# Isospin-asymmetric QCD matter

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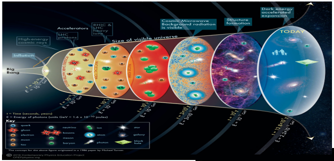
with Bastian Brandt and Gergely Endrődi

**Phase transitions in particle physics**
March 29th, 2022 - GGI

# Hot/dense/magnetized states of QCD matter

CRC-TR 211




**Early Universe (QCD epoch)**

$100\text{MeV} \lesssim T \lesssim 200\text{MeV}$

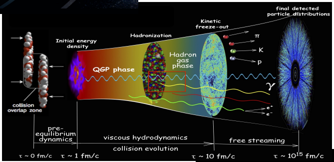
$b = (8.60 \pm 0.06) \times 10^{-11}$, $q = 0$

$|l| = |l_e + l_\mu + l_\tau| < 0.012$

$B? \longrightarrow 10^{-16} \lesssim B \lesssim 10^{-9}\text{Gauss}$ (EGMFs)

**Hotter**

**"diluter"**




**Heavy ion collision (HIC)**

$50\text{MeV} \lesssim T \lesssim 200\text{MeV}$

$n \lesssim 0.12\text{fm}^{-3}$

$\delta = N - Z/(N+Z) \lesssim 0.25$

$B \lesssim 10^{19}\text{Gauss} \sim 10^{20} B_{\text{Earth}}$

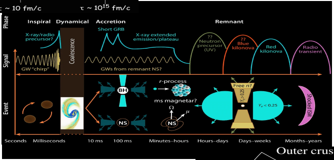
$T \in [50, 80]\text{MeV}$

$n \sim 2n_0, \quad n_0 = 0.16\text{fm}^{-3}$

$n_{\text{quark}} \neq 0$

$10^{10} \lesssim B \lesssim 10^{12}\text{Gauss} \rightarrow B \gtrsim 10^{16}\text{Gauss}$

**Binary Neutron Star (BNS) merger**



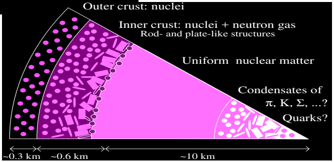

$T \lesssim 1\text{KeV}$

$0.3n_0 \lesssim n \lesssim 15n_0, \quad n_0 = 0.16\text{fm}^{-3}$

$n_p/n \sim 0.04$, at $n_0$

$10^8 \lesssim B \lesssim 10^{15}\text{Gauss}$

**Neutron star interior**

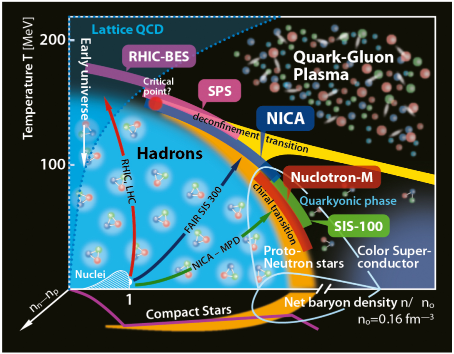



**Colder**

**denser**

$1\text{eV} \approx 12 \times 10^3 K; \quad B_{\text{Earth}} \approx 0.5\text{Gauss}$

Nonzero **baryon density** $n = \frac{n_u + n_d}{3}$
$\rightarrow$ excess of matter over antimatter




**Sign problem**

Nonzero **isospin density** $n_I = n_u - n_d$
$\rightarrow$ excess of neutrons over protons

# Densities we can(not) afford...

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**Chemical potentials basis**: $\mu_\mathrm{u} = \mu_\ell + \mu_\mathrm{I}, \quad \mu_\mathrm{d} = \mu_\ell - \mu_\mathrm{I}, \quad \mu_\mathrm{s}$

$\mu_\ell \neq 0$ and/or $\mu_s \neq 0$
$\longrightarrow$ unsuited for lattice simulations!

$\longleftrightarrow$

Nonzero **baryon density** $n = \frac{n_\mathrm{u}+n_\mathrm{d}}{3}$
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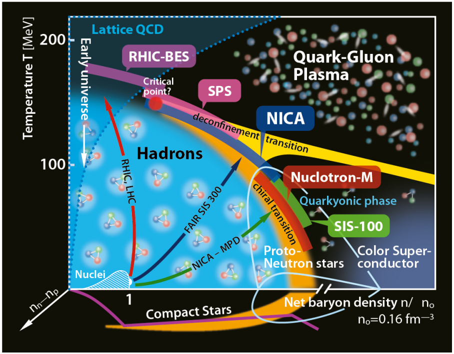


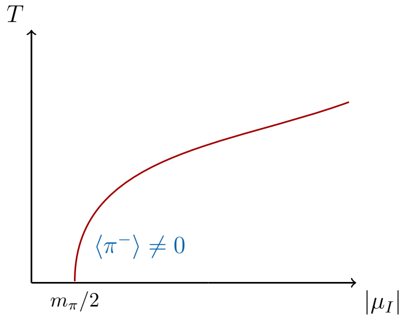

**Sign problem**

Nonzero **isospin density** $n_\mathrm{I} = n_\mathrm{u} - n_\mathrm{d}$
$\rightarrow$ excess of neutrons over protons

$\longleftrightarrow$

$\mu_\mathrm{I} = \mu_\mathrm{u} = -\mu_\mathrm{d} \neq 0; \quad \mu_\ell = \mu_\mathrm{s} = 0$
$\longrightarrow$ suited for lattice simulations!

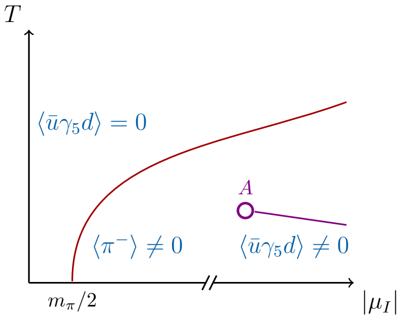
# The "analytical phase diagram"




**In the limit $n_I \to 0$, i.e. $|\mu_I| \ll m_\rho$ $\chi$PT applies** Son, Stephanov (2000)

- $\pi^\pm$ lightest hadrons coupling to $\mu_I$: $\chi$PT describes their effective dynamics
- At $T = 0$, $\mu_I \geq \mu_{I,c} = m_\pi/2$, sufficient energy to create $\pi^\pm$
- A Bose-Einstein condensate (BEC) is formed
- Second order (O(2) universality class) Hadronic/BEC transition

# The "analytical phase diagram"




**In the limit $n_I \to \infty$, i.e. $|\mu_I| \gg \Lambda_{QCD}$ p-QCD applies** Son, Stephanov (2000)

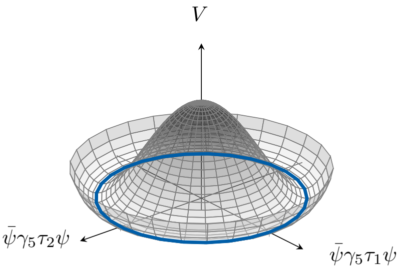
- Attractive gluon interaction leads to diquark pairing of the BCS type
  - **superfluid BCS phase** of pseudoscalar Cooper-pairs
- BEC/BCS analytic crossover (same symmetry breaking pattern)
- At asymptotically large $\mu_I$, decoupling of gluonic sector, first-order deconfinement transition

- $SU_V(2) \times U_V(1)$ flavor symmetry group for QCD with light quark matrix

$$\mathcal{M}_{ud}\big|_{\mu_i=\lambda=0} = \gamma_\mu(\partial_\mu + iA_\mu)\,\mathbb{1} + m_{ud}\mathbb{1}, \qquad \psi = (u,d)^\top$$

- At $\mu_I \neq 0 \quad \longrightarrow \quad \mathcal{M}_{ud} = \mathcal{M}_{ud}\big|_{\mu_i=\lambda=0} + \mu_I\gamma_4\tau_3$

$$SU_V(2) \times U_V(1) \longrightarrow U_{\tau_3}(1) \times U_V(1)$$




- **Spontaneous breaking** with pion condensate $\langle\bar{\psi}\gamma_5\tau_{1,2}\psi\rangle$
  $\longrightarrow$ Appearance of Goldstone mode

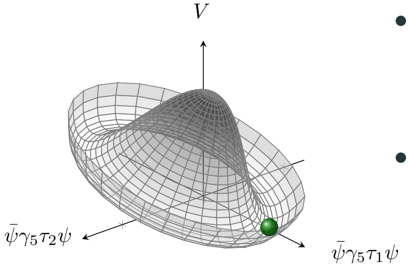
# $n_I$-QCD on the lattice - Symmetry breaking patterns

- $SU_V(2) \times U_V(1)$ flavor symmetry group for QCD with light quark matrix

$$\mathcal{M}_{ud}\big|_{\mu_i=\lambda=0} = \gamma_\mu(\partial_\mu + iA_\mu)\,\mathbb{1} + m_{ud}\mathbb{1}, \qquad \psi = (u,d)^\top$$

- At $\mu_I \neq 0$ , $\lambda \neq 0$ $\longrightarrow$ $\mathcal{M}_{ud} = \mathcal{M}_{ud}\big|_{\mu_i=\lambda=0} + \mu_I\gamma_4\tau_3 + i\lambda\gamma_5\tau_2$

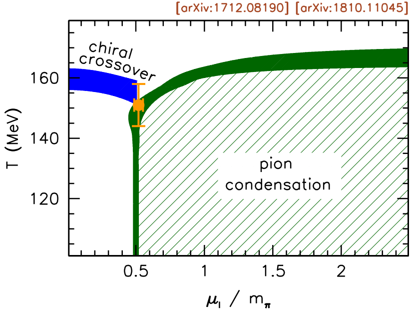
$$SU_V(2) \times U_V(1) \longrightarrow U_{\tau_3}(1) \times U_V(1) \longrightarrow \varnothing \times U_V(1)$$




- **Spontaneous breaking** with pion condensate $\langle\bar\psi\gamma_5\tau_{1,2}\psi\rangle$
  $\longrightarrow$ Appearance of Goldstone mode
- **Explicit breaking** via pionic source $\lambda$,
  $\longrightarrow$ pseudo-Goldstone boson
  ($\lambda$ necessary trigger for spontaneous breaking and I.R. regulator)

# The "numerical phase diagram"

$N_f = 2+1$ improved dynamical staggered quarks with physical quark masses




Looked at: Brandt et al. (2018)

- Renormalized pion condensate
  $\Sigma_\pi = \frac{m_{ud}}{m_\pi^2 f_\pi^2} \langle \pi^\pm \rangle_{T,\mu_I}$
- Renormalized chiral condensate
  $\Sigma_{\bar\psi\psi} = \frac{m_{ud}}{m_\pi^2 f_\pi^2} \left[ \langle \bar\psi\psi \rangle_{T,\mu_I} - \langle \bar\psi\psi \rangle_{0,0} \right] + 1$

- BEC boundary at $\mu_{I,c} = m_\pi/2$ up to $T \approx 140\,\mathrm{MeV}$, very flat at larger $\mu_I$
- Confirmed second order ($\mathrm{O}(2)$ universality class) of the BEC boundary
- ⚠ Dedicated improvement program to achieve reliable $\lambda \to 0$ extrapolations

# Outline

1. More phases to unveil at large nonzero isospin
2. Beyond the phase diagram
3. Pion condensation in the early Universe

# MORE PHASES TO UNVEIL AT LARGE $\mu_I$

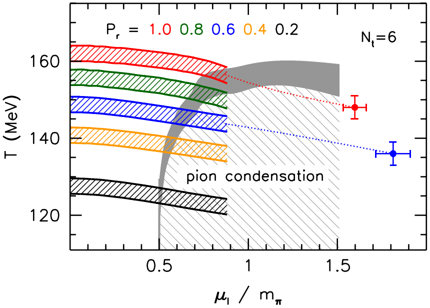
# BCS phases

We are after some quantity that is sensitive to the the BEC/BCS crossover...

**Signatures considered so far in alternative/effective theories**

- LO $\chi_{\mathrm{PT}}$: conformality of the system $\Longleftrightarrow$ crossover Carignano *et al.* '17
- PQM: "shift" minimum quark energies $\Longleftrightarrow$ crossover Adhikari *et al.* '18
- $\mathrm{Q_2CD}$: $\langle qq \rangle \sim \mu^2$ $\Longleftrightarrow$ BCS phase Boz *et al.* '20

**A conjecture based on lattice results at $\mu_I \neq 0$**




- Deconfinement crossover might continuously connect to the BEC/BCS crossover
  - Large Polyakov loops $P_{\mathrm{r}}$ within BEC phase
  - $T_c^{\mathrm{deconf.}}(\mu_I)$ insensitive to BEC boundary

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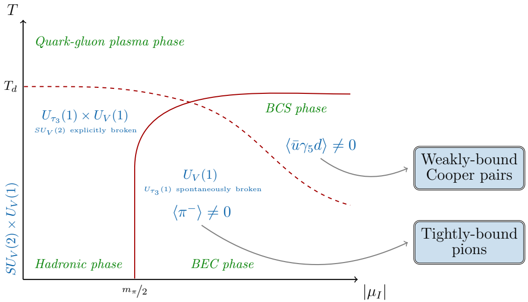


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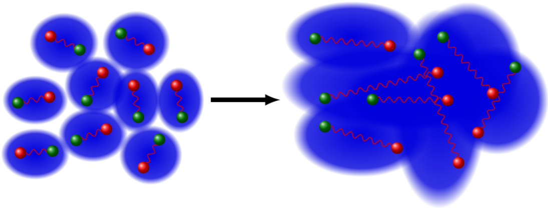


- Deconfinement crossover might continuously connect to the BEC/BCS crossover
  - Large Polyakov loops $P_{\mathrm{r}}$ within BEC phase
  - $T_c^{\mathrm{deconf.}}(\mu_I)$ insensitive to BEC boundary
- Superfluid ground state with "deconfined" quarks

# SIGNATURES OF THE BCS PHASE AT LARGE $\mu_I$ FROM THE DIRAC SPECTRUM

# Complex spectrum of the Dirac operator

- The Dirac operator has complex eigenvalues $\nu_n \in \mathbb{C}$
- $[\not{D}(\mu_I), \not{D}^\dagger(\mu_I)] \neq 0$, so left and right eigenvectors of $\not{D}(\mu_I)$ do not coincide
- $\forall$ eigenvalue $\nu_n$ in the up sector, complex conjugate $\nu_n^*$ in the down sector

$$\underbrace{[\not{D}(\mu_I)]\,\psi_n = \nu_n\,\psi_n}_{\text{up sector},\mu_I} \xleftrightarrow[\textit{chiral symmetry}]{\eta_5-\textit{hermiticity}} \underbrace{\widetilde{\psi}_n^\dagger\,[\not{D}(-\mu_I)] = \widetilde{\psi}_n^\dagger \nu_n^*}_{\text{down sector},-\mu_I,\widetilde{\psi}_n=\gamma_5\psi_n}$$

making the determinant of the total light quark matrix real and positive!

## Measurement

- Spectrum measured with SLEPc (Scalable Library for Eigenvalue Problem Computations), set up to obtain, via the Krylov-Schur method, ~150 complex eigenvalues of $\not{D}(\mu_I)$ (the closest, in modulo to the origin).

## Analysis

- Spectral density $\rho(\nu)$ extrapolated to $m_{ud}$, since we simulate at $m_{ud} \neq 0$, by
  - Using kernel density estimation (KDE) as a non-parametric way to estimate the multivariate probability density function from the measured spectrum.

# Why looking at the Dirac spectrum?

- Spontaneous chiral symmetry breaking reflected in the IR limit of the Dirac eigenvalue spectrum through the Banks-Casher relation for QCD at $\mu = 0$
  Banks, Casher (1980)

$$\left|\langle\bar{\psi}\psi\rangle\right| = \pi\ \rho(0)$$

Accumulation of near-zero eigenvalues

- At $\mu_{\rm B} \neq 0$, non positive-definiteness of the fermionic measure
  - renders $\rho(0)$ undefined
    Leutwyler, Smilga (1992)
  - complicates the connection between $\langle\bar{\psi}\psi\rangle$ and the **complex** Dirac spectrum
    Osborn, Splittorff, Verbaarschot (2005)

- At only $\mu_{\rm I} \neq 0$ though, the fermionic measure is still positive definite...
  - Generalization of the Banks-Casher relation is possible and was worked out at $T = 0$, $|\mu_I| \gg \Lambda_{QCD}$! Kanazawa, Wettig, Yamamoto (2013)

$$\Delta^2 = \frac{2\pi^3}{3N_C}\ \rho(0)$$

# RESULTS, SO FAR...

# Signatures of the BCS phase from $\not{D}(\mu_I)$ spectrum

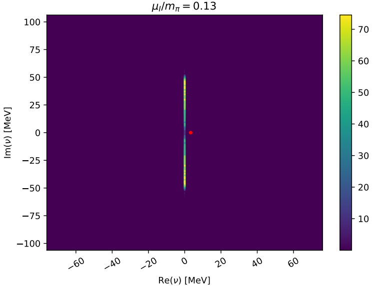
Banks-Casher relation at $\mu = 0$ Banks, Casher (1979)

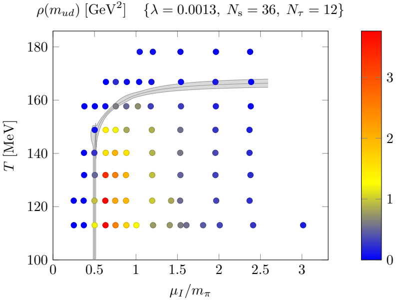
Chiral condensate order parameter for $\chi_{SB}$ ⟶ $|\langle\bar{\psi}\psi\rangle| = \pi\rho(0)$ ⟵ IR limit of imaginary $\not{D}$ spectral density

generalized to QCD at $T=0$, $|\mu_I| \gg \Lambda_{QCD}$ with complex Dirac spectrum...

Kanazawa, Wettig, Yamamoto (2013)

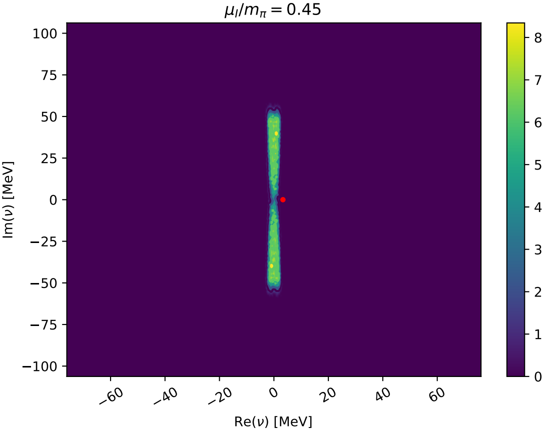
The BCS gap ⟶ $$\Delta^2 = \frac{2\pi^3}{3N_C}\,\rho(0)$$ ⟵ The complex $\not{D}(\mu_I)$ spectral density



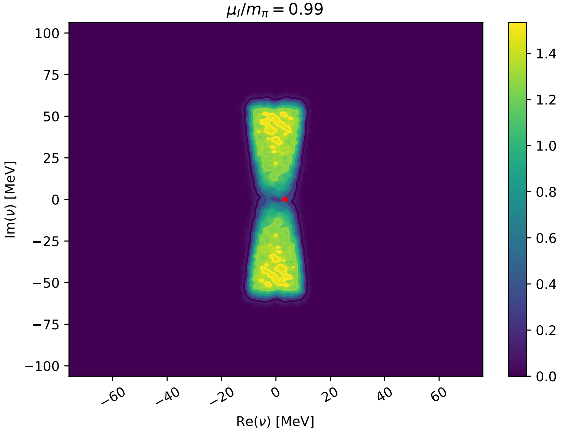

Brandt, Cuteri, Endrődi, Schmalzbauer (2020)

# Complex spectrum of $\not{D}(\mu_I)$ - Results, qualitatively




- Simulations are carried out away from the chiral limit $\rightarrow$ extract $\rho(m_{ud})$

- $\mu_I < m_\pi/2$: eigenvalues clustered along imaginary axis $\rightarrow \rho(m_{ud}) = 0$

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- $\mu_I > m_\pi/2$: spectrum 'wide' enough to include $m + i0 \rightarrow \rho(m_{ud}) \neq 0$

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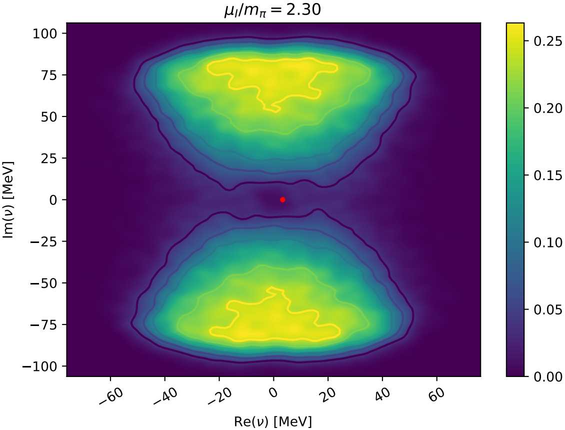


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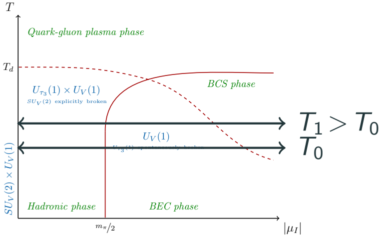
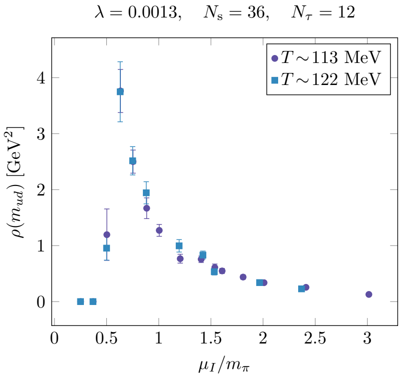
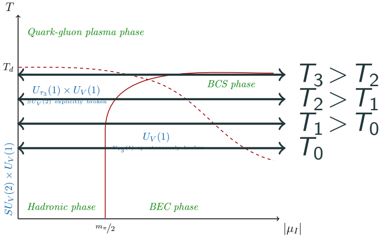
- $\mu_I < m_\pi/2$: eigenvalues clustered along imaginary axis $\rightarrow \rho(m_{ud}) = 0$
- $\mu_I > m_\pi/2$: spectrum 'wide' enough to include $m + i0 \rightarrow \rho(m_{ud}) \neq 0$
- Higher-$\mu_I$: eigenvalues drifting away from the real axis $\rightarrow \rho(m_{ud}) \rightarrow 0$

# Complex spectrum of $\not{D}(\mu_I)$ - Results, quantitatively




## $\mu_I$-scans, at fixed $T$



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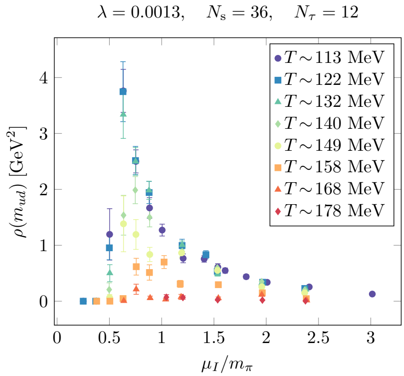

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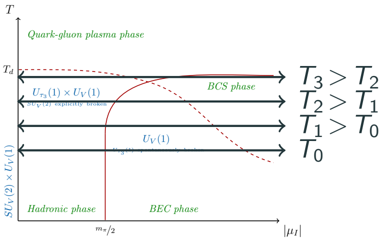
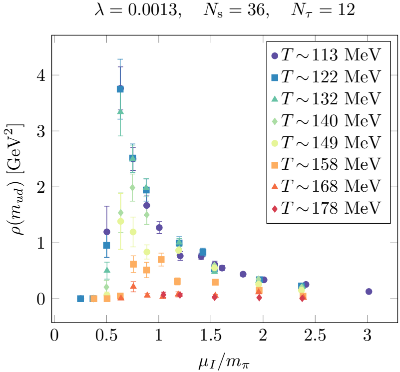



$\mu_I$**-scans, at fixed** $T$

Can we read

- $\mu_I^{\text{Hadr./BEC}}$
- $\mu_I^{\text{BEC/BCS}}$

off our $\rho(m_{ud})$?



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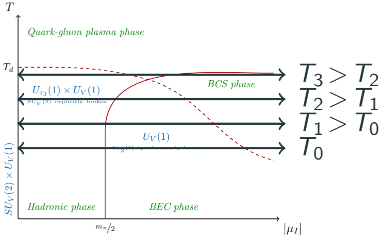


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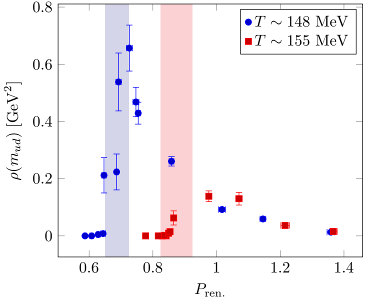


Matching $\rho(m_{ud})$ with $P_{\mathrm{ren.}}$, highlighting $\mu_I^{\mathrm{Hadr./BEC}}$

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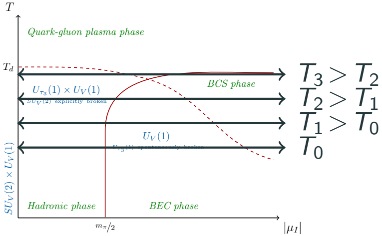


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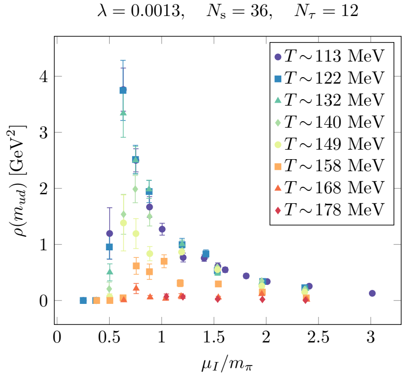


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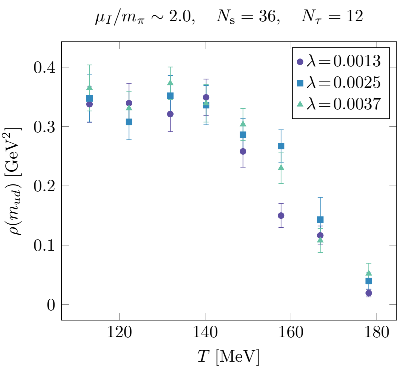
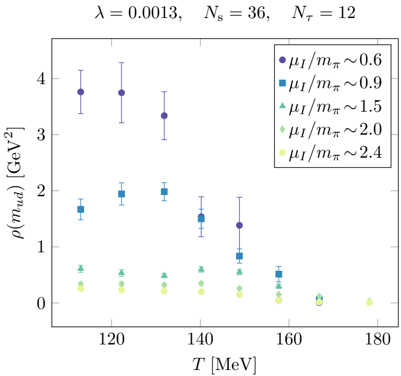
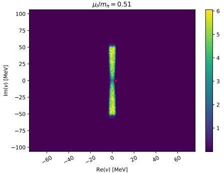


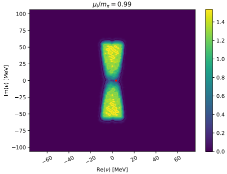



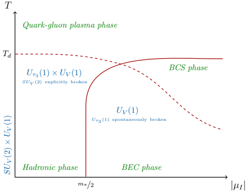

- Weak, if any, dependence of the extrapolated spectral density $\rho(m_{ud})$ on the value of the pionic source $\lambda$
- Weak, if any, dependence of the extrapolated spectral density $\rho(m_{ud})$ on the temperature $T$ within BEC phase, then rapidly drop at $(\mu_I^{\mathrm{BEC}}, T^{\mathrm{BEC}})$

# Complex spectrum of $\not{D}(\mu_I)$ - Summary

- Extrapolated spectral density $\rho(m_{ud})$ sensitive to the BEC boundary!



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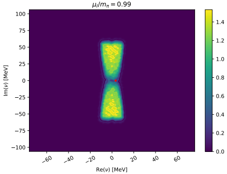
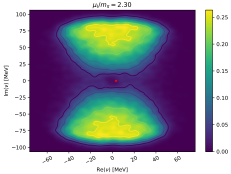
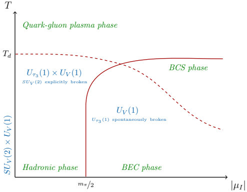



- Sensitivity to the BEC-BCS crossover? $\Delta \neq 0$ at high-$\mu_I$?
  More systematic analysis ongoing to disentangle signal for the BCS-BEC crossover from discretization errors ...
  ($a \to 0$, but also $V \to \infty$, $\lambda \to 0$)

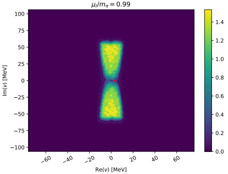


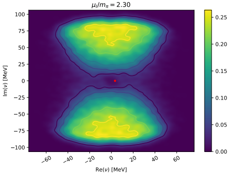
# Complex spectrum of $\not{D}(\mu_I)$ - Summary

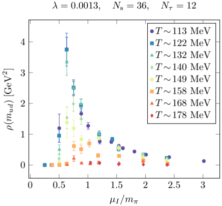
CRC-TR 211

- Extrapolated spectral density $\rho(m_{ud})$ sensitive to the BEC boundary!




- Sensitivity to the BEC-BCS crossover? $\Delta \neq 0$ at high-$\mu_I$?
  More systematic analysis ongoing to disentangle signal for the BCS-BEC crossover from discretization errors …
  ($a \to 0$, but also $V \to \infty$, $\lambda \to 0$)



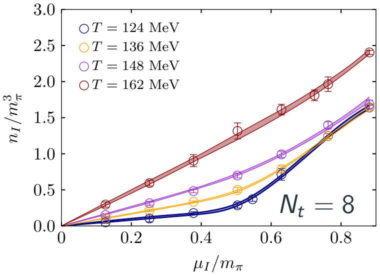

- Lacking generalization of Banks-Casher relation away from $T = 0$ & $|\mu_I| \gg \Lambda_{QCD}$

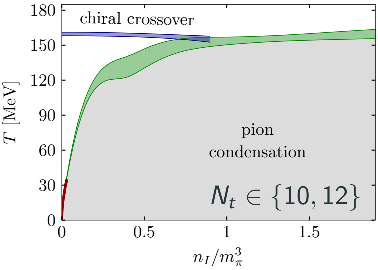
# Beyond the phase diagram

Use known $p(T,0)$, $I(T,0)$ Borsányi *et al.* '13 Bazavov *et al.* '14 and compute

$$\Delta p(T,\mu_I) \equiv p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} d\mu \, n_I(T,\mu) \qquad n_I(T,\mu_I)\text{: 2d spline}$$

$$\Delta I(T,\mu_I) \equiv \frac{I(T,\mu_I) - I(T,0)}{T^4} = \mu_I n_I(T,\mu_I) + \int_0^{\mu_I} d\mu_I' \left[T\frac{\partial}{\partial T} - 4\right] n_I(T,\mu_I')$$

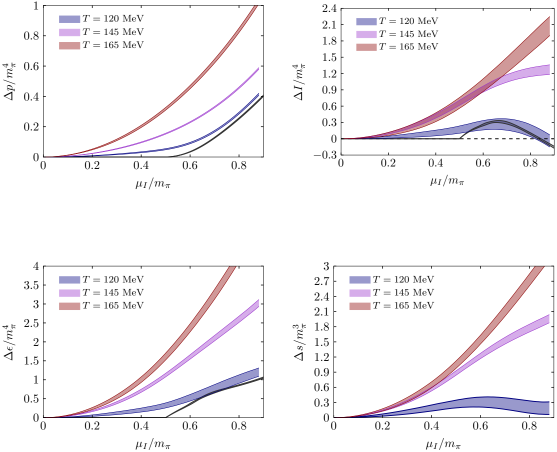


CRC-TR 211

...and from $n_I$, $p$ and $I$ then obtain $\epsilon = I + 3p$, and $s = (\epsilon + p - \mu_I n_I)/T$.
Results shown for $N_t = 8$:

- $T = 0$ results from
  Brandt *et al.* '18
- BEC at low T:
  Non-monotonous behavior of $\Delta s$ and $\Delta I$ with $\mu_I$ at low temperatures
  - Evidence for pion condensation



# Pion condensation in the early Universe

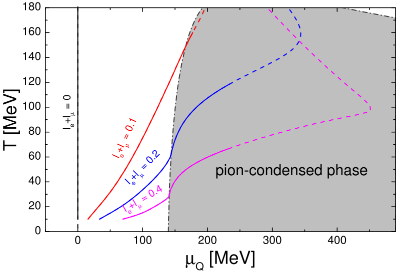
Results **calibrate** & **test** a variant of HRG model incorporating $\pi$-condensation, by replacing in the pressure the free pion gas with an interacting pion gas modelled by a quasiparticle (effective mass) approach. Other hadrons free.

Model $\rightarrow$ equation of state of cosmic matter fed to the conservation equations

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_e}{s} = l_e, \quad \frac{n_\mu}{s} = l_\mu, \quad \frac{n_\tau}{s} = l_\tau$$

that define the **cosmic trajectory** in the 6-d space of $T$, $\mu_B$, $\mu_Q$, $\mu_e$, $\mu_\mu$, $\mu_\tau$

Vovchenko, Stoecker (2019)



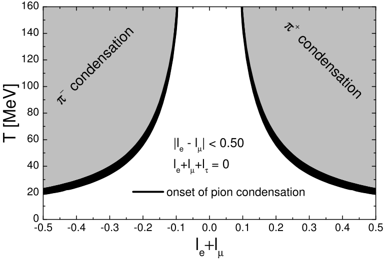

- Unequally distributed lepton asymmetries are sufficient condition for pion condensation Wygas et al. (2018) Middeldorf-Wygas et al. (2020)

# Pion condensation in the early Universe

## Working hypotheses

- Isentropic expansion of the Universe in cosmic QCD epoch: EoS mainly determined by QCD matter, QED interactions neglected
  $$p = p_{\rm QCD}(T,\mu_B,\mu_Q) + \underbrace{p_L(T,\mu_Q,\mu_e,\mu_\mu,\mu_\tau) + p_\gamma(T)}_{\text{ideal gases}}$$
- Empirical constraints on conserved charges per entropy ratios
  - $b = (8.60 \pm 0.06) \times 10^{-11}$ Planck Collaboration (2015)
  - $|l_e + l_\mu + l_\tau| < 0.012$ Oldengott, Schwarz (2017) via $l_\tau = -(l_e + l_\mu)$

## Result & phenomenological implications Vovchenko et al. (2020)




- Pion condensation for $|l_e + l_\mu| \gtrsim 0.1$
- Observable imprints of condensation
  - Enhanced relic density of primordial gravitational waves
  - Modified mass distribution of primordial black holes

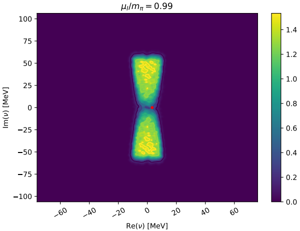
# Summary

Insights onto the $(T, \mu_I)$ phase diagram from Dirac spectrum

Towards the continuum extrapolated EoS at nonzero $T$

Pion condensation in the early Universe for $|l_e + l_\mu| \gtrsim 0.1$

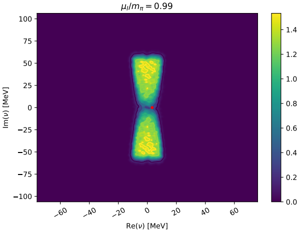
# Summary

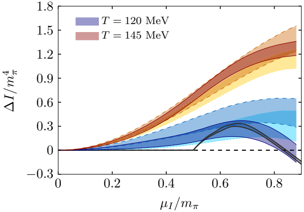



Towards the continuum extrapolated EoS at nonzero $T$

Pion condensation in the early Universe for $|l_e + l_\mu| \gtrsim 0.1$

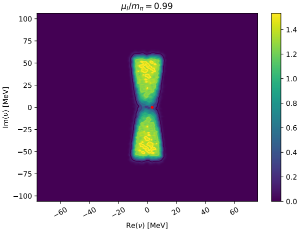
# Summary



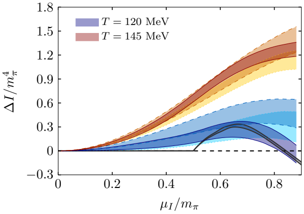




Pion condensation in the early Universe for $|l_e + l_\mu| \gtrsim 0.1$

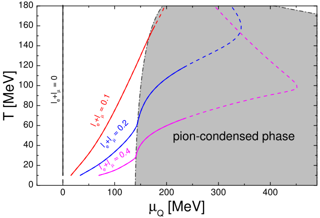
# Summary



$\mu_I/m_\pi = 0.99$
Im($\nu$) [MeV]
Re($\nu$) [MeV]

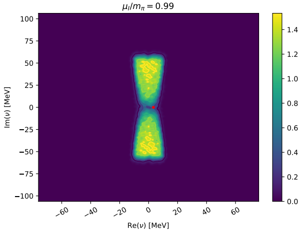



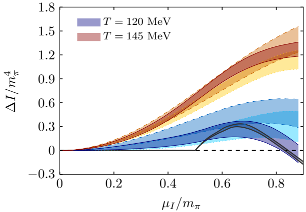
$T = 120$ MeV
$T = 145$ MeV
$\Delta I/m_\pi^4$
$\mu_I/m_\pi$

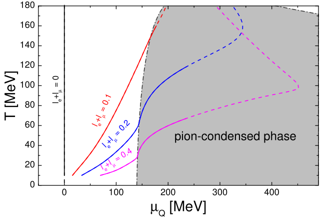



T [MeV]
$\mu_Q$ [MeV]
$l_e+l_\mu=0$
$l_e+l_\mu=0.1$
$l_e+l_\mu=0.2$
$l_e+l_\mu=0.4$
pion-condensed phase

# Summary




**Thank you**
**for your attention!**