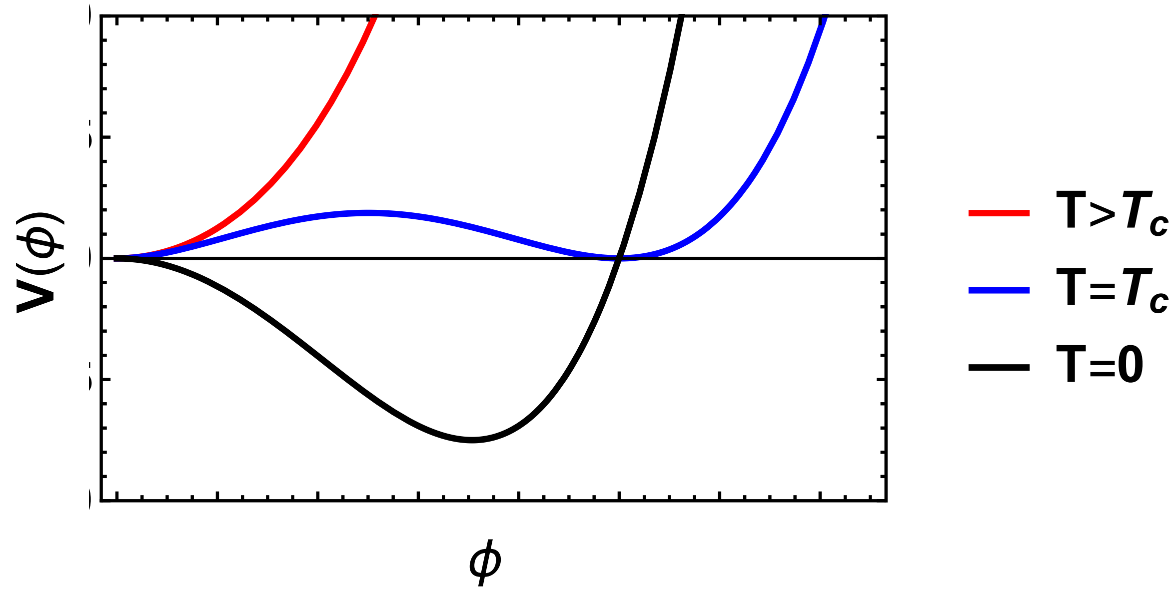
Dynamical Axions and Gravitational Waves

Rachel Houtz
Phase Transitions in Particle Physics
GGI Workshop
March 2022

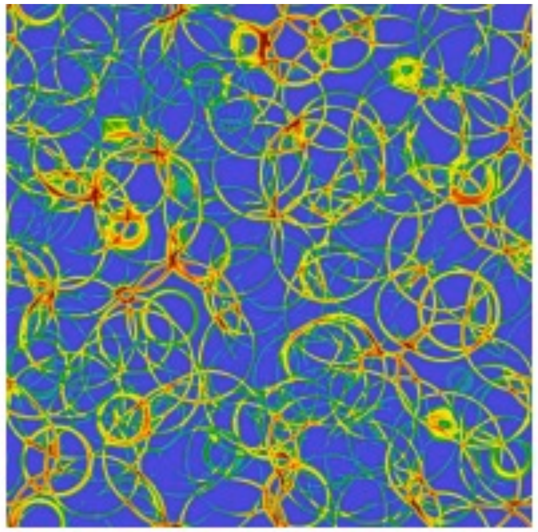
In Collaboration with D. Croon (TRIUMF & Durham) and V. Sanz (U. Sussex & IFIC)

Gravitational Waves and Confining Sectors



- First order phase transitions can give a stochastic gravitational wave background

  Witten (1984) Hogan (1986)



Hindmarsh, Huber, Rummukainen, Weir, 1504.0329

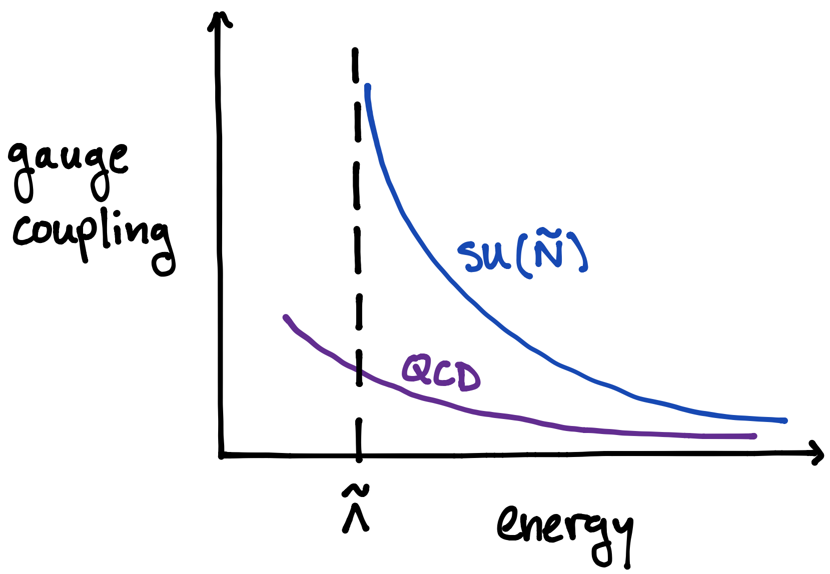
- Complementary probe of hidden sectors:
  - Spontaneous symmetry breaking
  - Confining exotic color sectors

    Helmboldt, Kubo, van der Woud, 1904.07891

    Halverson et al, 2012.04071

    Sannino, et al, 2012.11614, 2109.11552

Gravitational Waves and Confining Sectors



- First order phase transition at confinement if $N_F \geq 3$

  Pisarski, Wilczek (1984)

- Gravitational waves can probe confining exotic color sectors

- Use a low energy effective theory to try and parameterize the behavior of the potential at T_c Bigazzi, et al, 2011.08757 Helmboldt, Kubo, van der Woud, 1904.07891

- Improve with lattice results Sannino, et al, 2109.11552 Sannino, et al, 2012.11614 Halverson et al, 2012.04071

- Model building can motivate parameters in the low energy EFT

Dynamical Axions

- Solving the Strong CP problem by employing $U(1)_{PQ}$ results in the axion

$$\mathcal{L} \ni \frac{g^2}{32\pi^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

Peccei, Quinn (1977)
Wilczek (1978)
Weinberg (1978)

<u>Massless quark solution:</u>

If $m_\psi = 0$, under chiral rotation of ψ

$$\theta \frac{g^2}{32\pi^2}G\tilde{G} \rightarrow (\theta - 2\alpha)\frac{g^2}{32\pi^2}G\tilde{G}$$

➡ θ can be removed by field redefinition

- Massless quarks will form bound states, one of which is the dynamical axion

Motivation for Exotic Confining Groups

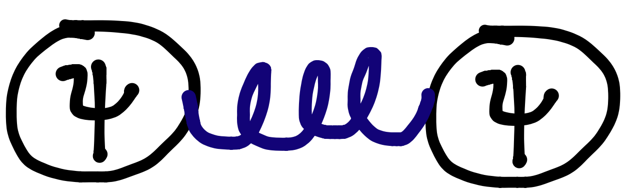
(1) Additional color interactions can alter the m_a, f_a relationship

$$m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \longrightarrow + \sim \Lambda_{\text{new}}^4$$

(2) Hide massless quark in bound states

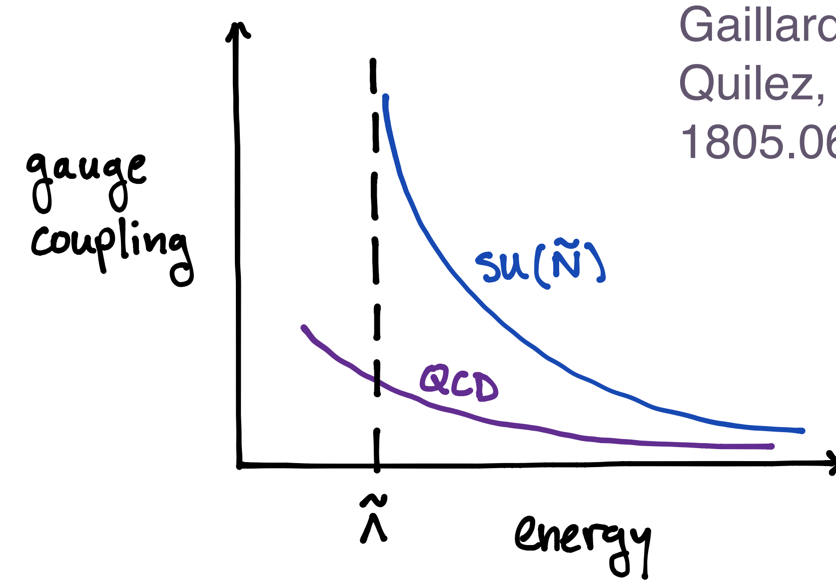
Choi, Kim (1985) Hook, 1411.3325

Gaillard, Gavela, RH, Quilez, del Rey, 1805.06465



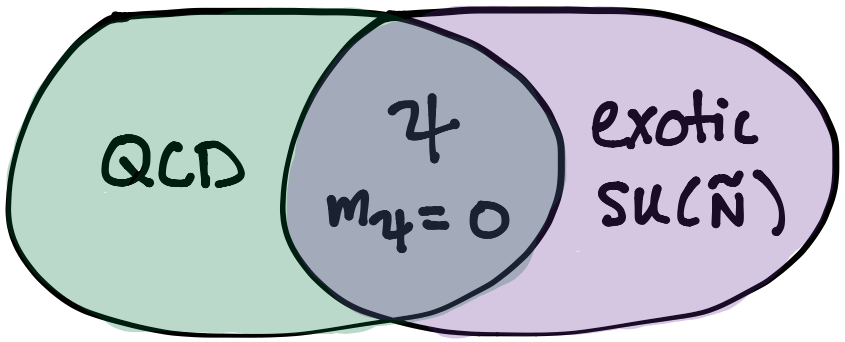
$\sim \Lambda_{\text{new}}$

$m_\psi = 0$



Generic Properties of Dynamical Axion Models

- Massless messenger fields



- $N_F \geq 3$ at $SU(\tilde{N})$ confinement
- First order PT at $\tilde{\Lambda} \sim 3 \text{ TeV}$

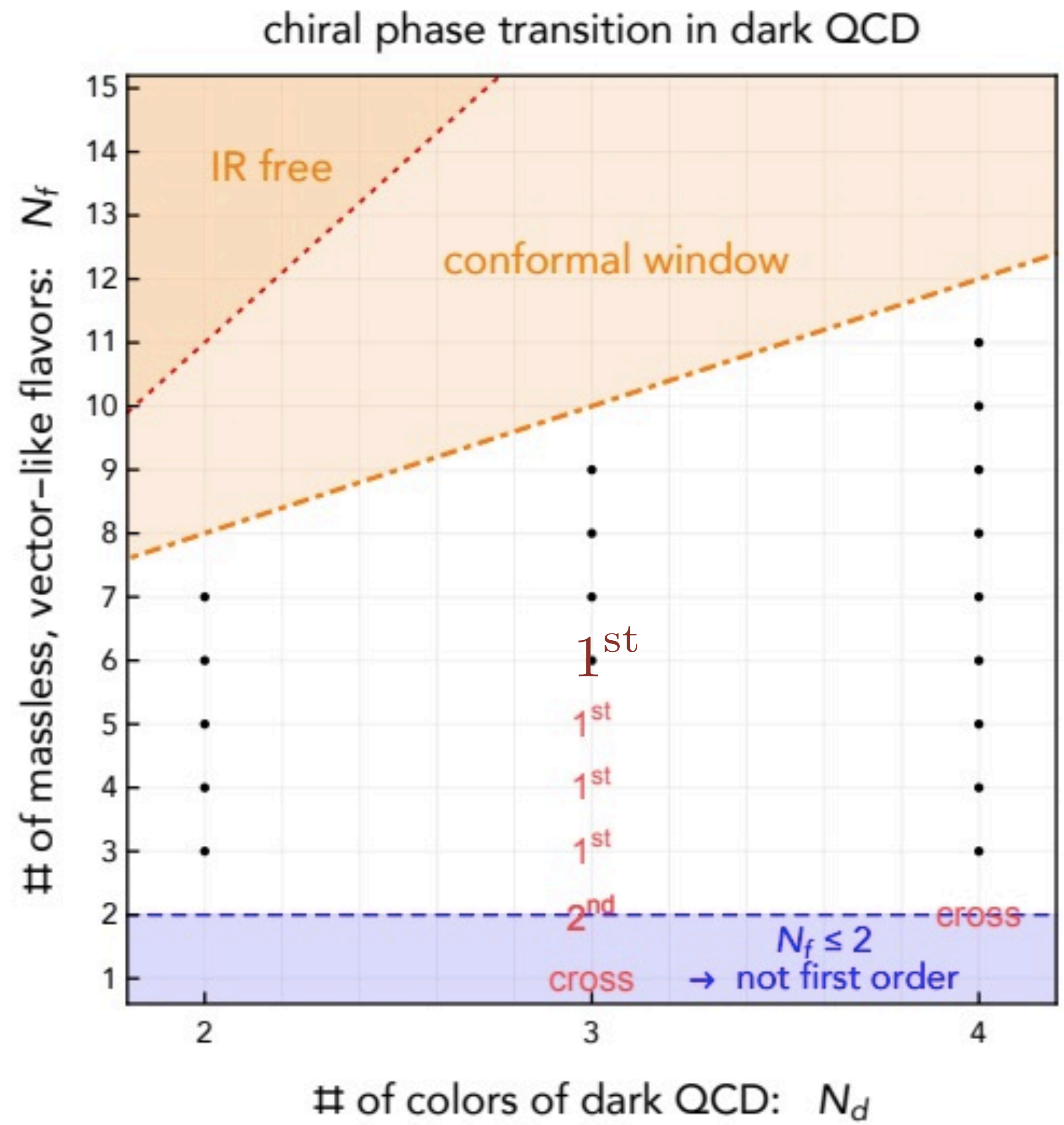
  Pisarski, Wilczek (1984)

- Quadratically divergent mass terms for pions

g_{QCD} $\quad m^2(\tilde{\pi}) \sim g_{\text{QCD}}^2 \tilde{\Lambda}^2$

Plot lifted from: Bai, Long, Lu, arXiv:1810.04360

$N_F = 6$: Iwasaki, Kanaya, Sakai, Yoshié, hep-lat/9504019



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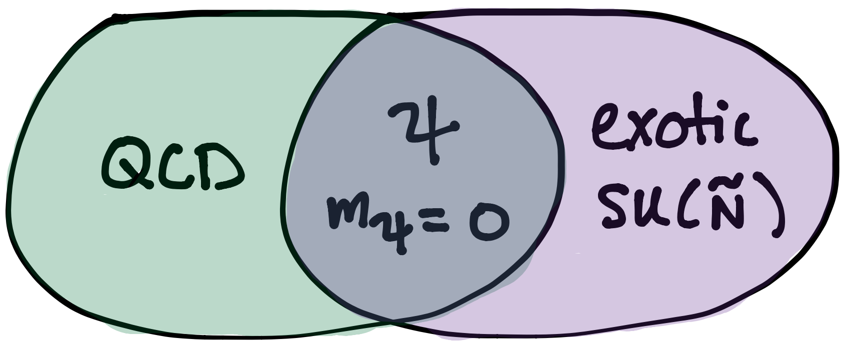

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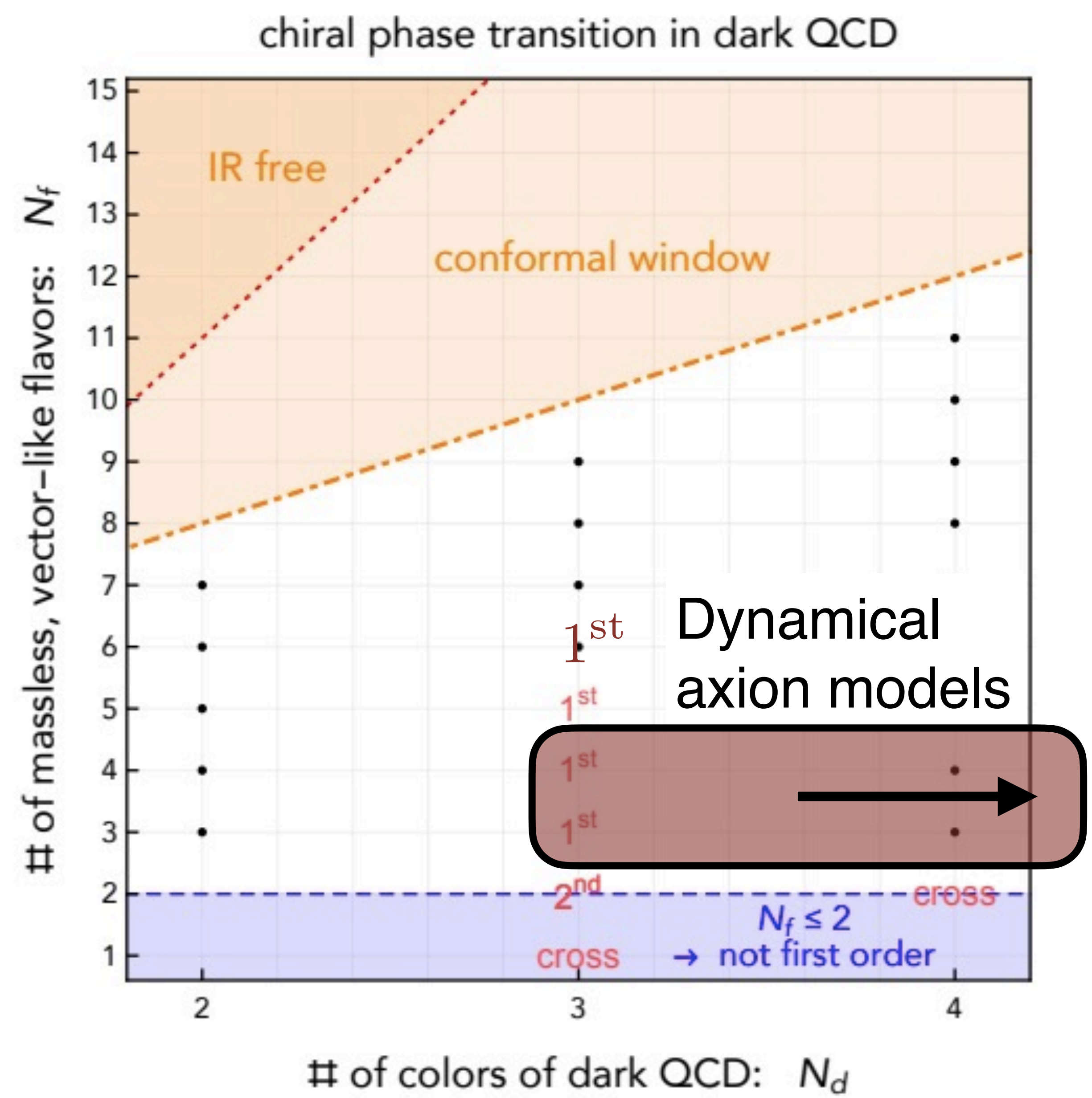
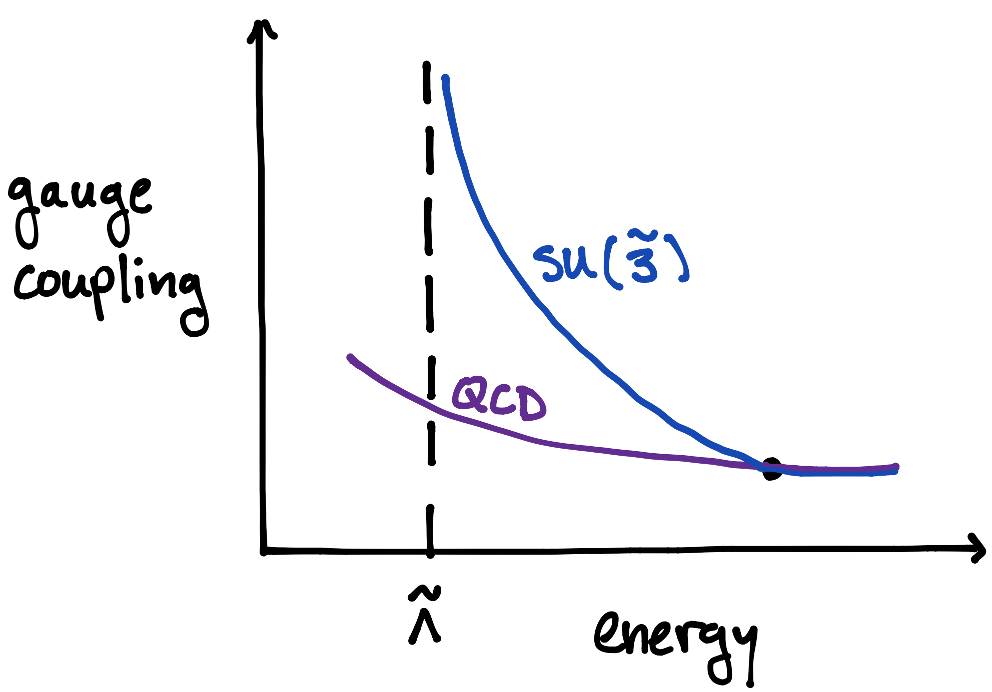


Dynamical Axion Models

$$N_F = 3$$

Hook, 1411.3325

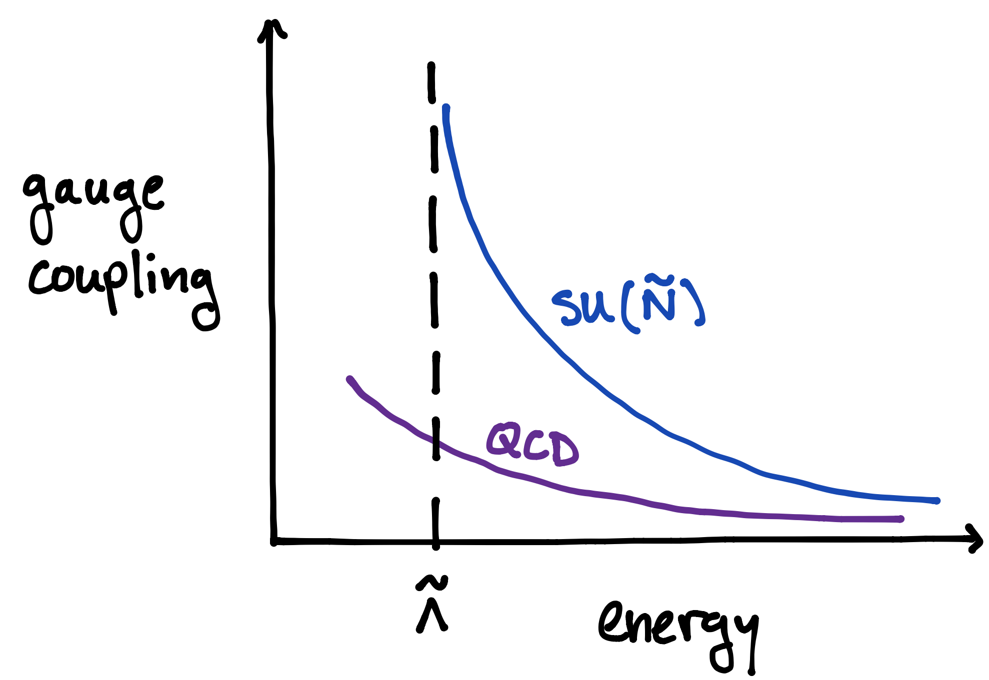
	$SU(3)_{QCD}$	$SU(\tilde{3})$
ψ	□	□



$$N_F = 4$$

Choi, Kim (1985)

	$SU(3)_{QCD}$	$SU(\tilde{N})$
ψ	□	□
χ	$\mathbb{1}$	□



Dynamical Axion Models

$N_F = 3$

Hook, 1411.3325

	$SU(3)_{QCD}$	$SU(\tilde{3})$
ψ	$\square$	$\square$

$N_F = 4$

Choi, Kim (1985)

	$SU(3)_{QCD}$	$SU(\tilde{N})$
ψ	$\square$	$\square$
χ	$\mathbb{1}$	$\square$

- Chiral symmetry breaking pattern at confinement:

$$SU(N_F)_L \times SU(N_F)_R \to SU(N_F)_V$$

➡ Bound states composed of massless quarks live at $\tilde{\Lambda}$

➡ Possible light pNGB states

$\mathbf{SU(\tilde{3})}$ Confinement, $\mathbf{N_F = 3}$

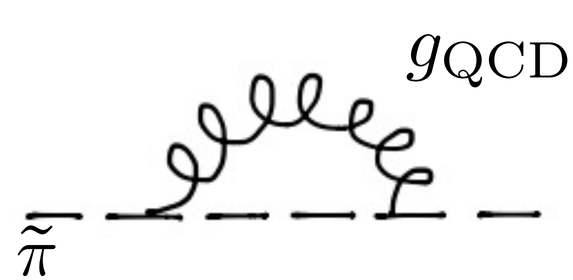
- Spontaneous chiral symmetry breaking:

$$SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R \rightarrow SU(3)_V \times U(1)_V$$

- Resulting Goldstone Bosons: $9 \rightarrow 8_c + 1_c = \tilde{\pi}_8 + a_\psi$

<u>Explicit symmetry breaking effects:</u>

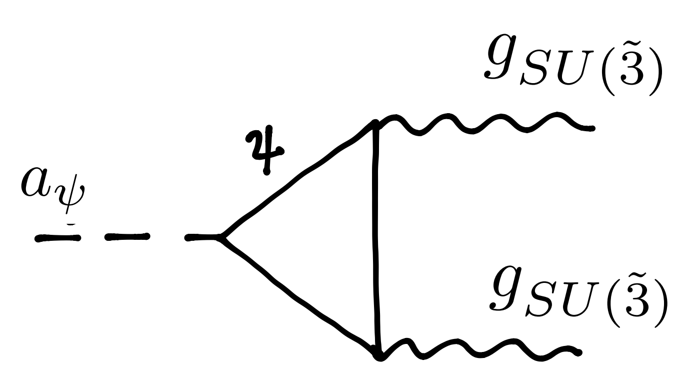
(1) QCD explicitly breaks $SU(3)_V$



$m^2(\tilde{\pi}_8) \sim g^2_{\text{QCD}}\tilde{\Lambda}^2$

(2) $G\tilde{G}$ explicitly breaks $U(1)_A$

➡ The a_ψ is the **visible dynamical axion**



$m^2(a_\psi) \sim \tilde{\Lambda}^2$

$\mathrm{SU}(\tilde{\mathrm{N}})$ Confinement, $\mathrm{N_F} = 4$

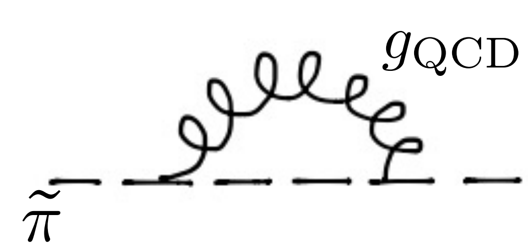
Spontaneous symmetry breaking: $U(4)_L \times U(4)_R \to U(4)_V$

- Resulting Goldstone Bosons: $15 + 1 \to 8_c + 3_c + \bar{3}_c + 1_c + 1_c$

$$= \tilde{\pi}_8 + \tilde{\pi}_3 + \bar{\tilde{\pi}}_3 + a_\psi + a_\chi$$

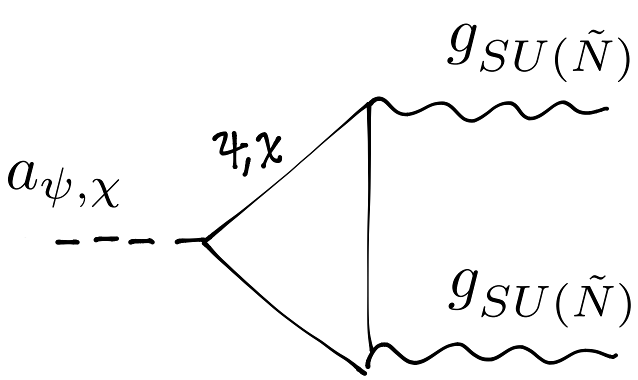
Explicit symmetry breaking effects:

(1) QCD explicitly breaks $SU(4)_V$



$m^2(\tilde{\pi}_{3,8}) \sim g^2_{\mathrm{QCD}} \tilde{\Lambda}^2$

(2) $G\tilde{G}$ explicitly breaks $U(1)_A$



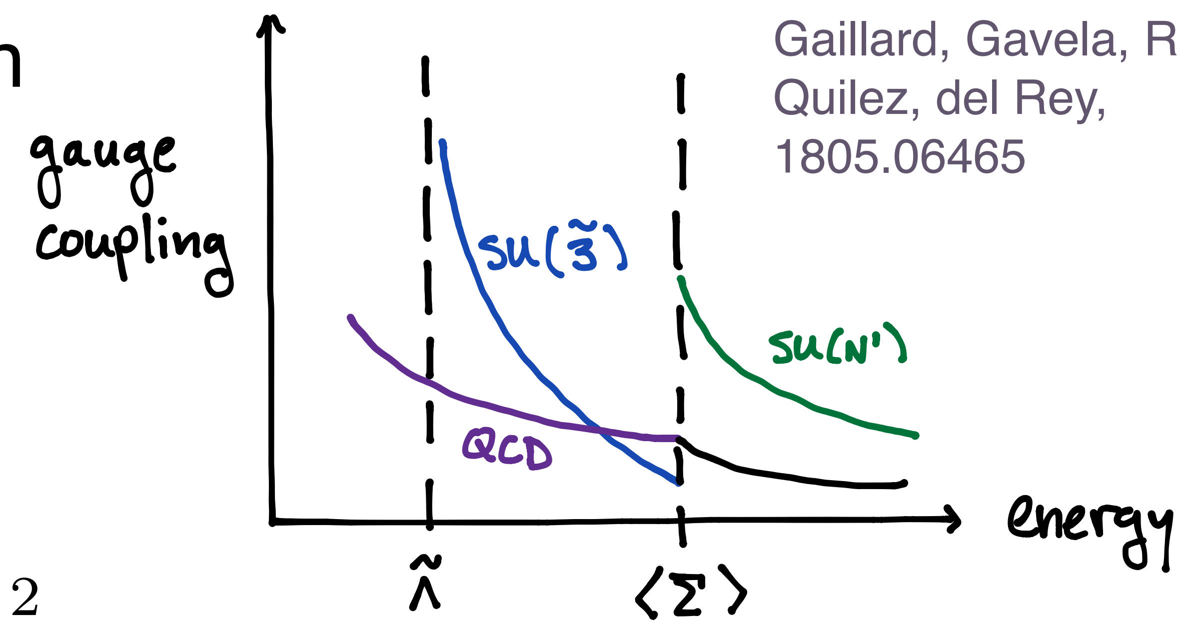
$m^2(a_\psi) \sim \tilde{\Lambda}^2$

$m(a_\chi)\tilde{f} \sim m_\pi f_\pi$

- The anomaly only gives mass to one axion a_ψ
- The light a_χ is an **invisible dynamical axion**

Visible Axion Models $\mathbf{N_F = 4}$

- New physics at high energies can induce sizable instanton corrections to the axion mass



Gaillard, Gavela, RH, Quilez, del Rey, 1805.06465

Agrawal, Howe, 1710.04213 & 1712.05803

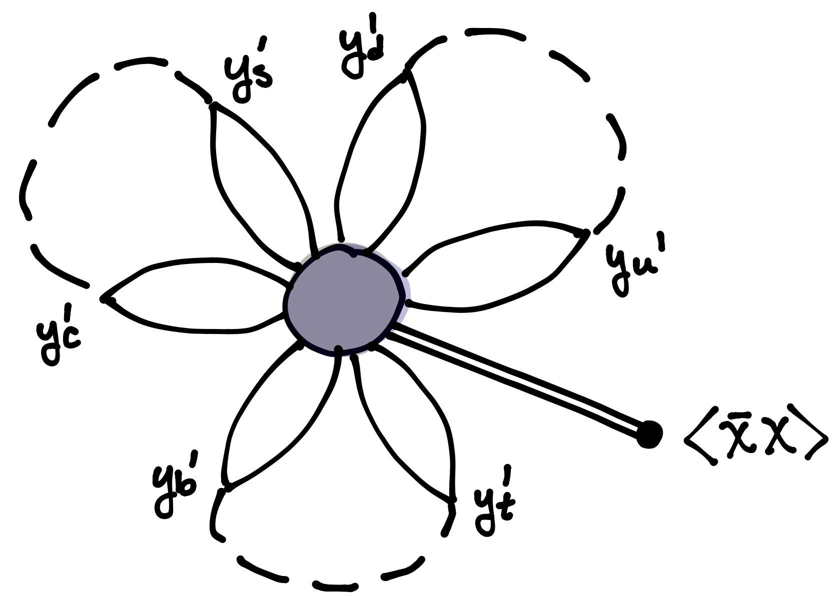
Fuentes-Martin, Reig, Vicente, 1907.02550

$$m^2(a_\psi) \sim \tilde{\Lambda}^2 \qquad m(a_\chi)\tilde{f} \sim m_\pi f_\pi$$

$$\rightarrow$$

$$m^2(a_\psi) \sim \tilde{\Lambda}^2 \qquad m^2(a_\chi) \sim \Lambda_{SSI}^2$$

- Possible to have a combination of anomalous effects raise the mass of the lightest $a_{\chi,\psi}$

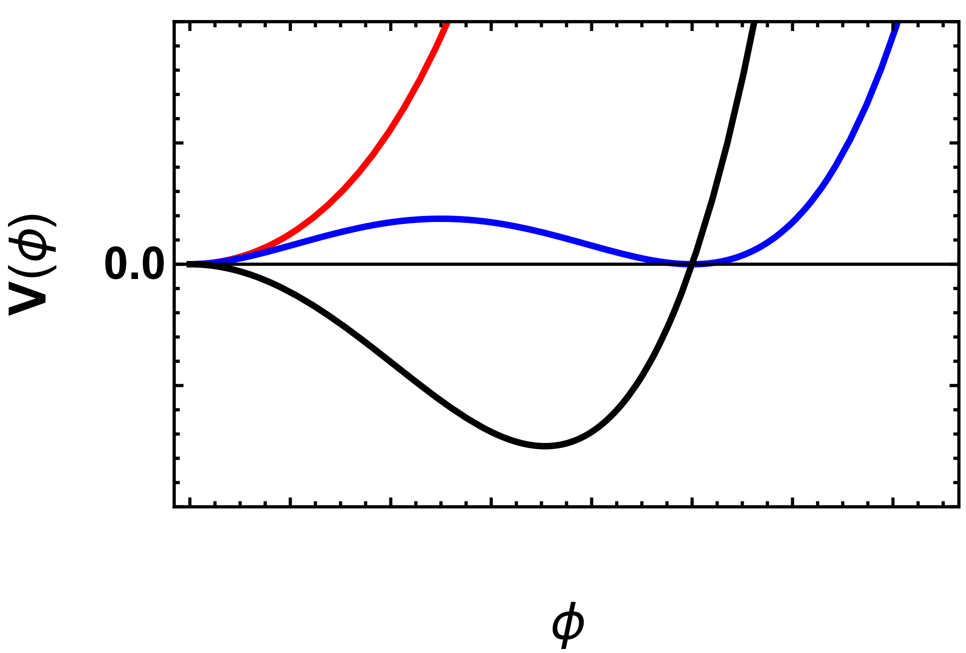


Phase Transition at Confinement

- Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \,\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) + \frac{\lambda}{2}\left[\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right]^2 + \frac{\kappa}{2}\left(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right),$$

- Spontaneous chiral symmetry breaking $\Sigma_{ij} \sim \langle \bar{\psi}_{R_j} \psi_{L_i} \rangle$



$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}}\delta_{ij} + X^a T^a_{ij} + i\pi^a T^a_{ij}$$

$\langle\varphi\rangle = 0, \quad T \gg 0$ Chiral symmetry restored

$\langle\varphi\rangle = f_\Sigma, \quad T \leq T_c$ ~~Chiral symmetry~~

Phase Transition at Confinement

- Model the phase transition using the linear sigma model

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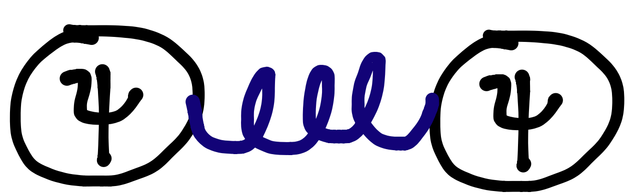
Heavy fields corresponding to unbroken generators



$\sim \tilde{\Lambda}^2$

Phase Transition at Confinement

- Model the phase transition using the linear sigma model

$$V(\Sigma) = -m_\Sigma^2 \, \mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) + \frac{\lambda}{2}\left[\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right]^2 + \frac{\kappa}{2}\left(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right),$$

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$$\Sigma_{ij} = \frac{\varphi + i\eta'}{\sqrt{2N_F}}\delta_{ij} + X^a T^a_{ij} + i\pi^a T^a_{ij}$$

(pseudo) Goldstone Boson fields

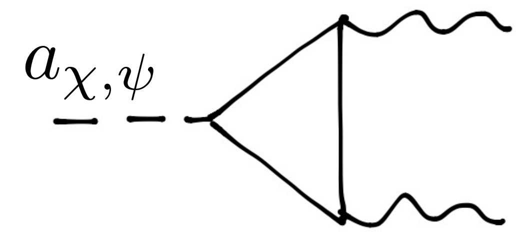
➡ Masses due explicit symmetry breaking effects

Phase Transition at Confinement

- Symmetry breaking parameters $\mu_\Sigma,\ \xi,\ \mu_{SSI}$ determine the masses of the pNGB's π and η'

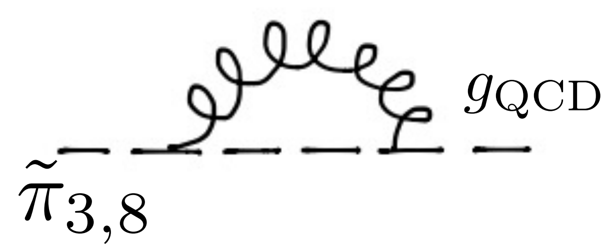
$$V(\Sigma) = -m_\Sigma^2 \,\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) + \frac{\lambda}{2}\left[\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right]^2 + \frac{\kappa}{2}\left(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right)$$

$$-\left(\mu_\Sigma \det\Sigma + h.c.\right)$$



~~$U(1)_A$~~

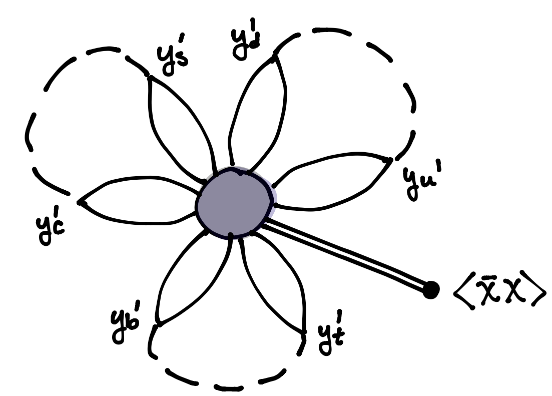
$$-\xi\ \mathrm{Tr}\ Q^a\Sigma\Sigma^\dagger Q^{a\dagger}$$



~~$SU(N_F)$~~

$$-\mu_{\mathrm{SSI}}\mathrm{Tr}\left(P_\chi\Sigma P_\chi\Sigma^\dagger\right)$$

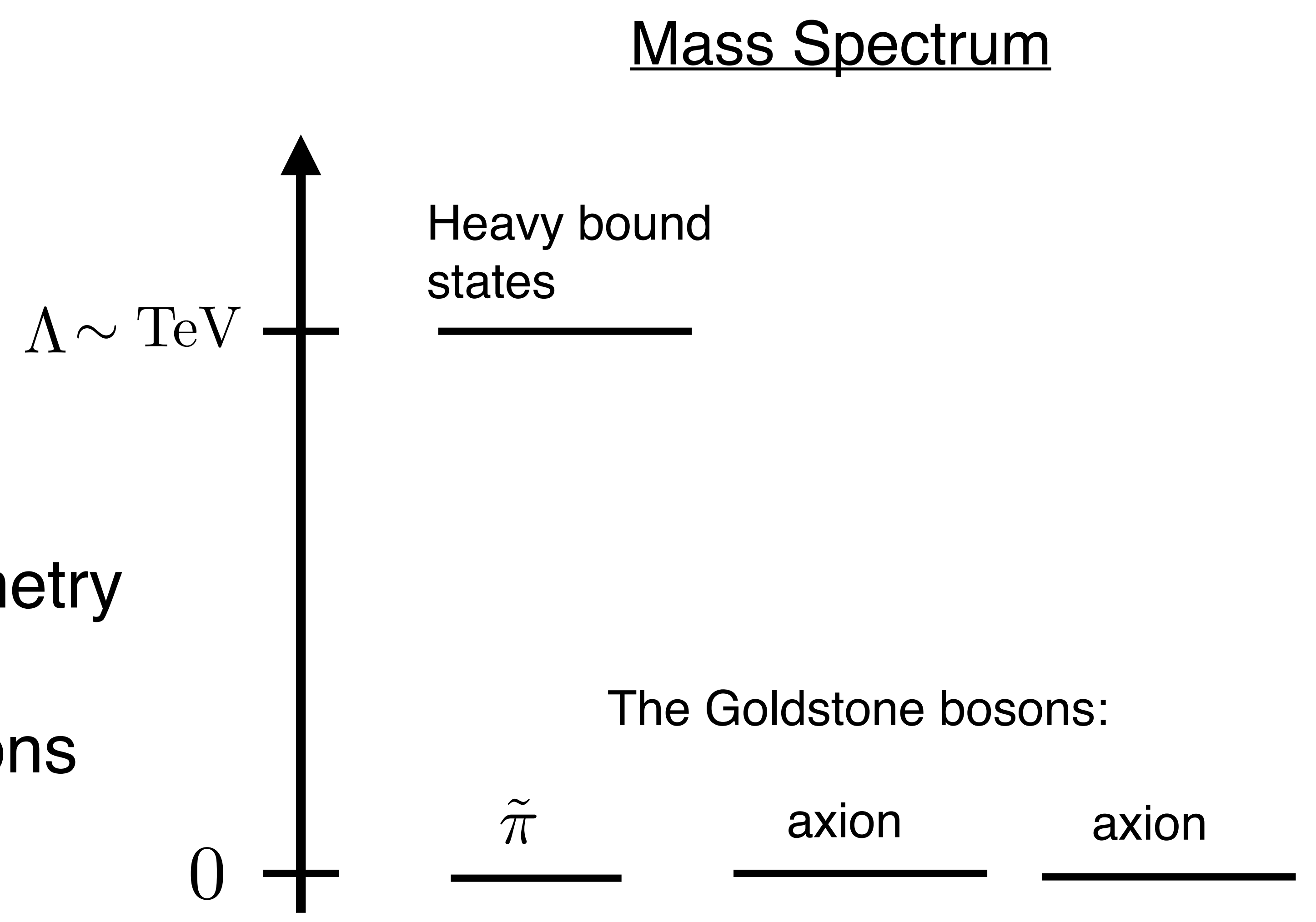
Include new mass contributions from small-sized instantons



$N_F = 4$ spontaneous chiral symmetry breaking

$$V(\Sigma) = -m_\Sigma^2 \,\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) + \frac{\lambda}{2}\left[\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right]^2 + \frac{\kappa}{2}\left(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right)$$

- Heavy bound states near confinement scale
- Spontaneous chiral symmetry breaking gives a set of massless Goldstone bosons

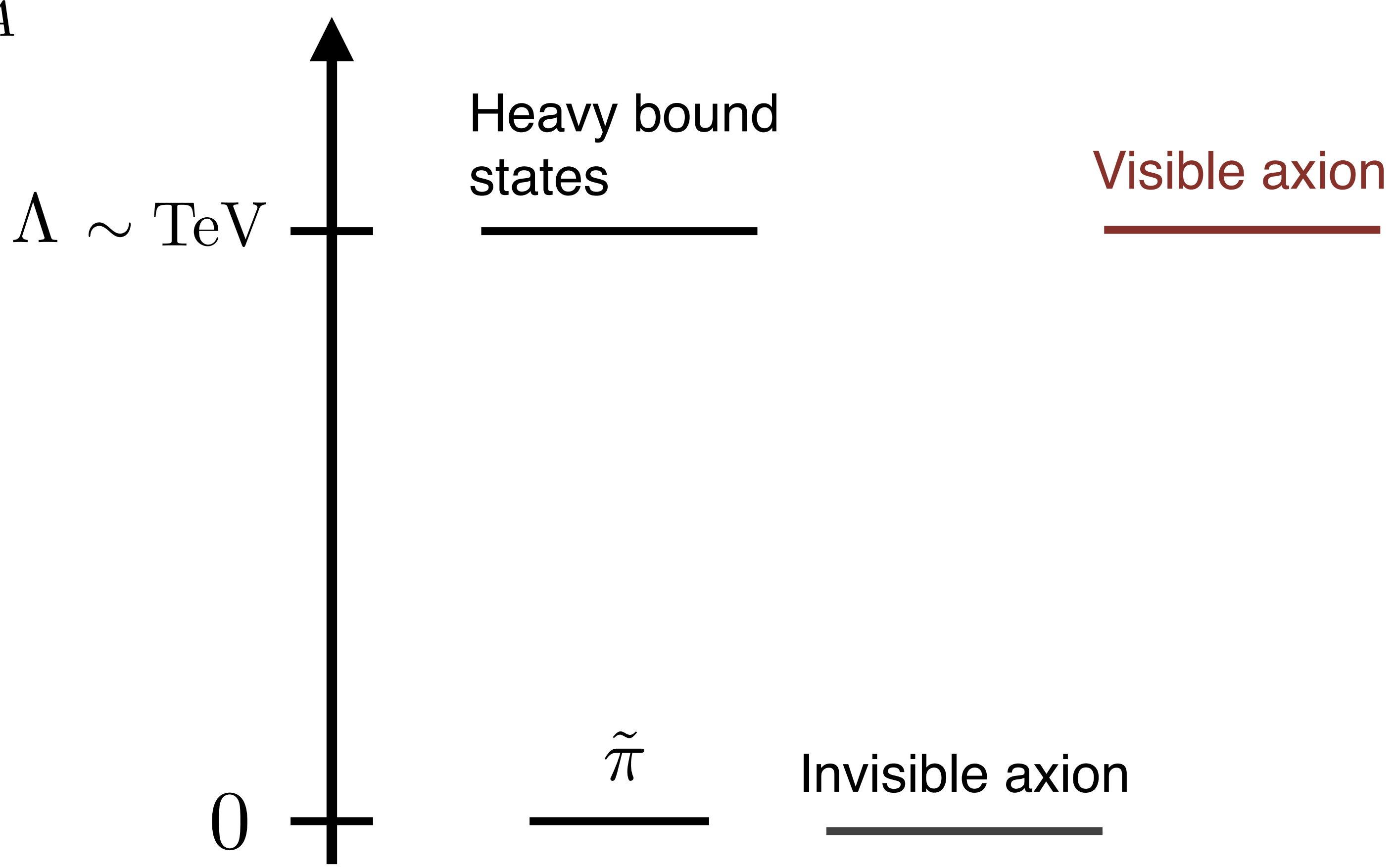


$N_F = 4$ explicit symmetry breaking effects

$$V(\Sigma) = -m_\Sigma^2 \,\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) + \frac{\lambda}{2}\left[\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right]^2 + \frac{\kappa}{2}\left(\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right)$$
$$- \left(\mu_\Sigma \det\Sigma + h.c.\right)$$

- Include the explicit $U(1)_A$ symmetry breaking from instanton effects

Mass Spectrum



$N_F = 4$ explicit symmetry breaking effects

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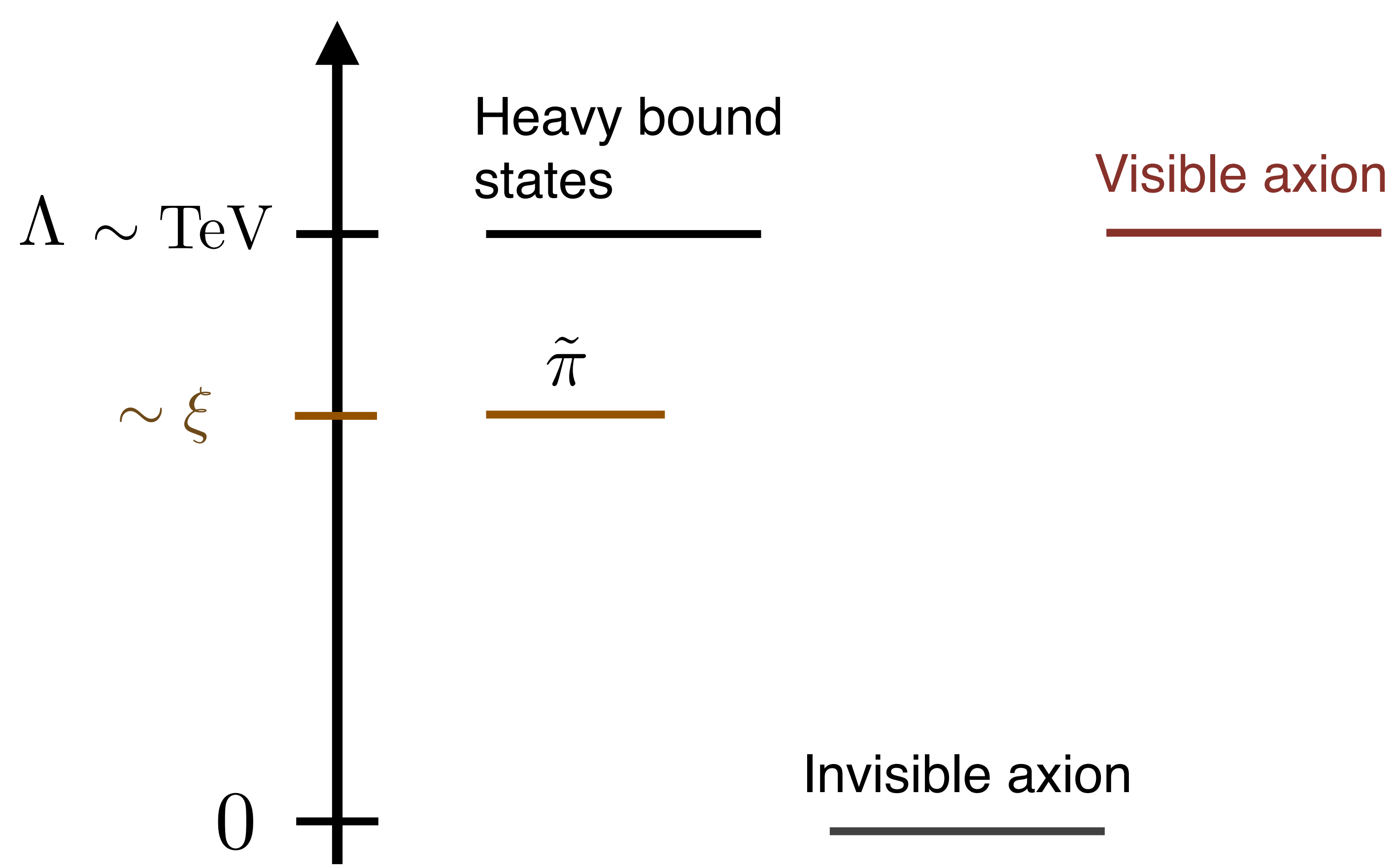
$$- \left(\mu_\Sigma \det\Sigma + h.c.\right) - \xi \;\mathrm{Tr}\; Q^a \Sigma\Sigma^\dagger Q^{a\,\dagger}$$

- Include the explicit $U(1)_A$ symmetry breaking from instanton effects
- Include the explicit symmetry breaking from QCD charges

Mass Spectrum



$N_F = 4$ explicit symmetry breaking effects

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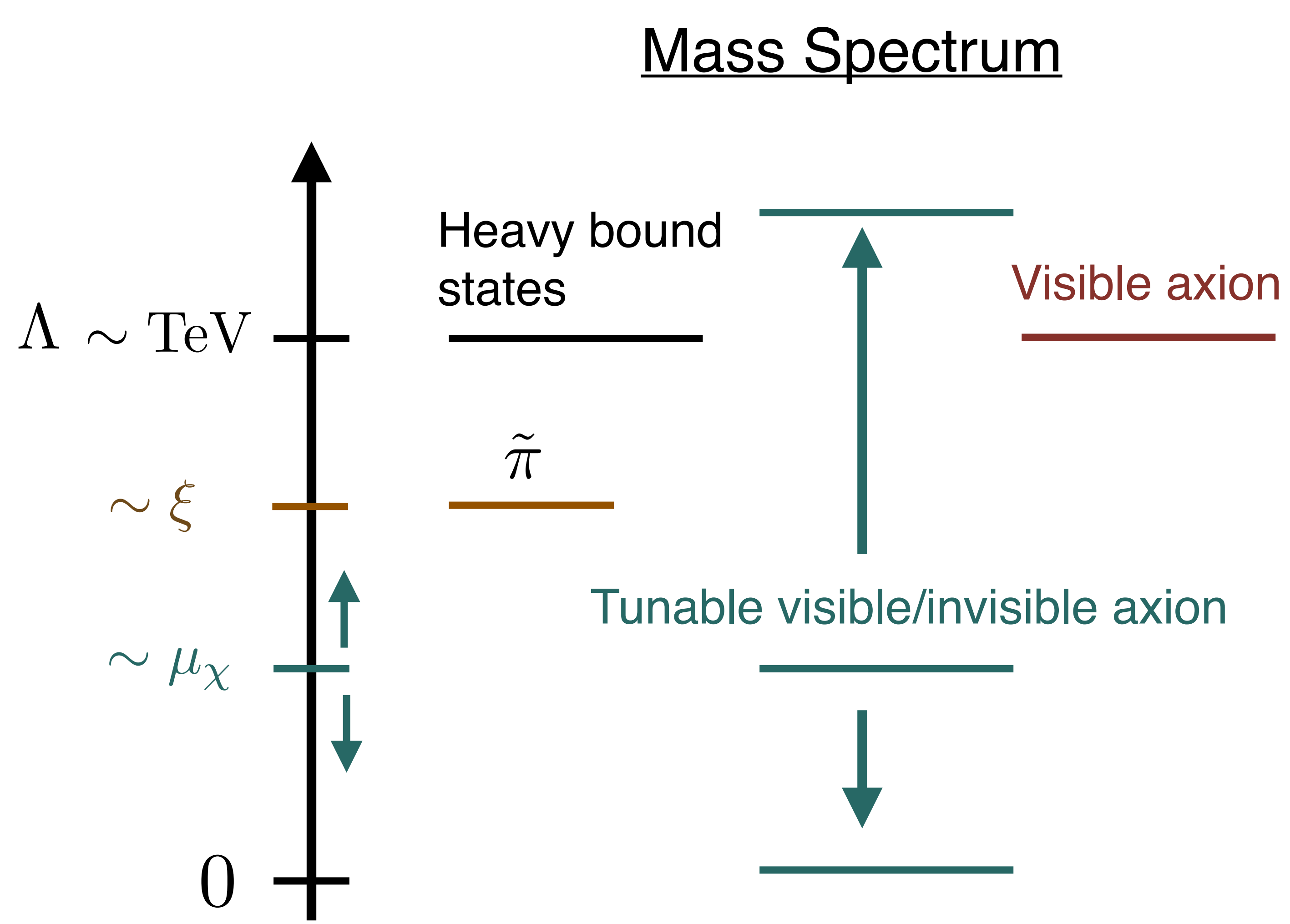
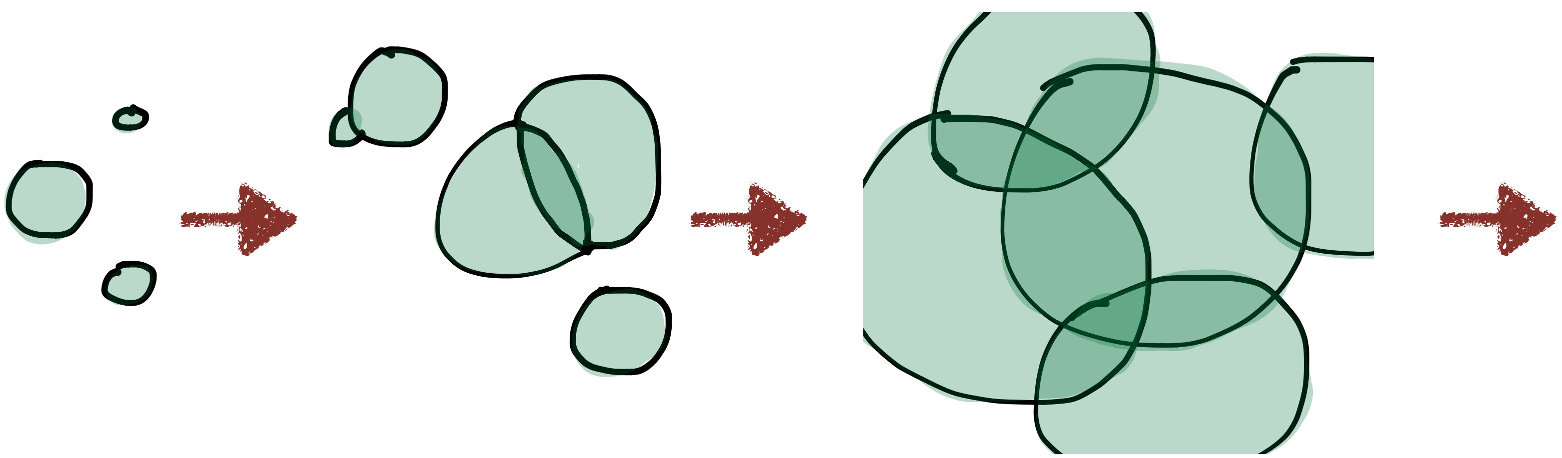
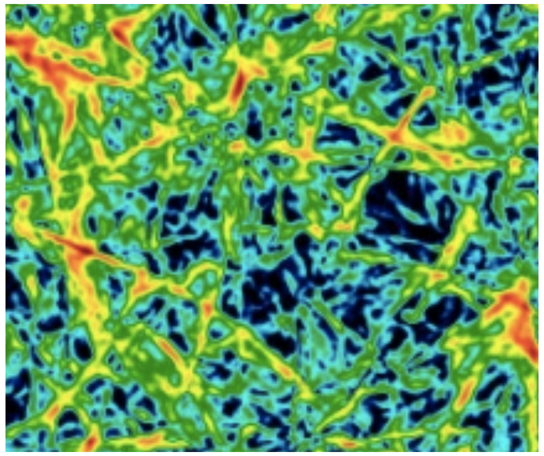
$$- \left(\mu_\Sigma \det\Sigma + h.c.\right) - \xi \;\mathrm{Tr}\; Q^a\Sigma\Sigma^\dagger Q^{a\,\dagger} - \mu_{SSI}\mathrm{Tr}\left(P_\chi \Sigma P_\chi \Sigma^\dagger\right)$$

- Include the explicit $U(1)_A$ symmetry breaking from instanton effects
- Include the explicit symmetry breaking from QCD charges
- Include the new mass contributions from small-sized instantons

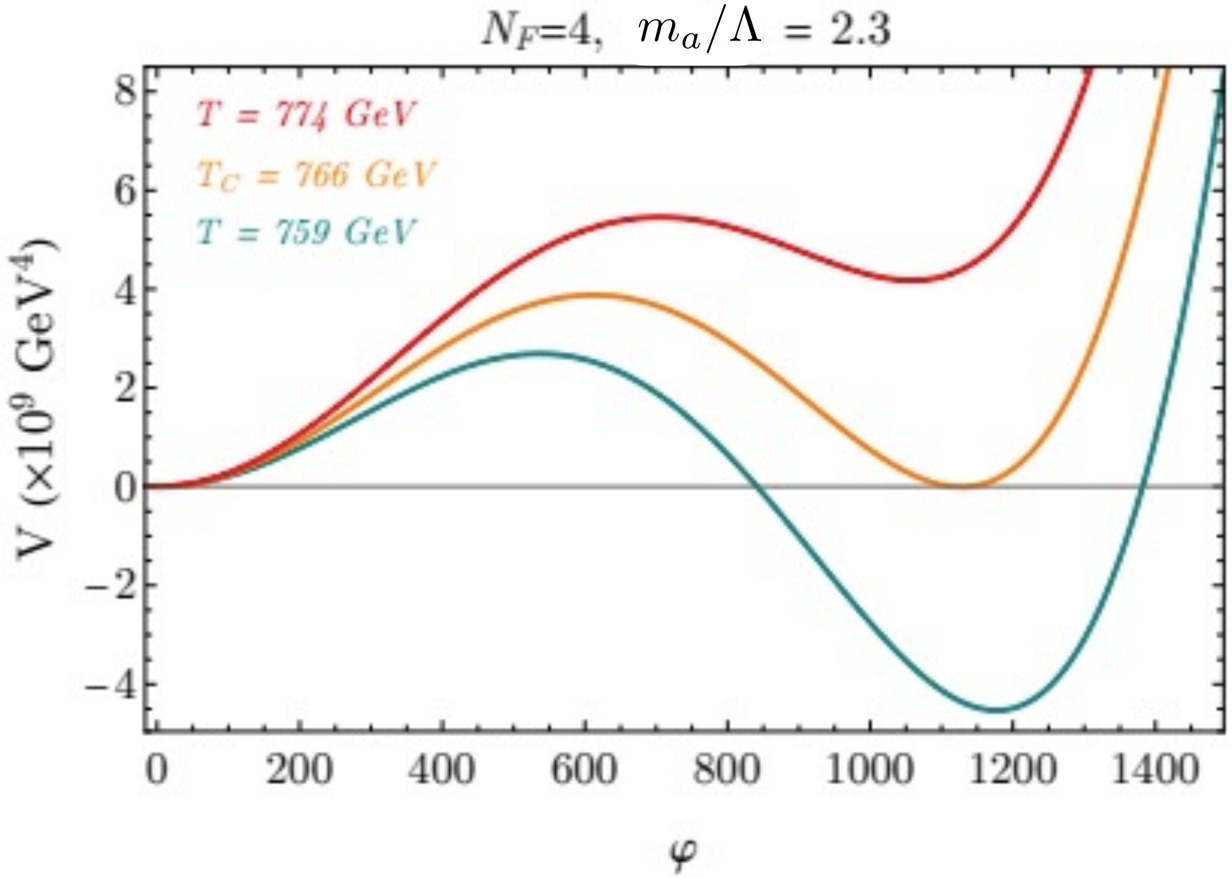


Phase Transition in the Early Universe





Weir, "The sound of gravitational waves from a [confinement] phase transition," saoghal.net/slides/ectstar/

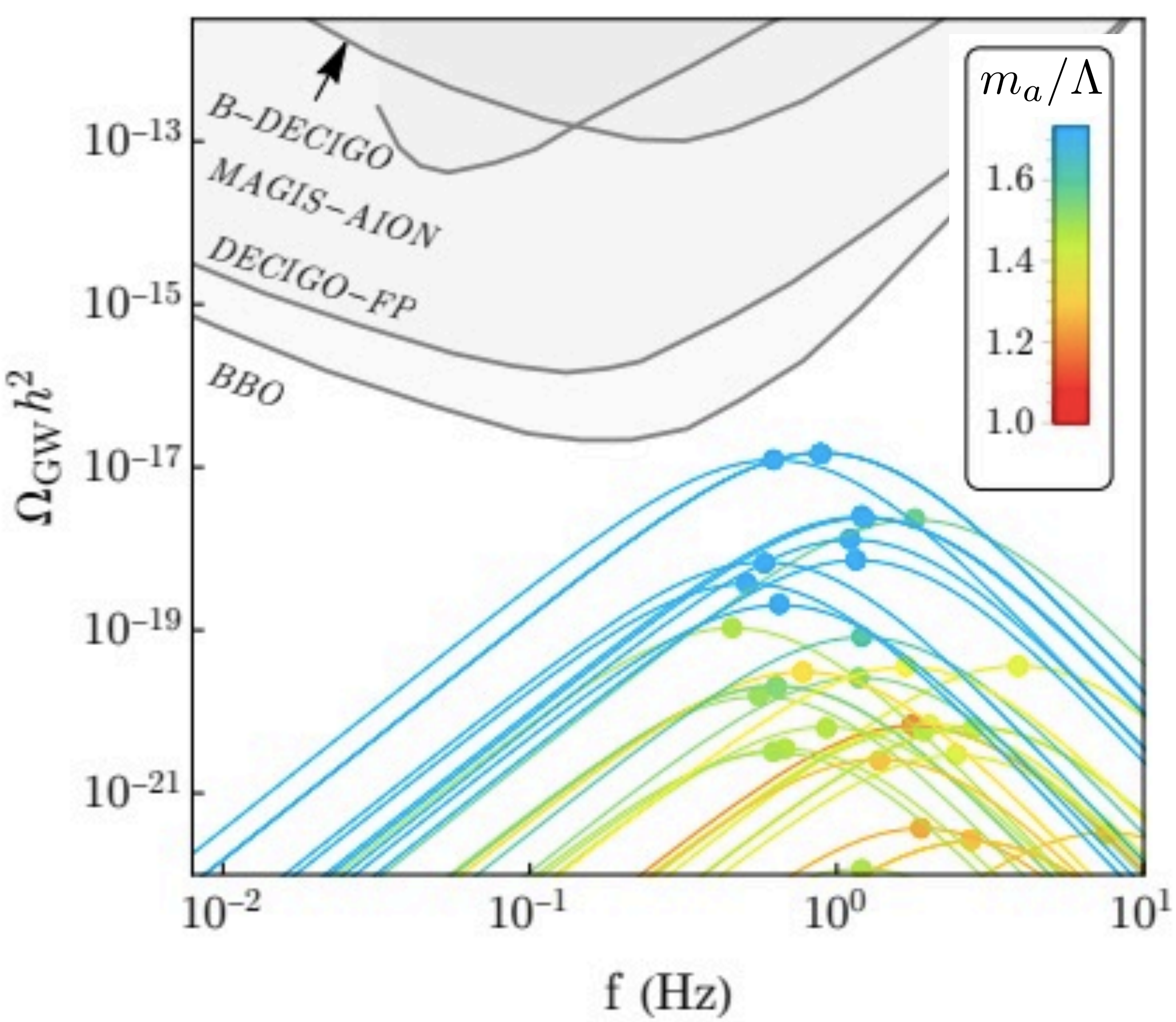


- Find the bounce solution to describe the tunneling from the false vacuum
- Calculate the frequency and power spectrum of the stochastic GW background

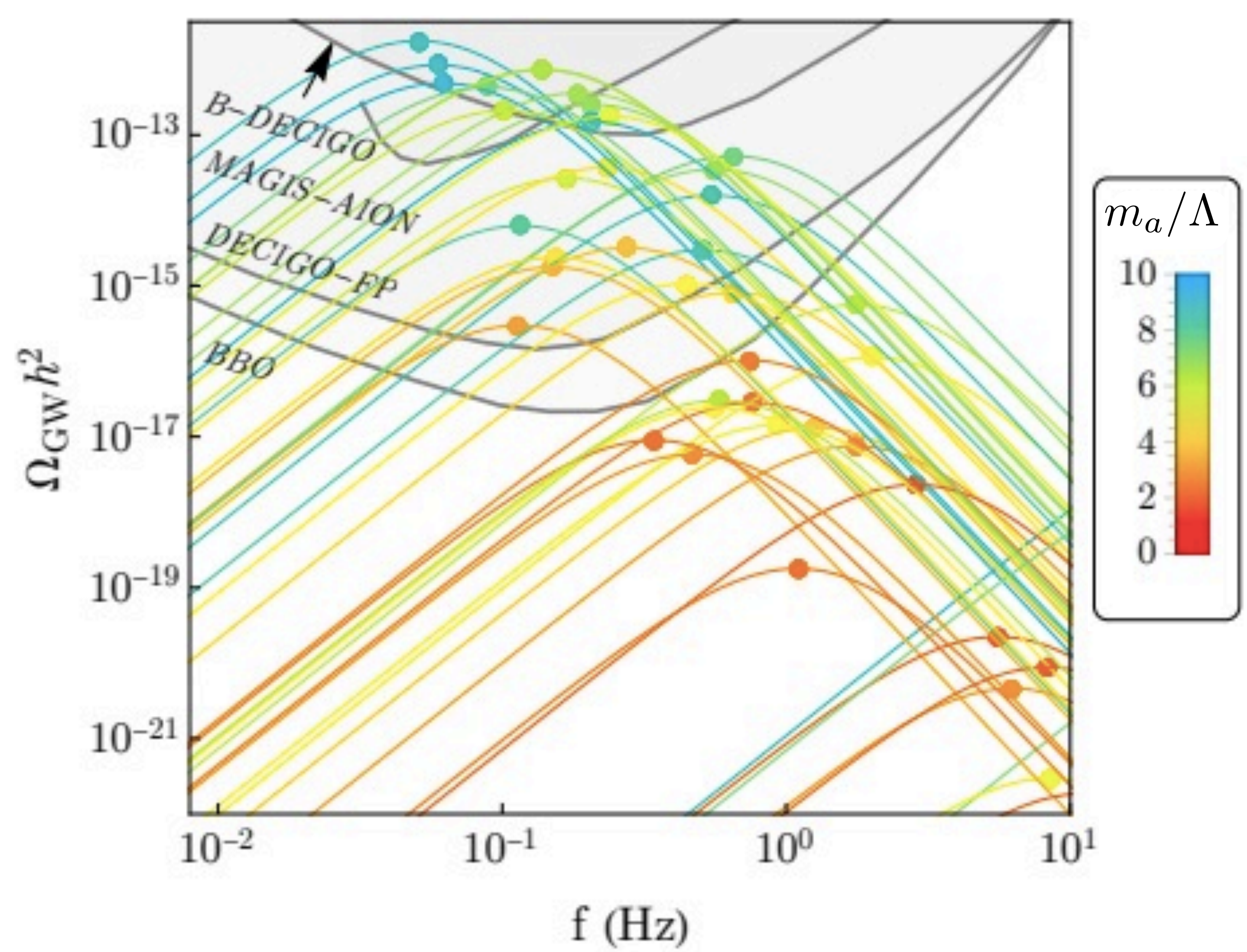
Gravitational Wave Signal

$\tilde{\Lambda} \sim 3$ TeV Croon, RH, Sanz, 1904.10967

$N_F = 3$

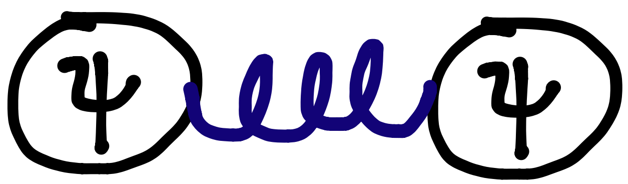


$N_F = 4$



Summary

- Prospects for gravitational wave signals for dynamical axion models with $\tilde{\Lambda} \sim 3\ \mathrm{TeV}$
- Features of dynamical axion models:

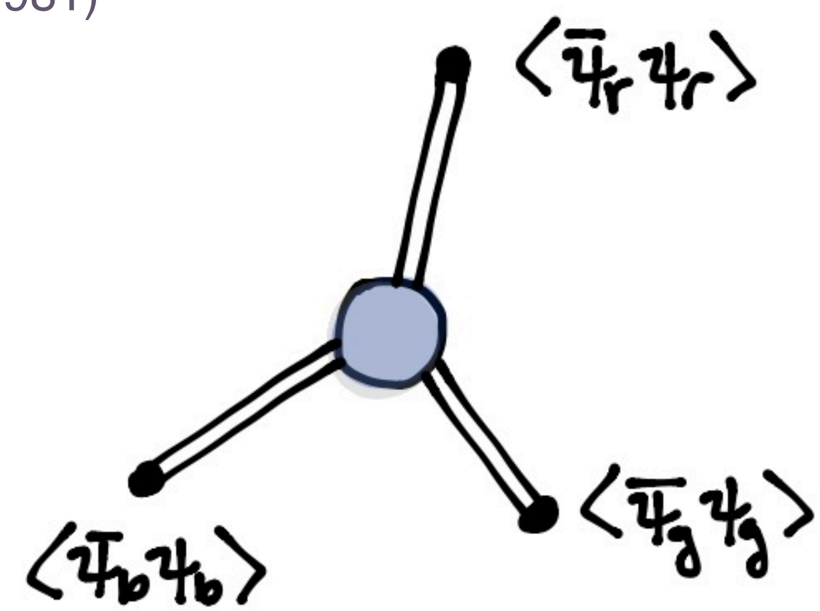


  - Massless quark messenger between QCD and $SU(\tilde{N})$
  - At least three light flavors and a first order phase transition
  - Generic parameters in the theory below $\tilde{\Lambda}$, for example the exotic pion masses due to gluon interactions
- The gravitational wave signature is sensitive to the anomalous effects that raise the axion mass in the linear sigma model

Temperature Dependence Axion Masses

- Axion masses are coming from instanton effects 't Hooft (1976)
- When $T \gg T_c$ and g_{QCD} is perturbative, the dilute instanton gas approximation holds Gross, Pisarski, Yaffe, (1981)
- In DGA, axion mass is lifted by small size instanton effects:

$$m_{a_\psi}^2 \sim \frac{\Lambda_{SSI}^4}{\tilde{f}^2} \sim \frac{1}{\tilde{f}^2} \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3}\pi^2 \rho^3 \langle \bar{\psi}\psi \rangle \right)^3$$

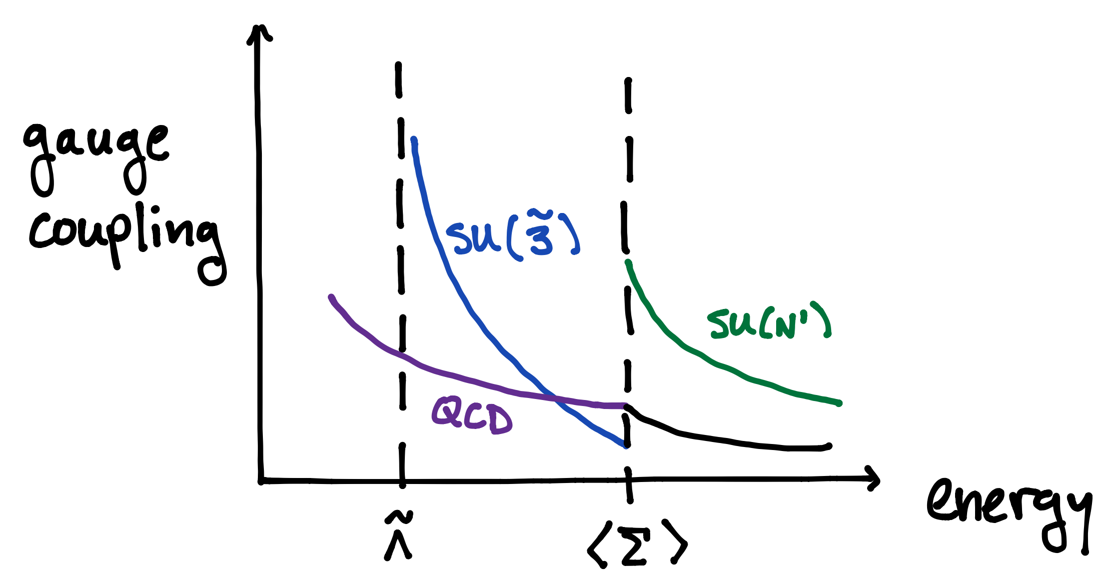


Shifman, Vainshtein, Zakharov (1980)
Callan, Dashen, Gross, (1978)

- Do μ_Σ and μ_{SSI} contribute significantly during the phase transition?

Conclusions

- Extra color groups allow for visible axions and dynamical axions
  - Visible axions $m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \longrightarrow + \sim \Lambda_{\rm new}^4$
- Gravitational waves can probe exotic confining color groups
  - Symmetries of UV model inform the low energy EFT
  - EFT's break down near the confining PT
- GW signal favors models where high energy effects of extra color groups provide another source of axion mass



Thank you!

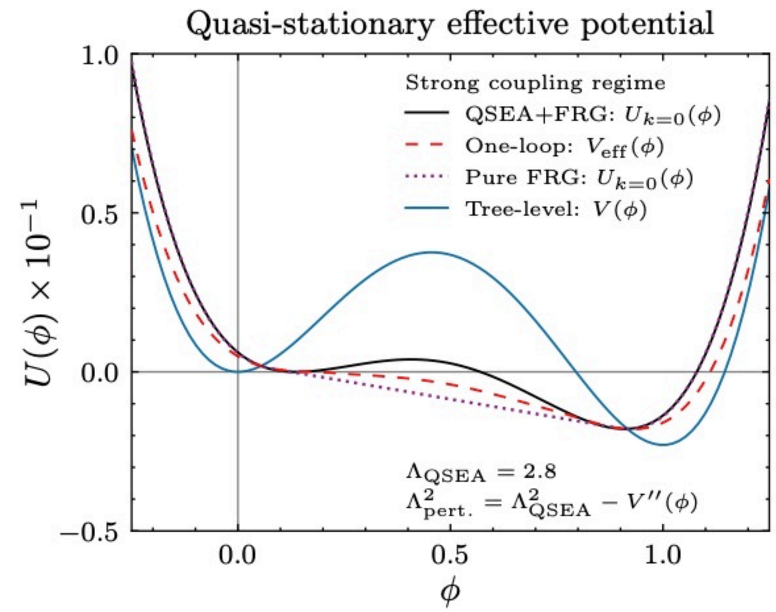
Back-up slides

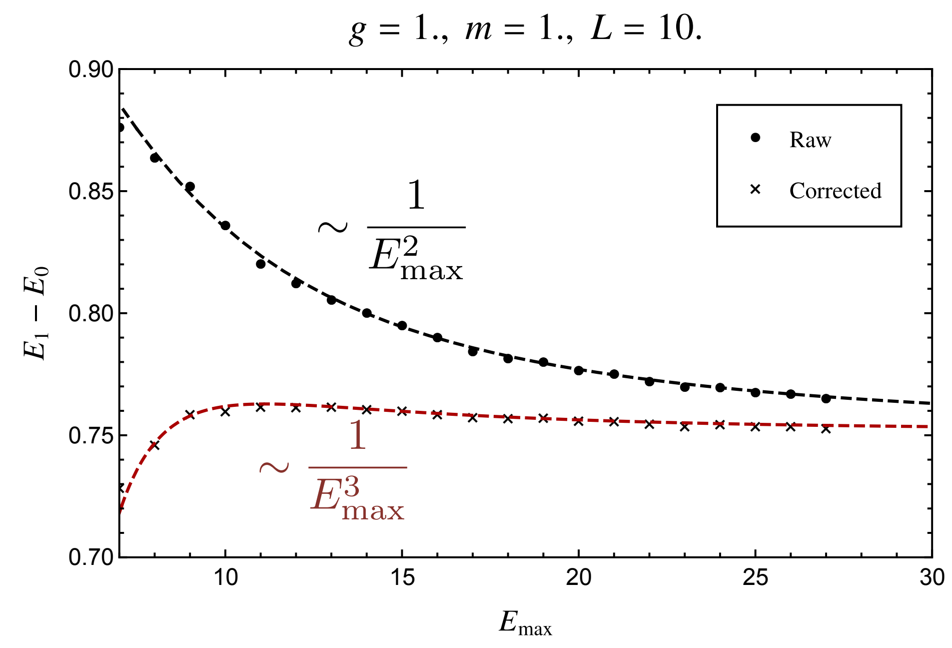
Strategies for Probing Confining Theories

- Nonperturbative methods like the quasi-stationary effective action
  - Using functional renormalization group methods to study axion couplings near QCD confinement
- Finite Energy Effective Field Theory
  - Consistent order-by-order scheme to improve Hamiltonian truncation to probe strongly coupled theories

Cohen, Farnsworth, RH, Luty, 21XX.XXXX

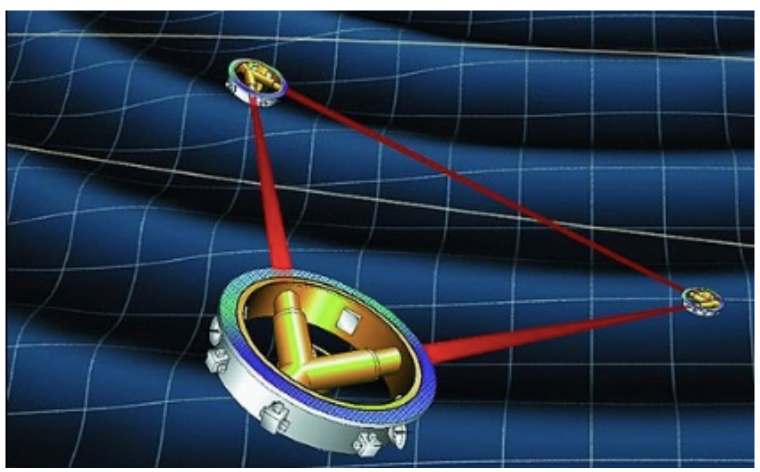
Plot lifted from Croon, Hall, Murayama, 2104.10687



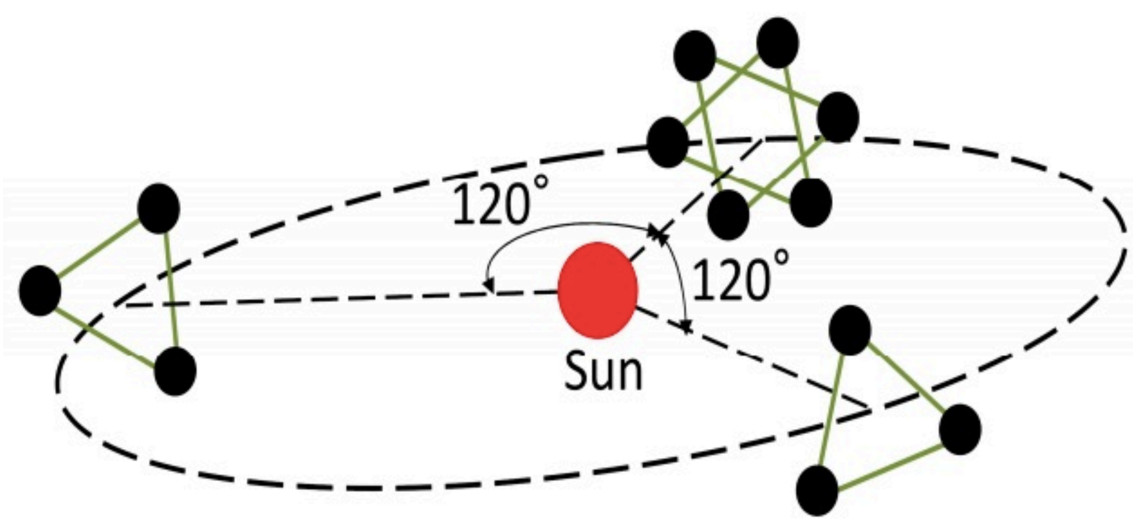


Future Gravitational Wave Detectors

- Laser interferometers LISA, B-DECIGO, DECIGO and BBO
- Atom interfermeters AION and MAGIS

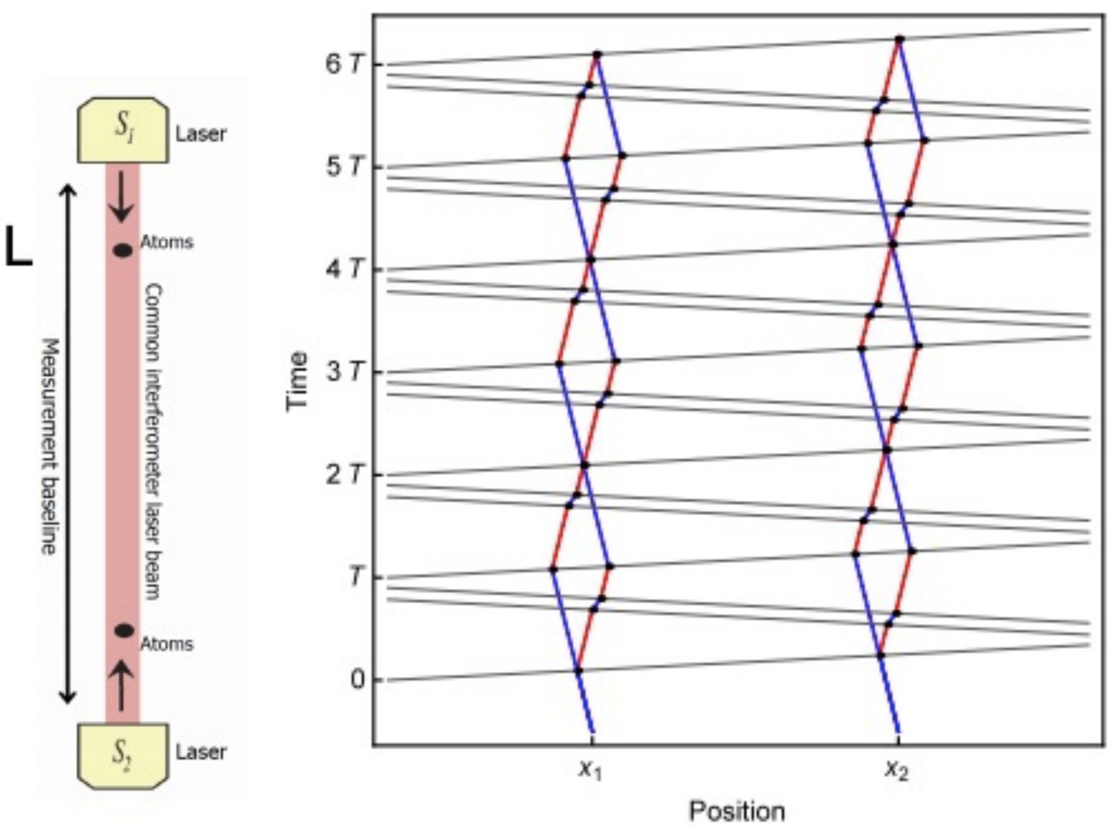


NASA illustration of LISA



Yagi (2013)

Graham, Hogan, Kasevich, Rajendran (2016)

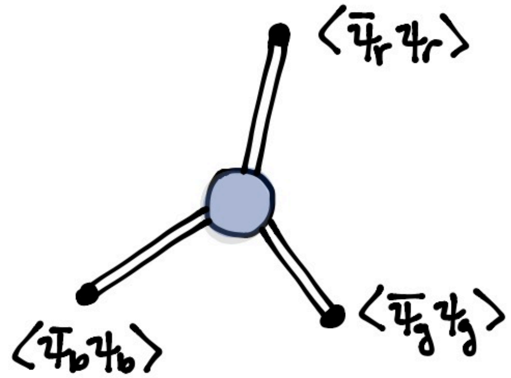


Buchmueller, “A UK AION for the exploration of ultra-light dark matter and mid-frequency gravitational waves” (2018)

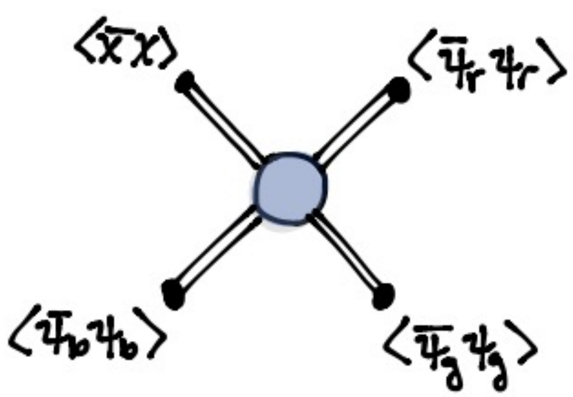
Small Size Instantons with Fermions

- Adding fermion effects gives an instanton suppression

$$d(\rho, T) = C_{inst} \left(\frac{2\pi}{\alpha(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha(1/\rho)}}$$



$$\Lambda_{SSI}^4 = \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3} \pi^2 \rho^3 \langle \bar\psi \psi \rangle \right)^3$$



$$\Lambda_{SSI}^4 = \int \frac{d\rho}{\rho^5} d(\rho, T) \left(\frac{2}{3} \pi^2 \rho^3 \langle \bar\psi \psi \rangle \right)^4$$

$$\mathcal{L}_{eff} = \Lambda_{SSI}^4 \cos\left(2 \frac{\eta'_\chi}{f_d} \right) + \Lambda_{\mathrm{diag}}^4 \cos\left(2 \frac{\eta'_\chi}{f_d} + \sqrt{6} \frac{\eta'_\psi}{f_d} \right) + \Lambda_{\mathrm{QCD}}^4 \cos\left(\sqrt{6} \frac{\eta'_\psi}{f_d} \right)$$

Plausible Range of Dynamical Axion Masses