The QCD chiral phase transition for different numbers of quark flavours

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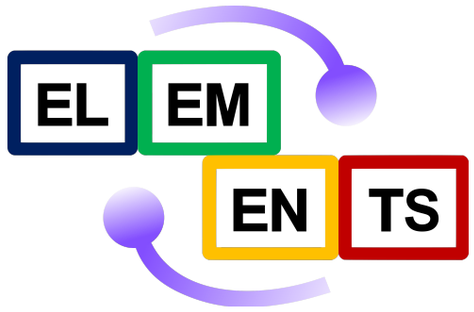
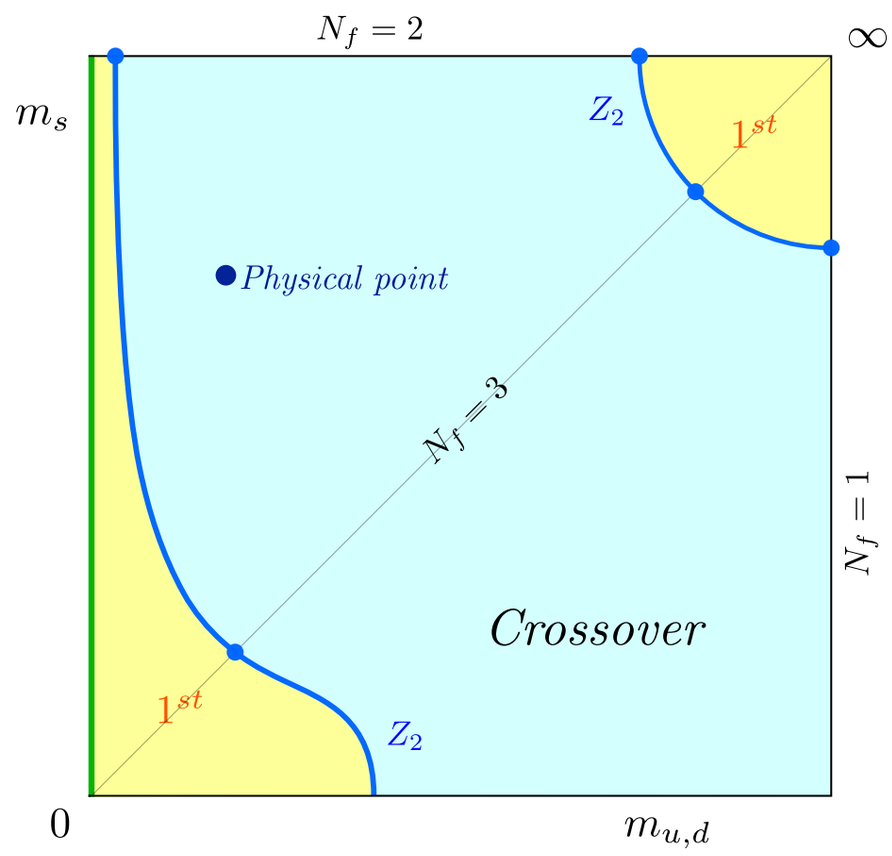
with Francesca Cuteri and Alessandro Sciarra

- Chiral phase transition in massless limit constrains the QCD phase diagram
- 40 years of common wisdom and inconclusive lattice results
- Resolution by a new approach + data [Cuteri, O.P., Sciarra JHEP 21]



The nature of the QCD thermal transition

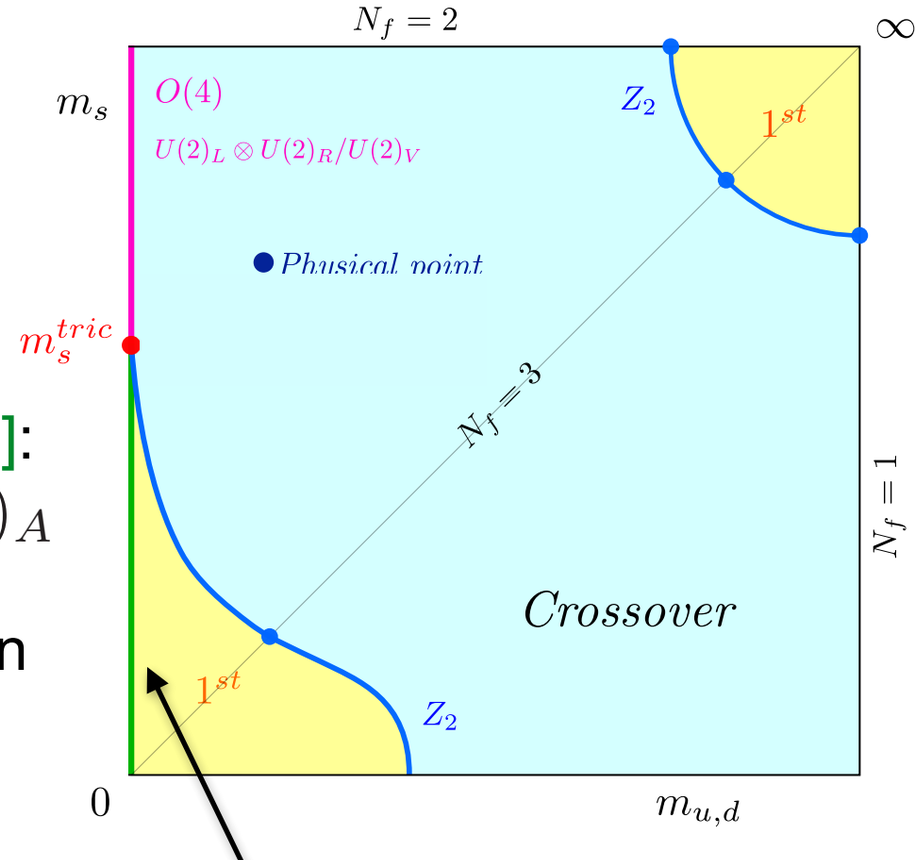
deconfinement p.t.:
breaking of global $Z(3)$ symmetry



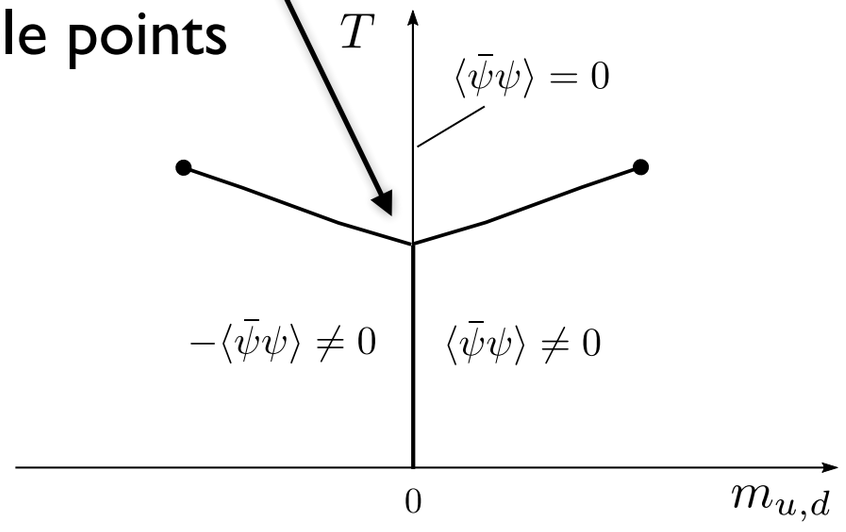
[Pisarski, Wilczek, PRD 84]:
$N_f = 2$ depends on $U(1)_A$

restored broken

$N_f \geq 3$ 1st order



triple points



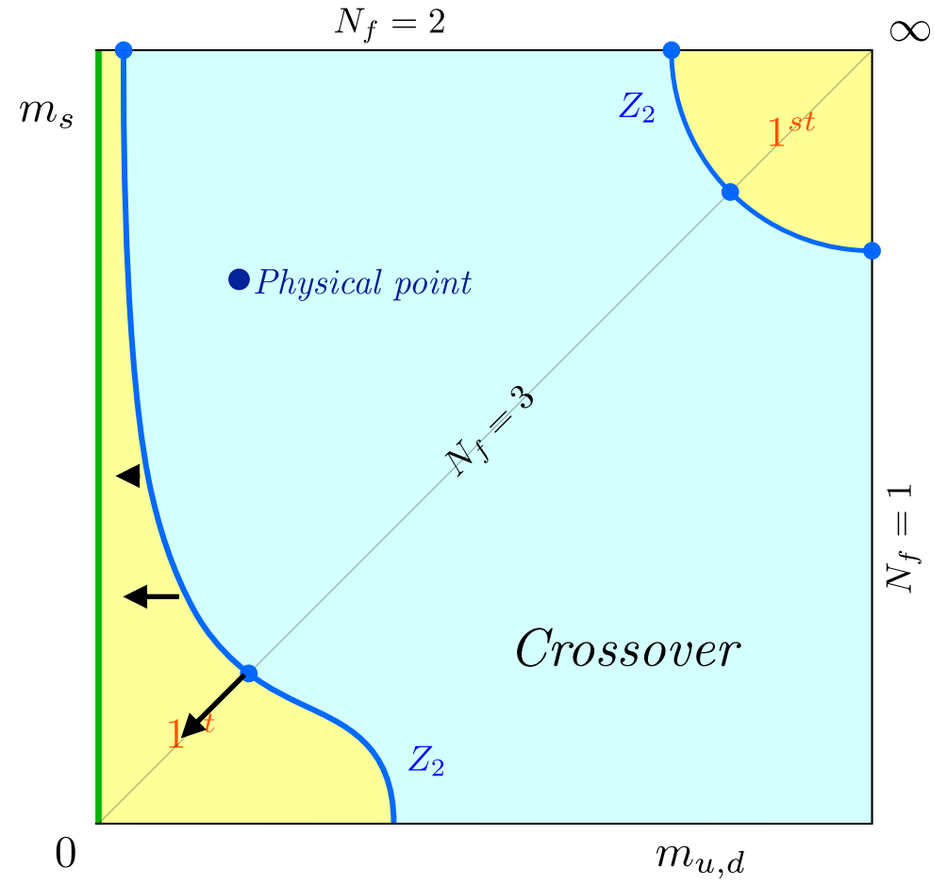
chiral p.t.
restoration of global symmetry in flavour space

$SU(2)_L \times SU(2)_R \times U(1)_A$

anomalous

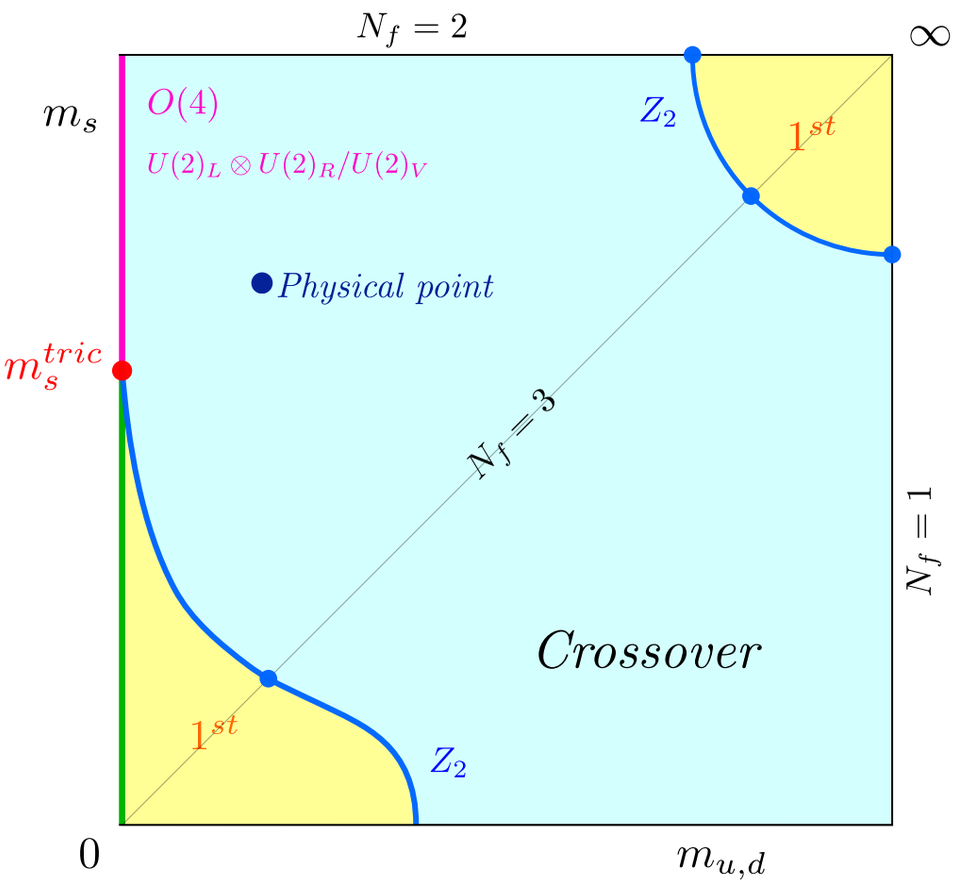
The nature of the QCD chiral transition

…is elusive, massless limit not simulable!



?

$a \to 0$

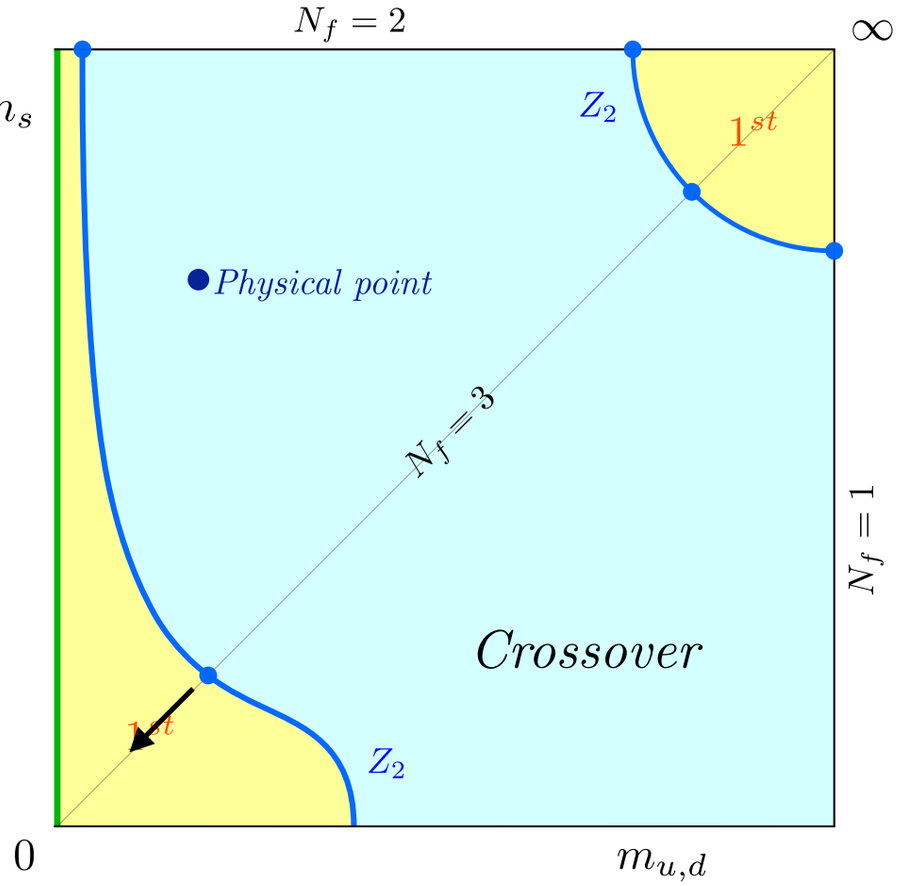


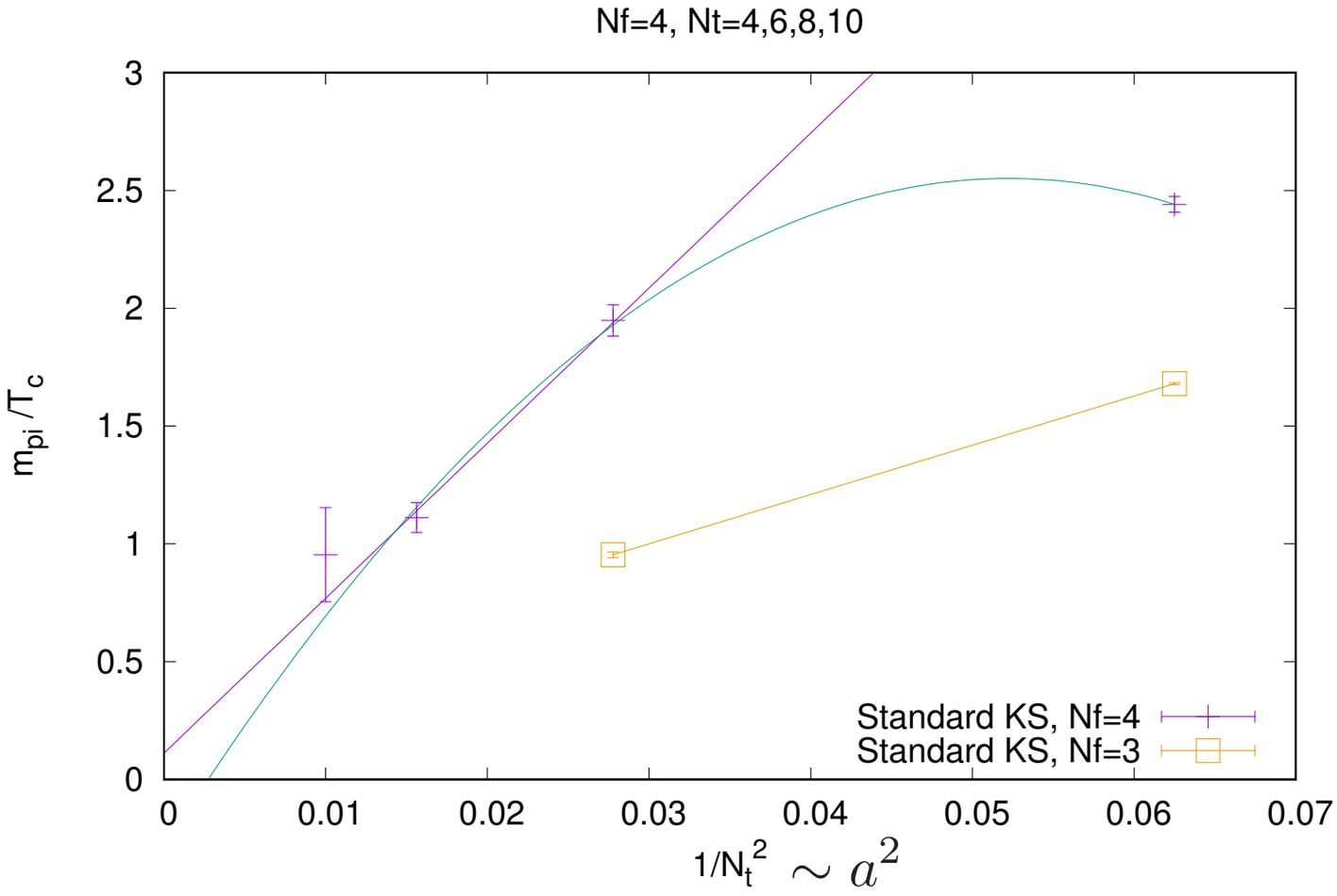
- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$
- Improved staggered actions: no 1st order region so far, even for $N_f = 3$ $m_{PS} > 45\text{MeV}$

Details and references: [O.P., Symmetry 13, 2021]

The nature of the QCD chiral transition, Nf=3,4

…has enormously large cut-off effects!



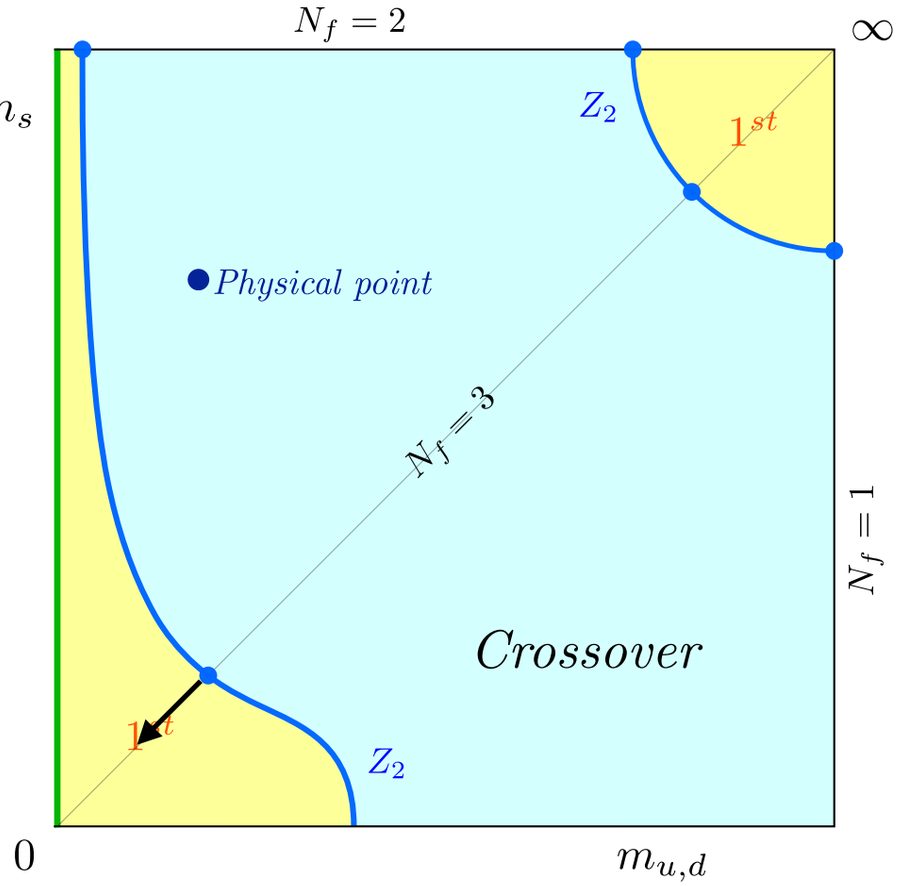


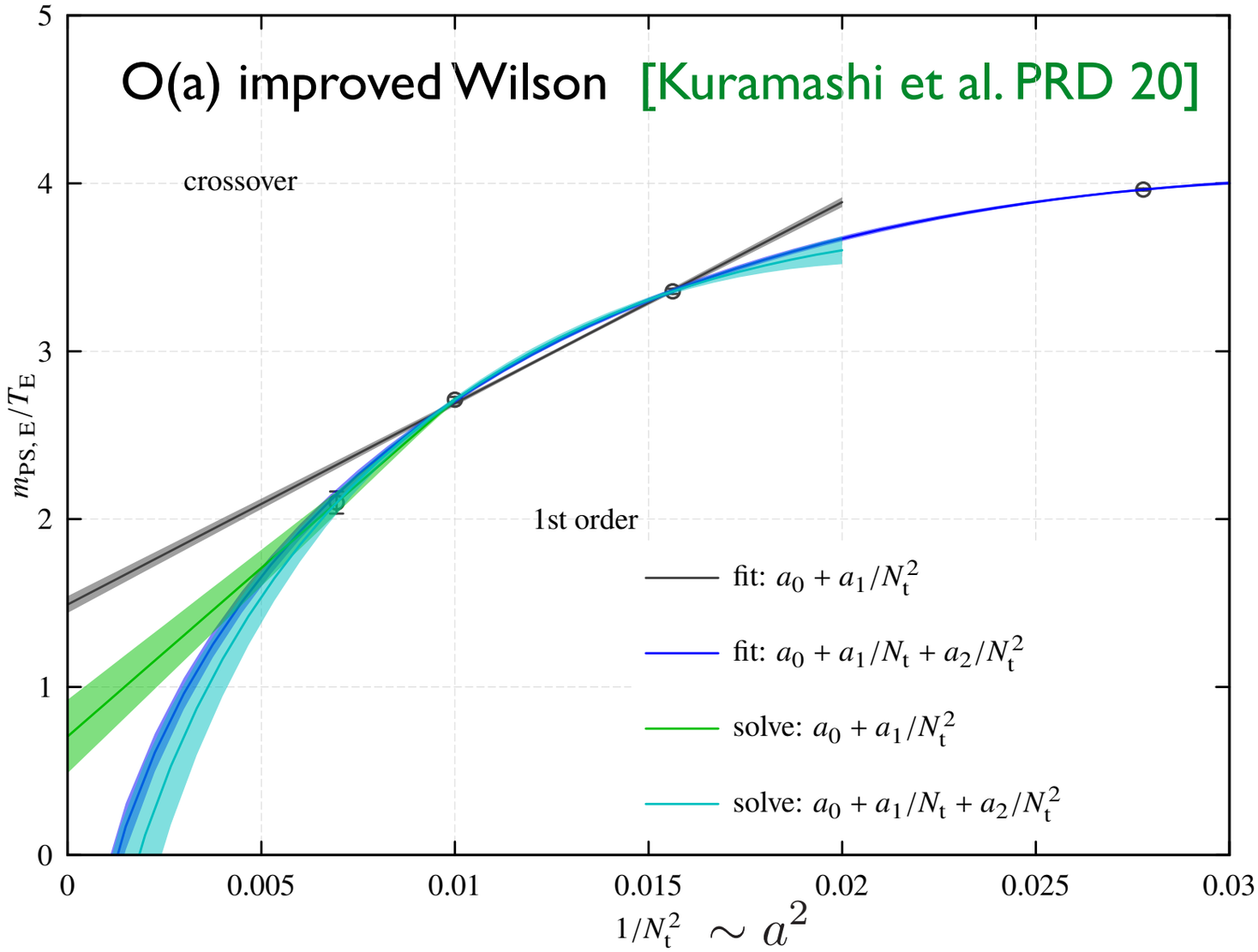
Unimproved staggered:
1st order region shrinks for $a \to 0$, both for $N_f = 3, 4$ [de Forcrand, D'Elia, PoS LAT 17]

No 1st order seen for improved staggered actions, even $N_f = 3$ [HotQCD PRD 17, PRD 22]

The nature of the QCD chiral transition, Nf=3

…has enormously large cut-off effects!

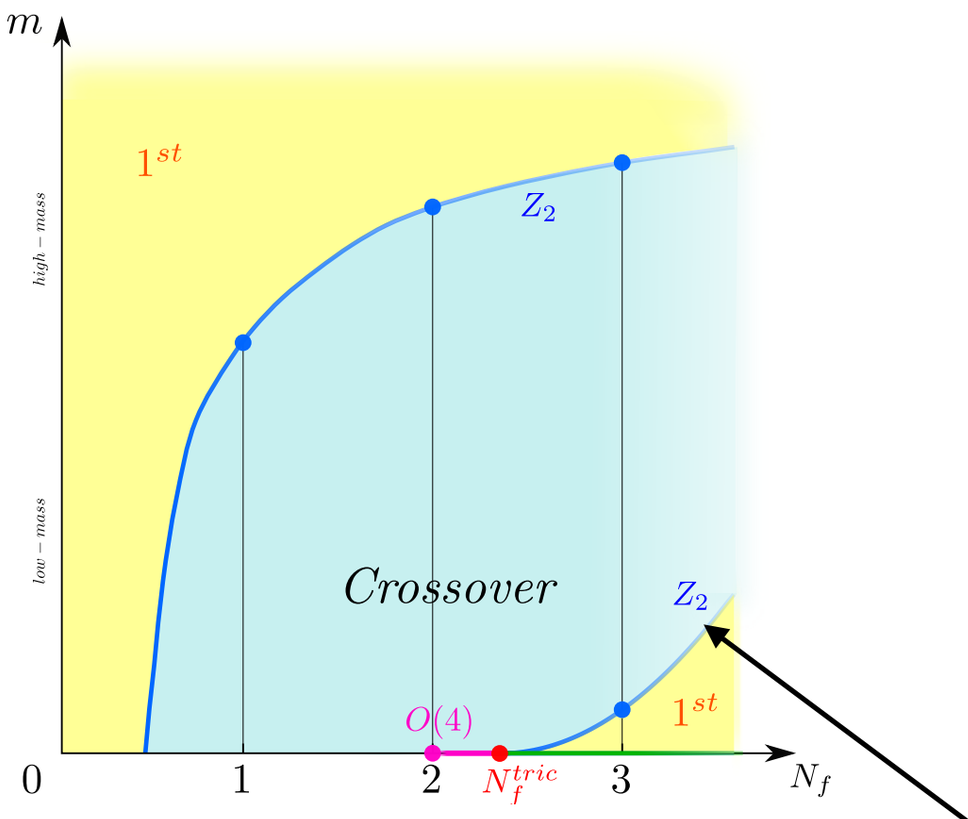


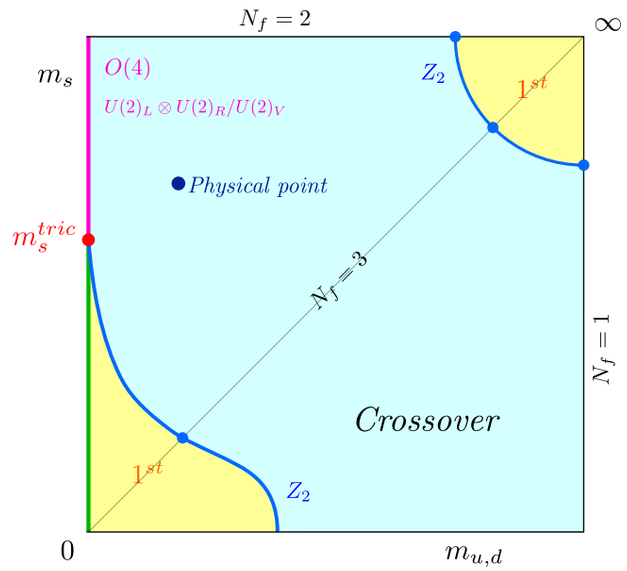


O(a)-improved Wilson:
1st order region shrinks for $a \to 0$

$m_\pi^c \leq 110$ MeV $\quad N_\tau = 4, 6, 8, 10, 12$

Different view point: mass degenerate quarks





$$Z(N_{\rm f}, g, m) = \int \mathcal{D}A_\mu \, (\det M[A_\mu, m])^{N_{\rm f}} \, e^{-\mathcal{S}_{\rm YM}[A_\mu]}$$

$$N_{\rm f}^c(am) = N_{\rm f}^{\rm tric} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}\big((am)^{4/5}\big)$$

- Consider analytic continuation to continuous N_f
- Tricritical point guaranteed to exist if there is 1st order at any N_f
- Known exponents for critical line entering tric. point!
- Continuation to $a \neq 0$: Z(2) surface ends in tricritical line

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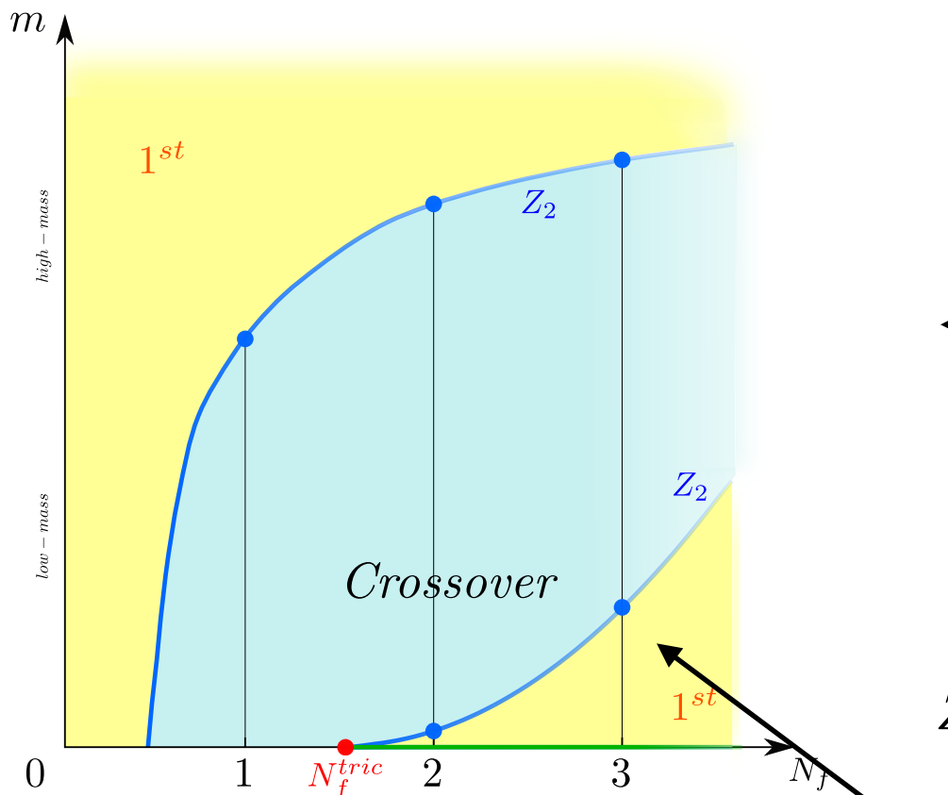
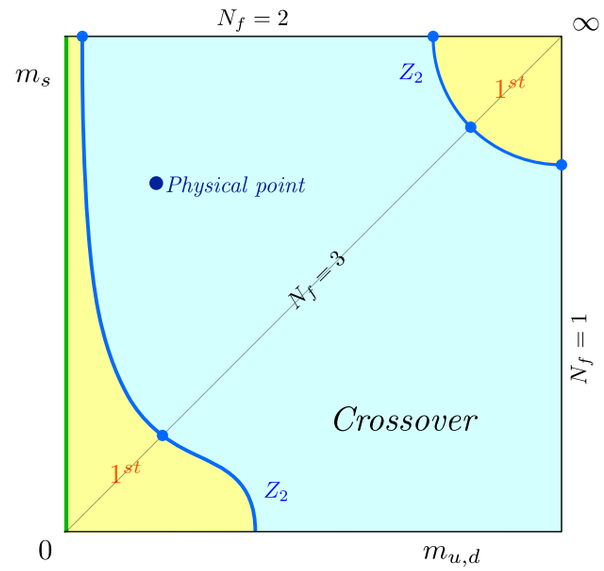
[Cuteri, O.P., Sciarra PRD 18]

Different view point: mass degenerate quarks





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[Cuteri, O.P., Sciarra PRD 18]

Methodology to determine order of transition

Finite size scaling of generalised cumulants

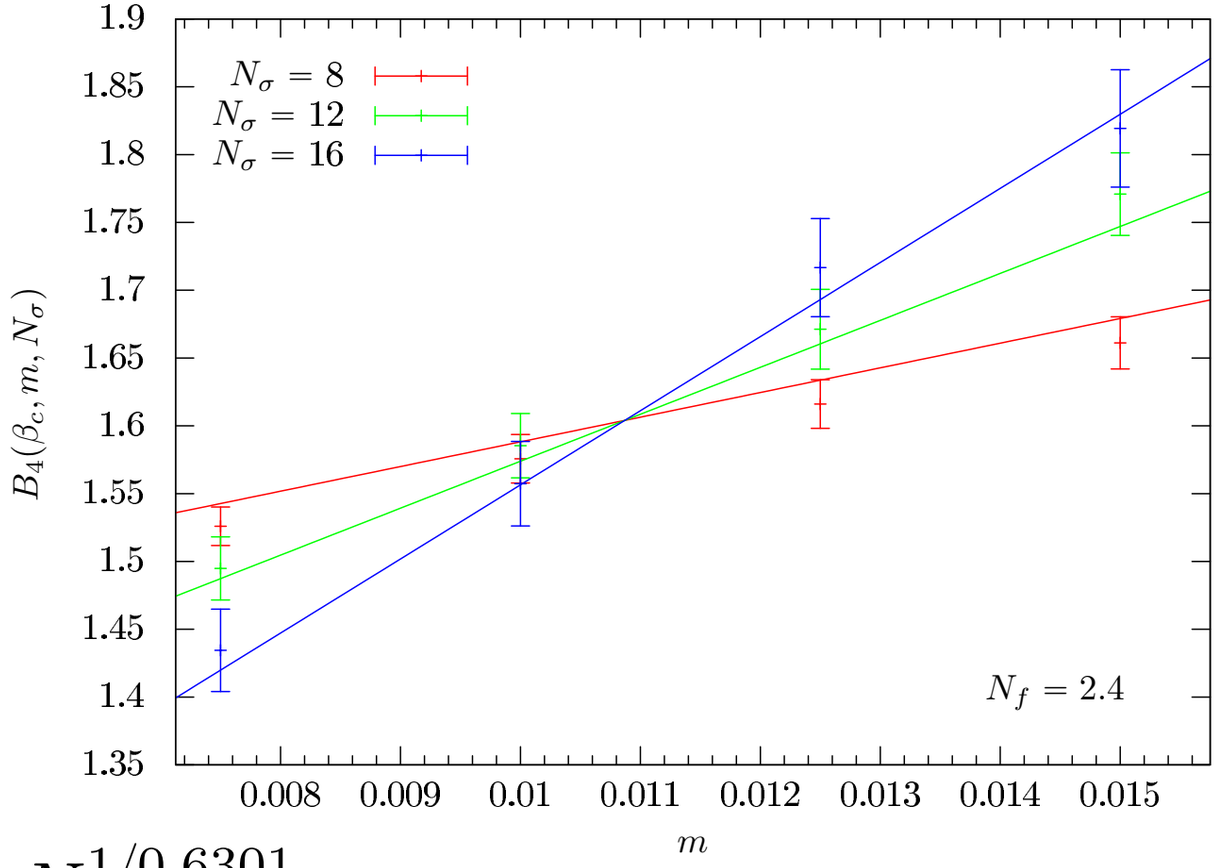
$$B_n = \frac{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^n \rangle}{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^2 \rangle^{n/2}}$$

Standard staggered fermions, bare parameters: β, am, N_f, N_τ

(Pseudo-critical) phase boundary: $B_3 = 0$ 3d manifold

Second-order 3D Ising:

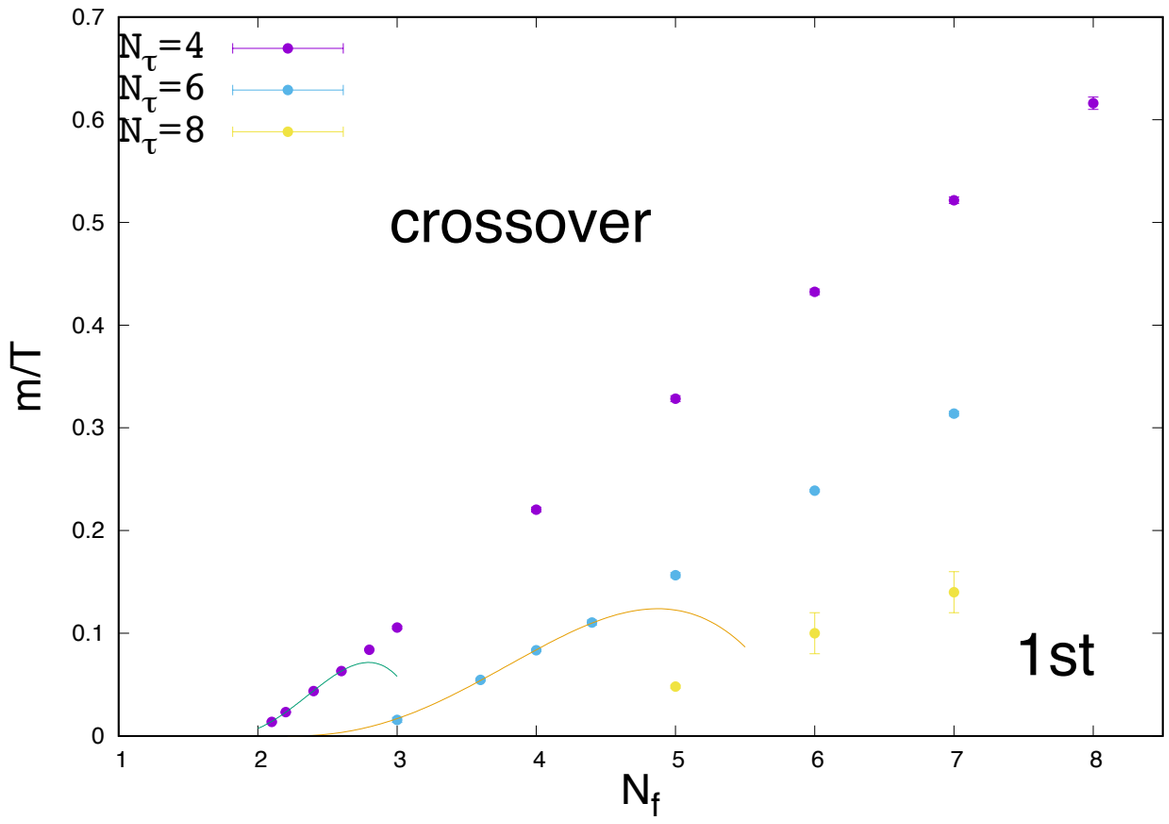
2d chiral critical surface separates 1st order from crossover

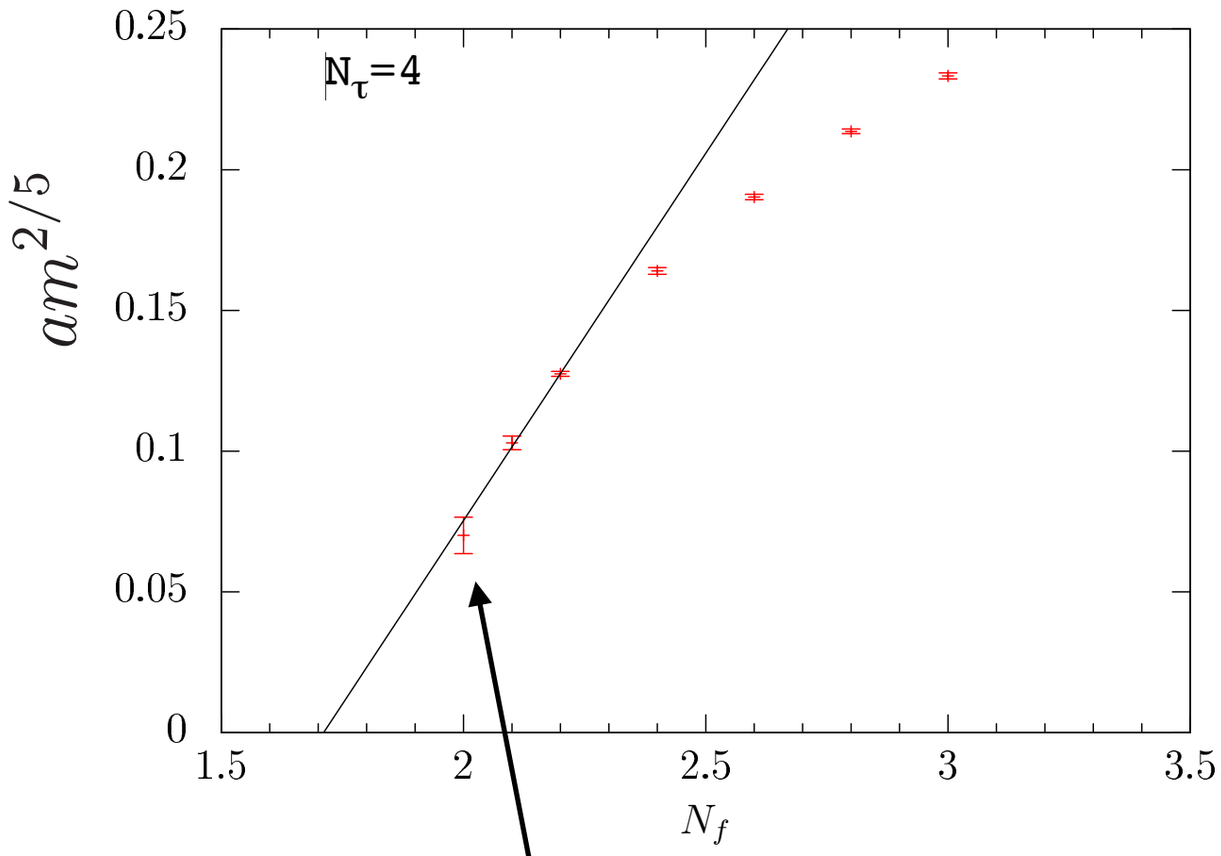


$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c\,(am - am_c)\, N_\sigma^{1/0.6301}$$

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21] ~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5



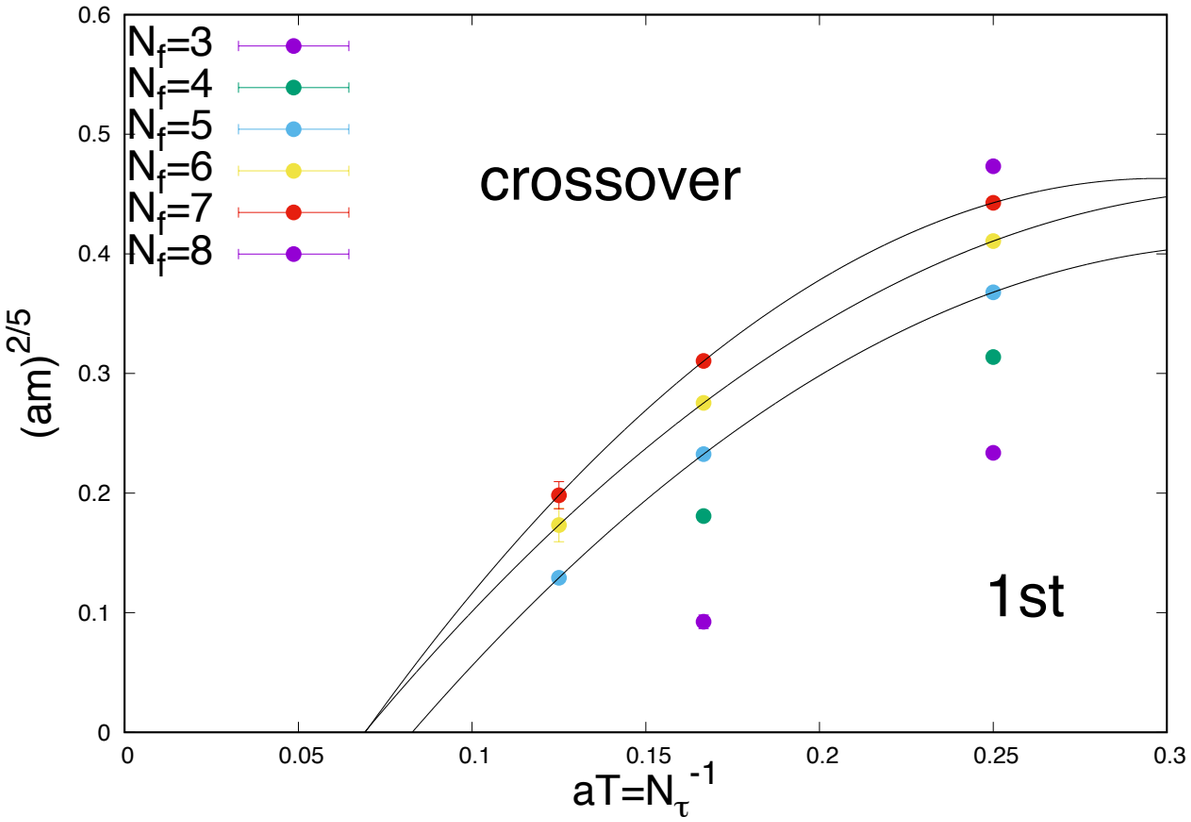


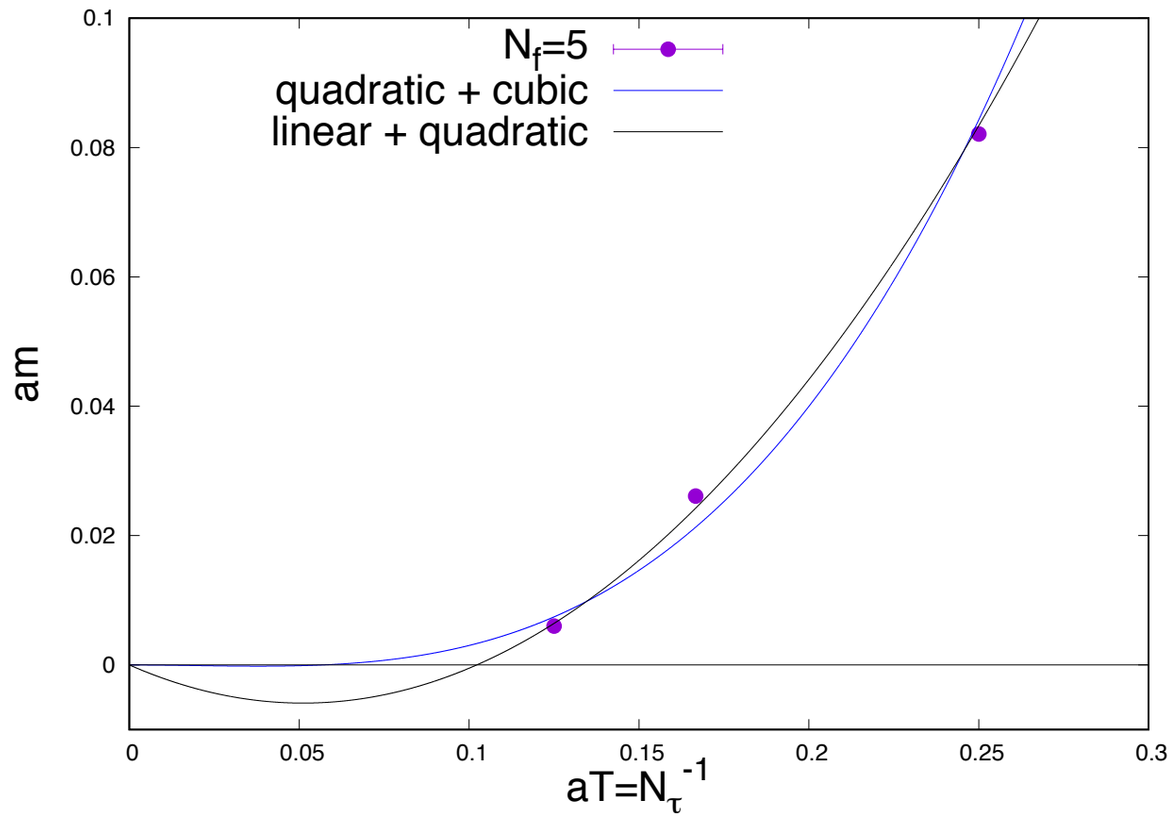
- Tricritical scaling observed also in different variable pairings
- Extrapolated by tric. scaling from finite imaginary μ [Bonati et al. PRD 14]
- Old question: $m_c/T = 0$ or $\neq 0$? Answered for $N_f = 2$
- New question: will $N_f^{\rm tric}$ slide beyond $N_f = 3$?

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]

1st order scenario does not fit!





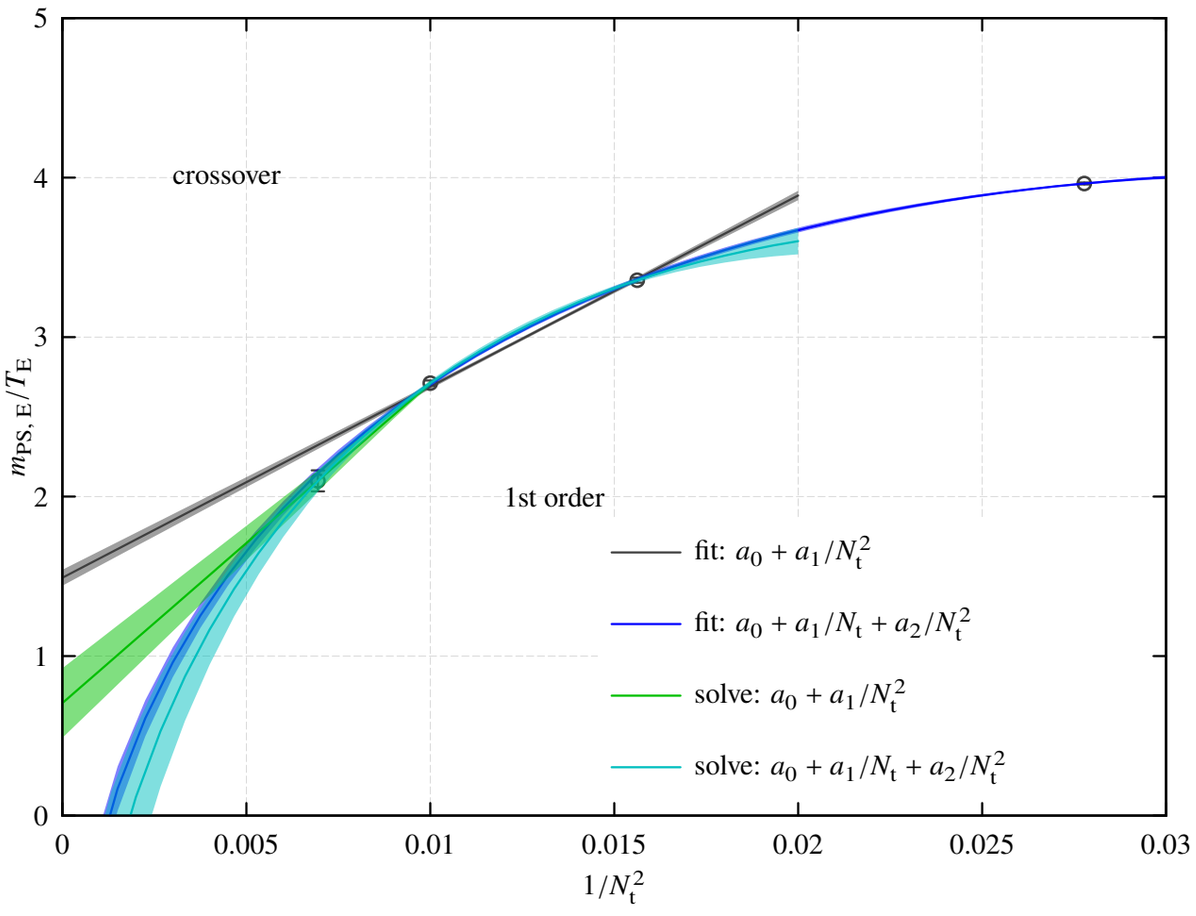
- Tricritical scaling observed also in plane of mass vs. lattice spacing
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\mathrm{tric}}(N_f)$
- 1st order scenario: $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \ldots$

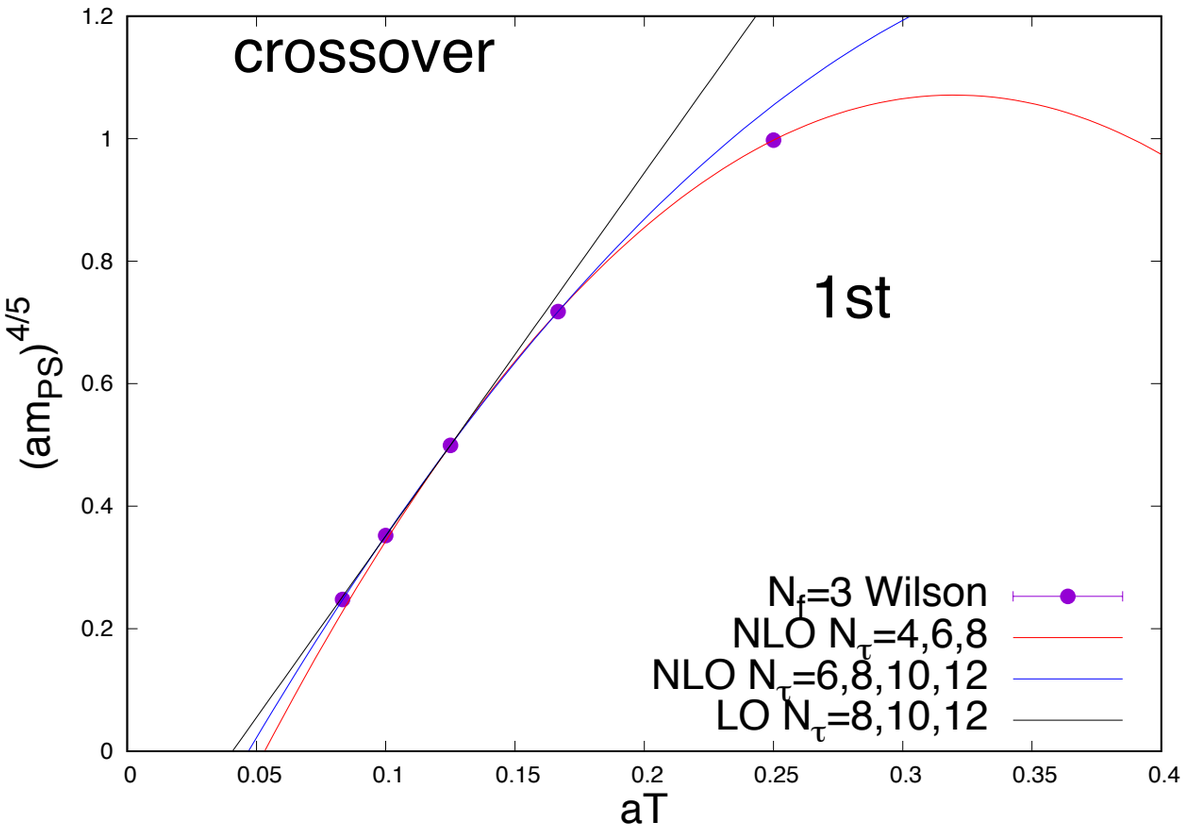
Incompatible with data! $\chi^2_{\mathrm{dof}} > 10$

Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110\ \mathrm{MeV} \quad N_\tau = 4, 6, 8, 10, 12$

Re-analysis using: $am_{PS}^2 \propto am_q$

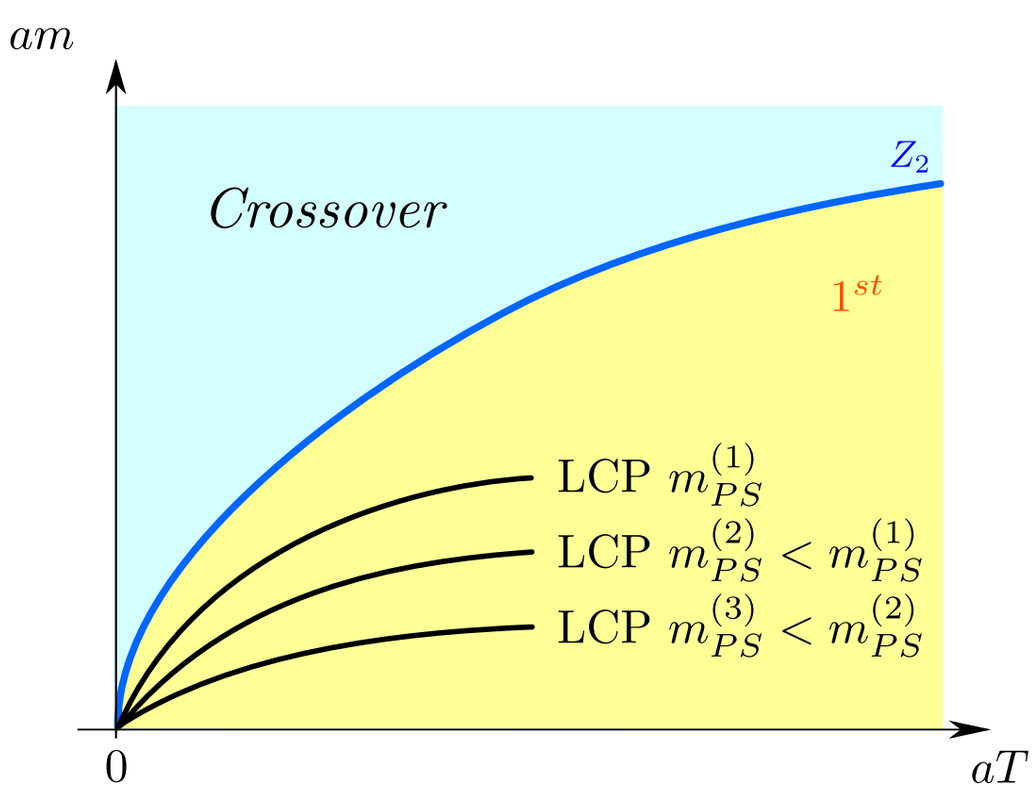




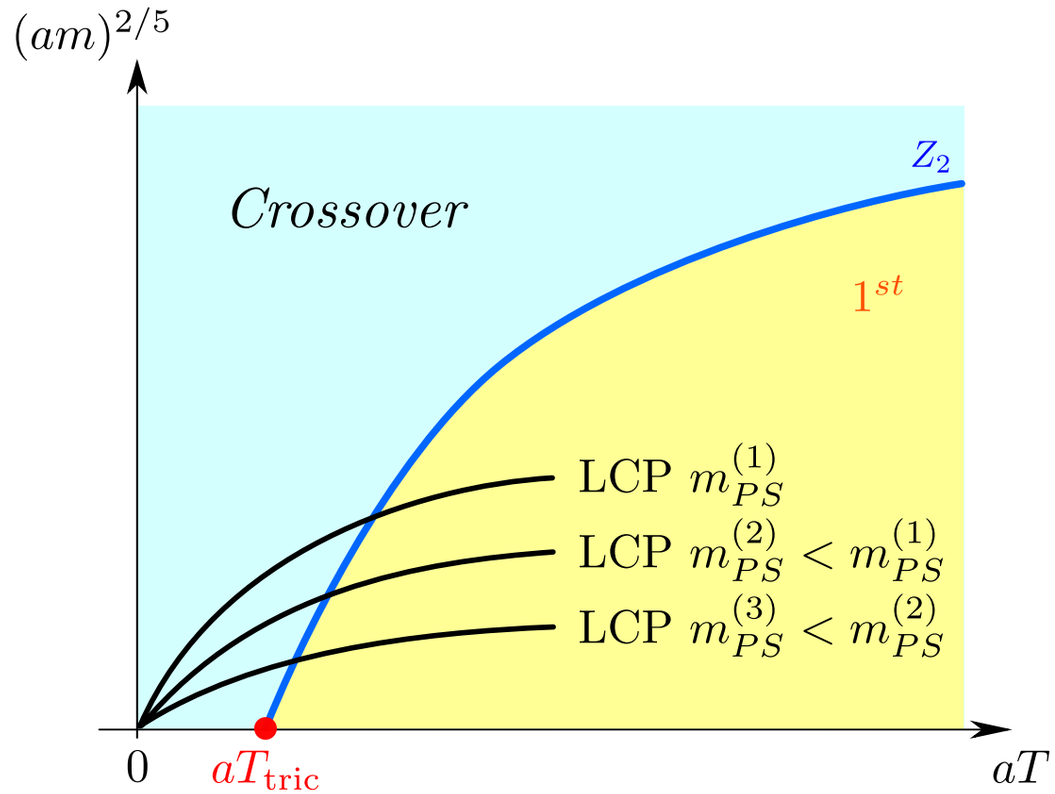
Tricritical scaling, Nf=3 consistent with unimproved staggered!

Implications for the continuum

- Finite $N_\tau^{\rm tric}(N_f)$ implies that 1st order transition is not connected to continuum
- Approaching continuum first, then chiral limit:
  Continuum chiral phase transition second-order!

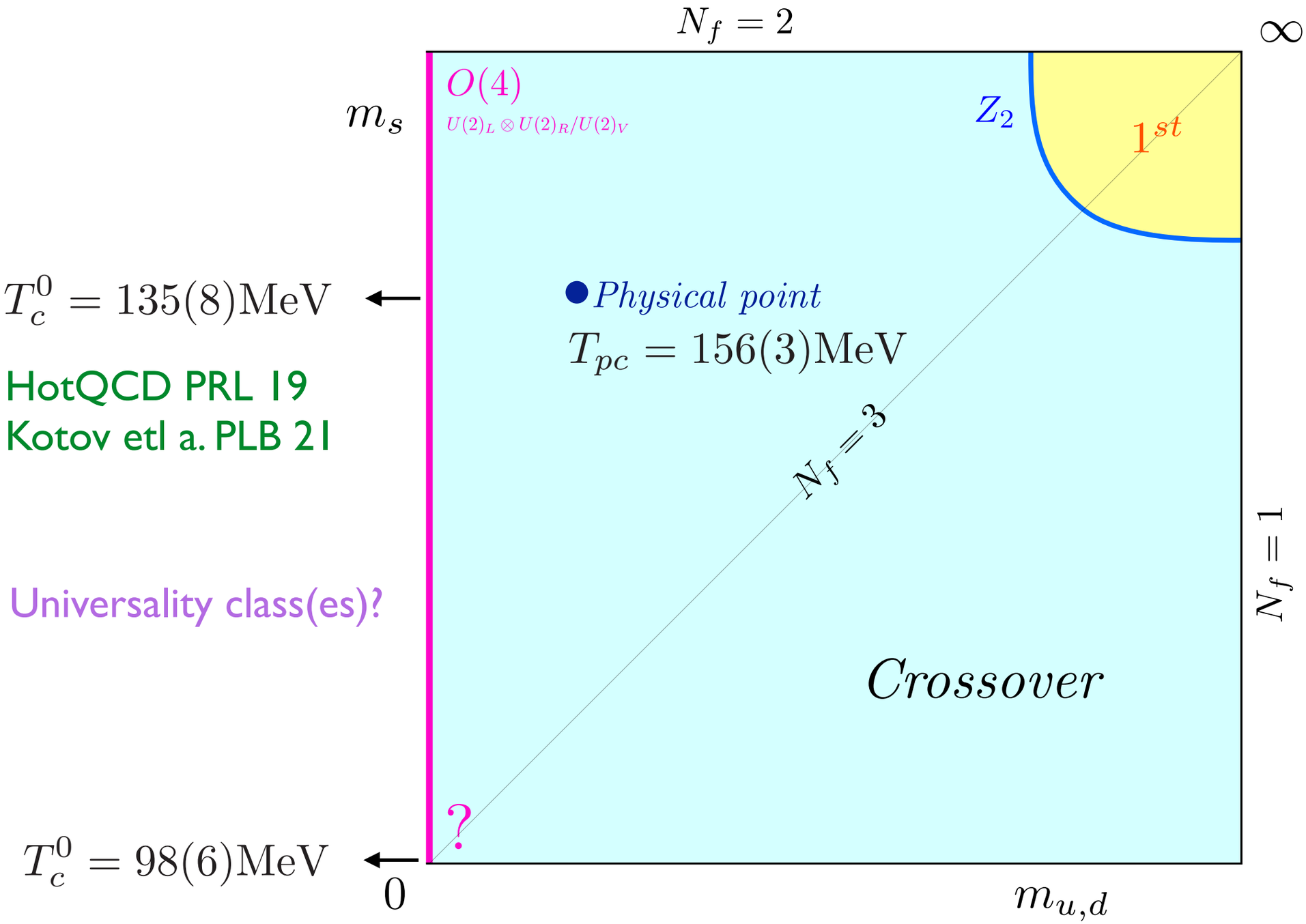


1st order scenario

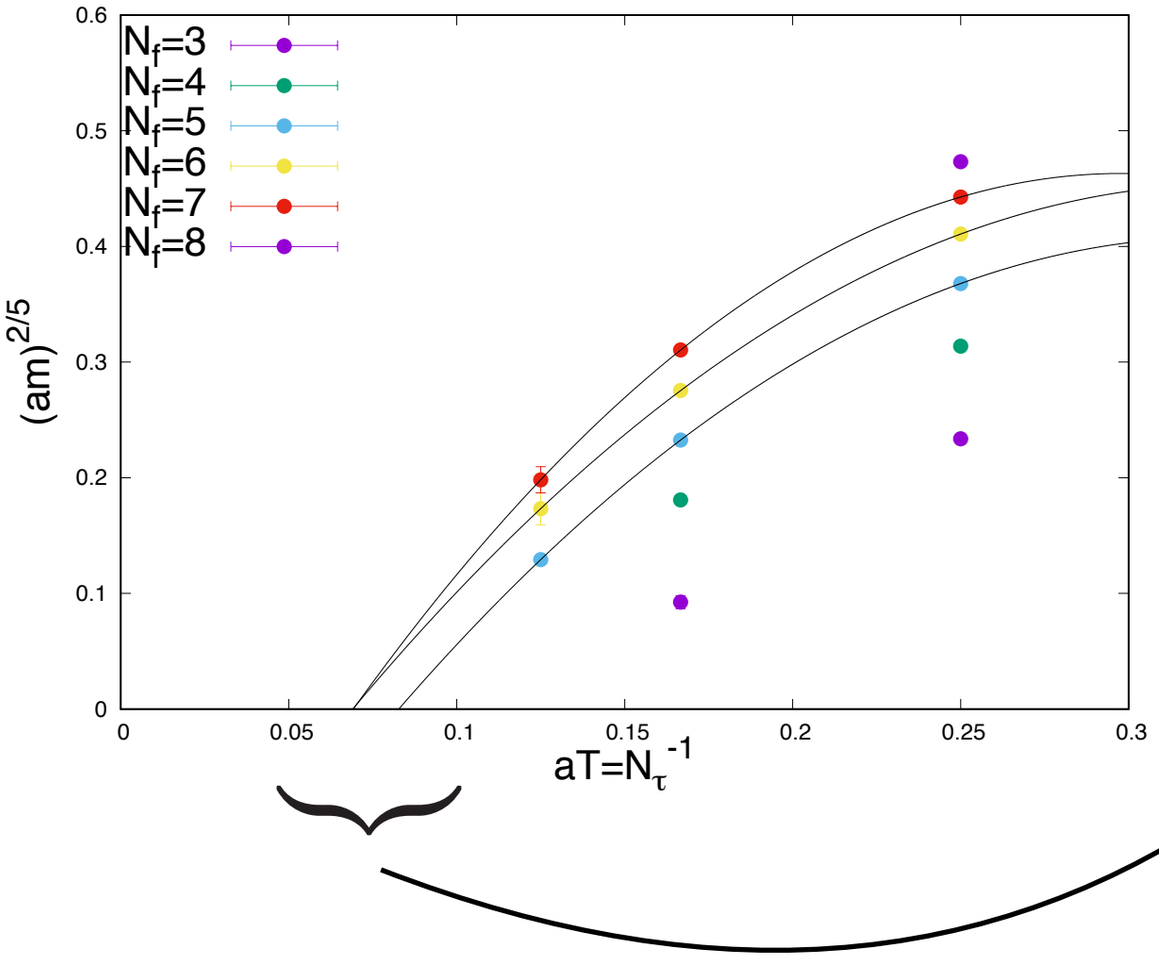


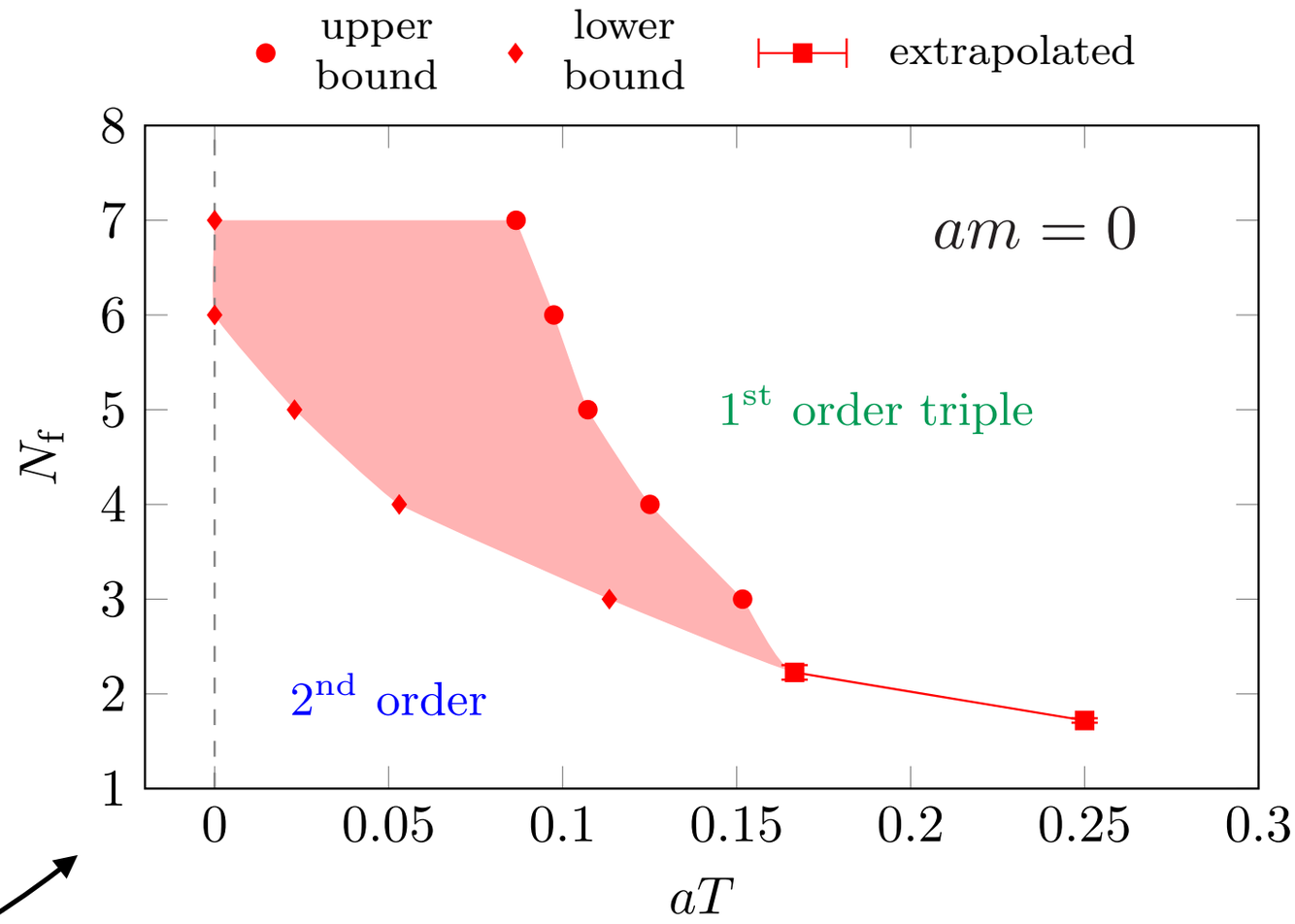
2nd order scenario

The Columbia plot in the continuum



Staggered: tricritical points as function of Nf

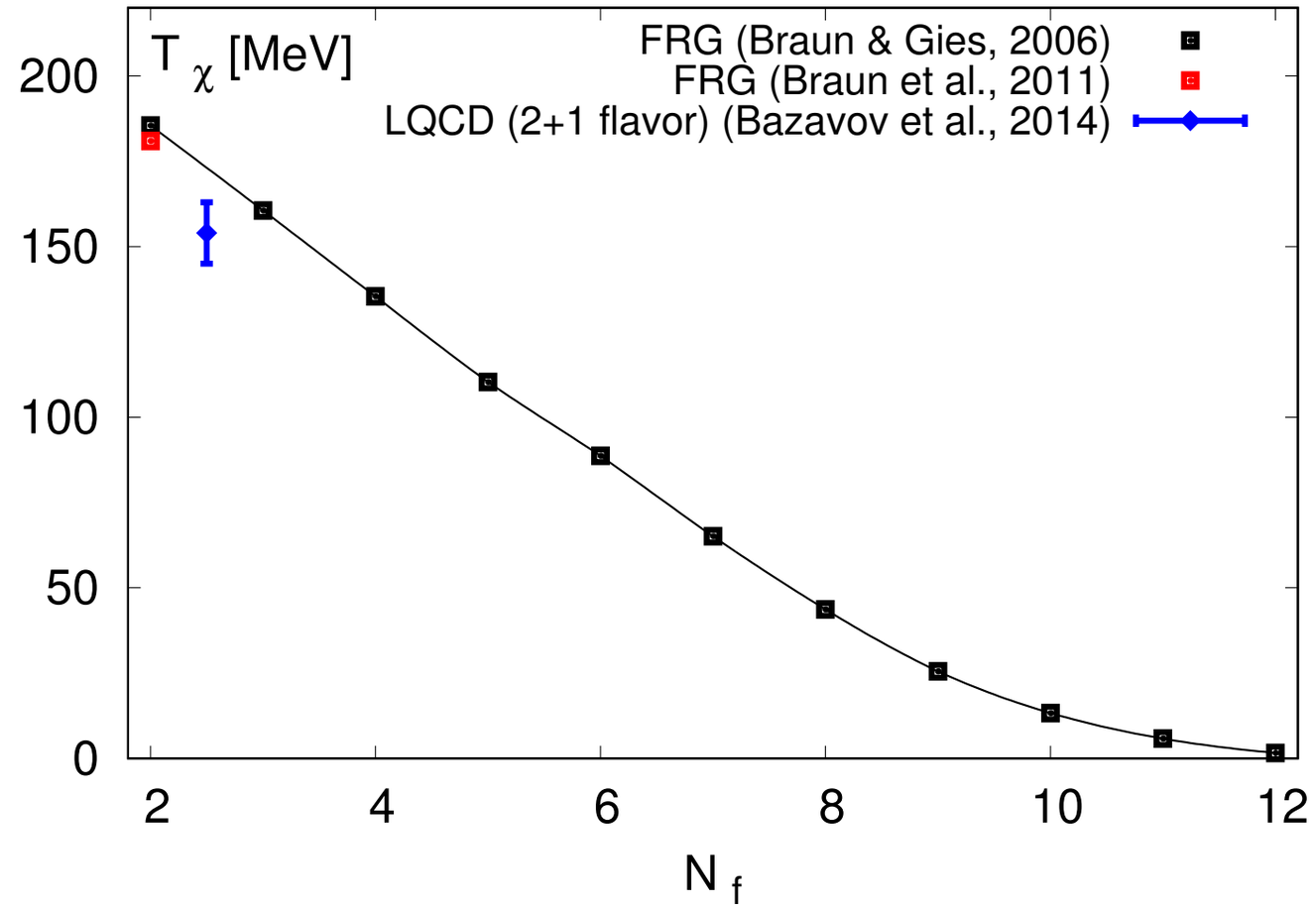




- $N_\tau^{\mathrm{tric}}(N_f)$ increasing function
- Tricritical line in the plane of the lattice chiral limit, separates 1st from 2nd

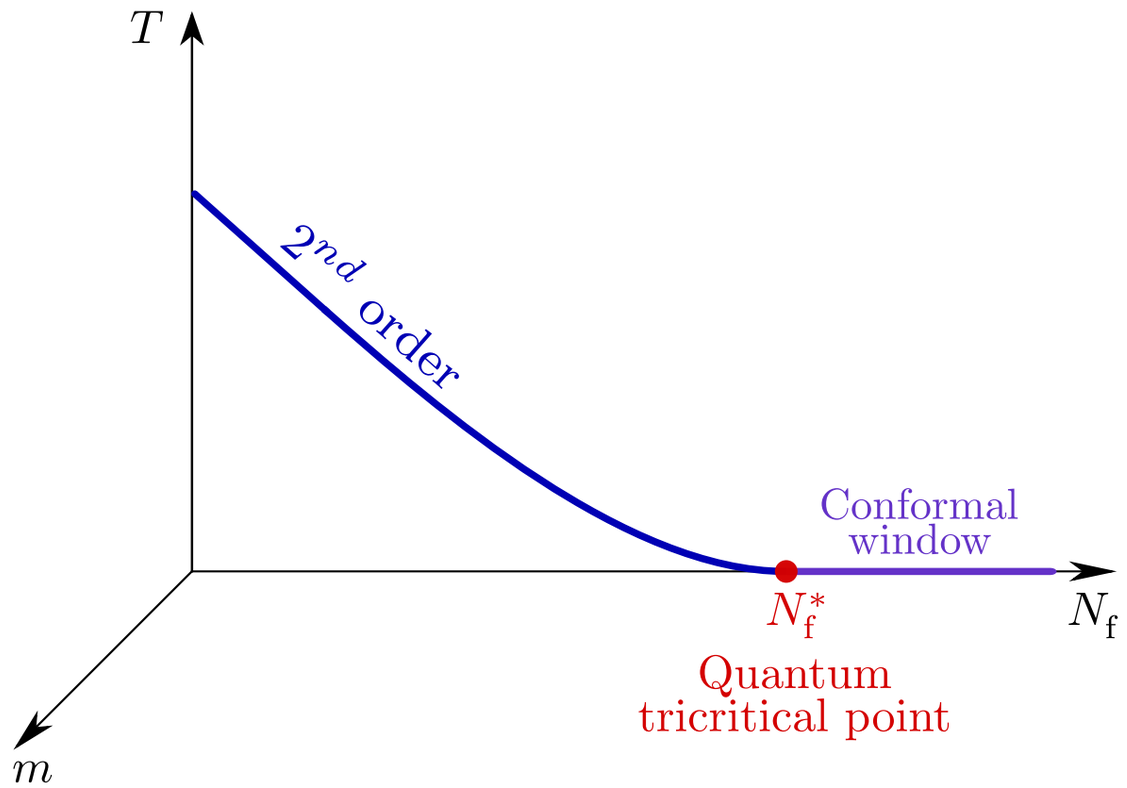
The chiral phase transition for different N_f

Temperature dependence:



For lattice, see [Miura, Lombardo, NPB 13]

Order of the transition:



[Cuteri, O.P., Sciarra 21]

The chiral phase transition in the massless limit is likely second-order for all N_f

What about Pisarski, Wilczek?

- Investigated 3d sigma model,
  i.e. ϕ^4 - Ginzburg-Landau-Wilson theory for chiral condensate
- Results based on epsilon expansion about $\epsilon = 1$
- All conclusions confirmed by [Butti, Pelisetto, Vicari, JHEP 03]
  (High order perturbative expansion in fixed d)
- Support also from simulation of 3d sigma model [Gausterer, Sanielovici, PLB 88]

Suggested resolution:

ϕ^6 term should be included in 3d, renormalisable

FRG: 3d ϕ^6 has infrared fixed points and 2nd order transitions [Litim, Tetradis, NPB 96]

[Fejos, arxiv:2201.07909] 3d ϕ^6 with t'Hooft term: 2nd order transition for restored anomaly!

Conclusions

- Zero density, unimproved staggered, Nf=2-6, O(a)-improved Wilson Nf=3:

  1st order chiral transition region not connected to continuum limit

- Chiral transition second order up to conformal window

- Check: Wilson Nf=4

- Domain wall and overlap fermions…… + FRG!

Long history, key results

- [Pisarski, Wilczek PRD 84]
  Universality, 3D sigma models: 1st order for $N_f \geq 3$
  $N_f = 2$ depends on $U(1)_A$

- Shrinking of 1st-order region towards continuum, will something remain?:
  Standard staggered $N_f = 2$ [Bonati et al. PRD 14, Cuteri et al. PoS LAT 18]
  $N_f = 3$ [de Forcrand, O.P. PoS LAT 07]
  $N_f = 4$ [de Forcrand, D'Elia PoS LAT 16]

  O(a)-improved Wilson $N_f = 3$ [Jin et al. PRD 15,17; Kuramashi et al. PRD 20]
  $N_f = 4$ [Ohno et al. PoS LAT 18]

- No 1st order transition seen at all:
  HISQ, $N_f = 3, m_{PS} \geq 50$ MeV [HotQCD. PRD 17, 21]