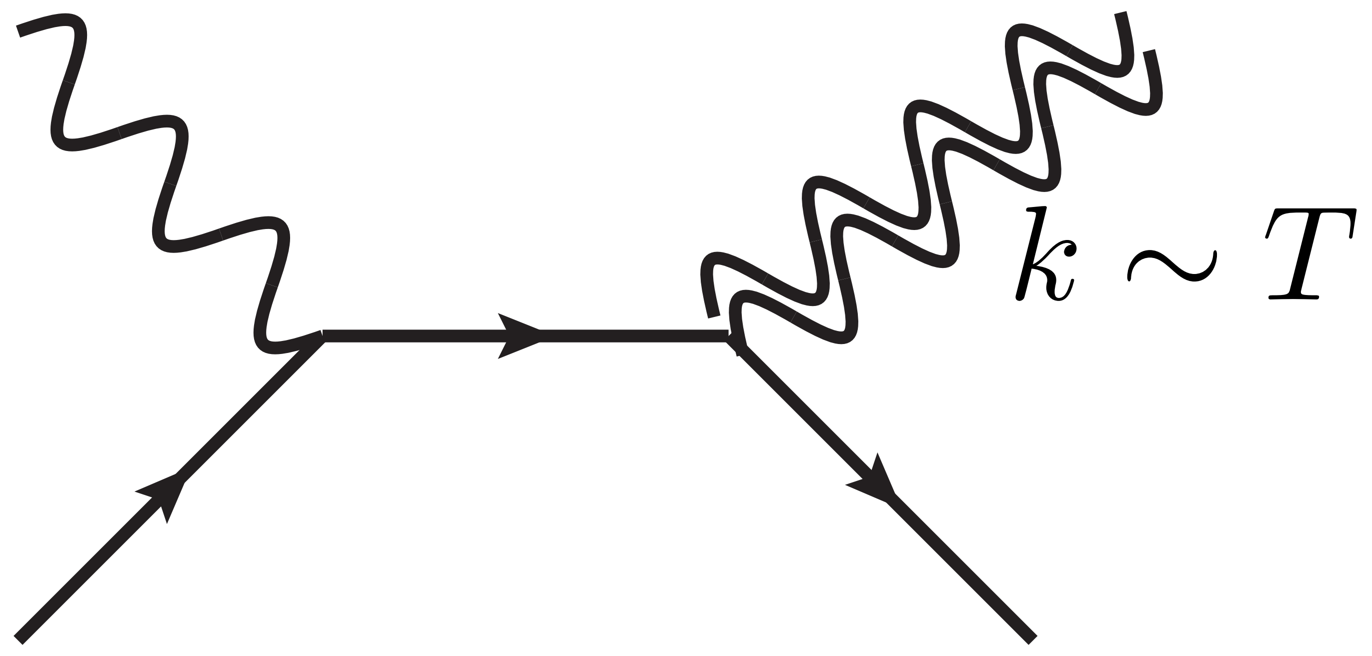



🤔 Longer-lived intermediate states, collinear and soft kinematics. Changes to simple particle picture

# Going to the IR

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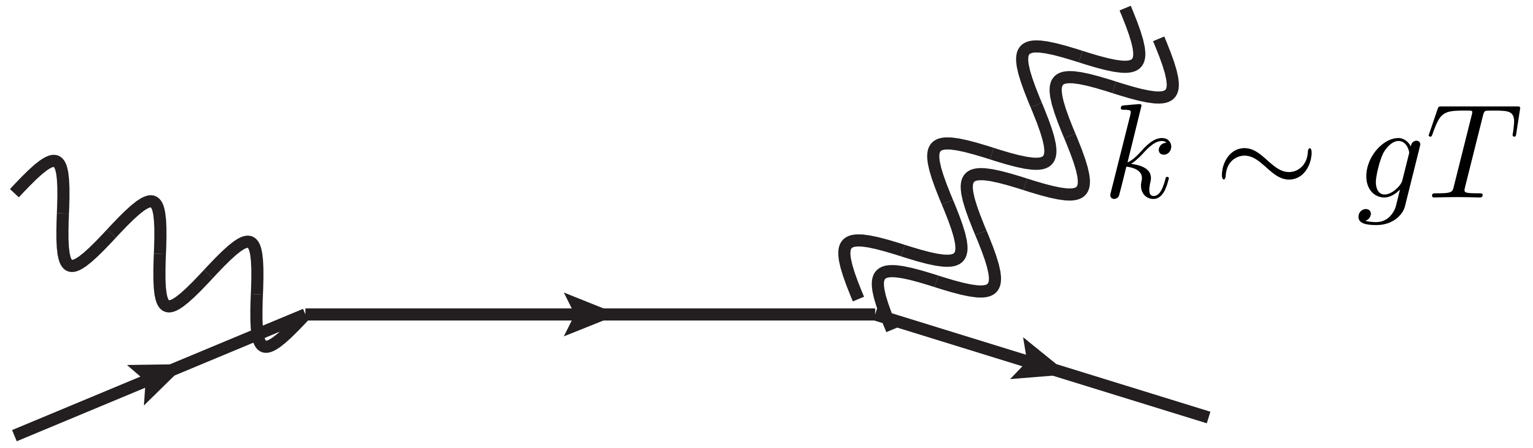


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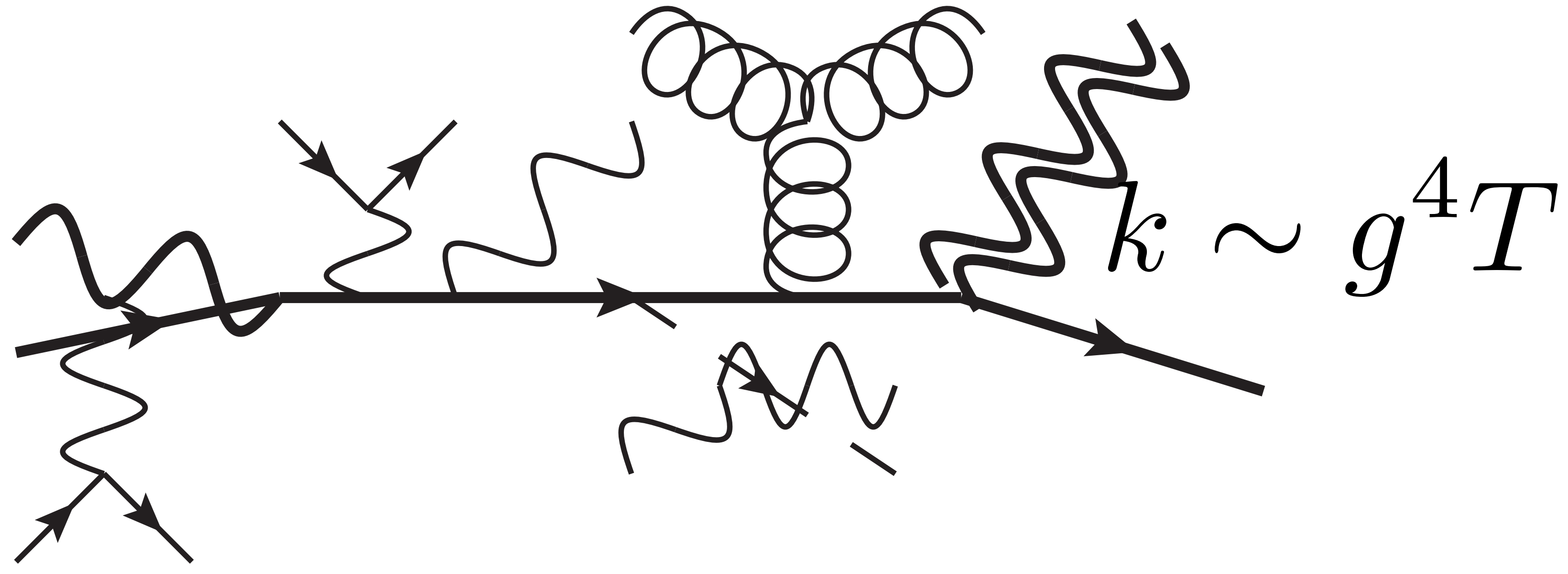
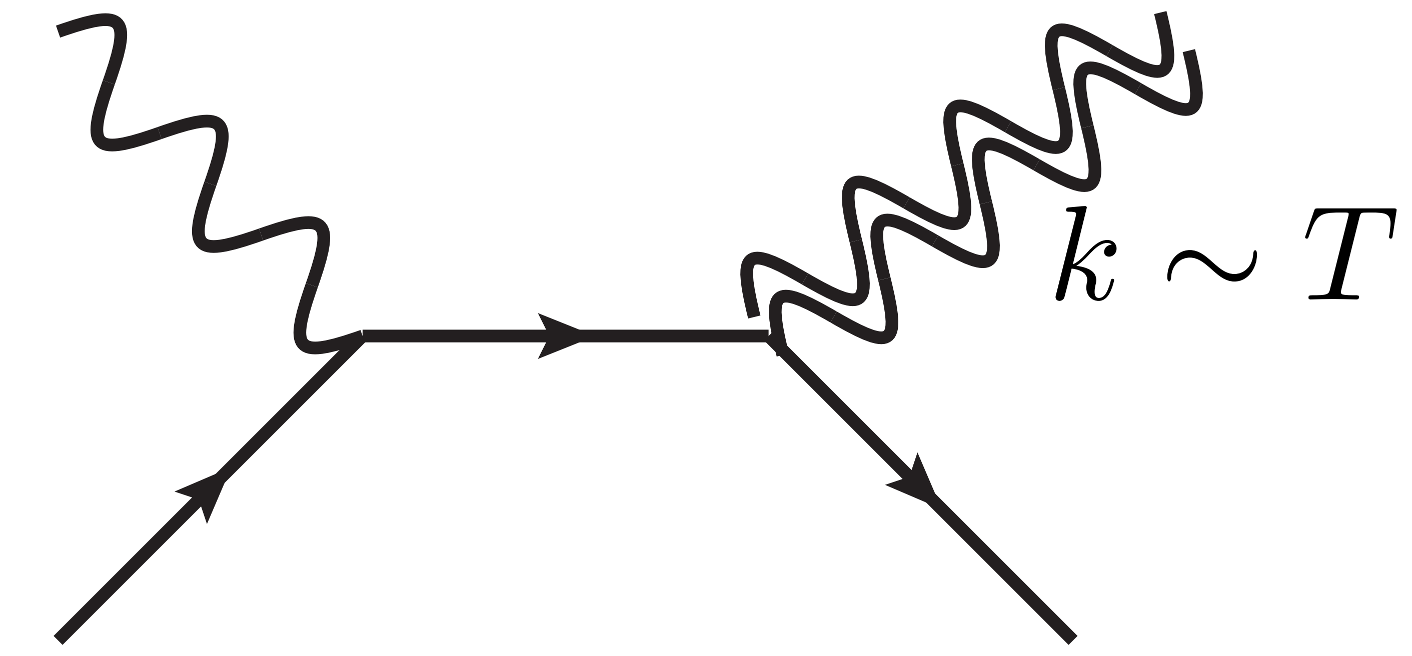



😳 Duration of order mean free time: scattering picture completely breaks down, GW does not resolve the microscopic scale

# Going to the IR

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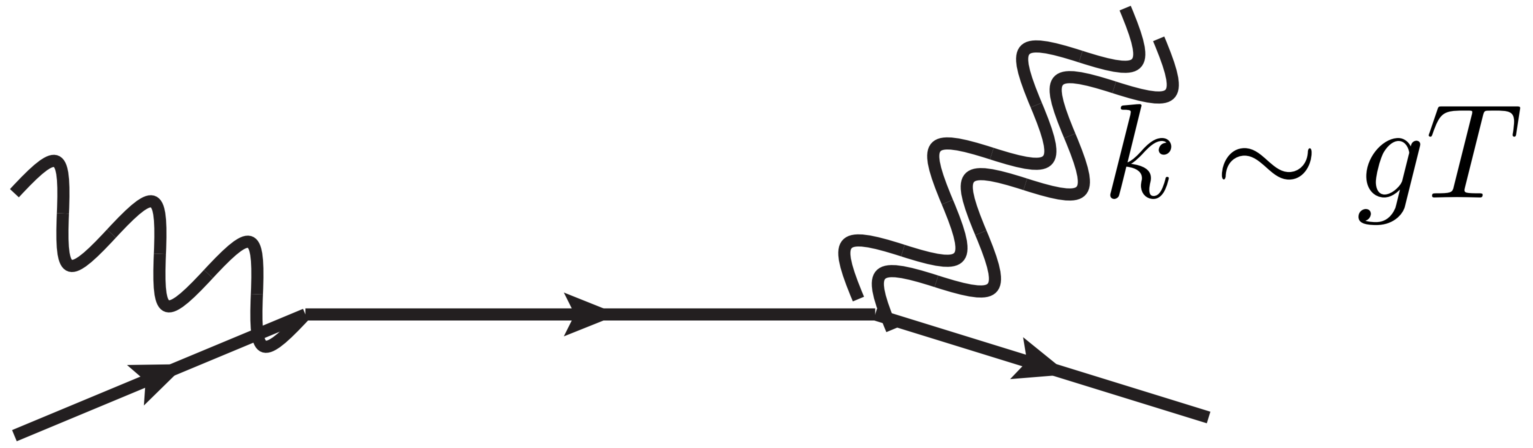


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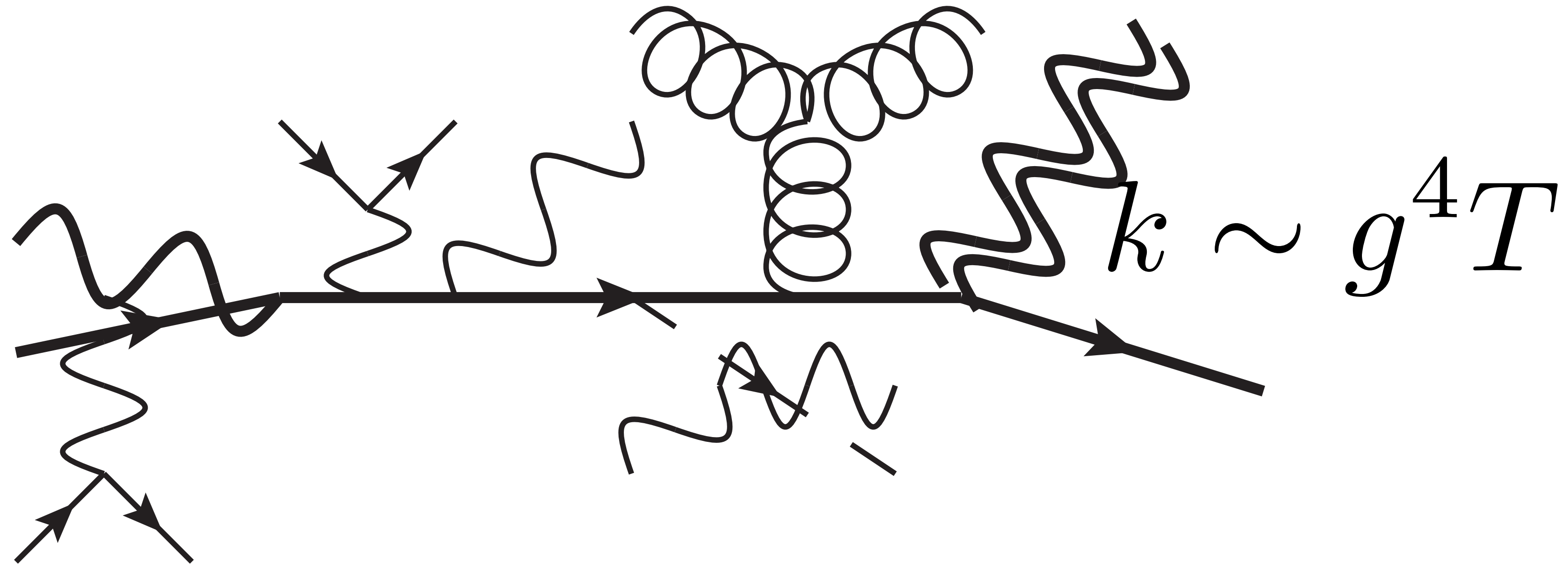
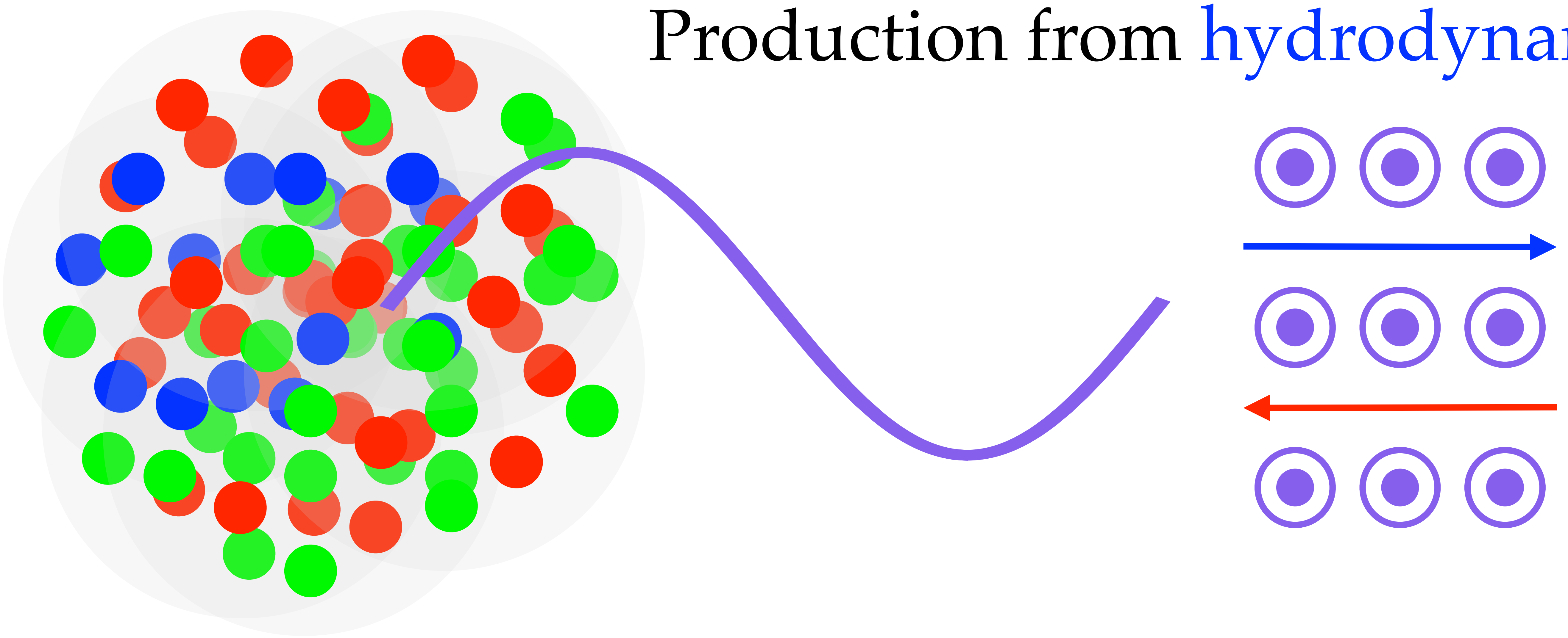



😳 Duration of order mean free time: scattering picture completely breaks down, GW does not resolve the microscopic scale

- Nothing here specific to GW

# Going to the IR

Production from hydrodynamic fluctuations



- TFT formalism shows that the IR rate is proportional to the *shear viscosity* of the plasma

$$\Gamma(k) = \frac{8\pi}{k\, m_{\rm Pl}^2} \int d^4X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle \qquad n_{\rm B}(k)\Gamma(k) \overset{k \lesssim g^4 T}{=} \frac{16\pi\, T\, \eta}{k\, m_{\rm Pl}^2}$$

JG Laine **JCAP1507** (2015)

# Going to the IR

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- For the SM at $T$>160 GeV $\eta$ is dominated by the slowest processes in eq., those involving right-handed leptons only

$$\eta \simeq \frac{16T^3}{g_1^4\ln(5T/m_{\rm D1})} \quad \rightarrow \quad \eta\simeq 400\,T^3$$

$g_1$ hypercharge coupling with screening mass $m_{\rm D1}=\sqrt{11/6}\,g_1T$

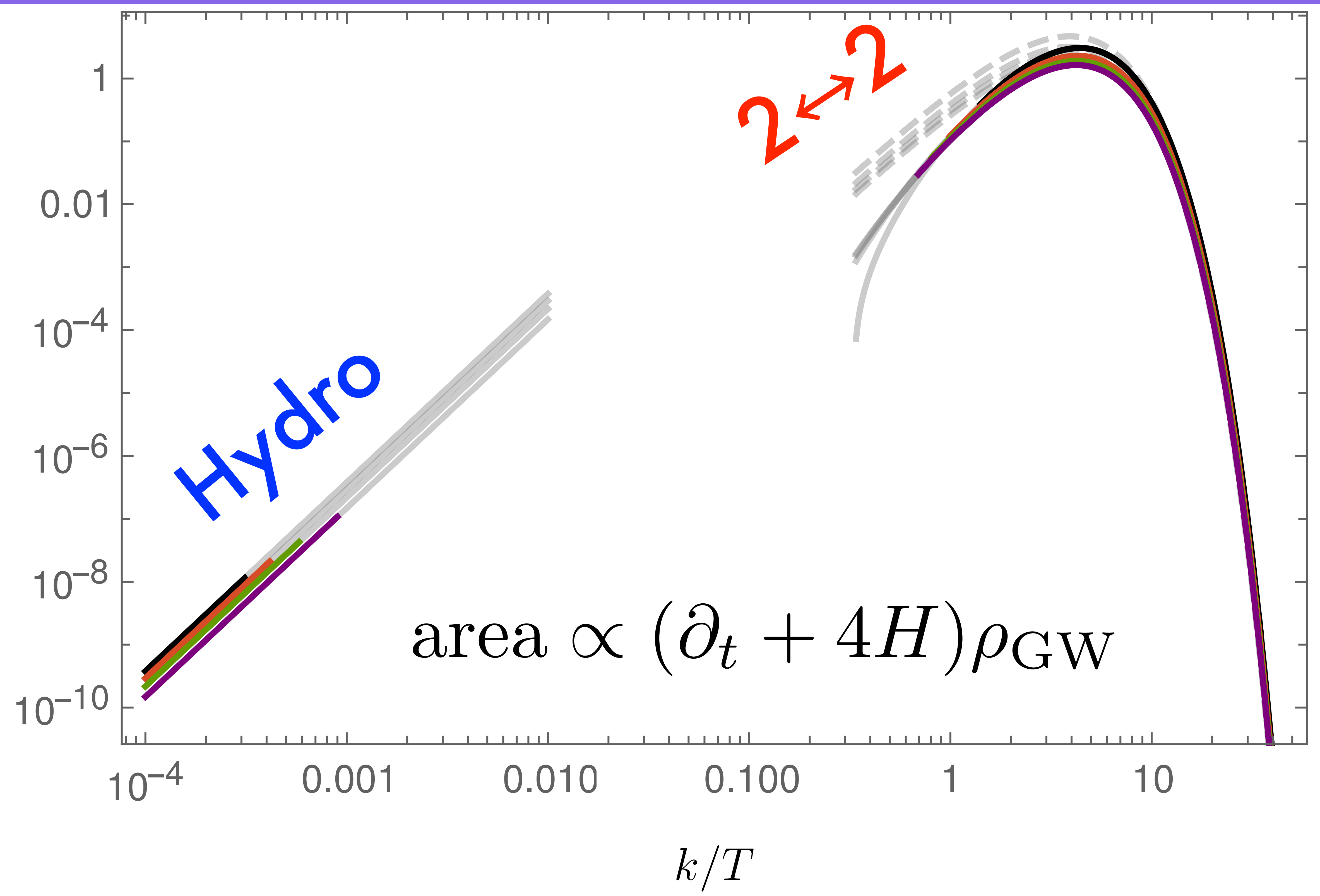
Only a leading-log estimate, no complete LO for $T$>160 GeV

Arnold Moore Yaffe (2000-2003)

JG Laine **JCAP1507** (2015)

# Cosmological implications

$$\frac{k^4 m_{\rm Pl}^2 n_{\rm B}(k)}{16\pi T^7}\Gamma(k)$$




- Peak: frequency at $k \approx 4T$. Redshifts at decoupling to $k_{\rm dec} \approx 4T_{\rm dec}(3.9/106.75)^{1/3} \sim T_{\rm dec}$. Today f≈100 GHz. Amplitude determined by $T_{\rm max}$.

JG Laine **JCAP1507** (2015) Ringwald Schütte-Engel Tamarit **JCAP2103** (2021)

# Cosmological implications

- Direct detection challenging in the medium term
- Thermal production stores energy in GWs. BBN and CMB observations constrain the energy density stored in radiation at those epochs: GW contribution to $N_{\mathrm{eff}}$
Smith Pierpaoli Kamionkowski **PRL97** (2006) Henrot-Versille *et al Class. Quant. Grav.* **32** (2015) Caprini Figueroa *Class. Quant. Grav.* **35** (2018)
- The SM predictions have $10^{-3}$ uncertainty, the experimental accuracy $10^{-1}$, expected to increase with next-generation detectors CMB-S4
- Requiring $\Delta N_{\mathrm{eff}}=10^{-3}$ yields $T_{\mathrm{max}} < 2\ 10^{17}$ GeV for a SM universe, 2x more than that for a MSSM scenario (the extra GW production from the larger number of thermal d.o.f.s is more than compensated by the extra dilution)

JG Jackson Laine Zhu **JCHEP2007** (2020) Ringwald Schütte-Engel Tamarit **JCAP2107** (2021)

# Massive particles

# Massive particles: right-handed neutrinos

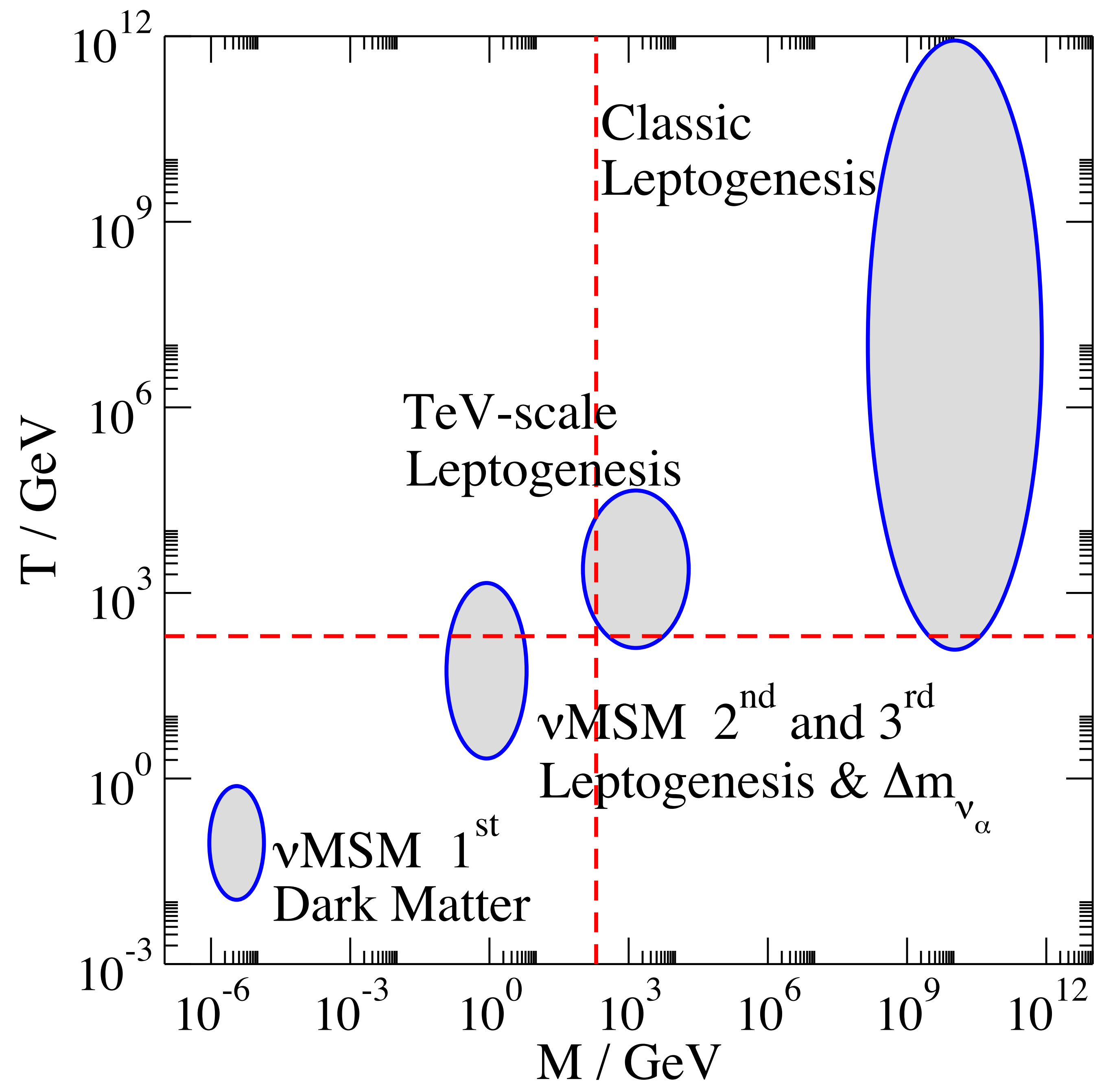
- $n$ sterile (SM gauge singlet), Majorana neutrinos coupling to the three active lepton flavours and the (conjugate) Higgs field

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\sum_I \bar{N}_I (i\gamma^\mu \partial_\mu - M_I) N_I - \sum_{I,a} (\bar{N}_I\, h_{Ia} \tilde{\phi}^\dagger a_L\, l_a + \bar{l}_a\, a_R\, \tilde{\phi}\, h^*_{Ia} N_I)$$

- Can address active neutrino masses (seesaw) and baryon asymmetry (leptogenesis) over a wide range of parameters
  Fukugita Yanagida **PLB174** (1986)

- A specific realisation (νMSM) can also provide a keV-scale DM right-handed neutrino
  Asaka Blanchet Shaposhnikov **PLB620**, **PLB631** (2005)

- Asymmetry generation and RHN production require **rates from $T \gg M_I$ to $T \ll M_I$**



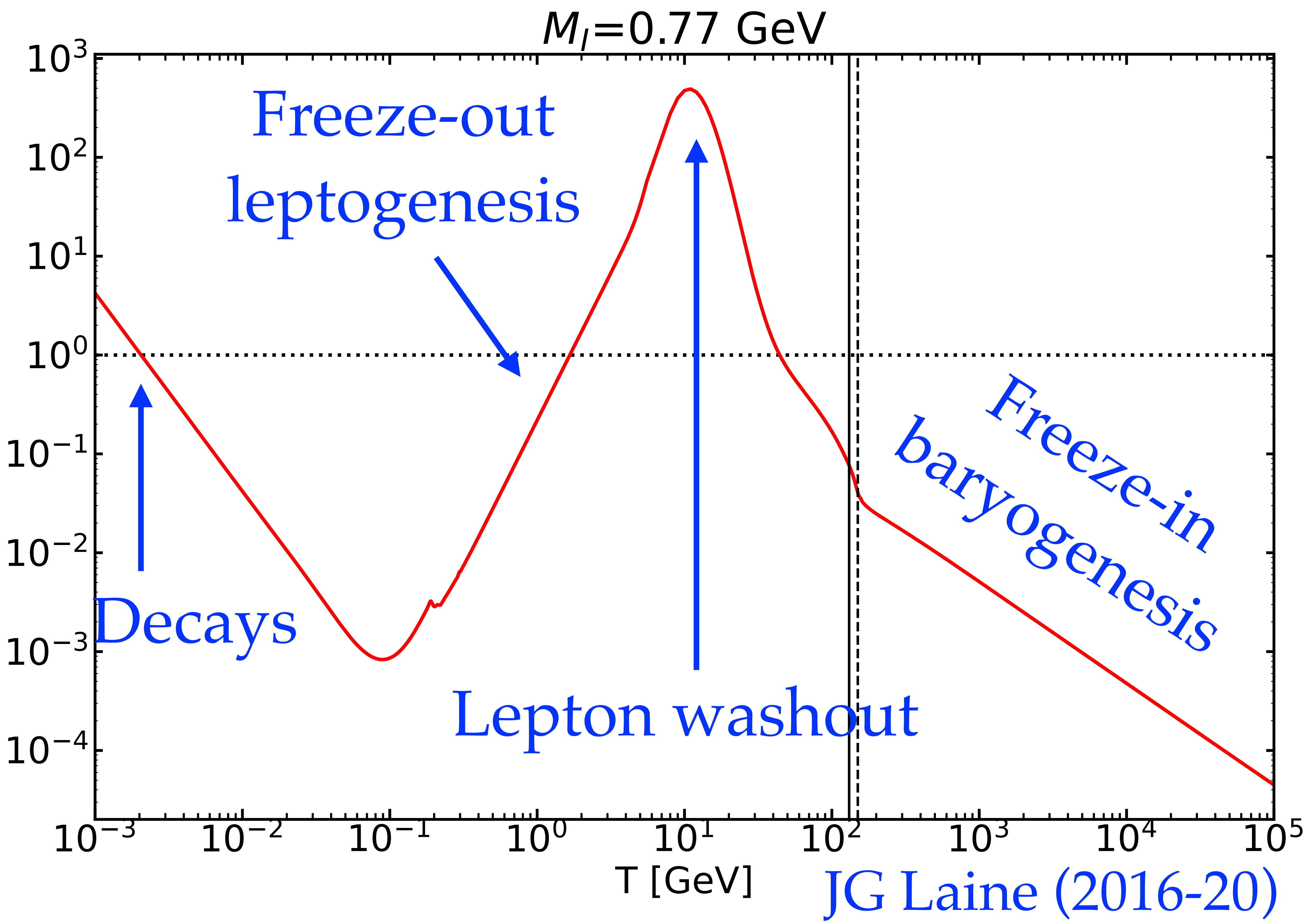

Plot by Mikko Laine

# Massive particles: right-handed neutrinos

$$(\partial_t - H\mathbf{k}\cdot\nabla_{\mathbf{k}})f_N(t,\mathbf{k}) = \Gamma(k)[n_{\mathrm{F}}(k^0) - f_N(t,\mathbf{k})] \qquad \Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0}\int d^4X e^{iK\cdot X}\langle[\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)]\rangle$$

- Production, equilibration, freeze-out and decay rates from the formalism over many decades

$$\frac{\langle\Gamma\rangle}{3c_s^2 H}$$




JG Laine (2016-20)

# Massive particles: right-handed neutrinos

- Symmetric phase $$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$
  - $T \ll M_I$ Salvio Lodone Strumia (2011), Laine Schröder (2012), Biondini Brambilla Escobedo Vairo (2012)
  - $T \sim M_I$ Laine (2013), Laine Jackson (2021)
  - $T \gg M_I$ Anisimov Besak Bödeker (2010-12), Ghisoiu Laine (2014), JG Laine (2021)
- Broken phase
  - $M_I \sim$ GeV JG Laine (2016-20), Jackson Laine (2019)
  - $M_I \sim$ keV Asaka Laine Shaposhnikov (2006), JG Laine (2015-20) Bödeker Klaus (2020)
- These calculations provide a pattern for models with many regimes to be followed. Review Laine **2203.05772**

# Massive particles: the ultrarelativistic regime

$T \gg M_I$

$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- In a first approximation mass seems negligible
- Just 2↔2 processes (with fermion HTL included)?

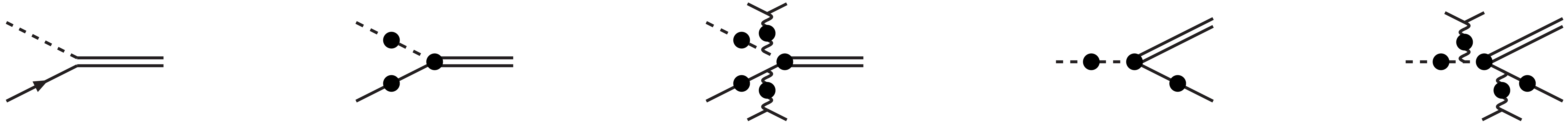
Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014)

# Massive particles: the ultrarelativistic regime

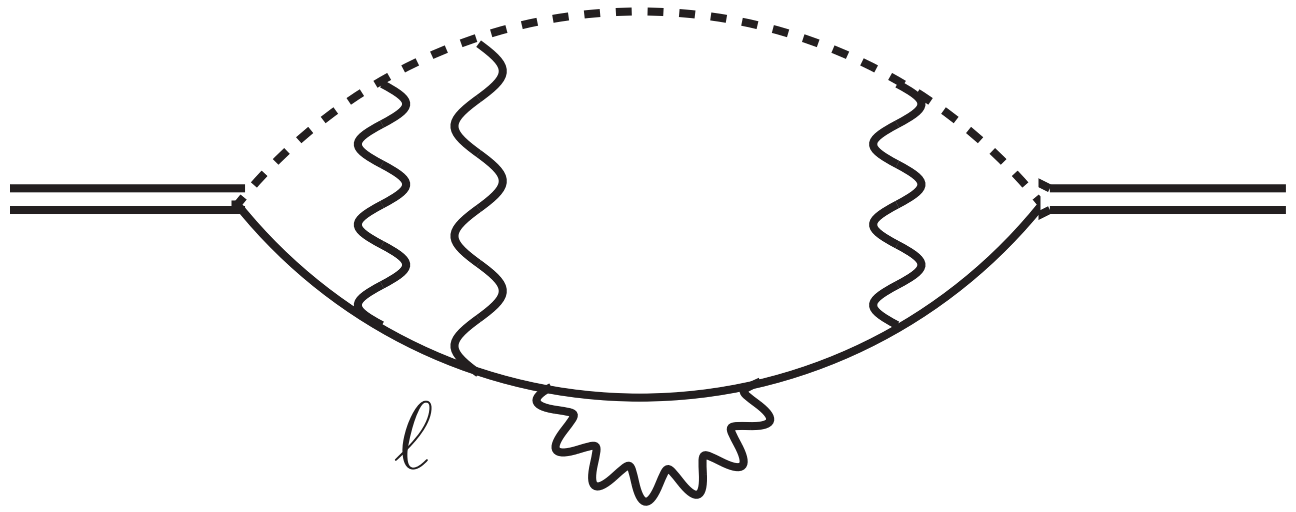
$T \gg M_I$

$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle[\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)]\rangle$$

- Effective 1↔2 processes



- Landau-Pomeranchuk-Migdal (LPM) interference of multiple soft scatterings, requires ladder resummation




- Borrow techniques from hot QCD to deal with LPM resummation
  Baier Dokshitzer Mueller Peigné Schiff (1995-97) Zakharov (1996-97) Arnold Moore Yaffe (2001-2003)

Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014) JG Laine (2021)

# Massive particles: the ultrarelativistic regime

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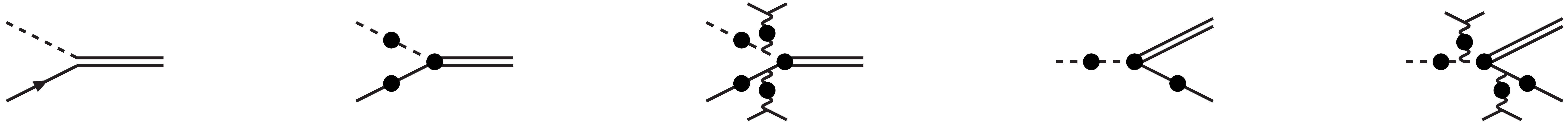
- Effective 1↔2 processes



- Absent from GW production calculation at LO (suppression in derivative coupling) and similar production calculations (gravitino, axion, etc…)
  Salvio Strumia Xue (2013)

Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014) JG Laine (2021)

# Massive particles: the ultrarelativistic regime

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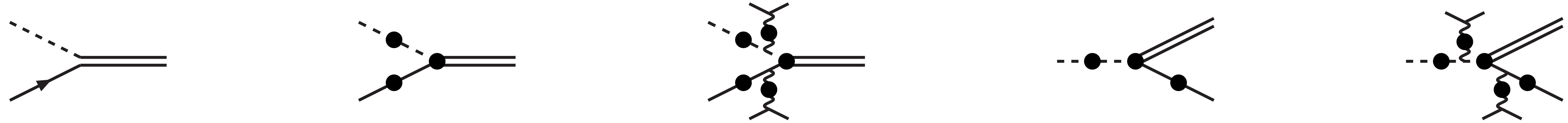
$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- Effective 1↔2 processes



- Very important beyond sterile neutrinos
  - Thermalisation during reheating (number-nonconserving and efficient energy equilibration) Davidson Sarkar (2001) Harigaya Mukaida (2014) Mukaida Yamada (2015) Drees Najjari (2021) Large body of literature on QCD thermalisation, review in Berges Heller Mazeliauskas Venugopalan (2020)

Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014) JG Laine (2021)

# Massive particles: the ultrarelativistic regime

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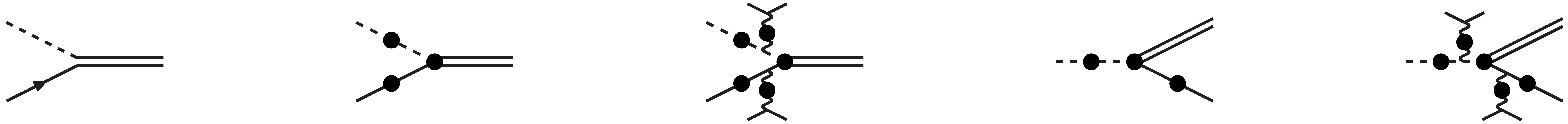
$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- Effective 1↔2 processes



- Very important beyond sterile neutrinos
  - Equilibration of the Yukawa interactions of right-handed electrons
    Bödeker Schröder (2019)

Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014) JG Laine (2021)

# Massive particles: the ultrarelativistic regime

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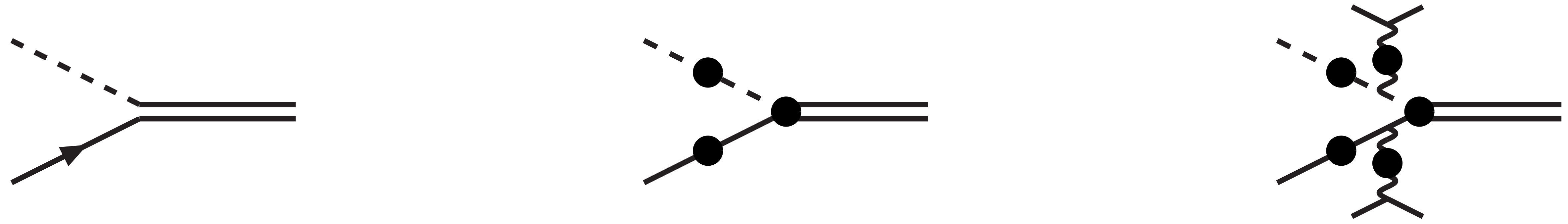
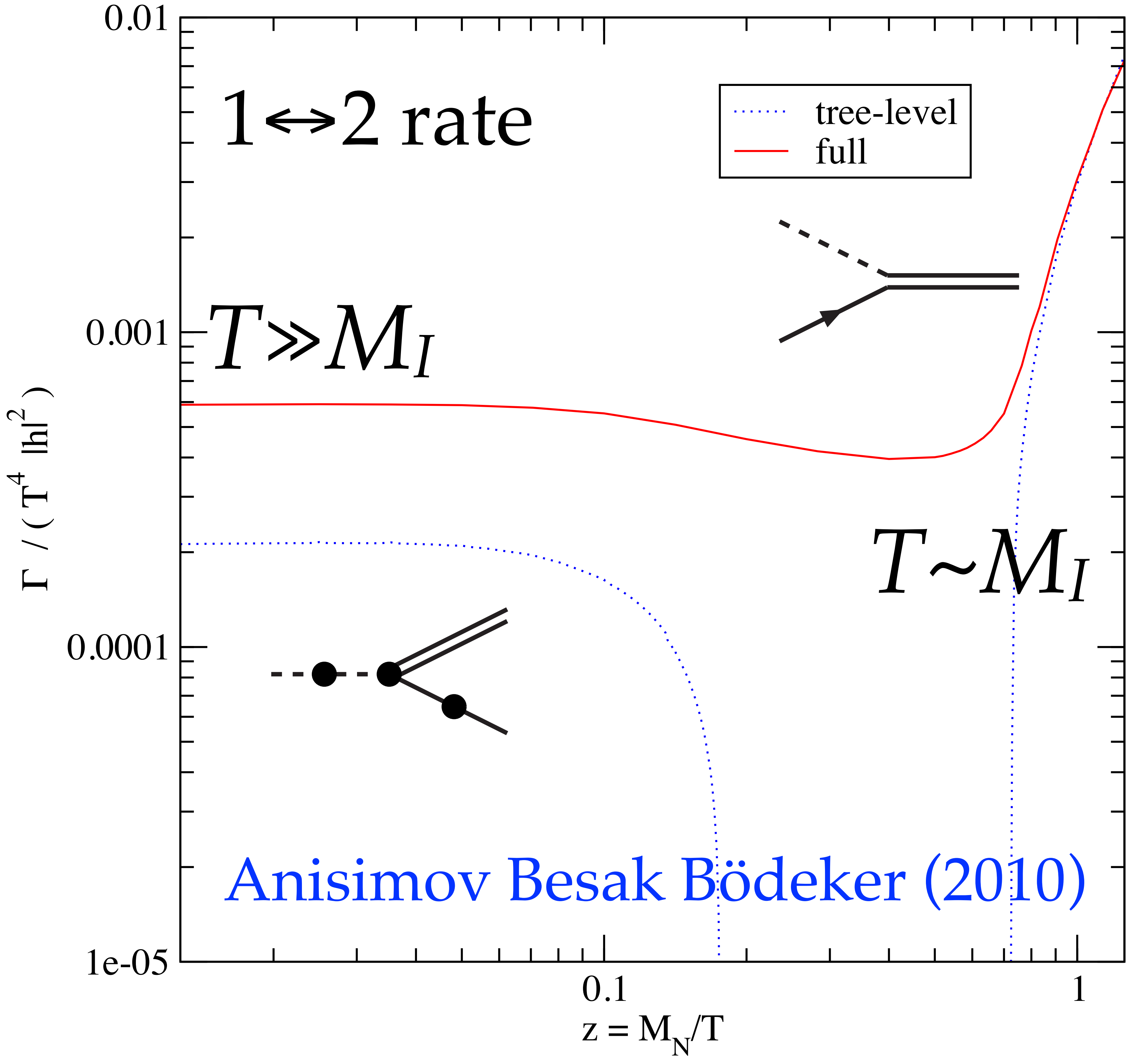
$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- Effective 1↔2 processes



- Large enhancement in the high-$T$ regime



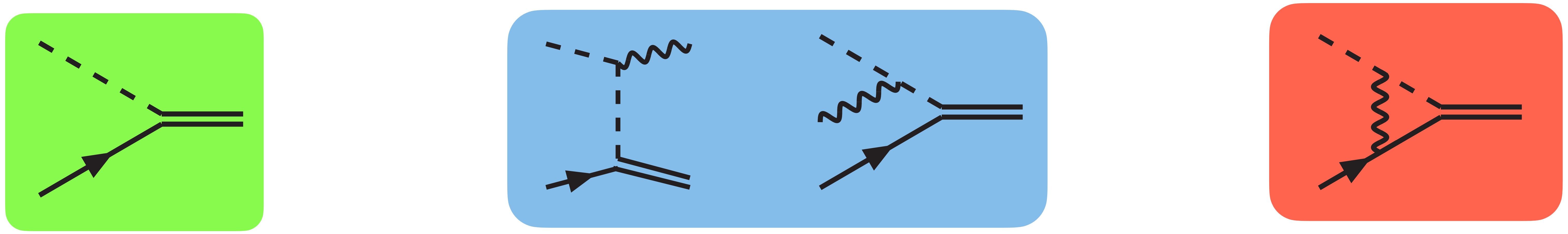

Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014) JG Laine (2021)

# Massive particles: the relativistic regime

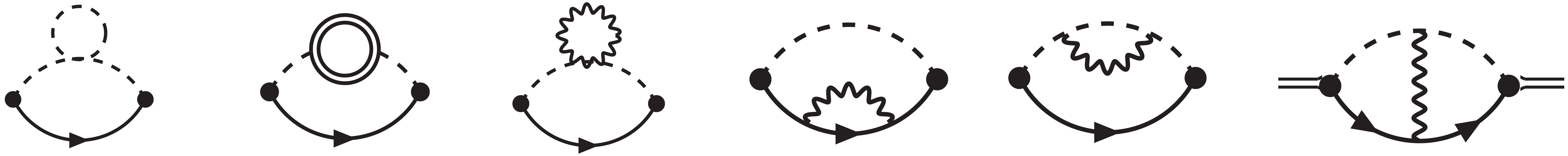
$T \sim M_I$

$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- A leading-order 2→1 process receiving real (2→2, 3→1) and virtual (2→1) **NLO** corrections that have been computed and merged with the $T \gg M_I$ range



- Real and virtual corrections are **individually IR divergent, only sum is physical**



Laine **JHEP1305 JHEP1308** (2013) Ghisoiu Laine **JCAP1412** (2014) Jackson Laine **JHEP09** (2021)

# Massive particles: the relativistic regime

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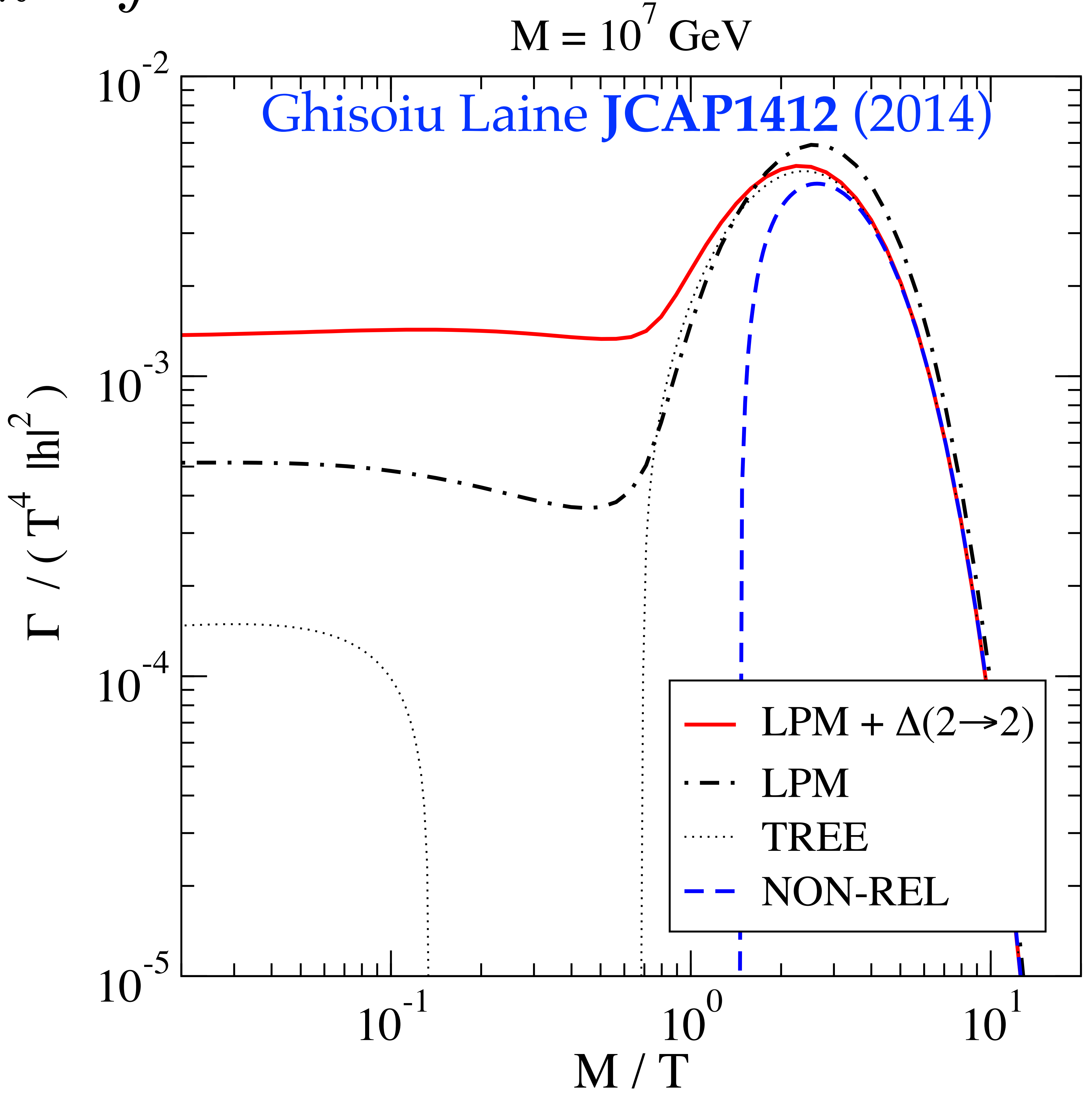
$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK\cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- A leading-order 2→1 process receiving real (2→2, 3→1) and virtual (2→1) **NLO** corrections that have been computed and **merged** with the $T \gg M_I$ range

- Agreement with OPE/EFT based calculations in the non-relativistic regime
  Salvio Lodone Strumia **JHEP1108** (2011)
  Laine Schröder **JHEP1202** (2012) Biondini Brambilla Escobedo Vairo **JHEP1312** (2013)




Laine **JHEP1305 JHEP1308** (2013) Ghisoiu Laine **JCAP1412** (2014) Jackson Laine **JHEP09** (2021)

# Conclusions

- TFT formalism for thermodynamics and phase transitions:
  - Well-tested for the SM
  - Being applied to BSM models with an outlook on first-order phase transitions and gravitational wave production

# Conclusions

- TFT formalism for thermal rates
  - Does not require quasi-particles, though it reproduces quasi-particle Boltzmann results where they apply
  - Relies on timescale separation
- Thermal production of gravitational waves: guaranteed to be there, contributes to $N_{\rm eff}$. No stringent bounds for SM-like universes. Methods applicable to light/massless states non-renomalizeably coupled to plasma
- Thermal production of massive particles: the case of heavy neutral leptons/sterile neutrinos. Many regimes to be examined, great progress with *interdisciplinary connections* to hot QCD and NLO available in some regimes